A primer on Unimodular gravity

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Мы даем базовое введение в унимодулярную гравитацию как на классическом, так и на квантовом уровне, обсуждая роль, которую она может сыграть в интерпретации проблемы космологической постоянной. Цель этой работы находится на базовом уровне, но представлены методы, используемые на исследовательском уровне. Цель статьи состоит в том, чтобы дать читателю возможность понять интерес к UG, сделав доступной современную литературу по этой теме.

We give a basic introduction to Unimodular Gravity both at the classical and quantum level, discussing the paper it might play in the interpretation of the Cosmological Constant problem. The aim of this work is at the basic level but techniques used at the research level are presented. The goal of the paper is to enable the reader to grasp the interest of UG while making the recent literature on the topic accessible.

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Introduction

Unimodular gravity (UG) was first considered by Einstein in 1919 [1] as an attempt to make a connection between gravity and Mie's theory of electromagnetic wave scattering published a few years prior. This original proposal was a far cry from the current status of UG. However, seven years later, in his book [2], Pauli discussed the topic in a much more modern flavor. This culminated when the attractive properties of UG in the context of a field theory were noticed in [3], which was soon followed by the works [4–6].

Our presentation of UG will be based on the construction of a theory whose particle content is that of a massless spin-two particle. In that sense, we follow the spirit of [3], where we review the widespread yet erroneous idea that GR is the only ghost-free theory of a massless spin-two particle. Only after this do we consider the fully non-linear theory and give a systematic approach to the quantization through the path integral formalism.

Let us precede this construction with a motivation for UG in the context of the Cosmological Constant Problem (CCP) see [7] for the original overview, or [8] for an updated and extensive bibliographic guide of the topic. The aspect of the CCP we want to address here is the fact that contrary to Minkowski space-time where normal ordering can set vacuum energy to zero.

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On curved spacetimes, vacuum energy is naively expected to contribute to the stress-energy tensor as

$$\langle T_\mu{}^\mu\rangle_0\, g_{\mu\nu}, \tag{1}$$

which has the same form as the CC term on the equations of motion (EoM) of GR, i.e.,

$$R_{\mu\nu} - \frac{1}{2}R g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu} + \langle T_\mu{}^\mu\rangle_0\, g_{\mu\nu}. \tag{2}$$

Therefore in GR one has that the CC term and vacuum energy have the same form.

However, any gross estimate for the order of magnitude of the contribution of vacuum energy yields results over 30 orders of magnitude above the observed value for the CC [8]. Thus there is a fine-tuning problem in which the different contributions to the vacuum energy, including those coming from phase transitions have to cancel to an accuracy way beyond thirty decimal places. It is in this sense that the CC is non-natural.

We will show in this report that, in the context of UG, the CC appears as an integration constant with no naturalness problem associated. Furthermore, the CC was shown to be stable under radiative corrections in UG [9].

In this work, we also introduce the necessary tools to apply the path integral formalism to UG.

Let us stress here that we do not aim for a full status report of UG, for that one can consult [10] and references therein. We aspire to a well-motivated introduction to the theory, its interest, and some calculations that try to solve the question of the equivalence of UG and GR in different scenarios.

Linear Field Theory.

In this section, we review the theory of a linear field that **only propagates a massless spin-two field**, that is, with two polarisation degrees of freedom (DOF). As originally discussed in [3] and later in [11], we show that the linear diffeomorphisms invariance, *LDiff*, is too restrictive in the sense that the subgroup of volume-preserving linear diffeomorphisms *LTDiff*, can successfully do the job.

The notion of a massless free particle in flat spacetimes is tied to the invariance under the Poincaré group, (in particular to the *proper orthochronous Lorentz group* $\mathcal{L}^+_\uparrow$), of the unitary representation of the covering group of the *little group* of the four-vector,

$$k = \begin{pmatrix} E, & 0, & 0, & E \end{pmatrix}. \tag{3}$$

To make the group finite-dimensional, gauge invariance introduces the equivalence class,

$$h_{\mu\nu}(k) \equiv h_{\mu\nu}(k) + 2k_{(\mu}\xi_{\nu)}(k), \tag{4}$$

with the two constraints,

$$k^2\xi_\mu(k) = 0 \qquad \text{and} \qquad k_\mu\xi^\mu(k) = 0. \tag{5}$$

As mentioned in [3] the first condition is too restrictive for interaction and hence, it must be discarded. At this point, if we also dropped the second condition we would get just,

