Probability of quantum transitions in a stoochastic process in the space of joint events
Вероятность квантовых переходов в стохастическом процессе в пространстве совместных событий

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Эволюция системы описывается как стохастический процесс в пространстве случайных совместных событий. В пространстве вводятся симметричная разность и симметричная сумма случайных совместных событий. Вероятность квантовых переходов системы между определенными случайными событиями представляется суммой вероятностей совместных событий (двух, трех, ...). Примечательно,что полученное выражение для вероятностей переходов совпадает с вероятностью перехода в квантовой теории, если в модели учесть только попарно совместные случайные события.

The evolution of the system is described as a stochastic process in the space of random joint events. A symmetric difference and a symmetric sum of random joint events are introduced in space. The probability of quantum transitions of the system between certain random events is represented by the sum of the probabilities of joint events (two, three, ...). It is noteworthy that the obtained expression for the transition probabilities coincides with the transition probability in quantum theory, if only pairwise joint random events are taken into account in the model.

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INTRODUCTION

The development of innovative technologies in quantum theory stimulate the development of its mathematical apparatus, and, in particular, from the point of view of probability theory. For example, articles [1]– [6], which present original research and reviews of works on the development of methods of probability theory in quantum mechanics.

Stochastic processes in classical physics are successfully described by probability theory in the space of incompatible events (for example, review [7]).

To describe quantum processes it was proposed to model them in the space of joint random events [8]– [14]. In the space of joint events, in accordance with Kolmogorov's axiomatics defines the symmetric difference

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of events. For an adequate description of quantum processes, an additional principle on the compatibility of quantum events is introduced in space. For quantum random joint events, a symmetric sum is introduced.

The paper proposes to simulate the evolution of the system by a stochastic process in the constructed space of joint random events. An expression is constructed for the probability of system transitions between certain events in the representation of functional integration.

PROBABILITY OF TRANSITION OF A QUANTUM SYSTEM BETWEEN STATES IN THE REPRESENTATION OF PATH INTEGRATION

The evolution of a quantum system is described by the probability $P(b|a)$ of the system's transition from the state $|a\rangle$ at time $t=0$ to the state $|b\rangle$ at time $t>0$. The explicit form of the transition probability is determined by the system evolution operator $\hat{U}(t)$:

$$P(b|a) = \langle b|\hat{U}(t)|a\rangle\langle a|\hat{U}^{+}(t)|b\rangle. \tag{1}$$

To construct an explicit form of $\langle b|\hat{U}(t)|a\rangle$, it is necessary to specify the quantum system model. Let us construct an explicit form of the transition probability in a form that is convenient for interpretation in terms of probability theory.

For a number of quantum systems, it is convenient to represent the amplitude of quantum transitions as an integral over virtual trajectories in real space [15]

$$\langle b|\hat{U}(t)|a\rangle = \int\limits_{-\infty}^{\infty} \sqrt{P_0}\exp[\imath S[b,\mathbf{r},a]]\mathcal{D}\mathbf{r}(t), \tag{2}$$

where $S[b,\mathbf{r},a]$ dimensionless action of a particle along a trajectory passing through points $a,\mathbf{r},b$; integration is carried out over all vertical trajectories. The transition probability is determined by the formula:

$$P(b,|a) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} P_f \exp[\imath(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])]\mathcal{D}\mathbf{r}(t)\mathcal{D}\mathbf{r}'(t). \tag{3}$$

In formulas (3) we represent: $\exp[\imath(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])] = \cos[(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])] + \imath\sin[(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])]$. The integrals (3) of $\sin[(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])]$ are equal to zero because the functional is antisymmetric. Expression (3) are represented by formula

$$P(b,|a) = \int\limits_{-\infty}^{\infty}\int\limits_{-\infty}^{\infty} P_f \cos[\imath(S[b,\mathbf{r},a]-S[b,\mathbf{r}',a])]\mathcal{D}\mathbf{r}(t)\mathcal{D}\mathbf{r}'(t). \tag{4}$$

Fig. 1. Transition probability diagram of a quantum system

Similarly, one can construct the probability of quantum transitions for other models of quantum processes. For example: a description of the evolution of a quantum system in terms of energy [16], diffraction of elementary particles during interaction with a crystal [11]. In the general case, the expression for the probabilities of quantum transitions has the form

$$P(b \mid a) = \sum_{n,m=1}^{N} \cos(S[b,n,a] - S[b,m,a]), \tag{5}$$

where actions are functionals of continuous or discrete trajectories, depending on the quantum model, respectively, summation is carried out (they can be multi-indexes) or integration over all pairs of the trajectory.

