Central exclusive diffractive production of axial-vector f_1 mesons in proton-proton collisions

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Contents

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Introduction | Motivation

- Central exclusive production (CEP) of axial-vector mesons $f_1(1285)$ and $f_1(1420)$ ($J^{PC} = 1^{++}$) was studied in proton-proton collisions by the WA102 Collaboration for c.m. energies 12.7 GeV and 29.1 GeV
  - *D. Barberis et al. (WA102 Collaboration), PLB 413 (1997) 217; PLB 413 (1997) 225; PLB 440 (1998) 225*
  - *A. Kirk (WA102 Collaboration), Nucl. Phys. A663 (2000) 608*

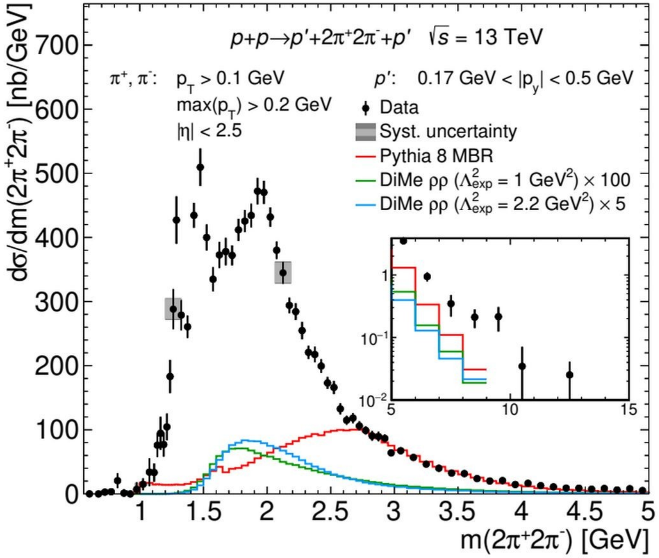
  The branching fractions of both mesons in all major decay modes were determined.
  $f_1(1285)$ was found to decay to $\eta\pi^+\pi^-$, 4π, $K\bar{K}\pi$, and $\rho\gamma$
  while $f_1(1420)$ to decay dominantly to $K\bar{K}\pi$, including $K^*(892)\bar{K}$ + *c.c.*

  - *D. Barberis et al. (WA102 Collaboration), PLB 471 (2000) 440,* arXiv:hep-ex/9912005

  A spin analysis of the 4π channel were performed and a clear peak associated with the $f_1(1285)$ meson in the $J^P = 1^+$ $\rho\rho$ wave was observed.
  There the $f_0(1370)$ was found to decay dominantly to $\rho\rho$ while the $f_0(1500)$ is found to decay to $\rho\rho$ and $\sigma\sigma$. The observed there $f_0(1500)$ and $f_2(1950)$ states are candidates to be scalar and tensor glueball, respectively.

- Current experiments (STAR, CMS, ATLAS, LHCb, ALICE) measured various central (exclusive or semiexclusive) hadronic systems, like $\pi^+\pi^-$, K^+K^-, $p\bar{p}$ and 4π.
  Double Pomeron Exchange processes are suitable to study low-mass resonances and to search for glueballs, because they provide a gluon-rich environment.

  Preliminary studies of CEP of the $\pi^+\pi^-\pi^+\pi^-$ channel, ATLAS-ALFA @ 13 TeV →



Preliminary ATLAS-ALFA data
Courtesy of R. Sikora,
CERN-THESIS-2020-235

- Experimental studies of single meson CEP reactions will give many information on pomeron-pomeron-meson (IP IP M) couplings. Their theoretical calculation is a challenging problem of nonperturbative QCD.

Introduction | Motivation

- What is the nature of f_1 mesons?
  The $f_1(1285)$ and the $f_1(1420)$ are well known experimentally but their internal structure (for instance, normal $q\bar{q}$ state, tetraquark or $\bar{K}K^*$ molecule) remains to be established

  - information from WA102 → both mesons are suppressed at small values of "glueball-filter variable"

    $$\boldsymbol{dP_t} = \boldsymbol{q_{t,1}} - \boldsymbol{q_{t,2}} = \boldsymbol{p_{t,2}} - \boldsymbol{p_{t,1}}\,, \quad \mathrm{dP_t} = |\boldsymbol{dP_t}|$$

    this behavior is consistent with the signals being due to standard $q\bar{q}$ states
    [F. Close, A. Kirk, Z. Phys. C76 (1997) 469; PLB397 (1997) 333]

  - analysis of the $f_1(1285) \to \rho\pi^+\pi^-$ decay mode in Nambu—Jona-Lasinio model favors a $q\bar{q}$ content of the $f_1(1285)$ [Osipov, Pivovarov, Volkov, PRD98 (2018) 014037], but a glue component for the $f_1(1285)$ is not excluded
    [Birkel, Fritzch, PRD53 (1996) 6195; Moreira, Silva, Nucl. Phys. A992 (2019) 121642]

  - the study done by Oset et al., PRD95 (2017) 034015, EPJC80 (2020) 407 proposes that the $f_1(1420)$ may not be a genuine $q\bar{q}$ resonance, but the manifestation of the $K^*(892)\bar{K}$ and $\pi a_0(980)$ decay modes of the $f_1(1285)$ resonance around 1420 MeV

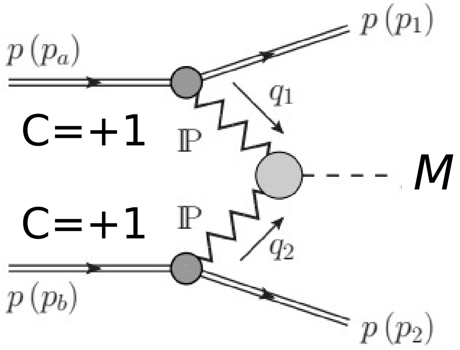
  In this talk we shell treat the $f_1(1285)$ and $f_1(1420)$ as separate objects, we can say, as two effective resonances

- What is underlying production mechanism for studies of f_1 CEP at high energies?

  We will be concerned with diffractive production of axial-vector f_1 mesons via IPIP-fusion mechanism.
  We shall discuss two ways to derive the nonperturbative IPIPf_1 couplings.
  We will try to describe the data measured by the WA102 Collaboration and provide predictions for the LHC experiments.

The tensor-pomeron concept for soft reactions in QCD

At high energies double pomeron exchange (DPE) is dominant production mechanism of resonances



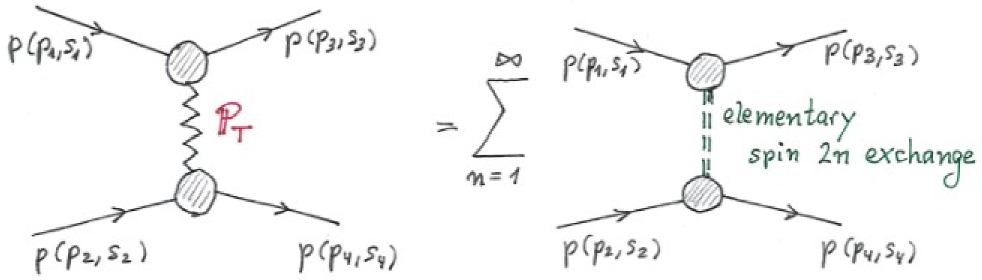
$I^G = 0^+$

$J^{PC} = 0^{-+}$ e.g., η, $\eta'(958)$
0^{++} $f_0(980)$, $f_0(1370)$, $f_0(1500)$
1^{++} $f_1(1285)$, $f_1(1420)$
2^{++} $f_2(1270)$, $f_2'(1525)$, $f_2(1950)$
4^{++} $f_4(2050)$
(…) and glueballs !

We treat soft reactions in the tensor-pomeron approach [Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31].
The pomeron and the C=+1 reggeons are described as effective rank 2 symmetric tensor exchanges,
the odderon and the C= -1 reggeons are described as effective vector exchanges.

This approach has a good basis from nonperturbative QCD considerations [Nachtmann, Ann.Phys.209 (1991) 436].
The IP exchange can be understood as a coherent sum of elementary spin 2+4+6+…exchanges.
Note that spin 0 exchange is missing!

A tensor character of the pomeron is also preferred
in holographic QCD, see e.g.,
Brower, Polchinski, Strassler, Tan, JHEP 12 (2007) 005
Domokos, Harvey, Mann, PRD 80 (2009) 126015
Iatrakis, Ramamurti, Shuryak, PRD 94 (2016) 045005



Applications of the model

- **Helicity in proton-proton elastic scattering and the spin structure of the soft pomeron**

Ewerz, P.L., Nachtmann, Szczurek, PLB 763 (2016) 382

Studying the ratio r_5 of single-helicity-flip to non-flip amplitudes we found that the STAR data are compatible with the tensor pomeron ansatz while they exclude a scalar character of the pomeron (the scalar-pomeron result is far outside the experimental error ellipse).

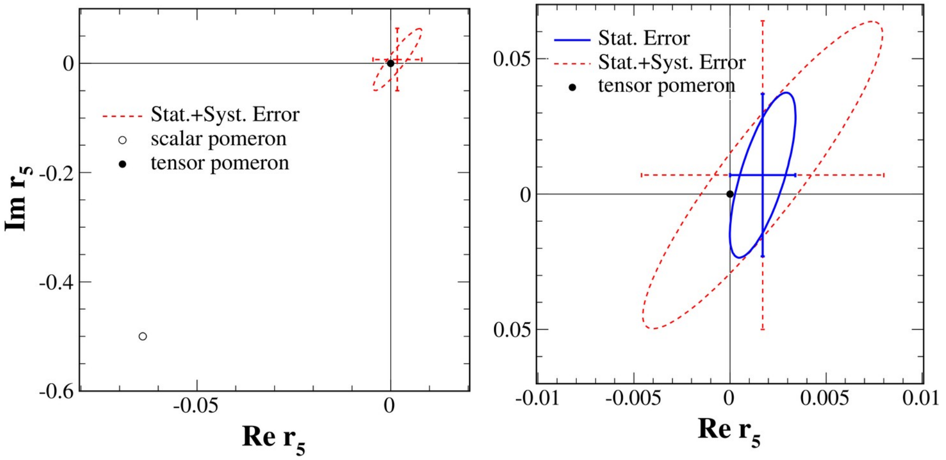
$$r_5(s,t) = \frac{2m_p\,\phi_5(s,t)}{\sqrt{-t}\,\mathrm{Im}[\phi_1(s,t)+\phi_3(s,t)]}$$

$$r_5^{I\!P_T}(s,t) = -\frac{m_p^2}{s}\left[i+\tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right],\; r_5^{I\!P_T}(s,0) = (-0.28 - i2.20)\times 10^{-5}$$

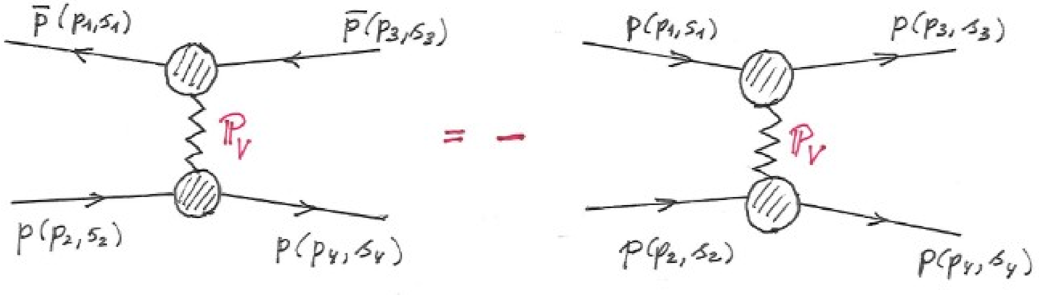
$$r_5^{I\!P_S}(s,t) = -\frac{1}{2}\left[i+\tan\left(\frac{\pi}{2}(\alpha_{I\!P}(t)-1)\right)\right],\; r_5^{I\!P_S}(s,0) = -0.064 - i0.500$$

STAR data: Adamczyk *et al.*, PLB 719 (2013) 62
[single spin asymmetry A_N in polarised pp → pp]

$$\sqrt{s} = 200\ \mathrm{GeV},\quad 0.003 \leqslant |t| \leqslant 0.035\ \mathrm{GeV}^2$$



Problem with the vector pomeron:



$$\sigma_{tot}^{pp} = \frac{1}{2\sqrt{s(s-4m_p^2)}}\,\mathrm{Im}\left[\phi_1(s,0)+\phi_3(s,0)\right]$$

Vector exchange has C = -1.
It follows

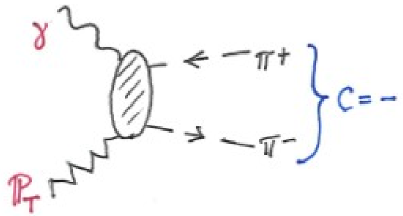
$$\sigma_{tot}^{\bar{p}p}|_{I\!P_V} = -\sigma_{tot}^{pp}|_{I\!P_V}$$

In our opinion a vector pomeron is not a viable option.