$$h_{\mu\nu}(k) \equiv h_{\mu\nu}(k) + 2k_{(\mu}\xi_{\nu)}(k), \tag{6}$$

which is known to correspond to linearized diffeomorphisms *LDiff*. But from the above discussion, we can see that keeping the transverse condition does the job. This would correspond in position space to,

$$h_{\mu\nu}(x) \equiv h_{\mu\nu}(x) + 2\partial_{(\mu}\xi_{\nu)}(x), \tag{7}$$

$$\partial_\mu \xi^\mu(x) = 0. \tag{8}$$

This characterizes linearized transverse diffeomorphisms, i.e., *LTDiff*.

Examining eqs. (7) and (8) we can see that, from a physical perspective, *LTDiff* transformations have three DOF. These three DOF are enough to pass from the five DOF of massive gravity to the two DOF of a theory that only propagates a massless spin-two particle.

Taking into account these observations we now build a linearized lagrangian that is second order in derivatives and such that it is ghost free while only propagating a spin-two massless particle. We start by writing the more general set of operators that satisfy the above conditions.

$$\mathcal{L} \equiv \sum_{i=0}^{4} C_i \mathcal{O}^{(i)}, \tag{9}$$

where, defining $h \equiv \eta^{\alpha\beta} h_{\alpha\beta} = h_\alpha{}^\alpha$

$$\mathcal{O}^{(1)} \equiv \frac{1}{4}\partial_\mu h_{\alpha\beta}\partial^\mu h^{\alpha\beta} \qquad \mathcal{O}^{(2)} \equiv -\frac{1}{2}\partial_\lambda h^{\mu\lambda}\partial_\rho h_\mu{}^\rho \tag{10}$$

$$\mathcal{O}^{(3)} \equiv \frac{1}{2}\partial_\mu h \partial_\lambda h^{\mu\lambda} \qquad \mathcal{O}^{(4)} \equiv -\frac{1}{4}\partial^\mu h \partial_\mu h \tag{11}$$

Setting $C_1 = 1$ for normalization and requiring invariance under *LTDiff* yields[1]

$$C_1 = C_2 = 1. \tag{12}$$

Additionally, imposing invariance under Weyl transformations results in,

$$C_3 = \frac{2}{n} \qquad \text{and} \qquad C_4 = \frac{n+2}{n^2}. \tag{13}$$

Thus we have that *LWTDiff*[2] can be obtained by the combination of the conditions in eqs. (9), (12) and (13).

[1]For Fierz-Pauli we have the more restrictive $C_i = 1$. We can see that *LTDiff* gives a less stringent condition.

[2]The reason to consider *LWTDiff* will be discussed in the next section.

It is interesting to note that we could have obtained these constraints for C_i starting from the Fierz-Pauli Lagrangian (with $C_i = 1$) by rewriting,

$$h_{\mu\nu} \to h_{\mu\nu} - \frac{1}{n} h \eta_{\mu\nu}. \tag{14}$$

Here, we stress that eq. (14) **does not represent a field redefinition**, as it is not invertible.

In the next section, we will consider the fully non-linear theories where *LTDiff* will be substituted by *TDiff*, which correspond to volume-preserving diffeomorphisms that are connected to the identity.

Non-Linear Field Theory.

In the previous section, we considered the theory at the linear level. One could always perform a bootstrap [12,13] to arrive at the complete non-linear theory. Alternatively, we present here the easiest[1] way to get the complete, non-linear theory.

We consider the Einstein-Hilbert action,

$$S[\hat{g}_{\mu\nu}] = -\frac{1}{2\kappa^2} \int d^n\, x \sqrt{-\hat{g}} R \left[\hat{g}_{\mu\nu}\right]. \tag{15}$$

Where the metric $\hat{g}_{\mu\nu}$ is unimodular. The EoM can be obtained considering only transverse variations of the metric [14]. Here consider a practical approach that will, later on, simplify the quantization. For this, we define, from the unimodular metric,

$$\hat{g}_{\mu\nu} \to g_{\mu\nu} |g|^{\frac{1}{n}}. \tag{16}$$

Since we are only considering an action that is invariant under volume-preserving diffeomorphisms, the allowed changes of coordinates will be such that the determinant of the metric behaves as a scalar and not as a density. Then, we can regard eq. (16) as a conformal transformation. The action (15), after integration by parts, reads,

$$S[g_{\mu\nu}] = -\frac{1}{2\kappa^2} \int d^n\, x |g|^{\frac{1}{n}} \left(R\left[g_{\mu\nu}\right] + \frac{(n-1)(n-2)}{4n^2} \frac{\nabla_\mu g \nabla^\mu g}{g^2} \right). \tag{17}$$

