

# The QCD chiral phase transition for different numbers of quark flavours

Owe Philipsen

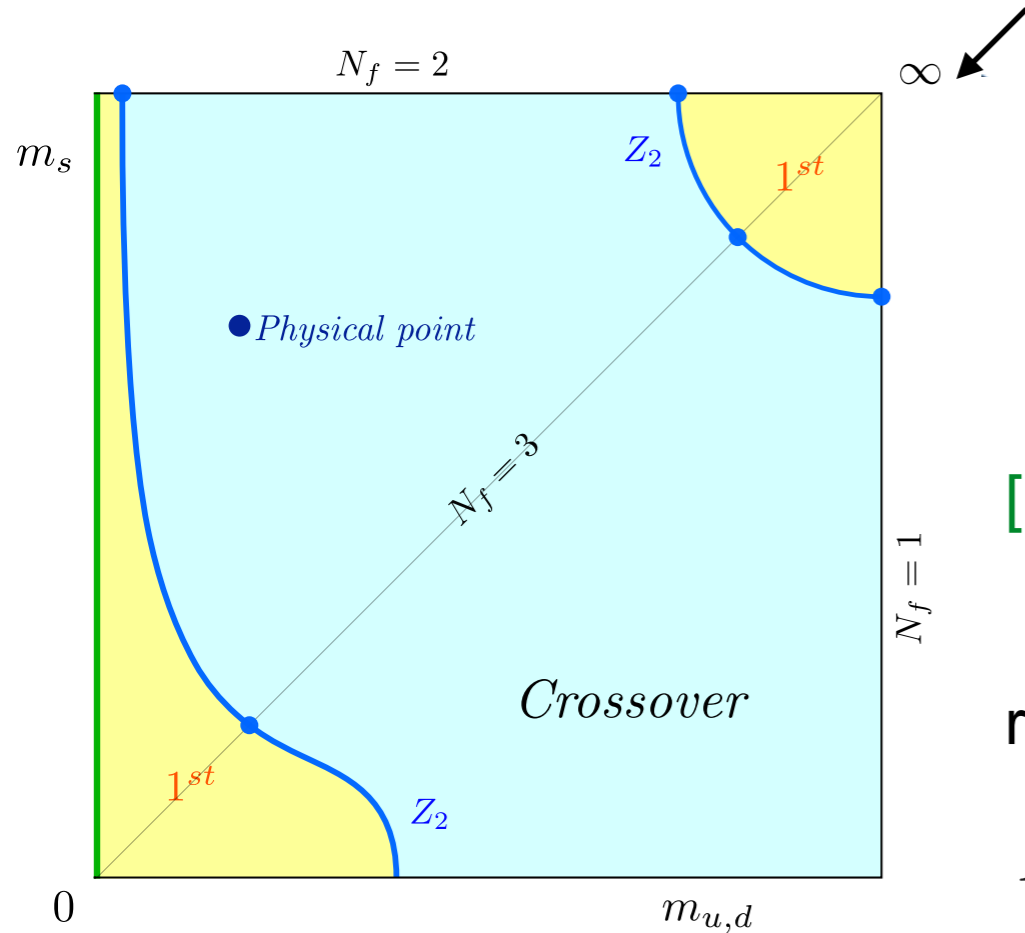
A truly hard problem: order of chiral phase transition at  $m=0$

- Light mass regime in the Columbia plot: contradicting results?
- No, new data suggest resolution and modified Columbia plot

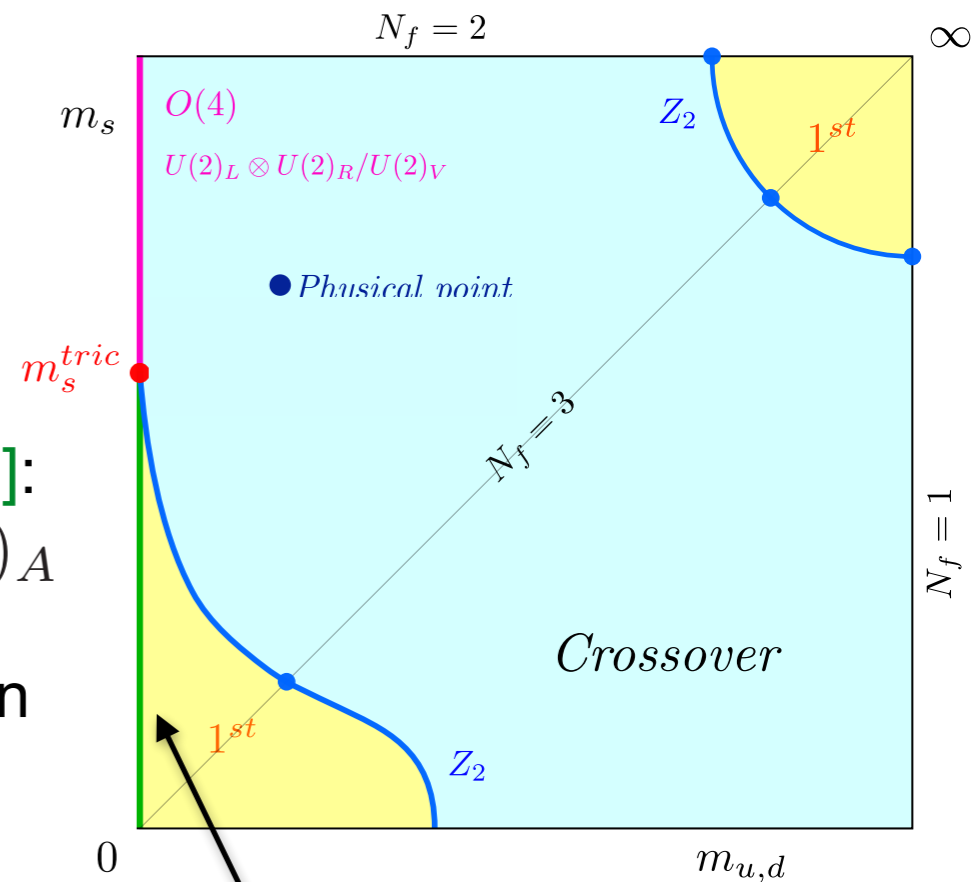
[Cuteri, O.P., Sciarra, arxiv2107.12739]

# The nature of the QCD thermal transition

deconfinement p.t.:  
breaking of global  $Z(3)$  symmetry



[Pisarski, Wilczek, PRD 84]:  
 $N_f = 2$  depends on  $U(1)_A$   
restored  
 $N_f \geq 3$  1st order



broken

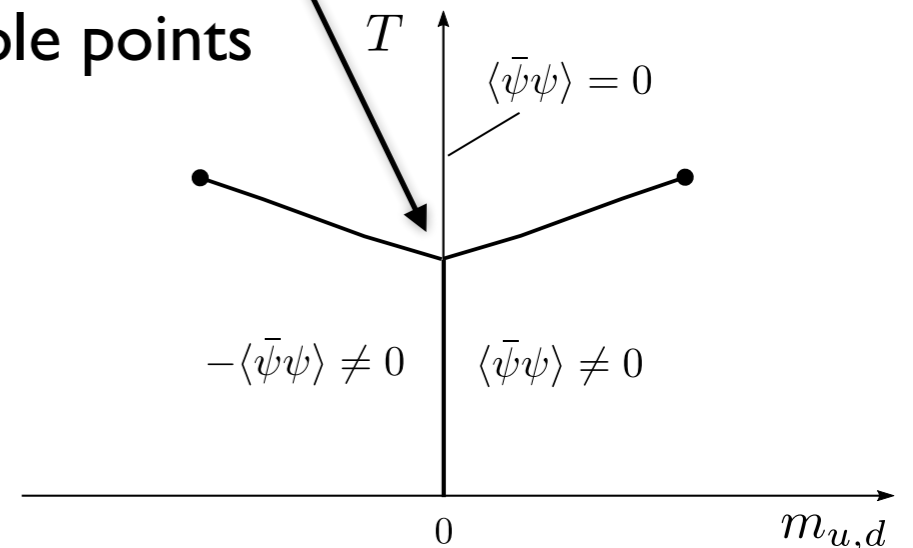
chiral p.t.

restoration of global symmetry in flavour space

$$SU(2)_L \times SU(2)_R \times U(1)_A$$

↑  
anomalous

triple points



# Long history, key results

- [Pisarski, Wilczek PRD 84]

Universality, 3D sigma models: 1st order for

$$N_f \geq 3$$

$N_f = 2$  depends on  $U(1)_A$

- Shrinking of 1st-order region towards continuum, will something remain?:

Standard staggered

$$N_f = 2 \quad [\text{Bonati et al. PRD 14, Cuteri et al. PoS LAT 18}]$$

$$N_f = 3 \quad [\text{de Forcrand, O.P. PoS LAT 07}]$$

$$N_f = 4 \quad [\text{de Forcrand, D'Elia PoS LAT 16}]$$

O(a)-improved Wilson

$$N_f = 3 \quad [\text{Jin et al. PRD 15,17; Kuramashi et al. PRD 20}]$$

$$N_f = 4 \quad [\text{Ohno et al. PoS LAT 18}]$$

1st order region wider with Wilson than with staggered

- No 1st order transition seen at all:

HISQ,  $N_f = 3, m_{PS} \geq 50 \text{ MeV}$

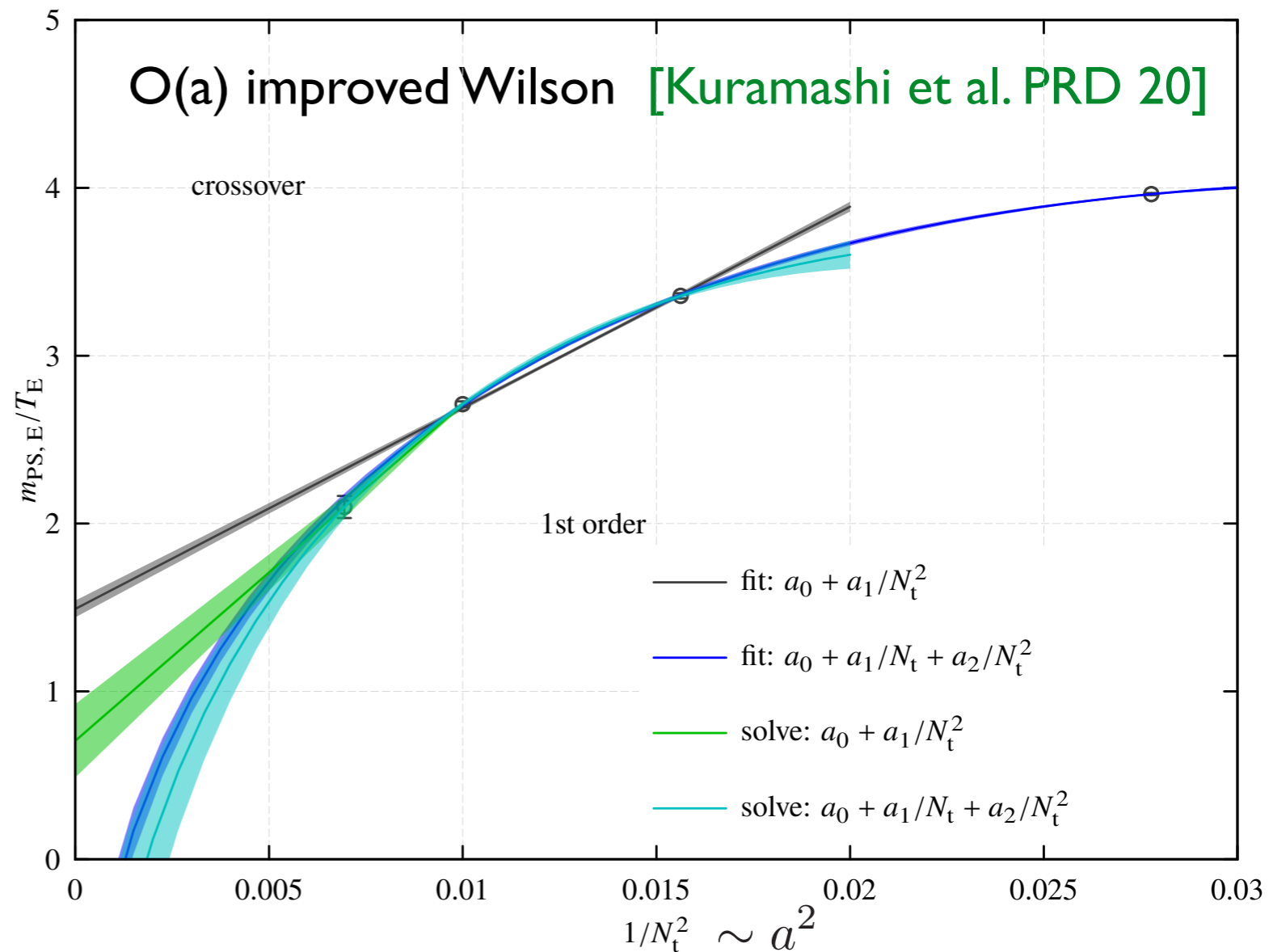
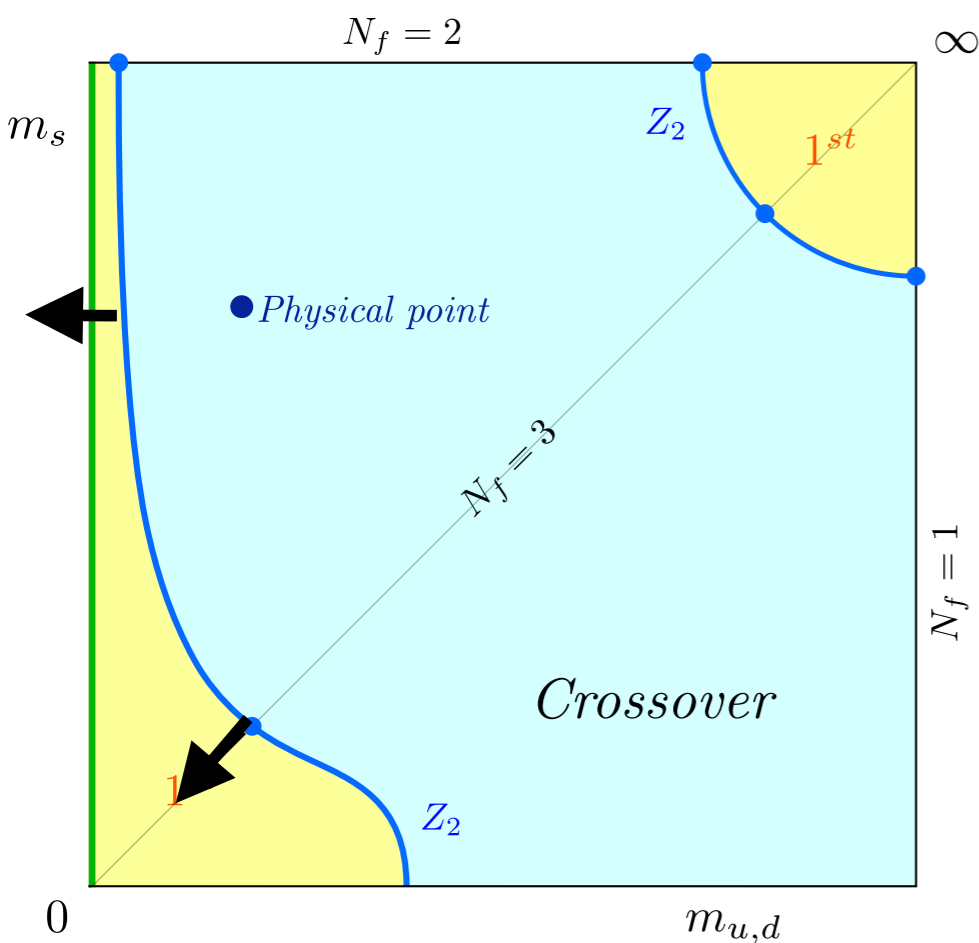
[Bazavov et al. PRD 17]

Nf=2+1

[HotQCD PRL 19]

# The nature of the QCD chiral transition, $N_f=3$

...has enormously large cut-off effects!



