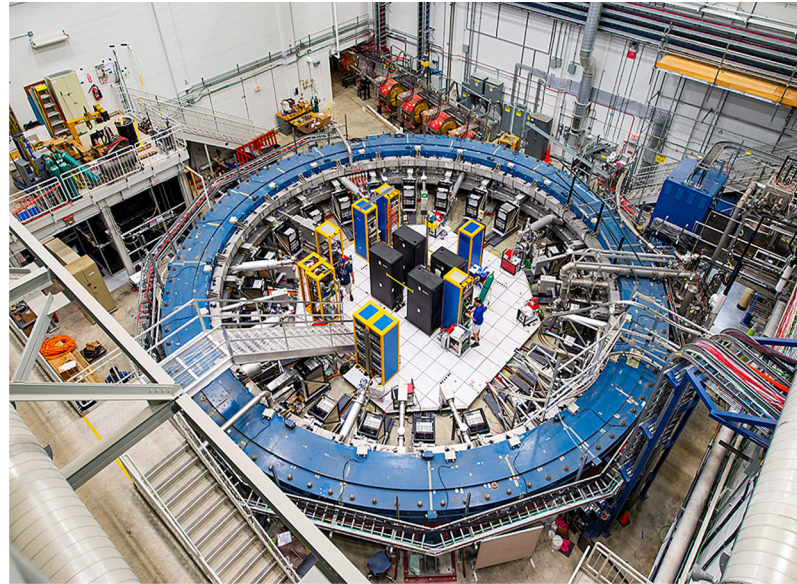


# Muon g-2: Hadronic Contributions



Alex Keshavarzi

 @AlexKeshavarzi

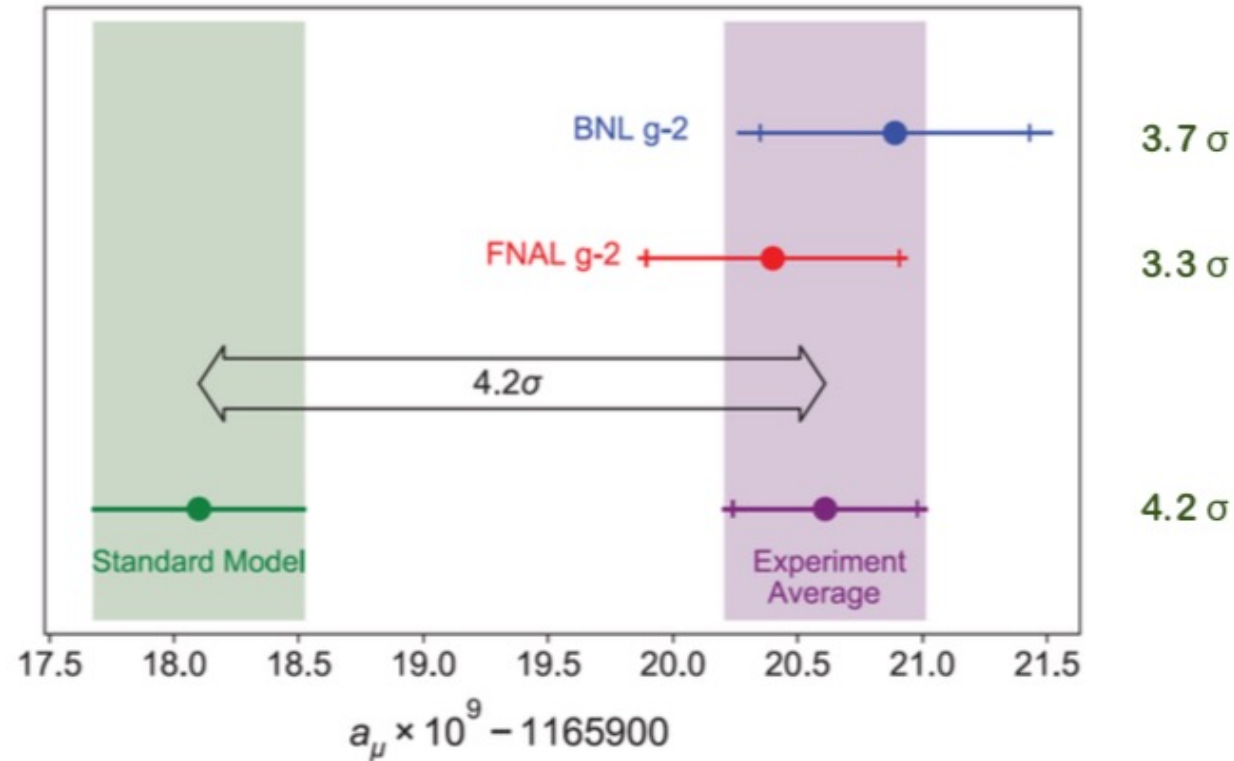
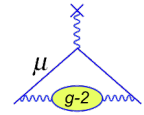
XXXIII International Workshop on High Energy Physics  
“Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quets”

8<sup>th</sup> November 2021



The University of Manchester

# Muon g-2: FNAL confirms BNL

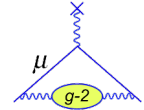


$$a_\mu^{\text{EXP}} = (116592089 \pm 63) \times 10^{-11} [0.54\text{ppm}] \text{ BNL E821}$$

$$a_\mu^{\text{EXP}} = (116592040 \pm 54) \times 10^{-11} [0.46\text{ppm}] \text{ FNAL E989 Run 1}$$

$$a_\mu^{\text{EXP}} = (116592061 \pm 41) \times 10^{-11} [0.35\text{ppm}] \text{ WA}$$

# Magnetic moments



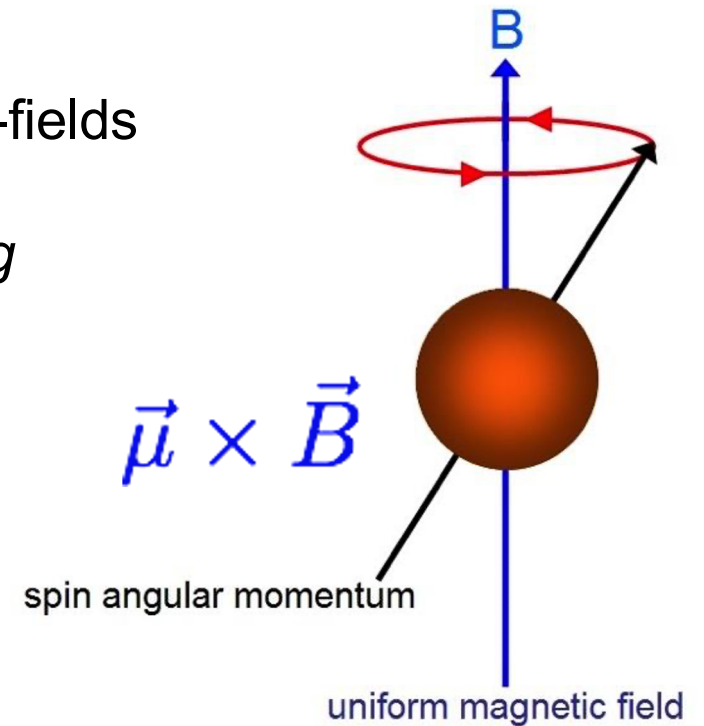
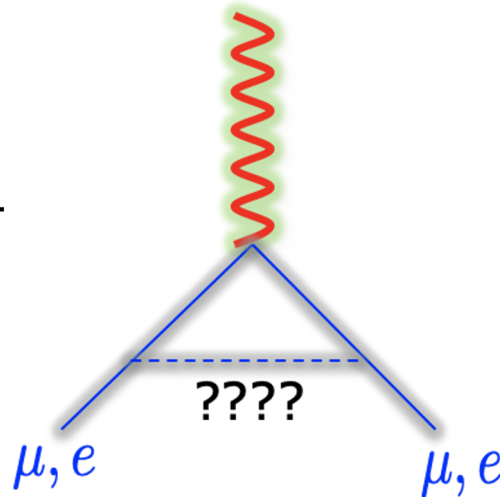
The muon has an intrinsic magnetic moment that is coupled to its spin via the gyromagnetic ratio  $g$ :

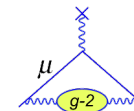
$$\vec{\mu} = g \frac{e}{2m_{\mu}} \vec{S}$$

Magnetic moment (spin) interacts with external B-fields

Makes spin precess at frequency determined by  $g$

$$a_{\mu} = \frac{g - 2}{2}$$





# Muon g-2 Theory

arXiv.org > hep-ph > arXiv:2006.04822

Search...  
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High Energy Physics – Phenomenology

[Submitted on 8 Jun 2020]

