Relativistic formulation of spin hydrodynamics framework based on GLW spin and energy-momentum tensors

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Related papers: arXiv:2103.02592, arXiv:2011.14907, arXiv:1901.09655

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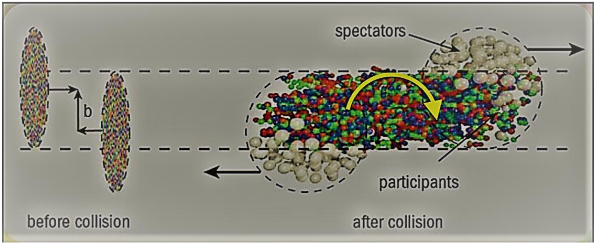
Heavy-ion collisions:

- Non-central relativistic heavy-ion collisions create global rotation of matter, which may induce spin polarization.
- Emerging particles are expected to be globally polarized with their spins on average pointing along the systems angular momentum.

  nucl-th/0410079, nucl-th/0410089, arXiv:0708.0035.

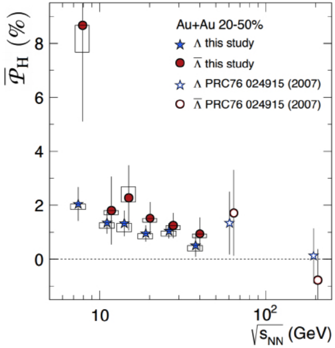


Source: CERN Courier

$$\boldsymbol{J}_{\text{initial}} = \boldsymbol{L}_{\text{initial}} = \boldsymbol{L}_{\text{final}} + \boldsymbol{S}_{\text{final}}$$

Global polarization:

First positive measurements of global spin polarization of Λ hyperons by STAR



thermal approach $\longrightarrow$ $P_\Lambda \approx \frac{1}{2}\frac{\omega}{T} + \frac{\mu_\Lambda B}{T}$ $\quad P_{\overline{\Lambda}} \approx \frac{1}{2}\frac{\omega}{T} - \frac{\mu_\Lambda B}{T}$

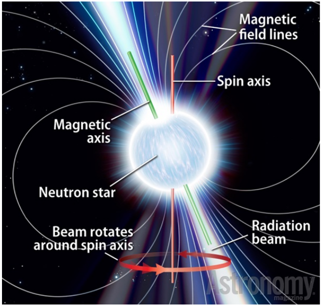
Becattini, F., Karpenko, I., Lisa, M., Upsal, I. , Voloshin, S. , PRC 95, 054902 (2017)

. . . the hottest, least viscous – and now, most vortical – fluid produced in the laboratory . . .

$$\omega = \left(P_\Lambda + P_{\overline{\Lambda}}\right) k_B T/\hbar \sim 0.6 - 2.7 \times 10^{22} \mathrm{s}^{-1}$$

L. Adamczyk et al. (STAR) (2017), Nature 548 (2017) 62-65

Even larger than...



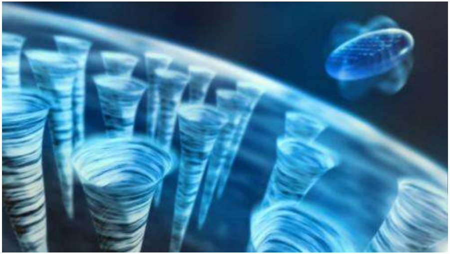


Figure: PSR J1748-2446ad ($716s^{-1}$) & Nanodroplets of superfluid helium ($10^7 s^{-1}$).

10.1126/science.1123430, Science 345, 906–909 (2014)

Global polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...to ∞

- But...

Longitudinal polarization:

- Good agreement between experiment and models based on local thermodynamic equilibrium of spin degrees of freedom.

  0711.1253, 1304.4427, 1303.3431, 1501.04468, 1610.02506, 1610.04717, 1605.04024, 1703.03770, etc...to ∞

- But...

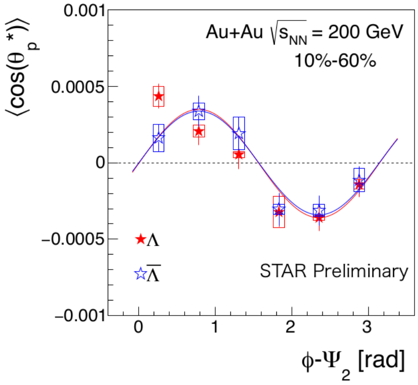


Figure: Longitudinal polarization of Λ-$\bar{\Lambda}$ (1905.11917)

Bigger picture:

- This study will help us to know the formation and characteristics of the QGP, a state of matter believed to exist at sufficiently high energy densities.

- Detecting and understanding the QGP allows us to understand better the universe in the moments after the Big Bang.

Our approach:

- Include spin degrees of freedom into the ideal standard hydrodynamics to form spin hydrodynamics formalism.
- $J^{\mu,\alpha\beta}(x) = x^{\alpha} T^{\mu\beta}(x) - x^{\beta} T^{\mu\alpha}(x) + S^{\mu,\alpha\beta}(x)$
- And, conservation of total angular momentum, $\partial_{\lambda} J^{\lambda,\mu\nu}(x) = 0$ gives $\partial_{\lambda} S^{\lambda,\mu\nu}(x) = T^{\nu\mu}(x) - T^{\mu\nu}(x)$
- For symmetric energy-momentum tensor, $T^{\nu\mu}_{\mathrm{GLW}}(x) = T^{\mu\nu}_{\mathrm{GLW}}(x)$, we have $\partial_{\lambda} S^{\lambda,\mu\nu}_{\mathrm{GLW}}(x) = 0$
- Hence conservation of the angular momentum implies the conservation of its spin part in the de Groot-van Leeuwen-van Weert (GLW) formulation.

