SU(3) hybrid static potentials at small quark-antiquark separations from fine lattices

XXXIII International Workshop on High Energy Physics “Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quests”

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Main goals / literature

(1) Compute hybrid static potentials, i.e. potentials of a static quark antiquark pair ($\bar{Q}Q$), where the flux tubes are excited with quantum numbers different from the ground state.
$\rightarrow$ SU(3) lattice gauge theory

(2) Use these potentials to approximately compute the spectra of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.
$\rightarrow$ Born-Oppenheimer approximation, effective field theories, quantum mechanics

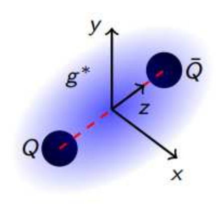
(3) Explore hybrid static potential flux tubes by computing the chromoelectric and chromomagnetic energy density.
$\rightarrow$ SU(2) and SU(3) lattice gauge theory

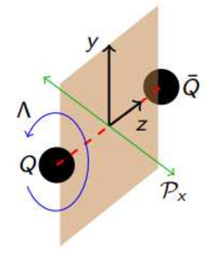
- The talk focuses on (1) and summarizes
[C. Schlosser, M.W., [arXiv:2111.00741]]

- Further references on (1):
[K. J. Juge, J. Kuti, C. J. Morningstar, Nucl. Phys. Proc. Suppl. **63**, 326 (1998) [hep-lat/9709131]
[C. Michael, Nucl. Phys. A **655**, 12 (1999) [hep-ph/9810415]
[G. S. Bali *et al.* [SESAM and TχL Collaborations], Phys. Rev. D **62**, 054503 (2000) [hep-lat/0003012]
[K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]
[C. Michael, Int. Rev. Nucl. Phys. **9**, 103 (2004) [hep-lat/0302001]
[G. S. Bali, A. Pineda, Phys. Rev. D **69**, 094001 (2004) [hep-ph/0310130]
[P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]]
[S. Capitani, O. Philipsen, C. Reisinger, C. Riehl. M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046 [hep-lat]]]

Hybrid static potentials: quantum numbers

- "(Hybrid) static potential states" can be characterized by the following quantum numbers:
  - Absolute total angular momentum with respect to the $\bar{Q}Q$ separation axis (z axis): $\Lambda = 0, 1, 2, \ldots \equiv \Sigma, \Pi, \Delta, \ldots$
  - Parity combined with charge conjugation: $\eta = +, - = g, u$.
  - Relection along an axis perpendicular to the $\bar{Q}Q$ separation axis (x axis): $\epsilon = +, -$.
- For $\Lambda \geq 1$ potentials are degenerate with respect to ϵ, i.e. $V_{\Lambda_\eta^+}(r) = V_{\Lambda_\eta^-}(r)$
  $\rightarrow$ use quantum numbers Λ_η^ϵ for $\Lambda = \Sigma$
  $\rightarrow$ use quantum numbers Λ_η for $\Lambda = \Pi, \Delta, \ldots$
- The ordinary static potential has quantum numbers $\Lambda_\eta^\epsilon = \Sigma_g^+$.
- In this talk I focus on the two lowest hybrid static potentials with quantum numbers $\Lambda_\eta^\epsilon = \Pi_u, \Sigma_u^-$.





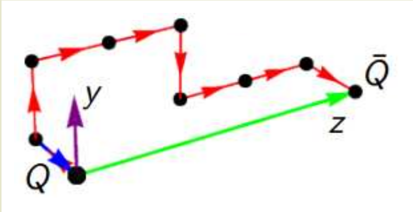
Hybrid static potentials: trial states (1)

- To determine the hybrid static potential with quantum numbers Λ_η^ϵ, compute the temporal correlation functions of suitable trial states,

$$W_{S,S';\Lambda_\eta^\epsilon}(r,t) \quad = \quad \langle \Psi_{\text{hybrid}}(t) |_{S;\Lambda_\eta^\epsilon} | \Psi_{\text{hybrid}}(0) \rangle_{S';\Lambda_\eta^\epsilon} \quad \sim_{t \rightarrow \infty} \quad \exp\Big(- V_{\Lambda_\eta^\epsilon}(r) t \Big).$$