## The anomalous magnetic moment of the muon in the Standard Model

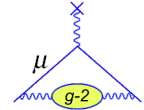
T. Aoyama, N. Asmussen, M. Benayoun, J. Bijnens, T. Blum, M. Bruno, I. Caprini, C. M. Carloni Calame, M. Cè, G. Colangelo, F. Curciarello, H. Czyż, I. Danilkin, M. Davier, C. T. H. Davies, M. Della Morte, S. I. Eidelman, A. X. El-Khadra, A. Gérardin, D. Giusti, M. Golterman, Steven Gottlieb, V. Gülpers, F. Hagelstein, M. Hayakawa, G. Herdoíza, D. W. Hertzog, A. Hoecker, M. Hoferichter, B.–L. Hoid, R. J. Hudspith, F. Ignatov, T. Izubuchi, F. Jegerlehner, L. Jin, A. Keshavarzi, T. Kinoshita, B. Kubis, A. Kupich, A. Kupść, L. Laub, C. Lehner, L. Lellouch, I. Logashenko, B. Malaescu, K. Maltman, M. K. Marinković, P. Masjuan, A. S. Meyer, H. B. Meyer, T. Mibe, K. Miura, S. E. Müller, M. Nio, D. Nomura, A. Nyffeler, V. Pascalutsa, M. Passera, E. Perez del Rio, S. Peris, A. Portelli, M. Procura, C. F. Redmer, B. L. Roberts, P. Sánchez-Puertas, S. Serednyakov, B. Shwartz, S. Simula, D. Stöckinger, H. Stöckinger–Kim, P. Stoffer, T. Teubner, R. Van de Water, M. Vanderhaeghen, G. Venanzoni, G. von Hippel, H. Wittig, Z. Zhang, M. N. Achasov, A. Bashir, N. Cardoso, B. Chakraborty, E.–H. Chao, J. Charles, A. Crivellin, O. Deineka, A. Denig, C. DeTar, C. A. Dominguez, A. E. Dorokhov, V. P. Druzhinin, G. Eichmann, M. Fael, C. S. Fischer, E. Gámiz, Z. Gelzer, J. R. Green, S. Guellati–Khelifa, D. Hatton, N. Hermansson–Truedsson et al. (32 additional authors not shown)

## The Muon g-2 Theory Initiative



# Muon g-2 in the SM

$$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65) \text{ ppm}$$



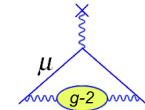
- $a_\mu$  arises due to quantum corrections / higher order interactions / loop contributions
- All SM particles contribute → Calculate and sum all sectors of the SM:

$$a_\mu^{\text{SM}} = a_\mu^{\text{QED}} + a_\mu^{\text{EW}} + a_\mu^{\text{HVP}} + a_\mu^{\text{HLbL}}$$

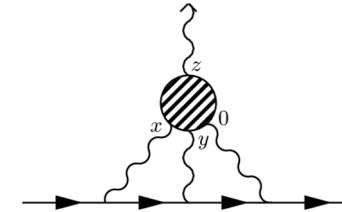
			$a_\mu^{\text{SM}}$ portion	$\delta a_\mu^{\text{SM}}$ portion
QED		Perturbative (Known to five-loop)	~ 99.99%	~0.001%
EW		Perturbative (Known to two-loop)	~ 1 ppm	~0.2%
HVP		Non-perturbative (Data-driven & lattice)	~ 59 ppm	~84%
HLbL		Non-perturbative (Data-driven & lattice)	~ 1 ppm	~16%

# Muon g-2 in the SM: HLbL

$$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65) \text{ ppm}$$



- HLbL scattering - hadronic blob coupled to 3 off-shell/1 on-shell photon.
- Four point function - notoriously difficult to calculate.
- Previously only calculated from models with large systematics.

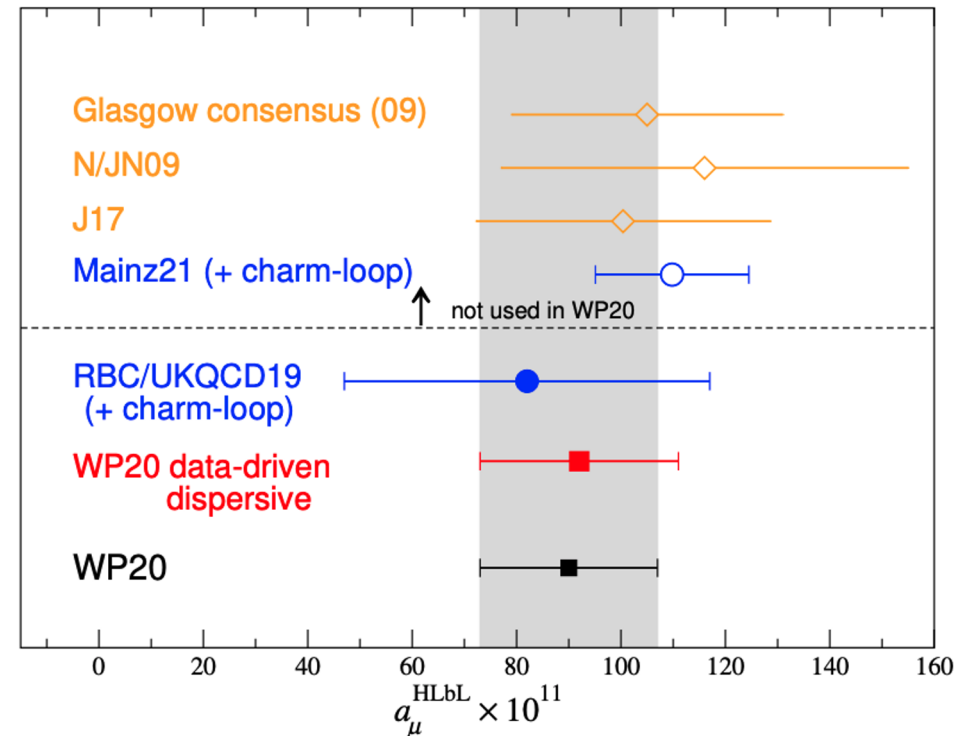


## Data-driven (error ~ 0.2 ppm of $a_\mu^{\text{SM}}$ )

- Model-independent dispersive evaluation, using data (e.g.  $\pi$ ,  $\eta$ ,  $\eta'$  TFFs) as input for hadronic insertions.

## Lattice (error ~ 0.3 ppm of $a_\mu^{\text{SM}}$ )

- Model-independent evaluation, computed on discretized Euclidean spacetime (lattice) in finite volume.



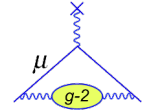
Recommended Muon g-2  
TI result (before Mainz):

$$a_\mu^{\text{HLbL}} = 92(18) \times 10^{-11}$$

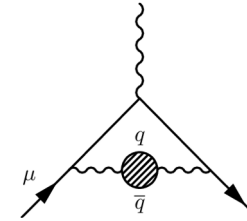
**Improved, but still evolving.**  
**Still systematics dominated**  
**(goal < 10% uncertainty)**

# Muon g-2 in the SM: HVP

$$\Delta a_\mu = 279(76) \times 10^{-11} \rightarrow 2.39(0.65) \text{ ppm}$$



- Hadronic Vacuum Polarisation - hadronic blob coupled to 2 photons.
- Two-point function - in principle, much easier than HLbL.
- Most precisely calculated from  $e^+e^- \rightarrow$  hadrons cross section data.



## Lattice (error ~ 1.6 ppm of $a_\mu^{\text{SM}}$ )

- Uncertainties dominated by finite volume, discretisation and isospin breaking systematics.