To interpret the equation (5) in terms of probability theory, it is convenient to represent it in the form

$$P(b \mid a) = \sum_{n,m=1}^{N} g_{nm} P(S[b,n,a], S[b,m,a]), \tag{6}$$

where $P(S[b,n,a], S[b,m,a]) = P_o \mid \cos[b,n,a] - S[b,m,a]] \mid$,
$g_{nm} = \cos[S[b,n,a] - S[b,m,a]] \mid \cos[S[b,n,a] - S[b,m,a]] \mid^{-1}$,
i.e. $g_{n,m} = +1, -1, 0, n \neq m$; $g_{n,n} = +1, n = m$. Equation (6) shows that the probability $P(b \mid a)$ is represented by the summation (or integration) of the probabilities P(S[b,n,a], S[b,m,a]) pairs of joint virtual random trajectories over all virtual pairs of these trajectories.

Equation (6) can be represented in terms of random events corresponding to random numbers. Let us replace random numbers in equations (19.1),(19.2) with random events: $a - A_{in}; b - B_f; S[b;n;a] - S_{fni}$. Let's write the equations for random events

$$P(B_f \mid A_{in}) = \sum_{n,m=1}^{N} g_{nm} P(S_{fni} \cap S_{fmi}), \tag{7}$$

where $P(S_{fni} \cap S_{fmi})$ - probability of random joint events.

The equations (6), (7) are presented in diagrammatic form in Figure 1.

Equations (6), (7) show that the quantum process occurs in the space of random pairwise compatible events (trajectories).

We believe that it is of interest to study the stochastic process in the space of random joint events. In this space, every 2 events are joint, every 3 events are joint, that is, each set of events is joint events.

In the following sections of the article, we will construct a space of joint events, a model of a stochastic process in this space, and the probability of a system transition between process events.

MODEL OF THE SPACE OF RANDOM JOINT EVENTS FOR QUANTUM SYSTEMS

We will construct a model of the space of random joint events, in which it is possible to describe both classical and quantum processes [12]– [14].

Consider the space N of random events: S_n^{kl}, where $n = 1, 2, ..., N$, symbol kl means that events and their probabilities obey Kolmogorov's axioms [17].

For the case when the events $S_n^{kl}, n = 1, 2, ..., N$, are incompatible, the probability of their union is given by [17] $P(\cup_{n=1}^{N} S_n^{kl}) =$
$= \sum_{n=1}^{N} P(S_n^{kl})$.

We will study the space in which all events are joint, that is, the probability of combining events is determined by the formula [18]

$$P\big(\bigcup_{n=1}^{N} S_n^{kl}\big) = \sum_{n=1}^{N} P(S_n^{kl}) - \sum_{n<m=1}^{N} P\big(S_n^{kl} \cap S_m^{kl}\big) +$$

$$+ \sum_{n<m<k=1}^{N} P\big(S_n^{kl} \cap S_m^{kl} \cap S_k^{kl}\big) - \ldots + (-1)^{N'-1} P(S_1^{kl} \cap S_2^{kl} \cap ... \cap S_N^{kl}), \quad (8)$$

where the probabilities of crossing these events are nonzero:

$$P(S_n^{kl} \cap S_m^{kl}) \neq 0, \;\; P(S_n^{kl} \cap S_m^{kl} \cap S_k^{kl}) \neq 0, \;\; \ldots \;\; P(S_1^{kl} \cap S_2^{kl} \cap ... \cap S_N^{kl}) \neq 0, \quad (9)$$

all indices $k, n, m, ...$ take integer values from 1 to N;N' - порядковый номер слагаемого.