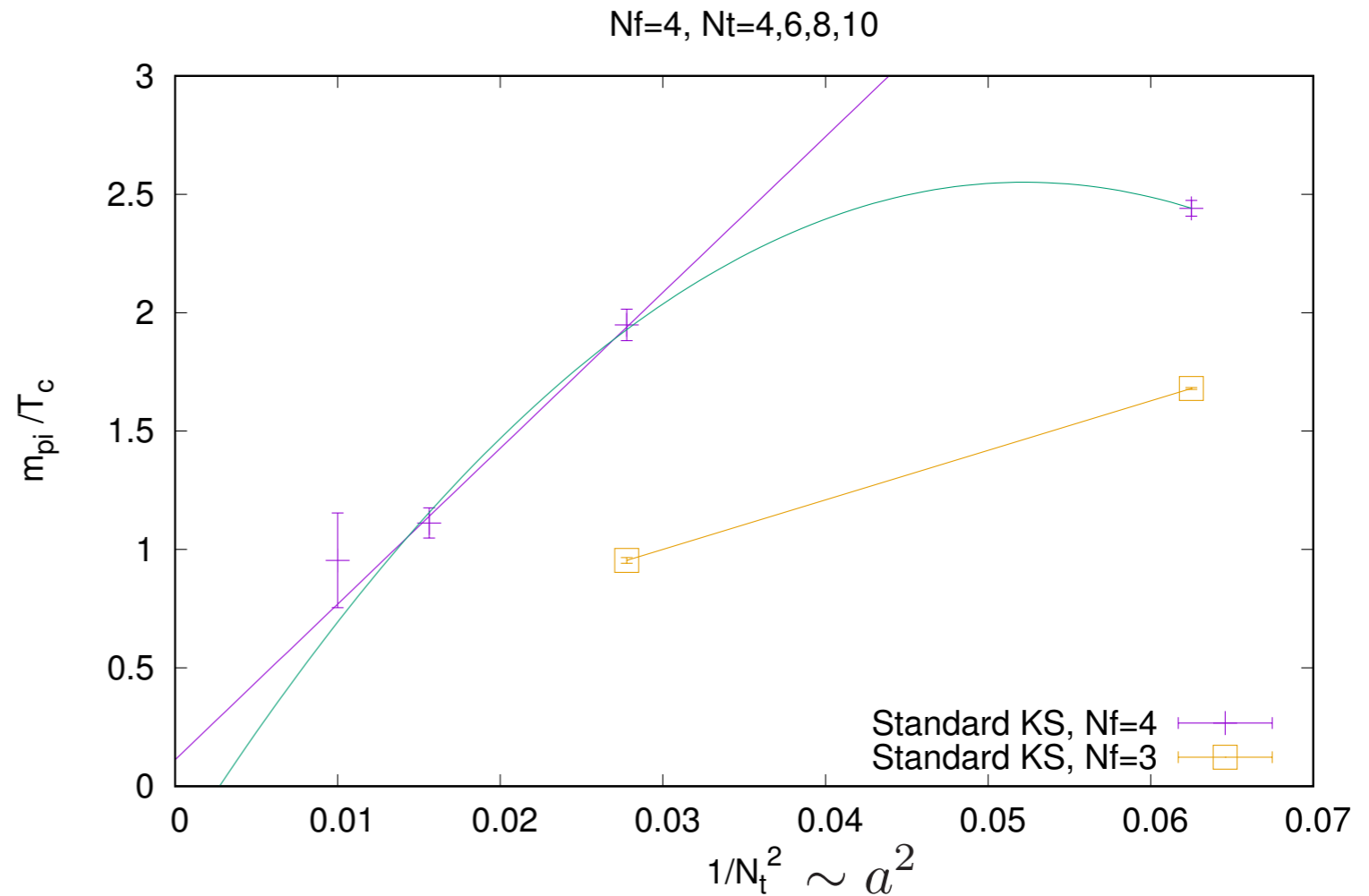
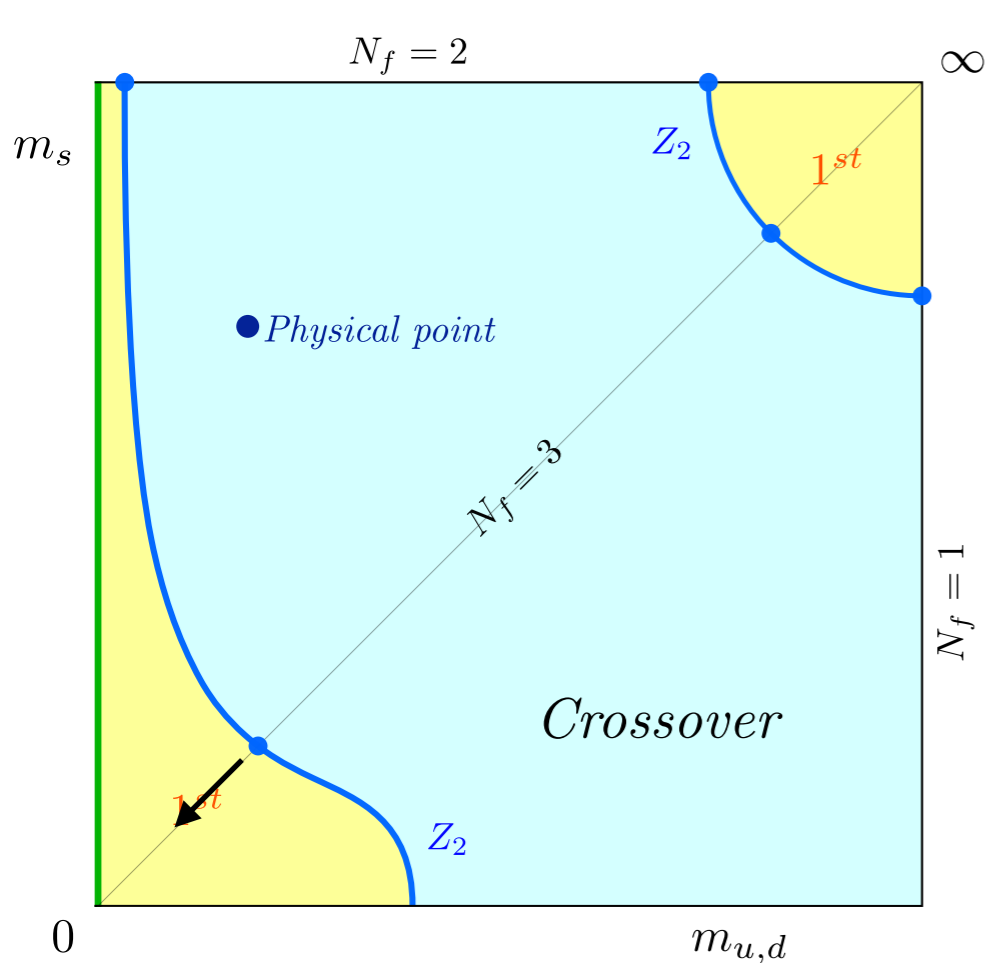
O(a)-improved Wilson:

1st order region shrinks for  $a \rightarrow 0$

$$m_\pi^c \leq 110 \text{ MeV} \quad N_\tau = 4, 6, 8, 10, 12$$

# The nature of the QCD chiral transition, $N_f=3,4$

...has enormously large cut-off effects!



Unimproved staggered:

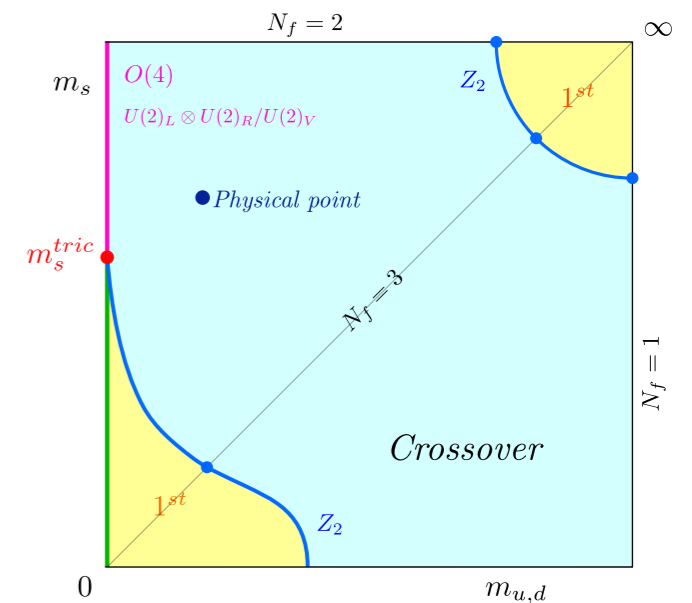
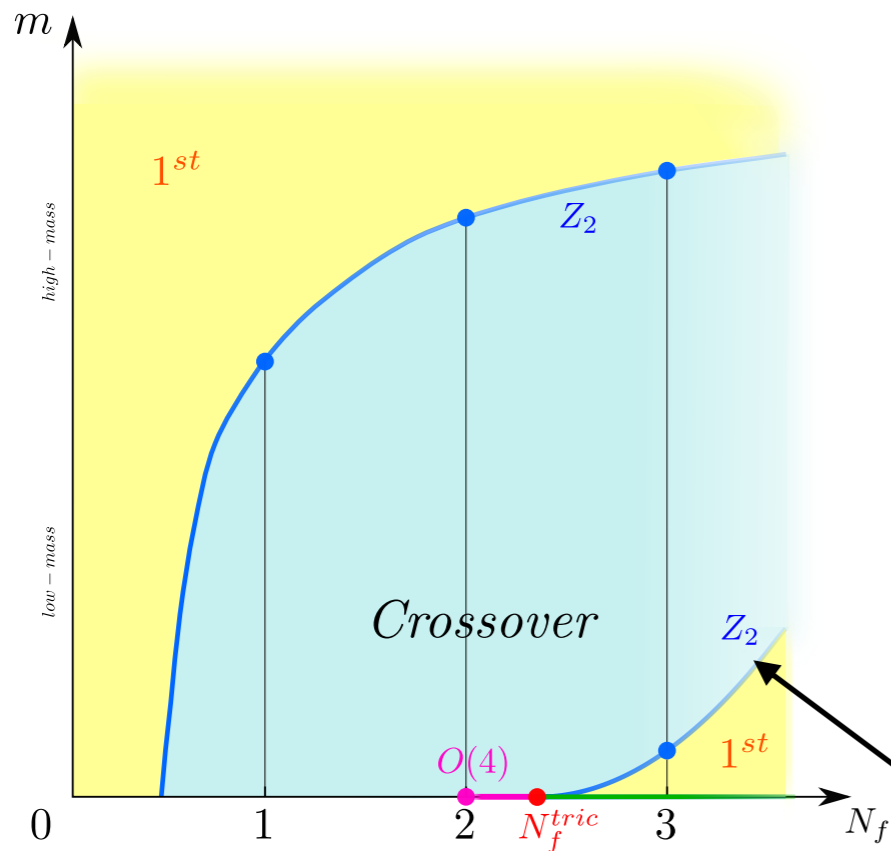
1st order region shrinks for  $a \rightarrow 0$ , both for  $N_f = 3, 4$

[de Forcrand, D'Elia, PoS LAT 17]

No 1st order seen for improved staggered actions, even  $N_f = 3$

[HotQCD PRD 17]