## Data-driven (error ~ 0.3 ppm of $a_\mu^{\text{SM}}$ )

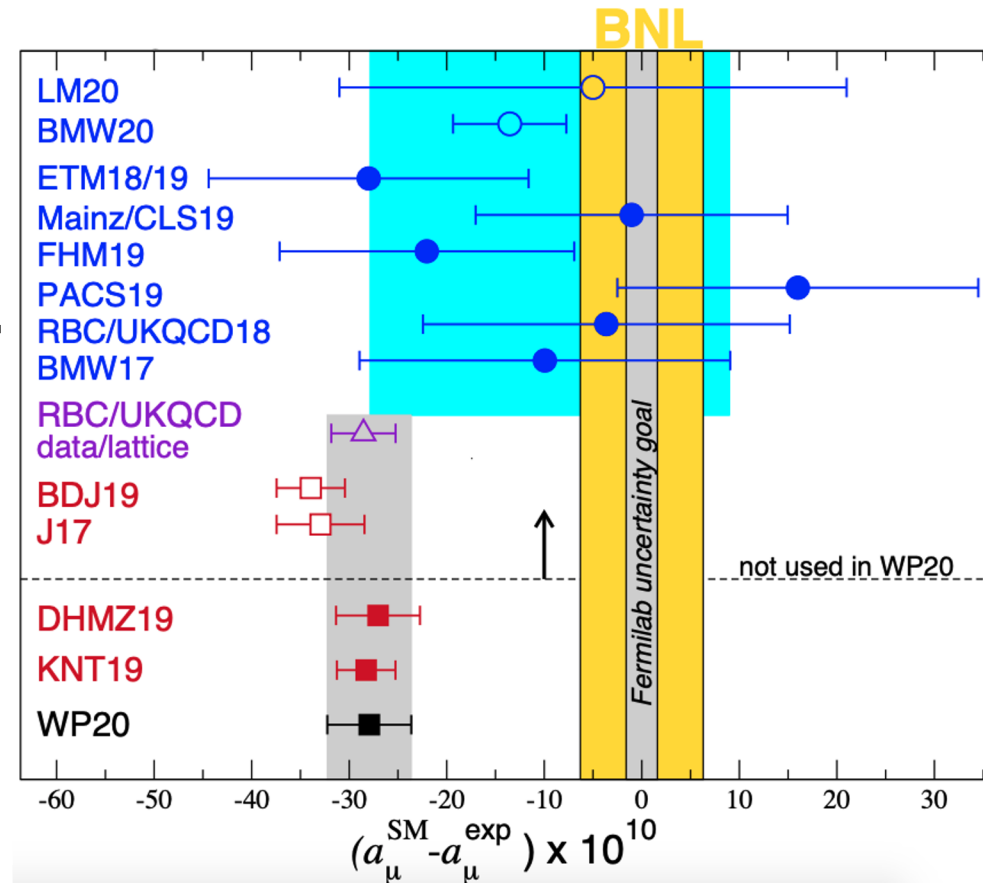
- Cross section data consistently combined and input into dispersion integral:

$$a_\mu^{\text{LOHVP}} = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds K(s) \sigma_{\text{had}}(s)$$

- Several groups have achieved this.

Recommended Muon g-2 TI value from data-driven result:

$$a_\mu^{\text{HVP}} = 6845(40) \times 10^{-11}$$



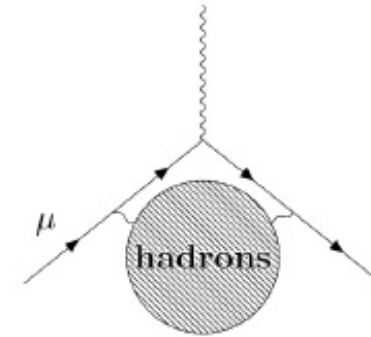
# Dispersive HVP: theoretical setup

⇒ We want to calculate the **leading order hadronic vacuum polarisation (HVP) contribution**

1) Feynman rules for **HVP insertion to photon propagator**:

$$\mu \text{---} q \text{---} \text{hadrons} \text{---} q \text{---} \nu = \frac{-ig^{\mu\alpha}}{(q^2 - i\varepsilon)} (-ie)i\Pi_{\alpha\beta}(q^2)(-ie) \frac{-ig^{\beta\nu}}{(q^2 - i\varepsilon)}$$

$\Pi_{\alpha\beta}(q^2)$



2) Employ **analyticity**:

$$\mu \text{---} q \text{---} \text{hadrons} \text{---} q \text{---} \nu = \frac{ie^2 g_{\mu\nu}}{(q^2 - i\varepsilon)^2} \frac{q^4}{\pi} \int_{s_{th}}^{\infty} ds \frac{\text{Im} \Pi(s)}{s(s - q^2 - i\varepsilon)}$$

$\Pi_{\alpha\beta}(q^2)$

3) **Insert to vertex correction**, solve for  $a_\mu$ :  $a_\mu^{\text{had, LO VP}} = \frac{\alpha}{\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} \text{Im} \Pi_{\text{had}}(s) K(s)$

4) Utilise **optical theorem**:

$$\text{Im} \left| \text{had} \right| \quad \Leftrightarrow \quad \left| \text{had} \right|^2$$

$\text{Im} \Pi_{\text{had}}(q^2) \qquad \sim \sigma_{\text{had}}(q^2)$

5) Arrive at **equation for  $a_\mu^{\text{had, LO VP}}$** :

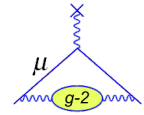
$$a_\mu^{\text{had, LO VP}} = \frac{1}{4\pi^3} \int_{s_{th}}^{\infty} ds \sigma_{\text{had},\gamma}^0(s) K(s)$$

$\sigma_{\text{had},\gamma}^0 =$  **bare cross section, FSR included**

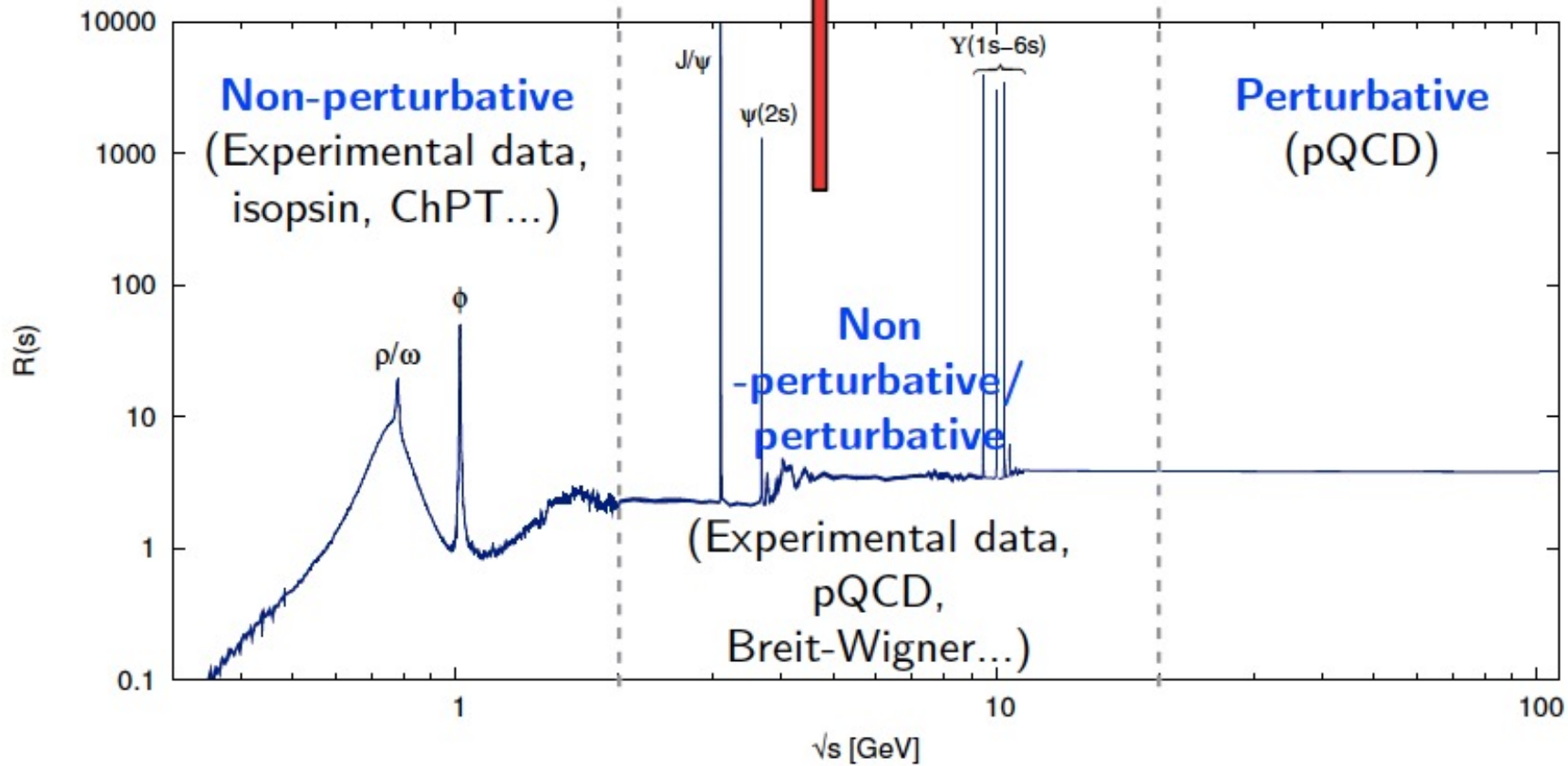
⇒ **Similar dispersion integrals for NLO and NNLO HVP**



# Building the hadronic R-ratio

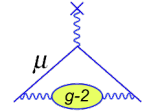


$$a_{\mu}^{\text{had, LO VP}} = \frac{\alpha^2}{3\pi^2} \int_{s_{th}}^{\infty} \frac{ds}{s} R(s) K(s), \text{ where } R(s) = \frac{\sigma_{\text{had},\gamma}^0(s)}{4\pi\alpha^2/3s}$$



