Trajectories of bright stars and shadows around supermassive black holes as tests of gravity theories

A. F. Zakharov[a,b][1],

[a] National Research Center "Kurchatov Institute Moscow, Russia

[b] Bogoliubov Laboratory for Theoretical Physics, JINR, 141980 Dubna, Russia

Общая теория относительности (ОТО), созданная более века назад, прошла проверку в различных экспериментальных и наблюдательных тестах. На ранней стадии своего развития предсказания ОТО проверялись в задачах, где гравитационное поле слабое, и релятивистские поправки можно рассматривать как малые возмущения ньютоновской теории гравитации. Однако в последние годы, в связи с развитием новых технологий, оказалось возможным проверить предсказания ОТО в пределе сильного гравитационного поля, как это было сделано для проверки предсказаний о профиле рентгеновской линии железа $K\alpha$, оценок гравитационного волнового сигнала при слиянии двойных черных дыр и/или нейтронных звезд и при восстановлении теней черных дыр в Sgr A* и M87*. Группы астрономов при помощи телескопов Keck и VLT (GRAVITY), подтвердили предсказания ОТО в первом пост-ньютоновском приближении для красного смещения спектральных линий звезды S2 вблизи прохождения ее перицентра. Ожидается, что в ближайшем будущем наблюдения ярких звезд с помощью больших телескопов VLT (GRAVITY), Keck, E-ELT и TMT позволят нам проверить предсказания ОТО в сильном гравитационном поле сверхмассивных черных дыр. Наблюдения ярких звезд в окрестностях Галактического Центра и реконструкции теней черных дыр позволяют не только проверить предсказания ОТО, но и получить ограничения на альтернативные теории гравитации.

1 General relativity (GR), created more than a century ago, has been tested in various experimental and observational tests. At an early stage of its development, GR predictions were tested in problems where the gravitational field is weak and relativistic corrections can be considered as small perturbations of the Newtonian theory of gravity. However, in recent years due to the progress of new technologies It turned out to be possible to verify the predictions of GR in the limit of a strong gravitational field, as it was done to verify predictions about the profile of the X-ray line of iron $K\alpha$, estimates of the gravitational wave signal during the merger of binary black holes and/or neutron stars and during the restoration of the shadows of black holes in Sgr A* and M87*. Groups of astronomers using the Keck and VLT (GRAVITY) telescopes confirmed the GR predictions for the redshift of the spectral lines of the S2 star near the passage of its pericenter (these predictions were done in the first post-Newtonian approximation). It is expected that in the near future, observations of bright stars using large telescopes VLT (GRAVITY), Keck, E-ELT and TMT will allow us to verify the predictions of GRT in the strong gravitational field of supermassive black holes. Observations of bright stars in the vicinity of the Galactic Center and reconstructions of the shadows of black holes allow not only to verify the predictions of the GR, but also to obtain restrictions on alternative theories of gravity.

2 PACS: 04.80.Cc; 04.20.-q; 04.25.Nx; 04.50.+h; 95.30.Sf; 96.12.Fe

---

[1] E-mail: alex.fed.zakharov@gmail.com

1. Introduction

After WWII an importance of development of theoretical and experimental aspects of general relativity (GR) started to be evident and the conference dedicated to the Jubilee of Relativity Theory was organized by W. Pauli in Bern in 1955.[2] In 1957 the next conference (GR1) was organized by B. De Witt in Chapel Hill [2] where he was the director of Institute of Field Theory at University of North Carolina. The goal of this conference was to provide a good platform where experimentalists could met theorists to boost research in gravitational physics. Intensive discussions of opportunity to detect gravitational waves were started by R. Feynman, H. Bondi, J. Weber at this conference and now we could see that this brain storm was very efficient.

In 1962 GR3 conference was organized in Warsaw and Jablonna (it was the first conference where a large group of Western scientists met with scientists from Eastern Europe (the conference was organized in times of Cold War and the Iron Curtain was very high). Among 114 participants 33 participants were from Eastern countries. The chairman was L. Infeld. Many outstanding scientists attended the conference including P.A. M. Dirac (who got his Nobel prize many years before the conference), and many others (including R. Feynman, S. Chandrasekhar, V. L. Ginzburg, P. Higgs, R. Penrose) who got the Nobel prizes later. This fact illustrates a high level of the conference organization and it is also the evidence that in 1962 GR studies are among the hottest topics in physics in spite of Feynman's criticism of GR conferences [3, 4]. A representative group of Soviet scientists (11 people) attended the activity and two of them (V. Ginzburg and V. A. Fock) presented plenary talks at the meeting. The title of Ginzburg's talk was "Experimental Verification of General Relativity Theory" [5] (it was the only talk devoted to observational tests of GR [6]). Later, Ginzburg wrote a review about the most important and interesting problems in physics and astrophysics and many versions of the review were published as journal articles, chapters of Ginzburg's books and separate booklets (see, versions of the review published in Physics Uspekhi [7, 8]. Ginzburg called the articles as realizations of the Project on Physical Minimum and he claimed that a basic knowledge of is necessary to increase a qualification of young physicists and astrophysicists. [1] These Ginzburg's articles provided a high impact on scientific community in Russia and abroad. Astrophysical black holes are directly connected with the following problems from the Ginzburg's list: 21. Experimental verification of general theory of relativity; 22. Gravitational waves and their detection; 25. Black holes. Cosmic strings (?);[2] 26. Quasars

