Recent Progress in Partial-Wave Analysis

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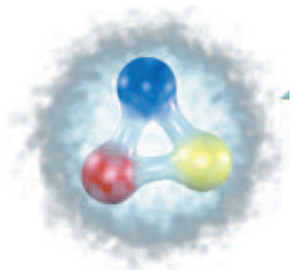
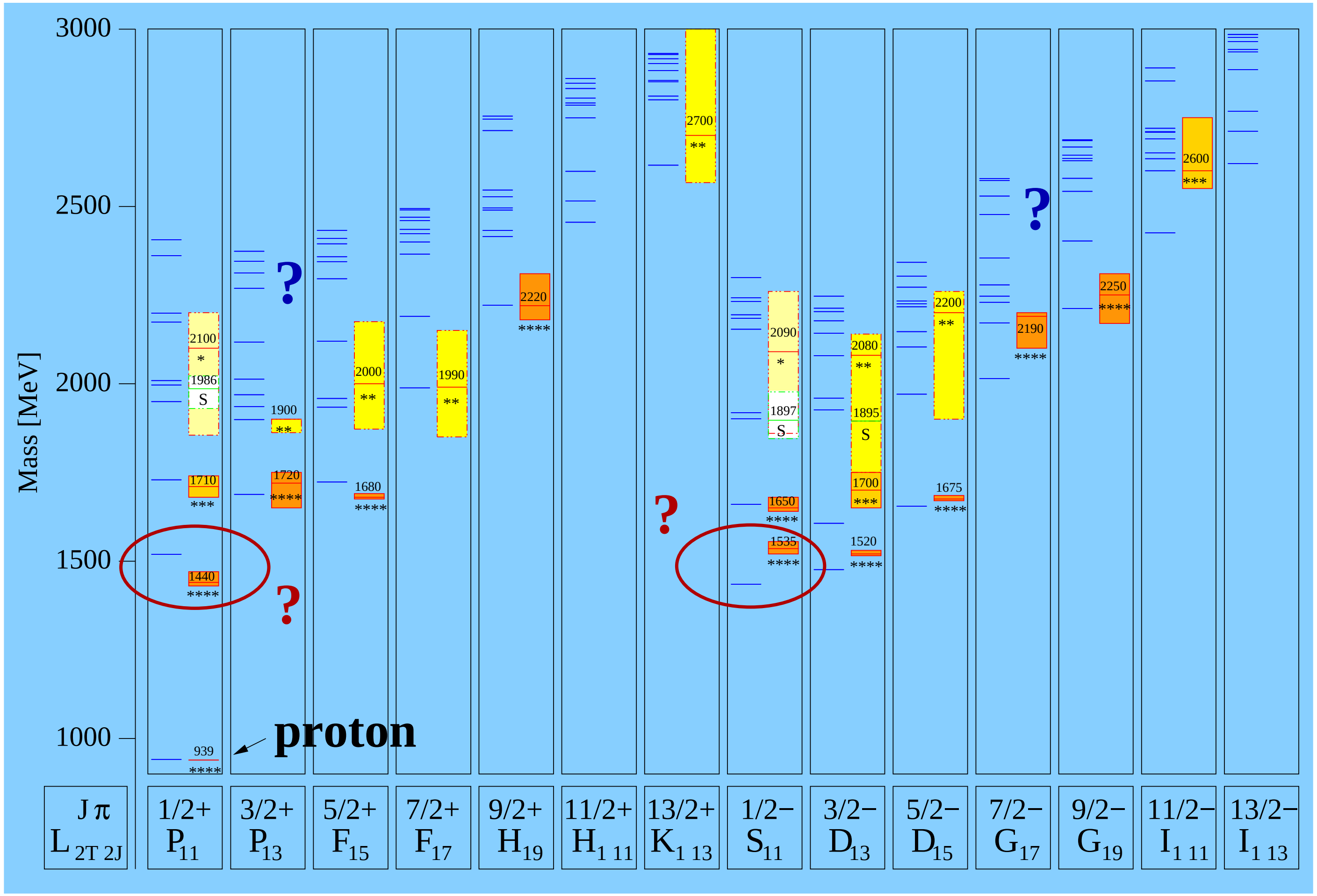
N^*- resonances in the quark model

Nukleon

10^{-15} m



U. Loering, B. Metsch, H. Petry et al. (Bonn)



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Constituent quarks

Confinement-potential

Residual interaction

Search for light baryon states

1. **Single and double meson production in pion-induced reactions:**

   The old data on $\pi N \to \pi N, \eta N, K\Lambda, K\Sigma$ **and**

   new HADES data on $\pi^- p \to \pi\pi N$

2. **Single and double meson photoproduction reactions.**

   $\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma, \pi\pi N, \pi\eta N$, ωp, $K^*\Lambda$,

   CB-ELSA, CLAS, MAMI GRAAL, LEPS.

3. **The decay of the heavy meson and baryon states** $\Psi' \to \pi^0 p\bar{p}, \eta p\bar{p}$ **(BES III),**

   $\Lambda_b \to J/\Psi K p$ **(LHCB).**

4. **The hyperon production in the kaon-nucleon collision reactions**

   $K^- p \to K^- p, K_0 n, \pi\Lambda, \pi\Sigma, \pi\pi\Lambda, \pi\pi\Sigma, K\pi N$

Meson Photoproduction experiments

- **GRAAL (Grenoble): Polarized beam. Ideal for the beam asymmetry and double polarization observables for hyperon final states.** $\gamma p \to \pi^0 p, \eta p, K\Lambda, K\Sigma, \pi^0\pi^0 p, \pi^0\eta p, \omega p$, $\gamma n \to \pi^0 n, \eta n$. **Energy up to 1.8 GeV. Good quality polarized beam.**

- **CLAS (JLAB): High statistic, very good detector of charged particles:** $\gamma p \to \pi^- n, K\Lambda, K\Sigma, \pi^+\pi^- p, \omega p$. **As missing mass data** $\gamma p \to \pi^0 p, \eta p$. **Polarized beam and target. Electro-production data. Data on deuterium target. Energy is up to W=2.7 GeV. Polarized beam and target.**

- **MAMI (Mainz): High statistic, very good detector of neutral particles: (Crystal Ball):** $\gamma p \to \pi^0 p, \eta p, K\Lambda, K\Sigma, \pi^0\pi^0 p, \pi^0\eta p$, $\gamma n \to \eta n, \pi^0 n, \pi^0\pi^0 n, \pi^0\eta n$. **Energy is only up to W=1.95 GeV. Polarized beam and target.**

- **CB-ELSA (Bonn): Moderate statistic, very good detector of neutral particles: (Crystal Barrel):** $\gamma p \to \pi^0 p, \eta p, \pi^0\pi^0 p, \pi^0\eta p, \omega p$, $\gamma n \to \eta n, \pi^0 n$. **Energy is up to W=2.3 GeV. Polarized beam and target.**

- **For the full reconstruction of the amplitude it is necessary to measure 8 observables.**
- **Single polarization data are available** Σ, T
- **Double polarization data are available** E, G, H, F, P **.**
- **For the production of the** Λ **and** Σ **hyperons all observables can be measured.**

Photon		**Target**			**Recoil**			**Target + Recoil**			
	$-$	$-$	$-$	$-$	x'	y'	z'	x'	x'	z'	z'
	$-$	x	y	z	$-$	$-$	$-$	x	z	x	z
unpol.	σ_0	0	T	0	0	P	0	$T_{x'}$	$-L_{x'}$	$T_{z'}$	$L_{z'}$
lin.pol.	$-\Sigma$	H	$-P$	$-G$	$O_{x'}$	$-T$	$O_{z'}$	$-L_{z'}$	$T_{z'}$	$-L_{x'}$	$-T_{x'}$
circ.pol.	0	F	0	$-E$	$-C_{x'}$	0	$-C_{z'}$	0	0	0	0

Baryon sector: the partial wave analysis groups

- **SAID (GWU,USA): Analysis of elastic πN data in energy independent method and then in the K-matrix approach. Fit of the $\gamma n \to \pi N, \eta N$ data in the framework of K-matrix/P-vector approach. Recently: including dispersion corrections to the analysis.**
- **MAID (Mainz): Energy dependent analysis of photoproduction data on γN to πN, ηp, $\eta' p$,$K\Lambda$, $K\Sigma$. Parameterization of partial waves as a sum of Breit-Wigner amplitudes with unitarisation procedure. Development of the approach which takes into account crossing symmetry and dispersion corrections.**
- **Bonn-Gatchina: Energy dependent analysis of pion induced (inelastic) and almost all photoproduction data. K-matrix/P-vector and now N/D-dispersion approach. Minimization: χ^2 for 2 body final state and maximum likelihood for multi-body final states. Combined fit of all available data.**
- **Juelich group: Energy dependent covariant approach. Pion induced data (elastic and inelastic), $\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma$. Unitarity, analyticity and chiral constraints. Solution of the Bethe-Salpeter equation.**

- **ANL-OSAKA Energy dependent covariant approach. Pion induced data (elastic and inelastic), $\gamma p \to \pi N, \eta N$ (all data) and $\gamma p \to K\Lambda, K\Sigma$. Unitarity and analyticity constraints. Solution of the Bethe-Salpeter equation. Analysis of the KN collision data with one meson in the final state.**

- **M. Manley (Kent Uni) Energy dependent covariant approach. Pion induced data (elastic and inelastic including 2pion production), $\gamma p \to \pi N$ (all data) and $\gamma p \to K\Lambda, K\Sigma$. Unitarity and analyticity constraints.**

Bonn-Gatchina partial wave analysis group:

A. Anisovich, E. Klempt, K. Nikonov, A. Sarantsev, U. Thoma.

http://pwa.hiskp.uni-bonn.de/

Petersburg Nuclear Physics Institute

Bonn-Gatchina Partial Wave Analysis

Address: Nussallee 14-16, D-53115 Bonn   Fax: (+49) 228 / 73-2505

Data Base   **Meson Spectroscopy**   **Baryon Spectroscopy**   **NN-interaction**   **Formalism**

Analysis of Other Groups

- SAID
- MAID
- Giessen Uni

BG PWA

- Publications
- Talks
- Contacts

Useful Links

- SPIRES
- PDG Homepage
- Durham Data Base
- Bonn Homepage

CB-ELSA Homepage

Responsible: Dr. V. Nikonov, E-mail: *nikonov@hiskp.uni-bonn.de*
Last changes: January 26th, 2010.

