Hydrodynamics with 50 particles. What does it mean and how to think about it?

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2007.09224 (JHEP), 2109.06389 (With M.Shokri,L.Gavassino,D.Montenegro)

Answers somewhat speculative... but I think I am asking good questions!

- The necessity to redefine hydro
  - Small fluids and fluctuations
  - Statistical mechanicists and mathematicians

- A possible answer:
  - Describing equilibrium at the operator level using the Zubarev operator
  - Definining non-equilibrium at the operator level using Crooks theorem

  Relationship to usual hydrodynamics analogous to "Wilson loops" vs "Chiral perturbation" regarding usual QCD

- The emergence of redundances and the reverse attractor

Some experimental data warmup (Why the interest in relativistic hydro ?)
(2004) Matter in heavy ion collisions seems to behave as a perfect fluid, characterized by a very rapid thermalization

RHIC Scientists Serve Up 'Perfect' Liquid

New state of matter more remarkable than predicted — raising many new questions

April 18, 2005

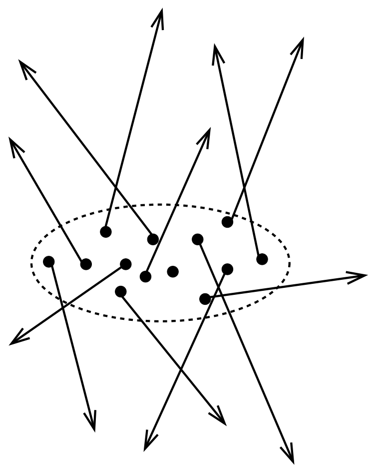
TAMPA, FL — The four detector groups conducting research at the Relativistic Heavy Ion Collider (RHIC) — a giant atom "smasher" located at the U.S. Department of Energy's Brookhaven National Laboratory — say they've created a new state of hot, dense matter out of the quarks and gluons that are the basic particles of atomic nuclei, but it is a state quite different and even more remarkable than had been predicted. In peer-reviewed papers summarizing the first three years of RHIC findings, the scientists say that instead of behaving like a gas of free quarks and gluons, as was expected, the matter created in RHIC's heavy ion collisions appears to be more like a *liquid*.

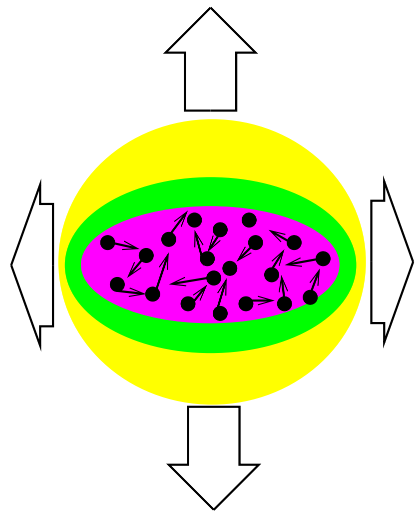
"Once again, the physics research sponsored by the Department of Energy is producing historic results," said Secretary of Energy Samuel Bodman, a trained chemical engineer. "The DOE is the principal federal funder of basic research in the physical sciences, including nuclear and high-energy physics. With today's announcement we see that investment paying off."

"The truly stunning finding at RHIC that the new state of matter created in the collisions of gold ions is more like a liquid than a gas gives us a profound insight into the earliest moments of the universe," said Dr. Raymond L. Orbach, Director of the DOE Office of Science.

The technical details

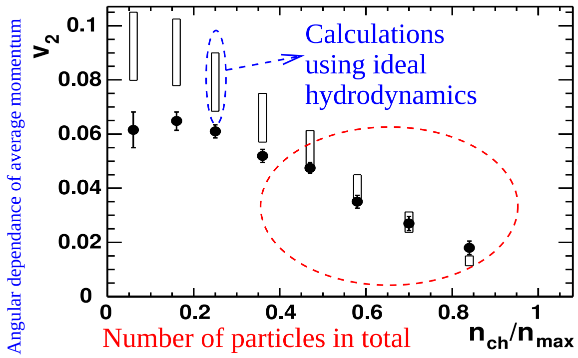
A "dust"
Particles ignore each other, their path is independent of initial shape



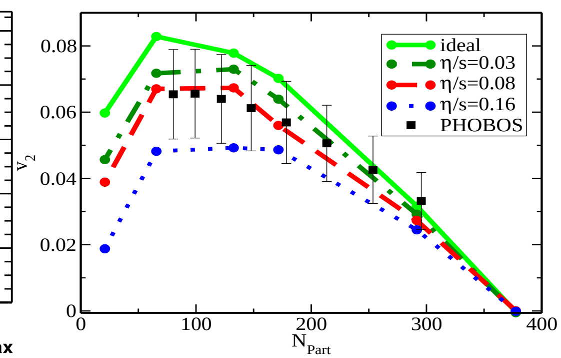


A "fluid"
Particles continuously interact. Expansion determined by density gradient (shape)

P.Kolb and U.Heinz,Nucl.Phys.A702:269,2002.



P.Romatschke,PRL99:172301,2007



The conventional widsom

Hydrodynamics is an "effective theory", built around coarse-graining and "fast thermalization". Fast w.r.t. Gradients of coarse-grained variables

If thermalization instantaneus, then isotropy,EoS enough to close evolution

$$T_{\mu\nu} = (e + P(e))u_\mu u_\nu + P(e) g_{\mu\nu}$$

In rest-frame at rest w.r.t. u^μ

$$T_{\mu\nu} = \mathrm{Diag}\left(e(p), p, p, p\right)$$

(NB: For simplicity we assume no conserved charges, $\mu_B = 0$)

If thermalization not instantaneus,

$$T_{\mu\nu} = T_{\mu\nu}^{eq} + \Pi_{\mu\nu} \quad , \quad u_\mu \Pi^{\mu\nu} = 0$$

$$\sum_n \tau_{n\Pi} \partial_\tau^n \Pi_{\mu\nu} = -\Pi_{\mu\nu} + \mathcal{O}\left(\partial u\right) + \mathcal{O}\left((\partial u)^2\right) + ...$$

A series whose "small parameter" (Barring phase transitions/critical points/... all of these these same order):

$$K \sim \frac{l_{micro}}{l_{macro}} \sim \frac{\eta}{sT} \nabla u \sim \frac{\mathrm{Det}\Pi_{\mu\nu}}{\mathrm{Det}T_{\mu\nu}} \sim ...$$

and the transport coefficients calculable from asymptotic correlators of microscopic theory

Navier-Stokes $\sim K$, Israel-Stewart $\sim K^2$ etc.