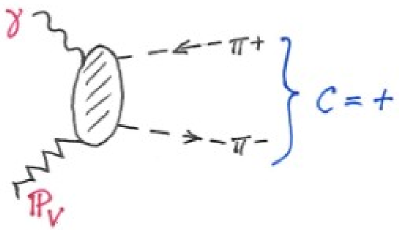
Applications of the model

- **Photoproduction and low *x* DIS** *Britzger, Ewerz, Glazov, Nachtmann, Schmitt, PRD100 (2019) 114007*
  “vector IP” decouples completely in the total photoabsorption cross section and in the structure functions of DIS

- $\boldsymbol{\gamma\, p \to \pi^+ \pi^- p}$ *Bolz, Ewerz, Maniatis, Nachtmann, Sauter, Schöning, JHEP 01 (2015) 151*
  interference betwenn $\gamma p \to (\rho^0 \to \pi^+\pi^-)p$ (pomeron exch.) and $\gamma p \to (f_2(1270) \to \pi^+\pi^-)p$ (odderon exch.) → $\pi^+\pi^-$ charge asymmetries



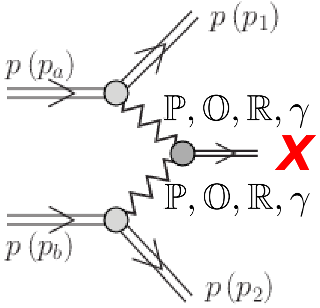
$\pi^+\pi^-$ in antisymmetric state



$\pi^+\pi^-$ in symmetric state

For a tensor (vector) pomeron the $\pi^+\pi^-$ pair is in antisymmetric (symmetric) state under the exchange $\pi^+ \longleftrightarrow \pi^-$. Since the pomeron has C = +1 the $\pi^+\pi^-$ pair must be in antisymmetric state. This gives a further clear evidence against a vector nature of the pomeron.

- **Central Exclusive Production (CEP),** $p\, p \to p\, p$ **X**, *P.L., Nachtmann, Szczurek:*



X: η, η', f_0 — *Ann. Phys. 344 (2014) 301*
ρ^0 — *PRD91 (2015) 074023*
$\pi^+\pi^-$ continuum, $f_2(1270) \to \pi^+\pi^-$ — *PRD93 (2016) 054015, PRD101 (2020) 034008*
$\pi^+\pi^-\pi^+\pi^-$, $\rho^0\rho^0$ — *PRD94 (2016) 034017*
ρ^0 with proton diss. — *PRD95 (2017) 034036*
$p\bar{p}$ — *PRD97 (2018) 094027*
K^+K^- — *PRD98 (2018) 014001*
$\phi \to K^+K^-, \mu^+\mu^-$ — *PRD101 (2020) 094012*
$\phi\phi \to K^+K^-K^+K^-$ — *PRD99 (2019) 094034*
odderon exchange
$f_1(1285)$, $f_1(1420)$ P.L., Leutgeb, Nachtmann, Rebhan, Szczurek, PRD102 (2020) 114003
$K^{*0}\bar{K}^{*0}$ continuum vs $f_2(1950)$ — *P.L., PRD103 (2021) 054039*

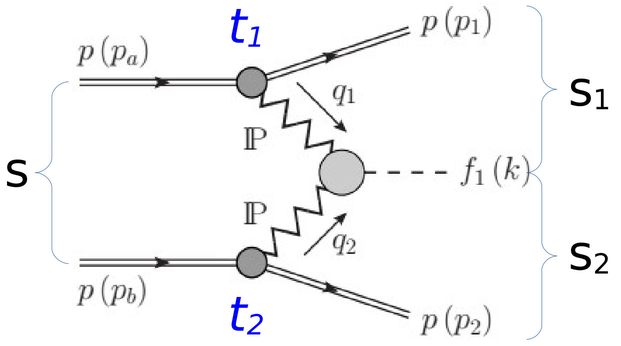
CEP of f_1 mesons in proton-proton collisions | Born-level amplitude

$$p(p_a,\lambda_a)+p(p_b,\lambda_b)\to p(p_1,\lambda_1)+f_1(k,\lambda)+p(p_2,\lambda_2)$$

where $p_{a,b}$, $p_{1,2}$ and $\lambda_{a,b}$, $\lambda_{1,2}=\pm\frac{1}{2}$ denote the four-momenta and helicities of the protons, and k and $\lambda=0,\pm1$ denote the four-momentum and helicity of the f_1 meson, respectively.

Kinematic variables:

$$q_1=p_a-p_1,\quad q_2=p_b-p_2,\quad k=q_1+q_2,$$
$$t_1=q_1^2,\quad t_2=q_2^2,\quad m_{f_1}^2=k^2,$$
$$s=(p_a+p_b)^2=(p_1+p_2+k)^2,$$
$$s_1=(p_a+q_2)^2=(p_1+k)^2,$$
$$s_2=(p_b+q_1)^2=(p_2+k)^2$$



The Born-level IPIP-fusion amplitude can be written as

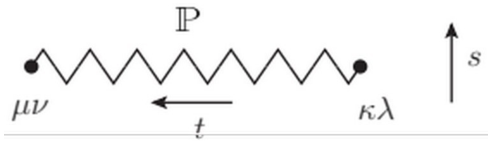
$$\begin{aligned}\mathcal{M}^{\rm Born}_{\lambda_a\lambda_b\to\lambda_1\lambda_2\lambda} \;=\;& (-i)\,(\epsilon^{\mu}(k,\lambda))^{*}\,\bar{u}(p_1,\lambda_1) i\Gamma^{(I\!Ppp)}_{\mu_1\nu_1}(p_1,p_a)u(p_a,\lambda_a)\\ &\times i\Delta^{(I\!P)\,\mu_1\nu_1,\alpha_1\beta_1}(s_1,t_1)\, i\Gamma^{(I\!PI\!Pf_1)}_{\alpha_1\beta_1,\alpha_2\beta_2,\mu}(q_1,q_2)\, i\Delta^{(I\!P)\,\alpha_2\beta_2,\mu_2\nu_2}(s_2,t_2)\\ &\times\bar{u}(p_2,\lambda_2) i\Gamma^{(I\!Ppp)}_{\mu_2\nu_2}(p_2,p_b)u(p_b,\lambda_b)\end{aligned}$$

in terms of the polarisation vector of the f_1 meson, the propagator and effective pomeron-proton vertex functions for the tensor-pomeron exchange, and the pomeron-pomeron-f_1 vertex.

Propagator and effective IPNN vertex

- Propagator for tensor-pomeron exchange

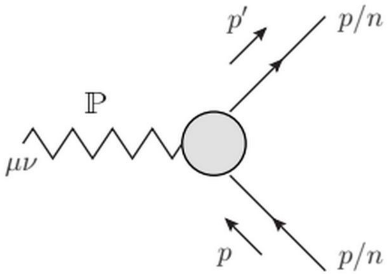
[Ewerz, Maniatis, Nachtmann, Ann. Phys. 342 (2014) 31]

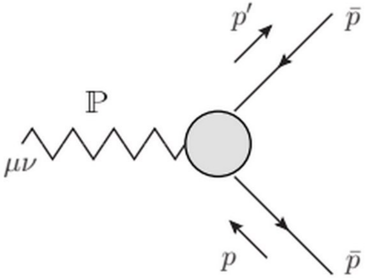


$$i\Delta^{(I\!P)}_{\mu\nu,\kappa\lambda}(s,t) = \frac{1}{4s}\left(g_{\mu\kappa}g_{\nu\lambda} + g_{\mu\lambda}g_{\nu\kappa} - \frac{1}{2}g_{\mu\nu}g_{\kappa\lambda}\right)(-is\alpha'_{I\!P})^{\alpha_{I\!P}(t)-1}$$

$$\alpha_{I\!P}(t) = \alpha_{I\!P}(0) + \alpha'_{I\!P}t\,, \quad \alpha_{I\!P}(0) = 1.0808, \quad \alpha'_{I\!P} = 0.25\,\mathrm{GeV}^{-2}$$ ← linear pomeron trajectory

- Effective pomeron-nucleon (IP N N) vertex





$$i\Gamma^{(I\!P pp)}_{\mu\nu}(p',p) = i\Gamma^{(I\!P \bar{p}\bar{p})}_{\mu\nu}(p',p) = -i3\beta_{I\!P NN}F_1\big((p'-p)^2\big)\left\{\frac{1}{2}[\gamma_\mu(p'+p)_\nu + \gamma_\nu(p'+p)_\mu] - \frac{1}{4}g_{\mu\nu}(\not{p}'+\not{p})\right\}$$

$$\beta_{I\!P NN} = 1.87\ \mathrm{GeV}^{-1}, \qquad F_1(t):\ \text{Dirac form factor of the proton}$$

$$F_1(0) = 1$$

The IP IP f1 coupling

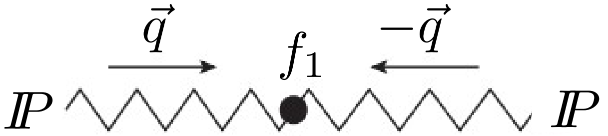
We follow two strategies for constructing coupling Lagrangian (vertex function) $\left\{ \begin{array}{l} \mathcal{L}^{(I\!\!P I\!\!P f_1)} \\ i\Gamma^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\text{bare}} \end{array} \right.$

(1) Phenomenological approach.

First we consider a fictitious process: the fusion of two "real spin 2 pomerons" of mass m giving f_1 meson

$$I\!\!P(m,\epsilon_1) + I\!\!P(m,\epsilon_2) \to f_1(m_{f_1},\epsilon)$$

$\epsilon_{1,2}$: polarisation tensors, $\quad \epsilon$: polarisation vector



We work in the rest system of the f_1.

The spin 2 of these two "real pomerons" can be combined to a total spin S ($0 \leq S \leq 4$).

This has to be combined with the orbital angular momentum ℓ to give $J^{PC} = 1^{++}$, the quantum numbers of f_1.

There are exactly two possibilities: $(\ell,S) = (2,2)$ and $(4,4)$ [see Ann. Phys. 344 (2014) 301]

The ℓ values of the couplings should be reflected by ℓ derivatives.

Corresponding covariant couplings are:

$$\mathcal{L}^{(2,2)}_{I\!\!P I\!\!P f_1} = \frac{g'_{I\!\!P I\!\!P f_1}}{32\,M_0^2}\left(I\!\!P_{\kappa\lambda}\,\overleftrightarrow{\partial}_\mu \overleftrightarrow{\partial}_\nu\, I\!\!P_{\rho\sigma}\right)\left(\partial_\alpha U_\beta - \partial_\beta U_\alpha\right)\Gamma^{(8)\,\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}$$

$$\mathcal{L}^{(4,4)}_{I\!\!P I\!\!P f_1} = \frac{g''_{I\!\!P I\!\!P f_1}}{24\times 32\,M_0^4}\left(I\!\!P_{\kappa\lambda}\,\overleftrightarrow{\partial}_{\mu_1} \overleftrightarrow{\partial}_{\mu_2} \overleftrightarrow{\partial}_{\mu_3} \overleftrightarrow{\partial}_{\mu_4}\, I\!\!P_{\rho\sigma}\right)\left(\partial_\alpha U_\beta - \partial_\beta U_\alpha\right)\Gamma^{(10)\,\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta}$$

Here $M_0 \equiv 1$ GeV, $g'_{I\!\!P I\!\!P f_1}, g''_{I\!\!P I\!\!P f_1}$: dimensionless coupling parameters,

$I\!\!P_{\kappa\lambda}$ effective pomeron field satisfies $I\!\!P_{\kappa\lambda} = I\!\!P_{\lambda\kappa}$ and $g^{\kappa\lambda} I\!\!P_{\kappa\lambda} = 0$,

U_α is f_1 meson field, $\overleftrightarrow{\partial}_\mu = \overrightarrow{\partial}_\mu - \overleftarrow{\partial}_\mu$ asymmetric derivative,

and $\Gamma^{(8)}, \Gamma^{(10)}$ are known tensor functions.

