



Studying mass generation in Landau-gauge Yang-Mills theory

GE, Pawłowski, Silva, 2107.05352
(accepted for publication in PRD)

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LIP & IST Lisboa

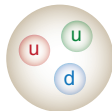
XIII International Workshop on High-Energy Physics
“Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quets”

Nov 11, 2021

Motivation

Three current quarks
do not make a proton:

$$2m_u + m_d \sim 10 \text{ MeV} \ll 1 \text{ GeV} ?$$



u	d	s	c	b	t
3	5	100	1000	4000	175000
350	350	350	350	350	350

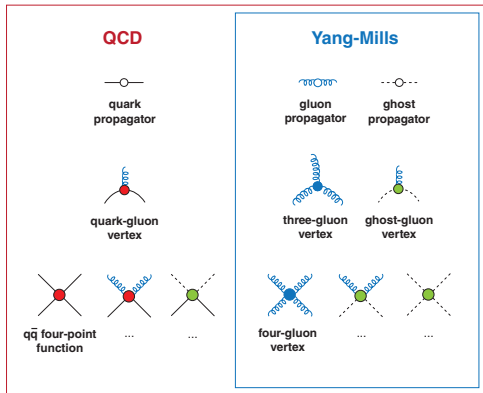
[MeV]



QCD's n-point functions

QCD is determined by its **n-point correlation functions**:

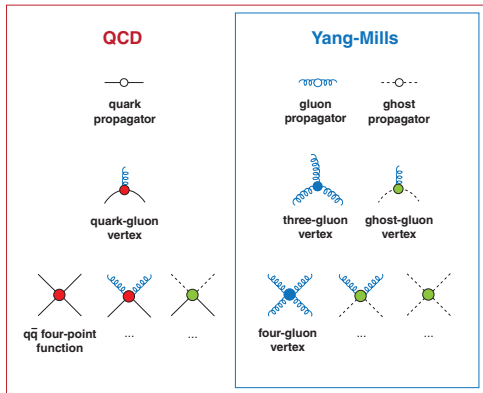
$$S_{\text{QCD}} = \int d^4x \left[\bar{\psi} (\not{D} + M) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} + \mathcal{L}_{\text{GF}} \right]$$



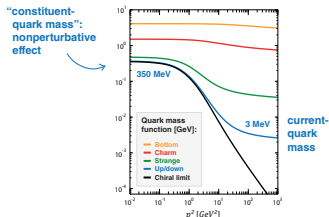
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- **Mass generation for quarks:**
Dynamical chiral symmetry breaking



- Outcome of **Dyson-Schwinger equations (DSEs)**, functional renormalization group (FRG), lattice QCD

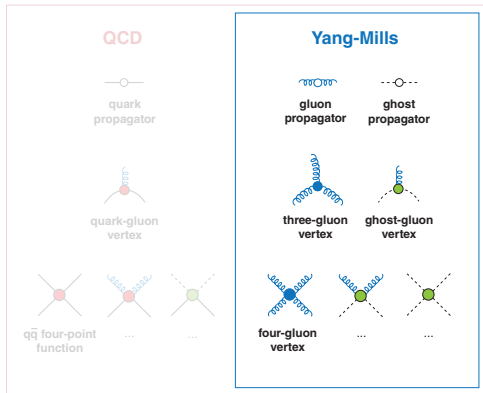
$$\text{quark propagator}^{-1} = \text{free quark propagator}^{-1} - \text{quark self-energy}$$

- Relies on underlying mass generation for **gluons**

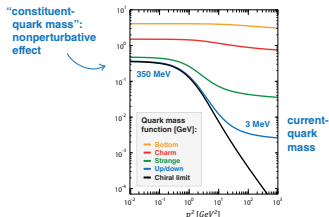
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$$\text{quark propagator}^{-1} = \text{free quark propagator}^{-1} - \text{quark self-energy}$$

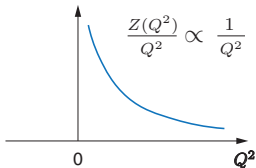
- Relies on underlying mass generation for **gluons**

Mass generation for gluons?