In the space of random joint events, we construct the events $S_1^-, S_2^-, ..., S_N^-$, each of which means that if an event with the corresponding number occurs, then no other event of space. It follows from the definition that the events $S_1^-, S_2^-, ..., S_N^-$ are not compatible. The equation for the symmetric difference of events is proved [19]:

$$P\big(\bigcup_{n=1}^{N} S_n^{-}\big) = \sum_{n=1}^{N} P(S_n^{kl}) - 2 \sum_{n<m=1}^{N} P\big(S_n^{kl} \cap S_m^{kl}\big) +$$

$$+4 \sum_{n<m<k=1}^{N} P\big(S_n^{kl} \cap S_m^{kl} \cap S_k^{kl}\big) - \ldots + 2^{N'-1} (-1)^{N'-1} P(S_1^{kl} \cap S_2^{kl} \cap ... \cap S_N^{kl}). \quad (10)$$

The analysis of experiments shows that equations (8)-(10) successfully describe joint events in the physics of the macrocosm and a large class of events in the physics of the microcosm. However, in the physics of the microcosm there are random joint events (for example, in experiments on electron diffraction), the description of which on the basis of equations (8)-(10) is impossible. To describe them, additional provisions on the compatibility of events are required.

We postulate that in the physics of the microworld, along with the events S_n^{kl}, there are joint events S_n^{qv} whose combination probability is $P(S_1^{qv} \cup S_2^{qv} \cup ... \cup S_N^{qv})$ is described by the equation:

$$P\Big(\bigcup_{n=1}^{N} S_n^{qv}\Big) = \sum_{n=1}^{N} P(S_n^{qv}) + \sum_{n<m=1}^{N} P\big(S_n^{qv} \cap S_m^{qv}\big) -$$

$$- \sum_{n<m<k=1}^{N} P\big(S_n^{qv} \cap S_m^{qv} \cap S_k^{qv}\big) - \ldots (-1)^{N'} P(S_1^{qv} \cap S_2^{qv} \cap ... \cap S_N^{qv}), \quad (11)$$

where the symbol qv means that a random event refers to the physics of the microworld (to processes in quantum mechanics); the sign before the first term is "+", before the other terms the signs are determined by the factors $(-1)^{N'}$, where N' is the ordinal number of the term. In the space of random joint events $S_1^{qv}, S_2^{qv}, ..., S_N^{qv}$ we construct the events $S_1^+, S_2^+, ..., S_N^+$, each of which means that when this event occurs, all S_n^{qv} events are realized with the corresponding probabilities.

For the space of two joint events S_1^{qv}, S_2^{qv}, the events S_1^+, S_2^+ are defined by the expressions

$$S_1^+ = S_1^{qv} \cup (S_1^{qv} \cap S_2^{qv}), \;\; S_2^+ = S_2^{qv} \cup (S_1^{qv} \cap S_2^{qv}).$$

From these definitions it follows:

$$P\Big(\bigcup_{n=1}^{2} S_n^+\Big) = \sum_{n=1}^{2} P(S_n^{qv}) + 2P\big(S_1^{qv} \cap S_2^{qv}\big). \quad (12)$$

Expression (12) is naturally called the symmetric sum of two events.

Probability of combining events $P(S_1^+ \cup S_2^+ \cup ... \cup S_N^+)$ is expressed in terms of event probabilities $S_n^{qv}, n = 1, 2, ..., N$, and the probabilities of their intersection by the equation

$$P\Big(\bigcup_{n=1}^{N} S_n^+\Big) = \sum_{n=1}^{N} P(S_n) + 2\sum_{n<m=1}^{N} P\big(S_n^{qv} \cap S_m^{qv}\big) -$$

$$-4 \sum_{n<m<k=1}^{N} P\big(S_n^{qv} \cap S_m^{qv} \cap S_k^{qv}\big) + \ldots + 2^{N'-1}(-1)^{N'} P(S_1^{qv} \cap S_2^{qv} \cap ... \cap S_N^{qv}), \quad (13)$$

where the sign in front of the term is determined in the same way as in formula (11). Equation (13) is proved by mathematical induction using formula (12). Expression (13) is a symmetric sum of N random joint events. Note that the interpretation of events in equations (11)-(13) by sets, as provided for by Kolmogorov's axiomatics, is unacceptable, it leads to a contradiction.