Energy dependent fully covariant approach

In many cases an unambiguous partial wave decomposition at fixed energies is impossible. Then the energy and angular parts should be analyzed together:

$$A(s,t) = \sum_{\beta\beta' n} A_n^{\beta\beta'}(s) Q^{(\beta)+}_{\mu_1\ldots\mu_n} F^{\mu_1\ldots\mu_n}_{\nu_1\ldots\nu_n} Q^{(\beta')}_{\nu_1\ldots\nu_n}$$

πN **interaction:**

$$Q^{(+n)}_{\mu_1\ldots\mu_n} = X^{(n)}_{\mu_1\ldots\mu_n} \qquad Q^{(-n)}_{\mu_1\ldots\mu_n} = i\gamma_\nu\gamma_5 X^{(n+1)}_{\nu\mu_1\ldots\mu_n}$$

$$X^0 = 1\,; \quad X^1_\mu = k^\perp_\mu\,; \quad X^2_{\mu\nu} = \frac{3}{2}\left(k^\perp_\mu k^\perp_\nu - \frac{1}{3}\,k^2_\perp g^\perp_{\mu\nu}\right);$$

$$X^3_{\mu\nu\alpha} = \frac{5}{2}\left[k^\perp_\mu k^\perp_\nu k^\perp_\alpha - \frac{k^2_\perp}{5}\left(g^\perp_{\mu\nu}k^\perp_\alpha + g^\perp_{\mu\alpha}k^\perp_\nu + g_{\nu\alpha}k^\perp_\mu\right)\right],$$

1. C. Zemach, Phys. Rev. 140, B97 (1965); 140, B109 (1965).

2. S.U.Chung, Phys. Rev. D 57, 431 (1998).

A. V. Anisovich, V. V. Anisovich, V. N. Markov, M. A. Matveev and A. V. Sarantsev, J. Phys. G 28, 15 (2002)

3. B. S. Zou and D. V. Bugg, Eur. Phys. J. A 16, 537 (2003)

Minimization methods

1. **The two body final states $\pi N, \gamma N \to \pi N, \eta N, K\Lambda, K\Sigma, \omega N, K^*\Lambda$: χ^2 method.**

   For n measured bins we minimize

$$\chi^2 = \sum_j^n \frac{(\sigma_j(PWA) - \sigma_j(exp))^2}{(\Delta\sigma_j(exp))^2}$$

   Present solution $\chi^2 = 54634$ for 33988 points. $\chi^2/N_F = 1.6$

2. **Reactions with three or more final states are analyzed with logarithm likelihood method. $\pi N, \gamma N \to \pi\pi N, \pi\eta N$. The minimization function:**

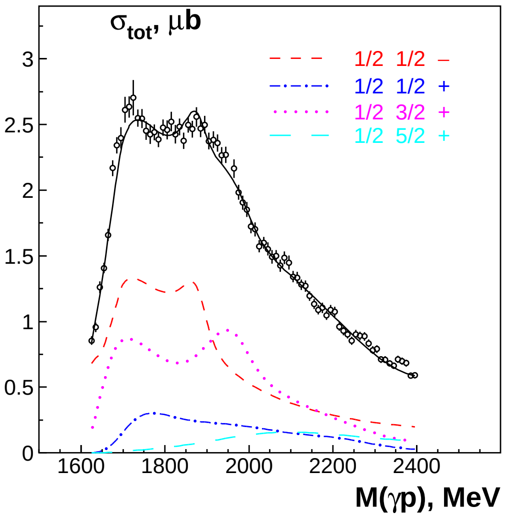
$$f = -\sum_j^{N(data)} ln \frac{\sigma_j(PWA)}{\sum\limits_m^{N(rec\,MC)} \sigma_m(PWA)}$$

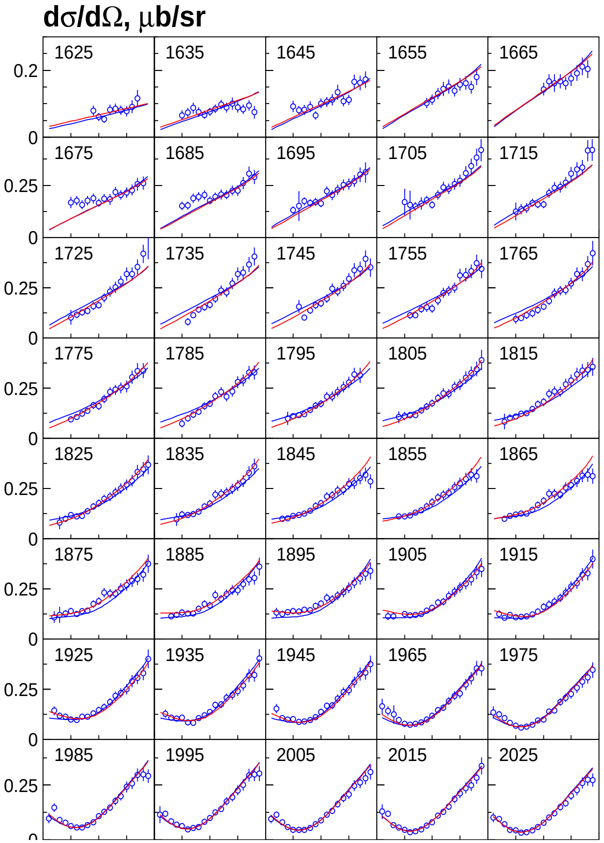
   This method allows us to take into account all correlations in many dimensional phase space. Above 1 000 000 data events are taken in the fit.

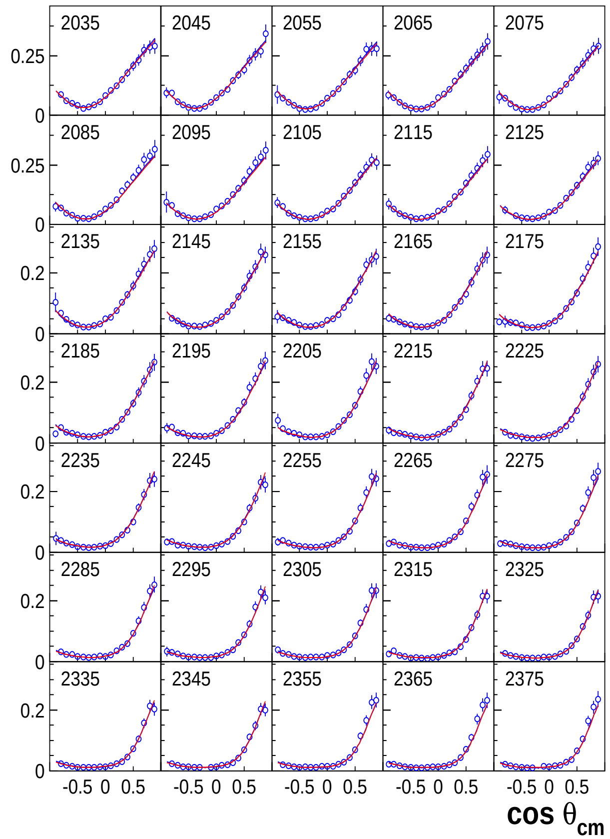
The included meson photoproduction data

DATA	2011-2016	added in 2016-2020
$\gamma n \to \Lambda K, \Sigma^- K$		$\frac{d\sigma}{d\Omega}$ **(CLAS), E (CLAS)**
$\gamma n \to \pi^- p$	$\frac{d\sigma}{d\Omega}, \Sigma, P$	E, Σ **(CLAS)**
$\gamma n \to \eta n$ $\gamma p \to \eta p$ $\gamma p \to \eta' p$	$\frac{d\sigma}{d\Omega}, \Sigma$ $\frac{d\sigma}{d\Omega}, \Sigma (GRAAL)$	$\frac{d\sigma}{d\Omega}$ **(MAMI)** $\frac{d\sigma}{d\Omega}(h = \frac{1}{2})$ **(CB-ELSA)** $\frac{d\sigma}{d\Omega}, F, T$ **(MAMI)** T, P, H, G,**(CB-ELSA)** E, Σ **(CB-ELSA,CLAS)** $\frac{d\sigma}{d\Omega}, \Sigma$
$\gamma p \to K^+ \Lambda$ $\gamma p \to K^+ \Sigma^0$	$\frac{d\sigma}{d\Omega}, \Sigma, P, T, C_x, C_z, O_{x'}, O_{z'}$ $\frac{d\sigma}{d\Omega}, \Sigma, P, C_x, C_z$	Σ, P, T, O_x, O_z **(CLAS)** Σ, P, T, O_x, O_z **(CLAS)**
$\pi^- p \to \pi^+ \pi^- n$ $\pi^- p \to \pi^- \pi^0 p$		$d\sigma/d\Omega$ **(HADES)** $d\sigma/d\Omega$ **(HADES)**
$\gamma p \to \pi^0 \pi^0 p$ $\gamma p \to \pi^+ \pi^- p$	$d\sigma/d\Omega, \Sigma, E, I_c, I_s$	T, P, H, F, P_x, P_y **(CB-ELSA)** $d\sigma/d\Omega, I_c, I_s$ **(CLAS)**
$\gamma p \to \omega p$	$d\sigma/d\Omega, \Sigma, \rho^k_{ij}, E, G$ **(CB-ELSA)**	Σ **(CLAS) P,T,F,H (CLAS)**
$\gamma p \to K^* \Lambda$		$d\sigma/d\Omega$, ρ_{ij}

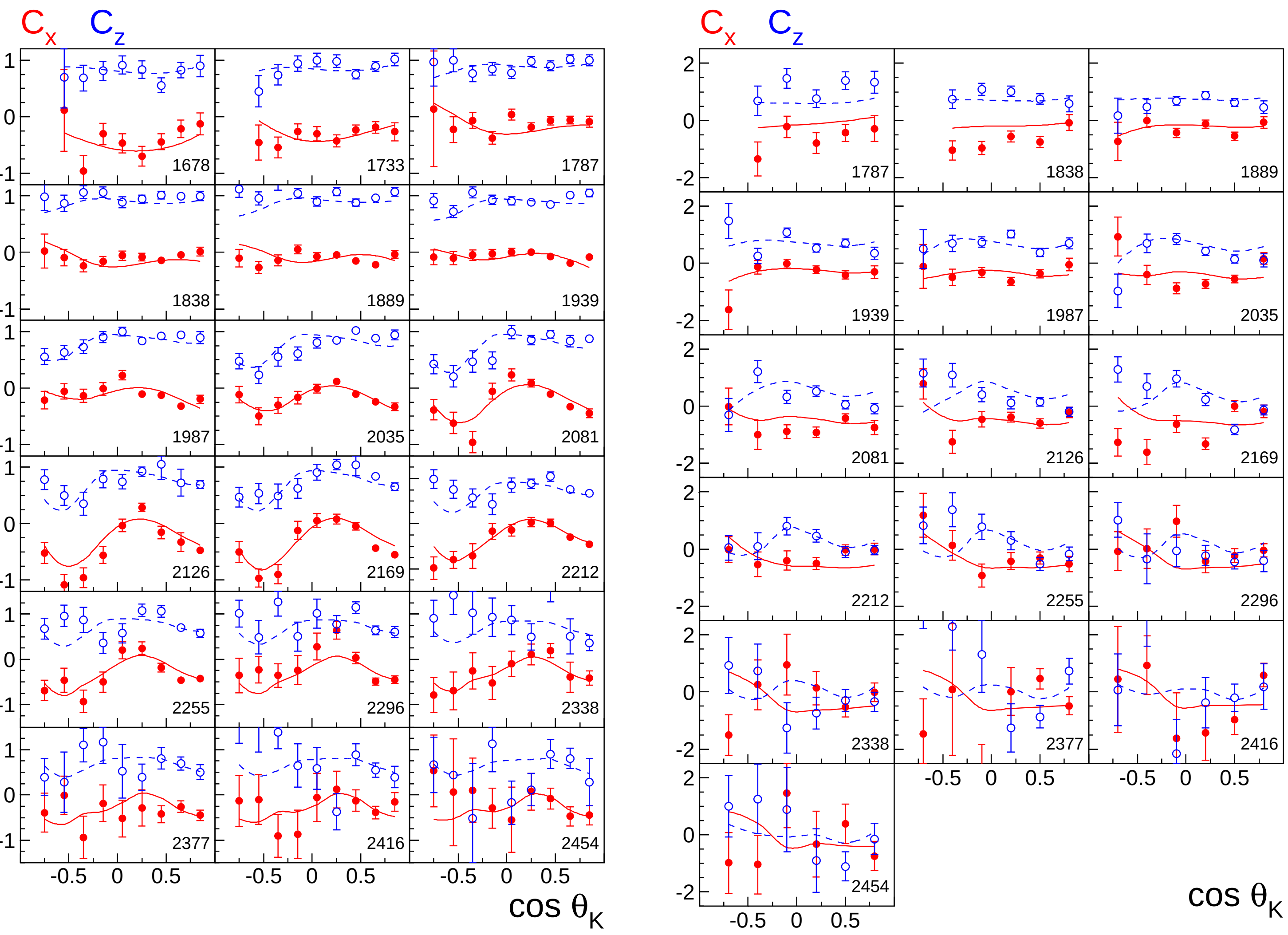
The $\gamma p \to K\Lambda$ reaction (CLAS 2009)







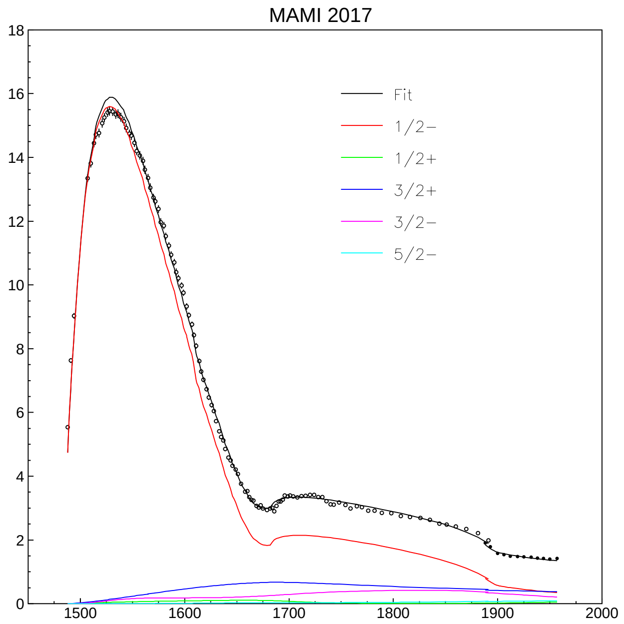
New S_{11} state with mass 1890 ± 10 MeV and width 90 ± 10 MeV improves description of the data.