At this point, we note that eq. (16) is such that the field $g_{\mu\nu}$ is Weyl-invariant. Casting the action in terms of the metric $g_{\mu\nu}$ results in an action whose gauge group is given, see [15], by the semidirect product

$$\text{WTDiff} = \text{Weyl} \ltimes \text{TDiff}. \tag{18}$$

[1] To our knowledge.

The equations of motion for this action read [16],

$$R_{\mu\nu} - \frac{1}{n}Rg_{\mu\nu} + \Theta_{\mu\nu} = T_{\mu\nu} - \frac{1}{n}Tg_{\mu\nu}, \tag{19}$$

$$\Theta_{\mu\nu} = \frac{(2-n)(2n-1)}{4n^2}\left(\frac{\nabla_\mu g \nabla_\nu g}{g^2} - \frac{1}{n}\frac{(\nabla g)^2}{g^2}g_{\mu\nu}\right) + + \frac{n-2}{2n}\left(\frac{\nabla_\mu \nabla_\nu g}{g} - \frac{1}{n}\frac{\nabla^2 g}{g}g_{\mu\nu}\right). \tag{20}$$

If we choose the gauge $g = 1$, in four dimensions eqs. (19) and (20) become the traceless EoM of GR, [1,17],

$$R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu} = T_{\mu\nu} - \frac{1}{4}Tg_{\mu\nu}. \tag{21}$$

Now in eq. (21), there seems to be no CC. However, if we assume the covariant conservation of the stress-energy tensor[1] and use the Bianchi identities, we have,

$$\begin{aligned} &\nabla^\mu T_{\mu\nu} = 0 \rightarrow \\ &\nabla^\mu\left(R_{\mu\nu} - \frac{1}{4}Rg_{\mu\nu}\right) = \frac{1}{4}\nabla^\nu R = -\frac{1}{4}\nabla^\nu T \\ &\nabla^\mu\left(T + R\right) = 0 \rightarrow T + R = -\mathcal{C}. \end{aligned} \tag{22}$$

Plugging in the constraint eq. (22) into the gauge fixed EoM,

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \mathcal{C}g_{\mu\nu} = T_{\mu\nu}\,. \tag{23}$$

In eq. (23) we can appreciate the different context in which a *Cosmological Constant term* appears in the EoM as in contrast to GR. Here it is an **integration constant** term, agnostic to the vacuum expectation value of fields. Furthermore, it was proven in [9], that it remains stable under radiative corrections.

We have just shown that eq. (21) can be reduced to those of GR[2]. In particular the line element,

$$ds^2 = a(t)^{-3/2}dt^2 - a^{1/2}(t)\delta_{ij}\, dx^i\, dx^j \tag{24}$$

satisfies the Cosmological principle[3], it is unimodular and it corresponds to an expanding solution where

$$a(t) = \mathcal{A}\left(3\left(t - t_0\right)\right)^{4/3}. \tag{25}$$

In this case, the expansion is geometry-driven.

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[1] This is in GR given by the invariance of the action under *Diff.* Nonetheless as discussed in [10], it is hard to find out a physically consistent theory that does not satisfy the covariant conservation of $T_{\mu\nu}$.

[2] Bearing in mind the different nature of the CC term.

[3] Being homogeneous and spatially isotropic at large scales.

Quantizing UG

Since we have just discussed that both UG and GR have the same EoM and propagate the same number of degrees of freedom, then it must be the case that any tree-level computation must be equivalent for both theories, as explored recently in [**?**].

Nevertheless, this equivalence at the tree level need not, in principle, extrapolate to loop-level computations. From the path integral approach to the quantization of UG, the path integral measure must incorporate the fact that the gauge group is not *Diff* but *WTDiff*, as discussed for example in [18].

Therefore, while on-shell states match for both UG and GR, since these two theories **do not share gauge groups**, while the GR gravitons running on loops are off-shell and as such need not be traceless, UG gravitons are traceless inside loops. That is, since in each case loops run over different states, lacking a reason to expect the cancellations of these differences at all orders in perturbation theory, we could expect differences to arise when comparing quantum effects for both UG and GR.