# Different view point: mass degenerate quarks

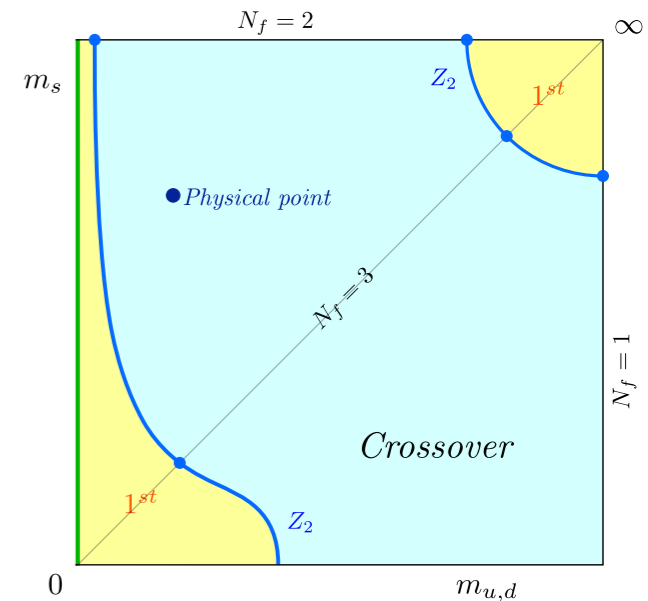
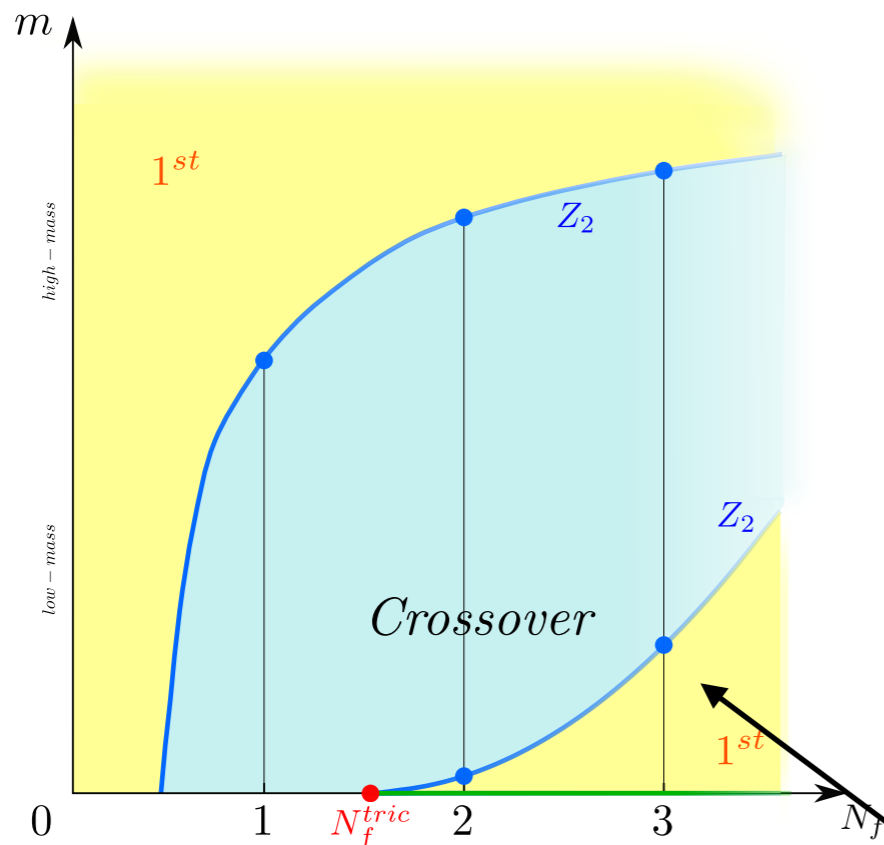


$$Z(N_f, g, m) = \int \mathcal{D}A_\mu (\det M[A_\mu, m])^{N_f} e^{-\mathcal{S}_{\text{YM}}[A_\mu]}$$

$$N_f^c(am) = N_f^{\text{tric}} + \mathcal{B}_1 \cdot (am)^{2/5} + \mathcal{O}((am)^{4/5})$$

- Consider analytic continuation to continuous  $N_f$
- Tricritical point **guaranteed** to exist if there is 1st order at any  $N_f$
- Known exponents for critical line entering tric. point!
- Continuation to  $a \neq 0$ :  $Z(2)$  surface ends in tricritical line

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# Methodology to determine order of transition

Finite size scaling of generalised cumulants

$$B_n = \frac{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^n \rangle}{\langle (\bar{\psi}\psi - \langle \bar{\psi}\psi \rangle)^2 \rangle^{n/2}}$$

Standard staggered fermions, bare parameters:

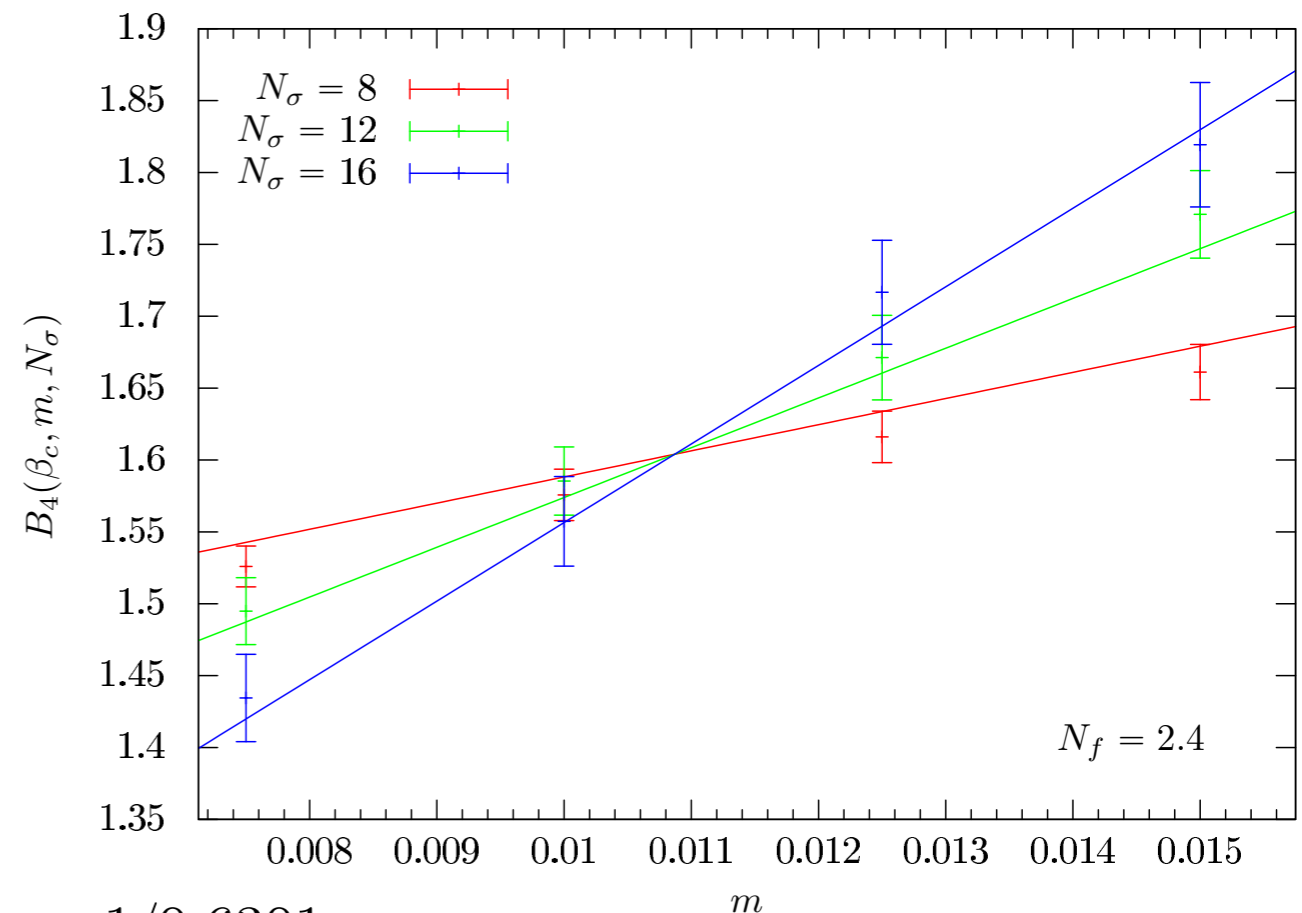
$$\beta, am, N_f, N_\tau$$

(Pseudo-critical) phase boundary:  $B_3 = 0$

3d manifold

Second-order 3D Ising:

2d chiral critical surface separates 1st order from crossover



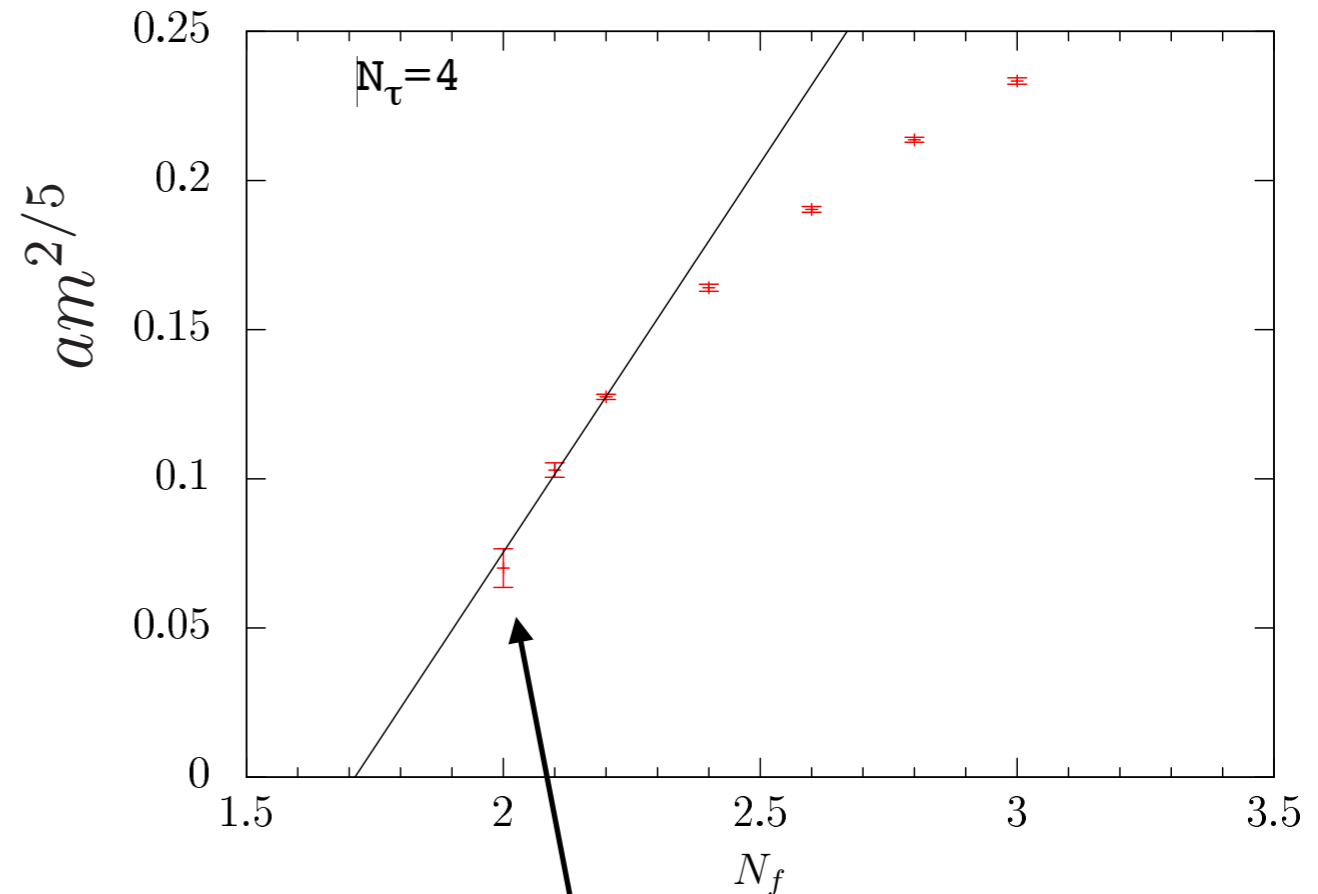
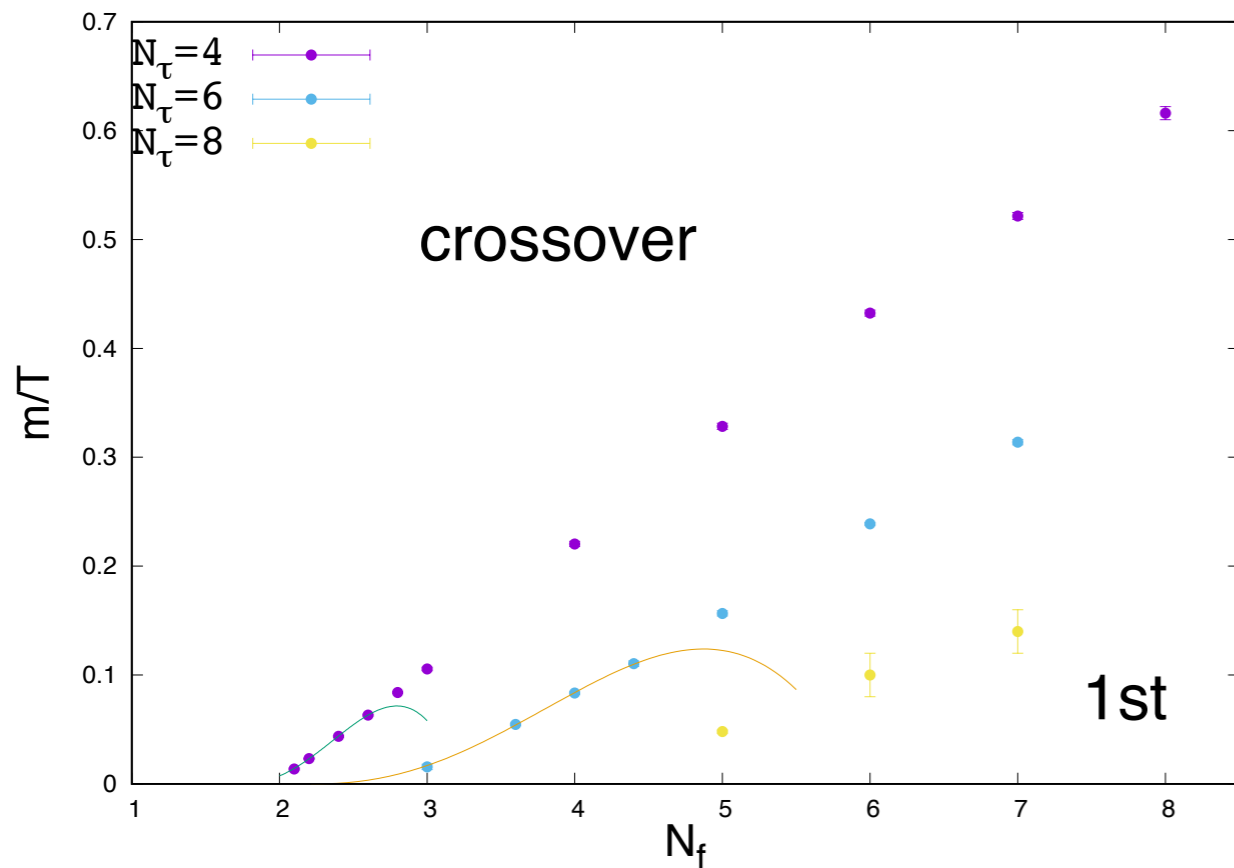
$$B_4(\beta_c, am, N_\sigma) \approx 1.604 + c(am - am_c) N_\sigma^{1/0.6301}$$



# Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]

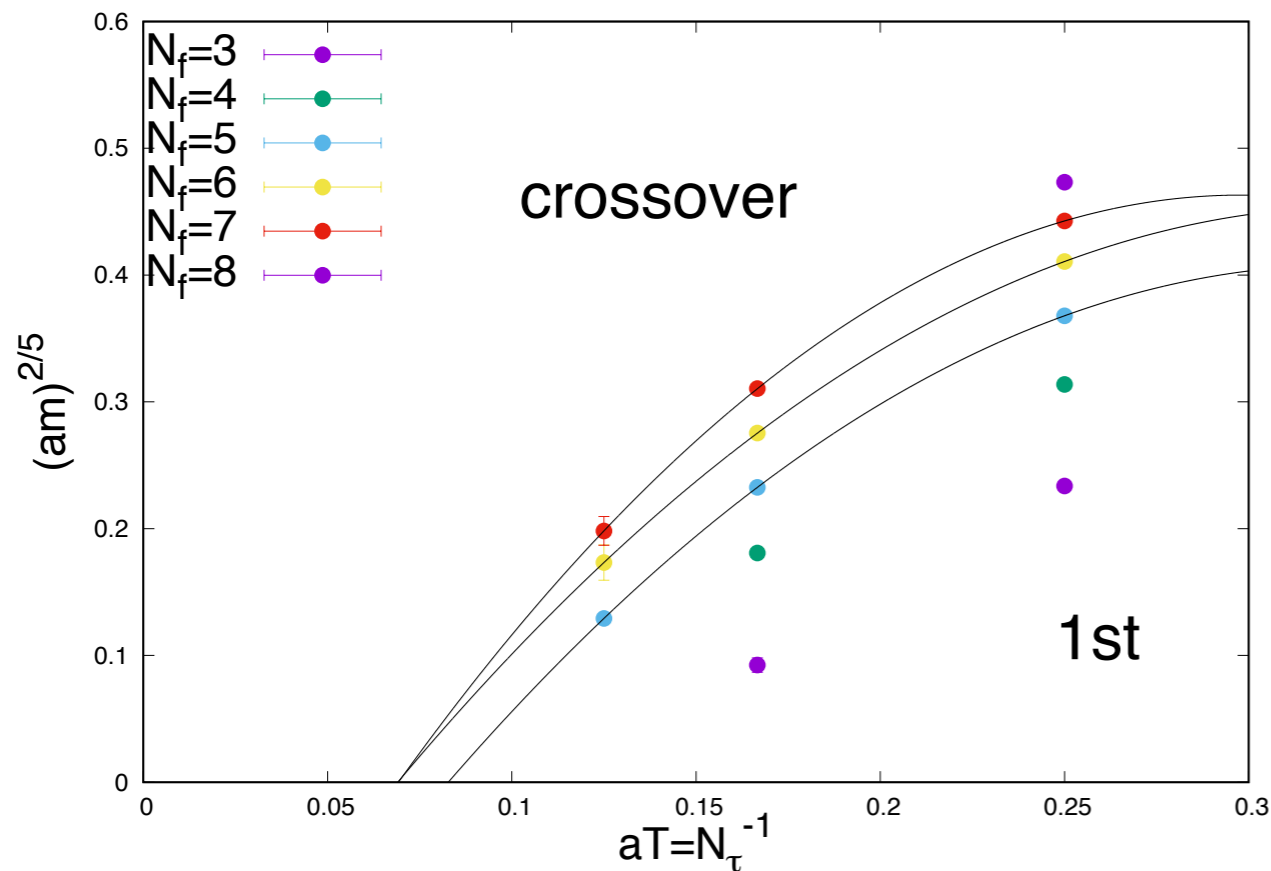
~120 M configurations, aspect ratios 3,4,5