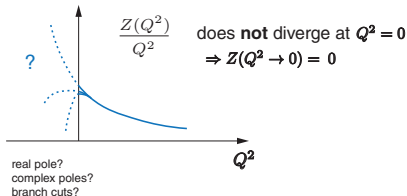
Gluon propagator:

$$D^{\mu\nu}(Q) = \underbrace{\frac{Z(Q^2)}{Q^2}}_{= D(Q^2)} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \frac{\overbrace{L(Q^2)}^{=1}}{Q^2} \frac{Q^\mu Q^\nu}{Q^2}$$

- **Perturbation theory:**
Massless pole in gluon propagator



- **Nonperturbative calculations:**
Massless pole disappears



$$\lim_{r \rightarrow \infty} \int \frac{d^3 Q}{(2\pi)^3} \frac{Z(Q^2)}{Q^2} e^{i\mathbf{x} \cdot \mathbf{Q}} \propto e^{-m_{\text{gluon}} r}$$

Mass generation for gluons?

Gluon propagator:

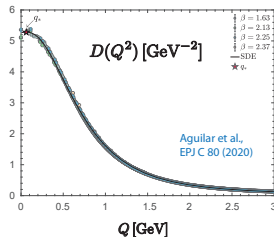
$$D^{\mu\nu}(Q) = \underbrace{\frac{Z(Q^2)}{Q^2}}_{= D(Q^2)} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \frac{L(Q^2)}{Q^2} \frac{Q^\mu Q^\nu}{Q^2}$$

$= 1$

- Massive (decoupling, “DC”) solution of DSEs & FRG

$$D(Q^2 \rightarrow 0) \sim \frac{1}{m^2} \Leftrightarrow \frac{1}{Z(Q^2)} \xrightarrow{Q^2 \rightarrow 0} \frac{m^2}{Q^2}$$

- Lattice QCD:



Aguilar, De Soto, Ferreira, Papavassiliou,
Rodríguez-Quintero, Zafeiropoulos,
EPJ C 80 (2020)

Duarte, Oliveira, Silva, PRD 94 (2016)

Maas, Phys. Rept. 524 (2013)

Bogolubsky, Ilgenfritz, Müller-Preussker,
Sternbeck, PLB 676 (2009)

Cucchieri, Maas, Mendes, PRD 77 (2008)

...

Mass generation for gluons?

Gluon propagator:

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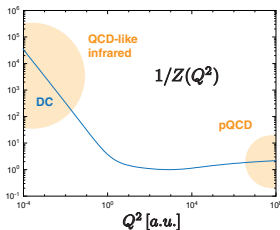
$$D(Q^2 \rightarrow 0) \sim \frac{1}{m^2} \Leftrightarrow \frac{1}{Z(Q^2)} \xrightarrow{Q^2 \rightarrow 0} \frac{m^2}{Q^2}$$

- For **gluon DSE**, this means somewhere on the r.h.s. there **must** be a $1/Q^2$ pole \rightarrow where?



- PT-BFM** (Pinch-technique/background-field method):
massless bound states in three-gluon vertex

[Aguilar, Binosi, Papavassiliou, PRD 78 \(2008\)](#), [Aguilar, Papavassiliou, PRD 91 \(2015\)](#)



Mass generation for gluons?

Gluon propagator:

$$D^{\mu\nu}(Q) = \frac{Z(Q^2)}{Q^2} \left(\delta^{\mu\nu} - \frac{Q^\mu Q^\nu}{Q^2} \right) + \xi \frac{L(Q^2)}{Q^2} \frac{Q^\mu Q^\nu}{Q^2}$$

= 1

$$= D(Q^2)$$

- Scaling (“SC”) solution of DSEs & FRG:

n-point functions scale with IR power laws:

$$Z \sim (Q^2)^{2\kappa}, G \sim (Q^2)^{-\kappa}, \dots, \kappa \sim 0.59$$

Smekal, Hauck, Alkofer, Ann. Phys. 267 (1998), Lerche, Smekal, PRD 65 (2002), Fischer, Alkofer, PLB 536 (2002), PRD 70 (2004), Zwanziger, PRD 67 (2003)

$q\bar{q}$ four-point function $\sim 1/Q^4$

Alkofer, Fischer, Llanes-Estrada, Mod. Phys. Lett. A 23 (2008)



DSEs, FRG: family of DC solutions with scaling solution as endpoint

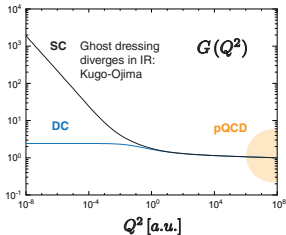
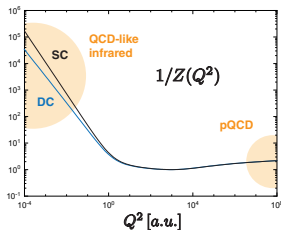
Boucaud, Leroy, Yaouanc, Micheli, Pène, Rodriguez-Quintero, JHEP 06 (2008)

Fischer, Maas, Pawłowski, Ann. Phys. 324 (2009),

Reinos, Serreau, Tissier, Wschebor, PRD 96 (2017)

Not seen on lattice (but different DC solutions?)