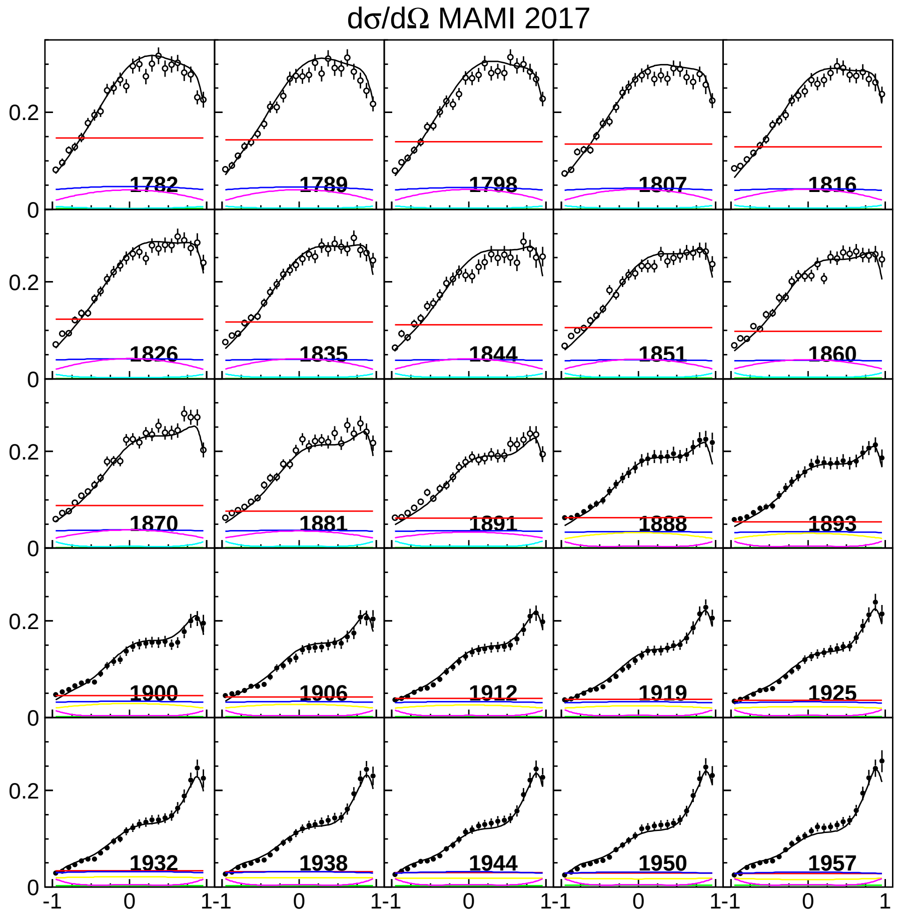


C_x (black circle) and C_z (open circle) for $\gamma \mathrm{p} \to \Lambda \mathrm{K}^+$. The solid and dashed curves are results of our fit obtained with solution 1 (left) and solution 2 (right) for C_x and C_z.

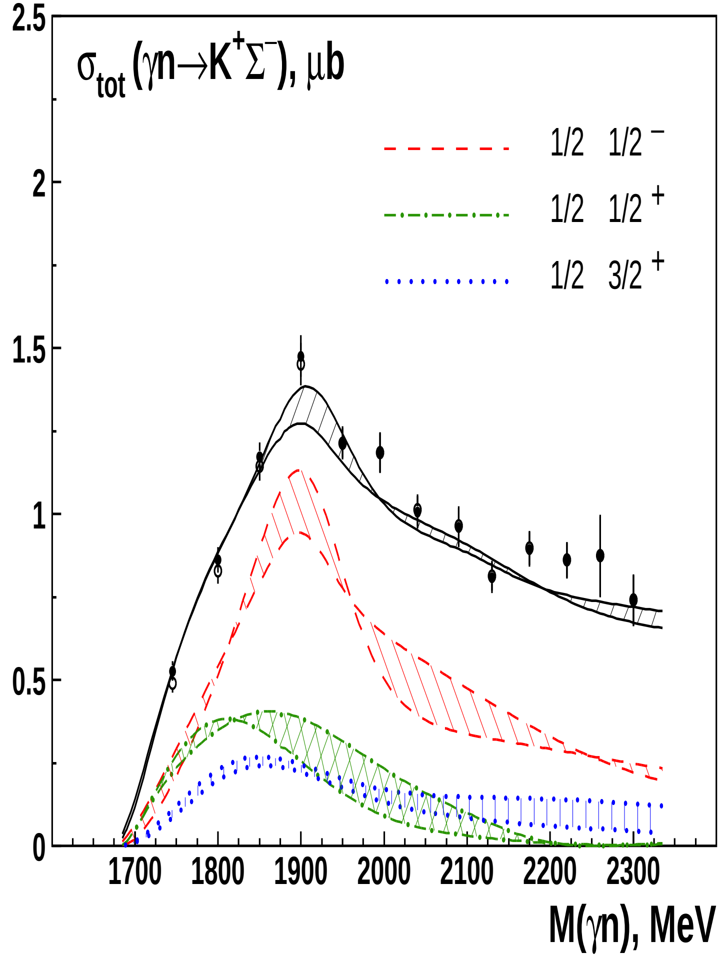
The analysis of the new $\gamma p \to \eta p$ data.

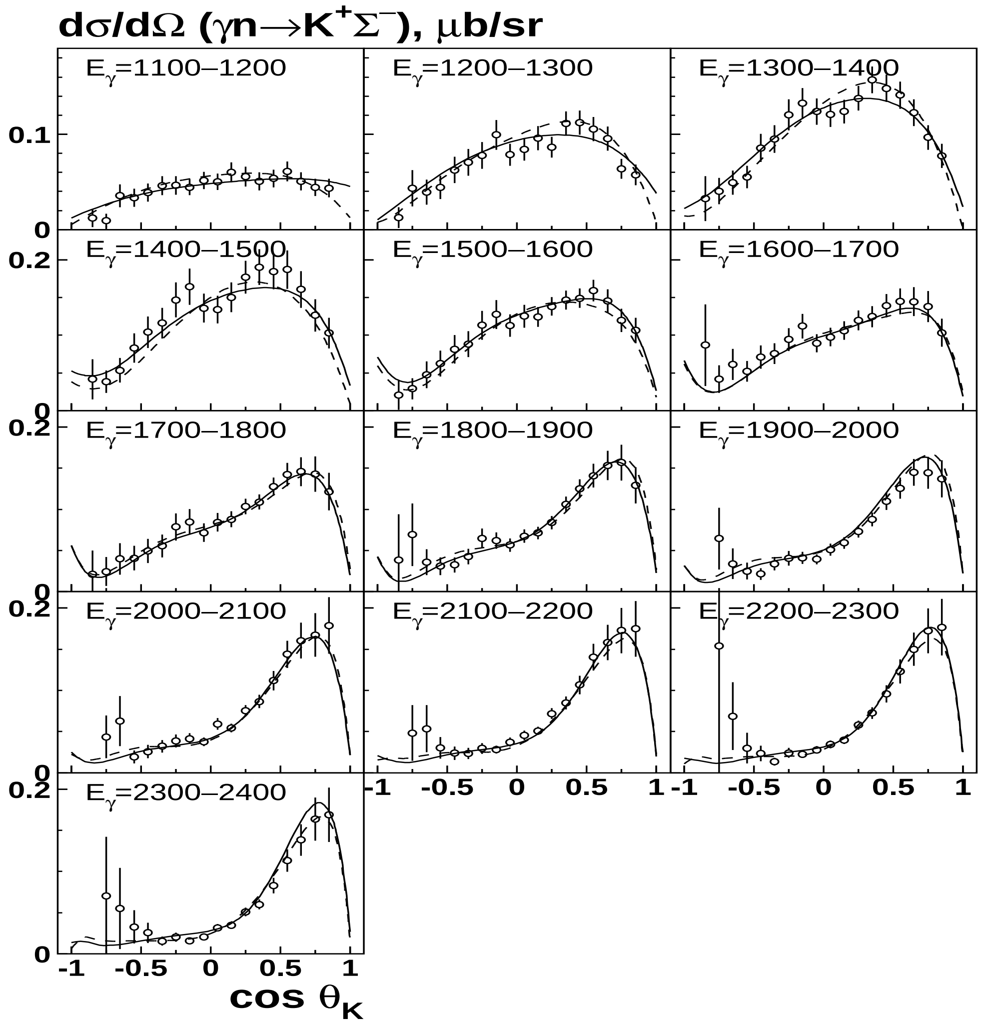
New MAMI data: a strong cusp effect from the $\eta' p$ channel





The analysis of the $\gamma n \to K^+\Sigma^-$ data (Practically no free parameters)

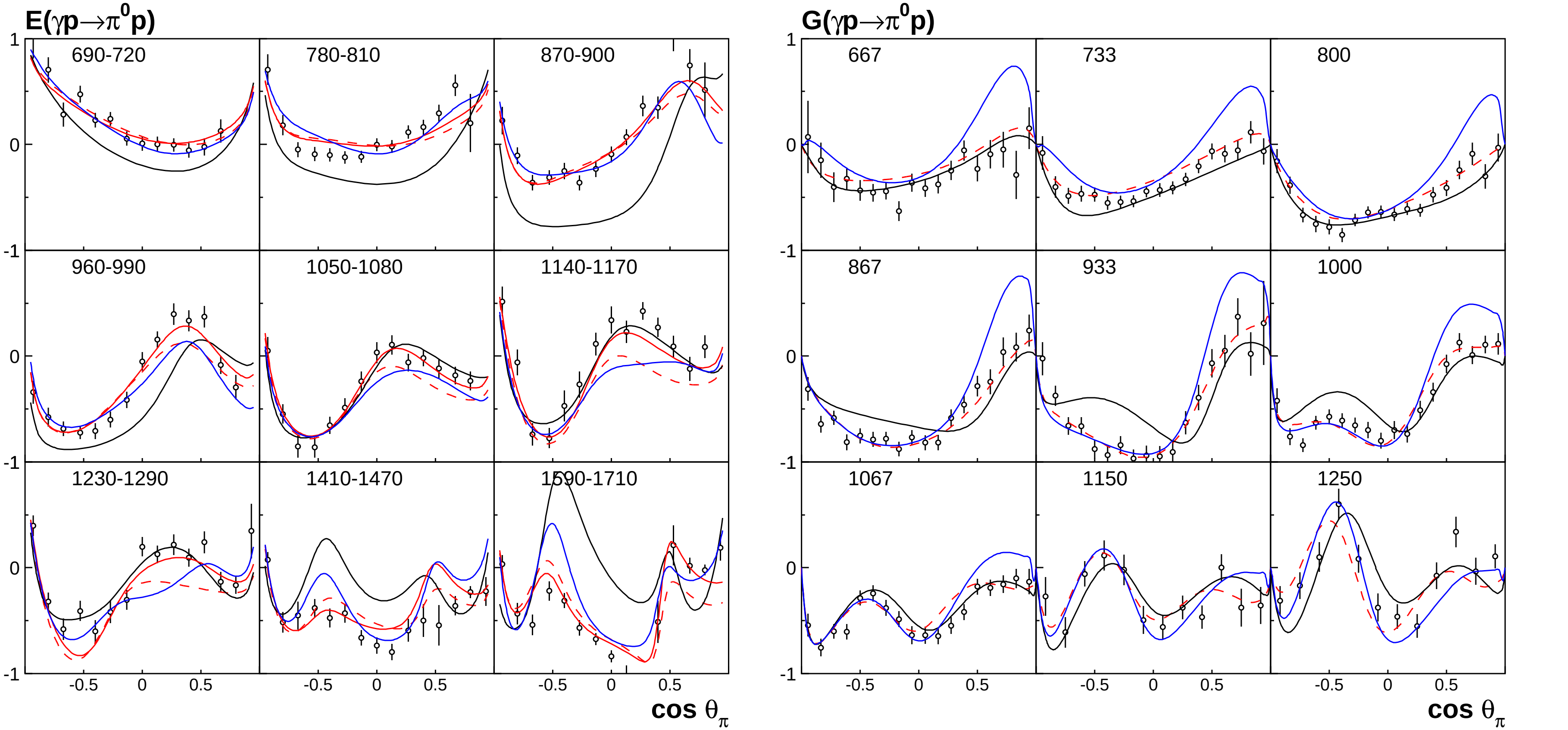




Clear contributions from the $S_{11}(1895)$ states.