Since the only possible difference[1] must be at the quantum level. We now present a particular formalism[2] that allows performing perturbative calculations in the path integral formalism.

We will consider the **background field expansion**, see [19] for a pedagogic introduction and further references. In this approach, we split the metric field into the sum of a *classical* and a *quantum* part,

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}. \tag{26}$$

Additionally, we assume the measure of the path integral is shift-invariant, i.e.,

$$\int [\mathcal{D}g_{\mu\nu}] \;\; \exp\left[i\, S[g] + T \cdot g\right] = \int [\mathcal{D}h_{\mu\nu}] \;\; \exp\left[i\, S[h] + T \cdot g\right], \tag{27}$$

where $T \cdot g$ represents the sources that are added to the partition function to obtain field expectation values by functional differentiation,

$$T \cdot g = \int d^4x \sqrt{-g} T^{\mu\nu} g_{\mu\nu}. \tag{28}$$

The beauty of this method is that it allows fixing the gauge of the quantum part while preserving the gauge invariance of the background. Then all computations are invariant under gauge transformations of the background. The same happens for the counterterms.

As it happens for other gauge theories, gauge freedom introduces an indeterminacy in the path integral. To resolve this one often uses the DeWitt-Feynman-Fadeev-Poppov method. This results in adding to the original action, a part coming from the gauge fixing and certain ghost content that

[1] Aside from the different role the Cosmological Constant plays on each theory.

[2] This choice is by no means unique, and there is an ongoing discussion on whether different approaches to the quantization might yield different results [10]. Despite its interest, we will not further discuss this equivalence here.

represents the jacobian of the gauge fixing condition.

$$S \to S + S_{\text{G.F.}} + S_{\text{Gh}}, \tag{29}$$

finding a gauge fixing condition for the quantum *WTDiff* that preserves the background's *WTDiff* invariance is an open problem [20].

For this reason, we consider the alternative BRST approach. It was found that the Gauge fixed action eq. (29) is invariant under the so-called BRST transformation [21, 22]. This implies that the physical content of the gauge theory is given by the cohomology class of the BRST nilpotent operator $\mathfrak{s}$. In this case, we can split it as,

$$\mathfrak{s} = \mathfrak{s}_W + \mathfrak{s}_D, \qquad \text{where} \tag{30}$$

$$\mathfrak{s}\bar{g}_{\mu\nu} = 0 \tag{31}$$

$$\mathfrak{s}h_{\mu\nu} = \mathfrak{s}_D h_{\mu\nu} + \mathfrak{s}_W h_{\mu\nu} = \mathcal{L}_{c^\mu}(\bar{g}_{\mu\nu} + h_{\mu\nu}) + 2c(\bar{g}_{\mu\nu} + h_{\mu\nu}) \tag{32}$$

where the action is given in terms of the original lagrangian density by,

$$S_{\text{BRST}} = \int d^4x\, \mathcal{L} + \mathfrak{s}\Psi. \tag{33}$$

Now, because the generator of the volume-preserving diffeomorphisms is transverse, so must be the associated ghost field c^μ,

$$D_\mu c^\mu = 0, \tag{34}$$

where the transversality condition is given by the Weyl covariant derivative.

The easiest way to implement this is using a projector [9, 20] Π^μ_ν acting on an unrestricted c^ν. This introduces a $U(1)$ invariance,

$$c^\nu \to D^\nu f. \tag{35}$$

A general treatment of such an algebra requires the introduction of the BV quantization techniques[1] [23, 24]. But in this case, the procedure can be carried out by enlarging the ghost content of the theory. We now give the guidelines to do so. A more crude discussion can be found in [9, 20].

We include the necessary set of anti-ghost and auxiliary fields that close the algebra.

$$\begin{aligned} & h_{\mu\nu}{}^{(0,0)},\ c_\mu^{(1,1)},\ b_\mu^{(1,-1)},\ f_\mu^{(0,0)},\ \phi^{(0,2)} \\ & \pi^{(1,-1)},\ \pi'^{(1,1)},\ \bar{c}^{(0,-2)},\ c'^{(0,0)} \\ & c^{(1,1)},\ b^{(1,-1)},\ f^{(0,0)}, \end{aligned} \tag{36}$$

where in the label (n, m), n denotes the Grasmann number (modulo 2) and m is the ghost number. The first line corresponds to the physical graviton and

[1]In the workshop S. Lyakhovich presented his work on Unfree Gauge Symmetries which covers the topic in full detail.

the ghost fields that one would naively need for *Diff*. In adition ϕ corresponds to the $U(1)$ transformation. The second line has the ghost content to fix that $U(1)$ and the last line corresponds to the Weyl invariance.