---

[2] Now this conference is called usually GR0, see for instance `isgrg.org/pastconf.php`. Outstanding Soviet scientists V. A. Fock and A. D. Alexandrov attended the activity where A. D. Alexandrov (who was the Leningrad State University rector at this time) delivered a plenary talk there [1]. It was a visible recognition of a high reputation and a significant contribution of Soviet scientists in GR development at this time.

[1] `https://ufn.ru/tribune/trib230107.pdf`

[2] In cited reviews Ginzburg marked *Cosmic strings* by the question mark perhaps due

and galactic nuclei. Formation of galaxies; 27. The problem of dark matter (hidden mass) and its detection.

Therefore, international conferences on GR played the extremely important role to settle and solve many theoretical and experimental aspects of gravity and to start intensive studies of theoretical and experimental aspects of gravitational wave detection[1] because even the question about an existence of gravitational waves was a subject of doubts among many researchers including A. Einstein and many others and an existence of gravitational waves started to much more clear in a result of efforts of many scientists. For example, in 1936 Einstein and Rosen submitted under the title "Do gravitational waves exist?"in Physical Review (it was his joint paper with N. Rosen) and the Einstein's answer for the title question was "No"as it was noted [9, 10]. Before the submission, Einstein wrote a letter to M. Born in letter "Together with a young collaborator, I arrived at the interesting result that gravitational waves do not exist through they had assumed certainly to the first approximation" [11]. Einstein received a negative referee's report from the Chief Editor of Physical Review (it was John Tate at this time). Einstein was very angry and he wrote to Tate: "Dear Sir, We (Mr Rosen and I) had sent you our manuscript for publication and had not authorized you to show it to specialists before it is printed... On the basis of this incident I prefer to publish the paper elsewhere..." [10]. After conversations with H. P. Robertson and L. Infeld (who were in Princeton in 1936) Einstein revised his conclusion and submitted the joint paper with Rosen in Journal of Franklin Institute [12] and at the end the paper Einstein wrote "The second part of this paper altered by ne after departure of Mr. Rosen for Russia since we had interpreted our formula erroneously. I wish to thank my colleague Professor Robertson for his friendly assistance in the clarification of the original error."As it was noted in [10] Robertson was the referee of the paper by Einstein and Rosen submitted in Physical Review.

2. Direct ways to evaluate gravitational potential near supermassive black holes

The most natural way to evaluate a gravitational potential is a consideration of test body trajectories in the potential and comparison of theoretical orbits calculated in the framework of a selected model with observational results. For solar system potential this way was passed due to efforts of scientific giants like Tycho Brahe, J. Kepler, R. Hooke and I. Newton. Really, Tycho Brahe collected observational data about trajectories of planets, Kepler formulated laws for planet motions, Hooke wrote a letter to Newton [13, 14] and later

---

to his doubts in existence of these objects in nature.

[1] Active studies of opportunities to detect gravitational waves in Soviet Union started after the GR3 Conference because Soviet participants of GR3 learned during the activity about works by J. Weber who was creating the first gravitational wave bar-detector. No we know that great efforts of international collaborations LIGO and Virgo lead to the discoveries of gravitational waves and binary black holes.

Newton proved that really this gravity law could explain Kepler's laws for solar system and established his gravity law for all celestial objects in the Universe. Similarly, Rutherford used trajectories of α-particles to investigate a structure of atoms. Similarly, we could use trajectories of bright stars, gas clouds to test gravitational field in centers of galaxies including our Galactic Center.

3. Bright stars as test bodies in centers of galaxies

Two groups of astronomers monitor bright stars near the Galactic Center (GC) for decades (see early publications on the subject in [15,16]). Currently the group led by A. Ghez uses the Keck twin telescope in Hawaii while the European group led by R. Genzel uses VLT facilities in Chile (currently, four VLT telescopes with 8 m diameter mirrors can form the interferometer which is called GRAVITY). Recently, relativistic predictions evaluated in the framework of the first post-Newtonian approximation about gravitational redshifts for S2 star near its pericenter passage in May 2018 have been confirmed in observations (these conclusions were done by both Keck and GRAVITY collaborations) [17–19]. The GRAVITY collaboration reported about a discovery of hot spot motions near the innermost stable circular orbit of supermassive black hole at the GC [20] and these achievements give an opportunity to investigate GR predictions in a strong gravitational field limit. In 2020 the GRAVITY collaboration found that the Schwarzschild precession corresponds to its relativistic estimates done in the first post-Newtonian approximation [21]. These observational results support the Newton's declaration that the gravity law is universal elsewhere including the Solar system and the Galactic Center. This conclusion is important since in last years theorists proposed a number of alternative theories of gravity and gravity laws may be different in different astronomical systems.