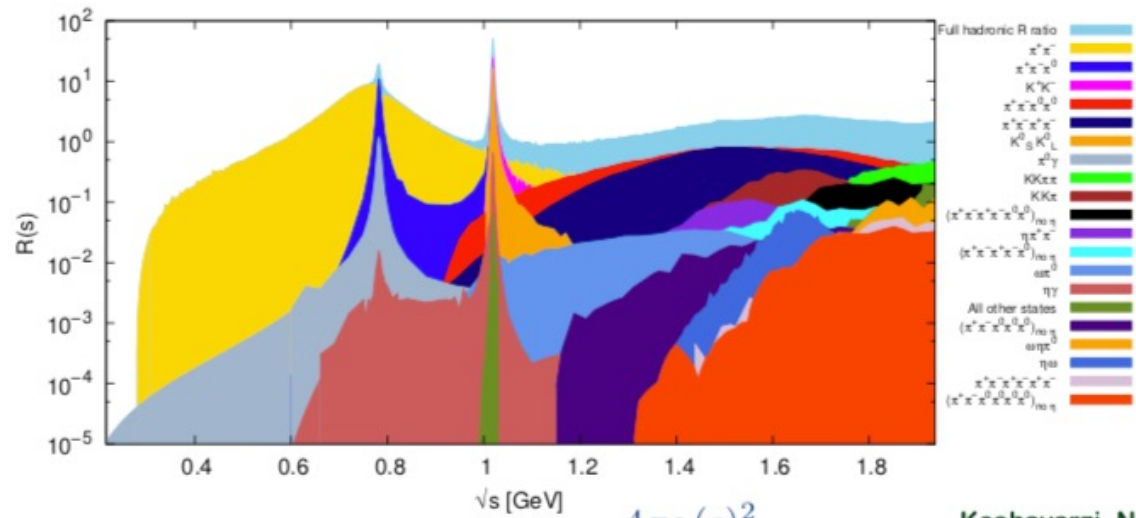
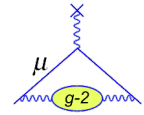
# Dispersive HVP

Slide content by Aida El-Khadra.



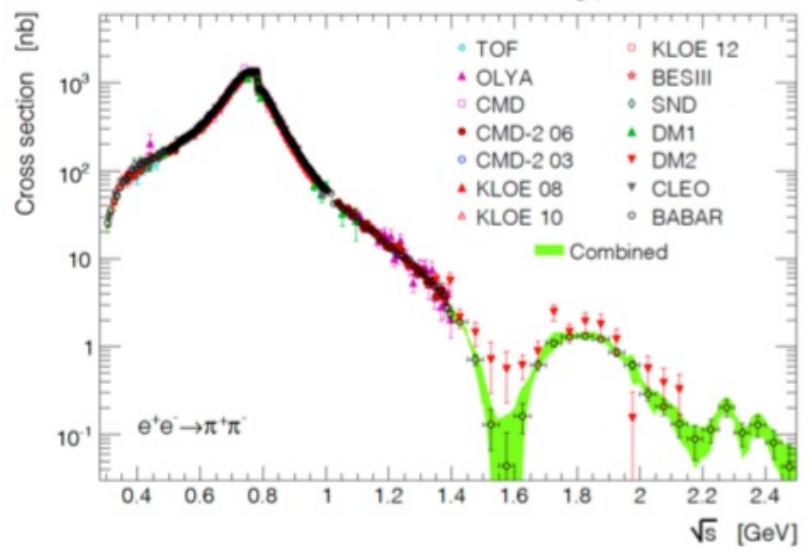
- ◆ Target:  $\sim 0.2\%$  total error
- ◆ Dispersion relation + experimental data for  $e^+e^- \rightarrow$  hadrons (and  $\tau$  data)
  - current uncertainty  $\sim 0.5\%$
  - can be improved with more precise experimental data
  - new experimental measurements expected/ongoing at BaBar, BES-III, Belle-II, CMD-3, SND, KEDR, KLOE,....
- ◆ Challenges:
  - below  $\sim 2$  GeV: sum  $> 30$  exclusive channels:  $2\pi, 3\pi, 4\pi, 5\pi, 6\pi, 2K, 2K\pi, 2K2\pi, \eta\pi, \dots$  (use isospin relations for missing channels)
  - above  $\sim 1.8$  GeV:
    - inclusive, pQCD (away from flavor thresholds)
    - + narrow resonances ( $J/\psi, \Upsilon, \dots$ )
  - Combine data from different experiments/measurements:
    - understanding correlations, sources of sys. error, tensions...
  - include FS radiative corrections

# Low energy hadronic cross section



$$R(s) = \sigma(e^+e^- \rightarrow \text{hadrons}) / \frac{4\pi\alpha(s)^2}{3s}$$

Keshavarzi, Nomura Teubner  
PRD 2018

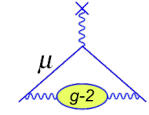


Davier, Hoecker, Malaescu, Zhang  
EPJC 2020

EPJ Dis

11

# Dispersive HVP from KNT



## The muon $g - 2$ and $\alpha(M_Z^2)$ : a new data-based analysis

Alexander Keshavarzi<sup>1</sup>, Daisuke Nomura<sup>2,3</sup> and Thomas Teubner<sup>4</sup>

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Email: a.i.keshavarzi@liverpool.ac.uk

<sup>2</sup>KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan

<sup>3</sup>Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan  
Email: dnomura@post.kek.jp

<sup>4</sup>Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom  
Email: thomas.teubner@liverpool.ac.uk

### Abstract

This work presents a complete re-evaluation of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the muon,  $a_\mu^{\text{had, VP}}$  and the hadronic contributions to the effective QED coupling at the mass of the Z boson,  $\Delta\alpha_{\text{had}}(M_Z^2)$ , from the combination of  $e^+e^- \rightarrow$  hadrons cross section data. Focus has been placed on the development of a new data combination method, which fully incorporates all correlated statistical and systematic uncertainties in a bias free approach. All available  $e^+e^- \rightarrow$  hadrons cross section data have been analysed and included, where the new data compilation has yielded the full hadronic  $R$ -ratio and its covariance matrix in the energy range  $m_\pi \leq \sqrt{s} \leq 11.2$  GeV. Using these combined data and perturbative QCD above that range results in estimates of the hadronic vacuum polarisation contributions to  $g - 2$  of the muon of  $a_\mu^{\text{had, LO VP}} = (693.26 \pm 2.46) \times 10^{-10}$  and  $a_\mu^{\text{had, NLO VP}} = (-9.82 \pm 0.04) \times 10^{-10}$ . The new estimate for the Standard Model prediction is found to be  $a_\mu^{\text{SM}} = (11\,659\,182.04 \pm 3.56) \times 10^{-10}$ , which is  $3.7\sigma$  below the current experimental measurement. The prediction for the five-flavour hadronic contribution to the QED coupling at the Z boson mass is  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2) = (276.11 \pm 1.11) \times 10^{-4}$ , resulting in  $\alpha^{-1}(M_Z^2) = 128.946 \pm 0.015$ . Detailed comparisons with results from similar related works are given.

2019 data update and applications of data  $\rightarrow$   
compilation to other observables.

Results for  $a_e$ ,  $a_\mu$ ,  $a_\tau$ ,  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$  and  $\Delta\nu_{\text{Mu}}^{\text{had, VP}}$ .  
*Phys.Rev.D* 101 (2020) 014029.

$\leftarrow$  Major 2018 update to data combination methodology and data input.  
Results for  $a_\mu^{\text{had, VP}}$  and  $\Delta\alpha_{\text{had}}^{(5)}(M_Z^2)$ .  
*Phys.Rev.D* 97 (2018) 114025.

## The $g - 2$ of charged leptons, $\alpha(M_Z^2)$ and the hyperfine splitting of muonium

Alexander Keshavarzi<sup>1,2</sup>, Daisuke Nomura<sup>3</sup> and Thomas Teubner<sup>4</sup>

<sup>1</sup>Department of Physics and Astronomy, The University of Manchester, Manchester M13 9PL, United Kingdom

<sup>2</sup>Department of Physics and Astronomy, The University of Mississippi, Mississippi 38677, U.S.  
Email: alexander.keshavarzi@manchester.ac.uk

<sup>3</sup>KEK Theory Center, Tsukuba, Ibaraki 305-0801, Japan  
Email: dnomura@post.kek.jp

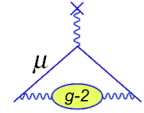
<sup>4</sup>Department of Mathematical Sciences, University of Liverpool, Liverpool L69 3BX, United Kingdom  
Email: thomas.teubner@liverpool.ac.uk

### Abstract

Following updates in the compilation of  $e^+e^- \rightarrow$  hadrons data, this work presents re-evaluations of the hadronic vacuum polarisation contributions to the anomalous magnetic moment of the electron ( $a_e$ ), muon ( $a_\mu$ ) and tau lepton ( $a_\tau$ ), to the ground-state hyperfine splitting of muonium and also updates the hadronic contributions to the running of the QED coupling at the mass scale of the Z boson,  $\alpha(M_Z^2)$ . Combining the results for the hadronic vacuum polarisation contributions with recent updates for the hadronic light-by-light corrections, the electromagnetic and the weak contributions, the deviation between the measured value of  $a_\mu$  and its Standard Model prediction amounts to  $\Delta a_\mu = (28.02 \pm 7.37) \times 10^{-10}$ , corresponding to a muon  $g - 2$  discrepancy of  $3.8\sigma$ .