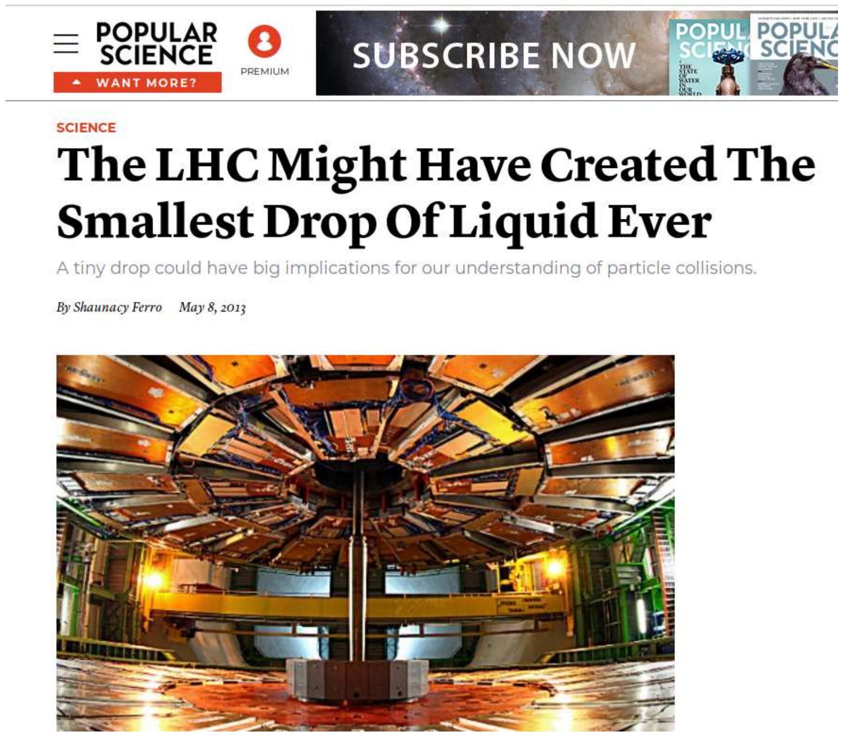
So hydrodynamics is an EFT in terms of K and correlators

$$\eta = \lim_{k\to 0} \frac{1}{k} \int dx \left\langle \hat{T}_{xy}(x)\hat{T}_{xy}(y) \right\rangle \exp\left[ik(x-y)\right] \quad , \quad \tau_\pi \sim \int e^{ikx} \left\langle TTT \right\rangle , ...$$

This is a classical theory , $\hat{T}_{\mu\nu} \rightarrow \langle T_{\mu\nu} \rangle$ Higher order correlators $\langle T_{\mu\nu}(x)...T_{\mu\nu} \rangle$ play role in transport coefficients, not in EoM (if you know equation and initial conditions, you know the whole evolution!)

Both top-down theories compared to hydrodynamics, Boltzmann and AdS/CFT, effectively "classical" (no fluctuations due to molecular chaos/large N_c!)

2011-2013 FLuid-like behavior has been observed down to very small sizes, $p-p$ collisions of 50 particles



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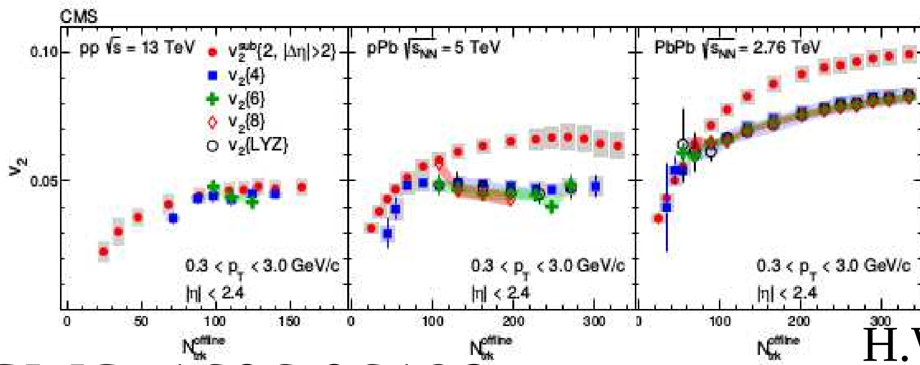
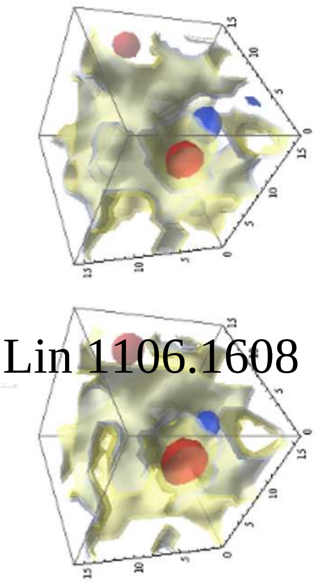
The LHC Might Have Created The Smallest Drop Of Liquid Ever

A tiny drop could have big implications for our understanding of particle collisions.

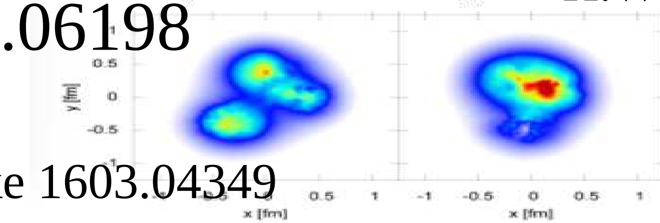
By Shaunacy Ferro May 8, 2013





H.W.Lin 1106.1608

CMS 1606.06198

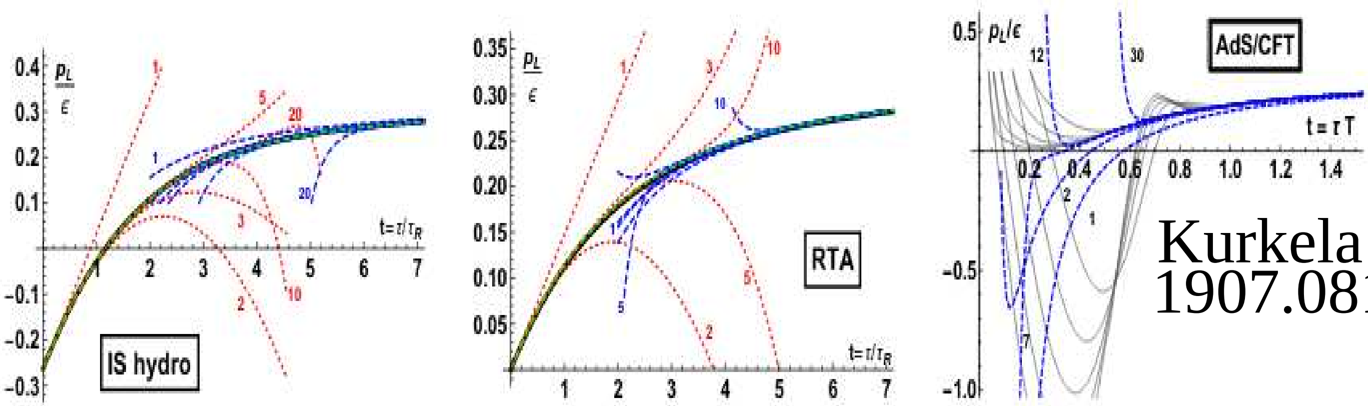


BSchenke 1603.04349

1606.06198 (CMS) : When you consider geometry differences, hydro with $\mathcal{O}(20)$ particles "just as collective" as for 1000. Thermalization scale $\ll$ color domain wall scale.

Little understanding of this in "conventional widsom"

Hydrodynamics in small systems: “hydrodynamization” /” fake equilibrium”
A lot more work in both AdS/CFT and transport theory about ” hydrodynamization” /” Hydrodynamic attractors”



Kurkela et al 1907.08101.

Fluid-like systems far from equilibrium (large gradients)! Usually from 1D solution of Boltzmann and AdS/CFT EoMs! “hydrodynamics converges even at large gradients with no thermal equilibrium”

But I have a basic question: ensemble averaging!

- What is hydrodynamics if $N \sim 50$ ...
  - Ensemble averaging , $\langle F(\{x_i\}, t)\rangle \neq F(\{\langle x_i\rangle\}, t)$ suspect for any non-linear theory. molecular chaos in Boltzmann, Large N_c in AdS/CFT, all assumed . But for $\mathcal{O}(50)$ particles?!?!
  - For water, a cube of length $\eta/(sT)$ has $\mathcal{O}\left(10^9\right)$ molecules,

$$P(N \neq \langle N\rangle) \sim \exp\left[-\langle N\rangle^{-1}(N-\langle N\rangle)^2\right] \ll 1$$

.

