Electromagnetic conductivity of quark-gluon plasma at non-zero baryon density

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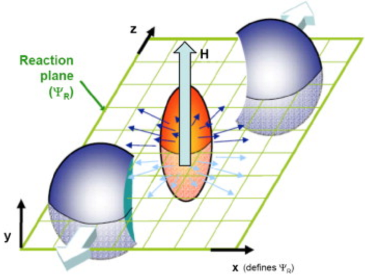
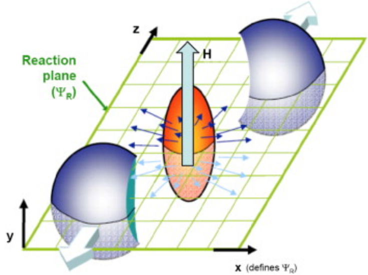
The first results are presented in e-Print: 2110.10727

Motivation



- Charge transport(σ) of QGP is important for dynamics of QGP
- QGP in heavy-ion collisions may have non-zero baryon density
- Baryon density introduces addition fermion states to QGP
- Baryon density might change σ significantly

Motivation



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How baryon density influences electromagnetic conductivity?

Theoretical studies of $\sigma(\mu)$

- T. Steinert and W. Cassing, Phys. Rev. C 89, 035203 (2014)
- P. K. Srivastava, L. Thakur, and B. K. Patra, Phys. Rev. C 91, 044903 (2015)
- G. Kadam, H. Mishra, and L. Thakur, Phys. Rev. D 98, 114001 (2018)
- O. Soloveva, P. Moreau, and E. Bratkovskaya, Phys. Rev. C 101, 045203 (2020)
- R.-A. Tripolt, C. Jung, N. Tanji, L. von Smekal, and J. Wambach, Nucl. Phys. A 982, 775 (2019)

Lattice studies of σ

- H. T. Ding, A. Francis, O. Kaczmarek, F. Karsch,E. Laermann, and W. Soeldner, Phys. Rev. D83, 034504(2011)
- A. Amato, G. Aarts, C. Allton, P. Giudice, S. Hands,and J.-I. Skullerud, Phys. Rev. Lett.111, 172001 (2013)
- G. Aarts, C. Allton, A. Amato, P. Giudice, S. Hands, and J.-I. Skullerud, JHEP02, 186 (2015)
- B. B. Brandt, A. Francis, B. Jager, and H. B. Meyer,Phys. Rev. D93, 054510 (2016)
- H.-T. Ding, O. Kaczmarek, and F. Meyer, Phys. Rev. D94, 034504 (2016)
- N. Astrakhantsev, V. V. Braguta, M. D'Elia, A. Y. Kotov, A. A. Nikolaev, and F. Sanfilippo, Phys. Rev. D102, 054516 (2020)
- P.V. Buividovich, D. Smith, L. von Smekal, Phys.Rev.D 102 (2020) 9, 094510

Conductivity in lattice simulations

- $J_i = \sigma_{ij} E_j$

- Electromagnetic conductivity
  $\sigma_{ij} = \lim_{\omega\to 0} \frac{1}{\omega} \int_0^\infty dt \int d^3x \; e^{i\omega t} \langle [J_i(x), J_j(0)] \rangle$
  $\rho_{ij} = -\frac{1}{\pi} Im G_R^{ij}(\omega, \vec{k} = 0)$
  $\sigma_{ij} = \pi \lim_{\omega\to 0} \frac{1}{\omega} \rho_{ij}(\omega)$

- Analytic continuation
  $G_E(\omega, \vec{p}) = -G_R(i\omega, \vec{p}), \quad \omega > 0$

- On lattice we measure
  $C_E(\tau) = \int d^3x \langle J_i(\tau, \vec{x}) J_j(0, \vec{0}) \rangle$
  $C_E(\tau) = \int_0^\infty d\omega \rho(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega\tau\right)}{sh\left(\frac{\omega}{2T}\right)}, \quad \tau \in \left(0, \frac{1}{T}\right)$

Conductivity with staggered fermions

- We account only connected diagrams
- Correlation function for staggered fermions

$$C_{ij}(\tau) = \frac{1}{L_s^3} \langle J_i(\tau) J_j(0) \rangle,$$

$$J_i(\tau) = \frac{1}{4} e \sum_f q_f \sum_{\vec{x}} \eta_i(x) (\bar{\Psi}_x^f U_{x,i} \Psi_{x+i}^f + \bar{\Psi}_{x+i}^f U_{x,i}^+ \Psi_x^f)$$

- Conserved current⇒renormalization is not required
- Two branches of staggered correlator