- Trial states are

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_\eta^\epsilon} \quad = \quad \bar{Q}(-r/2) a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2) Q(+r/2) |\Omega\rangle$$



with gluonic parallel transporters (on the lattice products of gauge links)

$$a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2) \quad =$$

$$= \quad \frac{1}{4} \sum_{k=0}^{3} \exp\Big(\frac{i \pi \Lambda k}{2}\Big) R\Big(\frac{\pi k}{2}\Big) \bigg(U(-r/2,r_1) \Big(S(r_1,r_2) + \epsilon S_{\mathcal{P}_x}(r_1,r_2) \Big) U(r_2,+r/2) + U(-r/2,-r_2) \Big(\eta S_{\mathcal{P} \circ \mathcal{C}}(-r_2,-r_1) + \eta \epsilon S_{(\mathcal{P} \circ \mathcal{C}) \mathcal{P}_x}(-r_2,-r_1) \Big) U(-r_1,+r/2) \bigg)$$

generating quantum numbers Λ_η^ϵ.

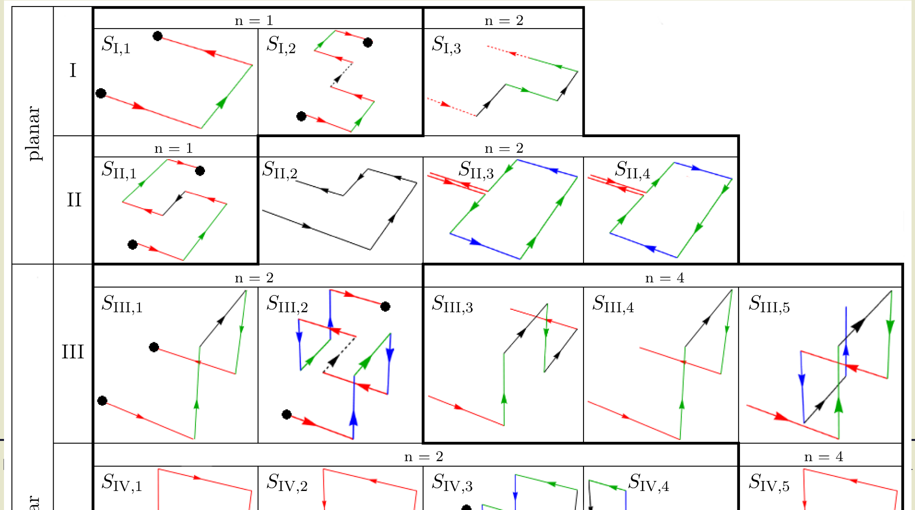
Hybrid static potentials: trial states (2)

- For $a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)$, which define the trial states

$$|\Psi_{\text{hybrid}}\rangle_{S;\Lambda_\eta^\epsilon} \quad = \quad \bar{Q}(-r/2)a_{S;\Lambda_\eta^\epsilon}(-r/2,+r/2)Q(+r/2)|\Omega\rangle,$$

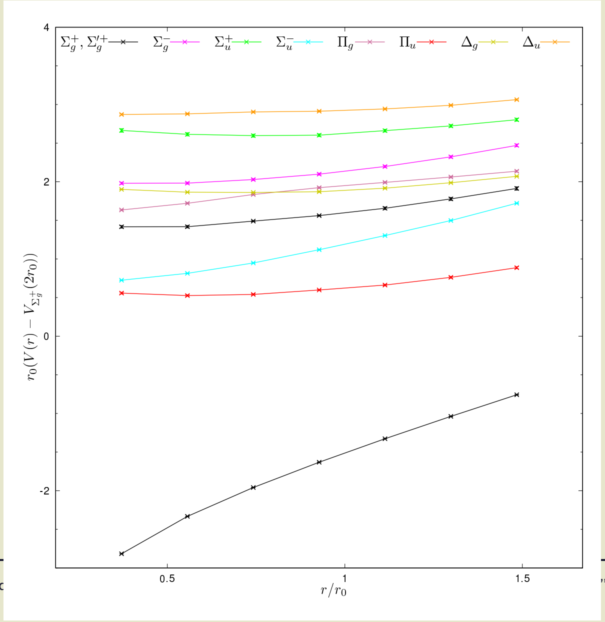
  we have explored many different shapes and variations of their extents.

- For the final computation of each hybrid static potential $V_{\Lambda_\eta^\epsilon}(r)$ we have used an optimized set of 3 to 4 creation operators and have solved generalized eigenvalue problems for the corresponding correlation matrices (arXiv:1811.11046) or the operator with the largest ground state overlap (arXiv:2111.00741).