- How do microscopic, macroscopic and quantum corrections talk to eac other? EoS is given by $p = T\ln Z$ but $\partial^2 \ln Z/\partial T^2, dP/dV$??

NB: nothing to do with equilibration timescale . Even "things born in equilibrium" locally via Eigenstate thermalization have fluctuations!

And there is more... How does dissipation work in such a "semi-microscopic system"?

- What does local and global equilibrium mean there?
- If $T_{\mu\nu} \to \hat{T}_{\mu\nu}$ what is $\hat{\Pi}_{\mu\nu}$ Second law fluctuations? Sometimes because of a fluctuation entropy decreases! What is the role of microstates?

The obvious conclusion is Fluctuations only help dissipation, they are random .

Perhaps $l_{mfp} \geq \mathcal{O}(1) \left(V/N_{dof}\right)^{1/3}$ or something like this.

Can this be wrong? Can fluctuations help thermalize so smaller systems thermalize faster? if $1/T \sim l_{mfp}$? PERHAPS...

Bottom line: Either hydrodynamics is not the right explanation for these observables (possible! But small/big systems similar!) or we are not understanding something basic about what's behind the hydrodynamics! What do fluctuations do? Just a lower limit to dissipation?

Every statistical theory needs a "state space" and an "evolution dynamics"
The ingredients

State space:Zubarev hydrodynamics Mixes micro and macro DoFs

Dynamics: Crooks fluctuation theorem provides the dynamics via a definition of $\Pi_{\mu\nu}$ from <u>fluctuations</u>

$\hat{T}^{\mu\nu}$ is an operator, so any decomposition, such as $\hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$ must be too!

Zubarev partition function for local equilibrium: think of Eigenstate thermalization...

Let us generalize the GC ensemble to a co-moving frame $E/T \to \beta_\mu T^\mu_\nu$

$$\hat{\rho}(T_0^{\mu\nu}(x), \Sigma_\mu, \beta_\mu) = \frac{1}{Z(\Sigma_\mu, \beta_\mu)} \exp\left[-\int_{\Sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}_0^{\mu\nu} \right]$$

Z is a partition function with a field of Lagrange multiplies $\beta_\mu = u_\mu/T$, with microscopic and quantum fluctuations included.

Effective action from $\ln[Z]$. Correction to Lagrangian picture?

All normalizations diverge but hey, it's QFT! (Later we resolve this!)

Entropy/Deviations from equilibrium

- In quantum mechanics Entropy function of density matrix

$$s = Tr(\hat{\rho} \ln \hat{\rho}) = \frac{d}{dT} (T \ln Z)$$

Conserved in quantum evolution, not coarse-graining/gradient expansion

- In IS entropy function of the dissipative part of E-M tensor

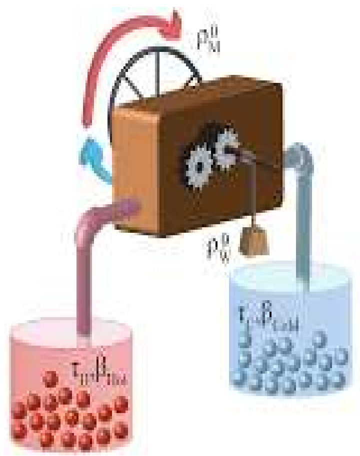
$$n^{\nu} \partial_{\nu} (s u^{\mu}) = n^{\mu} \frac{\Pi^{\alpha\beta}}{T} \partial_{\alpha} \beta_{\beta} \quad , \quad \geq 0$$

$n_{\mu} = d\Sigma_{\mu}/|d\Sigma_{\mu}|, \Pi_{\mu\nu}$ arbitrary. How to combine coarse-graining? if vorticity non-zero $n_{\mu} u^{\mu} \neq 0$

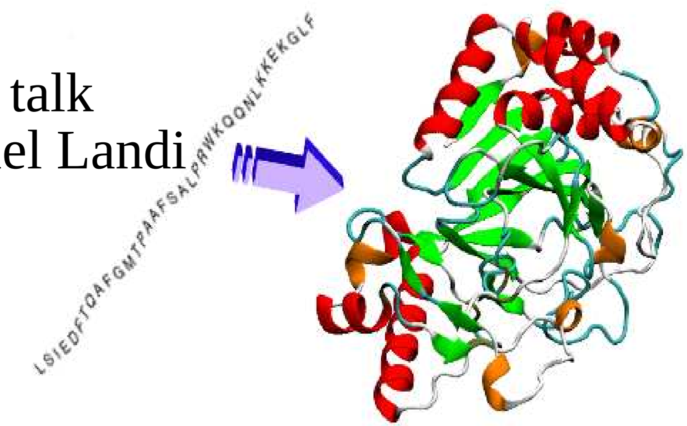
So we need

- a similarly probabilistic definition of $\hat{\Pi}^{\mu\nu} = \hat{T}^{\mu\nu} - \hat{T}_0^{\mu\nu}$ as an <u>operator</u>!!
- Probabilistic dynamics, to update $\hat{\Pi}_{\mu\nu}, \hat{T}_{\mu\nu}$!

Crooks fluctuation theorem!



From talk
Gabriel Landi



Relates fluctuations, entropy in <u>small</u> <u>fluctuating</u> systems (Nano,proteins)

Crooks fluctuation theorem!

$$P(W)/P(-W) = \exp\left[\Delta S\right]$$

P(W) Probability of a system doing some work in its usual thermal evolution

P(-W) Probability of the same system "running in reverse" and decreasing entropy due to a <u>thermal fluctuation</u>

ΔS Entropy produced by $P(W)$

Looks obvious but...

Is valid for systems very far from equilibrium (nano-machines, protein folding and so on)

Proven for Markovian processes and fluctuating systems in contact with thermal bath

Leads to irreducible fluctuation/dissipation: TUR (more later!)

Applying it to locally equilibrium systems within Zubarev's formalism is straight-forward . Since ratios of probabilities, divergences are resolved!

How is Crooks theorem useful for what we did? Guarnieri et al, arXiv:1901.10428 (PRX) derive Thermodynamic uncertainity relations from

$$\hat{\rho}_{ness} \simeq \hat{\rho}_{les}(\lambda) e^{\hat{\Sigma}} \frac{Z_{les}}{Z_{ness}} \quad , \quad \hat{\rho}_{les} = \frac{1}{Z_{les}} \exp\left[-\frac{\hat{H}}{T}\right]$$

$\hat{\rho}_{les}$ is Zubarev operator while Σ is calculated with a Kubo-like formula

$$\hat{\Sigma} = \delta_\beta \Delta \hat{H}_+ \quad , \quad \hat{H}_+ = \lim_{\epsilon \to 0^+} \epsilon \int dt e^{\epsilon t} e^{-\hat{H}t} \Delta \hat{H} e^{\hat{H}t}$$

Relies on

$$\lim_{w \to 0} \left\langle \left[\hat{\Sigma}, \hat{H}\right] \right\rangle \to 0 \equiv \lim_{t \to \infty} \left\langle \left[\hat{\Sigma(t)}, \hat{H}(0)\right] \right\rangle \to 0$$

This "infinite" is "small" w.r.t. hydro gradients. $\equiv$ Markovian as in Hydro with $l_{mfp} \to \partial$ but with operators$\to$ carries all fluctuations with it!