Equations (10), (13) can be written as a single equation. To this end we introduce random events $\tilde{S}_n$ each of which takes the value either S_n^- or S_n^+.

Probability of combining events $\tilde{S}_n$ is represented by a formula that is the union of the equations (11), (13):

$$P(\bigcup_{n=1}^{N}\tilde{S}_n)=\sum_{n=1}^{N}P(S_n)+2\sum_{n<m=1}^{N}g_{nm}P(S_n\cap S_m)+$$

$$+4\sum_{n<m<k=1}^{N}g_{nmk}P(S_n\cap S_m\cap S_k)+\ldots+2^{N-1}g_{12\ldots N}P(S_1\cap S_2\cap\ldots\cap S_N),\quad (14)$$

where the factors g_{nm}, g_{nkm} and others take one of the values $+1$, -1 depending on the events S_n (for example, $g_{nkm}=-g_{nm}$). The indices kl, qv of the events S_n^{kl}, S_n^{qv} in equation (14) are omitted, since they are taken into account by the signs of the coefficients $g_{nm}, g_{nkm}, \ldots$. Note that if in expression (14) we take events that are incompatible, then all coefficients are equal to zero: $g_{nm}=0, g_{nkm}=0, \ldots$

In the constructed space of random joint events, both the symmetric difference and the symmetric sum of events are realized. The space describes a wider range of systems and stochastic processes than the spaces in which only the symmetrical difference of events is realized.

Note that the number of events in the constructed space can be infinite, the corresponding series in equations (10),(13),(14) converge.

STOCHASTIC PROCESS IN THE SPACE OF RANDOM JOINT EVENTS AND AN EQUATION FOR THE PROBABILITY OF A SYSTEM TRANSITION BETWEEN STATES

Consider a system performing a stochastic process in the space of random joint events $S_n, n=1,2,...,N$ (the space is described in the previous part of this article). The evolution of the system is represented by its transition in the space of random events with a certain probability $P(b|a)$ from a state characterized by a certain event a at time $t=0$ to a state characterized by event b at time $t>0$.

Let us construct a model of the stochastic motion of the system in the given space of random joint events. The explicit form of the transition probability $P(b|a)$ will be constructed based on this model.

We will assume that the transition of the system from a specific initial event a to another specific event b is carried out through intermediate events $\tilde{S}_n$ of the event space. Based on this statement, we construct the desired stochastic equation:

$$b\cap a=\bigcup_{n=1}^{N}(b\cap\tilde{S}_n\cap a)=\bigcup_{n=1}^{N}\tilde{S}[b,n,a],\quad (15)$$

where $(b\cap\tilde{S}_n\cap a)=\tilde{S}[b,n,a]$, the events a and b are fixed in this equation, we represent them like indexes.

The desired transition probability $P(b|a)$, taking into account (15), is determined by the formula

$$P(b|a) = P(\bigcup_{n=1}^{N} \tilde{S}[b,n,a]). \tag{16}$$

Taking into account equation (14), we represent equation (16) in the form:

$$P(b|a) = \sum_{n=1}^{N} P(S[b,n,a]) + 2\sum_{n<m=1}^{N} g_{nm} P(S[b,n,a] \cap S[b,m,a]) +$$

$$+4\sum_{n<m<k=1}^{N} g_{nmk} P(S[b,n,a] \cap S[b,m,a] \cap S[b,k,a]) + \ldots +$$

$$+2^{N-1} g_{12\ldots N} P(S[b,1,a] \cap S[b,2,a] \cap \ldots \cap S[b,N,a]), \tag{17}$$

where $g_{n,m} = +1, -1, 0, n \neq m$; $g_{n,n} = +1, n = m$; $g_{nmk} = +1, -1, 0, or g_{nmk} = -g_{n,m}; \ldots$, the signs of the coefficients are determined by the ratios between events with different numbers $n, m, k, l, \ldots$. For equation (17) describing stochastic process in the space of random joint events, the principle correspondences: when the events are incompatible, only the first sum for $n = m$ remains, that is, it goes into the equation of Markov processes.