double polarization data from CB-ELSA:

Helicity asymmetry and G observable in $\gamma p \to \pi^0 p$

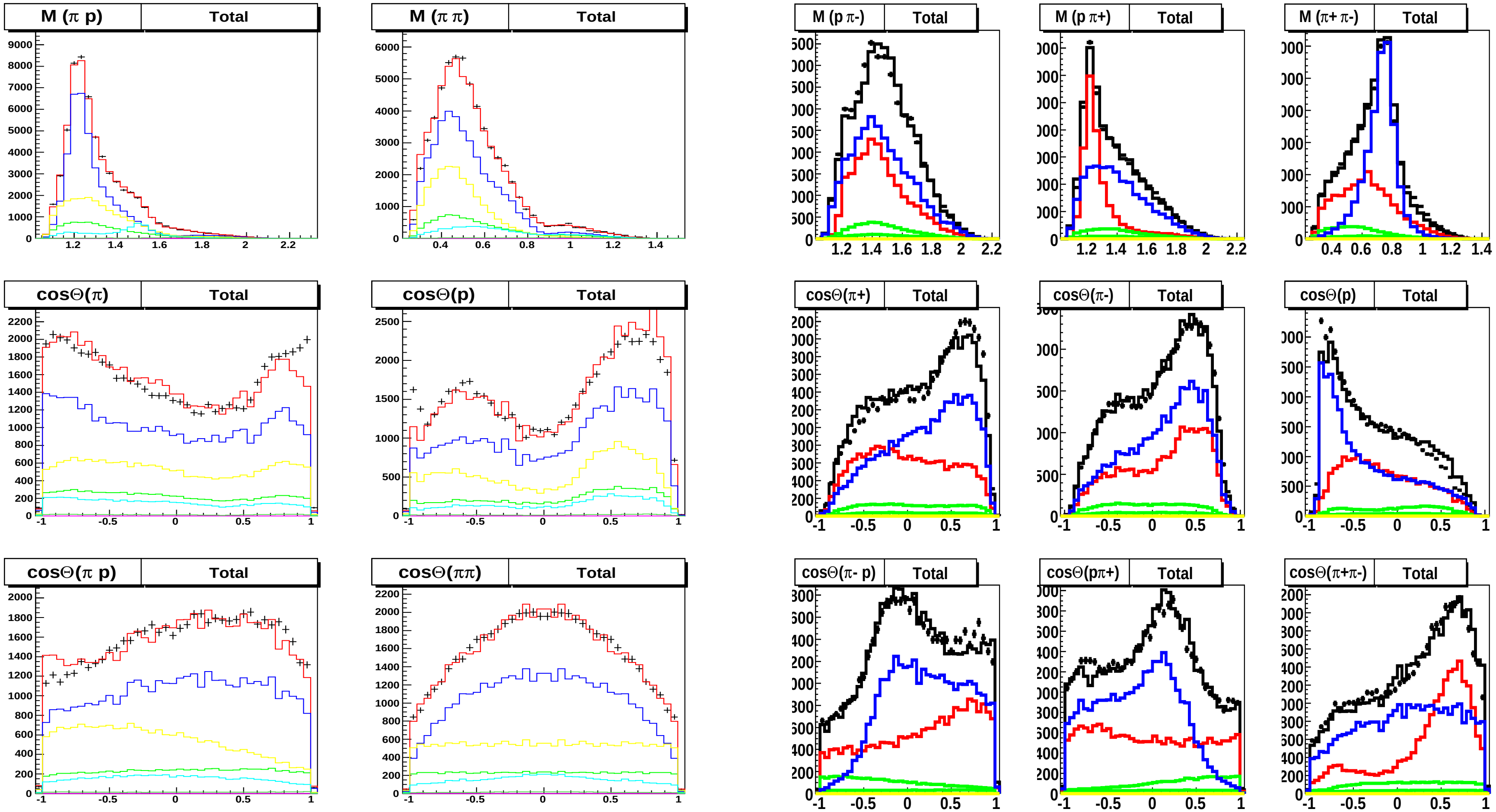


Bonn-Gatchina, SAID (CM12), MAID **Bonn-Gatchina, SAID (CM12), MAID dashed - predicted, full -fit**

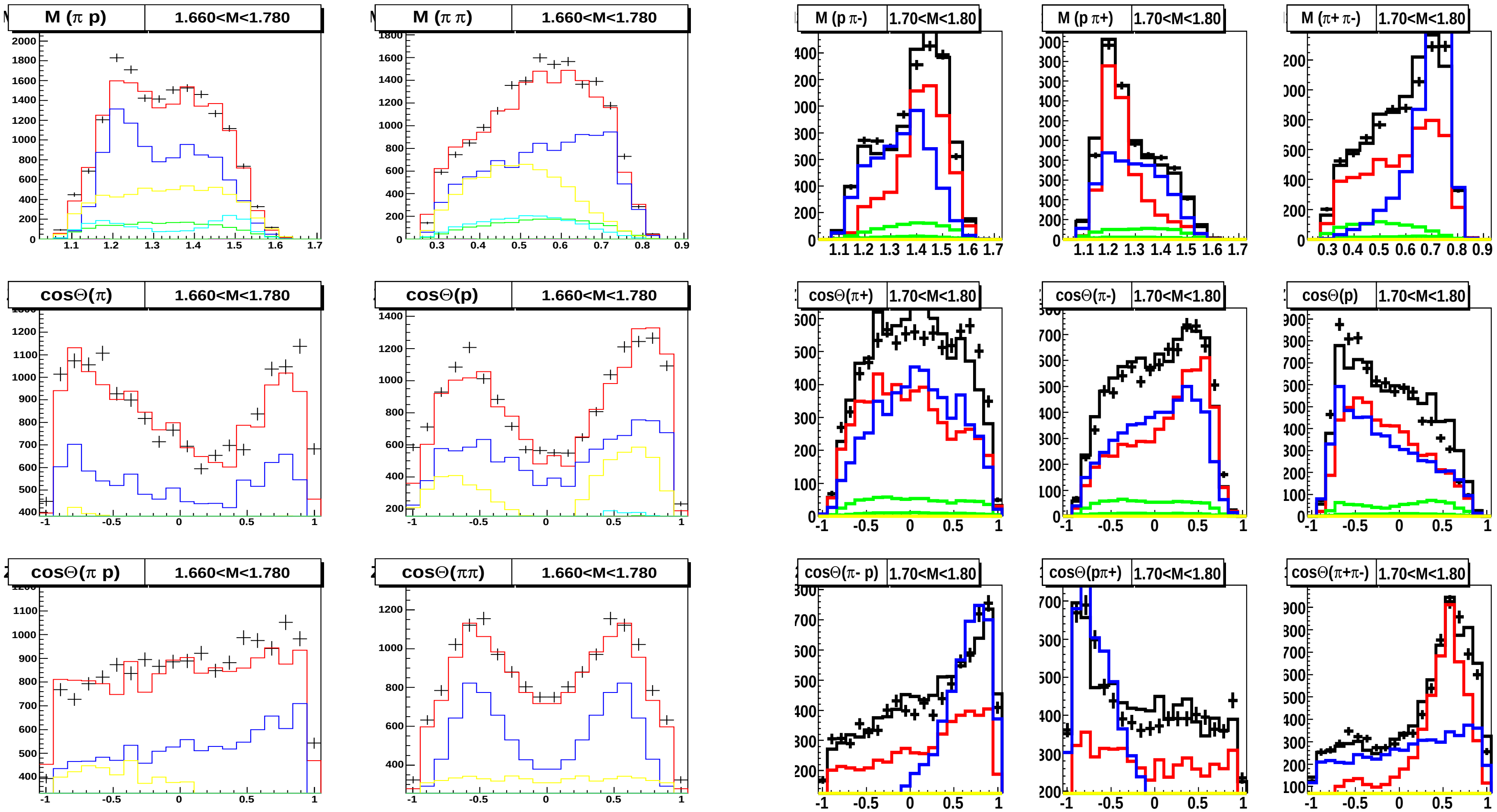
A. Thiel *et al.*, Phys. Rev. Lett. 109, 102001 (2012)

M. Gottschall *et al.*, "First measurement of the helicity asymmetry for $\gamma p \to p\pi^0$ ..."

The description of the $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^+\pi^- p$ data (preliminary)

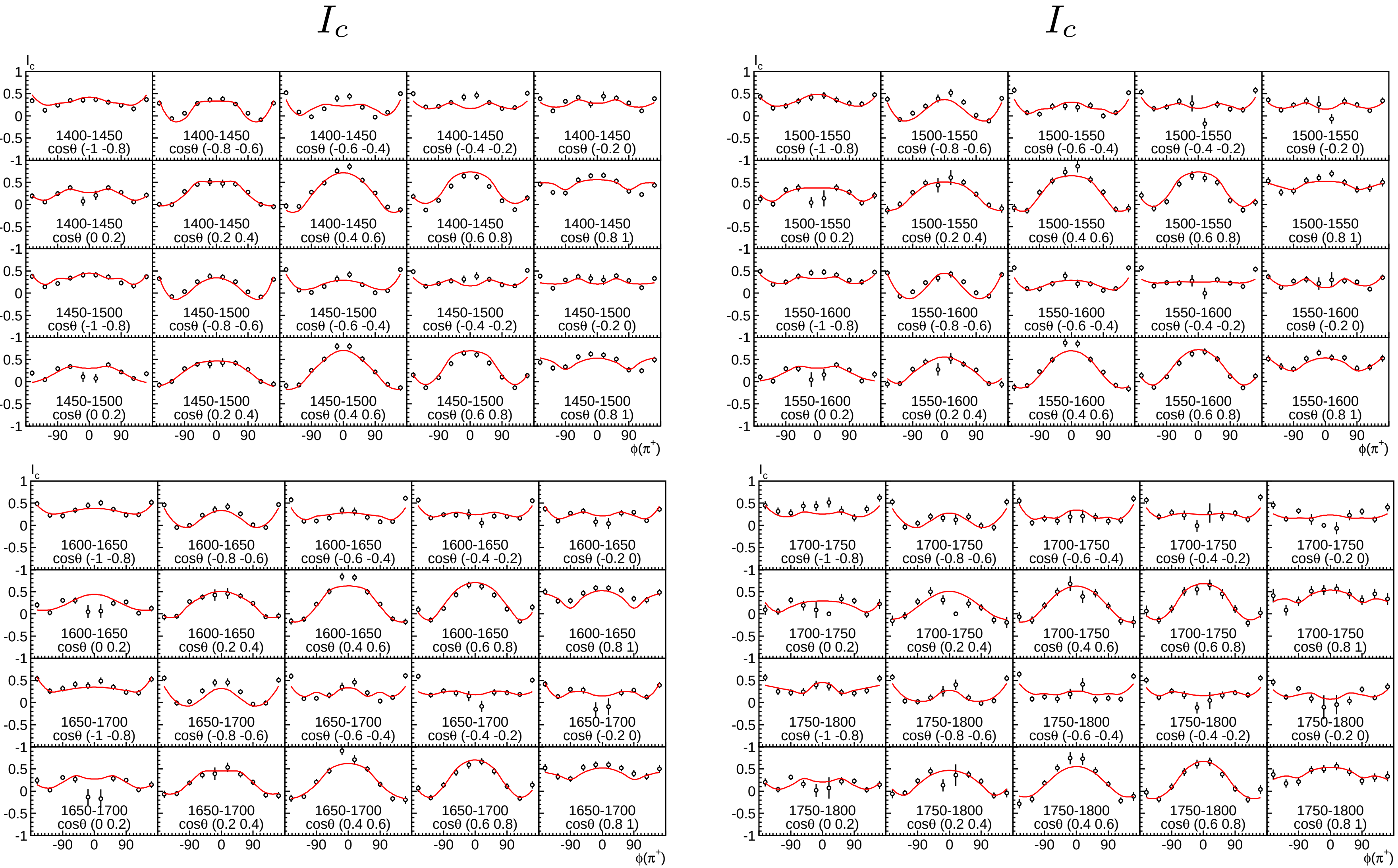


The description of the $\gamma p \to \pi^0\pi^0 p$ and $\gamma p \to \pi^+\pi^- p$ data for 1700 MeV$< W$,1800 MeV (preliminary)