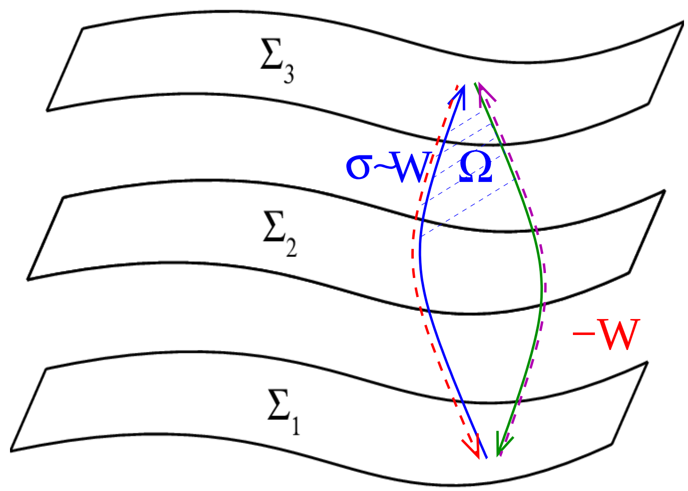
$$P(W)/P(-W) = \exp[\Delta S] \quad \text{Vs} \quad S_{eff} = \ln Z$$

KMS condition reduces the functional integral to a Metropolis type weighting, $\equiv$ periodic time at rest with β_μ

Markovian systems exhibit Crooks theorem, two adjacent cells interaction outcome probability proportional to number of ways of reaching outcome . The normalization divergence is resolved since ratios of probabilities are used . “instant decoherence/thermalization” within each step

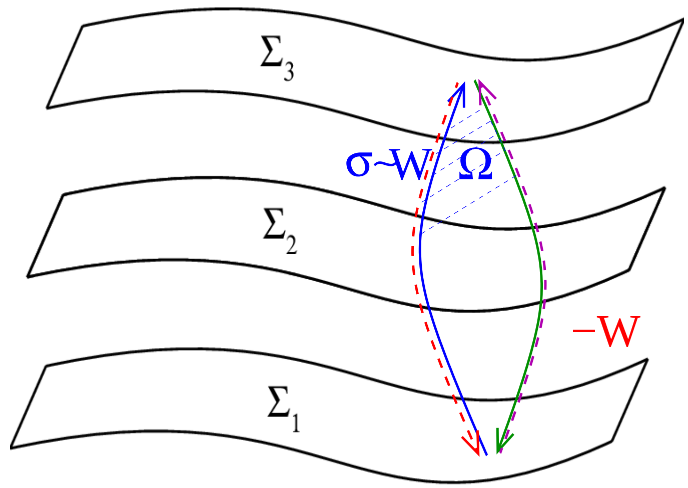
Relationship to gradient expansion similar to relationship between Wilson loop coarse-graining (Jarzynski's theorem, used on lattice ,Caselle et al, 1604.05544) with hadronic EFTs

Applying Crooks theorem to Zubarev hydrodynamics: Stokes theorem



$$-\int_{\Sigma(\tau_0)} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) = -\int_{\Sigma(\tau')} \mathrm{d}\Sigma_\mu \left(\widehat{T}^{\mu\nu}\beta_\nu\right) + \int_\Omega \mathrm{d}\Omega \left(\widehat{T}^{\mu\nu}\nabla_\mu\beta_\nu\right),$$

true for "any" fluctuating configuration.



Let us now invert one foliation so it goes "backwards in time" assuming Crooks theorem means

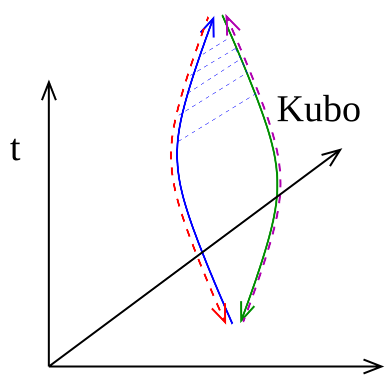
$$\frac{\exp\left[-\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]}{\exp\left[-\int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]} = \exp\left[\frac{1}{2}\int_\Omega d\Omega^\mu_\mu \left[\frac{\hat{\Pi}^{\alpha\beta}}{T}\right] \partial_\beta \beta_\alpha\right]$$

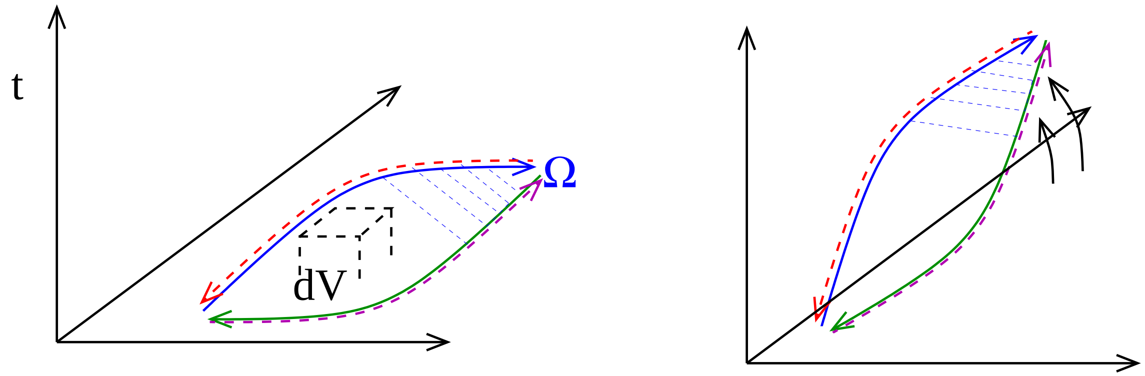
Small loop limit $\left\langle \exp\left[\oint d\Sigma_\mu \omega^{\mu\nu}\beta^\alpha \hat{T}_{\alpha\nu}\right]\right\rangle = \left\langle \exp\left[\int \frac{1}{2} d\Sigma_\mu \beta^\mu \hat{\Pi}^{\alpha\beta}\partial_\alpha \beta_\beta\right]\right\rangle$

A non-perturbative operator equation,divergences cancel out...

$$\left.\frac{\hat{\Pi}^{\mu\nu}}{T}\right|_\sigma = \left(\frac{1}{\partial_\mu \beta_\nu}\right)\frac{\delta}{\delta\sigma}\left[\int_{\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu} - \int_{-\sigma(\tau)} d\Sigma_\mu \beta_\nu \hat{T}^{\mu\nu}\right]$$

Note that a time-like contour produces a Kubo-formula

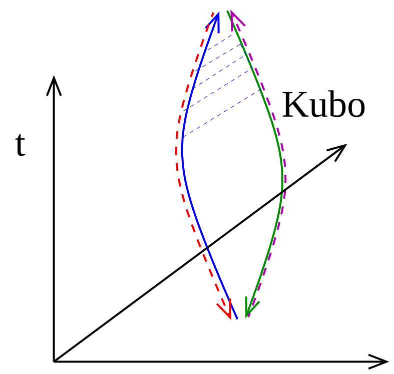




A sanity check: For a an equilibrium spacelike $d\Sigma_\mu = (dV, \vec{0})$ (left-panel) we recover Boltzmann's

$$\Pi^{\mu\nu} \Rightarrow \Delta S = \frac{dQ}{T} = \ln\left(\frac{N_1}{N_2}\right)$$

A sanity check



When $\eta \to 0$ *and* $s^{-1/3} \to 0$ (the first two terms in the hierarchy), Crooks fluctuation theorem gives $P(W) \to 1$ $P(-W) \to 0$ $\Delta S \to \infty$ so Crooks theorem reduces to δ-functions of the entropy current

$$\delta\left(d\Sigma_\mu\left(su^\mu\right)\right) \Rightarrow n^\mu \partial_\mu\left(su^\mu\right) = 0$$

