Relation between the chiral separation effect and the axial anomaly in real-time formalism.

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Показана связь между эффектом разделения киральностей и аксиальной аномалией в методе эпсилон-раздвижки в формализме реального времени. Как следствие показано различие механизмов защиты эффекта разделения киральностей и аксиальной аномалии.

The relation between the Chiral separation effect(CSE) and the axial anomaly in point-splitting method in real time formalism(RTF) is shown. As a consequence the difference of protection mechanism of CSE and of the axial anomaly is shown.

1. Introduction.

Chiral effects are associated with the appearance of nondissipative currents in media in thermodynamic equilibrium. The most famous examples in four dimensions are the chiral separation effect [1, 2], chiral magnetic effect [3], and chiral vortical effect [4]. Great attention is paid to the relation between these effects and anomalies in quantum field theory. It was assumed for a long time that all chiral effects occur in the infrared region and can thereby be calculated within quantum field theory without regularization. However, the authors of [5, 6] showed that this is not the case and that the chiral magnetic effect is absent. The authors of [7] earlier explained the universality of the experimental relation between the current in the effectively one-dimensional thin wires and the applied voltage. In the same way, an expression was obtained for the chiral magnetic effect [7], which makes the mentioned universality doubtful [5, 6].

2. Axial anomaly and point-splitting method in real time formalism.

Chiral anomaly is violation of a classical conservation law in QFTs. The axial current is conserved "on the classical level" for massless fermions

$$j^{\mu 5}(x) = \bar{\psi}(x)\gamma^{\mu}\gamma^{5}\psi(x), \tag{1}$$

$$\partial_{\mu} j^{\mu_5} = 2im\bar{\psi}\gamma^{5}\psi. \tag{2}$$

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but on the quantum level the situation changes. One needs to use regularization to obtain well defined answer. We will use point-splitting method:

$$j_5^{\mu(reg)}(x) = \bar{\psi}(x+\epsilon/2)U(x+\epsilon/2, x-\epsilon/2)\gamma^\mu\gamma^5\psi(x-\epsilon/2), \tag{3}$$

$$U(x,y) = \exp(-i\int_y^x A_\mu dx^\mu). \tag{4}$$

U is the parallel transporter that preserves the gauge invariance.In the real-time formalism the fermion propagator reads [8, 9]

$$S(p) = (\gamma^\mu p_\mu + m)G(p), \mu = 0..3 \tag{5}$$

$$G(p) = \frac{i}{p^2 - m^2 + i0} - 2\pi\delta(p^2 - m^2)n(|p_0| - sgn(p_0)\mu), \tag{6}$$

where $n(E) = (e^{E/T} + 1)^{-1}$ is the Fermi-Dirac distribution. We begin with the expression for the axial anomaly in the point splitting method.

$$\partial_\mu j^{\mu 5} = i2m\bar{\psi}(x+\frac{\epsilon}{2})\gamma^5 W(x+\frac{\epsilon}{2}, x-\frac{\epsilon}{2})\psi(x-\frac{\epsilon}{2}) + e\epsilon^\nu j^{5\mu}(x,\epsilon)F_{\nu\mu} \tag{7}$$

The regularized axial current reads

$$j_5^\mu(x,\epsilon) = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-ikx - i(p+k/2)\epsilon}\ A^\lambda(k)p^\nu k^\sigma\ 4i\epsilon_{\mu\nu\lambda\sigma}\ G(p)G(p+k) \tag{8}$$

One can decompose the last expression into convergent and divergent parts:

$$j_{5\mu}(x,\epsilon) = I_\mu^{(div)}(x,\epsilon) + I_\mu^{(conv)}(x,\epsilon), \tag{9}$$

$$I_\mu^{(div)}(x,\epsilon) = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-ikx - i(p+k/2)\epsilon}\ \frac{2p^\nu \tilde{F}_{\mu\nu}(k)}{(p^2 - m^2 + i0)((p+k)^2 - m^2 + i0)}, \tag{10}$$

$$\begin{aligned} I_\mu^{(conv)}(x,\epsilon) = &\int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4} e^{-ikx - i(p+k/2)\epsilon}\ (-2)p^\nu \tilde{F}_{\mu\nu}(k) \times \\ &\times\Bigg(\frac{(-2\pi i)\delta((p+k)^2 - m^2)n_{p+k}}{p^2 - m^2 + i0} + \frac{(-2\pi i)\delta(p^2 - m^2)n_p}{(p+k)^2 - m^2 + i0} \\ &+ (-2\pi)\delta((p+k)^2 - m^2)n_{p+k}(-2\pi)\delta(p^2 - m^2)n_p\Bigg) \end{aligned} \tag{11}$$

Lorentz symmetry dictates the UV properties of the theory, and thus fixes the anomaly expression[1]. On the other hand, the IR properties are model-specific (Dirac mass, interaction, and phase transitions!). Thus we argue for CSE sensitivity to interaction in the general case. For UV part of the current we can write

$$j_{5\mu}^{(div)}(x,\epsilon) \sim \frac{\epsilon_\mu}{\epsilon^2} \times (\text{finite value})|_{\epsilon \to 0} \tag{12}$$

[1] We remind that the axial anomaly could be calculated at any scale, we just prefer to use UV-domain.

The integral of convergent part gives CSE effect

$$j_{5\mu}^{(conv)}(x, \epsilon = 0) = \frac{\sqrt{\mu^2 - m^2}}{2\pi^2} \tilde{F}_{0\mu} \equiv << j_{\mu}^{5(CSE)}(x) >>, \quad (13)$$

within Landau levels picture this expression is just a charge density on the lowest landau level.We can write:

$$< \partial_\mu j_5^\mu(x, \epsilon) > \sim 2im < \bar{\psi}(x + \epsilon/2)\gamma_5\psi(x - \epsilon/2) > -iF_{\mu\nu}\epsilon^\nu I^{\mu(div)}(x, \epsilon). \quad (14)$$

and:

$$\epsilon_\nu I_\mu(x, \epsilon) = \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}(i\frac{\partial}{\partial p^\nu}e^{-ikx-i(p+k/2)\epsilon}) \; A^\lambda(k)p^\rho k^\sigma \; 4i\epsilon_{\mu\rho\lambda\sigma} \; G(p)G(p+k)(15)$$

$$= -i \int \frac{d^4p}{(2\pi)^4}\frac{d^4k}{(2\pi)^4}e^{-ikx-i(p+k/2)\epsilon} \; \frac{\partial}{\partial p^\nu}(A^\lambda(k)p^\rho k^\sigma \; 4i\epsilon_{\mu\rho\lambda\sigma} \; G(p)G(p+k))(16)$$

$$4i\epsilon^{\mu\sigma\lambda\rho} \int \frac{d^4k}{(2\pi)^4}e^{-ikx}k_\lambda A_{\rho(k)}\frac{d^4p}{(2\pi)^4}\frac{\partial}{\partial p^\nu}\frac{p_\sigma}{p^4}(17)$$

finally we obtain:

$$< \partial_\mu j_5^\mu(x, \epsilon) >_{\epsilon\to 0} = 2im < \bar{\psi}\gamma_5\psi > + \frac{e^2}{8\pi^2}\frac{\epsilon_{\mu\nu\lambda\sigma}}{2}F^{\mu\nu}F^{\lambda\sigma} \quad (18)$$

3. Conclusion and Discussions

With point-splitting regularization from the same expression we derived the Chiral Anomaly and Chiral Separation Effect. From our consideration it immediately follows the difference in protection mechanism of the axial anomaly and of the chiral separation effect.

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