Cucchieri, Mendes, PRD 88 (2013), Sternbeck, Müller-Preussker, PLB 726 (2013)



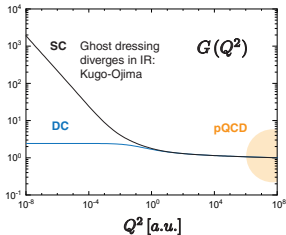
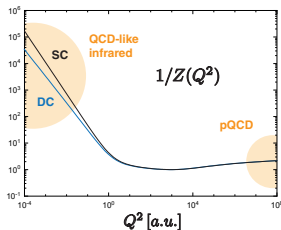
Mass generation for gluons?

Options:

- Only DC solution is physical? (Lattice, PT-BFM)
- Only SC solution is physical? (Kugo-Ojima, $1/Q^4$)
- All solutions are physically equivalent? (Hadron physics, $T > 0$, ...)

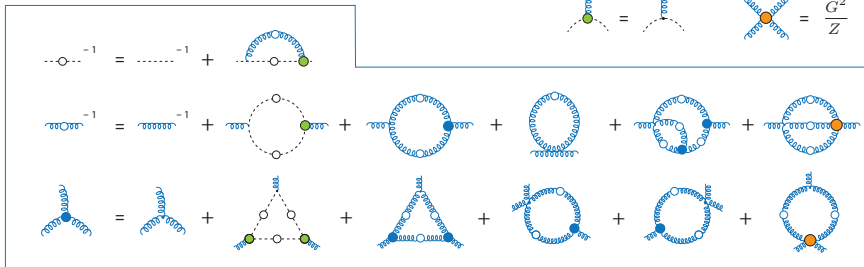
Questions:

- What actually distinguishes SC and DC?
- Where does the singularity in $1/Z$ come from?
- How is a gluon “mass” compatible with gauge invariance?



DSEs

This is what we solve:



Higher n-point functions:

DSEs & nPI: Huber, PRD 101 (2020) 114009

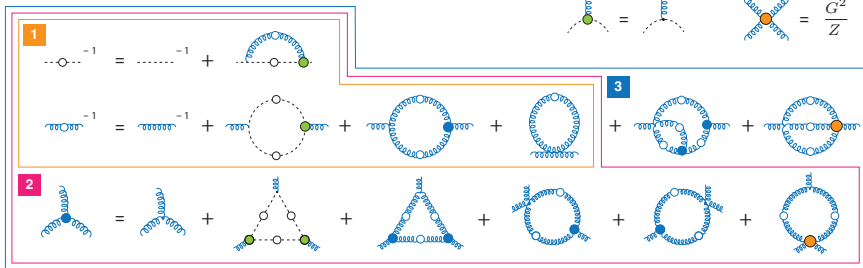
FRG: Cyrol, Fister, Mitter, Pawłowski, Strodthoff, PRD 94 (2016), PRD 97 (2018)

Binosi, Ibañez, Papavassiliou, JHEP 09 (2014)
Gracey, Pelaez, Reinos, Tissier, PRD 100 (2019),

...

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Binosi, Ibañez, Papavassiliou, JHEP 09 (2014)
 Gracey, Pelaez, Reinos, Tissier, PRD 100 (2019),
 ...

Different truncations do not change qualitative features.

Internal way to quantify **truncation error** (deviation from STI):

- 1** $c \sim 0.4$
- 2** $c \sim 0.9$
- 3** $c \sim 0.96 \rightarrow 4\% \text{ error}$ (Full: $c = 1$)

Mass generation

Gluon has T + L component, L = trivial.