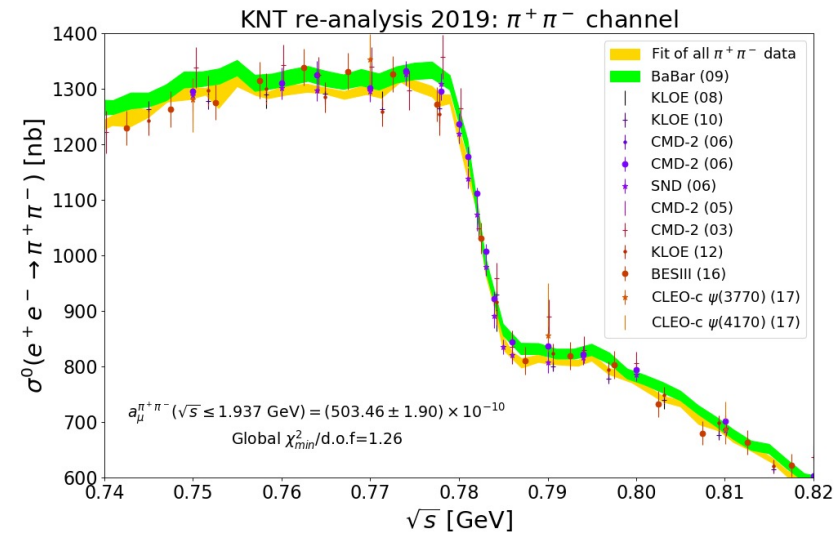
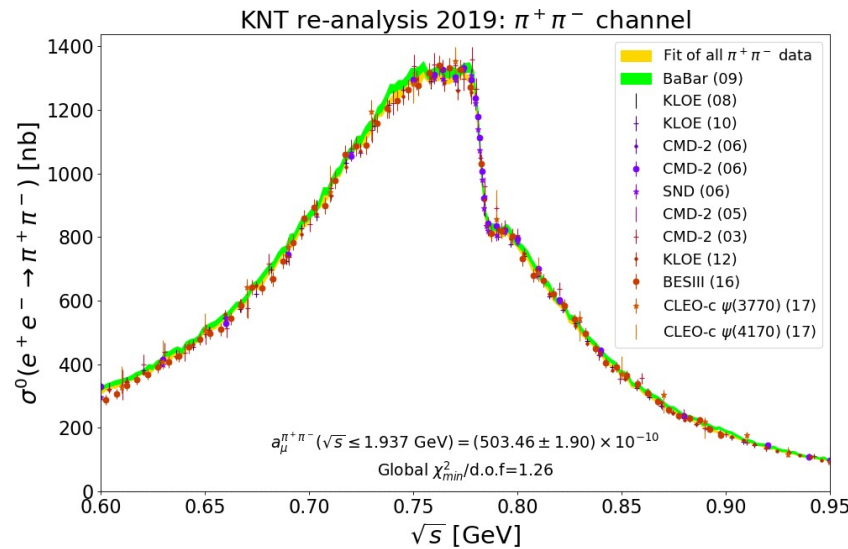
# The $\pi^+\pi^-$ channel

*Phys.Rev.D* 101 (2020) 014029.



$\pi^+\pi^-$  accounts for over 70% of  $a_\mu^{\text{had}}$ , LOVP

→ Combines ~30 measurement totalling over 1000 data points



→ Correlated & experimentally corrected  $\sigma_{\pi\pi(\gamma)}^0$  data entirely dominant

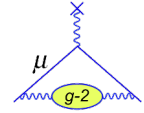
$$a_\mu^{\pi^+\pi^-} [0.305 \leq \sqrt{s} \leq 1.937 \text{ GeV}] = 503.46 \pm 1.14_{\text{stat}} \pm 1.52_{\text{sys}} \pm 0.05_{\text{vp}} \pm 0.14_{\text{fsr}}$$

$$= 503.46 \pm 1.91_{\text{tot}}$$

→ 14% local  $\chi^2_{\text{min}}/\text{d.o.f.}$  error inflation due to tensions in clustered data

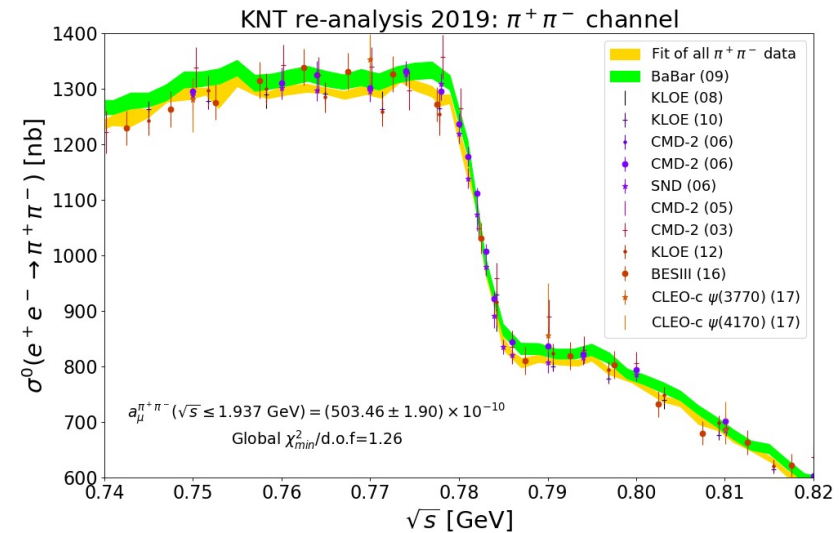
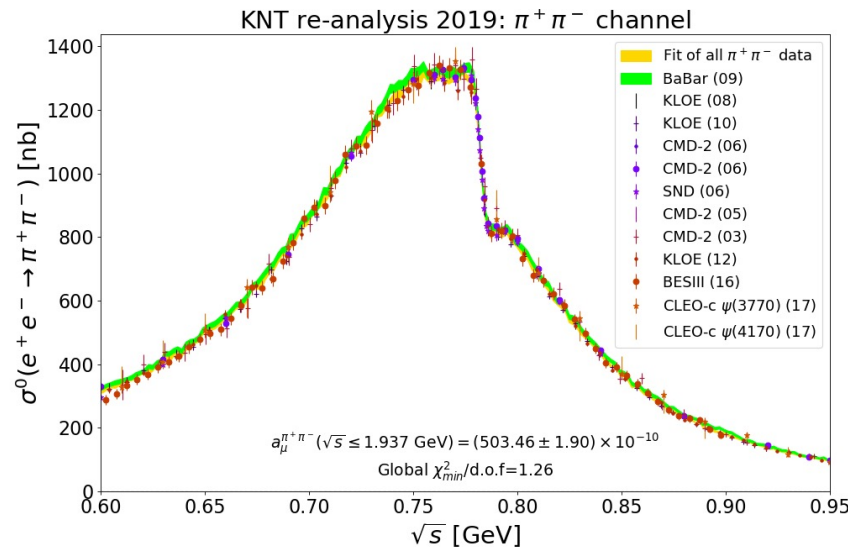
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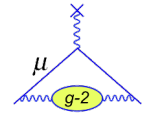
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$$= 503.46 \pm 1.91_{\text{tot}}$$