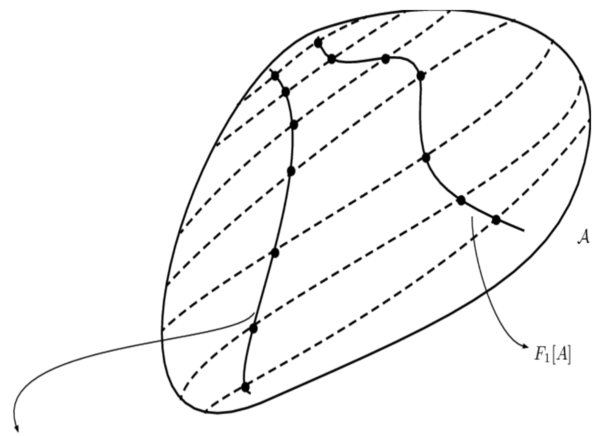
We therefore recover conservation equations for the entropy current, a.k.a. ideal hydro

So could fluctuations help thermalize? A key insight is redundances

Some qualitative developments: $T_0^{\mu\nu}, \Pi^{\mu\nu}, u^\mu$ are not actually experimental observables! Only total energy momentum tensor

$$\hat{T}^{\mu\nu} = \hat{T}_0^{\mu\nu} + \hat{\Pi}^{\mu\nu}$$

and its correlators are! Changing $d\Sigma_\mu$ in Zubarev $\equiv$ changing $\Pi^{\mu\nu}, T_0^{\mu\nu}$!



Analogy to choosing a gauge in gauge theory?

This is relevant for current hydrodynamic research

Causal relativistic hydrodynamics still contentious, with many definitions

Israel-Stewart Relaxing $\Pi_{\mu\nu}$.

Causal, but up to 9 additional DoFs (not counting conserved charges), blow-up possible (M.Disconzi, 2008.03841). $\Pi_{\mu\nu}$ "evolving" microstates!

BDNK,earlier Hiscock,Lindblom,Geroch,... $\Pi_{\mu\nu} \sim \partial u$ At a price of arbitrary (up to causality constaints) u_μ . If you care about statistical mechanics, price is steep! "special" time foliation from ergodic hypothesis/Poncaire cycles!

For phenomenology because of conservation laws "any" $\partial_\mu T^{\mu\nu}$ "can be integrated" but lack of link with equilibration and multiple definitions of "near-equilibrium" problematic. Could these be just "Gauge" choices?

What is a gauge theory,exactly?

$$\mathcal{Z} = \int \mathcal{D}A^\mu \exp\left[S[F_{\mu\nu}]\right] \equiv \int \mathcal{D}A_1^\mu \mathcal{D}A_2^\mu \exp\left[S[A_1^\mu]\right]$$

$A^\mu_{1,2}$ can be separated since physics sensitive to derivatives of $\ln \mathcal{Z}$

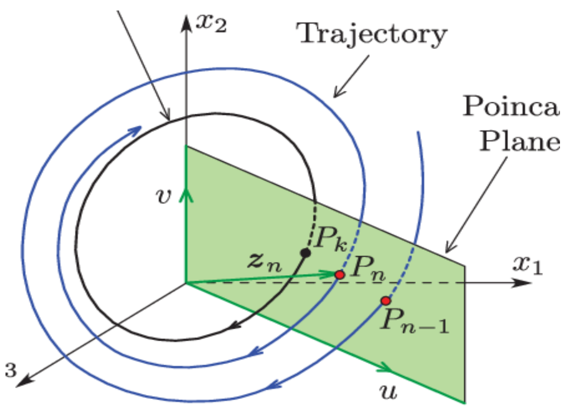
$$\ln \mathcal{Z} = \Lambda + \ln \mathcal{Z}_G \quad , \quad Z_G = \int \mathcal{D}\mathcal{A}^\mu \delta\left(G(A^\mu)\right) \exp\left[S(A_\mu)\right]$$

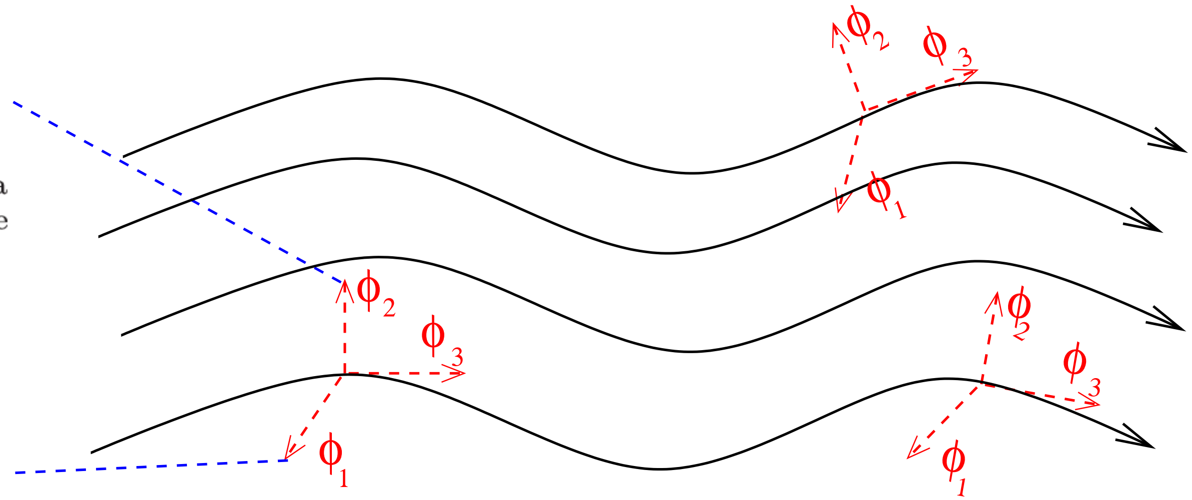
Ghosts come from expanding $\delta(...)$ term. In Zubarev

$$Z = \int \mathcal{D}\phi \quad , \quad "S" = d\Sigma_\nu \beta_\mu T^{\mu\nu}$$

Multiple $T_{\mu\nu}(\phi) \rightarrow$ Gauge-like configuration . Related to Phase space fluctuations of ϕ

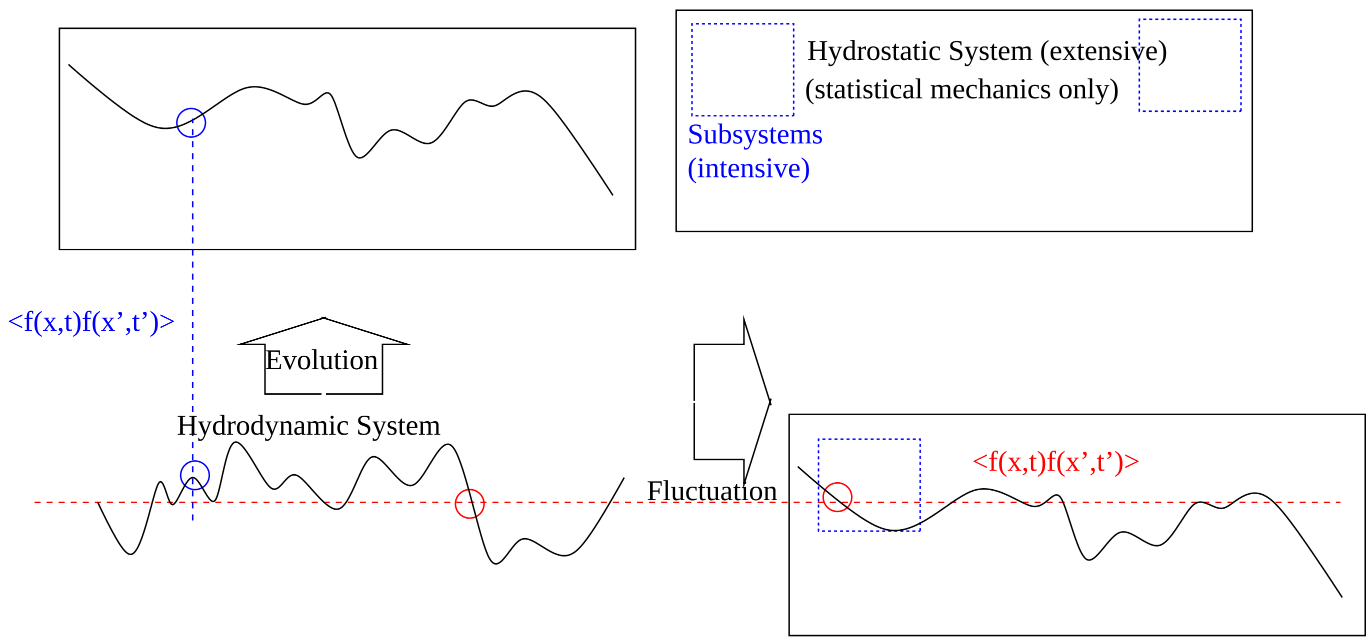
How to make physics fully "gauge"-invariant? Ergodicity/Poncaire cycles meet relativity slightly away from equilibrium!