4. Schwarzschild precession as a distance measure for GC model

Observations of bright stars at GC give an opportunity to choose the most suitable model for gravitational potential. If in the first approximation we can use a Newtonian approximation for gravity and we can assume that a gravitational potential is spherically symmetrical, so we can test any theoretical model in a spherical shell where the orbits of observed stars are located if observation accurate enough even in the case if observers monitor a small piece of an entire trajectory of a star. However, if we wish to rule out a set of models which do not correspond to observations we could theoretically evaluate the Schwarzschild precession for monitored stars for more than one period and after that we could reject models (or gravity theories) which are nor consistent with observations. Using such an approach we constrain GC models with dark matter based on properties of the S2 star trajectory [22].

5. Alternative theories of gravity

As the first case to constrain alternative theories of gravity we choose a version of $f(R)$ theory of gravity [23–25] where $f(R) = R^n$ (it is clear that for $n = 1$ the theory coincides with GR) and this approach was proposed in [26]. Later it was found that R^n theory can explain an accelerated expansion of the Universe [27] and flat rotation curves for spiral galaxies [28] or in other words for some cases the considered gravity change could fit dark energy (or Λ-term) and dark matter phenomena (naturally to explain DM and DE phenomena n must be significantly different from unity, since really, to fit supernovae type Ia data which was used to discover the accelerated expansion of the Universe n parameter must be around 3 [27], while to fit rotation curves for spiral galaxies we have to choose $n \approx 3.5$ [28]). However, Solar system data are not consistent with so big n numbers since we found that n must be very close to 1 [29]. From a comparison of theoretical estimates and observational data for the S2 star trajectory we concluded that n parameter must be very close to 1 otherwise the precession is much greater than its observed quantity [30]. Later we assumed a presence of R^n gravity and a bulk concentration of matter in GC we also obtained even more strict constraints on n parameter [31] since both R^n (for $n > 1$) and extended mass distribution cause retrograde orbital precession. Evaluating the Schwarzschild precession in the framework of Yukawa gravity and taking into account an absence of the precession for S2 star orbit we constrained Yukawa gravity parameters in [32] (it was useful result since there is a class of extended theories of gravity which have an Yukawa gravity as a weak gravitational field limit [33]). Constraints on parameters of extended gravity theory from observations of the S2 trajectory are presented in [34].

6. Theories of massive gravity

The first version of gravity theory where graviton is massive was proposed by Fierz and Pauli [35], however, later pathologies have been discovered in this theory, in particular, a presence of ghosts have found [36]. A version of massive theory of gravity have been developed by A. A. Logunov and his group in the framework of relativistic theory of gravity and using this approach astrophysical consequences were discussed, in particular, graviton mass have been constrained [37, 38] (see references therein as well) and the graviton mass constraint $m_g < 9 \times 10^{-34}$ eV obtained in [37] is still the strictest according to the last PDG review [39] (however, we have to note that it very hard to control systematic errors in different estimates of graviton mass bounds).

Several years ago C. de Rham and her co-authors found a way to construct a massive theory of gravity without ghosts [40] (see also a more extended review on the subject [41]). In the first publication on gravitational wave discovery [42] the LIGO and Virgo collaborations reported about detections of gravitational waves from binary black hole system and the authors considered a massive theory of gravity and found a constraint on graviton mass $m_g < 1.2 \times 10^{-22}$ eV. Assuming that massive gravity is valid at GC and considering the S2 star trajectory observed by Keck and VLT telescopes we obtained a graviton mass constraint $m_g < 2.9 \times 10^{-21}$ eV [43]. If we suppose that GR estimates concerning the orbital precessions of bright stars will be confirmed by future observations we showed that the current graviton mass estimate could be significantly improved at a level around 5×10^{-23} eV [44] which is comparable with constraints found by LIGO–Virgo collaborations.

7. Is it possible to substitute supermassive black hole with dark matter cloud in GC?

It was generally adopted that there supermassive black holes (SMBHs) in galactic nuclei (including our GC). However, other models are also proposed. For instance, it was suggested to substitute supermassive black hole in GC by dense core and diluted halo produced by dark matter [45] (it is now called RAR-model, since Ruffini, Argüelles and Rueda were the authors in [45]) it was also declared [46] that this model provided a better fit of trajectories of bright stars in comparison with the conventional model where SMBH is a key component. However, if we adopt RAR-model for GC we have a harmonic potential for the central core of dark matter and trajectories of bright stars are elliptical where GC coincides with centers of ellipses while in reality GC coincides with foci of observed trajectories of bright stars [47,48]. We should also mention that the RAR model for GC is not consistent the shadow reconstruction in Sgr A* [55] which was reproduced by the Event Horizon Telescope Collaboration.