The IP IP f1 coupling | Quantum numbers

l – orbital angular momentum
S – total spin, we have $S \in \{0, 1, 2, 3, 4\}$
J – total angular momentum (spin of the produced meson)
P – parity of meson
and Bose symmetry requires $l - S$ to be even

l	S	$\|l-S\| \leqslant J \leqslant l+S$	$P = (-1)^l$
0	0	0	+
	2	2	
	4	4	
1	1	0, 1, 2	−
	3	2, 3, 4	
2	0	2	+
	2	0,1,2,3,4	
	4	2,3,4,5,6	
3	1	2,3,4	−
	3	0,1,2,3,4,5,6	
4	0	4	+
	2	2,3,4,5,6	
	4	0,1,2,3,4,5,6,7,8	
5	1	4.5.6	−
	3	2,3,4,5,6,7,8	
6	0	6	+
	2	4,5,6,7,8	
	4	2,3,4,5,6,7,8,9,10	

The lowest (ℓ,S) term for a scalar meson $J^{PC} = 0^{++}$ is (0,0),
for pseudoscalar meson $J^{PC} = 0^{-+}$ is (1,1),
for a tensor meson $J^{PC} = 2^{++}$ is (0,2),
for axial-vector meson $J^{PC} = 1^{++}$ is (2,2).

For each value of ℓ, S, J, and P we can construct a covariant Lagrangian $\mathcal{L}^{(I\!PI\!PM)}$ coupling.

In general more than one coupling term is needed for the description of experimental results.

The rank-8 tensor function satisfies the following relations:

$$\begin{aligned}\Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta} \;=\;& g_{\kappa\rho}g_{\mu\sigma}\varepsilon_{\lambda\nu\alpha\beta} + g_{\lambda\rho}g_{\mu\sigma}\varepsilon_{\kappa\nu\alpha\beta} + g_{\kappa\sigma}g_{\mu\rho}\varepsilon_{\lambda\nu\alpha\beta} + g_{\lambda\sigma}g_{\mu\rho}\varepsilon_{\kappa\nu\alpha\beta}\\ &+g_{\kappa\rho}g_{\mu\lambda}\varepsilon_{\sigma\nu\alpha\beta} + g_{\sigma\kappa}g_{\mu\lambda}\varepsilon_{\rho\nu\alpha\beta} + g_{\rho\lambda}g_{\mu\kappa}\varepsilon_{\sigma\nu\alpha\beta} + g_{\sigma\lambda}g_{\mu\kappa}\varepsilon_{\rho\nu\alpha\beta}\\ &-g_{\kappa\lambda}g_{\mu\rho}\varepsilon_{\sigma\nu\alpha\beta} - g_{\kappa\lambda}g_{\mu\sigma}\varepsilon_{\rho\nu\alpha\beta} - g_{\kappa\mu}g_{\rho\sigma}\varepsilon_{\lambda\nu\alpha\beta} - g_{\lambda\mu}g_{\rho\sigma}\varepsilon_{\kappa\nu\alpha\beta}\\ &+(\mu\leftrightarrow\nu)\end{aligned}$$

$$\Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta} = \Gamma^{(8)}_{\lambda\kappa,\rho\sigma,\mu\nu,\alpha\beta} = \Gamma^{(8)}_{\kappa\lambda,\sigma\rho,\mu\nu,\alpha\beta} = \Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\nu\mu,\alpha\beta} = \Gamma^{(8)}_{\rho\sigma,\kappa\lambda,\mu\nu,\alpha\beta} = -\Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\beta\alpha}\,,$$

$$\Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}\,g^{\kappa\lambda} = 0\,,\quad \Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}\,g^{\rho\sigma} = 0\,,\quad \Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}\,g^{\mu\nu} = 0$$

For the Levi-Civita symbol we use the normalisation $\varepsilon_{0123} = +1$.

The rank-10 tensor function has the following properties:

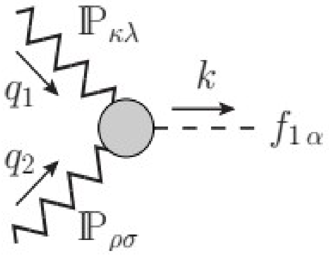
$$\begin{aligned}\Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} \;=\;& \Big\{\Big[\Big(g_{\kappa\mu_1}g_{\lambda\mu_2} - \frac{1}{4}g_{\kappa\lambda}g_{\mu_1\mu_2}\Big)\Big(g_{\rho\mu_3}\varepsilon_{\sigma\mu_4\alpha\beta} - \frac{1}{4}g_{\rho\sigma}\varepsilon_{\mu_3\mu_4\alpha\beta}\Big)\\ &+(\kappa\leftrightarrow\lambda) + (\rho\leftrightarrow\sigma) + (\kappa\leftrightarrow\lambda,\rho\leftrightarrow\sigma)\Big] + (\kappa,\lambda)\leftrightarrow(\rho,\sigma)\Big\}\\ &+ \text{all permutation of } \mu_1,\mu_2,\mu_3,\mu_4\end{aligned}$$

$$\Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} = \Gamma^{(10)}_{\lambda\kappa,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} = \Gamma^{(10)}_{\kappa\lambda,\sigma\rho,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} = \Gamma^{(10)}_{\rho\sigma,\kappa\lambda,\mu_1\mu_2\mu_3\mu_4,\alpha\beta} = -\Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\beta\alpha}\,,$$

$\Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta}$ is totally symmetric in μ_1,μ_2,μ_3,μ_4 ,

$$\Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta}\,g^{\kappa\lambda} = 0\,,\quad \Gamma^{(10)}_{\kappa\lambda,\rho\sigma,\mu_1\mu_2\mu_3\mu_4,\alpha\beta}\,g^{\rho\sigma} = 0$$

From the coupling Lagrangians we obtain the following (bare) IP IP f1 vertices



$k = q_1 + q_2$

(ℓ,S) = (2,2): $$i\Gamma'^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} = -\frac{g'_{I\!\!P I\!\!P f_1}}{8\,M_0^2}(q_1-q_2)^{\mu}(q_1-q_2)^{\nu}k^{\beta}\,\Gamma^{(8)}_{\kappa\lambda,\rho\sigma,\mu\nu,\alpha\beta}$$

(ℓ,S) = (4,4): $$i\Gamma''^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} = \frac{g''_{I\!\!P I\!\!P f_1}}{4\,M_0^4}(q_1-q_2)^{\mu_1}(q_1-q_2)^{\mu_2}(q_1-q_2)^{\mu_3}(q_1-q_2)^{\mu_4}k^{\beta} \times\Big[\big(g_{\kappa\mu_1}g_{\lambda\mu_2}-\frac{1}{4}g_{\kappa\lambda}g_{\mu_1\mu_2}\big)\big(g_{\rho\mu_3}\varepsilon_{\sigma\mu_4\alpha\beta}+g_{\sigma\mu_3}\varepsilon_{\rho\mu_4\alpha\beta}\big)+(\kappa,\lambda)\leftrightarrow(\rho,\sigma)\Big]$$

The values of the coupling constants in this approach are not known and are not easy to obtain from first principles of QCD, as they are of nonperturbative origin.

For the *f₁* CEP reaction we should multiply the “bare” vertices by a form factor:

$$i\Gamma^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2) = \Big(i\Gamma'^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} + i\Gamma''^{(I\!\!P I\!\!P f_1)}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare}\Big)\,\tilde{F}^{(I\!\!P I\!\!P f_1)}(q_1^2,q_2^2,k^2)$$

For the on-shell meson we have set $k^2 = m_{f_1}^2$, $\tilde{F}^{(I\!\!P I\!\!P f_1)}(0,0,m_{f_1}^2) = 1$

➡ $$\tilde{F}^{(I\!\!P I\!\!P f_1)}(t_1,t_2,m_{f_1}^2) = F_M(t_1)F_M(t_2), \quad F_M(t) = \frac{1}{1-t/\Lambda_0^2}, \quad \Lambda_0^2 = 0.5\ \mathrm{GeV}^2$$

➡ $$\tilde{F}^{(I\!\!P I\!\!P f_1)}(t_1,t_2,m_{f_1}^2) = \exp\left(\frac{t_1+t_2}{\Lambda_E^2}\right)$$

where the cutoff constant Λ_E should be adjusted to experimental data

(2) Holographic QCD approach using the Sakai-Sugimoto model.
There, the $I\!P I\!P f_1$ coupling can be derived from the bulk Chern-Simons (CS) term requiring consistency of supergravity and the gravitational anomaly.

$$\mathcal{L}^{\rm CS} = \varkappa' \, U_\alpha \, \varepsilon^{\alpha\beta\gamma\delta} \, I\!P^\mu{}_\beta \, \partial_\delta I\!P_{\gamma\mu} + \varkappa'' \, U_\alpha \varepsilon^{\alpha\beta\gamma\delta} \left(\partial_\nu P^\mu{}_\beta \right) \left(\partial_\delta \partial_\mu I\!P^\nu{}_\gamma - \partial_\delta \partial^\nu I\!P_{\gamma\mu} \right)$$

$$\varkappa' : \text{dimensionless}, \quad \varkappa'' : \text{dimension}\, \mathrm{GeV}^{-2}$$

- *Sakai, Sugimoto, Prog. Theor. Phys. 113 (2005) 843; 114 (2005) 1083*
- *Anderson, Domokos, Harvey, Mann, PRD90 (2014) 086010*
- *Leutgeb, Rebhan, PRD 101 (2020) 114015*

The $I\!P I\!P f_1$ vertex supplemented by suitable form factor is

$$i\Gamma^{(I\!P I\!P f_1)\,\rm CS}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2) = \left(i\Gamma'^{\rm CS}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} + i\Gamma''^{\rm CS}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} \right) \tilde{F}^{(I\!P I\!P f_1)}(q_1^2, q_2^2, k^2)$$

$$i\Gamma'^{\rm CS}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} = \varkappa' \, \varepsilon_{\alpha\beta\gamma\delta} \left(q_1^\delta \, g^{\kappa'\gamma} g^{\lambda'\rho'} g^{\sigma'\beta} + q_2^\delta \, g^{\kappa'\sigma'} g^{\lambda'\beta} g^{\rho'\gamma} \right) \tilde{R}_{\kappa\lambda\kappa'\lambda'} \, \tilde{R}_{\rho\sigma\rho'\sigma'}$$

$$i\Gamma''^{\rm CS}_{\kappa\lambda,\rho\sigma,\alpha}(q_1,q_2)\,|_{\rm bare} = \varkappa'' \, \varepsilon_{\alpha\lambda'\sigma'\delta} (q_1 - q_2)^\delta \left[q_{1\rho'} q_{2\kappa'} - (q_1 \cdot q_2) g_{\kappa'\rho'} \right] \tilde{R}_{\kappa\lambda}{}^{\kappa'\lambda'} \, \tilde{R}_{\rho\sigma}{}^{\rho'\sigma'}$$

$$\text{with } \tilde{R}_{\mu\nu\kappa\lambda} = \frac{1}{2} g_{\mu\kappa} g_{\nu\lambda} + \frac{1}{2} g_{\mu\lambda} g_{\nu\kappa} - \frac{1}{4} g_{\mu\nu} g_{\kappa\lambda}$$

The ratio between the two $I\!P I\!P f_1$ couplings in the Sakai-Sugimoto model is

$$\frac{\varkappa''}{\varkappa'} = -\frac{5.631}{M^2_{\rm KK}}$$

with a free parameter M_{KK} (Kaluza-Klein mass scale).