1705.00587, 1712.07676, 1806.02616, 1811.04409, S. R. De Groot *et. al.*, Relativistic Kinetic Theory: Principles and Applications (1980).

Steps of spin hydrodynamic framework:

- Solving the standard perfect-fluid hydrodynamic equations without spin.
- Determination of the spin evolution in the hydrodynamic background.
- Determination of the Pauli-Lubański (PL) vector on the freeze-out hypersurface.
- Then we calculate momentum dependent and momentum integrated spin polarization of particles in their rest frame which can be directly compared with the experimental findings.

  1901.09655

Conservation laws:

- $d_\alpha N^\alpha \equiv d_\alpha(\mathcal{N} U^\alpha) = 0 \to$ **Conservation of net baryon number**
  where $\mathcal{N} = 4\,\sinh(\mu/T)\,\mathcal{N}_{(0)}$.

- $d_\alpha T^{\alpha\beta} \equiv d_\alpha\left[(\mathcal{E}+\mathcal{P})\,U^\alpha U^\beta - \mathcal{P} g^{\alpha\beta}\right] = 0 \to$ **Conservation of EMT**
  where $\mathcal{E} = 4\cosh(\mu/T)\mathcal{E}_{(0)}$ and $\mathcal{P} = 4\cosh(\mu/T)\mathcal{P}_{(0)}$.

  These laws provide closed system of 5 eqns. for 5 funcs: μ, T, and three independent components of U^μ which need to be solved to get the perfect-fluid background evolution.

- $d_\alpha S^{\alpha,\beta\gamma} \equiv d_\alpha\left[\mathcal{A}_1 U^\alpha \omega^{\beta\gamma} + \mathcal{A}_2 U^\alpha U^{[\beta}\kappa^{\gamma]} + \mathcal{A}_3(U^{[\beta}\omega^{\gamma]\alpha} + g^{\alpha[\beta}\kappa^{\gamma]})\right] = 0$

  $\downarrow$

  Conservation of spin

  where $\mathcal{A}_1 = \cosh(\mu/T)\left(n_{(0)} - \mathcal{B}_{(0)}\right)$, $\mathcal{A}_2 = \cosh(\mu/T)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right)$,
  $\mathcal{A}_3 = \cosh(\mu/T)\,\mathcal{B}_{(0)}$
  with, $\mathcal{B}_{(0)} = -\frac{2}{(m/T)^2}(\mathcal{E}_{(0)} + \mathcal{P}_{(0)})/T$ and $\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{(0)}$.
  Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor.

1811.04409

Spin polarization tensor:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$.
U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X \; = \; Y \cdot Y \; = \; Z \cdot Z \; = \; -1.$

$$\omega_{\alpha\beta} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

Spin polarization (local and global):

$$\langle \pi_\mu \rangle_p = \frac{E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{E_p \frac{d\mathcal{N}(p)}{d^3p}} \quad \frac{\rightarrow Total\ \ Pauli\ \ Lubanski\ \ vector}{\rightarrow Momentum\ \ density\ \ of\ \ all\ \ particles} \sim \ \boldsymbol{p}\ dependent$$

$$\langle \pi_\mu \rangle = \frac{\int dP \langle \pi_\mu \rangle_p E_p \frac{d\mathcal{N}(p)}{d^3p}}{\int dP E_p \frac{d\mathcal{N}(p)}{d^3p}} = \frac{\int d^3p \ \ \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p}}{\int d^3p \ \ \frac{d\mathcal{N}(p)}{d^3p}} \sim \ \boldsymbol{p}\ integrated$$

where, $$E_p \frac{d\overset{*}{\Pi}_\mu(p)}{d^3p} = \frac{1}{(2\pi)^3 m} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p} \left(\tilde{\omega}_{\beta\mu} \ p^\beta\right)^*$$

$$E_p \frac{d\mathcal{N}(p)}{d^3p} = \frac{4}{(2\pi)^3} \int \cosh\left(\frac{\mu}{T}\right) \Delta\Sigma_\lambda p^\lambda \, e^{-\beta\cdot p}$$

* meaning quantities calculated in particle rest frame.

1901.09655

Non-Boost-invariant and transversely homogeneous flow

Initialization of spin components:

$\omega_{\mu\nu}$ is an anti-symmetric tensor of rank 2 and can be parameterized by the four-vectors κ^μ and ω^μ,

$$\boxed{\omega_{\mu\nu} = \kappa_\mu U_\nu - \kappa_\nu U_\mu + \epsilon_{\mu\nu\alpha\beta} U^\alpha \omega^\beta},$$

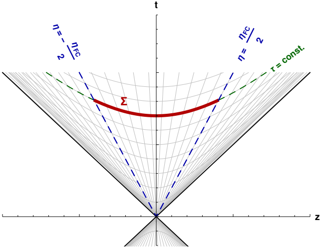
where $\kappa^\alpha = C_{\kappa X} X^\alpha + C_{\kappa Y} Y^\alpha + C_{\kappa Z} Z^\alpha, \quad \omega^\alpha = C_{\omega X} X^\alpha + C_{\omega Y} Y^\alpha + C_{\omega Z} Z^\alpha$.
U, X, Y and Z form a 4-vector basis satisfying the following normalization conditions: $U \cdot U = 1, \qquad X \cdot X = Y \cdot Y = Z \cdot Z = -1.$

$$\omega_{\alpha\beta} = \begin{bmatrix} 0 & e^1 & e^2 & e^3 \\ -e^1 & 0 & -b^3 & b^2 \\ -e^2 & b^3 & 0 & -b^1 \\ -e^3 & -b^2 & b^1 & 0 \end{bmatrix}$$

$$\begin{aligned} C_{\kappa X} &= e^1 \cosh(\vartheta + \eta) - b^2 \sinh(\vartheta + \eta), \\ C_{\omega Y} &= b^2 \cosh(\vartheta + \eta) - e^1 \sinh(\vartheta + \eta), \\ C_{\kappa Y} &= e^2 \cosh(\vartheta + \eta) + b^1 \sinh(\vartheta + \eta), \\ C_{\omega X} &= b^1 \cosh(\vartheta + \eta) + e^2 \sinh(\vartheta + \eta), \\ C_{\kappa Z} &= e^3, \quad C_{\omega Z} = b^3. \end{aligned}$$

$$\boxed{C_{\kappa X} = -C_{\omega Y} \tanh(\vartheta + \eta)}$$

Spin angular momentum components at the freeze-out:

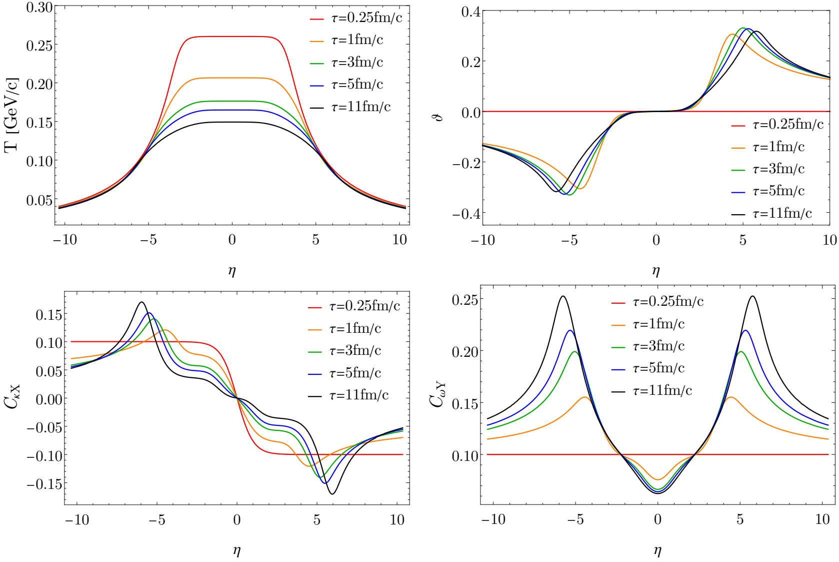


The hypersurface of the boost-invariant fire-cylinder (1901.09655).

$$S^{\mu\nu}_{\rm FC} = \int \Delta\Sigma_\lambda S^{\lambda,\mu\nu}_{\rm GLW} = \int dxdy\,\tau d\eta\, U^{\rm B}_\lambda S^{\lambda,\mu\nu}_{\rm GLW} = \pi R^2 \tau \int\limits_{-\eta_{\rm FC}/2}^{+\eta_{\rm FC}/2} d\eta\, U^{\rm B}_\lambda\; S^{\lambda,\mu\nu}_{\rm GLW}$$

$$S^{\rm FC}_{13} = \pi R^2 \tau \int\limits_{-\eta_{\rm FC}/2}^{+\eta_{\rm FC}/2} d\eta \Big[\mathcal{A}_1\, C_{\omega Y} \cosh(\eta) + \mathcal{A}_3\, C_{\kappa X} \sinh(\eta)\Big].$$

Perfect-fluid background and spin components evolution:



Momentum dependence of polarization:

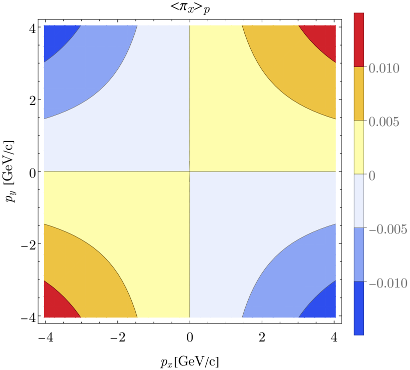


Figure: x-component of the PRF mean spin polarization three-vector of the Λ hyperons for $y_p = 0$ (left) and for $y_p = 2$ (right).

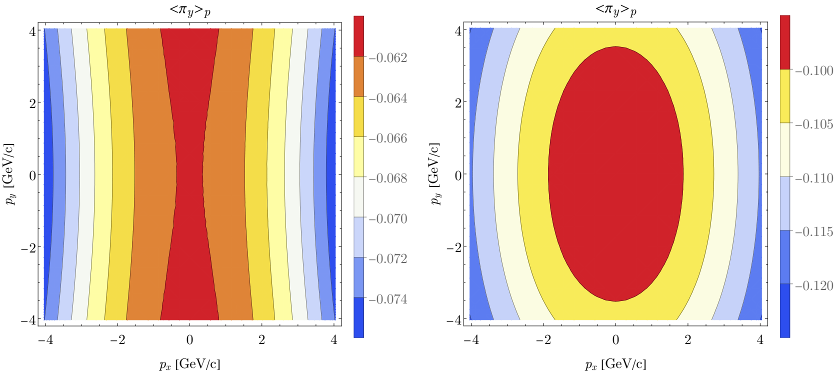


Figure: y-component of the PRF mean spin polarization three-vector of the Λ hyperons for $y_p = 0$ (left) and for $y_p = 2$ (right).