Hybrid static potentials: results (1)

- [S. Capitani, O. Philipsen, C. Reisinger, C. Riehl, M.W., Phys. Rev. D **99**, 034502 (2019) [arXiv:1811.11046]]
- Discrepancies to existing results for $V_{\Pi_g}(r)$ and $V_{\Delta_u}(r)$ at small $\bar{Q}Q$ separation $r \leq 0.25\,\text{fm}$.
  [K. J. Juge, J. Kuti, C. Morningstar, Phys. Rev. Lett. **90**, 161601 (2003) [hep-lat/0207004]]
- Observed degeneracies of $V_{\Sigma'^+_g}(r)$, $V_{\Pi_g}(r)$ and $V_{\Sigma^+_u}(r)$, $V_{\Delta_u}(r)$ at small r expected from pNRQCD.



Hybrid static potentials: results (2)

- [C. Schlosser, M.W., [arXiv:2111.00741]]
- Computations of the Π_u and the Σ_u^- hybrid static potentials (the two lowest hybrid static potentials) on four additional ensembles with lattice spacing a as small as $0.04\,\text{fm}$ (previously a single ensemble with $a = 0.093\,\text{fm}$).
  $\rightarrow$ Lattice data points at significantly smaller separations.
  $\rightarrow$ Determination and elimination of discretization errors possible ("continuum limit").

ensemble	β	a in fm	$(L/a)^3 \times T/a$
A	6.000	0.093	$12^3 \times 26$
B	6.284	0.060	$20^3 \times 40$
C	6.451	0.048	$26^3 \times 50$
D	6.594	0.040	$30^3 \times 60$

- Self energy is a-dependent, needs to be subtracted, before all lattice data point can be shown together in a meaningful plot.
  $\rightarrow$ Fits needed (see next slides).
- Unsmeared temporal links (previously: HYP2 static action).
- Multilevel algorithm (previously: without multilevel algorithm).

Hybrid static potentials: results (3)

- 8-Parameter-Fit for the ordinary (Σ_g^+) static potential:

$$V_{\Sigma_g^+}^{\text{fit},e}(r) = V_{\Sigma_g^+}(r) + C^e + \Delta V_{\Sigma_g^+}^{\text{lat},e}(r)$$

$$V_{\Sigma_g^+}(r) = -\frac{\alpha}{r} + \sigma r \quad \text{(Cornell ansatz)}$$

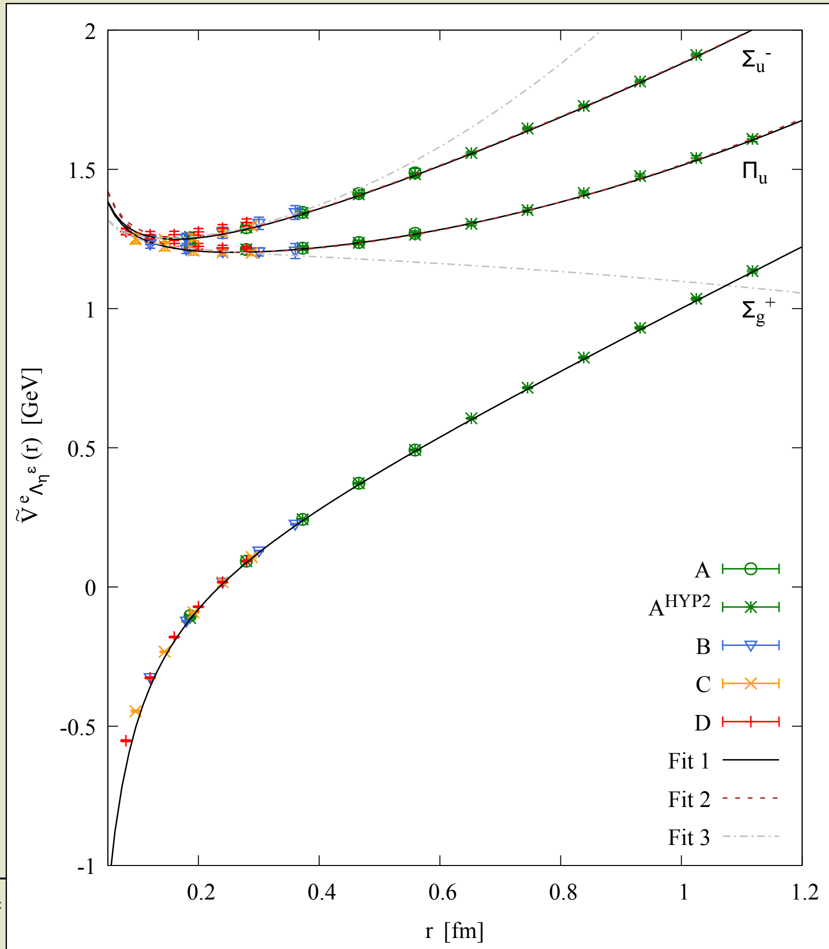
$$\Delta V_{\Sigma_g^+}^{\text{lat},e}(r) = \alpha' \left(\frac{1}{r} - \frac{G^e(r/a)}{a}\right) \quad \text{(difference of tree-level continuum and lattice result).}$$