$$(D^{-1})^{\mu\nu}(Q) = (D_0^{-1})^{\mu\nu}(Q) + \Pi^{\mu\nu}(Q)$$

$$\text{[Diagrammatic representation: a gluon loop with a ghost loop on top and a gluon loop on bottom, with a plus sign and an ellipsis following.]}$$

$$\Pi(Q^2) (Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu) + \tilde{\Pi}(Q^2) \delta^{\mu\nu}$$

$$\Rightarrow Z(Q^2)^{-1} = Z_A + \Pi(Q^2) + \frac{\tilde{\Pi}(Q^2)}{Q^2}$$

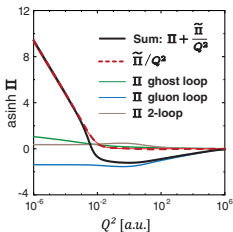
$$L(Q^2)^{-1} = 1 + \xi \tilde{\Pi}(Q^2)$$

- mass dimension 2
- violates gauge invariance \rightarrow must vanish
- vanishes in perturbation theory (dim. reg.), but not with cutoff \rightarrow quadratic divergences

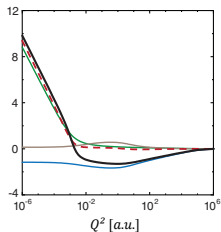
Usually:
drop longitudinal equation
($\tilde{\Pi}(Q^2)$ must be zero anyway),
solve transverse equation.

But is it zero in practice? No ...
Actually, this term is responsible
for **mass generation** \rightarrow ??

Decoupling:
 $\tilde{\Pi}(Q^2)$ generates mass?



Scaling:
ghost loop dominance



Mass generation

Gluon has T + L component, L = trivial.

$$(D^{-1})^{\mu\nu}(Q) = (D_0^{-1})^{\mu\nu}(Q) + \Pi^{\mu\nu}(Q)$$

Expand self-energy in overcomplete basis:

$$\Delta_T(Q^2) (Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu) + \Delta_0(Q^2) \delta^{\mu\nu} + \Delta_L(Q^2) Q^\mu Q^\nu$$

contains quadratic divergences, need to be subtracted

must contain longitudinal massless poles

$$\Delta_L = -\frac{\Delta_0}{Q^2}$$

Mass generation must come from here!

$$Z(Q^2)^{-1} = Z_A + \Delta_T + \frac{\Delta_0}{Q^2}$$

$$L(Q^2)^{-1} = 1 + \xi \left[\Delta_L + \frac{\Delta_0}{Q^2} \right]$$

$$\stackrel{!}{=} 0$$

$$\Pi = \Delta_T - \Delta_L$$

$$\frac{\tilde{\Pi}}{Q^2} = \frac{\Delta_0}{Q^2} + \Delta_L \stackrel{!}{=} 0$$

Two possibilities:

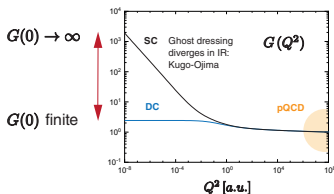
- **Scenario A:** $\Delta_L = 0 \Rightarrow \Delta_0$ must be artifact (from hard cutoff and/or truncation)
- **Scenario B:** $\Delta_L \neq 0 \Rightarrow$ Longitudinal consistency condition, requires longitudinal massless poles in the vertices (where?), does not affect transverse equation

$$\text{gluon}^{-1} = \text{gluon}^{-1} + \text{ghost-gluon vertex?} + \text{three-gluon vertex?} + \text{ghost loop} + \text{ghost-gluon loop} + \text{four-gluon vertex?}$$

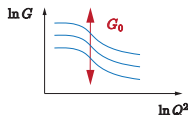
What distinguishes SC and DC solutions?

Renormalization

Boundary condition on ghost:



But $G(0) = G_0$ should only renormalize propagator...?



- n-point functions are related to bare ones by **renormalization constants**:

$$G^{(B)} = Z_c G, \quad \Gamma_{\text{gh}} = \tilde{Z}_\Gamma \Gamma_{\text{gh}}^{(B)},$$

$$Z^{(B)} = Z_A Z, \quad \Gamma_{3g} = Z_{3g} \Gamma_{3g}^{(B)},$$

$$g^{(B)} = Z_g g, \quad \Gamma_{4g} = Z_{4g} \Gamma_{4g}^{(B)},$$

- In Landau gauge, can set $\tilde{Z}_\Gamma = 1$
 \Rightarrow all ren. constants related to Z_A, Z_c

$$Z_g = \frac{1}{Z_A^{1/2} Z_c}, \quad Z_{3g} = \frac{Z_A}{Z_c}, \quad Z_{4g} = \frac{Z_A}{Z_c^2}.$$