Momentum dependence of polarization:

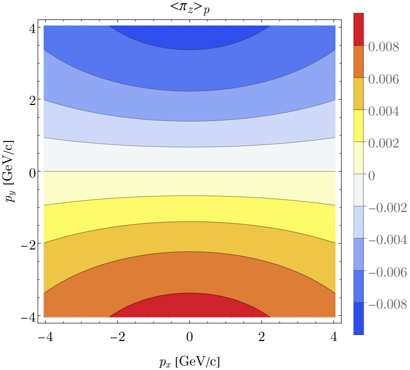


Figure: z-component of the PRF mean spin polarization three-vector of the Λ hyperons for $y_p = 2$

Global polarization:

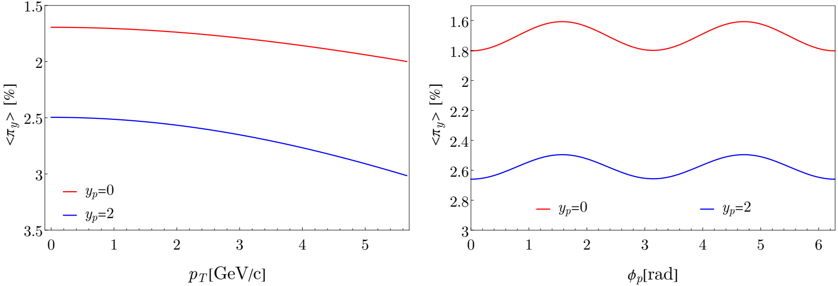
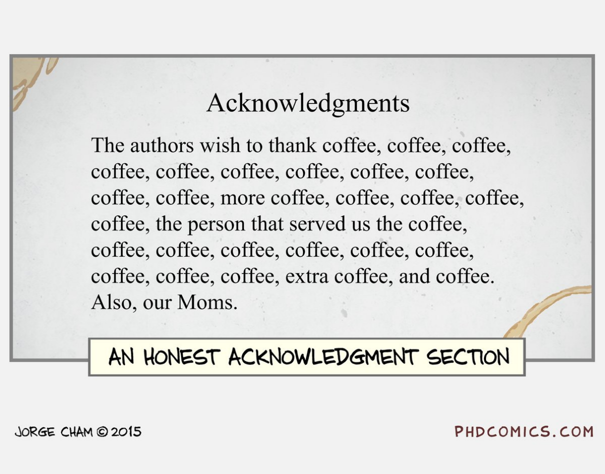


Figure: p_T (left) and ϕ_p (right) dependent $\langle \pi_y \rangle$ component of the global spin polarization.

Summary:

- Discussed relativistic hydrodynamics with spin based on the GLW formulation of energy-momentum and spin tensors.
- Showed how our formalism can be compared with the experiments.
- Obtained dynamics of spin polarization in the non-boost background.
- Incorporation of spin in full 3+1D hydro model required to address the problem of longitudinal polarization (which will be out pretty soon, stay tuned).

Thank you for your time!



Extra Slides

Perfect-fluid background dynamics:

- Conservation law of charge can be written as:

$$U^\alpha \partial_\alpha n + n \partial_\alpha U^\alpha = 0$$

Therefore, for Non-Boost type of flow we can write,

$$\dot{n} + n\ \theta_U = 0.$$

- Conservation law of energy-momentum can be written as:

$$U^\alpha \partial_\alpha \varepsilon + (\varepsilon + P) \partial_\alpha U^\alpha = 0$$

Hence for the Non-Boost flow,

$$\dot{\varepsilon} + (\varepsilon + P)\ \theta_U = 0$$

Conservation of spin:

$$\boxed{d_\alpha S^{\alpha,\beta\gamma}_{\rm GLW}(x) = 0}$$

GLW spin tensor in the leading order of $\omega_{\mu\nu}$ is:

$$S^{\alpha,\beta\gamma}_{\rm GLW} = \cosh(\tfrac{\mu}{T})\left(\mathcal{N}_{(0)} U^\alpha \omega^{\beta\gamma} + S^{\alpha,\beta\gamma}_{\Delta\rm GLW}\right)$$

Here, $\omega^{\beta\gamma}$ is known as spin polarization tensor, whereas the auxiliary tensor $S^{\alpha,\beta\gamma}_{\Delta\rm GLW}$ is:

$$\begin{gathered} S^{\alpha,\beta\gamma}_{\Delta\rm GLW} = \mathcal{A}_{(0)} U^\alpha U^\delta U^{[\beta} \omega^{\gamma]}{}_\delta \\ +\mathcal{B}_{(0)}\left(U^{[\beta}\Delta^{\alpha\delta}\omega^{\gamma]}{}_\delta + U^\alpha \Delta^{\delta[\beta}\omega^{\gamma]}{}_\delta + U^\delta \Delta^{\alpha[\beta}\omega^{\gamma]}{}_\delta\right), \end{gathered}$$

with,

$$\Delta^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$$

$$\mathcal{B}_{(0)} = -\tfrac{2}{(m/T)^2}(\mathcal{E}_{(0)} + \mathcal{P}_{(0)})/T$$

$$\mathcal{A}_{(0)} = -3\mathcal{B}_{(0)} + 2\mathcal{N}_{(0)}$$

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Initial baryon chemical potential $\mu_0 = 300$ MeV
Initial temperature $T_0 = 300$ MeV
Particle (Lambda hyperon) mass $m = 1116$ MeV

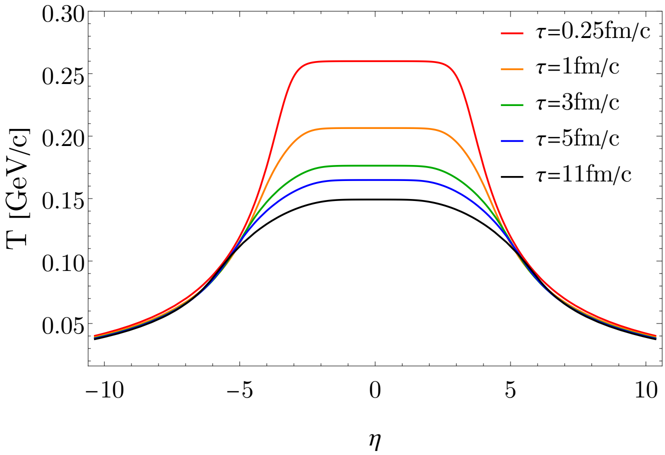

Figure: Temperature evolution.