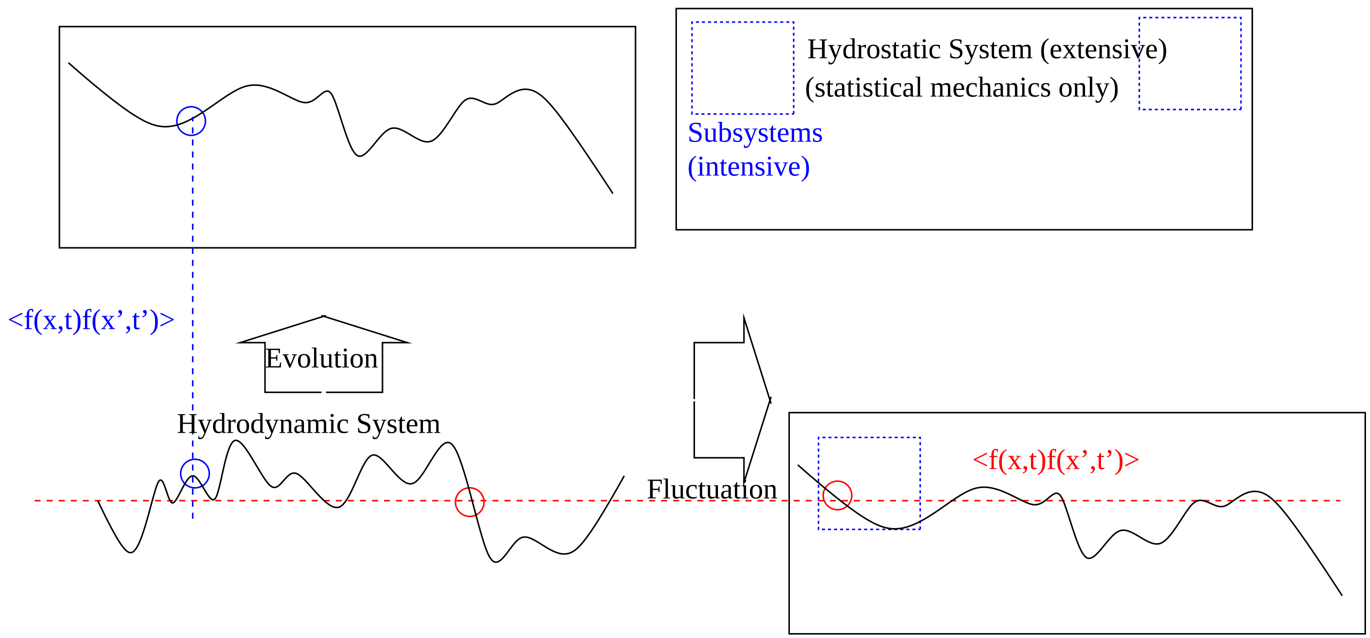


Gibbs entropy level+relativity : Lack of equilibrium is equivalent to "loss of phase" of Poncaire cycles. one can see a slightly out of equilibrium cell <u>either</u> as a "mismatched u_μ" (fluctuation) or as lack of genuine equilibrium (dissipation)

How to make physics fully “gauge”-invariant?



Fluctuation-dissipation at the cell level could do it! We don’t know if a ”step” is fluctuation ($T_0^{\mu\nu}$ or evolution ($\Pi_{\mu\nu}$)-driven!



But in hydro $T_0^{\mu\nu}, \Pi_{\mu\nu}$ treated very differently! "Sound-wave" $u \sim \exp[ik_\mu x^\mu]$ or "non-hydrodynamic Israel-Stewart mode?" $D\Pi_{\mu\nu} + \Pi_{\mu\nu} = \partial u$

Only in EFT $1/T \ll l_{mfp}$ they are truly different!

Infinitesimal transformation $dM_{\mu\nu}$ such that $dM_{\mu\nu}(x)\frac{\delta \ln \mathcal{Z}_E[\beta_\mu]}{dg^{\alpha\mu}(x)} = 0$

Change in microscopic fluctuation $\ln \mathcal{Z} \rightarrow \ln \mathcal{Z} + d\ln \mathcal{Z}$

$$d\ln \mathcal{Z} = \sum_{N=0}^{\infty} \int \prod_{j=1}^{N} d^4 p_j \delta \left(E_N(p_1, ...p_j) - \sum_j p_j^0 \right) \sqrt{|dM|} \exp\left(-\frac{dM_{0\mu} p^\mu}{T} \right)$$

Change in macroscopic dissipative term

$$\Pi_{\mu\nu} \rightarrow \Pi_{\alpha\gamma} \left(g_\mu^\alpha g_\nu^\gamma - g_\mu^\alpha dM_\nu^\gamma - g_\nu^\gamma dM_\mu^\alpha \right) \quad , \quad u_\mu \rightarrow u_\alpha \left(g_\mu^\alpha - dM_\mu^\alpha \right)$$

For $1/T \ll l_{mfp}$ probability of this vanishes, but for $1/T \sim l_{mfp}$ many "similar" probabilities!

The "gauge-symmetry" in practice

Generally $dM_{\mu\nu} = \Lambda^{-1}_{\alpha\mu} dU^{\alpha\beta} \Lambda_{\beta\mu}$

$$d\left[\ln \Pi_{\alpha\beta}\right] \Lambda^{\alpha\mu} \left(\Lambda^{\beta\nu}\right)^{-1} = \eta^{\mu\nu} d\mathcal{A} + \sum_{I=1,3} \left(d\alpha_I \hat{J}_I^{\mu\nu} + d\beta_I \hat{K}_I^{\mu\nu}\right)$$

$$K_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, K_3 = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

which move components from $\Pi_{\mu\nu}$ to Q_μ as well as $K_{1,2,3}$

$$J_1 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}, J_2 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, J_3 = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

Towards hydrodynamic Gibbsian entropy definition !

$$\int \mathcal{D}\phi e^{-S(\phi)} \underbrace{\rightarrow}_{coarse-grain} \int \mathcal{D}\alpha_{I=1,2,3}\mathcal{D}\beta_{I=1,2,3}\ \mathcal{D}\left[\mathcal{A}, e, p, u_\mu, \Pi_{\mu\nu}\right]$$

$$\delta\left(M_{\alpha\beta}\left[\mathcal{A}, \alpha_I, \beta_I\right] T^{\alpha\mu}\right)$$