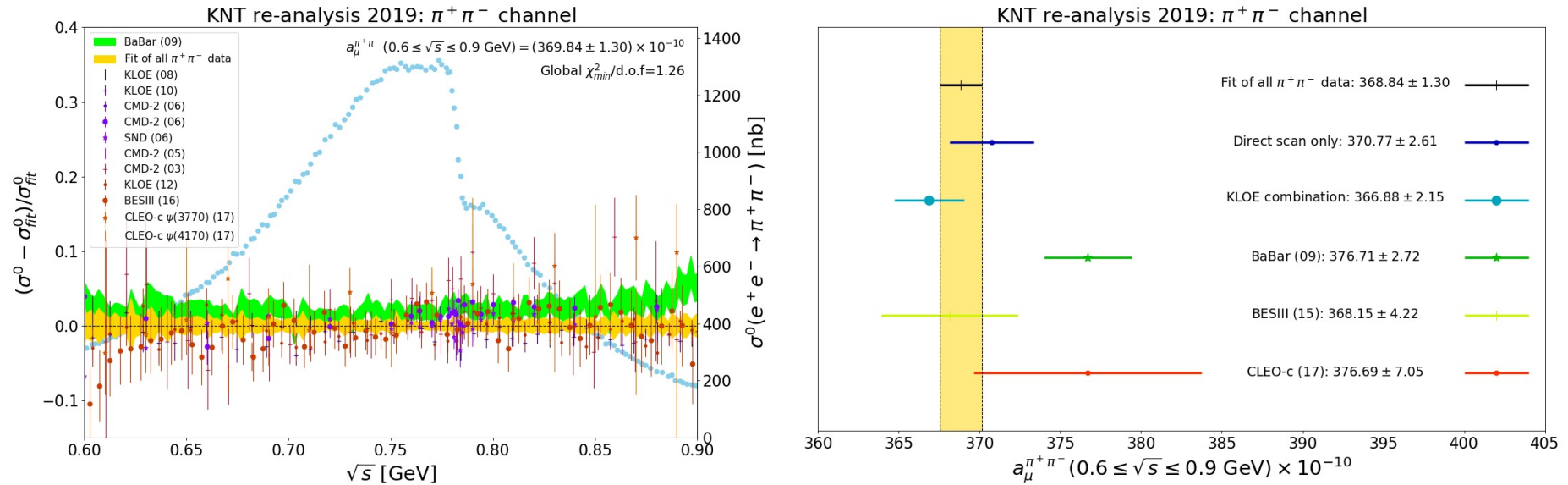
→ 14% local  $\chi^2_{\text{min}}/\text{d.o.f.}$  error inflation due to tensions in clustered data

# The $\pi^+\pi^-$ channel

*Phys.Rev.D* 101 (2020) 014029.



Large difference between KNT vs. BaBar and KLOE vs. BaBar is still evident.



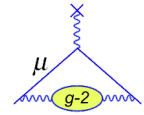
Compared to  $a_{\mu}^{\pi^+\pi^-} = 503.5 \pm 1.9 \rightarrow a_{\mu}^{\pi^+\pi^-}$  (BaBar data only) =  $513.2 \pm 3.8$

Simple weighted average of all data  $\rightarrow a_{\mu}^{\pi^+\pi^-}$  (weighted average) =  $509.2 \pm 2.9$   
(i.e. – no correlations in determination of mean value)

BaBar data dominate when no correlations are accounted for in the mean value.

➤ Highlights the importance of incorporating available correlated uncertainties in fit.

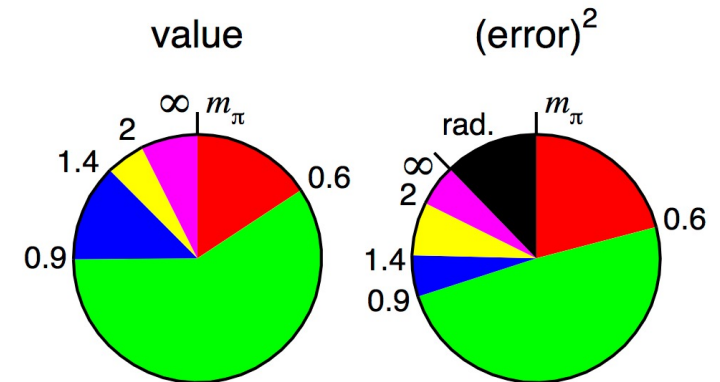
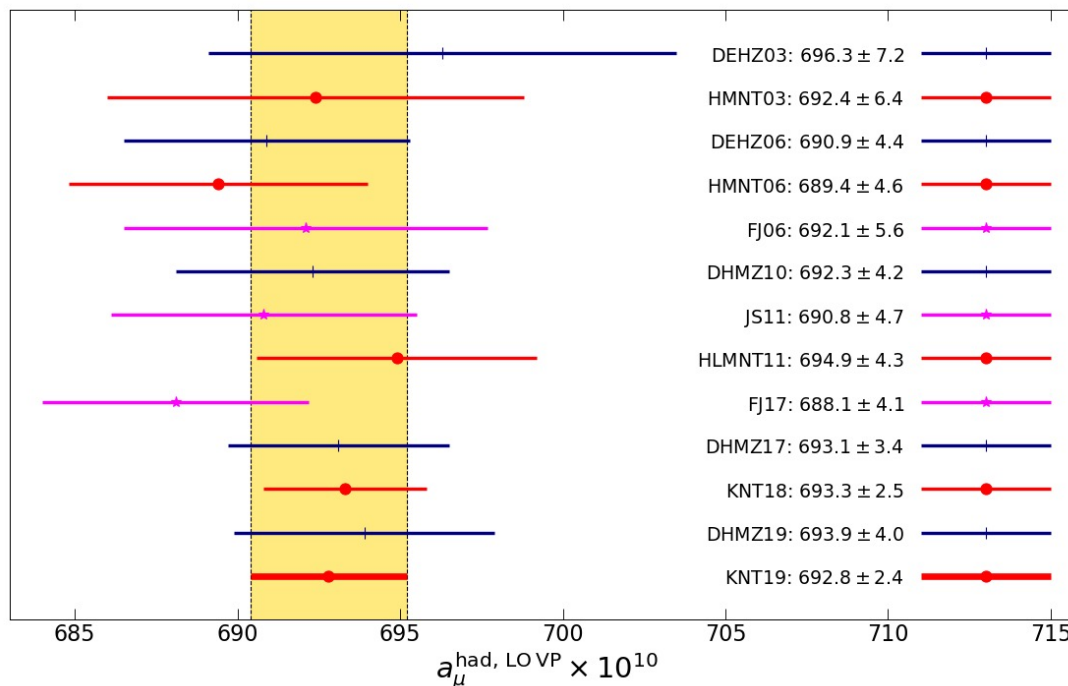
# $a_\mu^{\text{had, LO VP}}$ from KNT *Phys.Rev.D 101 (2020) 014029.*



$$\text{KNT18: } a_\mu^{\text{had, LOVP}} = 693.26 \pm 2.46_{\text{tot}}$$

$$\begin{aligned} a_\mu^{\text{had, LOVP}} &= 693.84 \pm 1.19_{\text{stat}} \pm 1.96_{\text{sys}} \pm 0.22_{\text{vp}} \pm 0.71_{\text{fsr}} \\ &= 693.84 \pm 2.29_{\text{exp}} \pm 0.74_{\text{rad}} \\ &= 692.78 \pm 2.42_{\text{tot}} \end{aligned}$$

➤ Precision better than 0.4% (uncertainties include all available correlations and  $\chi^2$  inflation)



➤ Clear  $\pi^+\pi^-$  dominance



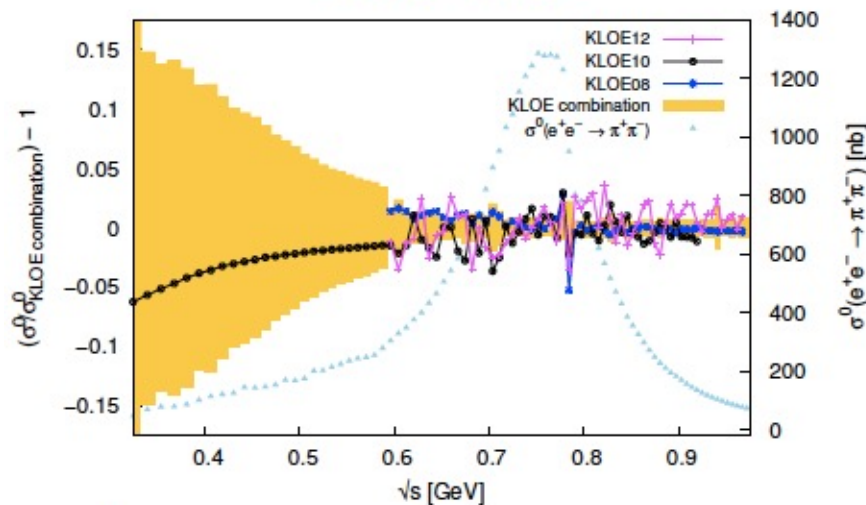
# KNT vs. DHMZ: the use of correlations

Differences are dominated by the choices of how to use the correlations

⇒ Consider the combination of the KLOE  $\pi^+\pi^-$  data sets:

## KNT18

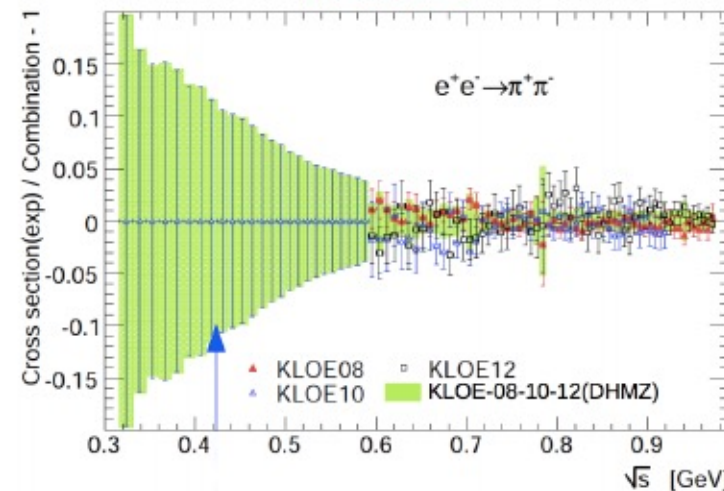
Use available uncertainty and correlation information to **fully influence over entire energy range**



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 503.74 \pm 1.96$$

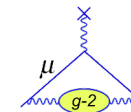
## DHMZ17

Only allow correlations to only influence the combination **locally inside defined energy intervals**



$$a_\mu^{\pi^+\pi^-}(\sqrt{s} \leq 2.0 \text{ GeV}) = 507.14 \pm 2.58$$

**Take-home message:** correlations are important and the choices of how to use them are not trivial

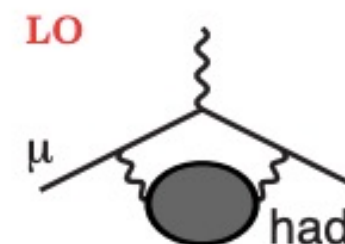


# Data-driven HVP Slide content by Aida El-Khadra.

First-time agreement between various groups...