8. Shadows as black hole fingerprints

Thought experiments were popular at the dawn of general relativity and quantum mechanics development. Around 50 years ago when people started to analyze consequences of an existence of astrophysical black holes James Maxwell Bardeen proposed to consider a bright screen behind an astrophysical rotating black hole assuming that photons propagated along geodesics (without scattering) and in this case he concluded that a virtual observer could detect a small spot in the sky [50] (and later this spot was called *shadow*). However, in these times people did not discuss the Bardeen's consideration as a GR test or a test for BH existence in an observed astronomical object, since first there is no a bright screen behind a selected black hole, second, for known black hole candidates sizes of these shadows are extremely small to be detected. Later, it was understood that secondary images should be concentrated near shadows [51] and shapes and sizes of shadows could be reconstructed from bright structure distributions around shadows and we declared that the shadow for Sgr A* could be reconstructed from global (or and ground – space) VLBI observations in mm or/and sub-mm bands (or X-ray band) [52] (simplifying our proposal we said that for theorists black holes are vacuum solutions of Einstein equations while for observers black holes are small spots (shadows) in the sky). These spectral bands were chosen since H. Falcke at al. showed in numerical simulations [53] that for 1 cm or longer wave lengths scatter of photons on electrons could spoil bright images around shadows while for 1.3 mm wave lengths or shorter shadows could be detectable. Consequent studies confirmed our predictions in [52] since the Event Horizon Telescope (EHT) Collaboration reconstructed the shadow around Sgr A* observed in April 2017 at 1.3 mm wavelength [55] (earlier, the EHT reported about shadow reconstruction for M87* [54]). In spite of great differences in masses and distances for Sgr A* and M87* their shadow diameters are comparable since as it was found we have 52 μas for Sgr A* and 42 μas for M87*. We showed that a black hole spin could be evaluated from an analysis of shadow shape [52].

A cosmic plasma is quasi-neutral it is natural to expect that astrophysical black hole has a very small electric charge. In spite of these expectations we derived an analytical expression for a shadow size as a function of charge [56] (we followed an approach used earlier in [57,58]). It means that photons could measure a black hole charge since a charge changes the Schwarzschild metric with the Reissner – Nordström one. We also should to note that Reissner – Nordström metric is a solution in Randall – Sundrum gravity theory with an extra dimension [59]. Really, this solution looks like Reissner – Nordström metric but it is a generalization of this solution since parameter q^2 may be negative (q is a black hole charge) and Dadhich et al. called it a Reissner – Nordström metric with a tidal charge since this additional parameter was caused by an existence of an extra dimension [59]. Later, it was proposed to adopt a Reissner – Nordström metric with a tidal charge for the GC [60], however, it was shown that a significant negative tidal charge is inconsistent with current estimates of a shadow size in Sgr A* [61].

Earlier we found allowed intervals for tidal charges based on EHT estimates of shadow sizes in M87* [54] and Sgr A* [55]. We will remind expression for a Reissner – Nordström black hole with a tidal charge in natural units ($G = c = 1$) in a form

$$ds^2 = -\left(1 - \frac{2M}{r} + \frac{\mathcal{Q}^2}{r^2}\right) dt^2 + \left(1 - \frac{2M}{r} + \frac{\mathcal{Q}^2}{r^2}\right)^{-1} dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2), \tag{1}$$

where M is a black hole mass, $\mathcal{Q}$ is its charge. Constants E and L are connected with photon and they are describe photon geodesics, namely E is photon's energy, L is its angular momentum. If we introduce normalized radial coordinate, impact parameter and charge $\hat{r} = r/M, \xi = L/(ME)$, $\hat{\mathcal{Q}} = \mathcal{Q}/M$. We introduce also variables $l = \xi^2, q = \hat{\mathcal{Q}}^2$, then critical impact parameter corresponding to shadow radius [62]

$$l_{\rm cr} = \frac{(8q^2 - 36q + 27) + \sqrt{D}}{2(1-q)}, \tag{2}$$

where $D=-512\left(q-\dfrac{9}{8}\right)^3$. As we noted earlier parameter q may be negative for a Reissner – Nordström black hole with a tidal charge (or for Horndeski scalar-tensor theory of gravity [63, 64]).

The EHT Collaboration evaluated the shadow radius in M87* and estimated parameters of several spherically symmetric metrics which may be considered as alternatives for Schwarzschild metric in M87* [65]. In [66] we generalizes results [65] for a Reissner – Nordström black hole with a tidal charge assuming similarly to [65], that angular diameter of a shadow in M87* $\theta_{\rm sh\ M87^*}\approx 3\sqrt{3}(1\pm 0.17)\,\theta_{\rm g\ M87^*}$, at confidence level around 68% or $\theta_{\rm sh\ M87^*}\in[4.31,6.08]\theta_{{\rm g}\ M87*}$, where $\theta_{{\rm g}\ M87*}\approx 8.1\ \mu as$, since $\theta_{\rm g\ M87^*}=2M_{M87*}/D_{M87*}$ ($M_{M87*}=6.5\times 10^9M_{\odot}$ and $D_{M87*}=17$ Mpc, we found $q\in[-1.22,0.814]$ from Eq. (2). In this case an upper limit for q parameter ($q_{upp}=0.814$) corresponds to an upper parameter $\mathcal{Q}_{upp}=\sqrt{q_{upp}}\approx 0.902$, which corresponds to quantity calculated numerically and shown in Fig. 2 in [65].

