Bigravity and all that
Бигравитация и ее особенности

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Загадки темной энергии и темной материи породили много новых вариантов теории гравитации. Все они безусловно должны удовлетворять принципу соответствия по отношению к общей теории относительности (ОТО), которая не противоречит всем до сих пор известным тестам. Бигравитация является одной из модификаций ОТО, сохраняющей лоренц-инвариатность.

The puzzles of dark energy and dark matter provided a lot of new variants of the theory of gravity. All of them must fulfill the correspondence principle in relation to the General Relativity (GR) as the GR fulfills all known experimental and observational tests. The bigravity is one of the GR modifications that preserves Lorentz-invariance.

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Introduction

The idea to work with two spacetime metrics is old enough. N. Rosen [1,2] was the first who introduced the Minkowskian metric to define the gravitational energy-momentum tensor. This flat metric also was used by Fierz and Pauli [3] to develop a theory of massive tensor field. Later some authors attempted to elaborate nonlinear massive gravity [4] and some others tried to invent a new gravitational theory based on two metrics [5]. The great progress was achieved after discovering a new potential for massive gravity by de Rham, Gabadadze and Tolley (dRGT) [6] and application of this potential to bigravity by Hassan and R. Rosen [7].

This article provides a brief introduction to main ideas of the bigravity theory. We discuss the Lagrangian, the massive and massless perturbations and the simplest cosmological scenario in this theory.

Let us comment on the notations. $f_{\mu\nu}$ and $g_{\mu\nu}$ are two spacetime metrics, M_f and M_g are the corresponding Planck masses, $\mathcal{L}_M^f$ and $\mathcal{L}_M^g$ are Lagrangian densities for the two species of matter minimally coupled to the corresponding metrics.

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The Lagrangian

If we start with an action being a sum of two copies of the GR Lagrangian

$$\begin{aligned} S_0 &= \int d^4x \left(\frac{M_f^2}{2} \sqrt{-f} f^{\mu\nu} R_{\mu\nu}^{(f)} + \mathcal{L}_M^{(f)}(\psi^A, f_{\mu\nu}) \right) & (1) \\ &+ \int d^4y \left(\frac{M_g^2}{2} \sqrt{-g} g^{\mu\nu} R_{\mu\nu}^{(g)} + \mathcal{L}_M^{(g)}(\phi^A, g_{\mu\nu}) \right), & (2) \end{aligned}$$

the new theory will have two diffeomorphism invariances

$$x^\mu \to x'^\mu(x^\alpha), \qquad y^\mu \to y'^\mu(y^\alpha). \tag{3}$$

But after introducing a potential of interaction

$$S = S_0 - m^2 M_g^2 \int d^4z \sqrt{-g} U(g_{\mu\nu}, f_{\mu\nu}), \tag{4}$$

we stay with only one (diagonal) diffeomorhism invariance. Let us suppose the potential U is composed of invariants of matrix $\mathsf{Y} = ||g^{\mu\alpha} f_{\alpha\nu}||$. It is possible to take the symmetric polynomials expressed through eigenvalues or through traces of powers of Y

$$\begin{aligned} &e_0 = 1, \\ &e_1 = \lambda_1 + \lambda_2 + \lambda_3 + \lambda_4 \equiv \mathrm{Tr}\mathsf{Y}, \\ &e_2 = \lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_4 + \lambda_1\lambda_3 + \lambda_1\lambda_4 + \lambda_2\lambda_4 \equiv \frac{1}{2}\left((\mathrm{Tr}\mathsf{Y})^2 - \mathrm{Tr}\mathsf{Y}^2 \right), \\ &e_3 = \lambda_1\lambda_2\lambda_3 + \lambda_2\lambda_3\lambda_4 + \lambda_1\lambda_3\lambda_4 + \lambda_1\lambda_2\lambda_4 \equiv \frac{1}{6}\left((\mathrm{Tr}\mathsf{Y})^3 - 3\mathrm{Tr}\mathsf{Y}\mathrm{Tr}\mathsf{Y}^2 + 2\mathrm{Tr}\mathsf{Y}^3 \right), \\ &e_4 = \lambda_1\lambda_2\lambda_3\lambda_4 \equiv \det \mathsf{Y}. \end{aligned}$$

For example, the RTG [4] (Relativistic Theory of Gravitation) by A.A. Logunov and his collaborators is a theory of massive gravity with the potential

$$\sqrt{-g} U = \left(\sqrt{-g} \left(\frac{1}{2} \mathrm{Tr}\mathsf{Y} - 1 \right) - \sqrt{-f} \right), \tag{5}$$

where the second metric $f_{\mu\nu}$ is treated as a non-dynamical Minkowskian background.