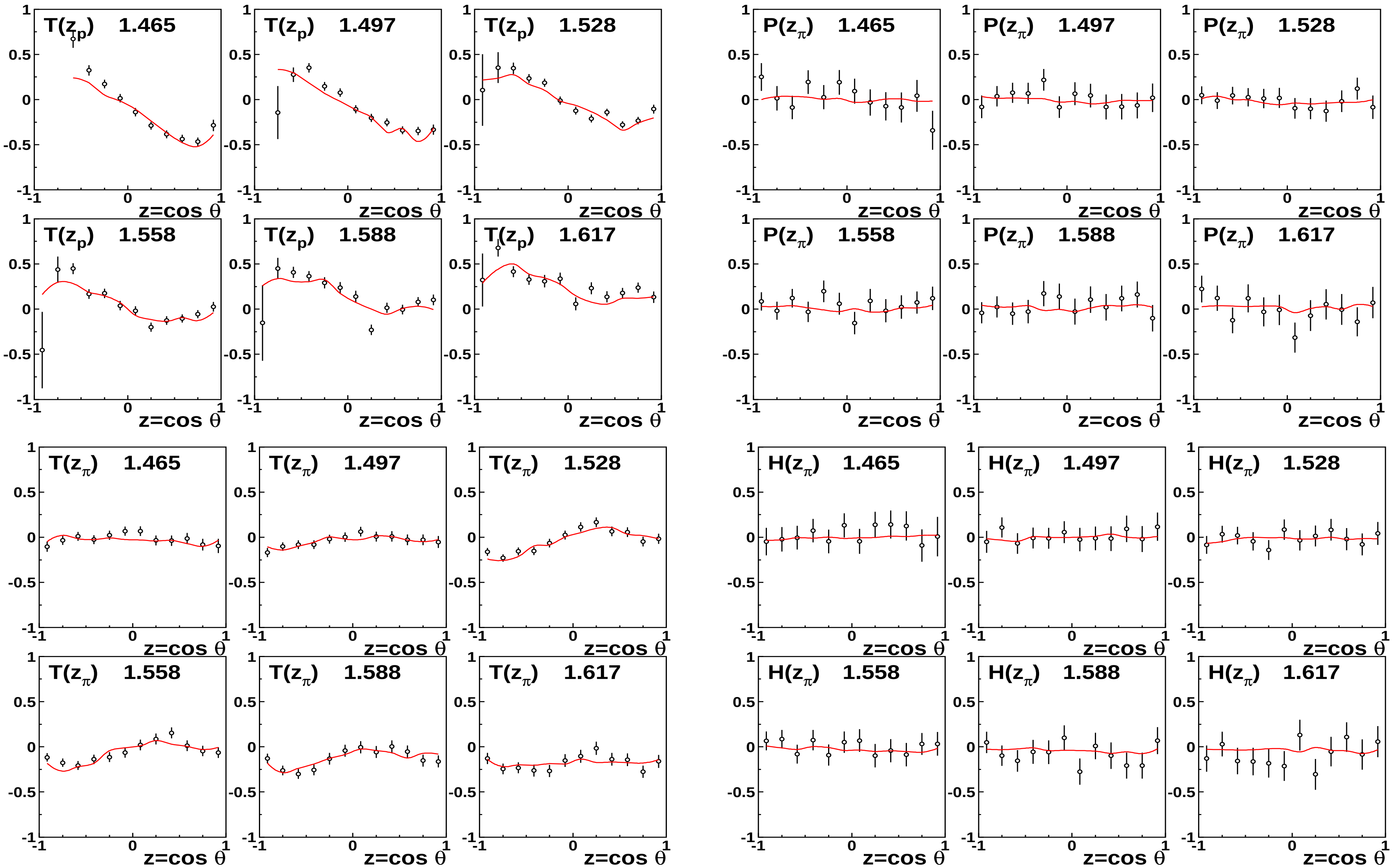


I_c and I_s for $\gamma p \to \pi^+\pi^- p$ from CLAS

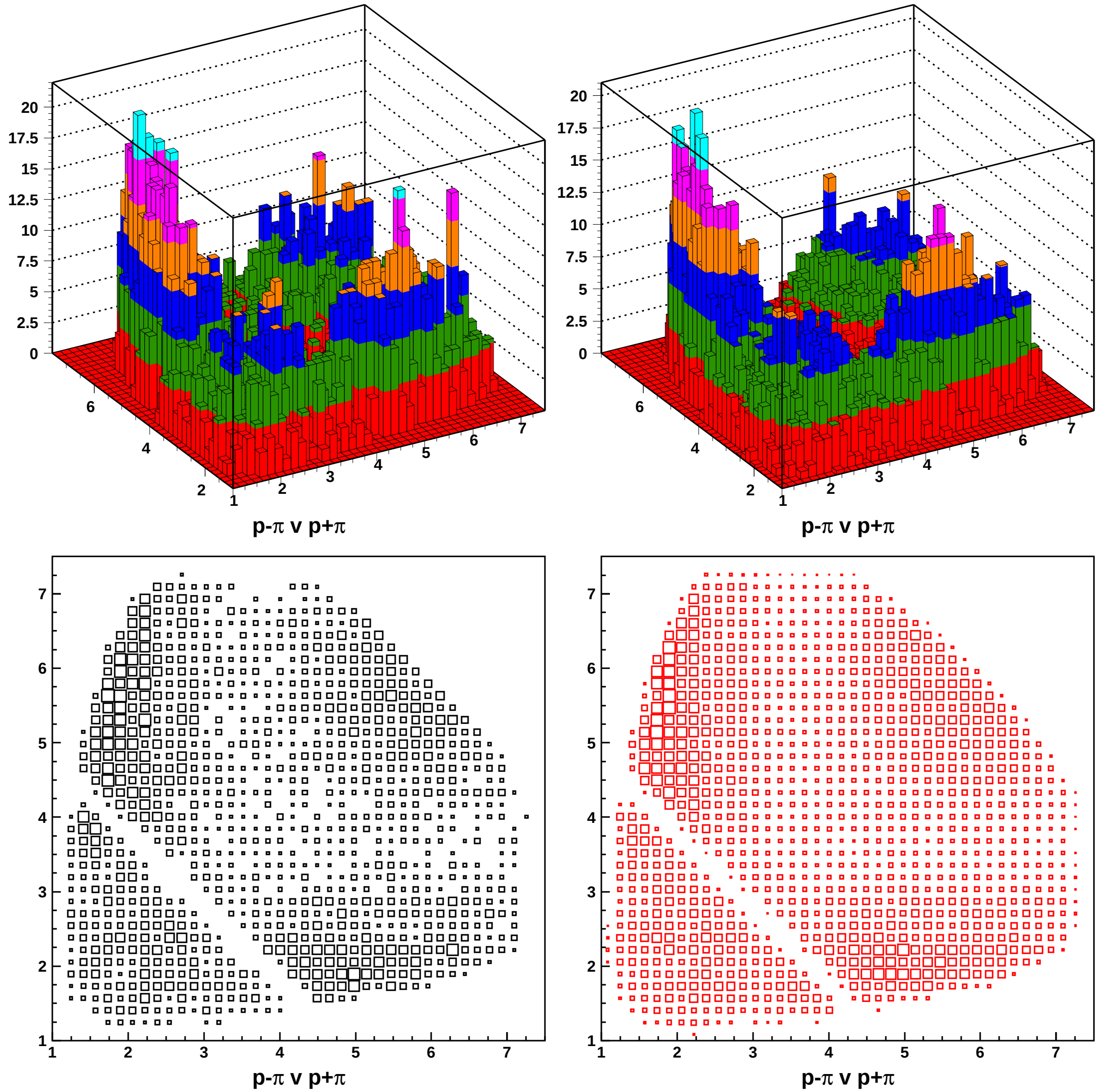
Courtesy of V. Crede, Florida State U



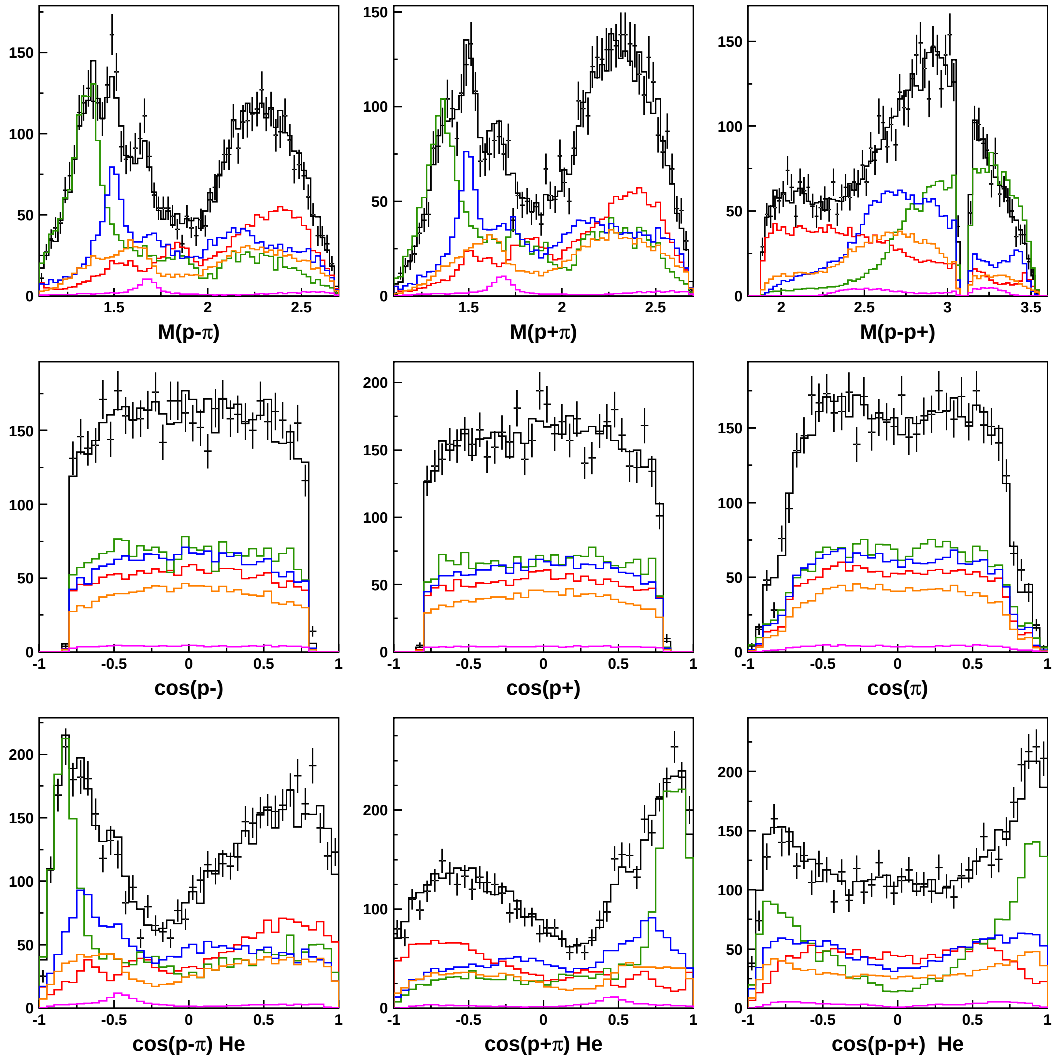
Prediction for H, P, T ($\gamma p \to \pi^0\pi^0 p$) from CB-ELSA (Preliminary)