Similarly to our previous estimates for tidal charge in M87* in paper [67] we estimated a tidal charge for the black hole in GC. We used estimates of shadow radius in GC from [55]. Following these studies, we assume that the shadow diameter in GC is $\theta_{\rm sh\ M87^*}\approx(51.8\pm 2.3)\ \mu as$ at C. L. 68% and in this case we obtain constraints for a tidal charge $-0.27<q<0.25$ at the same confidence level.

These results may be used for analytical estimates of charge for Kazakov – Solodukhin (KS) black hole. Really Kazakov and Solodukhin considered a Schwarzschild black hole perturbed by quantum fluctuations [68]. We should note that black hole with a negative tidal charge (or scalar-tensor charge in Horndeski gravity) could treated as a good approximation for KS black hole for a small KS charge, really according to Eq. (3.21) in [68] we have

$$g(r)=-\frac{2M}{r}+\frac{1}{r}\left(r^2-{q_{KS}}^2\right)^{1/2}\approx 1-\frac{2M}{r}-\frac{{q_{KS}}^2}{r^2}, \tag{3}$$

where q_{KS} is a KS charge. For small parameter q_{KS} approximation we could use previous estimates for a KS charge in Sgr A* $(q_{KS})^2<0.27$ ($(q_{KS})<0.52$). As we see in Fig. 2 in [65] the shadow radius is growing as q_{KS} is growing and it corresponds to the shadow diameter dependence of a tidal charge given in Eq. (2).

9. Conclusion

Observations of bright stars near the GC confirmed predictions of GR in the first post-Newtonian approximation for gravitational redshift for S2 star trajectory near its pericenter passage in May 2018. GRAVITY collaboration found that the Schwarzschild precession for S2 star corresponds to GR predictions. Several alternative theories of gravity were constrained with observations of bright stars. Reconstructions of shadows in M87* and Sgr A* give an opportunity to check GR predictions in these objects and to constrain parameters of alternative models for these objects [69, 70].

Acknowledgements

The author thanks the organizers of the XXXIV International Workshop on High Energy Physics "From Quarks to Galaxies: Elucidating Dark Sides"(IHEP, Protvino) for the invitation to participate at this event and their attention to the research presented in the paper.

Список литературы

1. *Alexandrov A. D.* The Space-time of the Theory of Relativity // Jubilee of Relativity Theory / Ed. by A. Mercier and M. Kervaire, Helvetica Physica Acta, Supplementum IV Basel: Birkhauser Verlag —— 1956. — P. 44–45.

2. The Role of Gravitation in Physics. Report from the 1957 Chapel Hill Conference / Ed. by C. M. DeWitt and Dean Rickles, Max Planck Institute for the History of Science Edition Open Access — 2017. — `www.edition-open-access.de`.

3. *Feynman R. P.* What Do You Care What Other People Think?— London: Penguin Group Ltd, 1988.

4. *Feynman R. P., Morinigo F. B. and Wagner W. G.* Feynman Lectures on Gravitation.— Boca Raton: Taylor & Francis Group Ltd, 2003.

5. *Ginzburg V. L.* Experimental Verification of General Relativity Theory // Relativistic Theories of Gravitation. Proceedings of the Conference held in Warsaw and Jablonna, July 1962/ Ed. by L. Infeld, — 1962. — P. 55–69.

6. *Demianski M.* The Jablonna conference on gravitation: continuing source of inspiration // Gen. Relativ. Grav. — 2014. — V. 46. Iss. 9. — ID. 1718 (10 pp.).

7. *Ginzburg V. L.* What problems of physics and astrophysics seem now to be especially important and interesting (thirty years later, already on the verge of XXI century)? // Phys. Uspekhi — 1999. — V. 42. Iss. 6. — P. 353–373.

8. *Ginzburg V. L.* "Physical minimum" – what problems of physics and astrophysics seem now to be especially important and interesting at the beginning of the XXI century? // Phys. Uspekhi — 2007. — V. 50. Iss. 4. — P. 332–332.

9. *Kennefick D.* Controversies in the History of the Radiation Reaction Problem in General Relativity // arXiv:gr-qc/9704002.

10. *Kennefick D.* Einstein versus the Physical Review // Phys. Today — 2005. — V. 58. Iss. 9. — P. 43.

11. *Born M. and Einstein A.* The Born – Einstein Letters. Correspondence between Albert Einstein and Max and Hedwig Born from 1916 to 1955 with commentaries by Max Born.— London: Macmillan Press Ltd, 1971. — P. 125.

12. *Einstein A and Rosen N.* On Gravitational Waves // J. Franklin Inst. — 1937. — V. 223. — P. 43–54.