  - C^e: a-dependent self energies.
  - $\Delta V_{\Sigma_g^+}^{\text{lat},e}(r)$: lattice discretization errors at tree-level.
  - $V_{\Sigma_g^+}(r)$: parameterization of the ordinary static potential; useful to set the energy scale for b and for c quarks (via a Born-Oppenheimer computation of the quarkonium ground state $\eta_b(1S) \equiv \Upsilon(1S)$ or $\eta_c(1S) \equiv J/\Psi(1S)$ and identification with the corresponding experimental result).

- Fit parameters allow to define data points, with the self-energy subtracted and discretization errors at tree-level removed:

$$\tilde{V}_{\Sigma_g^+}^e(r) = V_{\Sigma_g^+}^e(r) - C^e - \Delta V_{\Sigma_g^+}^{\text{lat},e}(r).$$

Hybrid static potentials: results (4)



Hybrid static potentials: results (5)

- 10-Parameter-Fit for the Π_u and Σ_u^- hybrid static potentials:

$$V_{\Lambda_\eta^\epsilon}^{\text{fit},e}(r) = V_{\Lambda_\eta^\epsilon}(r) + C^e + \Delta V_{\text{hybrid}}^{\text{lat},e}(r) + A_{2,\Lambda_\eta^\epsilon}^{\prime e} a^2 \quad , \quad \Lambda_\eta^\epsilon = \Pi_u, \Sigma_u^-$$

$$V_{\Pi_u}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 \quad , \quad V_{\Sigma_u^-}(r) = \frac{A_1}{r} + A_2 + A_3 r^2 + \frac{B_1 r^2}{1 + B_2 r + B_3 r^2}$$

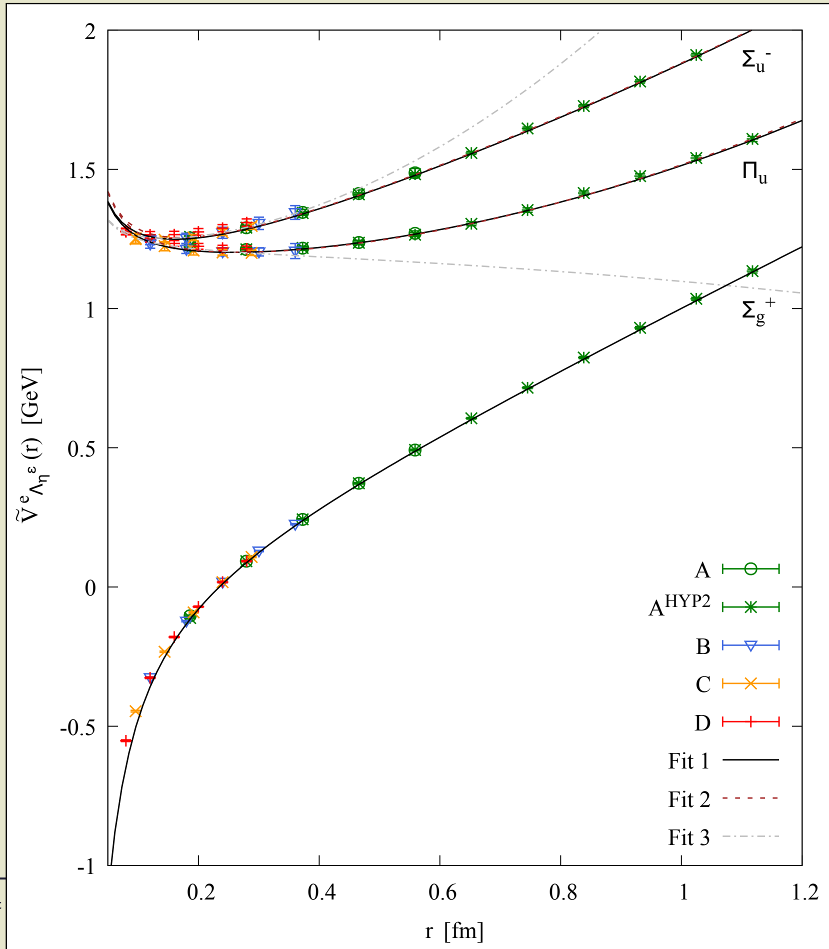
$$\Delta V_{\text{hybrid}}^{\text{lat},e}(r) = -\frac{1}{8} \Delta V_{\Sigma_g^+}^{\text{lat},e}(r).$$