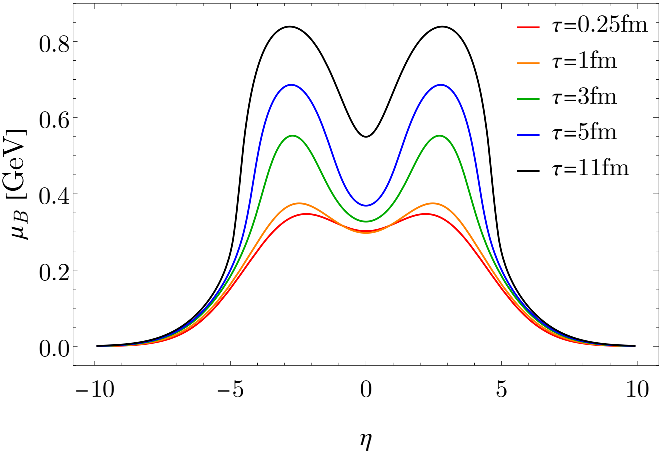

Figure: Baryon chemical potential evolution.

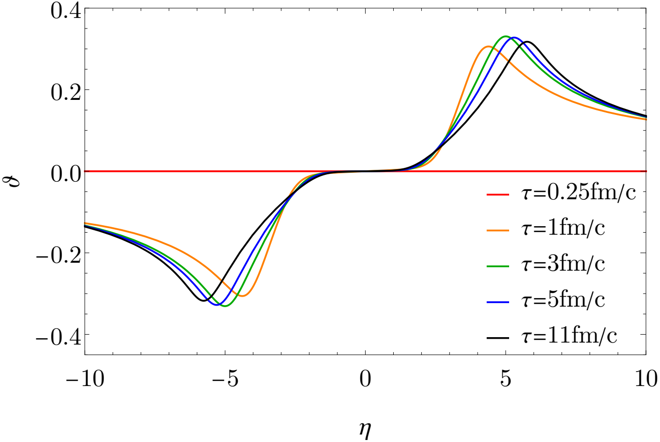

Figure: Fluid rapidity evolution.

Spin polarization coefficients evolution for Bjorken flow:

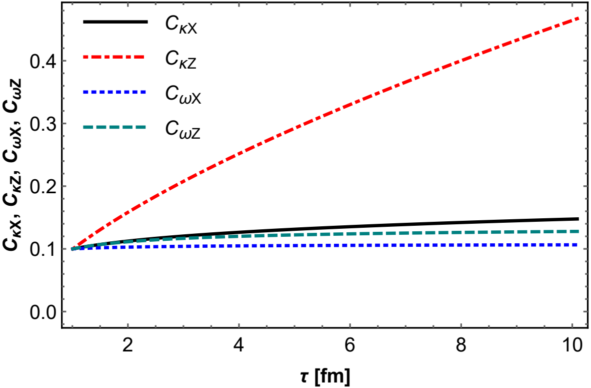

Figure: Proper-time dependence of the coefficients $C_{\kappa X}$, $C_{\kappa Z}$, $C_{\omega X}$ and $C_{\omega Z}$. The coefficients $C_{\kappa Y}$ and $C_{\omega Y}$ satisfy the same differential equations as the coefficients $C_{\kappa X}$ and $C_{\omega X}$.

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Spin polarization coefficients evolution for Non-boost flow:

For non-boost invariant set-up: $C_{\kappa X}$, $C_{\omega Y}$ and $C_{\kappa Y}$, $C_{\omega X}$ are coupled to each other.

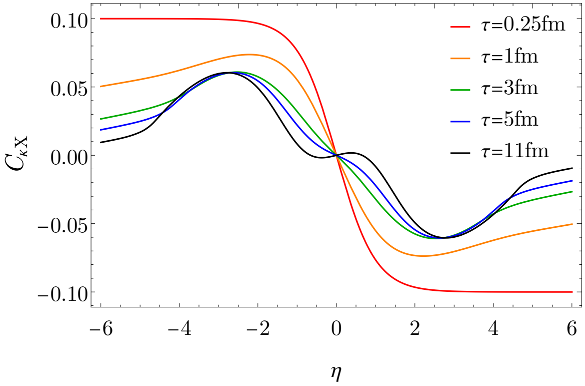


Figure: $C_{\kappa X}$ evolution

Spin polarization coefficients evolution for Non-boost flow:

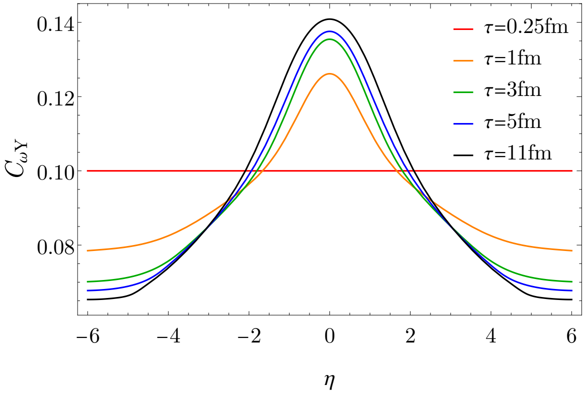
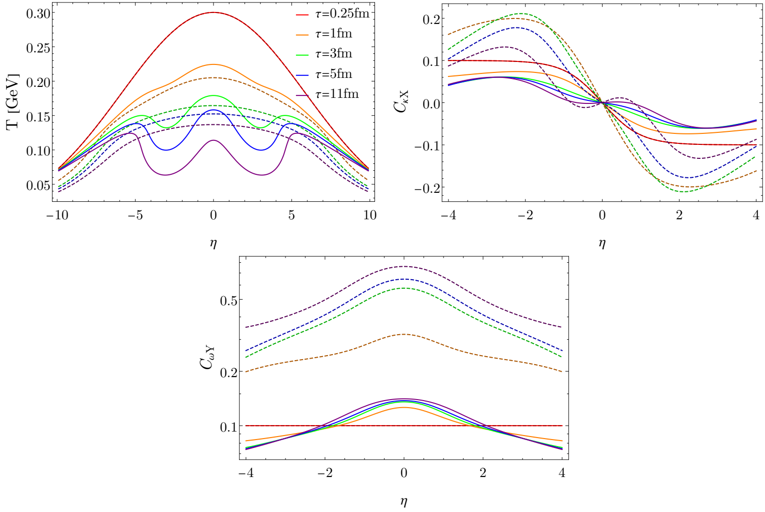


Figure: $C_{\omega Y}$ evolution

QCD EoS in Background:



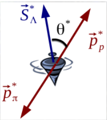
Measuring polarization in experiment:

Parity-violating decay of hyperons

Daughter baryon is preferentially emitted in the direction of hyperon's spin (opposite for anti-particle)

$$\frac{dN}{d\Omega^*} = \frac{1}{4\pi}(1 + \alpha_{\mathrm{H}}\mathbf{P}_{\mathrm{H}} \cdot \mathbf{p}_{\mathbf{p}}^*)$$

P_H: Λ polarization
p_p^*: proton momentum in the Λ rest frame
α_H: Λ decay parameter
($\alpha_\Lambda = -\alpha_{\bar{\Lambda}} = 0.642 \pm 0.013$)



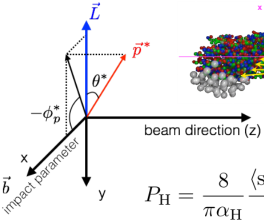
$$\Lambda \to p + \pi^-$$

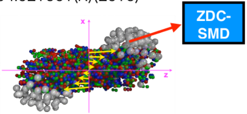
(BR: 63.9%, $c\tau$ ~7.9 cm)

C. Patrignani et al. (PDG), Chin. Phys. C 40, 100001 (2016)

Projection onto the transverse plane

Angular momentum direction can be determined by spectator deflection (spectators deflect outwards)
- S. Voloshin and TN, PRC94.021901(R)(2016)



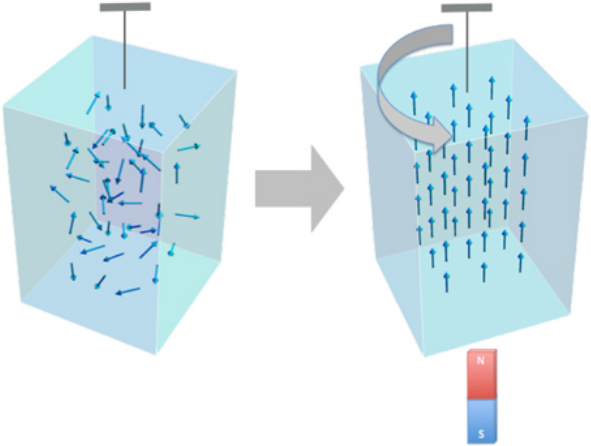


$$P_{\mathrm{H}} = \frac{8}{\pi\alpha_{\mathrm{H}}}\frac{\langle\sin(\Psi_1 - \phi_p^*)\rangle}{\mathrm{Res}(\Psi_1)}$$

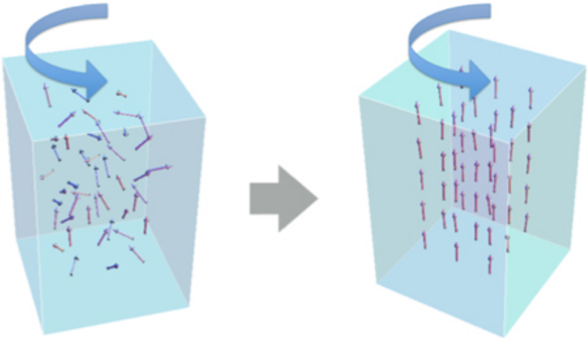
Ψ_1: azimuthal angle of b
ϕ_p^*: ϕ of daughter proton in Λ rest frame
STAR, PRC76, 024915 (2007)