The analysis of the BESIII data on $\Psi' \to p\bar{p}\pi^0$. BESII reported the observation of 2 new states: $N(2300)1/2^+$,$N(2570)5/2^-$

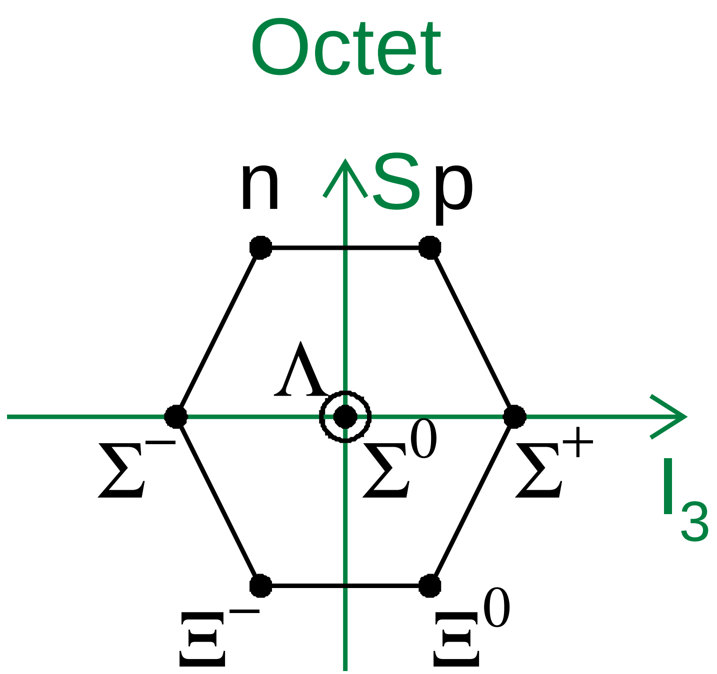


The analysis of the BESIII data on $\Psi' \to p\bar{p}\pi^0$



Resonance	Rating	N_{pp}	Resonance	Rating	N_{pp}	Resonance	Rating	N_{pp}
$N(1440)1/2^+$	****	13	$N(1520)3/2^-$	****	17	$N(1535)1/2^-$	****	15
$N(1650)1/2^-$	****	18	$N(1675)5/2^-$	****	14	$N(1680)5/2^+$	****	17
$N(1685)$	*		$N(1700)3/2^-$	***	15	$N(1710)1/2^+$	***	14
$N(1720)3/2^+$	****	17	$N(1860)5/2^+$	**	9	$N(1875)3/2^-$	***	16
$N(1880)1/2^+$	***	20	$N(1895)1/2^-$	****	17	$N(1900)3/2^+$	****	18
$N(1990)7/2^+$	**	9	$N(2000)5/2^+$	**	11	$N(2040)3/2^+$	*	
$N(2060)5/2^-$	**	13	$N(2100)1/2^+$	*		$N(2150)3/2^-$	**	11
$N(2190)7/2^-$	****	11	$N(2220)7/2^-$	****	7	$N(2250)9/2^-$	****	
$N(2600)11/2^-$	***		$N(2700)13/2^+$	**				
$\Delta(1232)$	****	8	$\Delta(1600)3/2^+$	***	12	$\Delta(1620)1/2^-$	****	10
$\Delta(1700)3/2^-$	****	11	$\Delta(1750)1/2^+$	*		$\Delta(1900)1/2^-$	**	13
$\Delta(1905)5/2^+$	****	11	$\Delta(1910)1/2^+$	****	13	$\Delta(1920)3/2^+$	***	21
$\Delta(1930)5/2^-$	***		$\Delta(1940)3/2^-$	*	5	$\Delta(1950)7/2^+$	****	13
$\Delta(2000)5/2^+$	**		$\Delta(2150)1/2^-$	*		$\Delta(2200)7/2^-$	*	
$\Delta(2300)9/2^+$	**		$\Delta(2350)3/2^-$	*		$\Delta(2390)7/2^+$	*	
$\Delta(2420)11/2^+$	****		$\Delta(2400)9/2^-$	****		$\Delta(2750)13/2^-$	**	
$\Delta(2950)15/2^+$	**							

$$3 \otimes 3 \otimes 3 = 10_S \oplus 8_M \oplus 8_M \oplus 1_A$$



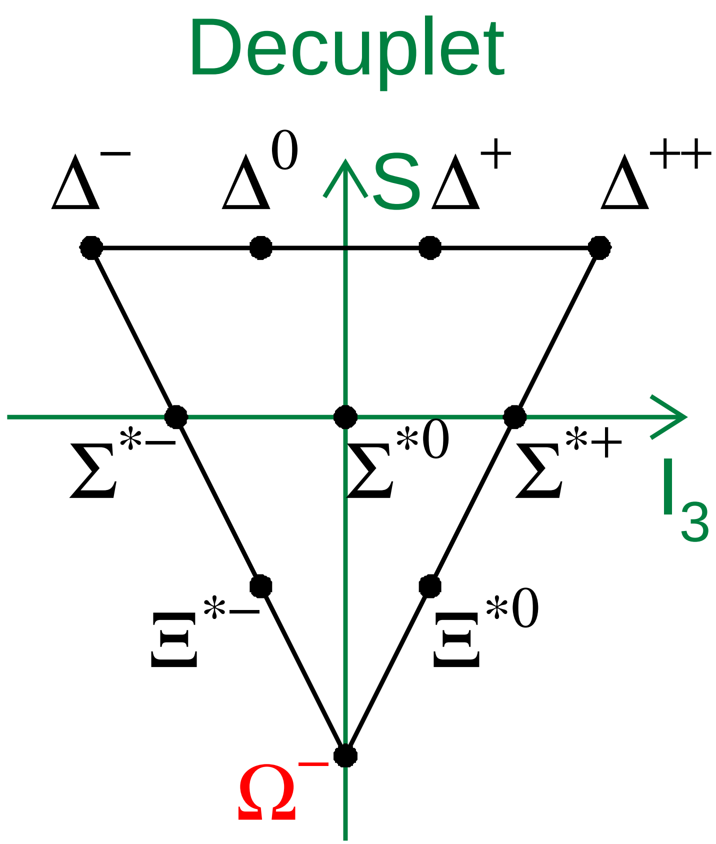


Tabelle 1: Σ-Hyperons used in the first fit of the data.

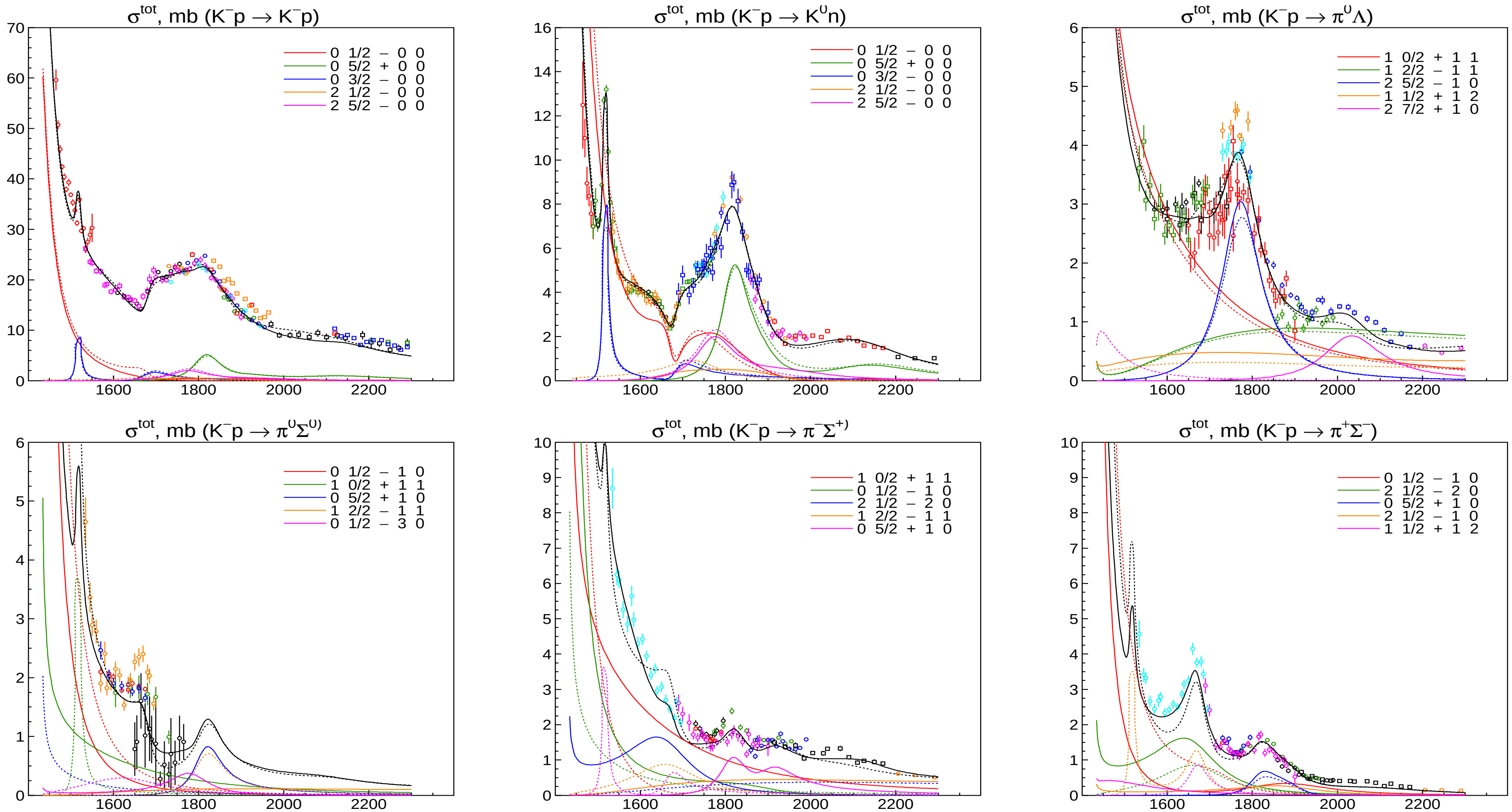
		J^P	**Status**	**Mass**	**Width**
$N(1440)$	$\Sigma(1660)$	$1/2^+$	***	$1630 - 1690$	$40 - 200$
$\Delta(1230)$	$\Sigma(1385)$	$3/2^+$	****	1382.80 ± 0.35	36.0 ± 0.7
$N(1680), \Delta(1905)$	$\Sigma(1915)$	$5/2^+$	****	$1900 - 1935$	$80 - 160$
$N(1990), \Delta(1950)$	$\Sigma(2030)$	$7/2^+$	****	$2025 - 2040$	$150 - 200$
$N(1520)$	$\Sigma(1670)$	$3/2^-$	****	$1665 - 1685$	$40 - 80$
$N(1535), \Delta(1620), N(1650)$	$\Sigma(1750)$	$1/2^-$	***	$1730 - 1800$	$60 - 160$
$N(1675)$	$\Sigma(1775)$	$5/2^-$	****	$1770 - 1780$	$105 - 135$
$N(1700), \Delta(1700)$	$\Sigma(1940)$	$3/2^-$	***	$1900 - 1950$	$150 - 300$