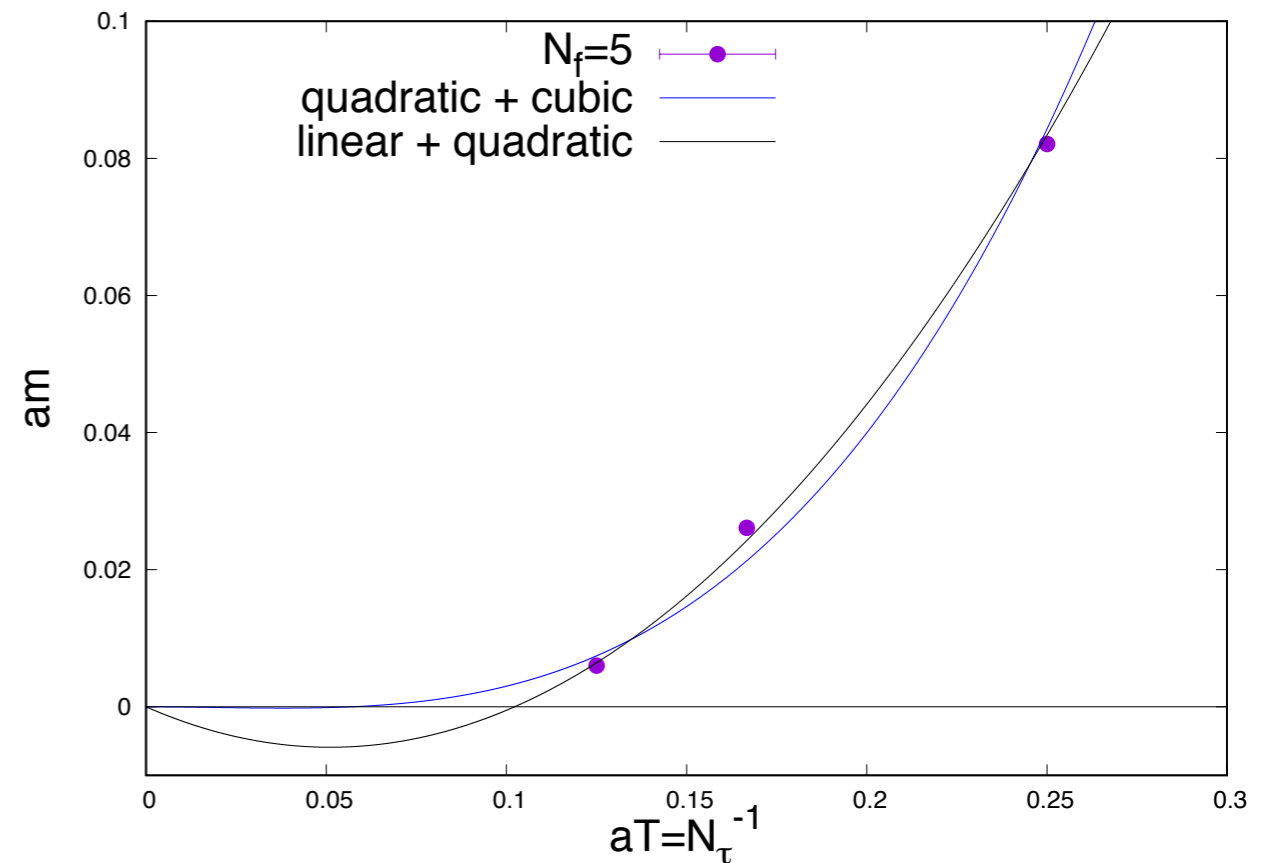
- Tricritical scaling observed also in different variable pairings
- Extrapolated by tric. scaling from finite imaginary  $\mu$  [Bonati et al. PRD 14]
- Old question:  $m_c/T = 0$  or  $\neq 0$  ? Answered for  $N_f = 2$
- New question: will  $N_f^{\text{tric}}$  slide beyond  $N_f = 3$  ?

# Bare parameter space of unimproved staggered LQCD

[Cuteri, O.P., Sciarra 21]



1st order scenario does not fit!

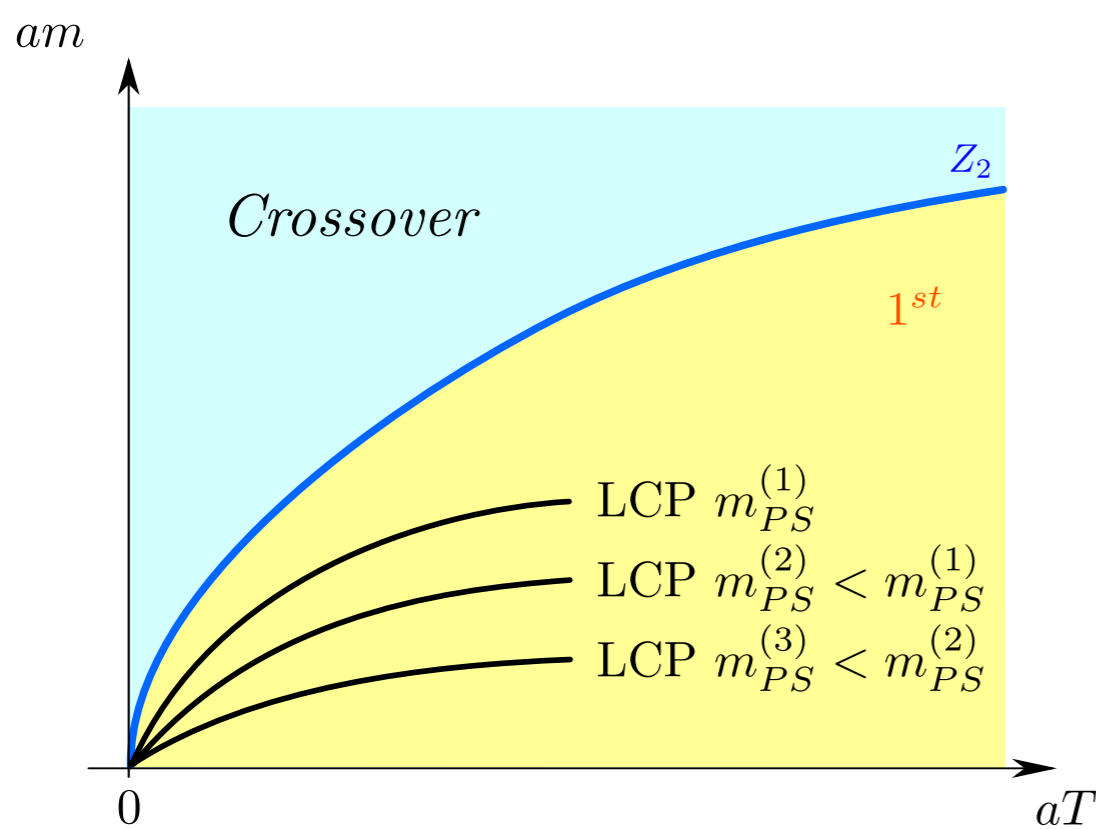


- Tricritical scaling observed also in plane of mass vs. lattice spacing
- Allows extrapolation to lattice chiral limit, tricritical points  $N_\tau^{\text{tric}}(N_f)$
- 1st order scenario:  $m_c(a) = m_c(0) + c_1(aT) + c_2(aT)^2 + \dots$

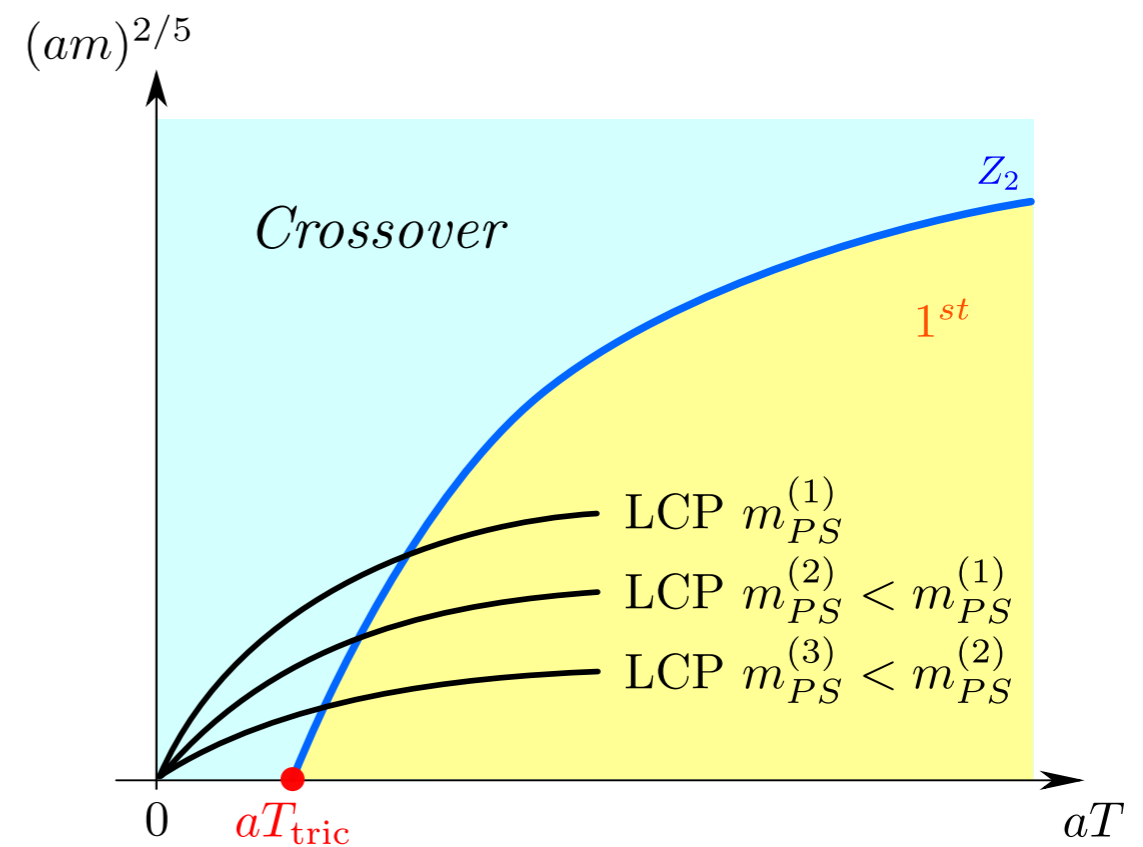
Incompatible with data!  $\chi_{\text{dof}}^2 > 10$

# Implications for the continuum

- Finite  $N_{\tau}^{\text{tric}}(N_f)$  implies that 1st order transition is not connected to continuum
- Approaching continuum first, then chiral limit:  
**Continuum chiral phase transition second-order!**