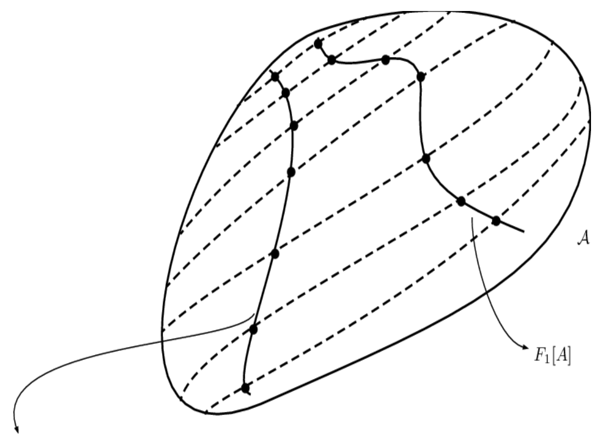
rotate "Gradient expansion" in $1/T, l_{mfp}$ parameter space.
Away from Boltzmann equation regime, $f(x,p) \rightarrow$ Functional

lagrangian , $\ln \mathcal{Z}$ subject to $\delta(...)$ constraint.

Causality also defined via correlator $[T_{\mu\nu}(x), T_{\mu\nu}(x')]$ $e, u_\mu \Pi_{\mu\nu}$ could be non-causal!

Cool but what about thermalization in small systems?

Initial and final state described by many equivalent trajectories



One of them could be close to an ideal-looking one. "reverse" attractor Few particles with strong interaction (Eigenstate thermalization?) correspond to many hydro like-configurations $\{u_\mu, \Pi_{\mu\nu}\}$ with fluctuations , within same Gibbs entropy class. some closer to ideal? No symmetries necessary!

Irrelevant in everyday liquids since $l_{mfp} \gg 1/T$ or AdS/CFT since $N_c \ll \infty$ but perhaps not for QGP!

Conclusions

- Linking hydrodynamics to statistical mechanics is still an open problem
  Only top-down models (Boltzmann,AdS/CFT) rather than bottom-up theory
  Is hydro universal? what are its limits of applicability? still open question

  The observation of hydro-like behavior in small systems liable to fluctuations makes this explicit!

- Crooks fluctuation theorem could provide such a link!

- redundances play crucial role in fluctuations, could mean small systems achieve "thermalization" quicker! inverse attractor!

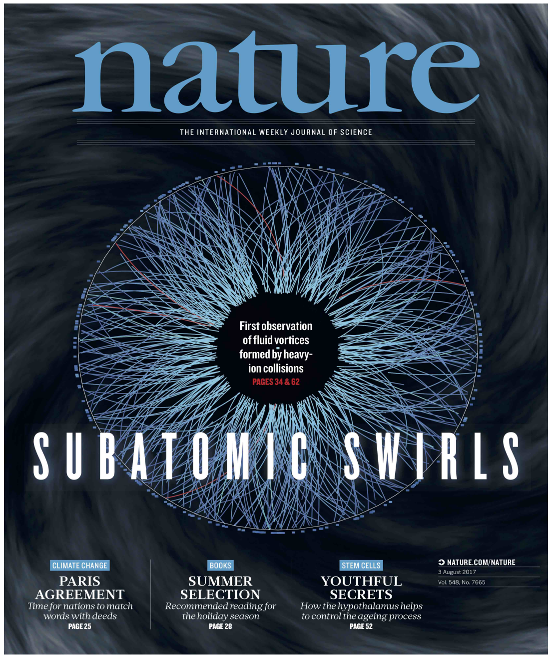
- An obvious extension/application is...

PS: transfer of micro to macro DoFs experimentally proven!

STAR collaboration 1701.06657

NATURE August 2017

Polarization by vorticity in heavy ion collisions



Could give new talk about this, but will mention hydro with spin not developed and a lot of conceptual debates Pseudo-gauge dependence if both spin and angular momentum present in fluid? Gauge symmetry "ghosts"? GT,1810.12468 (EPJA) . redundances?

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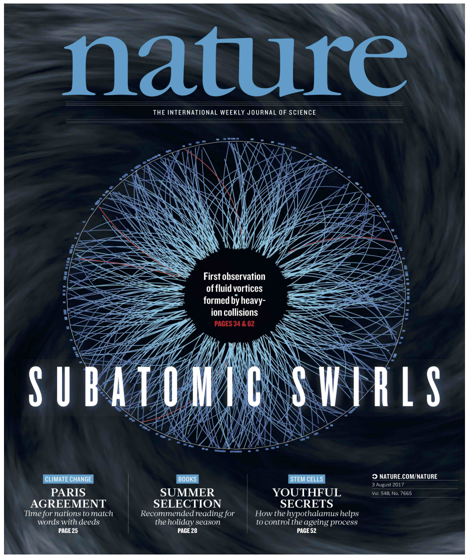
NATURE August 2017

Polarization by vorticity in heavy ion collisions



Pseudo-gauge symmetries physical interpretation: T.Brauner, 1910.12224

$$x^\mu \to x^\mu + \epsilon \zeta^\mu(x) \quad , \quad \psi_a \to \psi_a + \epsilon \psi'_a \to \mathcal{L} \to \mathcal{L}$$

$\ln \mathcal{Z}$ Invariant, but $\langle O \rangle$ generally is not. Spin $\leftrightarrow$ fluctuation, need equivalent of DSE equations! $D \langle O \rangle = 0 \to D \langle O \rangle = \langle O_I O_J \rangle$

Vlasov equation contains all classical correlations, instability-ridden

Boltzmann equation "Classical UV-completion" ov Vlasov equation, first term in BBGK hyerarchy, written in terms of Wigner functions.

Finite number of particles: $f(x,p)$ not a function but a functional ($\mathcal{F}(f(x,p)) \underbrace{\to}_{Boltzmann} \delta\left(f' - f(x,p)\right)$), incorporating continuum of functions and all correlations. Perhaps solvable!

$$\frac{p^\mu}{\Lambda}\frac{\partial}{\partial x^\mu} f(x,p) = \left\langle \underbrace{\hat{C}[\tilde{W}(\tilde{f}_1,\tilde{f}_2)] - g\frac{p^\mu}{\Lambda}\hat{F}^{\mu\nu}[\tilde{f}_1,\tilde{f}_2]\frac{\delta}{\delta \tilde{f}_{1,2}}\tilde{W}\left(\tilde{f}_1,\tilde{f}_2\right)}_{How \quad many \quad A-B=0?} \right\rangle$$

The difference in collision-term redundancy-ridden!

Landau and Lifshitz (also D.Rishke,B Betz et al): Hydrodynamics has three length scales

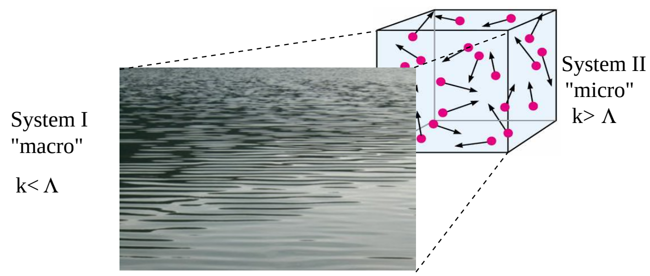
$$\underbrace{l_{micro}}_{\sim s^{-1/3}, n^{-1/3}} \ll \underbrace{l_{mfp}}_{\sim \eta/(sT)} \ll L_{macro}$$

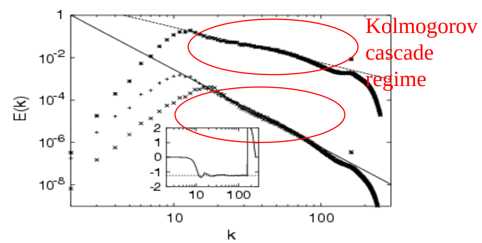
Weakly coupled: Ensemble averaging in Boltzmann equation good up to $\mathcal{O}\left((1/\rho)^{1/3}\partial_\mu f(...)\right)$
Strongly coupled: classical supergravity requires $\lambda \gg 1$ but $\lambda N_c^{-1} = g_{YM} \ll 1$ so

$$\frac{1}{TN_c^{2/3}} \ll \frac{\eta}{sT} \quad \left(\quad or \quad \frac{1}{\sqrt{\lambda}T}\right) \ll L_{macro}$$

QGP: $N_c = 3 \ll \infty$,so $l_{micro} \sim \frac{\eta}{sT}$. Cold atoms: $l_{micro} \sim n^{-1/3} > \frac{\eta}{sT}$?