This is enough to make $\mathfrak{s}$ nil-potent to the one-loop level, ensuring BRST invariance of the total action. For completeness, the counterterm reads,

$$S^{\text{TDiff}}_{\text{BRST}} + S^{\text{Weyl}}_{\text{BRST}} = \int d^n x\, b^\mu \Big(\Box^2 c^{(1,1)}_\mu - 2R_{\mu\rho}\nabla^\rho\nabla^\nu c^{(1,1)}_\nu - \Box R_\mu{}^\rho c^{(1,1)}_\rho - \\ - 2\nabla_\sigma R_\mu{}^\rho \nabla^\sigma c^{(1,1)}_\rho - R_{\mu\rho}R^{\rho\nu}c^{(1,1)}_\nu \Big) - \overline{c}^{(0,-2)}\Box\phi^{(0,2)} + \pi^{(1,-1)}\Box\pi'^{(1,1)} - \\ - \frac{1}{\rho_1}\left(F_\mu F^\mu + \nabla_\mu c'^{(0,0)}\nabla^\mu c'^{(0,0)} + 2F_\mu\nabla^\mu c'^{(0,0)}\right) - f^{(0,0)}\Box f^{(0,0)} + \frac{\alpha}{2} f^{(0,0)}\Box h + \\ + \frac{\alpha}{2} h\Box f^{(0,0)} + 2n\alpha b^{(1,-1)}\Box c^{(1,1)}, \qquad \text{where} \quad F_\mu \equiv \nabla^\nu h_{\mu\nu} - \frac{1}{n}\nabla_\mu h. \tag{37}$$

Once the ghost content issue is solved, one can apply the Schwinger-DeWitt proper time expansion[1] to calculate the divergences of UG. Using the procedure above described, it was found in [9] that the CC does not receive radiative corrections in UG.

Summary and conclusions.

In this work, we have provided an introduction to UG. First, we have discussed the interest the theory has from the point of view of the naturalness of the Cosmological Constant. We have paid special attention to its motivation from the particle content side of a linearized field theory. After this, a general discussion on the fully non-linear EoM and their solutions is presented. Here we show the equivalence of UG and GR at the classical level.

Lastly, we have given an overview of a procedure that allows performing QFT from the path integral. In doing so, we give references to works that construct or apply the described techniques to perform calculations in the context of UG. A remarkable result in this context is the fact that the CC is **stable under radiative corrections in UG**.

Some remarkable topics that have not found a place in this work are the question of the equivalence of both theories, the construction of a hamiltonian, the conformal factor in the path integral and the GHP prescription for it and a discussion on the Noether charges.

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[1] See [19] for an introduction to these techniques.

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REFERENCES

1. *Einstein A.* "Spielen Gravitationsfelder imAufbau der materiellen Elementarteilcheneine wesentliche Rolle?" // Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.). — 1919. — P. 349–356.

2. *Pauli W.* Theory of Relativity. — Dover, New York, 1981.

3. *Van Der Bij J., Van Dam H., Ng Y.J.* The exchange of massless spin-two particles // Physica A: Statistical Mechanics and its Applications. — 1982. — 11. — V. 116. — P. 307–320. — URL: https://linkinghub.elsevier.com/retrieve/pii/0378437182902473.

4. *Unruh W.G.* Unimodular theory of canonical quantum gravity // Physical Review D. — 1989. — 8. — V. 40. — P. 1048–1052. — URL: https://link.aps.org/doi/10.1103/PhysRevD.40.1048.

5. *Buchmüller W., Dragon N.* Einstein gravity from restricted coordinate invariance // Physics Letters B. — 1988. — 6. — V. 207. — P. 292–294. — URL: https://linkinghub.elsevier.com/retrieve/pii/0370269388905771.

6. *Buchmüller W., Dragon N.* Gauge fixing and the cosmological constant // Physics Letters B. — 1989. — 6. — V. 223. — P. 313–317. — URL: https://linkinghub.elsevier.com/retrieve/pii/0370269389916080.

7. *Weinberg S.* The cosmological constant problem // Reviews of Modern Physics. — 1989. — 1. — V. 61. — P. 1–23. — URL: https://link.aps.org/doi/10.1103/RevModPhys.61.1.