For the fictitious reaction $I\!P(q_1,\epsilon_1) + I\!P(q_2,\epsilon_2) \to f_1(k,\epsilon)\,, \quad q_1 + q_2 = k\,, \quad q_1^2 = q_2^2 = m^2$

with real pomerons there is strict equivalence $\mathcal{L}^{\rm CS} \,\widehat{=}\, \mathcal{L}^{(2,2)} + \mathcal{L}^{(4,4)}$

if the couplings satisfy: $g'_{I\!P I\!P f_1} = -\varkappa' \dfrac{M_0^2}{k^2} - \varkappa'' \dfrac{M_0^2(k^2 - 2m^2)}{2k^2}\,, \quad g''_{I\!P I\!P f_1} = \varkappa'' \dfrac{2M_0^4}{k^2}$

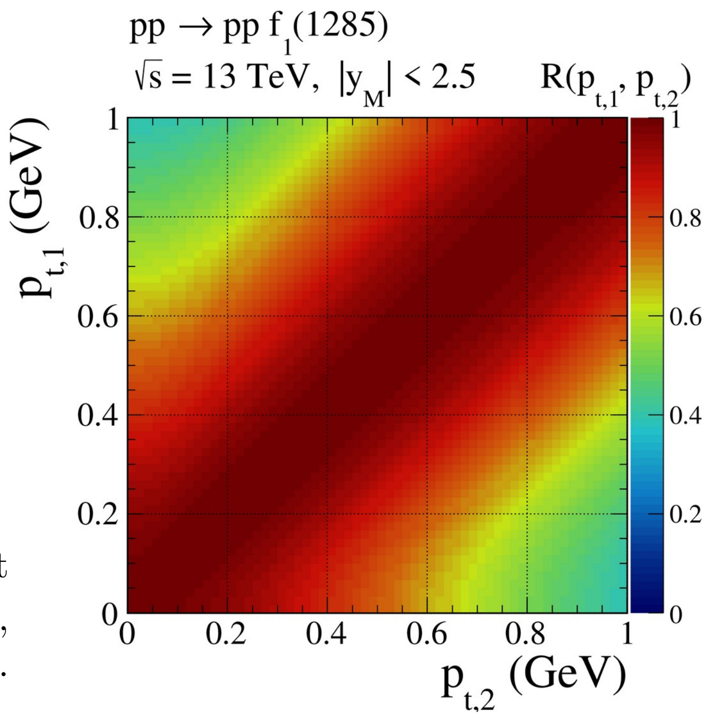
Here k^2 is invariant mass squared of the resonance f_1.

For the CEP reaction the pomerons have invariant mass squared t_1, $t_2 < 0$ instead of m^2 and, in general, $t_1 \neq t_2$. Replacing above $2m^2 \to t_1 + t_2$ we expect for small $|t_1|$ and $|t_2|$ still approximate equivalence to hold.

Plotted is the ratio

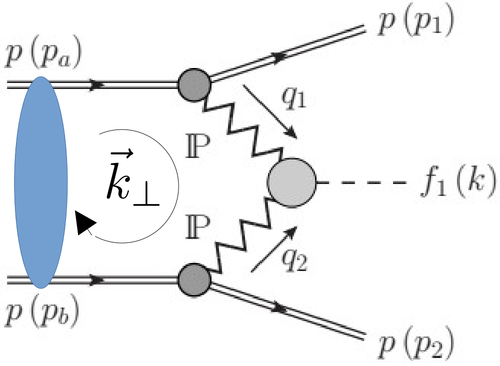
$$R(p_{t,1}, p_{t,2}) = \frac{d^2\sigma_{\varkappa'}/dp_{t,1}dp_{t,2}}{d^2\sigma_{(2,2)}/dp_{t,1}dp_{t,2}} \text{ for the pp} \to pp f_1(1285) \text{ reaction,}$$

for the case $g''_{f_1(1285)} = 0$, $\varkappa' = -g'_{f_1(1285)} \frac{m^2_{f_1(1285)}}{M_0^2}$. The ratio 1 occurs at $p_{t,1} = p_{t,2}$. In the limited range of transverse momenta of the outgoing protons, $p_{t,1} \lesssim 0.6$ GeV and $p_{t,2} \lesssim 0.6$ GeV, both approaches give similar contributions. But clear differences can be seen if one p_t is large and the other one is small. We note that by adjusting the $t_{1,2}$ dependent form factors we could, presumably, obtain the ratio $R(p_{t,1}, p_{t,2}) \sim 1$ for a larger range of $p_{t,1}$ and $p_{t,2}$.



No absorption effects were included here.

CEP of f_1 mesons in proton-proton collisions | Absorption



Absorption effects:

$$\mathcal{M}_{pp\to ppf_1} = \mathcal{M}^{\rm Born}_{pp\to ppf_1} + \mathcal{M}^{pp-{\rm rescattering}}_{pp\to ppf_1}$$

$$\mathcal{M}^{pp-{\rm rescattering}}_{pp\to ppf_1}(s,\vec{p}_{1\perp},\vec{p}_{2\perp}) = \frac{i}{8\pi^2 s}\int d^2\vec{k}_\perp \mathcal{M}^{\rm Born}_{pp\to ppf_1}(s,\vec{p}_{1\perp}-\vec{k}_\perp,\vec{p}_{2\perp}+\vec{k}_\perp)\,\mathcal{M}^{I\!P-{\rm exchange}}_{pp\to pp}(s,-\vec{k}^2_\perp)$$

where $\vec{k}_\perp$ is the transverse momentum carried around the loop

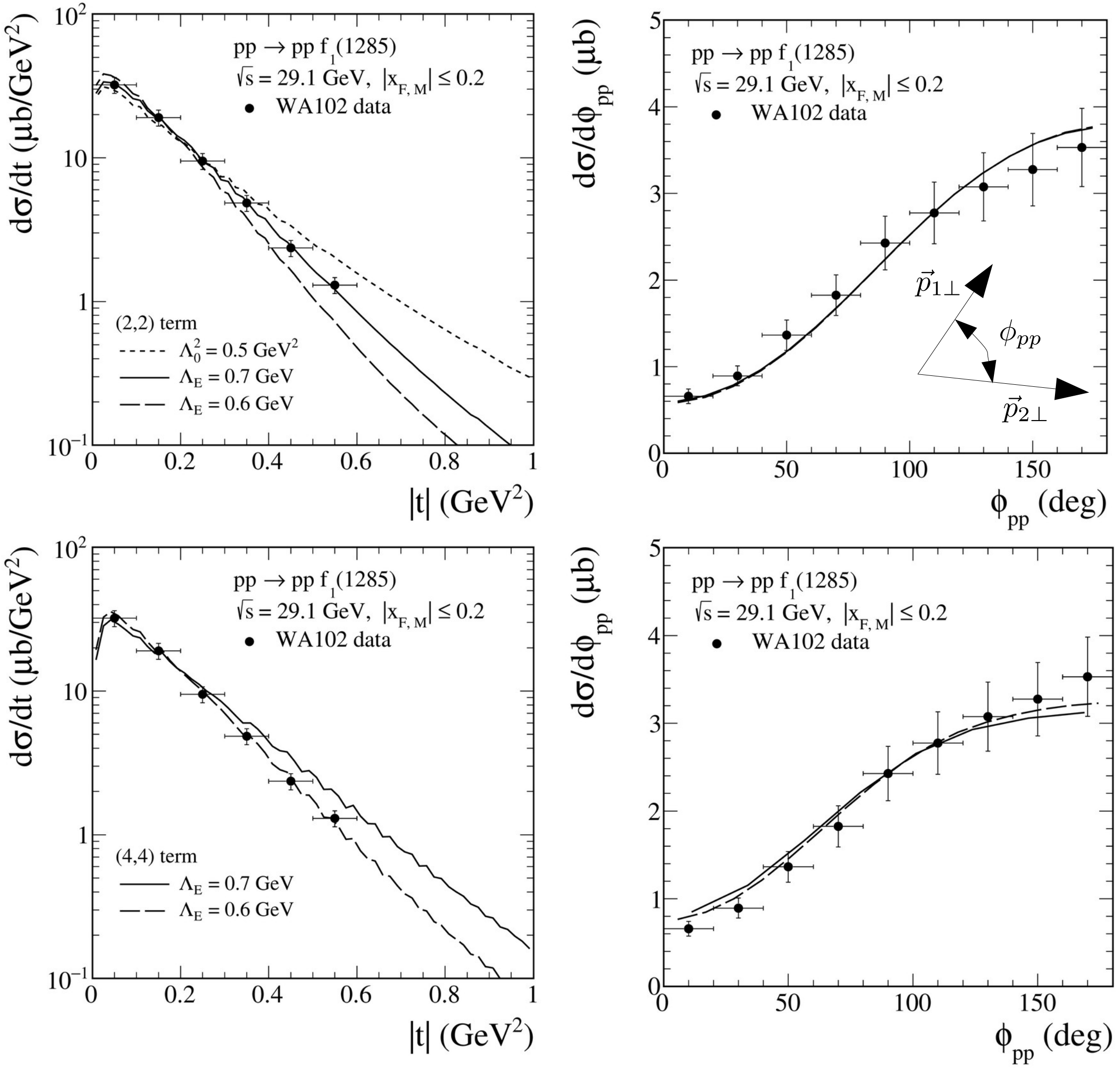
We include the absorptive corrections within the one-channel-eikonal approach.
In practice we work with the amplitudes in the high-energy approximation, i.e., assuming s-channel helicity conservation in IPNN vertices.

A more sophisticated absorption model was discussed in:

- Gotsman, Levin, Maor, PRD60 (1999) 094011
- Khoze, Martin, Ryskin, EPJC 18 (2000) 167, EPJC 24 (2002) 581, EPJC 73 (2013) 2503
- Petrov, Ryutin, Sobol, Guillaud, JHEP 06 (2005) 007
  Ryutin, EPJC 79 (2019) 12, 981

Comparison with experimental results from WA102@CERN

Phenomenological approach for CEP of $f_1(1285)$



Data: D. Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225

	$\sqrt{s} = 29.1$ GeV, $\lvert x_{F,M}\rvert \leqslant 0.2$
$f_1(1285)$	$\sigma_{\rm exp} = (6919 \pm 886)$ nb

The theoretical results and the WA102 data points have been normalised to the mean value of total cross section

← (ℓ,S) = (2,2) term only
$|g'_{I\!PI\!Pf_1}| = 4.89$

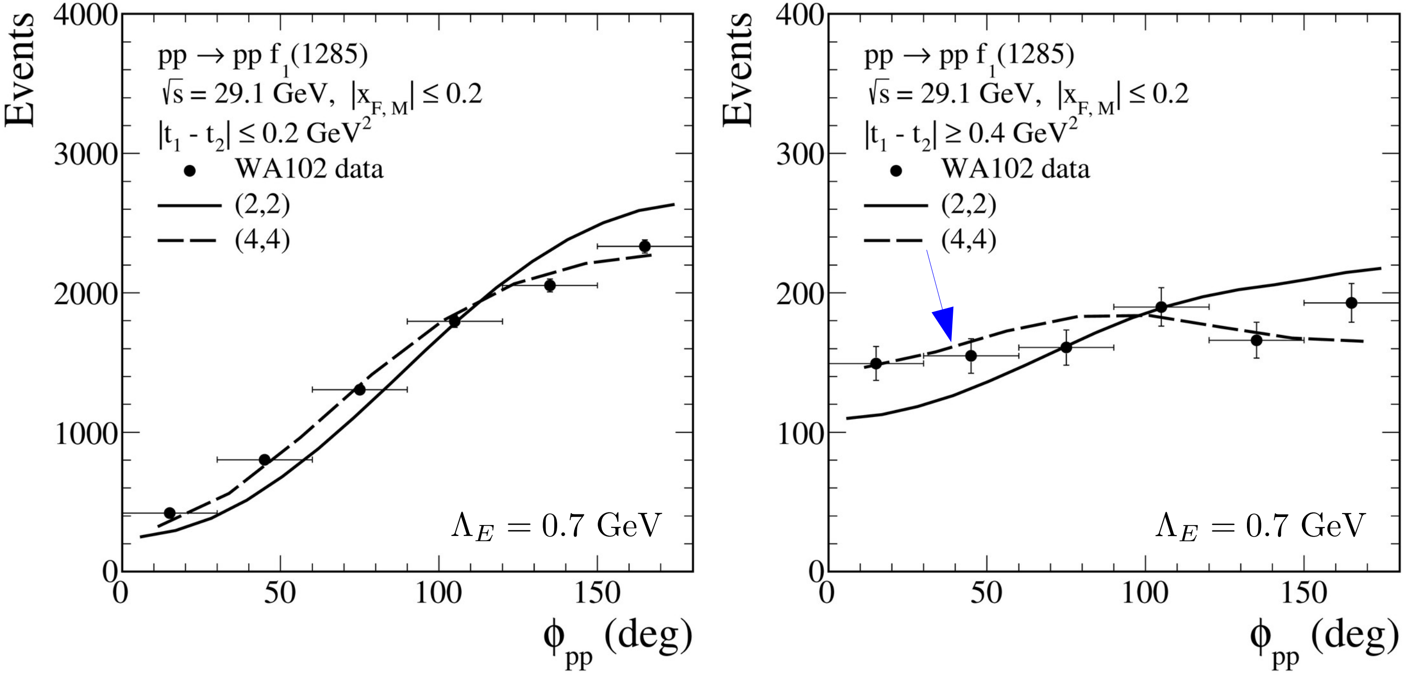
← (ℓ,S) = (4,4) term only
$|g''_{I\!PI\!Pf_1}| = 10.31$