Many Σ-states are missing, although there are number of BUMPS

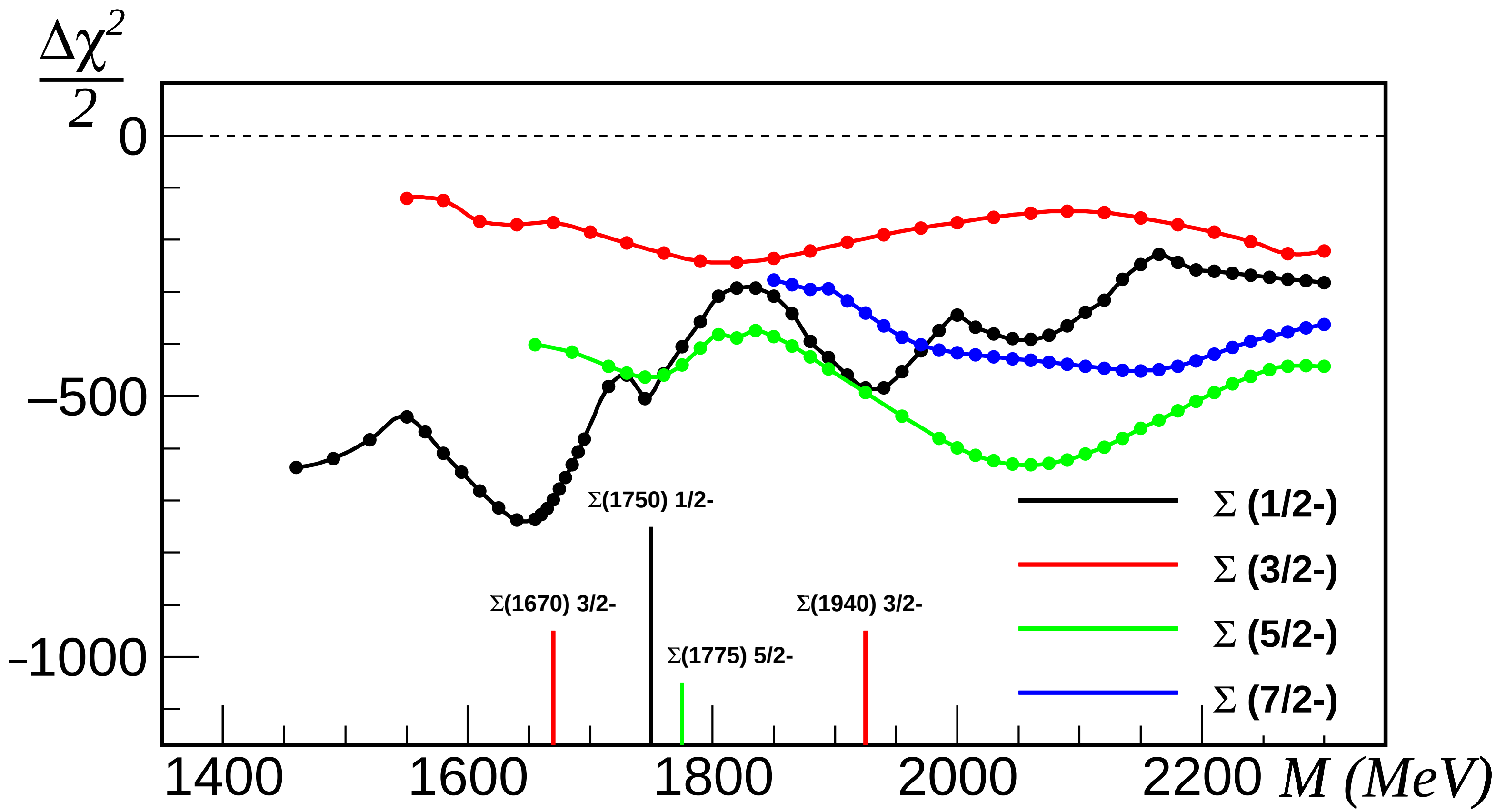
Tabelle 2: List of reactions used in the partial wave analysis.

$K^-p \to K^0 n$	$K^-p \to K^-p$	$K^-p \to \pi^0 \Lambda$
$K^-p \to \pi^0 \Sigma^0$	$K^-p \to \pi^- \Sigma^+$	$K^-p \to \pi^+ \Sigma^-$
$K^-p \to \omega \Lambda$	$K^-p \to \eta \Lambda$	$K^-p \to \pi^0 \pi^0 \Lambda$
$K^-p \to K^+ \Xi^-$	$K^-p \to K^0 \Xi^0$	$K^-p \to \pi^0 \pi^0 \Sigma^0$
$K^-p \to \pi^0 \Lambda(1520)$	$K^-p \to K^- \Delta^+(1232)$	$K^-p \to \pi^\pm \Sigma^\mp(1385)$
$K^-p \to K^*(890) N$		

Analysis of the Kp collision reactions



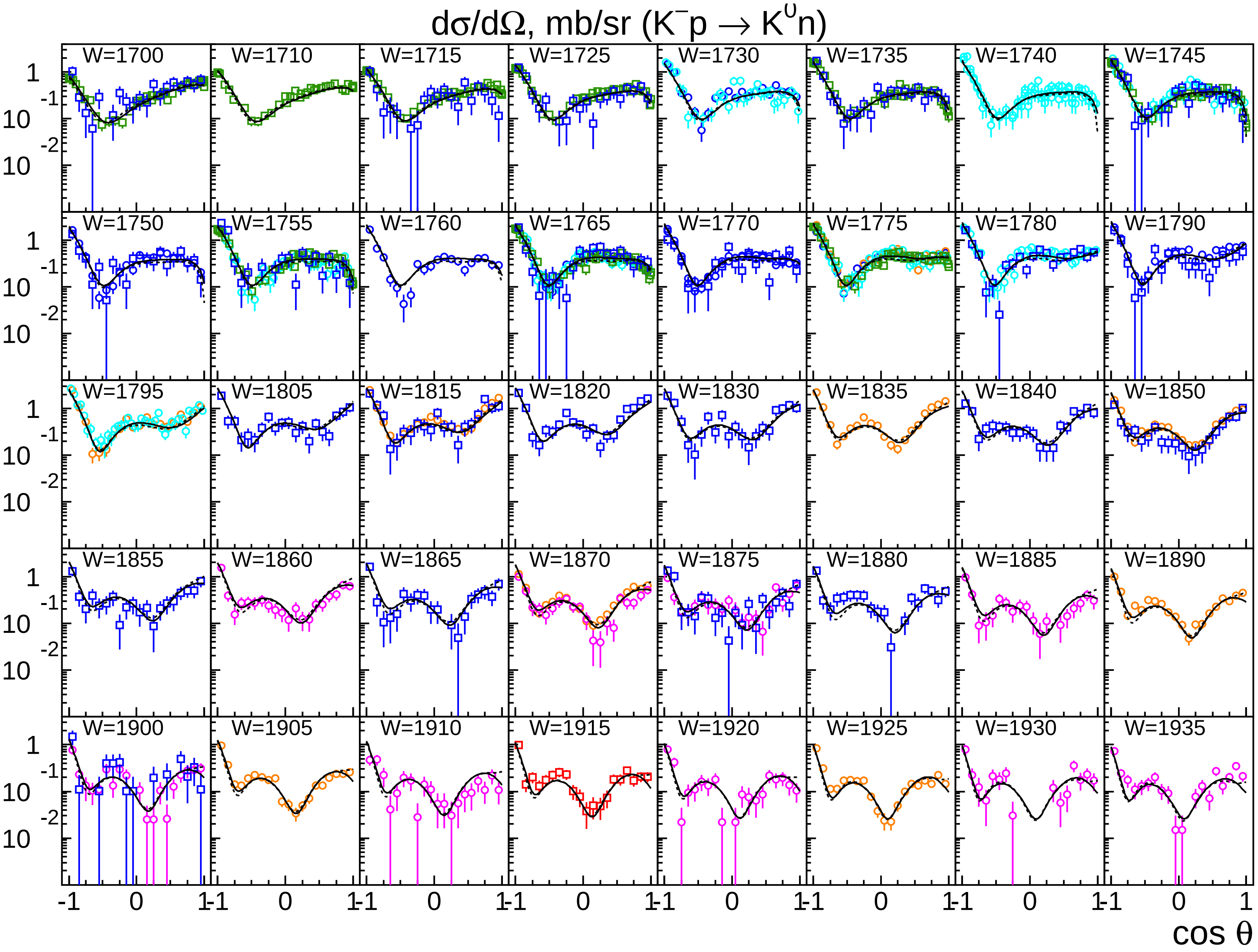
Mass scan of additional states

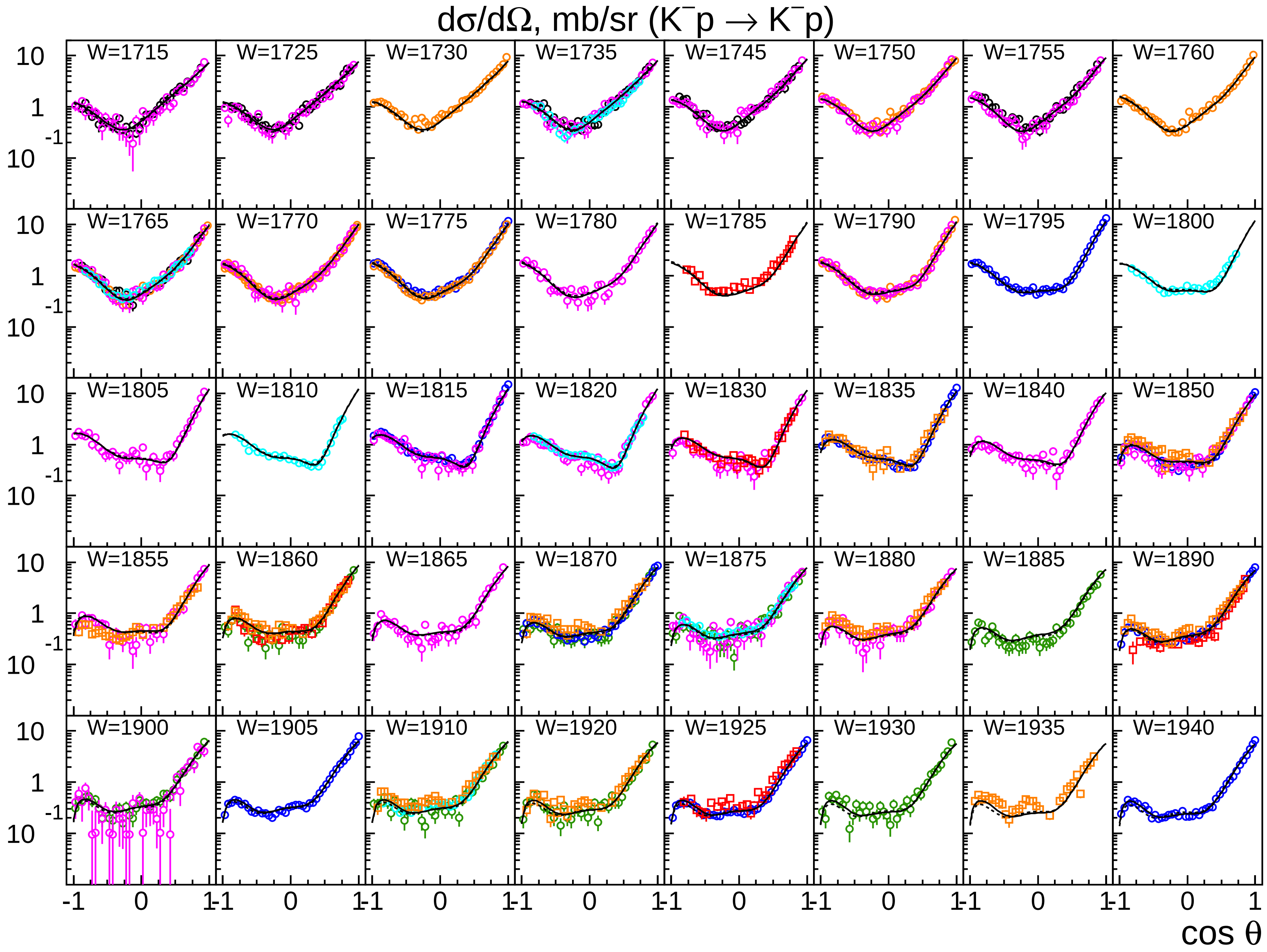


Λ and Σ resonances seen in early analyses (PDG)and by the BnGa group.