Source: T. Niida, WWND 2019

Einstein-De Haas Effect (1915): Rotation induced by Magnetization



Barnett Effect (1915): Magnetization induced by Rotation



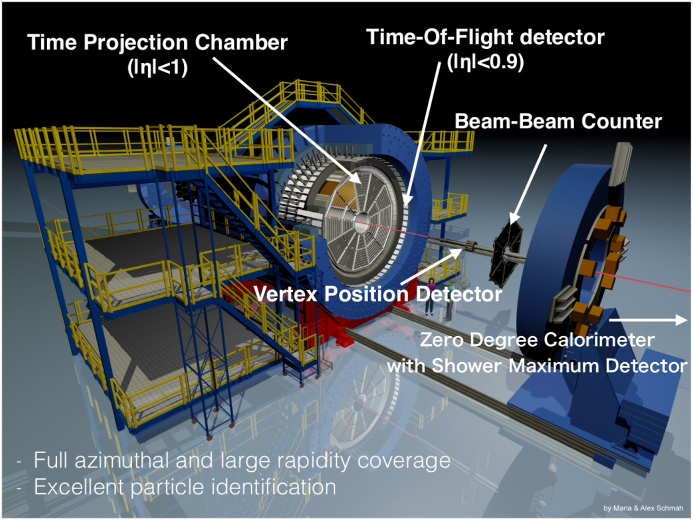


Figure: Schematic view of STAR Detector

Pseudo-gauge transformations

$$\begin{aligned}\hat{T}^{\mu\nu} &= \hat{T}_{\mathrm{C}}^{\mu\nu} + \frac{1}{2}\partial_\lambda(\hat{\Phi}^{\lambda,\mu\nu} + \hat{\Phi}^{\nu,\mu\lambda} + \hat{\Phi}^{\mu,\nu\lambda}) \\ \hat{S}^{\lambda,\mu\nu} &= \hat{S}_{\mathrm{C}}^{\lambda,\mu\nu} - \hat{\Phi}^{\lambda,\mu\nu} + \partial_\rho \hat{Z}^{\mu\nu,\lambda\rho}\end{aligned}$$

where, $\hat{\Phi}^{\lambda,\mu\nu}$ and $\hat{Z}^{\mu\nu,\lambda\rho}$ are arbitrary differentiable operators called super-potentials satisfying
$\hat{\Phi}^{\lambda,\mu\nu} = -\hat{\Phi}^{\lambda,\nu\mu}$ and $\hat{Z}^{\mu\nu,\lambda\rho} = -\hat{Z}^{\nu\mu,\lambda\rho} = -\hat{Z}^{\mu\nu,\rho\lambda}$

$\rightarrow$ The newly defined tensors preserve the total energy, linear momentum, and angular momentum after integrated over the freeze-out hypersurface.
$\rightarrow$ Conservation laws are unchanged.

Spin polarization coefficient evolution equations:

Contracting the spin conservation equation with $U_\beta X_\gamma, U_\beta Y_\gamma, U_\beta Z_\gamma, Y_\beta Z_\gamma, X_\beta Z_\gamma$ and $X_\beta Y_\gamma$.

$$\begin{bmatrix} \mathcal{L}(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{L}(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{L}(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{P}(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{P}(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{P}(\tau) \end{bmatrix} \begin{bmatrix} \dot{C}_{\kappa X} \\ \dot{C}_{\kappa Y} \\ \dot{C}_{\kappa Z} \\ \dot{C}_{\omega X} \\ \dot{C}_{\omega Y} \\ \dot{C}_{\omega Z} \end{bmatrix} = \begin{bmatrix} \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 & 0 \\ 0 & \mathcal{Q}_1(\tau) & 0 & 0 & 0 & 0 \\ 0 & 0 & \mathcal{Q}_2(\tau) & 0 & 0 & 0 \\ 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 & 0 \\ 0 & 0 & 0 & 0 & \mathcal{R}_1(\tau) & 0 \\ 0 & 0 & 0 & 0 & 0 & \mathcal{R}_2(\tau) \end{bmatrix} \begin{bmatrix} C_{\kappa X} \\ C_{\kappa Y} \\ C_{\kappa Z} \\ C_{\omega X} \\ C_{\omega Y} \\ C_{\omega Z} \end{bmatrix},$$

where,

$\mathcal{L}(\tau) = \mathcal{A}_1 - \frac{1}{2}\mathcal{A}_2 - \mathcal{A}_3,$

$\mathcal{P}(\tau) = \mathcal{A}_1,$

$\mathcal{Q}_1(\tau) = -\left[\dot{\mathcal{L}} + \frac{1}{\tau}\left(\mathcal{L} + \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{Q}_2(\tau) = -\left(\dot{\mathcal{L}} + \frac{\mathcal{L}}{\tau}\right),$

$\mathcal{R}_1(\tau) = -\left[\dot{\mathcal{P}} + \frac{1}{\tau}\left(\mathcal{P} - \frac{1}{2}\mathcal{A}_3\right)\right],$

$\mathcal{R}_2(\tau) = -\left(\dot{\mathcal{P}} + \frac{\mathcal{P}}{\tau}\right).$

$$\mathcal{A}_1 = \cosh(\xi)\left(n_{(0)} - \mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_2 = \cosh(\xi)\left(\mathcal{A}_{(0)} - 3\mathcal{B}_{(0)}\right),$$
$$\mathcal{A}_3 = \cosh(\xi)\,\mathcal{B}_{(0)}$$

Conformal symmetry:

In general, for a system to respect conformal symmetry, its dynamics should be invariant under Weyl rescaling. It implies that the (m, n)-type tensors (including scalars with $(m, n) = (0, 0)$) transform homogeneously, namely

$$A^{\mu_1 \cdots \mu_m}_{\nu_1 \ldots \nu_n}(x) \rightarrow \Omega^{\Delta_A} A^{\mu_1 \cdots \mu_m}_{\nu_1 \ldots \nu_n}(x)$$

where $\Omega \equiv e^{-\varphi(x)}$ with $\varphi(x)$ being function of space-time coordinates and $\Delta_A = [A] + m - n$ is the conformal weight of the quantity A, where $[A]$ is its mass dimension, and m and n being the number of contravariant and covariant indices, respectively.