- Renormalization conditions for the propagators eliminate Z_A, Z_c in favor of $Z(\mu^2) = Z_\mu, G(\nu^2) = G_\nu$, can always renormalize ghost at $\nu = 0$

\Rightarrow YM eqs. depend on 3 "parameters": g, Z_μ, G_0

$$G(Q^2)^{-1} = G_0^{-1} + \Sigma(Q^2) - \Sigma(0),$$

$$Z(Q^2)^{-1} = Z_\mu^{-1} + \Pi(Q^2) - \Pi(\mu^2), \quad \Pi = \Delta_T + \frac{\Delta_0}{Q^2}$$

$$F_{3g}(Q^2) = Z_{3g} + \mathcal{M}(Q^2).$$

Keep g & Z_μ fixed $\Rightarrow G_0$ distinguishes SC & DC

Renormalization

- $\Delta_0(Q^2)$ contains quadratic divergences which must be subtracted. Arbitrariness in subtraction compensated by parameter β :

$$\Delta_0(Q^2) \rightarrow \Delta_0(Q^2) - \Delta_0(Q_0^2) + \frac{g^2}{4\pi} G_0^2 \beta \mu^2$$

This introduces effective **mass term** into eqs:

$$\frac{\Delta_0}{Q^2} \rightarrow (\dots) + (\dots) \beta \frac{\mu^2}{Q^2}$$

- g, Z_μ, G_0 are actually not independent, but only appear in **combination**:

$$\alpha = Z_\mu G_0^2 \frac{g^2}{4\pi} \in \mathbb{R}_+$$

To see this, redefine $Z(Q^2) \rightarrow \frac{z(Q^2)}{Z_\mu}$, $G(Q^2) \rightarrow \frac{G(Q^2)}{G_0}$, same for Z_A, Z_c, F_{3g}, F_{4g}

Furthermore: $G(x) \rightarrow \sqrt{\alpha} G(x)$, $z = \frac{G^2}{\beta \mu^2}$

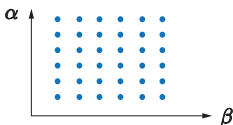
- ⇒ Yang-Mills eqs. depend on **two parameters** α, β (independent of truncation):

$$\begin{aligned} G(x)^{-1} &= \frac{1}{\sqrt{\alpha}} + \Sigma(x) - \Sigma(0) \\ Z(x)^{-1} &= 1 + \Pi(x) - \Pi\left(\frac{1}{\beta}\right) \\ F_{3g}(x) &= Z_{3g} + \mathcal{M}(x) \end{aligned}$$

- ⇒ α only appears in ghost DSE: $G(0) = \sqrt{\alpha}$

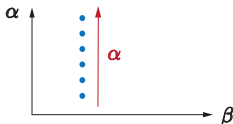
- ⇒ β only appears in subtraction point

- ⇒ Solve DSEs in (α, β) plane:



Family of solutions

Fixed β , vary $\alpha \in (0, \infty)$:



- **Running coupling** $\bar{\alpha}(x)$ is RG-invariant, sets scale (so far x is arbitrary)
- SC = envelope of DC solutions for $\alpha \rightarrow \infty$

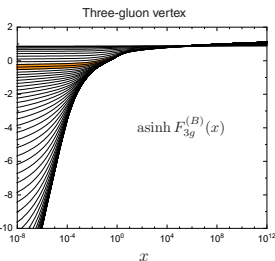
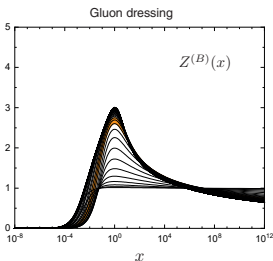
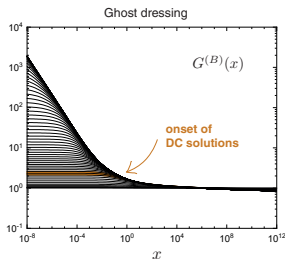
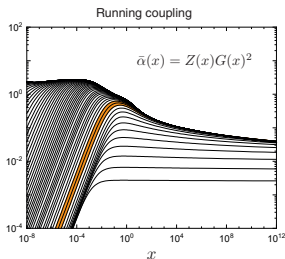
$$Z \sim (Q^2)^{2\kappa}$$