1st order scenario

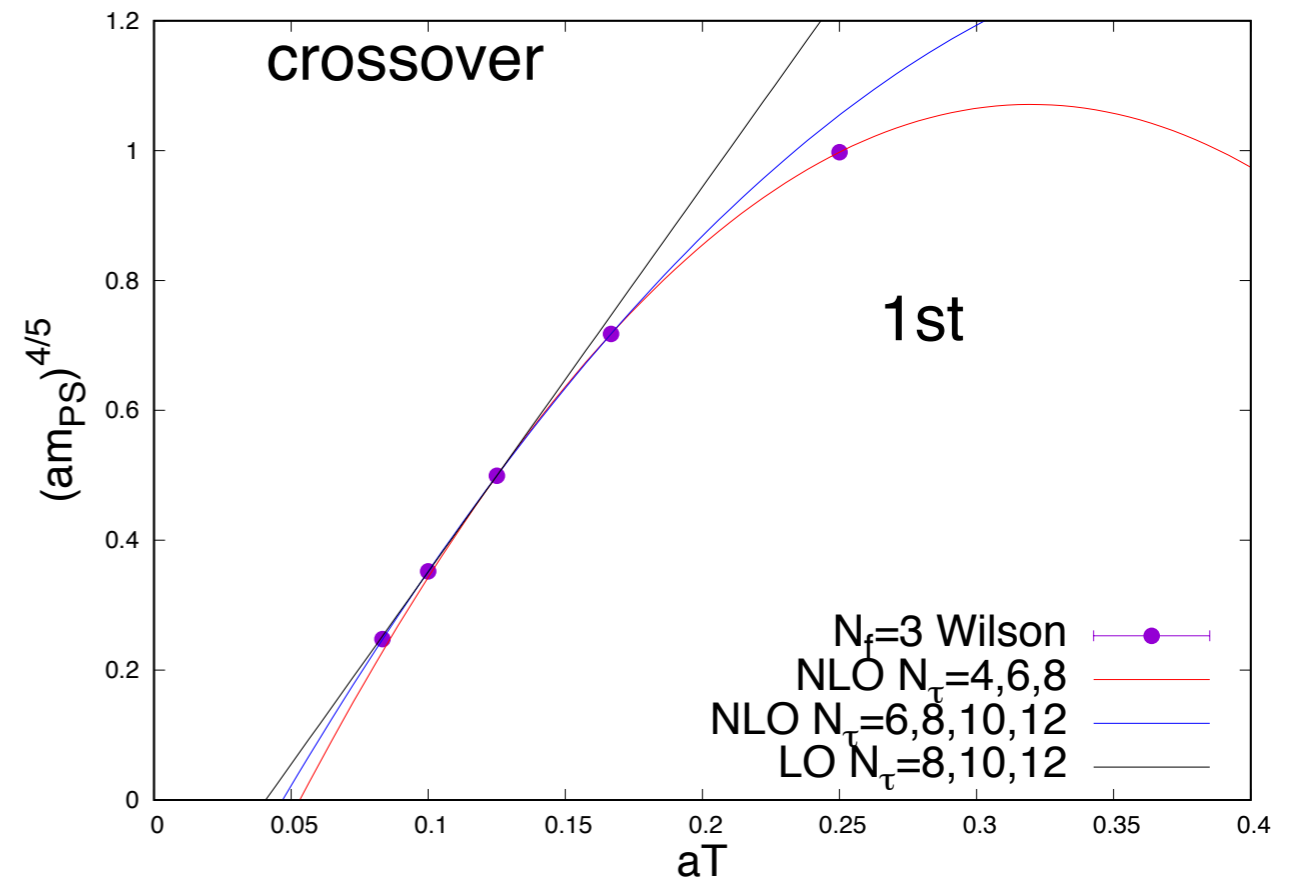
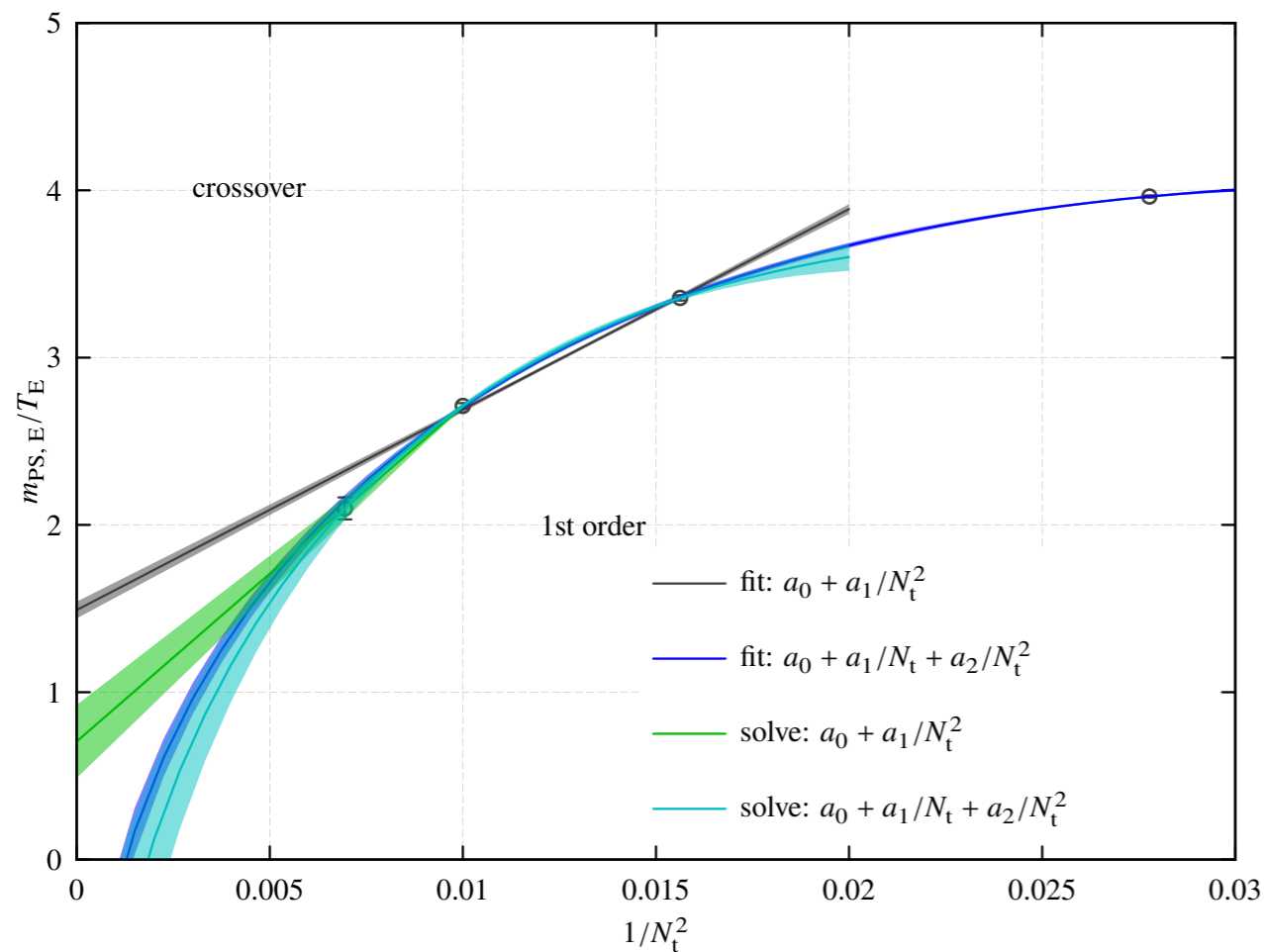


2nd order scenario

# Nf=3 O(a)-improved Wilson fermions

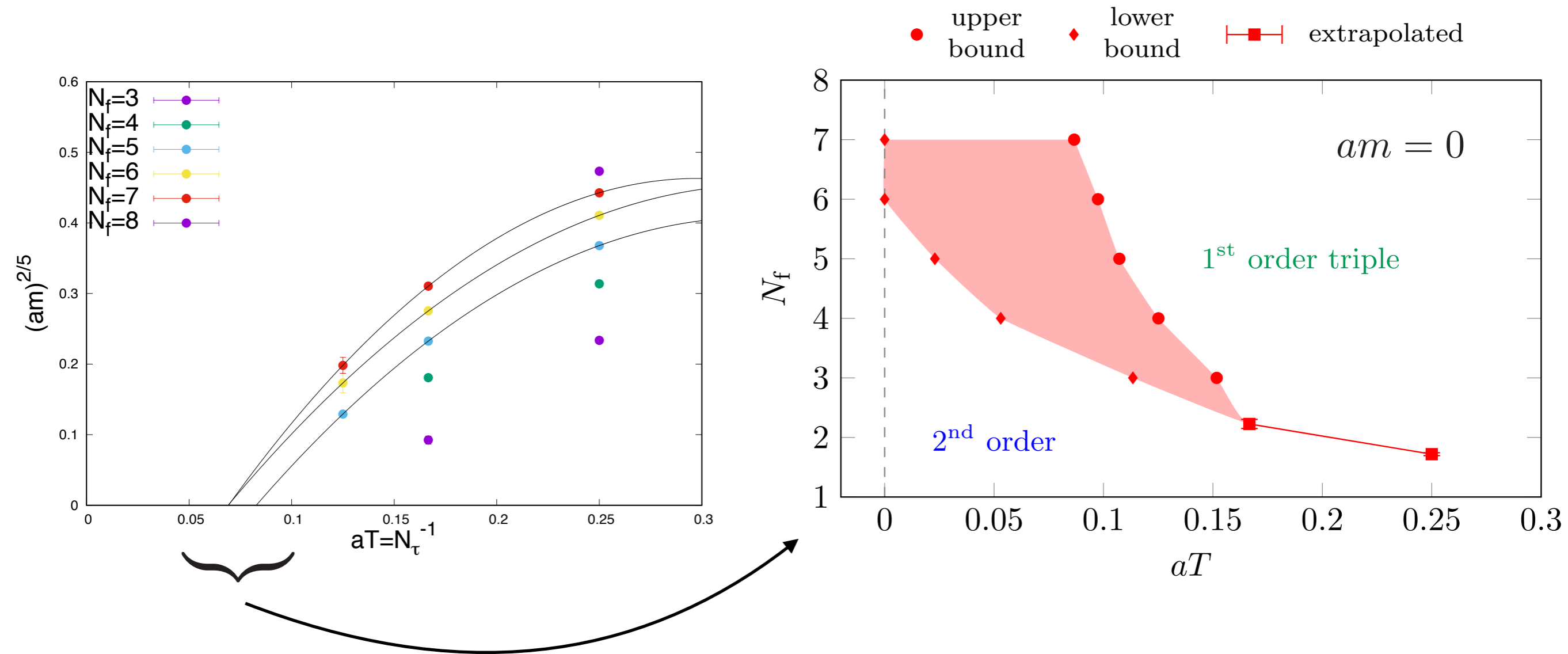
[Kuramashi et al. PRD 20]  $m_\pi^c \leq 110$  MeV  $N_\tau = 4, 6, 8, 10, 12$

Re-analysis using:  $am_{PS}^2 \propto am_q$



Tricritical scaling, Nf=3 consistent with unimproved staggered!