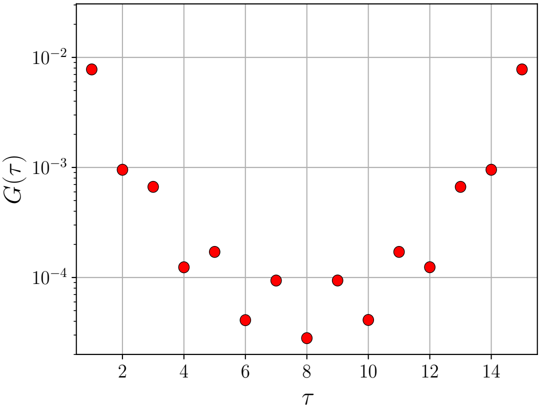
$$C_{ij}^e(\tau = 2n \times a) = \int d^3 y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle - \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$C_{ij}^o(\tau = (2n+1) \times a) = \int d^3 y (\langle A_i(\tau, \vec{y}) A_j(0, \vec{0}) \rangle + \langle B_i(\tau, \vec{y}) B_j(0, \vec{0}) \rangle)$$

$$A_i = e \sum_f q_f \bar{\psi}^f \gamma_i \psi^f, \quad B_i = e \sum_f q_f \bar{\psi}^f \gamma_5 \gamma_4 \gamma_i \psi^f$$

Conductivity with staggered fermions

- Typical plot for the staggered correlation function



Conductivity with staggered fermions

The strategy of the calculation

- Measure $C_E^{even,odd}(\tau)$ on two branches
- Reconstruct the $\rho^{even,odd}(\omega)$ (Backus-Gilbert method)

$$C_E^{even,odd}(t) = \int_0^\infty d\omega \rho^{even,odd}(\omega) \frac{ch\left(\frac{\omega}{2T} - \omega t\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Calculate $\rho(\omega) = \frac{1}{2}(\rho^{even}(\omega) + \rho^{odd}(\omega))$
  (what corresponds to the $\langle J_{el}(\tau) J_{el}(0) \rangle$)
- Calculate the conductivity $\sigma = \pi \frac{\rho(\omega)}{\omega}\Big|_{\omega \sim 0}$

Backus-Gilbert method for the spectral function

- Problem: find $\rho(\omega)$ from the integral equation

$$C(x_i) = \int_0^\infty d\omega \rho(\omega) K(x_i, \omega), \quad K(x_i, \omega) = \frac{ch\left(\frac{\omega}{2T} - \omega x_i\right)}{sh\left(\frac{\omega}{2T}\right)}$$

- Define an estimator $\tilde{\rho}(\bar{\omega})$ ($\delta(\bar{\omega}, \omega)$ - resolution function):

$$\tilde{\rho}(\bar{\omega}) = \int_0^\infty d\omega \hat{\delta}(\bar{\omega}, \omega) \rho(\omega)$$

- Let us expand $\delta(\bar{\omega}, \omega)$ as

$$\delta(\bar{\omega}, \omega) = \sum_i b_i(\bar{\omega}) K(x_i, \omega) \quad \tilde{\rho}(\bar{\omega}) = \sum_i b_i(\bar{\omega}) C(x_i)$$

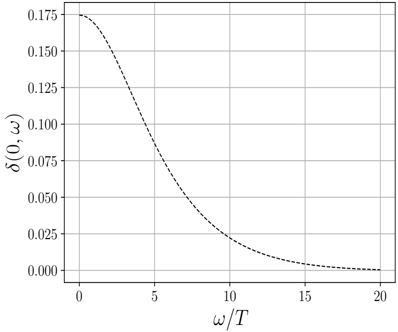
- Goal: minimize the width of the resolution function

$$b_i(\bar{\omega}) = \frac{\sum_j W_{ij}^{-1} R_j}{\sum_{ij} R_i W_{ij}^{-1} R_j},$$
$$W_{ij} = \int d\omega K(x_i, \omega)(\omega - \bar{\omega})^2 K(x_j, \omega), R_i = \int d\omega K(x_i, \omega)$$

- Regularization by the covariance matrix S_{ij}:

$$W_{ij} \to \lambda W_{ij} + (1 - \lambda) S_{ij}, \quad 0 < \lambda < 1$$

Backus-Gilbert method for the spectral function



- We calculate the estimator of the spectral function

$$\bar{\rho}(\bar{\omega}) = \int d\omega \delta(\omega, \bar{\omega}) \rho(\omega)$$