Let us represent equation (17) in a form more convenient for describing stochastic processes. We combine the first two terms in accordance with the principles of probability theory, equation (17) takes the form:

$$P(b|a) = \sum_{n,m=1}^{N} g_{nm} P(S[b,n,a] \cap S[b,m,a]) +$$

$$+4\sum_{n<m<k=1}^{N} g_{nmk} P(S[b,n,a] \cap S[b,m,a] \cap S[b,k,a]) + \ldots +$$

$$+2^{N-1} g_{12\ldots N} P(S[b,1,a] \cap S[b,2,a] \cap \ldots \cap S[b,N,a]). \tag{18}$$

To describe a physical system on the basis of equation (18), it is necessary to associate random events with random numbers characterizing the system.

Note that for the case when the events are only pairwise compatible, equation (18) takes the form (7), i.e., it coincides with the equation for the transition probability of a system in quantum mechanics. This fact allows us to assert that the proposed model of stochastic processes in the space of pairwise joint events describes quantum processes. This fact allows the events $S[b,n,a]$ to associate dimensionless (i.e., in units of $\hbar$) actions of the system $S[b,n,a]$, along these random trajectories (numbers n trajectories are numbered and can be multi-indices, the parameters a, b define the beginning and end of the trajectories).

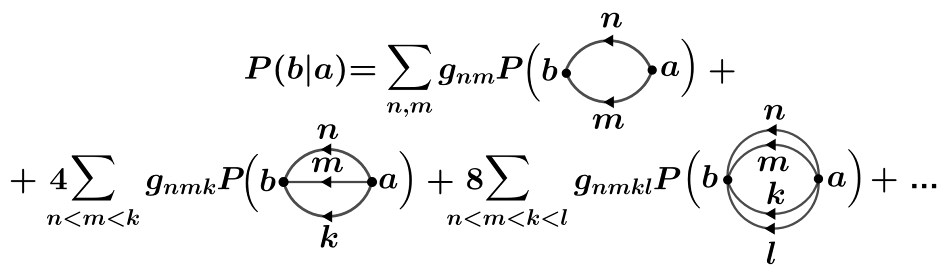

Fig. 2. Diagrams of the transition probability of a system in a stochastic process in the space of joint states.

We believe that the proposed method of matching random events of the system's action can be applied to equation (18). The equation takes the form:

$$P(b \mid a) = \sum_{n,m=1} g_{nm} P(S[b,n,a], S[b,m,a]) + 4 \sum_{n<m<k=1} g_{nmk} \times$$
$$\times P(S[b,n,a], S[b,m,a], S[b,k,a]) + 8 \sum_{n<m<k<l=1} g_{nmkl} \times$$
$$\times P(S[b,n,a], S[b,m,a], S[b,k,a], S[b,l,a]) + \dots . \qquad (19)$$

Equation (19) can be extended to any value of N for continuous trajectories as well.

CONCLUSION

A model of the space of random joint events with a certain symmetric difference and symmetric sum of events is constructed. The stochastic process of the system is considered in space. The probability of a system transition between states is determined by an equation that takes into account the compatibility of events. For the equation, the principle of correspondence is fulfilled, that is, if the events are not compatible, it goes into the equation of the Markov process. For the case when the events are only pairwise compatible equation is identical to the equation of quantum mechanics.

It is of interest to study systems for describing the evolution of which, on the basis of equation (19), it is required to take into account not only the probabilities of paired compatibility of events, but also the probabilities of compatibility of three, four or more events (the second, third and subsequent terms).

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