# Staggered: tricritical points as function of $N_f$



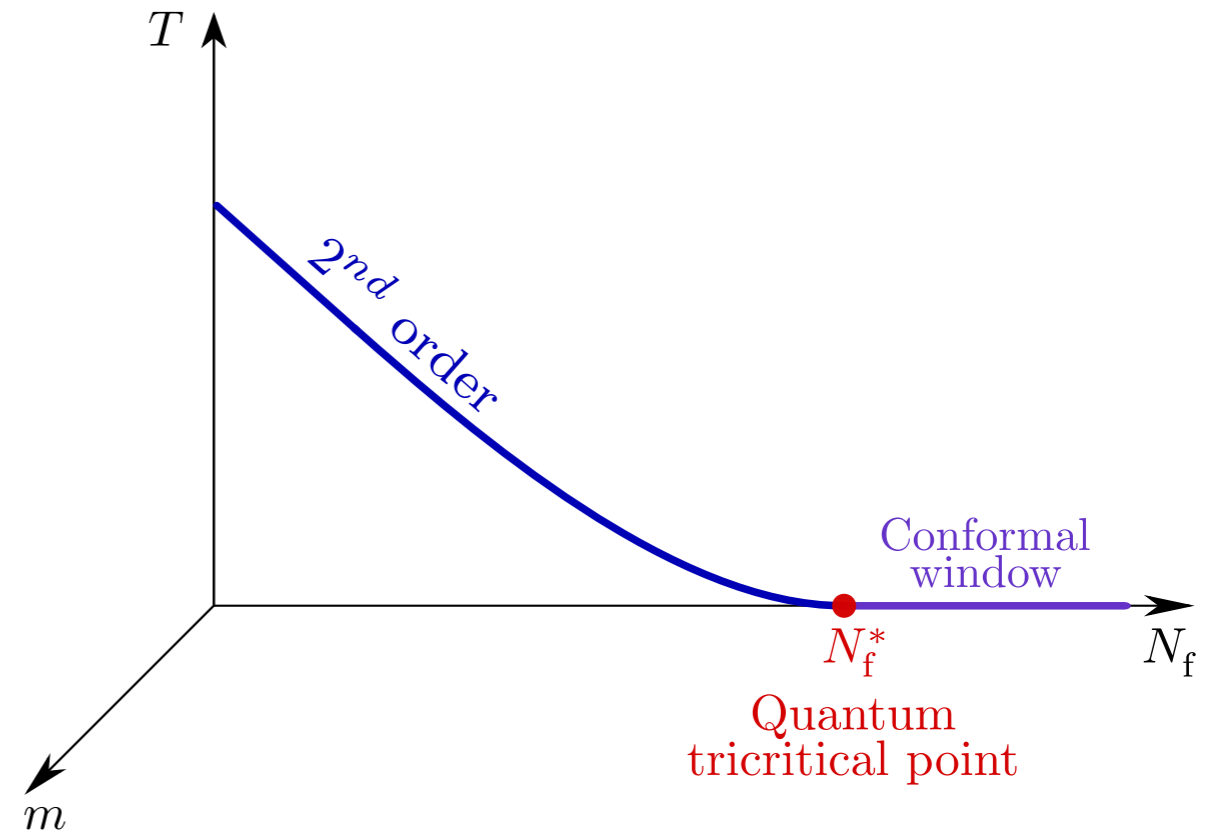
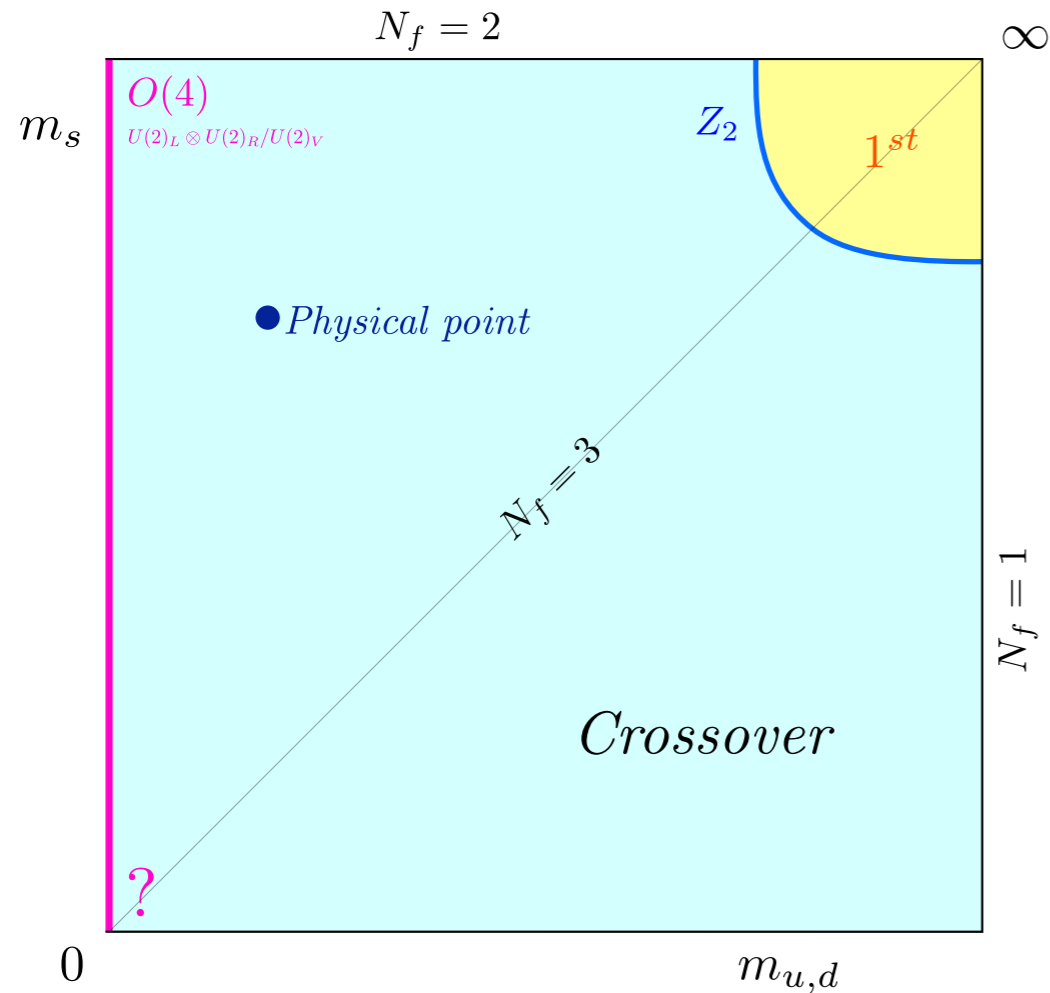
●  $N_\tau^{\text{tric}}(N_f)$  increasing function (approximately linear?)

● Tricritical line in the plane of the lattice chiral limit, separates 1st from 2nd

# The chiral phase transition for different $N_f$

[Cuteri, O.P., Sciarra 21]

Conjecture:



The chiral phase transition in the massless limit is likely second-order for all  $N_f$

# What about Pisarski, Wilczek?

- Investigated 3d sigma model,  
i.e.  $\phi^4$  - Ginzburg-Landau-Wilson theory for chiral condensate
- Results based on epsilon expansion about  $\epsilon = 1$
- All conclusions confirmed by [Butti, Pelissetto, Vicari, JHEP 03]  
High order perturbative expansion in fixed d
- Support also from simulation of 3d sigma model [Gausterer, Sanielevici, PLB 88]

Also doable by conformal bootstrap methods [Nakayama, Ohtsuki, PRD 14]

Suggested resolution:

$\phi^6$  term should be included in 3d, renormalisable!

FRG: 3d  $\phi^6$  has infrared fixed points and 2nd order transitions [Litim, Tetradis, NPB 96]

# Conclusions

- Modified strategy to study Columbia plot: lattice bare parameter space
- Knowing order of chiral transition in bare parameter space:  
Conclusions for continuum approach in correct order of limits
- For unimproved staggered,  $N_f=2-6$ ,  $O(a)$ -improved Wilson  $N_f=3$ :  
1st order transition region not connected to continuum limit
- Chiral transition second order up to conformal window?
- Check: Wilson  $N_f=4$ !
- Domain wall and overlap fermions.....