Why is $l_{micro} \ll l_{mfp}$ necessary? microscopic fluctuations (which have nothing to do with viscosity) will drive fluid evolution. $\Delta\rho/\rho \sim C_V^{-1} \sim N_c^{-2}$



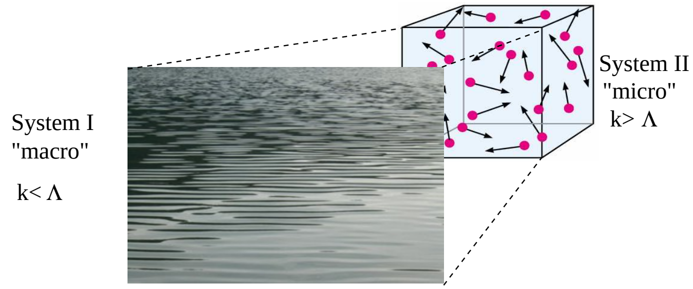


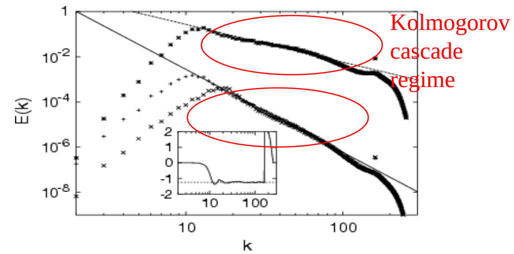
A classical low-viscosity fluid is turbulent. Typically, low-k modes cascade into higher and higher k modes In a non-relativistic incompressible fluid

$$\eta/(sT) \ll L_{eddy} \ll L_{boundary} \quad , \quad E(k) \sim \left(\frac{dE}{dt}\right)^{2/3} k^{-5/3}$$

For a classical ideal fluid, no limit! since $\lim_{\delta\rho\to 0, k\to\infty} \delta E(k) \sim \delta\rho k c_s \to 0$ but quantum $E \geq k$ so energy conservation has to cap cascade.

More fundamentally: take stationary slab of fluid at local equilibrium.

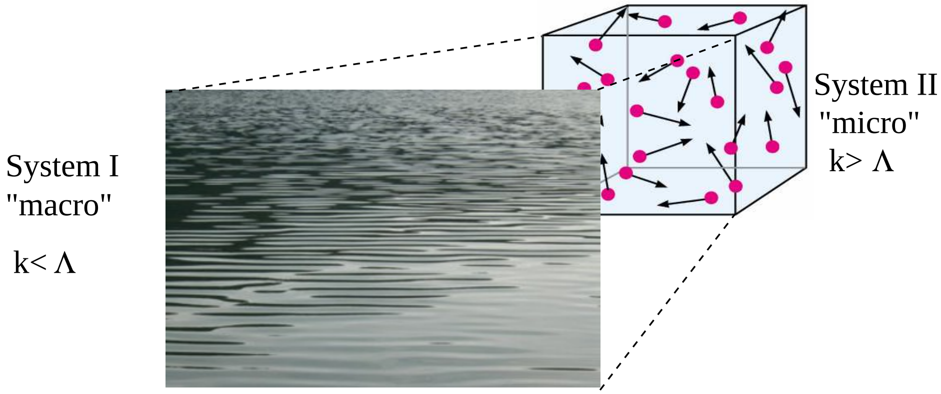


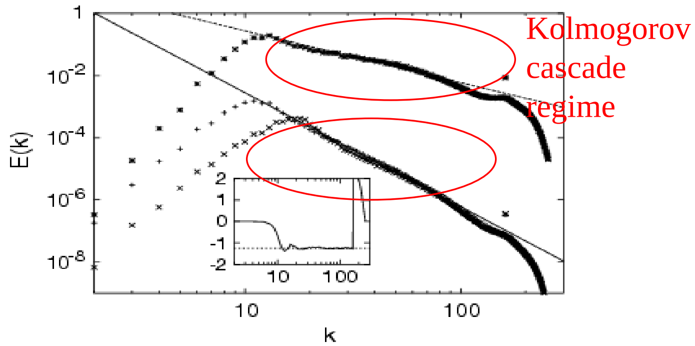


Statistical mechanics: This is a system in global equilibrium, described by a partition function $Z(T, V, \mu)$, whose derivatives give expectation values $\langle E \rangle$,fluctuations $\langle (\Delta E)^2 \rangle$ etc. in terms of conserved charges. All microstates equally likely, which leads to preferred macrostates!

Fluid dynamics: This is the state of a field in local equilibrium which can be perturbed in an infinity of ways. The perturbations will then interact and dissipate according to the Euler/N-S equations. What are micro/macrostates?

More fundamentally: take stationary slab of fluid at local equilibrium.





To what extent are these two pictures the same?

- Global equilibrium is also local equilibrium, if you forget fluctuations
- Dissipation scale in local equilibrium $\eta/(Ts)$, global equilibration timescale $(Ts)/\eta$

Some insight from maths

Millenium problem: existence and smoothness of the Navier-Stokes equations

Important tool are "weak solutions" , similar to what we call "coarse-graining".

$$F\left(\frac{d}{dx}, f(x)\right) = 0 \Rightarrow F\left(\int \frac{d}{dx}\phi(x)..., f(x)\right) = 0$$

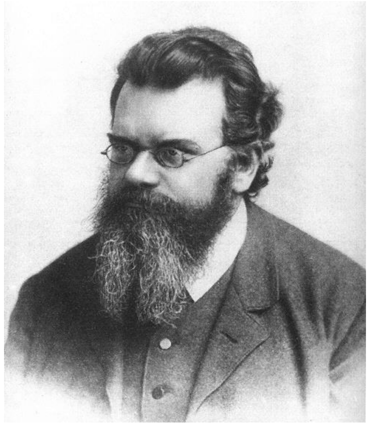
$\phi(x)$ "test function", similar to coarse-graining!

Existance of Wild/Nightmare solutions and non-uniqueness of weak solutions shows this tension is non-trivial, coarse-graining “dangerous”

I am a physicist so I care little about the ”existence of ethernal solutions” to an approximate equation, Turbulent regime and microscopic local equilibria need to be consistent

Thermal fluctuations could both ”stabilize” hydrodynamics and ”accellerate” local thermalization
But where do microstates,”local” microstates fit here?

the battle of the entropies



Boltzmann entropy is usually a property of the "DoF", and is "kinetic" subject to the H-theorem which is really a consequence of the not-so-justified molecular chaos assumption. Gibbsian entropy is the log of the area of phase space, and is justified from coarse-graining and ergodicity , but hard to define it in non-equilibrium . The two are different even in equilibrium, with interactions! Note, Von Neumann $\langle ln\hat{\rho}\rangle$ Gibbsian