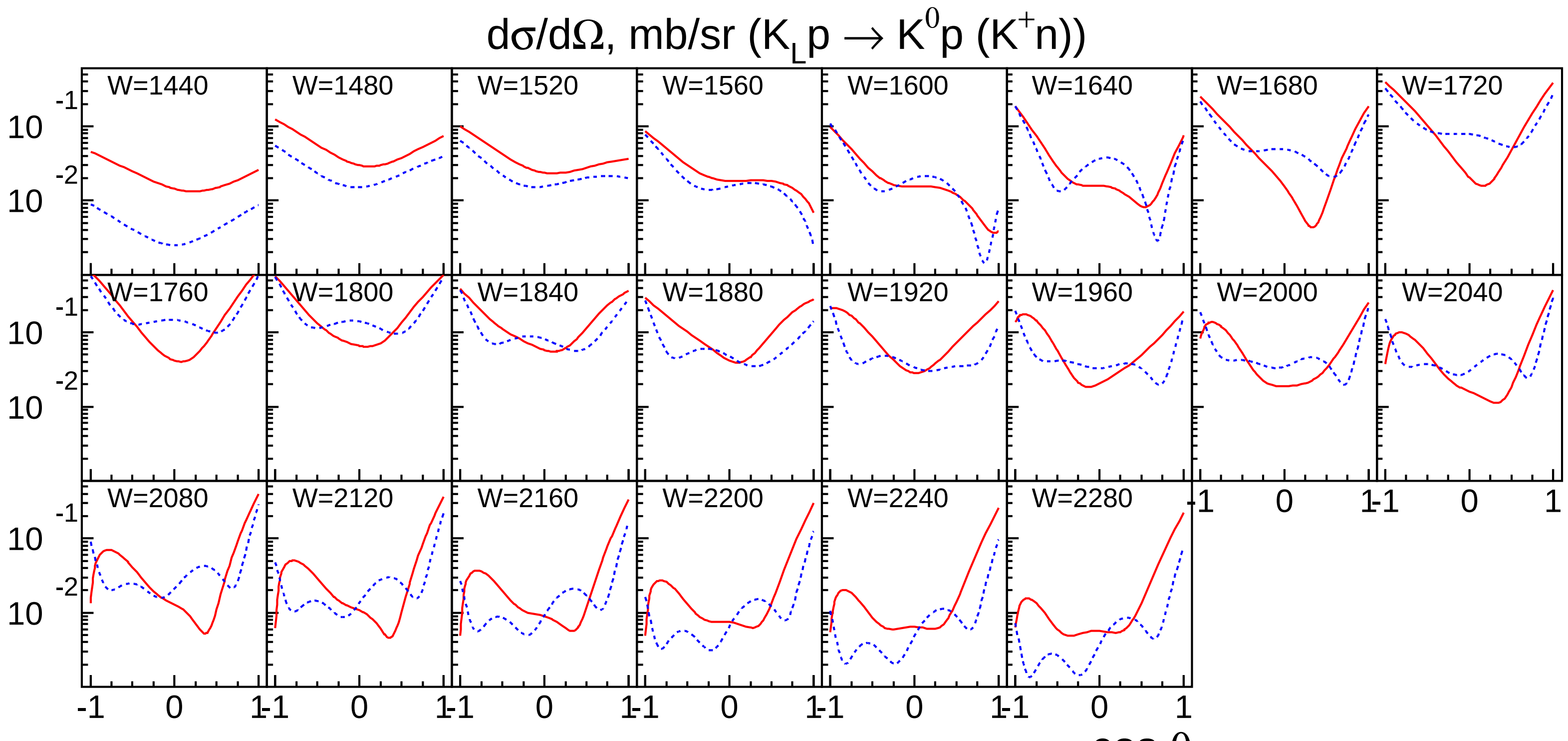
$\Lambda(1380)\ 1/2^-$		$\Sigma(1580)\ 3/2^-$	
$\Lambda(1405)\ 1/2^-$		$\Sigma(1620)\ 1/2^-$	**Confirmed**
$\Lambda(1520)\ 3/2^-$		$\Sigma(1660)\ 1/2^+$	
$\Lambda(1600)\ 1/2^+$		$\Sigma(1670)\ 3/2^-$	
$\Lambda(1670)\ 1/2^-$		$\Sigma(1750) 1/2^-$	
$\Lambda(1690)\ 3/2^-$		$\Sigma(1775)\ 5/2^-$	
$\Lambda(1710)\ 1/2^+$	**Not seen**	$\Sigma(1780)\ 3/2^+$	**Not seen**
$\Lambda(1800)\ 1/2^-$		$\Sigma(1880)\ 1/2^+$	**Not seen**
$\Lambda(1810)\ 1/2^+$		$\Sigma(1900)\ 1/2^-$	
$\Lambda(1820)\ 5/2^+$		$\Sigma(1910)\ 3/2^-$	
$\Lambda(1830)\ 5/2^-$		$\Sigma(1915)\ 5/2^+$	

$\Lambda(1890)\ 3/2^+$		$\Sigma(1940)\ 3/2^+$	**Not seen**
$\Lambda(2000)\ 1/2^-$	**Not seen**	$\Sigma(2010)\ 3/2^-$	**New**
$\Lambda(2050)\ 3/2^-$	**Not seen**	$\Sigma(2030)\ 7/2^+$	
$\Lambda(2070)\ 3/2^+$	**New**	$\Sigma(2070)\ 5/2^+$	**Not seen**
$\Lambda(2080)\ 5/2^-$		$\Sigma(2080)\ 3/2^+$	**Not seen**
$\Lambda(2085)\ 7/2^+$	**Not seen**	$\Sigma(2100)\ 7/2^-$	
$\Lambda(2100)\ 7/2^-$		$\Sigma(2110)\ 1/2^-$	**New**
$\Lambda(2110)\ 5/2^+$		$\Sigma(2230)\ 3/2^+$	
$\Lambda(2325)\ 3/2^-$	**Not seen**		
$\Lambda(2350)\ 9/2^+$	**Not seen**		

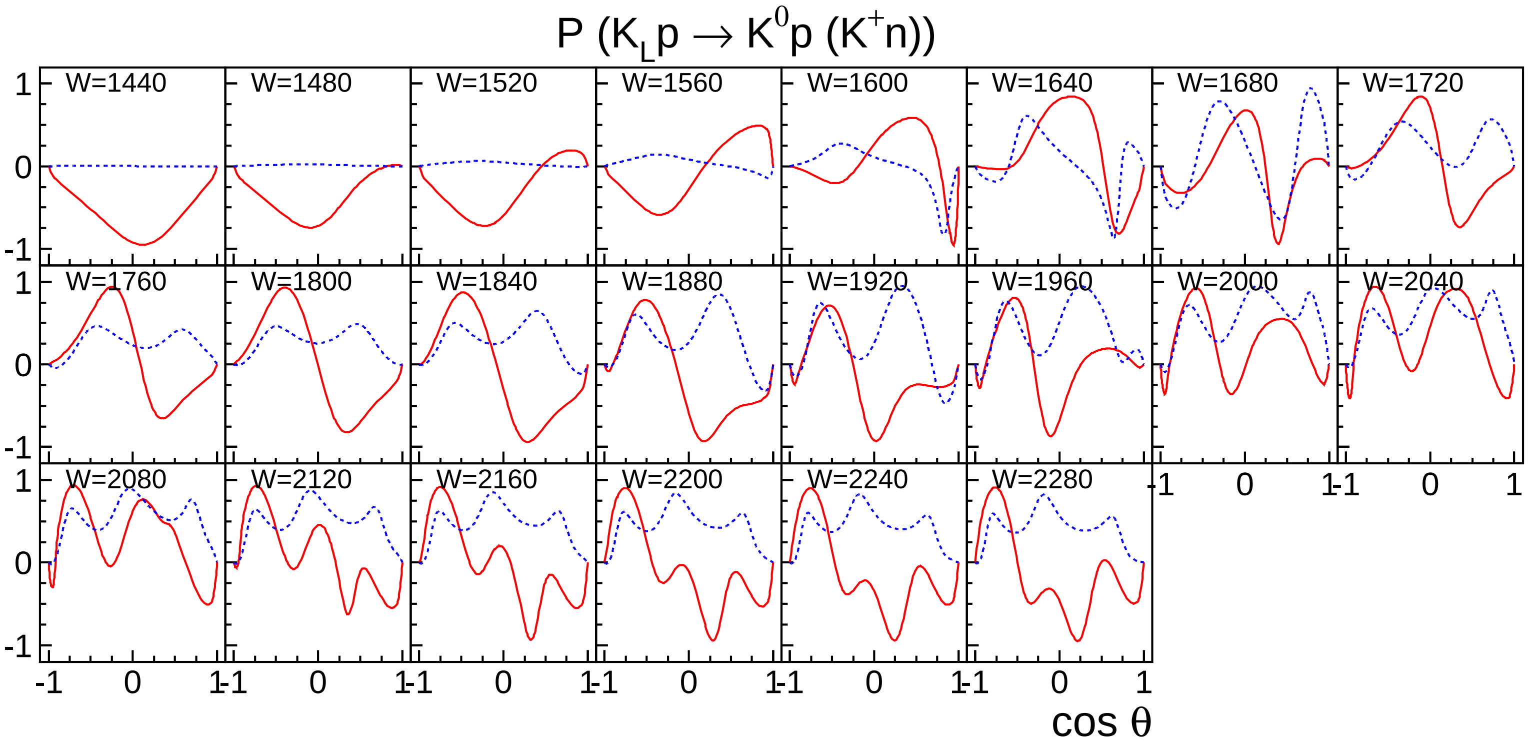

dσ/dΩ, mb/sr (K⁻p → K⁰n)
W=1700
W=1710
W=1715
W=1725
W=1730
W=1735
W=1740
W=1745
W=1750
W=1755
W=1760
W=1765
W=1770
W=1775
W=1780
W=1790
W=1795
W=1805
W=1815
W=1820
W=1830
W=1835
W=1840
W=1850
W=1855
W=1860
W=1865
W=1870
W=1875
W=1880
W=1885
W=1890
W=1900
W=1905
W=1910
W=1915
W=1920
W=1925
W=1930
W=1935
1
10^-1
10^-2
-1
0
1
cos θ


dσ/dΩ, mb/sr (K⁻p → K⁻p)
W=1715
W=1725
W=1730
W=1735
W=1745
W=1750
W=1755
W=1760
W=1765
W=1770
W=1775
W=1780
W=1785
W=1790
W=1795
W=1800
W=1805
W=1810
W=1815
W=1820
W=1830
W=1835
W=1840
W=1850
W=1855
W=1860
W=1865
W=1870
W=1875
W=1880
W=1885
W=1890
W=1900
W=1905
W=1910
W=1920
W=1925
W=1930
W=1935
W=1940
10
1
10^-1
-1
0
1
cos θ

Prediction for $\frac{d\sigma}{d\Omega}(K_L p \to K^0 p(K^+ n))$



Prediction for the recoil asymmetry $K_L p \to K^0 p (K^+ n)$



SUMMARY

- The combined analysis of the pion induced and meson photo-production data discovered 6 new N^* states.
- The large number of the baryon decay properties were defined with a good quality.
- In many partial waves the unitarity limit was reached in the energy region below 2 GeV.
- The present solution has a strong prediction power, but some states still can be found in the double polarization experiments.
- The analysis of the data on the $K^-p \to KN, \pi\Lambda, \pi\Sigma$ reactions is notably improved after adding to 4-star and 3-star resonances 5 states. Three of these states are Σ-hyperons with $J^P = \frac{1}{2}^-$
- The 11 states observed in earlier analyses was not confirmed by our analysis.
- The planned K_Lp experiment provides a unique possibility to perform the full amplitude decomposition of the observables measured in the K^-p collision reactions and to determine the spectrum and properties of Σ hyperons.