In general the bigravity theory constructed according to Eq.(4) in 4-dimensional spacetime has 8 degrees of freedom, i.e. one more than required for one massless and one massive fields of spin-2. Also it has been shown [8] that there is a negative kinetic energy for one degree of freedom, so it behaves as a ghost. Therefore for a long period of time it was believed that the bigravity (and also nonlinear massive gravity) was impossible to construct, nevertheless, some attempts were made [5].

The dRGT potential

As a result of interesting preliminary work on the higher dimensional models, Galileons and decoupling limits de Rham, Gabadadze and Tolley have found a new formula for the potential. It can be expressed as an arbitrary linear combination of the symmetric polynomials for a square root matrix $\mathsf{X} = \sqrt{\mathsf{Y}}$

$$U_{\text{dRGT}} = \sum_{i=0}^{i=4} \beta_i e_i(\mathsf{X}).$$

It was proved in different ways in publications [9–16] and some others that this potential allows to obtain a theory of bigravity (of massive gravity also) with a correct number of degrees of freedom and free of ghosts.

Evidently, the bigravity occurs a more complicated theory than the GR. Besides, it introduces some arbitrary new constants. And problems with the causality appear as it has two different lightcones in spacetime defined by two metric tensors. Nevertheless, the bigravity allows to consider nonlinear interaction of massive and massless gravity, and pretends to solve the dark energy problem.

When we speak about massless and massive gravitational fields we are to remember that this classification is possible only for metric perturbations on the specific backgrounds. Let us consider the background of proportional metrics

$$\bar{f}_{\mu\nu}(x^\alpha) = c^2(x^\alpha)\bar{g}_{\mu\nu}(x^\alpha), \tag{6}$$

as a solution of the vacuum (free of matter) bigravity equations

$$G_{\mu\nu}(\bar{g}) + V^g_{\mu\nu} = 0, \quad G_{\mu\nu}(\bar{f}) + V^f_{\mu\nu} = 0. \tag{7}$$

As usual, there are Bianchi identities for the Einstein tensors

$$\nabla^\mu_g G^g_{\mu\nu} \equiv 0, \qquad \nabla^\mu_f G^f_{\mu\nu} \equiv 0, \tag{8}$$

and therefore we get on-shell equations

$$\nabla^\mu_g V^g_{\mu\nu} \equiv 0, \qquad \nabla^\mu_f V^f_{\mu\nu} \equiv 0. \tag{9}$$

It follows from the above that c must be a constant.

Expressions for matrices Y and X become very simple in this case

$$f_{\mu\nu} = c^2 g_{\mu\nu}, \ \to \ \mathsf{Y} = ||g^{-1}f|| = c^2\mathsf{I}, \quad \mathsf{X} = \sqrt{\mathsf{Y}} = c\mathsf{I}. \tag{10}$$

And as a result

$$V^g_{\mu\nu} \ = \ g_{\mu\nu}\Lambda_g, \quad \Lambda_g = m^2 B_0(c), \tag{11}$$

$$V^f_{\mu\nu} \ = \ f_{\mu\nu}\Lambda_f, \quad \Lambda_f = \frac{m^2}{\alpha^2}\frac{B_1(c)}{c^3}, \tag{12}$$

where we use the following notations

$$\alpha = \frac{M_f}{M_g}, \quad B_i(c) = \beta_i + 3\beta_{i+1}c + 3\beta_{i+2}c^2 + \beta_{i+3}c^3. \tag{13}$$

In order to discuss the graviton mass we are to consider small perturbations on maximally symmetric space-times. Then for the background of proportional metrics we have

$$G_{\mu\nu}(\bar{g}) + \bar{g}_{\mu\nu}\Lambda_g = 0, \quad G_{\mu\nu}(\bar{f}) + \bar{f}_{\mu\nu}\Lambda_f = 0, \tag{14}$$

and as the Einstein tensor does not change when metric is multiplied by constant we must have

$$\Lambda_g = c^2\Lambda_f. \tag{15}$$

This provides the following equation for c:

$$\alpha^2\beta_3c^4 + (3\alpha^2\beta_2 - \beta_4)c^3 + 3(\alpha^2\beta_1 - \beta_3)c^2 + (\alpha^2\beta_0 - 3\beta_2)c - \beta_1 = 0. \tag{16}$$