- We get a good description of WA102 data with $\Lambda_E = 0.7$ GeV
- Absorption effects were included

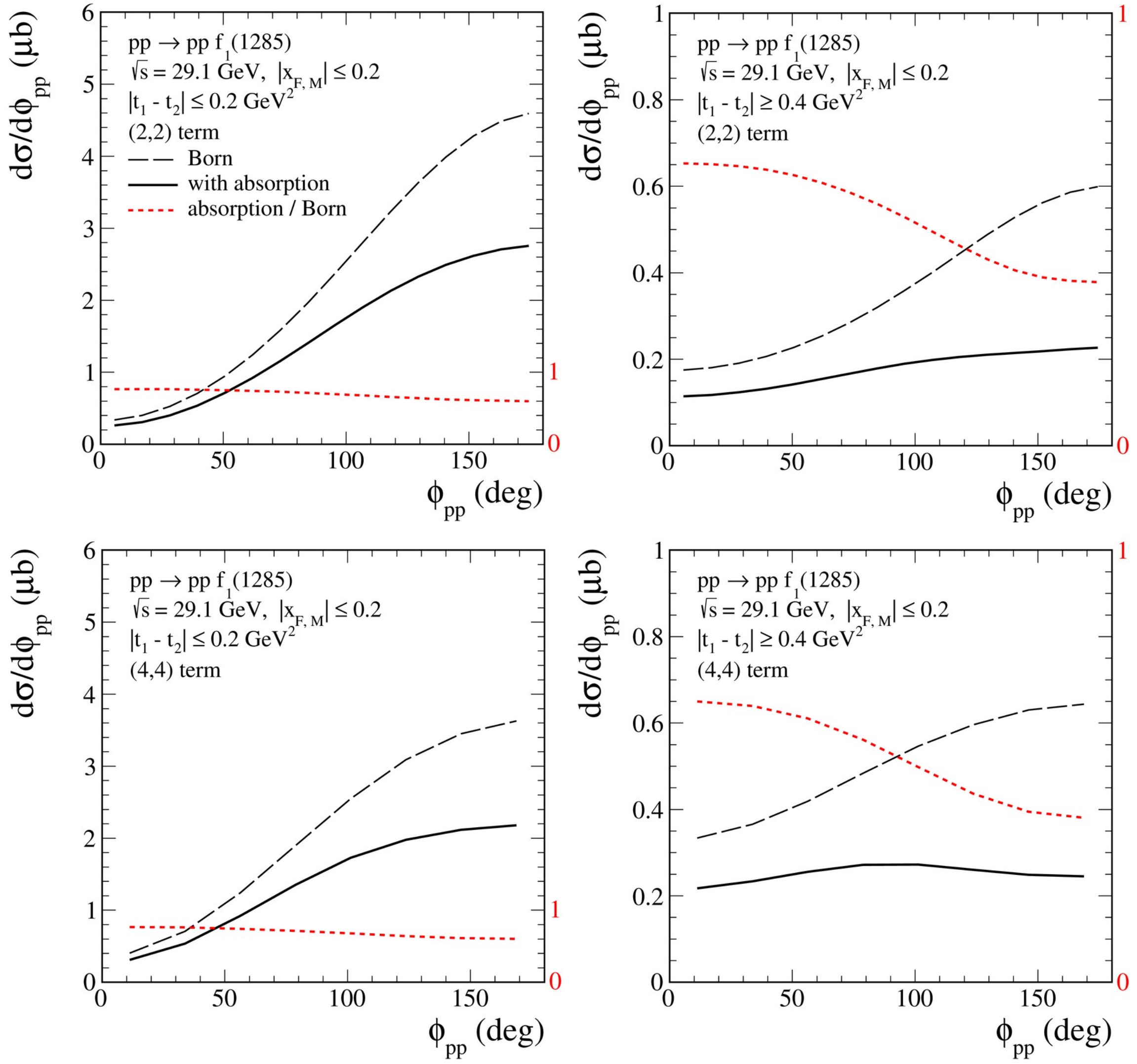
Comparison with experimental results from WA102@CERN

Data from: A. Kirk (WA102 Collaboration), Nucl. Phys. A 663 (2000) 608
The theoretical results have been normalised to the mean value of the number of events

Phenomenological approach for CEP of $f_1(1285)$



- An almost ‘flat’ distribution at large values of $|t_1 - t_2|$
  → absorption effects play a significant role there, large damping of cross section at higher values of ϕ_{pp}
- It seems that the $(\ell,S) = (4,4)$ term best reproduces the shape of the WA102 data



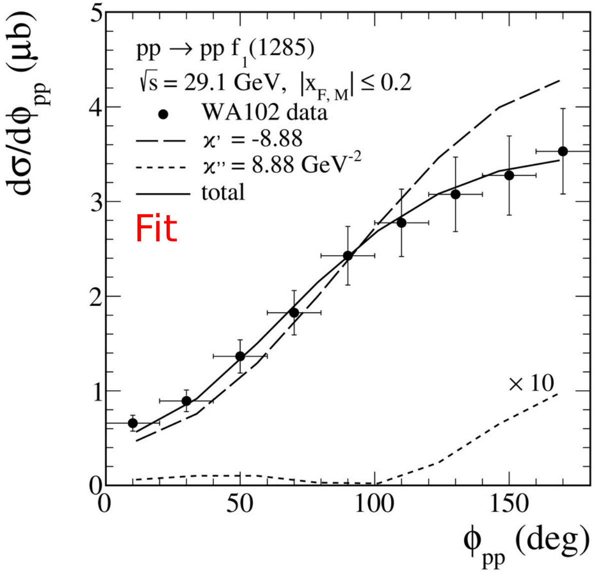
- The results for the two (ℓ,S) couplings are shown for different cuts on $|t_1 - t_2|$ without and with the absorption effects included in the calculations

  The long-dashed black lines represent the Born results and the solid black lines correspond to the results with the absorption effects included

  The dotted red lines represent the ratio of full and Born cross sections on the scale indicated by the red numbers on the r.h.s. of the panels

- We obtain the ratio of full and Born cross sections, the survival factor, as
  $\langle S^2 \rangle = \sigma_{abs}/\sigma_{Born} \approx 0.5 - 0.7$
  depending on the kinematics

- Larger damping of the cross section in back-to-back region ($\phi_{pp} \sim \pi$), especially for $|t_1 - t_2| > 0.4$ GeV^2



Holographic QCD approach for CEP of $f_1(1285)$

← Fit to WA102 data using the Chern-Simons (CS) coupling.

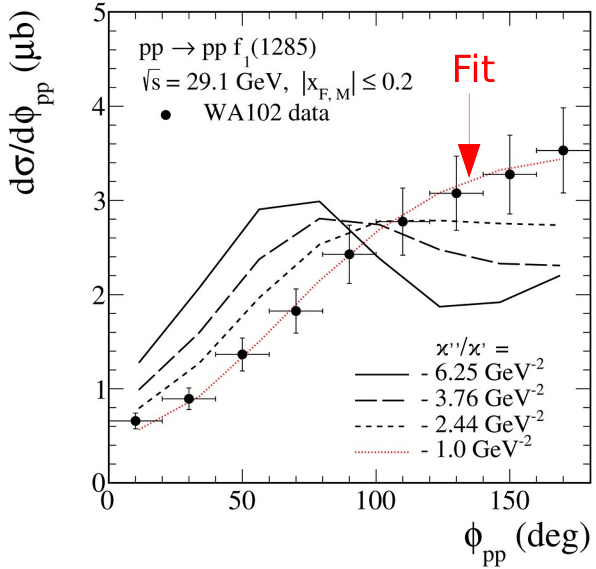
The relation between the (ℓ,S) and CS forms of the couplings:

With $\varkappa' = -8.88, \quad \varkappa''/\varkappa' = -1.0\ \mathrm{GeV}^{-2}$

and setting $t_1 = t_2 = -0.1\ \mathrm{GeV}^2$

we get: $g'_{I\!PI\!Pf_1} = 0.42, \quad g''_{I\!PI\!Pf_1} = 10.81$

The CS coupling corresponds practically to a pure (ℓ,S) = (4,4) coupling.



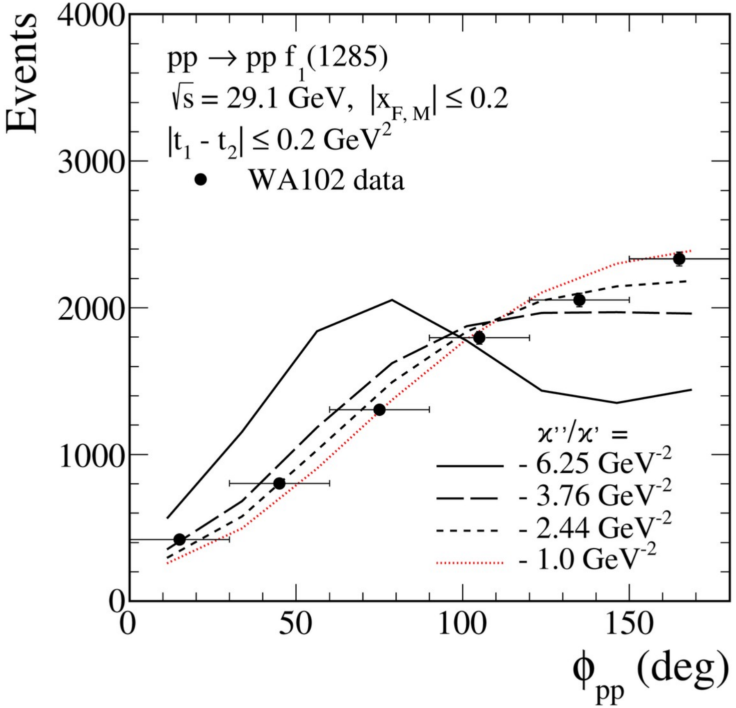
← The predictions for $\varkappa''/\varkappa'$ obtained in the Sakai-Sugimoto model:

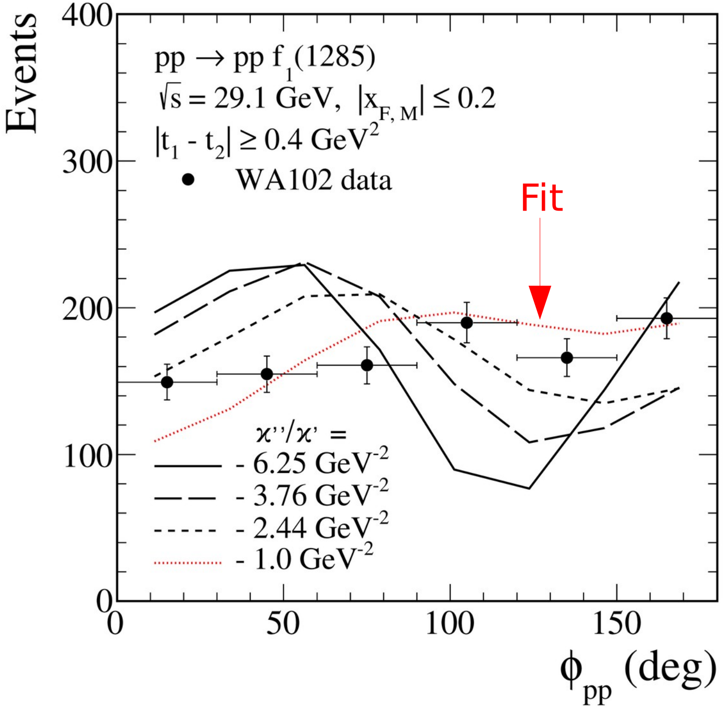
$$\varkappa''/\varkappa' = -5.631/M_{KK}^2 = -(6.25, 3.76, 2.44)\ \mathrm{GeV}^{-2}$$

$$\text{for}\quad M_{KK} = (949, 1224, 1519)\ \mathrm{MeV}$$

Usually M_{KK} (Kaluza-Klein mass scale) is fixed by matching the mass of the lowest vector meson to that of the physical ρ meson, leading to M_{KK} = 949 MeV.
However, this choice leads to tensor glueball mass which is too low, $M_T \approx 1.5$ GeV.
The standard pomeron trajectory corresponds to $M_T \approx 1.9$ GeV (M_{KK} = 1224 MeV),
whereas lattice gauge theory indicates $M_T \geq 2.4$ GeV (M_{KK} = 1519 MeV).