Transformation rules:

The transformation rules to map the quantities expressed in de Sitter coordinates back to the polar Milne coordinates can be written as

$$
\begin{aligned}
U_\mu(\tau, r) &= \tau \frac{\partial \hat{x}^\nu}{\partial x^\mu}\, \hat{U}_\nu(\rho)\,, \\
\mathcal{E}(\tau, r) = \frac{\hat{\mathcal{E}}(\rho)}{\tau^4}, \quad \mathcal{P}(\tau, r) &= \frac{\hat{\mathcal{P}}(\rho)}{\tau^4}, \quad \mathcal{N}(\tau, r) = \frac{\hat{\mathcal{N}}(\rho)}{\tau^3}\,, \\
T(\tau, r) = \frac{\hat{T}(\rho)}{\tau}, &\qquad \mu(\tau, r) = \frac{\hat{\mu}(\rho)}{\tau}.
\end{aligned}
$$

Conformal transformation of conservation equations:

For the 4D spacetime the conservation law for net baryon number is already conformal-frame independent, i.e. net baryon number is conserved in both Minkowski and de Sitter space-times. In this case, one can write

$$d_\alpha N^\alpha = \Omega^4 \hat{d}_\alpha \hat{N}^\alpha$$

Conservation of energy and linear momentum transforms as

$$d_\alpha T^{\alpha\beta} = \Omega^6 \left[\hat{d}_\alpha \hat{T}^{\alpha\beta} - \hat{T}^\lambda{}_\lambda \hat{g}^{\beta\delta} \partial_\delta \varphi \right]$$

We see that $\hat{T}^{\alpha\beta}$ needs to be traceless in order to be conserved in de Sitter spacetime. Therefore, the breaking of conformal invariance is characterized only by the trace of the energy-momentum tensor

Conformal transformation of the conservation law for spin takes the form

$$d_\alpha S^{\alpha\beta\gamma} \;\; = \;\; \Omega^6 \left[\hat{d}_\alpha \hat{S}^{\alpha\beta\gamma} - (\hat{S}_\lambda{}^{\lambda\gamma} \hat{g}^{\beta\sigma} + \hat{S}^{\alpha\beta}{}_\alpha \hat{g}^{\sigma\gamma}) \partial_\sigma \varphi \right].$$

We find that the conformal invariance of the spin conservation law requires the spin tensor to satisfy the condition $\hat{S}_\alpha{}^{\alpha\beta} = 0$.

Gubser flow

- Solve the perfect-fluid hydrodynamical equations using the Gubser flow.

- Obtain analytical solutions for T and μ.

- Derive the equations of motion for spin polarization components in de Sitter coordinates.

- The background solutions are not spoiled by the breaking of the symmetry at the level of angular momentum conservation.

- The coupling between the spin polarization coefficients emerge due to the conformal symmetry breaking.

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Space-time evolution of Temperature:

$T(\tau_0 = 1\,\mathrm{fm}, r = 0) = 1.2\,\mathrm{fm}^{-1}$

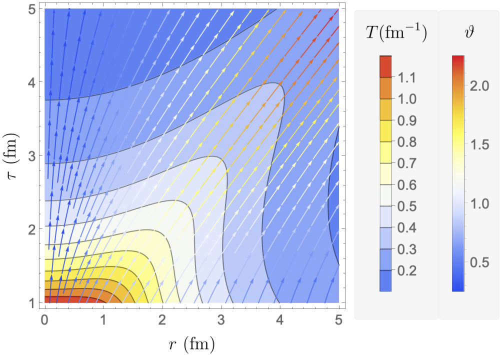

Figure: The space-time dependence of temperature (contours) and flow-vector components $(U^\tau, U^r)/\sqrt{(U^\tau)^2 + (U^r)^2}$ (stream lines – the coloring of arrows is given by the rapidity ϑ).

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Spin polarization coefficients:

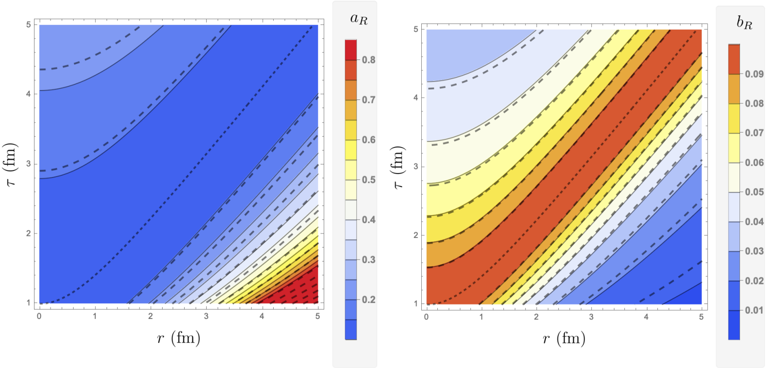

Figure: Numerical solutions for a_R and b_R components of the spin polarization tensor as functions of proper time τ and radial distance r.

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Bjorken-expanding resistive MHD background

Spin polarization dynamics:

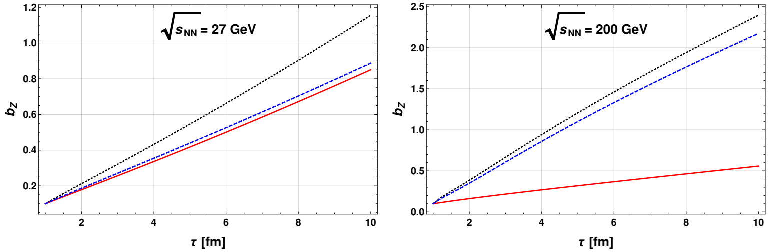

Figure: Spin polarization coefficient b_Z profile for $\sqrt{s_{\rm NN}} = 27\,\mathrm{GeV}$ (left panel) and $\sqrt{s_{\rm NN}} = 200\,\mathrm{GeV}$ (right panel) with initial value $b_Z^0 = 0.1$. The modification of the b_Z evolution slope due to electric field is much more pronounced when μ_0/T_0 is small as can be seen in the right panel. Dotted black line is for $\alpha = -8$, red line is for $\alpha = 0$ and dashed blue line is for $\alpha = 8$.

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