Let us consider the simplest case when both tensors $g_{\mu\nu}$ and $f_{\mu\nu}$ are close to the common flat background

$$\Lambda_f = 0 = \Lambda_g, \qquad c = 1, \qquad \bar{g}_{\mu\nu} = \eta_{\mu\nu} = \bar{f}_{\mu\nu}. \tag{17}$$

Linear perturbations $h_{\mu\nu}$ and $\ell_{\mu\nu}$ are as follows

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_g}h_{\mu\nu}, \qquad f_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_f}\ell_{\mu\nu}, \tag{18}$$

and for simplest case ($\beta_0 = 3,\ \beta_1 = -1,\ \beta_2 = 0 = \beta_3,\ \beta_4 = 1$) we get the following Fierz-Pauli term in the Lagrangian

$$-\frac{m^2M_{\text{eff}}^2}{4}\left[v_{\mu\nu}v^{\mu\nu} - \left(v^\mu_\mu\right)^2\right], \tag{19}$$

where

$$v_{\mu\nu} = M_{\text{eff}}\left(\frac{h_{\mu\nu}}{M_g} - \frac{\ell_{\mu\nu}}{M_f}\right), \qquad M_{\text{eff}} = \sqrt{M_g^2 + M_f^2}. \tag{20}$$

Let us now consider a more general case [17] when $c \neq 1$ and two species of matter with proportional energy-momentum tensors are allowed $\bar{T}^f_{\mu\nu} = \alpha^2\bar{T}^g_{\mu\nu}$. The metric perturbations on this background can be defined as follows

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \frac{1}{M_g}h_{\mu\nu}, \qquad f_{\mu\nu} = c^2\bar{g}_{\mu\nu} + \frac{c}{M_f}\ell_{\mu\nu}. \tag{21}$$

It can be shown that the massive $M_{\mu\nu}$ and massless $G_{\mu\nu}$ fluctuations are the following

$$G_{\mu\nu} = \frac{1}{1+c^2\alpha^2}\left(h_{\mu\nu} + c\alpha\ell_{\mu\nu}\right), \tag{22}$$

$$M_{\mu\nu} = \frac{1}{1+c^2\alpha^2}\left(\ell_{\mu\nu} - c\alpha h_{\mu\nu}\right), \tag{23}$$

and the Fierz-Pauli mass is

$$m_{\mathrm{FP}}^2 = m^4 \left(\frac{1}{c^2 M_f^2} + \frac{1}{M_g^2} \right) (\beta_1 c + 2\beta_2 c^2 + \beta_3 c^3). \tag{24}$$

The effective Planck mass now is given by the following formula

$$M_p = M_g \sqrt{1 + c^2 \alpha^2} \equiv \sqrt{M_g^2 + c^2 M_f^2}. \tag{25}$$

The linearized equations for massless and massive fluctuations of metrics on this background with account for the matter fluctuations are as follows

$$\bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} G_{\rho\sigma} + \Lambda_g G_{\mu\nu} = \frac{1}{M_p} \left(\delta T_{\mu\nu}^{(g)} + c^2 \delta T_{\mu\nu}^{(f)} \right), \tag{26}$$

$$\begin{aligned} \bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} M_{\rho\sigma} + \Lambda_g M_{\mu\nu} &= \frac{c}{M_p \alpha} \left(\delta T_{\mu\nu}^{(f)} - \alpha^2 \delta T_{\mu\nu}^{(g)} \right) - \\ &- \frac{m_{\mathrm{FP}}^2}{2} (M_{\mu\nu} - \bar{g}_{\mu\nu} \bar{g}^{\rho\sigma} M_{\rho\sigma}), \end{aligned} \tag{27}$$