Holographic QCD approach for CEP of $f_1(1285)$

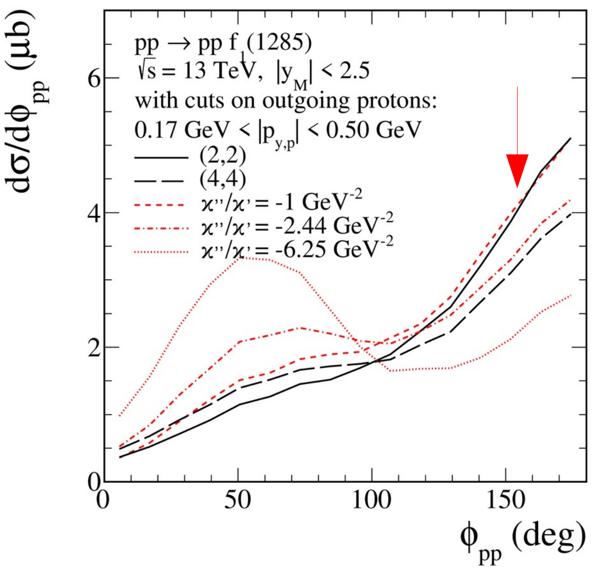


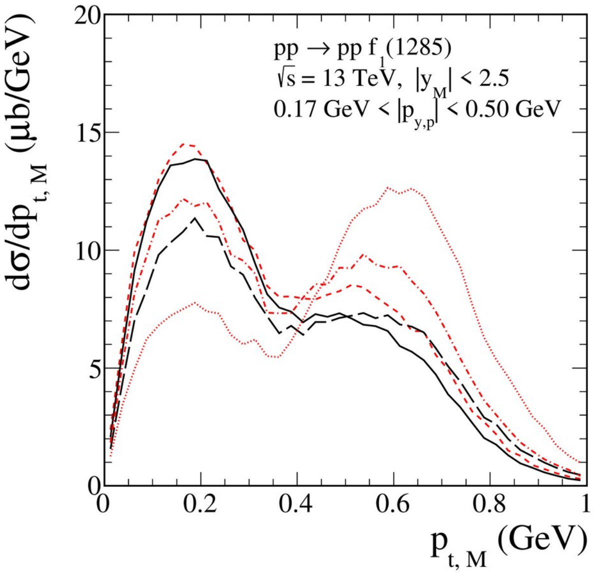


The discrepancy could be partly due to important contributions from subleading reggeon exchanges (IRIR, IRIP, IPIR).

A similar situation is for CEP of $f_1(1420)$.

Predictions for the LHC experiments for the *pp → pp f₁*





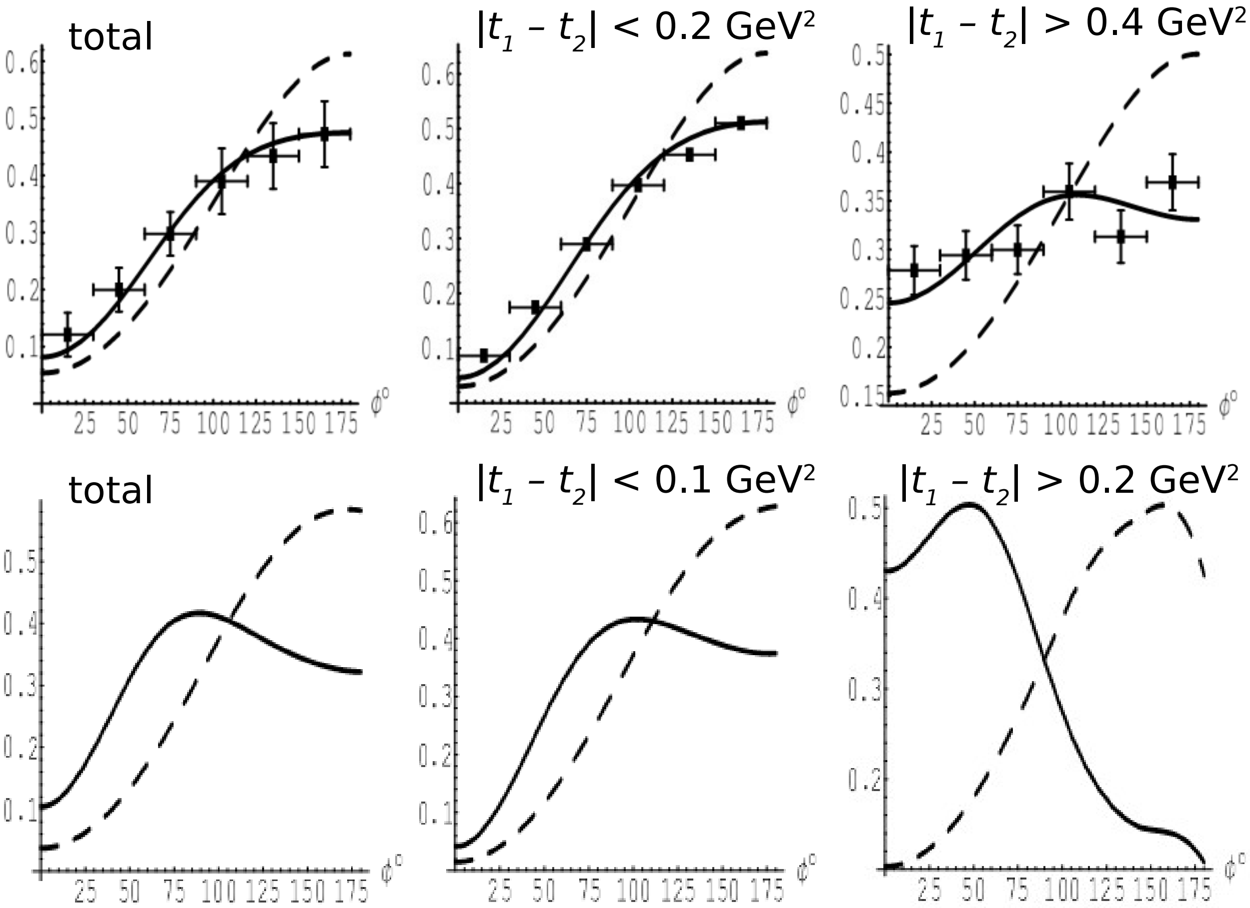
- The contribution with $\varkappa''/\varkappa' = -6.25\ \mathrm{GeV}^{-2}$ gives a significantly different shape
- The absorption effects are included, $\langle S^2 \rangle \approx 0.35$. They decrease the cross section mostly at higher values of ϕ_{pp} and at smaller values of $p_{t,M}$ (also $|t|$). This could be tested in ATLAS-ALFA experiment (both protons are measured)
- If at the WA102 energies there are important contributions from subleading reggeon exchanges, the cross sections at LHC energies could be significantly smaller.
  We estimate that the reduction could be by a factor of up to 4

Cross sections in μb for $\sqrt{s} = 13$ TeV:

Reaction	Contribution	Parameters $\Lambda_E = 0.7$ GeV,	$\lvert y_{f_1}\rvert < 1.0$	$\lvert y_{f_1}\rvert < 2.5$	$\lvert y_{f_1}\rvert < 2.5$, $0.17 < \lvert p_{y,p}\rvert < 0.50$ GeV	$2.0 < y_{f_1} < 4.5$
$pp \to pp f_1(1285)$	$(l,S) = (2,2)$	$g'_{I\!PI\!Pf_1} = 4.89$	14.8	37.5	6.46	18.9
	$(l,S) = (4,4)$	$g''_{I\!PI\!Pf_1} = 10.31$	13.8	34.0	6.06	18.1
	$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -6.25\ \mathrm{GeV}^{-2}$	18.6	45.8	7.14	23.1
	$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -2.44\ \mathrm{GeV}^{-2}$	17.5	43.4	7.10	22.1
	$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -1.0\ \mathrm{GeV}^{-2}$	16.6	41.0	7.09	20.5
$pp \to pp f_1(1420)$	$(l,S) = (2,2)$	$g'_{I\!PI\!Pf_1} = 2.39$	3.7	9.2	1.8	4.7
	$(\varkappa', \varkappa'')$	$\varkappa''/\varkappa' = -1.0\ \mathrm{GeV}^{-2}$	4.1	9.9	1.7	4.9

Results from other approach for the *pp* → *pp f₁*(1285)

- ***V. A. Petrov, R. A. Ryutin, A. E. Sobol, J.-P. Guillaud, JHEP 06 (2005) 007, arXiv:hep-ph/0409118*** *"Azimuthal Angular Distributions in EDDE [Exclusive Double Diffractive Events] as spin-parity analyser and glueball filter for LHC"* ← CEP of f_1 mesons in the framework of Regge picture based on Lorentz tensor reggeized exchanges with account of absorption effects (solid lines) both in initial and final state



Plotted is: $\frac{d\sigma}{d\phi_{pp}} \frac{1}{\sigma}$

← **WA102**

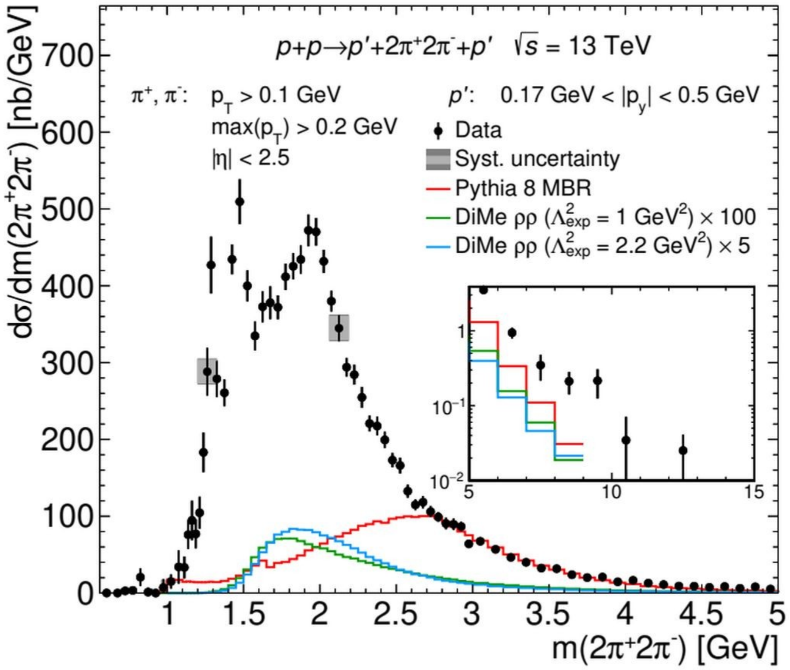
$\langle S^2 \rangle = 0.25 - 0.3$

← **LHC**

$\langle S^2 \rangle = 0.05 - 0.1$

more stronger suppresion particularly for large angles

Exclusive $pp \to pp\ 4\pi$ reaction



Courtesy of R. Sikora, CERN-THESIS-2020-235

One of the most prominent decay modes of $f_1(1285)$ is $2\pi+2\pi-$ channel.