13. *Arnold V. I.* Huygens and Barrow, Newton and Hooke.— Basel: Birkhäuser Verlag, 1990.

14. *Cohen I. B.* The Newtonian Revolution.— Cambridge: Cambridge University Press, 1983.

15. *Eckart A. and Genzel R.* Observations of stellar proper motions near the Galactic Centre // Nature — 1996. — V. 383. Iss. 6599. — P. 415–417.

16. *Ghez A. M., Klein B. L., Morris M., Becklin E. E.* High Proper-Motion Stars in the Vicinity of Sagittarius A*: Evidence for a Supermassive Black Hole at the Center of Our Galaxy // Astrophys. J. — 1998. — V. 509. Iss. 2. — P. 678–686. — arXiv:astro-ph/9807210.

17. *Abuter R. et al.* [GRAVITY Collaboration], Detection of the gravitational redshift in the orbit of the star S2 near the Galactic centre massive black hole // Astron. Astrophys. — 2018. — V. 615. — ID. L15 (10 PP.). — arXiv:1807.09409.

18. *Abuter R. et al.* [GRAVITY Collaboration], A geometric distance measurement to the Galactic Center black hole with 0.3% uncertainty // Astron. Astrophys. — 2019. — V. 625. — ID. L10 (9 PP.). — arXiv:1904.05721.

19. *Do T., Hees A., Ghez, A. et al.* Relativistic redshift of the star S0-2 orbiting the Galactic Center supermassive black hole // Science — 2019. — V. 365. ISS. 6454. — PP. 664 – 668. — arXiv:1907.10731v1 [astro-ph.GA].

20. *Abuter R. et al.* [GRAVITY Collaboration], Detection of orbital motions near the last stable circular orbit of the massive black hole Sgr A* // Astron. Astrophys. — 2018. — V. 618. — ID. L10 (15 PP.). — arXiv:1810.12641.

21. *Abuter R. et al.* [GRAVITY Collaboration], Detection of the Schwarzschild precession in the orbit of the star S2 near the Galactic Centre massive black hole // Astron. Astrophys. — 2020. — V. 636. — ID. L5 (14 PP.). — arXiv:2004.07187.

22. *Zakharov A. F., Nucita A. A., de Paolis F. and Ingrosso G.* Apoastron shift constraints on dark matter distribution at the Galactic Center // Phys. Rev. D — 2007. — V. 76. — ID. 062001 (6 PP.). — arXiv:0707.4423.

23. *Capozziello S.* Curvature Quintessence // Int. J. Mod. Phys. D — 2002. — V. 11. Iss. 4. — ID. 483–491. — arXiv:gr-qc/0201033.

24. *Capozziello S., Carloni S. and Troisi A.* Recent Res. Develop. Astron. Astrophys. 2003. — V. 1. — P. 625–671. — arXiv:astro-ph/0303041.

25. Carroll S. M., Duvvuri V., Trodden M. and Turner M. S. Is cosmic speed-up due to new gravitational physics? // Phys. Rev. D — 2004. — V. 90 — ID. 043528 (5 PP.). — arXiv:astro-ph/0306438.

26. *Capozziello S., Cardone V. F., Carloni S. and Troisi A.* Can higher order curvature theories explain rotation curves of galaxies? // Phys. Lett. A — 2004. — V. 326. ISS. 5–6. — P. 292–296. — arXiv:astro-ph/0306438.

27. *Borowiec A., Godlowski W. and Szydlowski M.* Accelerated cosmological models in modified gravity tested by distant supernovae SNIa data // Phys. Rev. D — 2006. — V. 74. — ID. 043502 (9 PP.). — arXiv:astro-ph/0602526.

28. *Capozziello S., Cardone V. F. and Troisi A.* Low surface brightness galaxy rotation curves in the low energy limit of R^n gravity: no need for dark matter? // Mon. Not. R. Astron. Soc. Lett. — 2007. — V. 375. — P. 1423–1440. — arXiv:astro-ph/0603522.

29. *Zakharov A. F., Nucita A. A., de Paolis F. and Ingrosso G.* Solar system constraints on R^n gravity // Phys. Rev. D — 2006. — V. 74. — ID. 107101 (4 PP.). — arXiv:astro-ph/0611051.

30. *Borka D., Jovanović P., Borka Jovanović V. and Zakharov A. F.* Constraints on R^n gravity from precession of orbits of S2-like stars // Phys. Rev. D — 2012. — V. 85. — ID. 124004 (11 PP.). — arXiv:1206.0851.

31. *Zakharov A. F., Borka D., Borka Jovanović V. and Jovanović P.* Constraints on R^n gravity from precession of orbits of S2-like stars: A case of a bulk distribution of mass // 2014. — V. 54. ISS. 6. — P. 1108–1112. — arXiv:1407.0366.