  - $A_{2,\Lambda_\eta^\epsilon}^{\prime e} a^2$: leading order ($\propto a^2$) lattice discretization error in the difference to the ordinary static potential.
  - $V_{\Pi_u}(r)$, $V_{\Sigma_u^-}(r)$: parameterizations of the Π_u and Σ_u^- hybrid static potentials; consistent with and motivated by the pNRQCD prediction at small r.

- Fit parameters allow to define data points, with the self-energy subtracted and discretization errors to a large part removed:

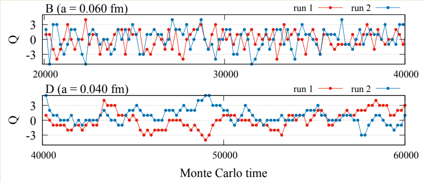
$$\tilde{V}_{\Lambda_\eta^\epsilon}^e(r) = V_{\Lambda_\eta^\epsilon}^e(r) - C^e - \Delta V_{\text{hybrid}}^{\text{lat},e}(r) - A_{2,\Lambda_\eta^\epsilon}^{\prime e} a^2.$$

Hybrid static potentials: results (6)



Hybrid static potentials: results (7)

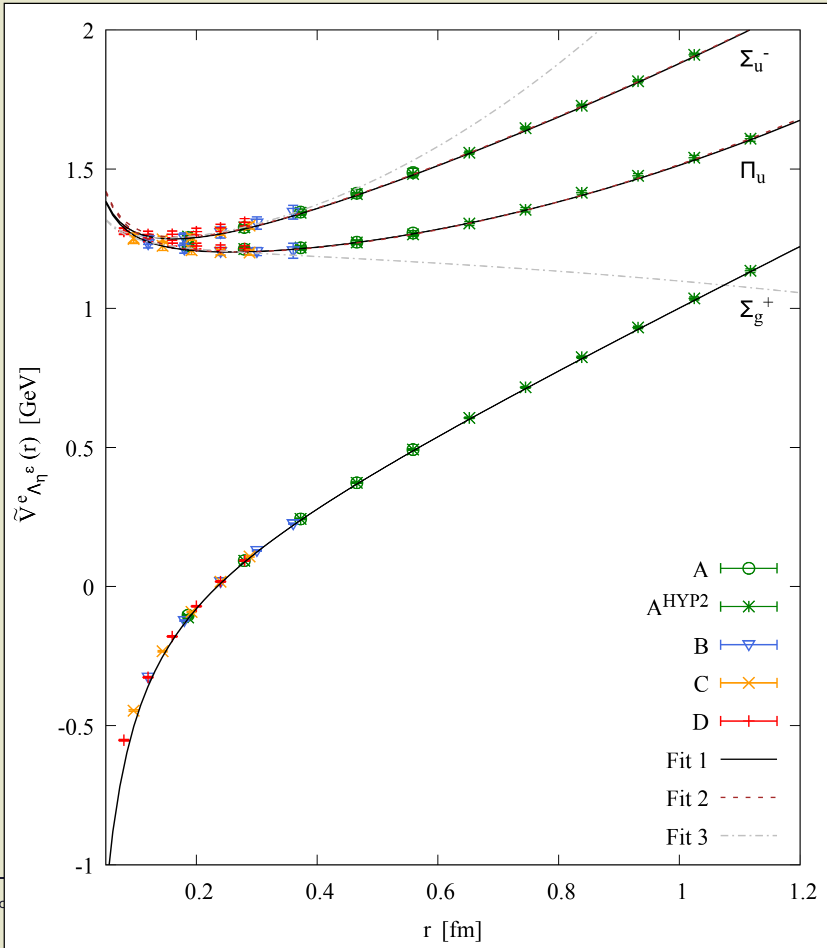
- Exclusion of the following types of systematic errors:
  - **Topological freezing:**
    - Monte Carlo algorithms have difficulties changing the topological charge Q for lattice spacings $a \lesssim 0.05\,\mathrm{fm}$.
    - Monte Carlo histories of Q need to be checked.
    - Lengthy simulations necessary for small a.