$$G \sim (Q^2)^{-\kappa}$$

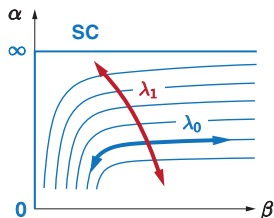
$$F_{3g} \sim (Q^2)^{-3\kappa}$$

$$\bar{\alpha} \sim \text{const.}$$

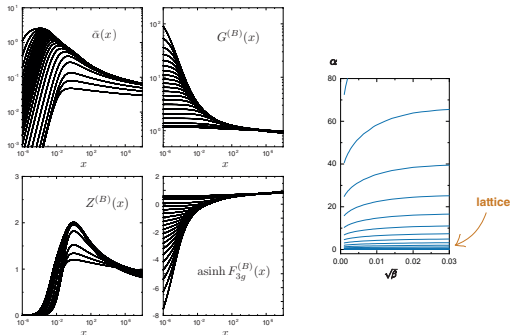
- Onset of DC solutions \leftrightarrow **lattice** solutions



Lines of constant physics



Solutions are identical along **lines of constant physics**:
each curve is superposition of **30 curves** along trajectories:



⇒ Two combinations of α and β determine solutions:

- $\lambda_0(\alpha, \beta)$ rescales system
- $\lambda_1(\alpha, \beta)$ distinguishes SC + DC

Family of solutions due to presence of **mass term!**

Where does the singularity in $1/Z$ come from?

Mass generation

$$\Pi(x) = \underbrace{\Delta_T(x)} + \underbrace{\frac{\Delta_0(x)}{x}}$$

⇒ Need $\Delta_L \neq 0$ to satisfy longitudinal consistency condition

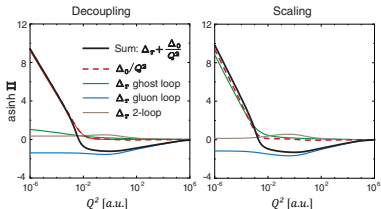
DC: ghost loop diverges logarithmically, other diagrams go to constant
diverges like $1/x$: **mass generation**

SC: diverges like $1/x^{2\kappa}$ (from ghost loop)
diverges like $1/x^{2\kappa}$ (from ghost loop)

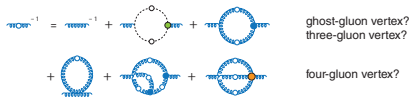
$$Z(Q^2)^{-1} = Z_A + \Delta_T + \frac{\Delta_0}{Q^2}$$

$$L(Q^2)^{-1} = 1 + \xi \left[\Delta_L + \frac{\Delta_0}{Q^2} \right]$$

$\stackrel{!}{=} 0$

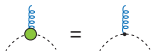


⇒ requires **longitudinal massless poles** in either of the vertices



Longitudinal poles?

So far, we employed tree-level **ghost-gluon vertex**:



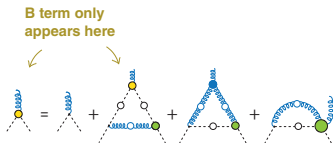
But general vertex has 2 tensors:

$$\Gamma_{\text{gh}}^\mu(p, Q) = -igf_{abc} [(1 + A) p^\mu + B Q^\mu]$$

$A(p^2, p \cdot Q, Q^2)$
 $B(p^2, p \cdot Q, Q^2)$

tree level small contributes to Δ_L ,
 does it have a $1/Q^2$ pole?

DSE for ghost-gluon vertex:



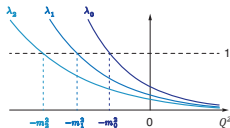
This can be read as **inhomogeneous BSE** for B , to determine if B has pole, sufficient to solve **homogeneous BSE**

Bethe-Salpeter amplitude

$$\int dx' \mathcal{K}(x, x') \varphi(x') = \lambda_0 \varphi(x),$$

$$\mathcal{K}(x, x') = \frac{N_c}{(2\pi)^2} x G(x')^2 \int_{-1}^1 dy (1-y^2)^{\frac{1}{2}} \frac{Z(w)}{w^2}$$

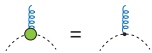
Typical **eigenvalue spectrum** of a homogeneous BSE:



If $\lambda_0(Q^2 = 0) = 1 \Leftrightarrow$ **Ghost-gluon vertex has longitudinal massless pole**

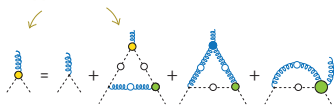
Longitudinal poles?