← Preliminary ATLAS-ALFA result

differential cross section for CEP of $2\pi+2\pi-$ state as a function of $M_{4\pi}$

Above 1 GeV the cross section rises rapidly, and a narrow peak is visible around 1.25 – 1.3 GeV. Small width of the peak suggests, that narrow $f_1(1285)$ resonance is responsible for this structure.
There $f_1(1285)$ and $f_2(1270)$ are close in mass. We obtain:

$$\sigma_{pp\to ppf_1(1285)} \times \mathcal{BR}(f_1(1285) \to 2\pi^+2\pi^-) = 34.0\ \mu\text{b} \times 0.109 = 3.7\ \mu\text{b}$$
$$\sigma_{pp\to ppf_2(1270)} \times \mathcal{BR}(f_2(1270) \to 2\pi^+2\pi^-) = 11.3\ \mu\text{b} \times 0.028 = 0.3\ \mu\text{b}$$
$$\left\{ \begin{array}{l} \sqrt{s} = 13\ \text{TeV} \\ |y_M| < 2.5 \end{array} \right.$$

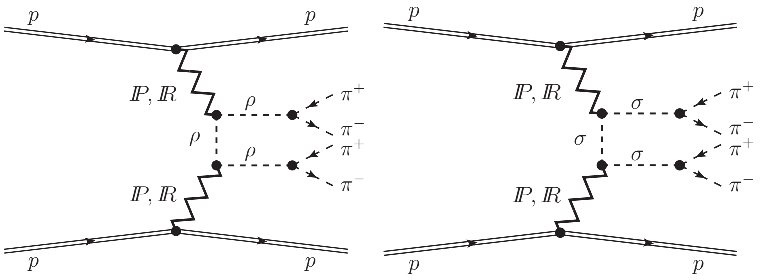
CEP of $f_2(1270)$: PRD 93 (2016) 054015, PRD 101 (2020) 034008

Between 1.3 – 1.6 GeV another structure is visible: a peak with maximum around 1.45 GeV, followed by a dip around 1.55 GeV. These structures can possibly be attributed to the $f_0(1370)$, $f_0(1500)$ or $f_2(1565)$ resonances.
The enhancement and suppresion of the cross section, below and above the resonance mass, might result from interference terms.
There may be also the $\eta(1405)$ resonance.

Around M ~ 1.95 GeV another peak is visible, whose nature is not obvious.
Potential resonances which could be produced in DPE are $f_2(1950)$, $f_0(2020)$.
However, it is also possible that lower mass resonances interfere with four-pion continuum leading to the observed structure.

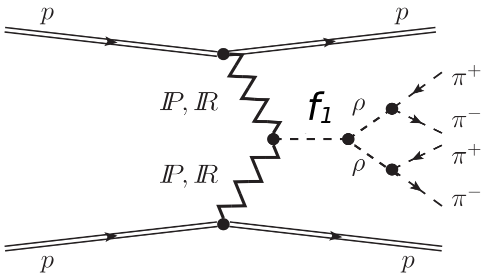
The $f_0(1500)$ and $f_2(1950)$ states are candidates to be scalar and tensor glueball, respectively.
Identification of glueball-like states in this channel requires estimation both of resonant and continuum contributions.

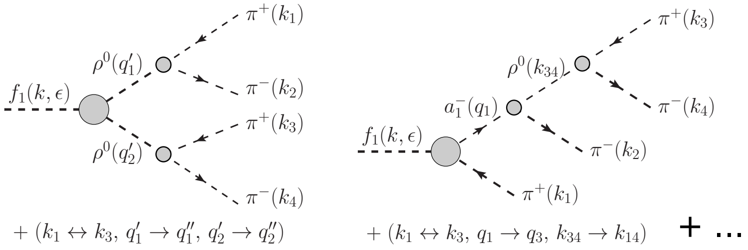
CEP of $2\pi^+2\pi^-$ can proceed in several ways



- *via the intermediate $\sigma\sigma$* and *$\rho\rho$ state*
  [Lebiedowicz, Nachtmann, Szczurek, PRD 94 (2016) 034017]

- production of a single resonance [e.g. $f_1(1285)$] that decays to $\pi^+\pi^-\pi^+\pi^-$
  → requires, for example, an analysis of decay processes (1 → 4)





and other processes: *f_0, f_2 → $\rho^0\rho^0$, $\sigma\sigma$ → $\pi^+\pi^-\pi^+\pi^-$* in progress

Theoretical/phenomenological studies of the 2 → 6 reaction (*pp* → *pp* $\pi^+\pi^-\pi^+\pi^-$) including both resonances and continuum contributions in consistent model is chellenging task.

Hopefully possible using MC generators for exclusive reactions, e.g., GenEx MC:

Kycia, Chwastowski, Staszewski, Turnau, Commun. Comput. Phys. 24 (2018) 860
Kycia, Turnau, Chwastowski, Staszewski, Trzebiński, Commun. Comput. Phys. 25 (2019) 5

and DECAY MC library for the decay of a particle with ROOT compatibility:

Kycia, Lebiedowicz, Szczurek, Commun. Comput. Phys. 30 (2021) 942

Conclusions

- We have discussed CEP of f_1 mesons in pp collisions at high energies in the tensor-pomeron approach. Different forms of the $IP\,IP\,f_1$ coupling are possible. Tests of the Sakai-Sugimoto model are possible.

- We obtain a good description of the WA102 data for the $pp \to pp\, f_1$ reactions assuming that the reactions are dominated by IP exchange.

  We have given predictions for experiments at the LHC.

  Experimental studies of single meson CEP reactions will give many $IP\,IP\,M$ coupling parameters.
  Their theoretical calculation is a challenging problem of nonperturbative QCD.

- Detailed investigations of ϕ_{pp} the azimuthal angle between the transverse momenta of the outgoing protons can help to solve several important problems
  → to check different models for soft processes and to study the real pattern of the interaction (absorption)

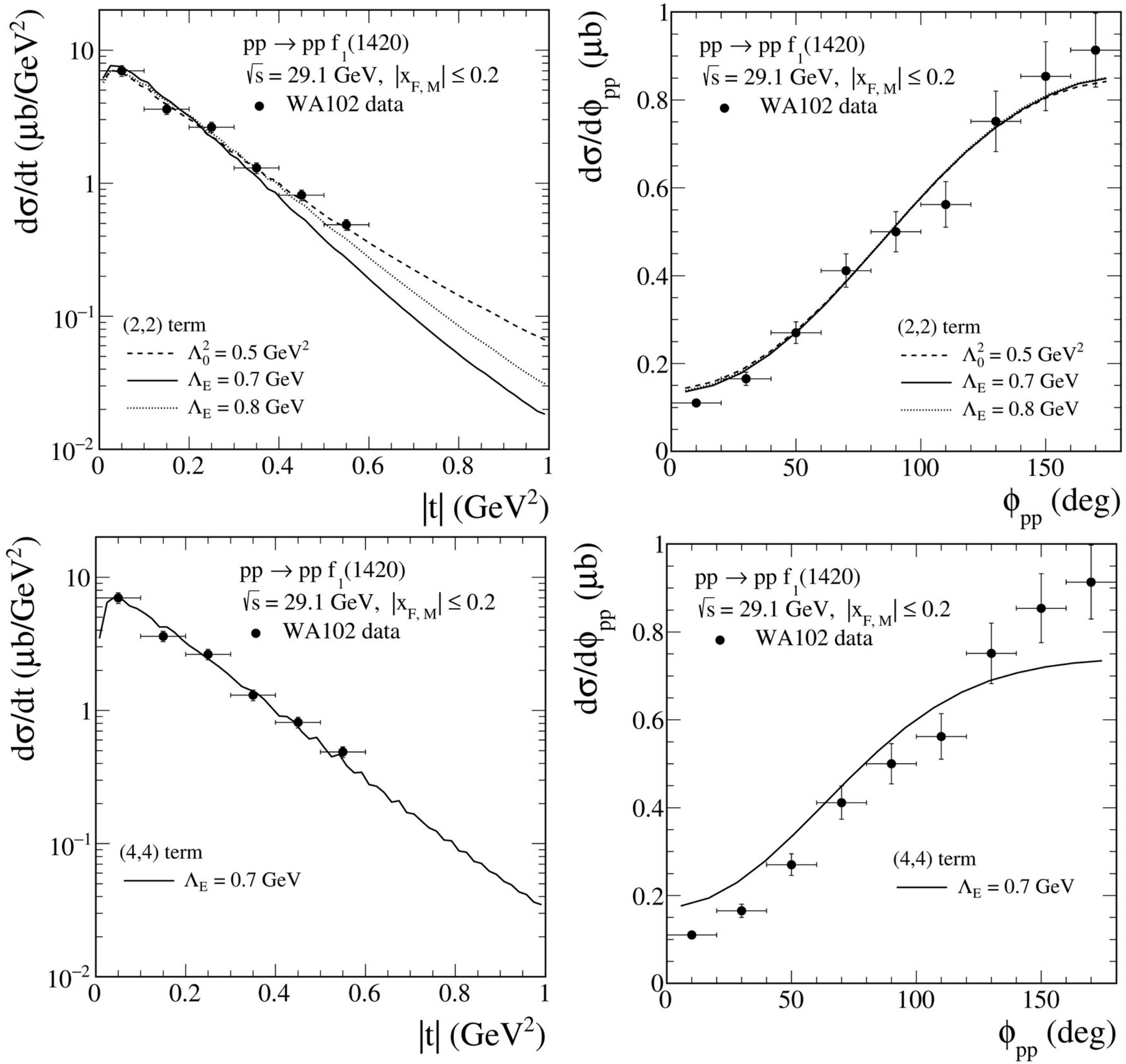
  → to understand the difference in the dynamics of production of $q\bar{q}$ and non-$q\bar{q}$ (glueballs) states possible in the reaction $pp \to pp\, \pi^+\pi^-\pi^+\pi^-$

  → to distinguish or to disentangle f_1- and η-type resonances (contributions). For CEP of an η-type meson at high energies the ϕ_{pp} must vanish for 0 and π. For CEP of f_1 meson there is no such restriction.

Thank you for your attention

Comparison with experimental results from WA102@CERN

Phenomenological approach for CEP of $f_1(1420)$



Data: D. Barberis et al. (WA102 Collaboration), PLB 440 (1998) 225

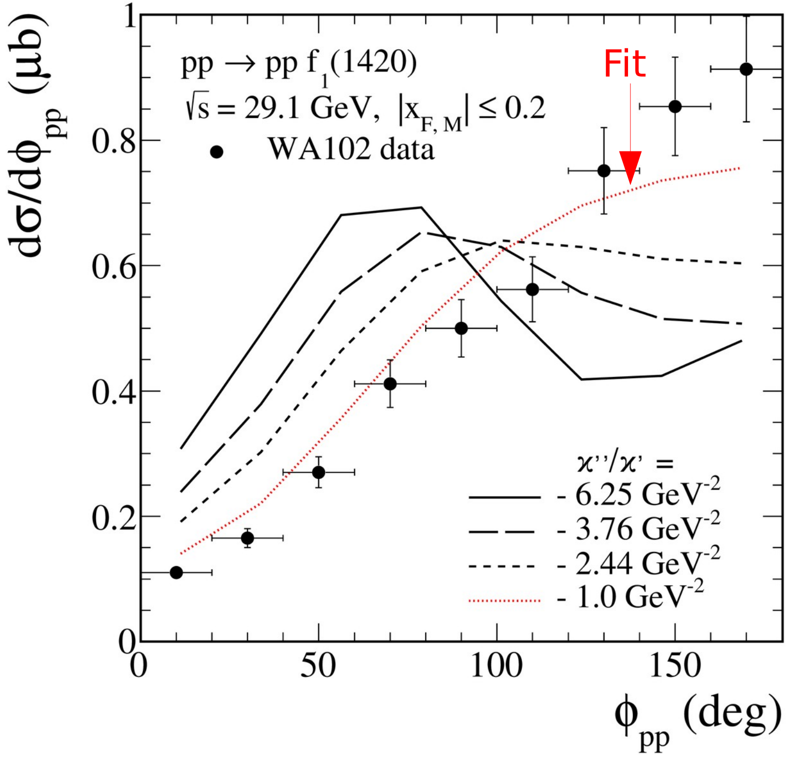
	$\sqrt{s} = 29.1$ GeV, $\lvert x_{F,M} \rvert \leqslant 0.2$
$f_1(1420)$	$\sigma_{\rm exp} = (1584 \pm 145)$ nb

← (ℓ,S) = (2,2) term only
$|g'_{I\!P I\!P f_1}| = 2.39$

← (ℓ,S) = (4,4) term only
$|g''_{I\!P I\!P f_1}| = 4.20$

- We get a good description of WA102 data with $\Lambda_E = 0.7$ GeV
- Absorption effects were included

Holographic QCD approach for CEP of $f_1(1420)$



- As for the $f_1(1285)$ CEP a resonable fit is obtained for

  $\varkappa''/\varkappa' = -1.0\ \mathrm{GeV}^{-2}$ but with $\varkappa' = -5.23$

  Using $t_1 = t_2 = -0.1\ \mathrm{GeV}^2$ we find that this coupling also corresponds practically to a pure $(\ell,S) = (4,4)$ term.