32. *Borka D., Jovanović P., Borka Jovanović V. and Zakharov A. F.* Constraining the range of Yukawa gravity interaction from S2 star orbits // J. Cosmology Astropart. Phys. — 2013. — Iss. 11. — ID. 050 (16 pp). — arXiv:1311.1404.

33. *Cardone V. F. and Capozziello S.* Systematic biases on galaxy haloes parameters from Yukawa-like gravitational potentials // Mon. Not. R. Astron. Soc. Lett. — 2011. — V. 414. — P. 1301–1313. — arXiv:1102.0916.

34. *Borka D., Borka Jovanović V., Capozziello S. et al.* Estimating the Parameters of Extended Gravity Theories with the Schwarzschild Precession of S2 Star // Universe — 2021. — V. 7. ISS. 11. — ID. 407 (18 PP.). — arXiv:2111.00578.

35. *Fierz M. and Pauli W.* On Relativistic Wave Equations for Particles of Arbitrary Spin in an Electromagnetic Field // Proc. R. Soc. London Ser. A — 1939. — V. 173. — P. 211–231.

36. *Boulware D. G. and Deser S.* Can Gravitation Have a Finite Range? // Phys. Rev. — 1972. — V. 6. ISS. 12. — P. 3368–3382.

37. *Gershtein S. S., Logunov A. A., Mestvirishvili M. A. and Tkachenko N. P.* Graviton mass, quintessence, and oscillatory character of Universe evolution // Physics Atomic Nucl. 2004. — V. 67. ISS. 8. — P. 1596–1604. — arXiv:astro-ph/0305125.

38. *Gershtein S. S., Logunov A. A. and Mestvirishvili M. A.* Gravitational field self-limitation and its role in the Universe // Physics – Uspekhi 2006. — V. 49. ISS. 11. — P. 1179–1195.

39. *Workman R. L. et al.* [Particle Data Group], The Review of Particle Physics // Progr. Theor. Exper. Phys. — 2022. — V. 2023. — ID. 083C01 (2270 PP.).

40. *de Rham C., Gabadadze G. and Tolley A. J.* Resummation of Massive Gravity // Phys. Rev. Lett. — 2017. — V. 89. — ID. 231101 (4 PP.). — arXiv:1011.1232.

41. *de Rham C., Deskins J. T., Tolley A. J. and Zhou S.-Y.* Graviton mass bounds // Rev. Mod. Phys. — 2017. — V. 89. ISS. 2 — ID. 025004 (29 PP.). — arXiv:1606.08462.

42. *Abbott B. P. et al.* [LIGO Scientific Collaboration and Virgo Collaboration], Observation of Gravitational Waves from a Binary Black Hole Merger // Phys. Rev. Lett. — 2016. — V. 116. — ID. 061102 (16 PP.). — arXiv:1602.03837.

43. *Zakharov A. F., Jovanović P., Borka D. and Borka Jovanović V.* Constraining the range of Yukawa gravity interaction from S2 star orbits II: bounds on graviton mass // J. Cosmology Astropart. Phys. — 2016. — Iss. 05. — ID. 045 (11 pp). — arXiv:1605.00913.

44. *Zakharov A. F., Jovanović P., Borka D. and Borka Jovanović V.* Constraining the range of Yukawa gravity interaction from S2 star orbits III: improvement expectations for graviton mass bounds // J. Cosmology Astropart. Phys. — 2018. — Iss. 04. — ID. 050 (21 pp). — arXiv:1801.04679.

45. *Ruffini R., Argüelles C. R. and Rueda J. A.* On the core-halo distribution of dark matter in galaxies // Mon. Not. R. Astron. Soc. — 2015. — V. 451. — P. 622–628. — arXiv:1409.7365.

46. *Becerra-Vergara E. A., Argüelles C. R., Krut A. , Rueda J. A. and Ruffini R.* Hinting a dark matter nature of Sgr A* via the S-stars // Mon. Not. R. Astron. Soc. Lett. — 2022. — V. 505. — P. L64–L68. — arXiv:2105.06301.

47. *Zakharov A. F.* Testing the Galactic Centre potential with S-stars // Mon. Not. R. Astron. Soc. Lett. — 2022. — V. 513. — P. L6–L9. — arXiv:2108.09709.

48. *Zakharov A. F.* Orbits of Bright Stars Near the Galactic Center as a Tool to Test Gravity Theories // Moscow Univ. Phys. — 2022. — V. 77. — P.341–348.

49. *Akiyama K., Alberdi A., Alef W. et al.* [Event Horizon Telescope Collaboration] First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way // Astrophys. J. Lett. — 2022. — V. 930 — L12 (21 pp).

50. *Bardeen J.* Timelike and null geodesics in the Kerr metric // Black Holes (Les Astres Occlus) / Ed. by S. DeWitt and C. DeWitt-Morette, New York: Gordon & Breach — 1973. — P. 215–239.

51. *Holz D. E. and Wheeler J. A.* Retro-MACHOs: π in the Sky? // Astrophys. J. — 2002. — V. 578, Iss. 1. — P. 330–334. — arXiv:astro-ph/0209039.