- Average spectral function(conductivity) over the width $\sim$ *few* $\times$ T

Backus-Gilbert method for the spectral function

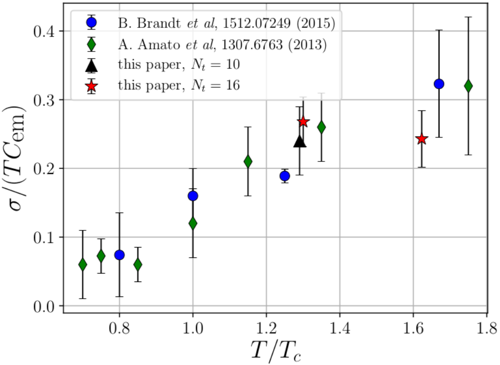
- Width of the resolution function is $\sim 4 \times T$
- For very narrow spectral density BG method underestimates conductivity
- But lattice studies give the width $\sim 4T$ or larger
  - G. Aarts et al, JHEP02, 186 (2015)
  - B. B. Brandt et al,Phys. Rev.D93, 054510 (2016)
  - H.-T. Ding, et al, Phys. Rev.D94, 034504 (2016)

Details of lattice simulations

- Stout smeared staggered $2 + 1$ fermions
- Physical pion m_π and strange m_s quark masses
- $T \approx 200,\ 250$ MeV
- $\mu_u = \mu_d = \mu_B/3,\ \mu_s = 0$
- Bacause of the sign problem the simulations are carried out at imaginary $\mu_B = I\mu_I$
- $\frac{\mu_I}{3\pi T} = 0.0,\ 0.14,\ 0.20, 0.245,\ 0.285$
- Lattice parameters:

a, fm	L_s	N_t	T, fm
0.0988	48	10	200
0.0788	48	10	250
0.0820	48	12	200
0.0657	48	12	250

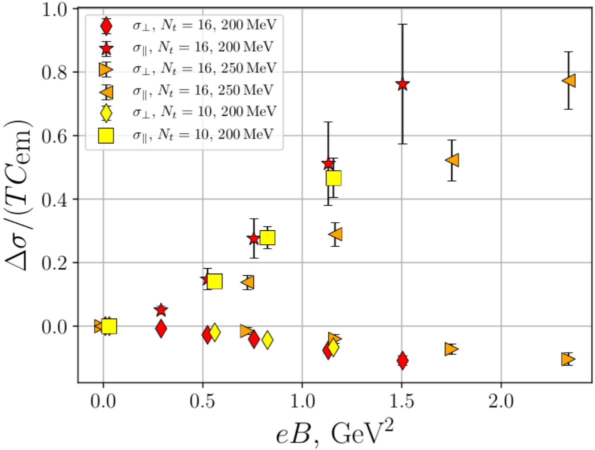
Our previous study: e.m. conductivity at finite T



- First calculation of the conductivity at physical pion mass
- Agreement with previous papers

The results from paper Phys.Rev. D102, 054516 (2020)

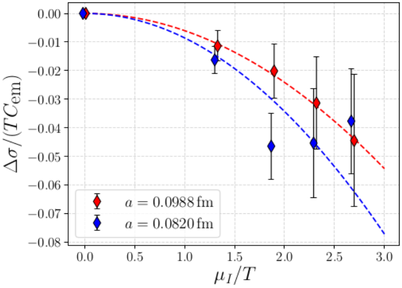
Our previous study: e.m. conductivity at finite eB

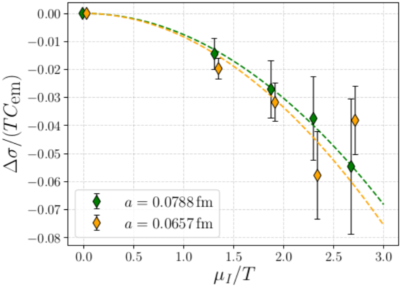


- $\Delta\sigma = \sigma(eB) - \sigma(eB = 0)$
- We observed CME and magnetoresistance in QGP

The results from paper Phys.Rev. D102, 054516 (2020)

E.m. conductivity at finite baryon density





- $\Delta\sigma = \sigma(\mu_I) - \sigma(\mu_I = 0)$ (to subtract UV contribution)
- Discretization effects are under control
- Our results can be well described by

$$\frac{\Delta\sigma}{TC_{em}} = -c(T)\left(\frac{\mu_I}{T}\right)^2 \Rightarrow \frac{\Delta\sigma}{TC_{em}} = c(T)\left(\frac{\mu_B}{T}\right)^2, \quad C_{em} = e^2 \sum_f q_f^2$$

- $c(T) \sim 0.007 \Rightarrow$ BARYON DENSITY ENHANCES E.M. CONDUCTIVITY
- $c(T)$ weakly depends on temperature
- At $\mu_q \sim T \quad \frac{\Delta\sigma}{\sigma} \sim 30\%$
  reasonable agreement with Phys. Rev. C 89, 035203 (2014), Phys. Rev. C 91, 044903 (2015)

Conclusion:

- E.m. conductivity at finite baryon density was calculated
- **Baryon density enhances e.m. conductivity**
- Our data are well described by

$$\frac{\Delta\sigma}{TC_{em}} = c(T)\left(\frac{\mu_B}{T}\right)^2$$

- Rather strong dependence of e.m. conductivity on chemical potential