8. *Carroll S.M.* The Cosmological Constant // Living Reviews in Relativity. — 2001. — 12. — V. 4. — P. 1. — URL: http://link.springer.com/10.12942/lrr-2001-1.

9. *Álvarez E., González-Martín S., Herrero-Valea M., Martín C.P.* Quantum corrections to unimodular gravity // Journal of High Energy Physics. — 2015. — 8. — V. 2015. — P. 78. — URL: http://link.springer.com/10.1007/JHEP08(2015)078.

10. *Carballo-Rubio R., Garay L.J., García-Moreno G.* Unimodular gravity vs general relativity: a status report // Classical and Quantum Gravity. — 2022. — 12. — V. 39. — P. 243001. — URL: https://iopscience.iop.org/article/10.1088/1361-6382/aca386.

11. *Álvarez E., Blas D., Garriga J., Verdaguer E.* Transverse Fierz–Pauli symmetry // Nuclear Physics B. — 2006. — 11. — V. 756. — P. 148–170. — URL: https://linkinghub.elsevier.com/retrieve/pii/S0550321306006614.

12. *Feynman R.* Feynman Lectures on Gravitation. — CRC Press, 2018. — 5. — ISBN: 9780429502859. — URL: https://www.taylorfrancis.com/books/9780429502859.

13. *Delhom A., García-Moreno G., Hohmann M., Jiménez-Cano A., Koivisto T.S.* Bootstrapping gravity and its extension to metric-affine theories. — 2022. — 11.

14. *Álvarez E.* Can one tell Einstein's unimodular theory from Einstein's general relativity? // Journal of High Energy Physics. — 2005. — 3. — V. 2005. — P. 002–002. — URL: http://stacks.iop.org/1126-6708/2005/i=03/a=002?key=crossref.2211d85084ebbdd933a0cf9bdb69a530.

15. *Álvarez E., González-Martín S., Martín C.P.* Note on the gauge symmetries of unimodular gravity // Physical Review D. — 2016. — 6. — V. 93. — P. 123018. — URL: https://link.aps.org/doi/10.1103/PhysRevD.93.123018.

16. *Álvarez E., Vidal R.* Weyl transverse gravity and the cosmological constant // Physical Review D. — 2010. — 4. — V. 81. — P. 084057. — URL: https://link.aps.org/doi/10.1103/PhysRevD.81.084057.

17. *Ellis G.F.R., van Elst H., Murugan J., Uzan J.P.* On the trace-free Einstein equations as a viable alternative to general relativity // Classical and Quantum Gravity. — 2011. — 11. — V. 28. — P. 225007. — URL: https://iopscience.iop.org/article/10.1088/0264-9381/28/22/225007.

18. *de León Ardón R., Ohta N., Percacci R.* Path integral of unimodular gravity // Physical Review D. — 2018. — 1. — V. 97. — P. 026007. — URL: https://link.aps.org/doi/10.1103/PhysRevD.97.026007.

19. *Álvarez E., Anero J.* Covariant Techniques in Quantum Field Theory. — Springer International Publishing, 2022. — ISBN: 978-3-031-13725-9. — URL: https://link.springer.com/10.1007/978-3-031-13726-6.

20. *Herrero-Valea M., Santos-García R.* Non-minimal tinges of Unimodular Gravity // Journal of High Energy Physics. — 2020. — 9. — V. 2020. — P. 41. — URL: https://link.springer.com/10.1007/JHEP09(2020)041.

21. *Becchi C., Rouet A., Stora R.* Renormalization of the abelian Higgs-Kibble model // Communications in Mathematical Physics. — 1975. — 6. — V. 42. — P. 127–162. — URL: http://link.springer.com/10.1007/BF01614158.

22. *Tyutin I.* // Lebedev Institute Preprint N(39). — 1975.

23. *Batalin I., Vilkovisky G.* Gauge algebra and quantization // Physics Letters B. — 1981. — 6. — V. 102. — P. 27–31. — URL: https://linkinghub.elsevier.com/retrieve/pii/0370269381902057.

24. *Kaparulin D.S., Lyakhovich S.L.* Unfree gauge symmetry in the BV formalism // The European Physical Journal C. — 2019. — 8. — V. 79. — P. 718. — URL: http://link.springer.com/10.1140/epjc/s10052-019-7233-2.