So far, we employed tree-level **ghost-gluon vertex**:



DSE for ghost-gluon vertex:

B term only appears here



This can be read as **inhomogeneous BSE** for B, to determine if B has pole, sufficient to solve **homogeneous BSE**

⇒ **Only SC solution** can satisfy consistency condition

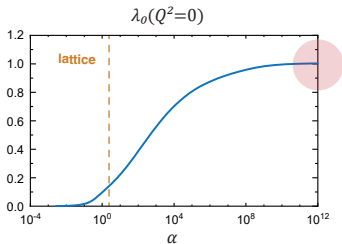


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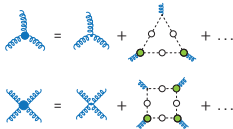
$$\mathcal{K}(x, x') = \frac{N_c}{(2\pi)^2} x G(x')^2 \int_{-1}^1 dy (1-y^2)^{\frac{1}{2}} \frac{Z(w)}{w^2}$$

If $\lambda_0(Q^2 = 0) = 1 \Leftrightarrow$ **Ghost-gluon vertex has longitudinal massless pole**

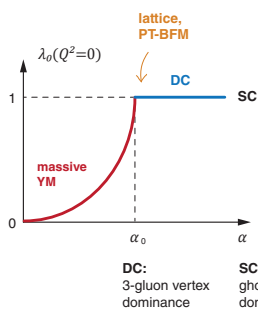
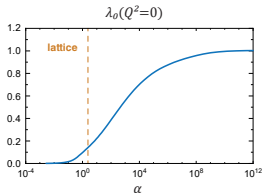


Longitudinal poles?

- But then what about lattice solutions?
- PT-BFM: longitudinal poles in 3-gluon vertex
 Aguilar, Ibanez, Mathieu, Papavassiliou, PRD 85 (2012),
 Aguilar, Binosi, Papavassiliou, PRD 95 (2017)
- Indeed, pole in ghost-gluon vertex would appear in any ghost loop



⇒ longitudinal poles in 3-gluon, 4-gluon vertex etc.



SC & DC physically equivalent, but generated by different mechanisms (?)

Summary

- Yang-Mills DSEs admit **family of solutions**, which depend on two parameters α, β

$\lambda_0(\alpha, \beta)$ only rescales solutions

$\lambda_1(\alpha, \beta)$ distinguishes SC + DC

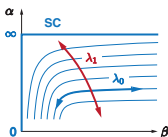
- Mass generation** through $\Delta_0 \Leftrightarrow \beta$, is entirely non-perturbative

$$Z(Q^2)^{-1} = Z_A + \Delta_T + \frac{\Delta_0}{Q^2}$$

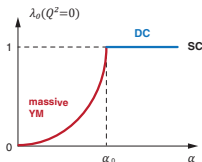
$$L(Q^2)^{-1} = 1 + \xi \left[\Delta_L + \frac{\Delta_0}{Q^2} \right]$$

$$\stackrel{!}{=} 0$$

- Gauge consistency requires **longitudinal massless poles** in vertices



- Ghost-gluon vertex has longitudinal massless pole \Rightarrow **SC solution consistent**
- Does not exclude consistency of DC solutions, if longitudinal poles in three-gluon vertex (PT-BFM)



Then presumably **all** solutions are physically equivalent (?)

Thank you!

Backup slides

Mass generation

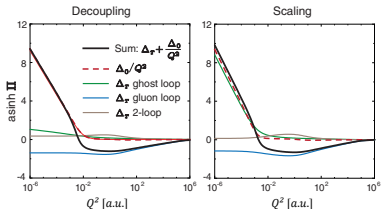
$$\Pi(x) = \underbrace{\Delta_T(x)}_{\text{DC}} + \underbrace{\frac{\Delta_0(x) - \Delta_0(x_0)}{x}}_{\text{SC}} + \frac{\beta}{x}$$

DC: ghost loop diverges logarithmically, other diagrams go to constant

diverges like $1/x$:
mass generation

SC: diverges like $1/x^{2n}$ (from ghost loop)

diverges like $1/x^{2n}$ (from ghost loop)



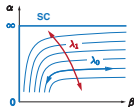
- **Scenario A:** Δ_0 is artifact from hard cutoff (like in QED) and/or truncation

→ Without this term, no QCD-like solutions

→ Take limit $\beta \rightarrow 0$:

$$\frac{\Delta_0(x) - \Delta_0(x_0)}{x} + \frac{\beta}{x} \rightarrow \lim_{\beta \rightarrow 0} \frac{\beta}{x}$$