where

$$\begin{aligned} \bar{\mathcal{E}}_{\mu\nu}^{\rho\sigma} = -\frac{1}{2} \big(& \delta_\mu^\rho \delta_\nu^\sigma \bar{\nabla}^2 + \bar{g}^{\rho\sigma} \bar{\nabla}_\mu \bar{\nabla}_\nu - \delta_\mu^\rho \bar{\nabla} \sigma \bar{\nabla}_\nu - \delta_\nu^\rho \bar{\nabla} \sigma \bar{\nabla}_\mu - \\ & - \bar{g}^{\rho\sigma} \bar{g}_{\mu\nu} \bar{\nabla}^2 + \bar{g}_{\mu\nu} \bar{\nabla}^\rho \bar{\nabla}^\sigma - \bar{g}_{\mu\nu} \bar{R}^{\rho\sigma} + \delta_\mu^\rho \delta_\nu^\sigma \bar{R} \big). \end{aligned} \tag{28}$$

Cosmology

Einstein was the first who tried to apply the GR equations to the whole Universe. But he was not brave to let it be dynamical. It was Alexander Friedmann [18] who predicted the expantion of the Universe in 1922.

In standard notations the homogeneous and isotropic Universe is described by metric

$$ds^2 = -dt^2 + R^2(t) \left(\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2\theta d\phi^2) \right) \tag{29}$$

where only one of the opportunities $k = -1, 0, +1$ must be chozen. The matter, i.e. a source of gravitation, is also supposed to be homogeneous isotropic and at rest. Usually, the ideal fluid with energy-momentum tensor

$$T_{\mu\nu} = \mathrm{diag}(\rho(t), p(t), p(t), p(t)) \tag{30}$$

is chozen.

The main Friedmann equation is a constraint from the Hamiltonian point of view, because it is 1st order in time derivatives

$$3 \left(\frac{\dot{R}}{R} \right)^2 + \frac{3k}{R^2} = \frac{\rho}{M_g^2} + \Lambda \equiv \frac{1}{M_g^2} \left(\rho + M_g^2 \Lambda \right), \tag{31}$$

therefore the data taken in arbitrary instant of time $H(t_0)$, $\rho(t_0)$, $R(t_0)$ must fulfil it. In the l.h.s of the Friedmann equation we see the Hubble constant $H = \frac{\dot{R}}{R}$ and the curvature parameter k. For brevity here we consider only the flat space case $k = 0$.

The second Friedmann equation contains 2nd order time derivatives $\ddot{R}$

$$\dot{H} = -\frac{1}{M_g^2}(\rho + p). \tag{32}$$

The energy-momentum conservation law

$$\dot{\rho} = -3H(\rho + p). \tag{33}$$

follows from Eqs. (31,32).

In bigravity, see for example [19–21], Friedmann's ansatz is similar to the GR case. As the starting point one usually chooses the two diagonal metrics with the same curvature parameter k

$$ds_g^2 = -dt^2 + R_g^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{34}$$

$$ds_f^2 = -N(t)^2dt^2 + R_f^2(t)\left(\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right), \tag{35}$$

below we consider only flat spatial geometry $k = 0$. The spatial scale factors R_g and R_f and the time scales in general are different, the ratio $y = R_f/R_g$ is a suitable dynamical variable.

The 1st order Friedmann equations

$$3H_g^2 = \frac{\rho_g}{M_g^2} + \Lambda_g(y), \tag{36}$$

$$3H_f^2 = \frac{\rho_f}{M_f^2} + \Lambda_f(y), \tag{37}$$

provide us with a dynamical dark energy, i.e. we get two functions of y instead of cosmological constants

$$\Lambda_g(y) = m^2\left(\beta_0 + 3\beta_1 y + 3\beta_2 y^2 + \beta_3 y^3\right), \tag{38}$$

$$\Lambda_f(y) = m^2\frac{M_g^2}{M_f^2}\left(\frac{\beta_1}{y^3} + 3\frac{\beta_2}{y^2} + 3\frac{\beta_3}{y} + \beta_4\right). \tag{39}$$