Detailed comparisons by-channel and energy range between direct integration results:

	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	-0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	-0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	-0.12
$K^+K^-$	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
$K_S K_L$	12.82(0.06)(0.18)(0.15)	13.04(19)	-0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	-0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$ )	33.45(71)	34.45(56)	-1.00
$J/\psi, \psi(2S)$	7.76(12)	7.84(19)	-0.08
[3.7, $\infty$ ) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP,LO}}$	694.0(1.0)(3.5)(1.6)(0.1) $_{\psi(0.7)}\text{DV+QCD}$	692.8(2.4)	1.2



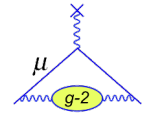
+ evaluations using unitarity & analyticity constraints for  $\pi\pi$  and  $\pi\pi\pi$  channels

[CHS 2018, HHKS 2019]

Conservative merging to obtain a realistic assessment of the underlying uncertainties:

- account for differences in results from the same experimental inputs
- include correlations between systematic errors

$$\Rightarrow a_\mu^{\text{HVP,LO}} = 693.1 (4.0) \times 10^{-10}$$



Calculate  $a_\mu^{\text{HVP}}$  in Lattice QCD:

$$a_\mu^{\text{HLO}} \equiv a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

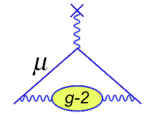
Note: almost always  $m_u = m_d$

$$\sum_f \left( \text{quark loop with } \bar{f} \text{ and } f \text{ lines} \right) + \left( \text{quark loop with } f \text{ line} \right) + \left( \text{quark loop with } f' \text{ line} \right) \quad f = ud, s, c, b$$

- need to add QED and strong isospin breaking ( $\sim m_u - m_d$ ) corrections:

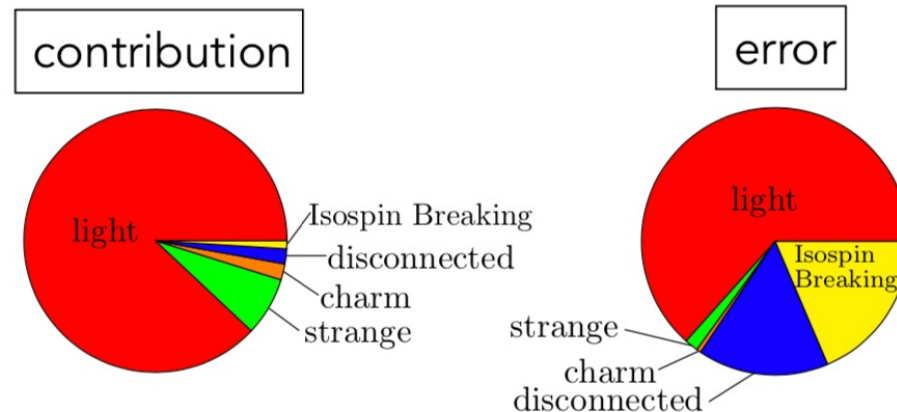
$$\left( \text{quark loop with gluon exchange} \right) + \dots$$

- either perturbatively on isospin symmetric QCD background
- or by using QCD + QED ensembles with  $m_u \neq m_d$



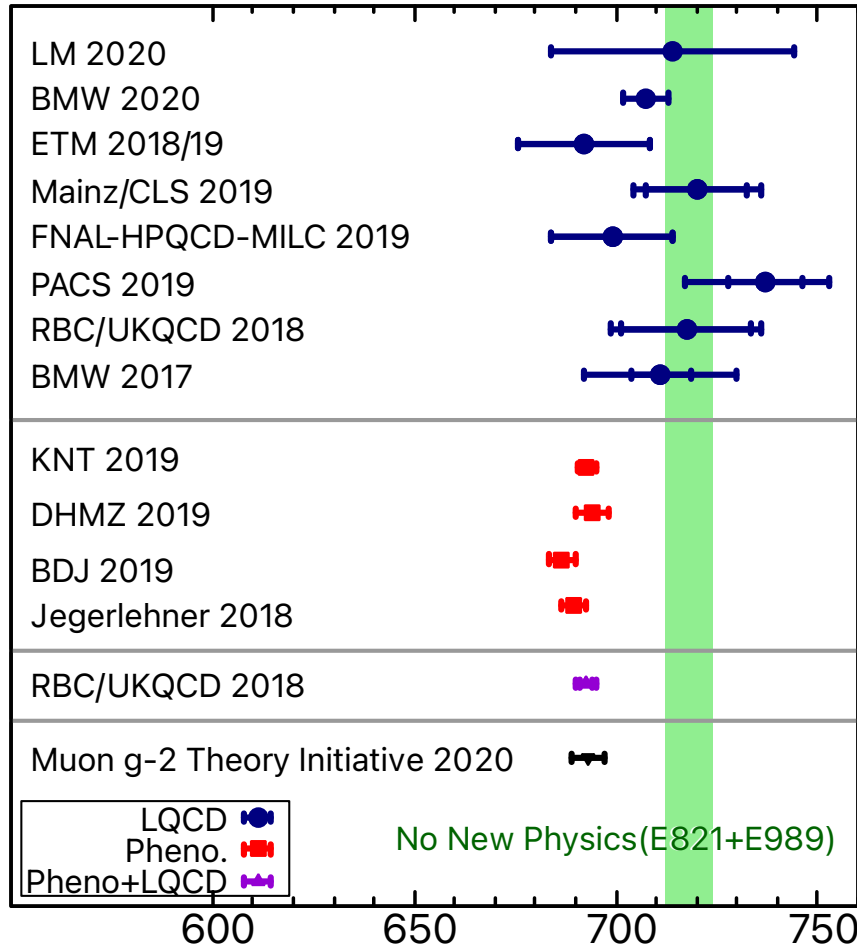
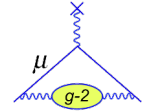
- Target: < 0.5% total error
- light-quark connected contribution,  $a_{\mu,ud}^{\text{HLO}}$ :  
~90% of total, with 1-3% error
- “heavy” flavor contributions,  $a_{\mu,s}^{\text{HLO}}$ ,  $a_{\mu,c}^{\text{HLO}}$ ,  $a_{\mu,b}^{\text{HLO}}$ :  
~8%, 2%, 0.05% of total  $a_{\mu}^{\text{HLO}}$ , can be calculated with sufficient precision
- disc. contribution:  
~2% of total  $a_{\mu}^{\text{HLO}}$ , contributes ~0.3-1% error to  $a_{\mu}^{\text{HLO}}$
- Isospinbreaking (QED +  $m_u \neq m_d$ ) corrections:  
~1% of total  $a_{\mu}^{\text{HLO}}$ , contribute ~0.3-1% error

[V. Gülpers, adapted for WP from talk @ Lattice 2019, arXiv:2001.11898]



# Lattice HVP

Slide content by Tom Blum.