  - **Finite volume corrections:**
    - Negative energy shifts, because of virtual glueballs traveling around the far side of the periodic spacetime volume.
    - Positive energy shifts, because of squeezed wave functions.
    - Corrections turned out to be negligible for $L \gtrsim 1.2\,\mathrm{fm}$.
  - **Glueball decays:**
    - At small r hybrid flux tubes can decay into Σ_g^+ flux tubes and glueballs.
    - Σ_u^- flux tube: protected by symmetries from decays into a 0^{++} glueball.
    - Π_u: decays into a 0^{++} glueball possible for $r \lesssim 0.11\,\mathrm{fm}$; however, no indication that $V^e_{\Pi_u}(r)$ is contaminated by such decays (the Π_u and Σ_u^- potentials approach each other for small r).

Hybrid static potentials: results (8)

- Summary of improvements:
  - Separations as small as $0.08\,\mathrm{fm}$.
  - Self energies subtracted.
  - Lattice discretization errors removed to a large extent.
  - Various systematic errors checked and excluded.
- Improvements important: Born-Oppenheimer predictions of heavy hybrid meson masses change by $\mathcal{O}(10\,\mathrm{MeV} \ldots 45\,\mathrm{MeV})$.
- Bare lattice data, improved lattice data and parmeterizations provided. [C. Schlosser, M.W., [arXiv:2111.00741]]



$\bar{b}b$ and $\bar{c}c$ hybrid meson masses: BO

- Born-Oppenheimer approximation: Compute $\bar{b}b$ and $\bar{c}c$ hybrid meson masses in two steps.

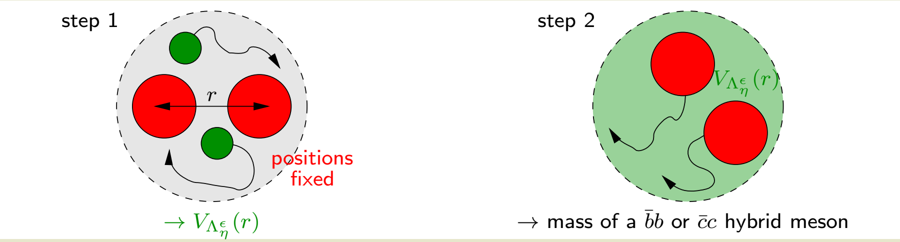
  (1) **Compute potentials of two static quarks ($\bar{b}b$ or $\bar{c}c$) in the presence of excited gluons generating quantum numbers Λ_η^ϵ using lattice gauge theory.**

  (2) **Solve Schrödinger equations for the relative coordinate of $\bar{b}b$ or $\bar{c}c$ using the potentials from (1) and the mass of either the b or the c quark, e.g.**

$$\bigg(-\frac{1}{2\mu}\frac{d^2}{dr^2} + \frac{L(L+1) - 2\Lambda^2 + J_{\Lambda_\eta^\epsilon}(J_{\Lambda_\eta^\epsilon}+1)}{2\mu r^2} + V_{\Lambda_\eta^\epsilon}(r)\bigg) u_{\Lambda_\eta^\epsilon;L,n}(r) \quad = \quad E_{\Lambda_\eta^\epsilon;L,n} u_{\Lambda_\eta^\epsilon;L,n}(r).$$

Energy eigenvalues $E_{\Lambda_\eta^\epsilon;L,n}$ correspond to masses of $\bar{b}b$ and $\bar{c}c$ hybrid mesons.