52. *Zakharov A. F., Nucita A. A., De Paolis F. and Ingrosso G.* Measuring the black hole parameters in the galactic center with RADIOASTRON // New Astron. — 2005. — V. 10. Iss. 6. — P. 479–489. — arXiv:astro-ph/0411511.

53. *Falcke H., Melia F. and Agol E.* Viewing the Shadow of the Black Hole at the Galactic Center // Astrophys. J. Lett. — 2000. — V. 528, Iss. 1. — L13–L16. — arXiv:astro-ph/9912263.

54. *Akiyama K., Alberdi A., Alef W. et al.* [Event Horizon Telescope Collaboration], First M87 Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole // Astrophys. J. Lett. — 2019. — V. 875. — L1 (17 pp). — arXiv:1906.11238.

55. *Akiyama K., Alberdi A., Alef W. et al.* [Event Horizon Telescope Collaboration], First Sagittarius A* Event Horizon Telescope Results. I. The Shadow of the Supermassive Black Hole in the Center of the Milky Way // Astrophys. J. Lett. — 2022. — V. 930 — L12 (21 pp).

56. *Zakharov A. F., De Paolis F., Ingrosso G. and Nucita A. A.* Direct measurements of black hole charge with future astrometrical missions // Astron. & Astrophys. — 2005. — V. 442. — P. 795–799. — arXiv:astro-ph/0505286.

57. *Zakharov A. F.* Capture Cross Sections for a Charged Reissner-Nordstrom Black-Hole - Photons Ultrarelativistic Uncharged Particles and Slow Uncharged Particles // Sov. Astron. — 1991. — V. 35. no. 2. — P. 147–150.

58. *Zakharov A. F.* Particle capture cross sections for a Reissner–Nordström black hole // Class. Quant. Grav. — 1994. — V. 11. Iss. 4. — P. 1027–1033.

59. *Dadhich N., Maartens R., Papadopoulos P. and Rezania V.* Black holes on the brane // Phys. Lett. B — 2000. — V. 487. Iss. 1–2. — P. 1–16. BibDash arXiv:hep-th/0003061.

60. *Bin-Nun A. Y.* Gravitational lensing of stars orbiting Sgr A* as a probe of the black hole metric in the Galactic center // Phys. Rev. — 2010. — V. 83. Iss. 6. — P. 064009 (8 PP.). BibDash arXiv:hep-th/0003061.

61. *Zakharov A. F., Nucita A. A., De Paolis F., Ingrosso G* Shadows as a tool to evaluate black hole parameters and a dimension of spacetime // New Astron. Rev. — 2012. — V. 56. Iss. 2–3. — P. 64–73.

62. *Zakharov A. F.* Constraints on a charge in the Reissner-Nordström metric for the black hole at the Galactic Center // Phys. Rev. D — 2014. — V. 90. Iss. 6. — ID. 062007. (8 pp). — arXiv:1411.5911 [gr-qc].

63. *Babichev E., Charmousis C. and Lehébel A.* Asymptotically flat black holes in Horndeski theory and beyond // J. Cosmology Astropart. Phys. — 2017. — Iss. 4. — ID. 027 (21 pp). — arXiv:1702.01938.

64. *Zakharov A. F.* Constraints on tidal charge of the supermassive black hole at the Galactic Center with trajectories of bright stars // European Phys. J. C — 2018. — V. 78. Iss. 8. — ID. 689. (7 pp). — arXiv:1804.10374.

65. *Kocherlakota P., Rezzolla L., Falcke H. et al* [Event Horizon Telescope Collaboration] Constraints on black-hole charges with the 2017 EHT observations of M87* // Phys. Rev. D — 2021. — V. 103 — ID. 104047 (15 pp). — arXiv:2105.09343.

66. *Zakharov A. F.* Constraints on a Tidal Charge of the Supermassive Black Hole in M87* with the EHT Observations in April 2017 // Universe — 2022. — V. 8. Iss. 3. — ID. 141 (10 pp). — arXiv:2108.01533.

67. *Zakharov A. F.* Shadows around Sgr A* and M87* as a tool to test gravity theories // Astron. Astrophys. Trans. — 2022. — V. 33. Iss. 3. — P. 285–296. — arXiv: arXiv:2208.06805[astro-ph.GA].

68. *Kazakov D. I. and Solodukhin S. N.* On the quantum deformation of the Schwarzschild solution // Phys. Lett. B. — 1994. — V. 429. Iss. 1. — P. 153-176.

69. *Akiyama K., Alberdi A., Alef W. et al.* [Event Horizon Telescope Collaboration] First Sagittarius A* Event Horizon Telescope Results. VI. Testing the Black Hole Metric // Astrophys. J. Lett. — 2022. — V. 930 — L17 (44 pp).

70. *Vagnozzi S., Roy R., Tsai, Y.-D. et al.* Horizon-scale tests of gravity theories and fundamental physics from the Event Horizon Telescope image of Sagittarius A^* — arXiv:2205.07787.