There is a secondary constraint following from dynamics

$$H_g = yH_f. \tag{40}$$

To simplify the situation let us put $\rho_f = 0 = p_f$, then

$$3H_g^2 = \frac{\rho_g}{M_g^2} + \Lambda_g(y), \tag{41}$$

$$3H_f^2 = \Lambda_f(y). \tag{42}$$

We also can derive the evolutionary equation for the matter

$$\dot{\rho}_g = -3H_g(\rho_g + p_g), \tag{43}$$

and for variable y

$$\dot{y} = (N - y)H_g. \tag{44}$$

By combining equations (38) – (42) it is possible to express ρ_g or H_g as functions of y. The inverse problem has up to 4 real solutions because in order to find y we are to solve the algebraic equation:

$$\frac{M_g^2}{M_f^2}\left(\frac{\beta_1}{y} + 3\beta_2 + 3\beta_3 y + \beta_4 y^2\right) - \beta_0 - 3\beta_1 y - 3\beta_2 y^2 - \beta_3 y^3 = \frac{\rho_g}{m^2 M_g^2}. \tag{45}$$

If we believe in the Big Bang scenario starting from the almost infinite matter density ρ_g and asymptotically going to the amost zero ρ_g, then we classify the solutions of Eq. (45) $y = y(\rho_g(t))$ as follows:

- The finite branch $y \to 0$ when $\rho_g \to \infty$. The ratio of scale factors y then is monotonically increasing and tends to a constant value when ρ_g tends to zero.
- The infinite branch $y \to \infty$ when $\rho_g \to \infty$. This ratio is monotonically decreasing up to finite value.
- Two other cases are treated as exotic branches.

From Eq. (44) we see that $y = \text{const}$ in two cases:

1. when the two metrics are proportional: $N = y$,
2. or when cosmology is static: $H_g = 0 = H_f$,

the last case is treated as unphysical.

The bigravity cosmological solution with proportional metrics

$$ds_f^2 = \left(\frac{R_f}{R_g}\right)^2 ds_g^2, \qquad H_g = NH_f = \text{const}, \tag{46}$$

provides de Sitter solutions for both metrics. The matter should be dissolved completely or should have vacuum equation of state according to Eq. (43).

If we start from current data $H(t_0)$, $\rho_g(t_0)$ where t_0 is supposed to be unknown we are able to calculate the whole evolution both in the past and in future directions. Let us take the finite branch solution of Eq. (45). Then the age of Universe t_0 can be found from equation $y(0) = 0$. The algorithm of calculations is the following. We are solving the evolution equation for $y(t)$, i.e. Eq. (44) with the initial condition $y(t_0)$ found from Eq. (45) given initial data for observable $\rho_g(t_0)$. $N = N(y)$ can be found from the requirement of conservation the secondary constraint $H_g = yH_f$ during evolution, and $H_g(y)$ is given by the Friedmann equation.

Conclusion

There is a hope that the bigravity can explain accelerated expansion of the Universe and the cosmological constant problem. Some people believe that the dark matter may also find an explanation in this theory. In recent works [22–25] it was shown that there is a domain in the space of parameters where the bigravity is in full correspondence with all the observation data.

Of course, one may find a lot of interesting questions that are not answered yet. For example, is there only one form of matter or maybe two of them? In the first case, an open question is how the coupling of matter with the two metrics is organized? What is the best metric for matter? Does the matter minimally interacts with only one metric field or with a combination of the two metrics called an effective metric? Is there any combination of $g_{\mu\nu}$ and $f_{\mu\nu}$ which is minimally coupled to matter? We address a reader to some articles [11, 15, 20] where this problem is discussed. One proposal was an effective metric

$$\mathcal{G}^{\text{eff}}_{\mu\nu} = (E^A_\mu + \xi F^A_\mu)(E_{\nu A} + \xi F_{\nu A}), \tag{47}$$

but it was proved that this theory would not be ghost-free. Another idea is to get a new (spatial) metric G_{ij} from the algebra of Hamiltonian constraints where we have an equation

$$\{\mathcal{R}(x), \mathcal{R}(y)\} = \mathcal{G}^{ij}(\mathcal{R}_j(x)\delta_{,i}(x-y) - \mathcal{R}_j(y)\delta_{,i}(y-x)), \tag{48}$$

but it is found [15] that both spatial metrics may appear in the above formula.

We come to the following conclusions.

- Bigravity is motivated theoretically and observationally
- Bigravity is not so nice as General Relativity and contains 6 arbitrary parameters
- Bigravity provides solutions for self-accelerated Universe
- Bigravity maybe has relations to dark matter
- Bigravity survives all the cosmological and local tests (as also ΛCDM model does)
- Bigravity has limits to GR and to dRGT massive gravity
- But questions prevail over answers

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