[E. Braaten, C. Langmack, D. H. Smith, Phys. Rev. D **90**, 014044 (2014) [arXiv:1402.0438]]
[M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]
[R. Oncala, J. Soto, Phys. Rev. D **96**, 014004 (2017) [arXiv:1702.03900]]
[N. Brambilla, G. Krein, J. Tarrus Castella, A. Vairo, Phys. Rev. D **97**, 016016 (2018) [arXiv:1707.09647]]
[N. Brambilla, W. K. Lai, J. Segovia, J. Tarrus Castella, A. Vairo, Phys. Rev. D **99**, 014017 (2019) [arXiv:1805.07713]]



Hybrid flux tubes: computation

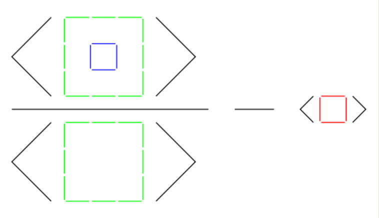
- We are interested in

$$\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x}) \quad = \quad \langle 0_{\Lambda_\eta^\epsilon}(r)|F^2_{\mu\nu}(\mathbf{x})|0_{\Lambda_\eta^\epsilon}(r)\rangle - \langle\Omega|F^2_{\mu\nu}|\Omega\rangle .$$

  - $F^2_{\mu\nu}(\mathbf{x})$, $F^2_{\mu\nu}$: squared chromoelectric/chromomagnetic field strength.
  - $|0_{\Lambda_\eta^\epsilon}(r)\rangle$: "hybrid static potential (ground) state" (r denotes the $\bar{Q}Q$ separation).
  - $|\Omega\rangle$: vacuum state.

- The sum over the six independent $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$ is proportional to the chromoelectric and -magnetic energy density of hybrid flux tubes.

- With lattice gauge theory $\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x})$ can be computed via



$$\Delta F^2_{\mu\nu,\Lambda_\eta^\epsilon}(r;\mathbf{x}) \quad = \quad \pm\frac{\langle\tilde{W}(r,t_2,t_0)P_{\mu\nu}(\mathbf{x},t_1)\rangle_U}{\langle\tilde{W}(r,t_2,t_0)\rangle_U} \mp \langle P_{\mu\nu}\rangle_U .$$

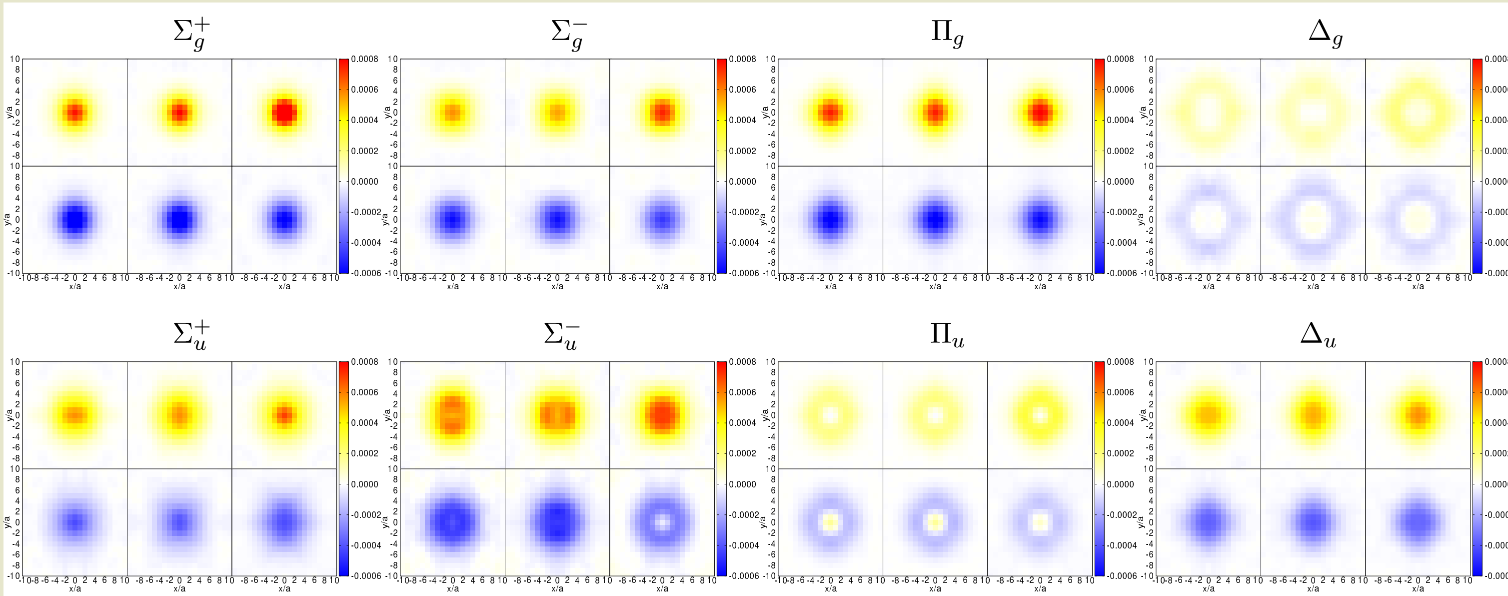
  - $P_{\mu\nu}(\mathbf{x})$, $P_{\mu\nu}$: plaquette, i.e. lattice gauge theory expression for $F^2_{\mu\nu}(\mathbf{x})$.
  - $\tilde{W}(r,t_2,t_0)$: "Wilson loop" (spatial extent r, temporal extent $t_2 - t_1$), with spatial parallel transporters as in the "hybrid static potential trial states".