⇒ No DC solutions,
only SC solution



However, singularity in $1/Z$ is triggered by β/x

- ⇒ could as well use arbitrary power β/x^{2n}
- ⇒ SC solution with infrared exponent $\kappa = n$

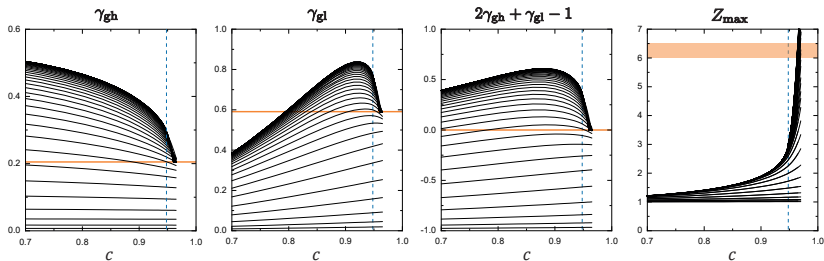
⇒ Ambiguity in SC solution,
disfavors Scenario A

Truncation error

- Set $Z_{3g} \rightarrow c Z_{3g}$... quantifies deviation from STI (without truncation: $c = 1$), same effect from “over-renormalizing” 3-gluon vertex
- YM system only converges up to $c_{\max} < 1$
- Anomalous dimensions reproduced for

1	$c \sim 0.4$
2	$c \sim 0.9$
3	$c \sim 0.96$

 } \Rightarrow identifies “physical point” for each truncation



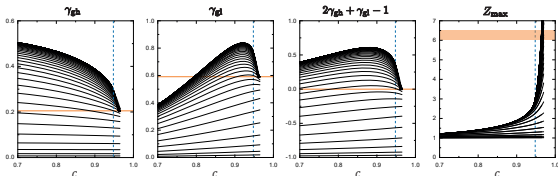
Truncation error

- Close to c_{\max} , convergence is lost; strong curvature in gluon dressing \Rightarrow do not reach lattice directly

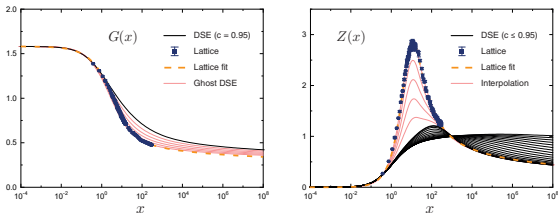
Back-coupling of 3-gluon vertex sensitive to **4-gluon vertex**

Workaround: solve nPI system (all internal vertices dressed)

[Huber, PRD 101 \(2020\) 114009](#)



- Still, extrapolation **does** reproduce lattice peak for $Z(x)$
- Standalone **ghost DSE** at c_{\max} with lattice gluon reproduces lattice ghost



\Rightarrow **Lattice & DSE results match** for particular α (or generally λ_1)



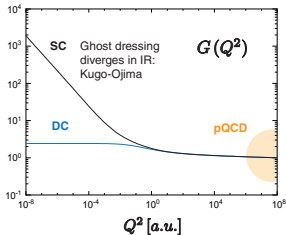
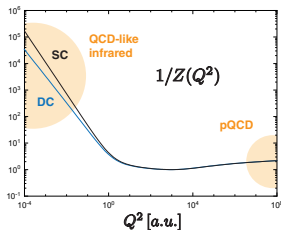
Mass generation for gluons?

Options:

- Only DC solution is physical? (Lattice, PT-BFM)
- Only SC solution is physical? (Kugo-Ojima, $1/Q^4$)
- All solutions are physically equivalent? (Hadron physics, $T > 0$, ...)

Questions:

- What actually distinguishes SC and DC?
- Where does the singularity in $1/Z$ come from?
- How is a gluon “mass” compatible with gauge invariance?



Summary

- Yang-Mills DSEs admit **family of solutions**, which depend on two parameters α, β

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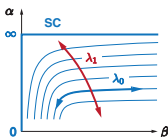
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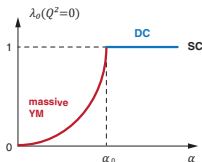
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- Mass generation triggered by **longitudinal massless poles** in vertices



- Ghost-gluon vertex has longitudinal massless pole \Rightarrow **SC solution consistent**
- Does not exclude consistency of DC solutions, if longitudinal poles in three-gluon vertex (PT-BFM)



Then presumably **all** solutions are physically equivalent (?)