The f_1 mixing angle and relations between the IPIPf_1(1285) and IPIPf_1(1420) coupling constants

The different magnitude of the coupling constants for the IPIPf_1(1285) and IPIPf_1(1420) interactions can be expected to be related to the internal structure of the mesons. A commonly used model (assuming that these mesons are $q\bar{q}$ states) is given by:

$$f_1(1285) = \cos\phi_f \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} - \sin\phi_f \; s\bar{s}$$

$$f_1(1420) = \sin\phi_f \frac{u\bar{u}+d\bar{d}}{\sqrt{2}} + \cos\phi_f \; s\bar{s}$$

with ϕ_f a mixing angle parametrising the deviation from "ideal" mixing ($\phi_f = 0°$), where the heavier f_1 meson would be purely $s\bar{s}$

Ideal mixing is often assumed as a first approximation to account for the fact that f_1(1420) decays dominantly into $K\bar{K}\pi$. Radiative processes, however, indicate a deviation from ideal mixing ϕ_f =20° [Leutgeb, Rebhan, PRD101 (2020) 114015], consistent with the LHCb result [PRL112 (2014) 091802] and [Dudek, PRD83 (2011) 111502], [Cheng, PLB707 (2012) 116].

In the chirally symmetric Sakai-Sugimoto model the IPIPf_1 couplings come exclusively from the axial-gravitational anomaly which involves only the flavour-singlet combination $(u\bar{u}+d\bar{d}+s\bar{s})/\sqrt{3}$.

The assumption that this also holds in real QCD would give:

$$\frac{g'_{I\!PI\!Pf_1(1420)}}{g'_{I\!PI\!Pf_1(1285)}} = \frac{\sqrt{2}\sin\phi_f + \cos\phi_f}{\sqrt{2}\cos\phi_f - \sin\phi_f}$$

and likewise for the couplings g'', $\varkappa'$, and $\varkappa''$.
Ideal mixing thus corresponds to

$$\frac{g'_{I\!PI\!Pf_1(1420)}}{g'_{I\!PI\!Pf_1(1285)}}\bigg|_{\phi_f=0°} = \frac{1}{\sqrt{2}} \approx 0.71$$

while $\phi_f \simeq +20°$ gives ratios larger than unity,

$$\frac{g'_{I\!PI\!Pf_1(1420)}}{g'_{I\!PI\!Pf_1(1285)}}\bigg|_{\phi_f\simeq+20°} \simeq 1.44$$

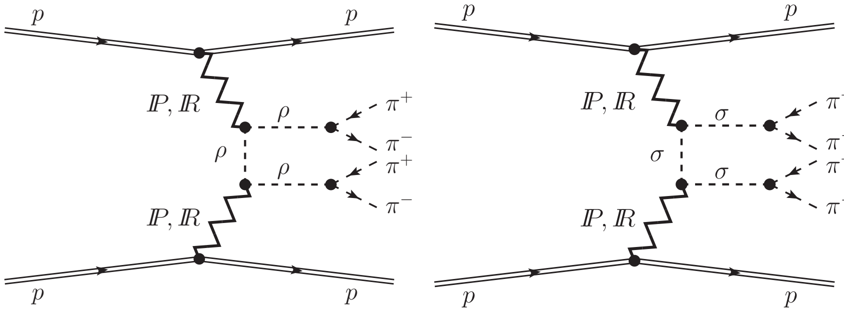
We get (from comparison to WA102 data)

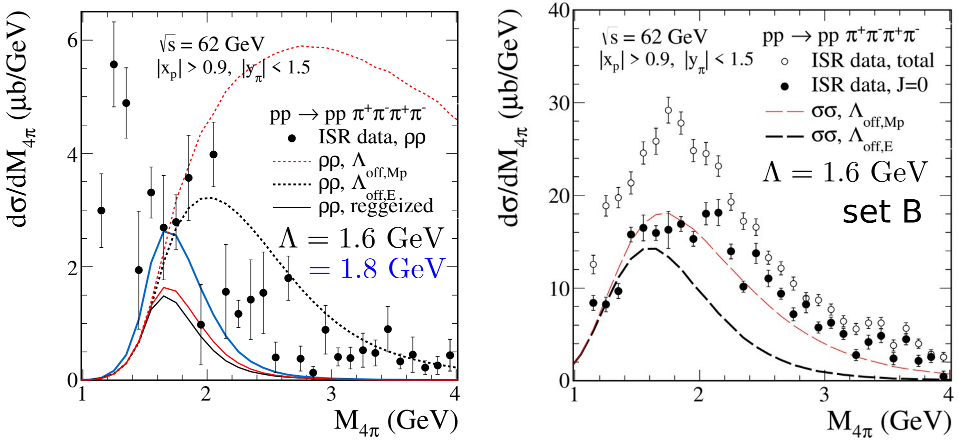
$$\frac{g'_{I\!PI\!Pf_1(1420)}}{g'_{I\!PI\!Pf_1(1285)}} = 0.49\,, \quad \frac{g''_{I\!PI\!Pf_1(1420)}}{g''_{I\!PI\!Pf_1(1285)}} = 0.41\,, \quad \frac{\varkappa'{}^{,}{}''_{I\!PI\!Pf_1(1420)}}{\varkappa'{}^{,}{}''_{I\!PI\!Pf_1(1285)}} = 0.59$$

If at the WA102 energy of $\sqrt{s} = 29.1$ GeV only $I\!PI\!P$ fusion contributes to the CEP of both f_1 mesons, this means that pomerons do not couple predominantly to the flavour-SU(3) singlet components that are involved in the axial-gravitational anomaly. However, if the breaking of the SU(3) flavour symmetry by the strange quark mass has a large effect for $I\!PI\!Pf_1$ couplings, this presents a problem for the chiral Sakai-Sugimoto model. The discrepancy could, however, be partly due to important contributions from subleading reggeon exchanges at WA102 energies. [Another possibility would be that the $f_1(1420)$ is not a separate resonance, but rather the manifestation of the opening of additional decay channels in the tail of the $f_1(1285)$]

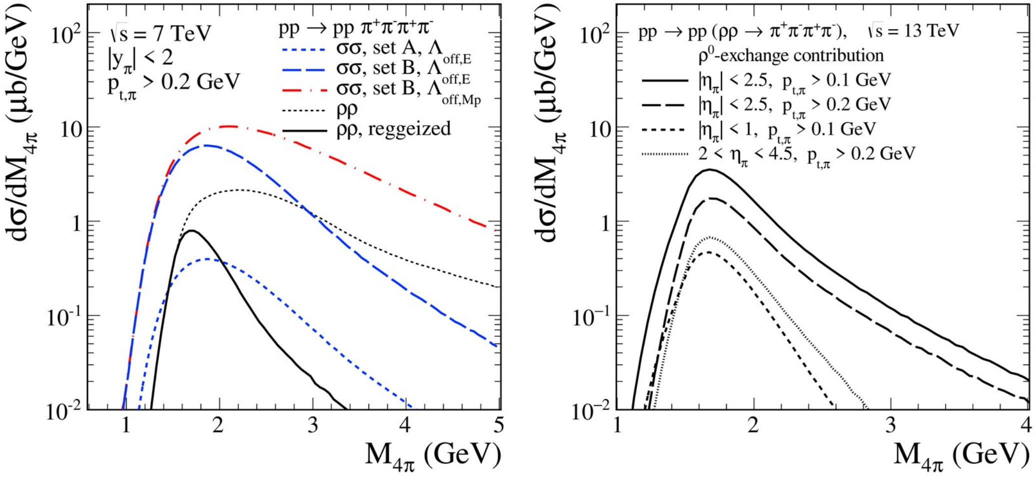
pp → pp 4π reaction | continuum processes

- *via the intermediate σσ and ρρ state* [Lebiedowicz, Nachtmann, Szczurek, PRD 94 (2016) 034017]





data: Breakstone et al., (ABCDHW Collaboration)
Z. Phys. C58 (1993) 251]



$$\sigma_{2\to6} = \int\int \sigma_{2\to4}(\ldots, m_{X_3}, m_{X_4})\, f_M(m_{X_3})\, f_M(m_{X_4})\, dm_{X_3}\, dm_{X_4}$$

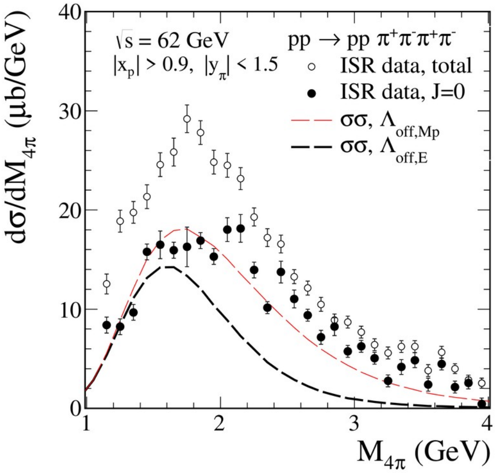
set A : $\beta_{I\!P\sigma\sigma} = 2\beta_{I\!P\pi\pi}\,,\; g_{f_{2I\!R}\sigma\sigma} = g_{f_{2I\!R}\pi\pi}$

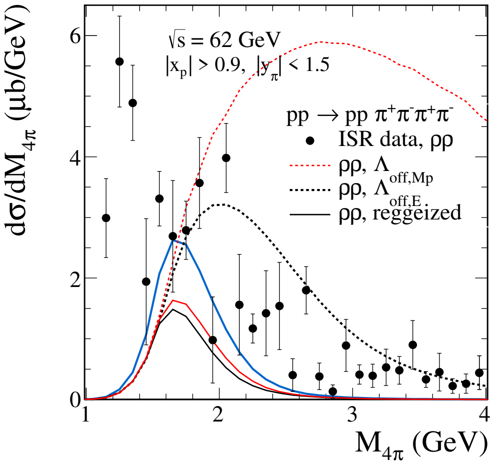
set B : $\beta_{I\!P\sigma\sigma} = 2\times(2\beta_{I\!P\pi\pi})\,,\; g_{f_{2I\!R}\sigma\sigma} = 2\times g_{f_{2I\!R}\pi\pi}$

- large cross sections for *σσ,* with set B (enhanced couplings)
- results very sensitive on type of form factor and cutoff param.
- important reggeization effect for the t/u-channel *ρ* exchange

- via continuum, "three-gap mechanism": proton – IP/IR – (*π*+*π*-) – IP/IR – (*π*+*π*-) – IP/IR – proton
  → contributes to large four-pion invariant masses [Kycia, Lebiedowicz, Szczurek, Turnau, PRD 95 (2017) 094020]
  → phenomenological Regge approach performed with GenEx MC

- Results from *Lebiedowicz, Nachtmann, Szczurek, PRD 94 (2016) 034017* on CEP of 4π continuum via the intermediate σσ and ρρ state compared to the ISR data





- The central 4*π* system in proton-proton collisions was measured also by the ABCDHW Collaboration at the CERN ISR [Breakstone et al., Z. Phys. C58 (1993) 251] (before WA102 experiment)

  A spin-parity decomposition of the 4*π*, ρ*ππ*, and ρρ states as a function of $M_{4\pi}$ was performed with the assumption that the dominant contributions arise from $J^P = 0^+$ and $J^P = 2^+$ states.
  Five contributions to the four-pion spectrum were identified: a 4*π* phase-space term with total angular momentum J=0, two ρ*π*π terms (J=0, 2), and two ρρ terms (J=0, 2).
  There an enhancement observed in the region $M_{4\pi}$ ~1300 MeV for the J=2 terms was assigned to the $f_2(1270)$ meson and for the J=0 ρ*ππ* term to the $f_0(1370)$ meson.
  However, the $J^P = 0^-$ and $J^P = 1^+$ terms, possible in this process (e.g., via IPIP fusion), were not considered!