Hybrid flux tubes: results (1)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), mediator plane (x-y plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\text{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

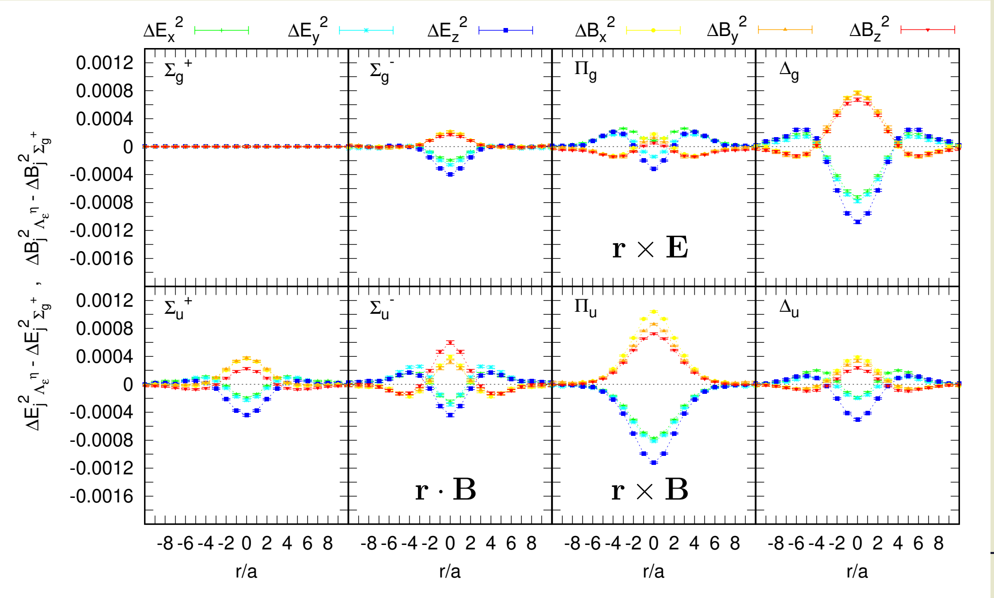
- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$.

ΔE^2_x	ΔE^2_y	ΔE^2_z
ΔB^2_x	ΔB^2_y	ΔB^2_z



Hybrid flux tubes: results (2)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x}) - \Delta F^2_{\mu\nu,\Sigma^+_g}(r;\mathbf{x})$, SU(2), mediator axis ($x$ axis with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,\text{fm}$.
  [L. Müller, O. Philipsen, C. Reisinger, M. Wagner, Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]
- Chromoelectric and chromomagnetic field stengths reflect typical operators used to study hybrid static potentials, e.g. in pNRQCD.
  [M. Berwein, N. Brambilla, J. Tarrus Castella, A. Vairo, Phys. Rev. D **92**, 114019 (2015) [arXiv:1510.04299]]



Hybrid flux tubes: results (3)

- $\Delta F^2_{\mu\nu,\Lambda^\epsilon_\eta}(r;\mathbf{x})$, SU(2), separation plane (x-z plane with Q, $\bar{Q}$ at $(0,0,\pm r/2)$), $r \approx 0.8\,$fm.
[L. Müller, O. Philipsen, C. Reisinger, M. Wagner, Phys. Rev. D **100**, 054503 (2019) [arXiv:1907.014820]]]

- See also [P. Bicudo, N. Cardoso and M. Cardoso, Phys. Rev. D **98**, 114507 (2018) [arXiv:1808.08815 [hep-lat]]] for results for $\Lambda^\epsilon_\eta = \Sigma^+_g, \Sigma^+_u, \Pi_u$.

ΔE_x^2	ΔE_y^2	ΔE_z^2
ΔB_x^2	ΔB_y^2	ΔB_z^2