HVP (BMW-20):  $a_\mu = 707.5 (5.5) \times 10^{-10}$   
(0.75%)

HVP (Lattice, WP):  $a_\mu = 711.6 (18.4) \times 10^{-10}$   
(2.6%)

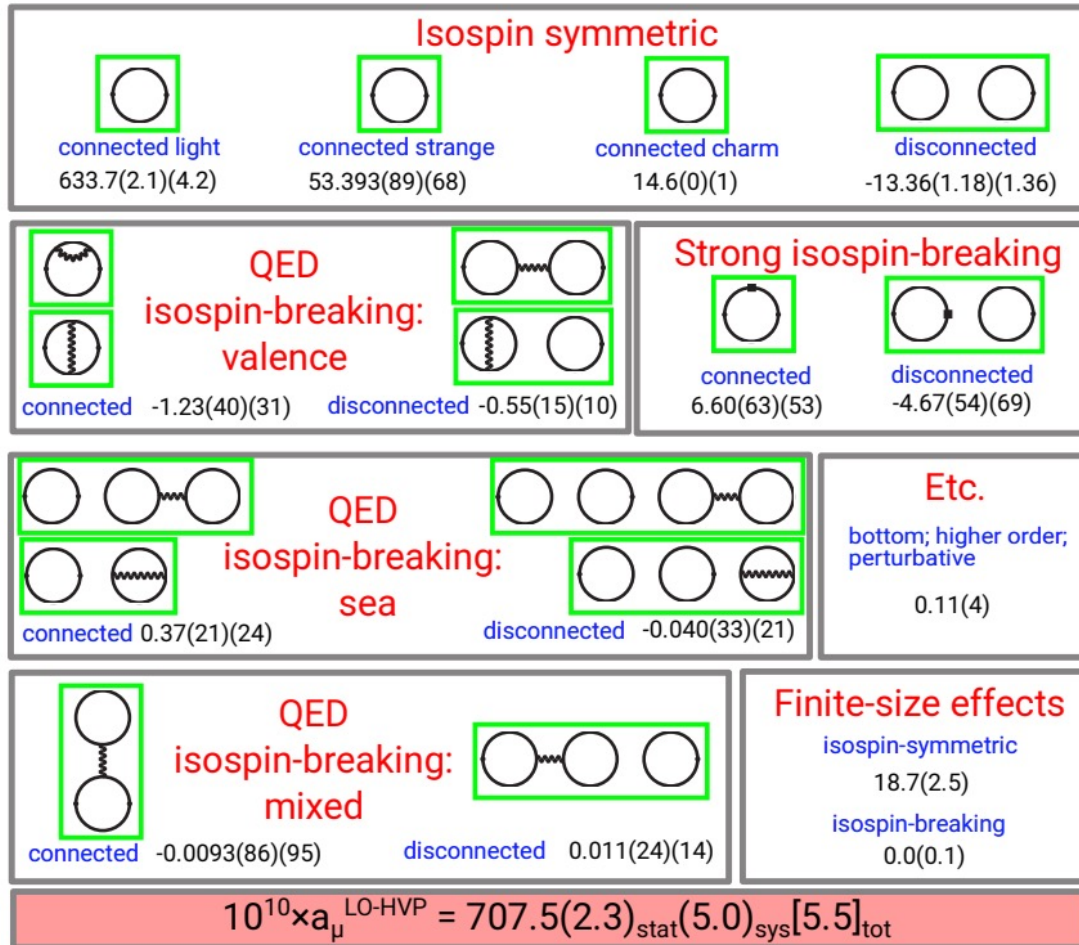
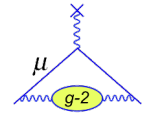
HVP (data driven):  $a_\mu = 693.1 (4.0) \times 10^{-10}$   
(0.58%)

Lattice, WP – data driven  $\approx 18.5 (18.8)$

BMW-20 – data driven  $\approx 14.4 (6.8)$

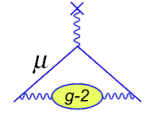
# The BMW result

Borsanyi et al. Nature 2021

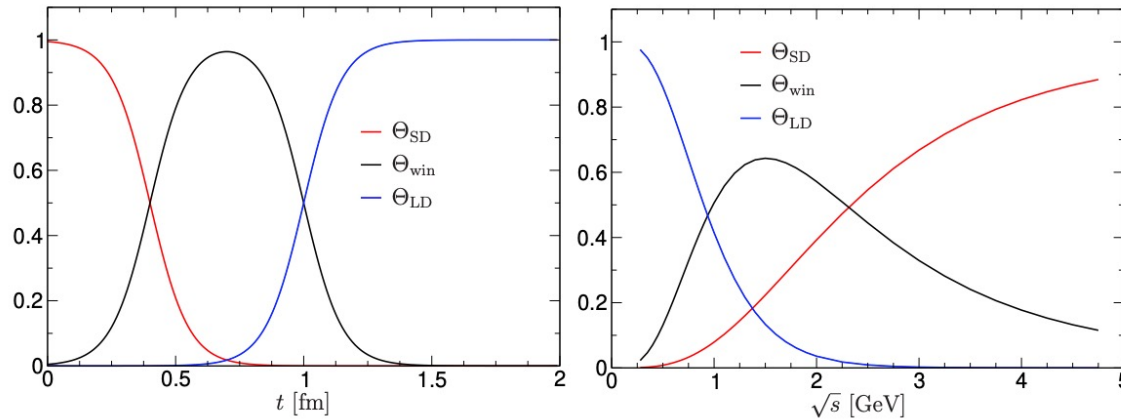


State-of-the-art lattice calculation of based on:

- current-current correlator, summed over all distances, integrated in time with appropriate kernel function
- using staggered fermions on an  $L \sim 6$  fm lattice ( $L \sim 11$ fm used for finite volume corrections)
- at (and around) physical quark masses
- including isospin-breaking effects



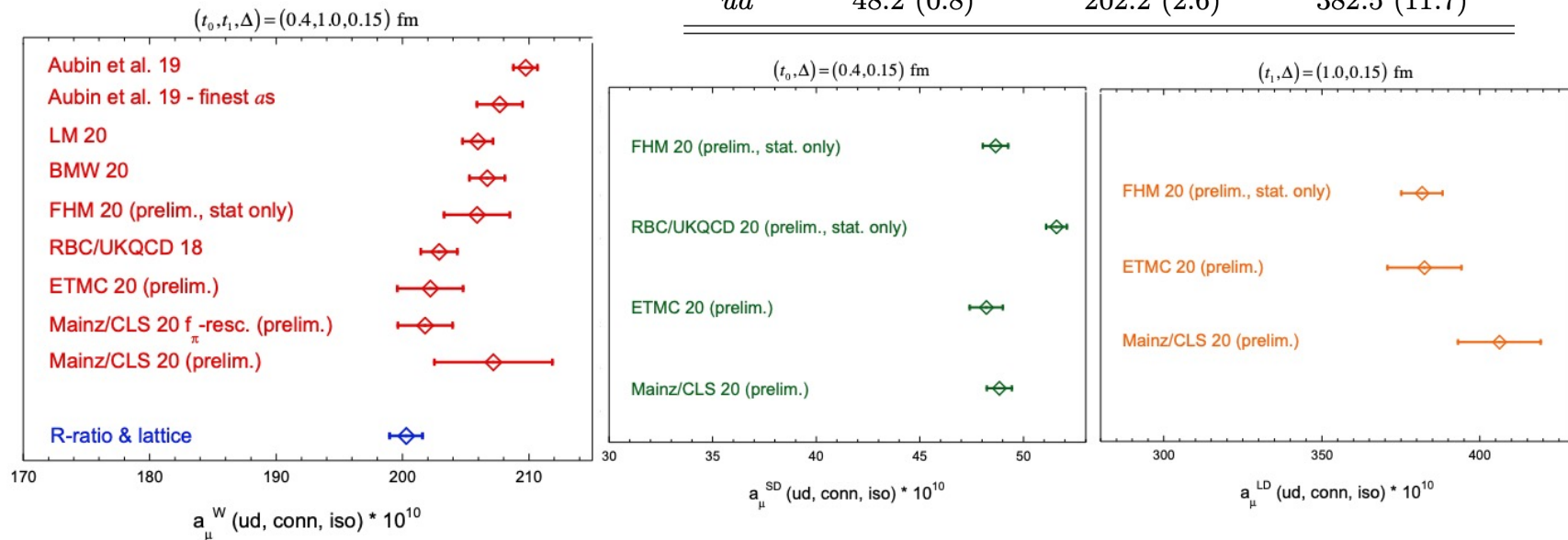
# Comparing evaluations D. Giusti, talk at Lattice 2021



Weight functions for window quantities to compare lattice with lattice, and lattice with data

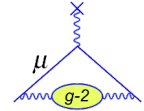
Comparing ud contributions:

$f$	$a_\mu^{SD}(f) \cdot 10^{10}$	$a_\mu^W(f) \cdot 10^{10}$	$a_\mu^{LD}(f) \cdot 10^{10}$
$ud$	48.2 (0.8)	202.2 (2.6)	382.5 (11.7)



# Lattice HVP status

Slide content by Tom Blum.



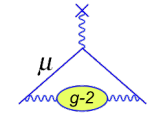
To reach desired precision (2-5 per-mil):

- Strange, charm, (bottom) contributions in good shape.  
(will not resolve issues)
- FV corrections ( $L > 6$  fm) reliable (NNLO  $\chi$ PT, LLGS, HP).  
Important to have a big box (BMW, PACS use  $L = 10$  fm)
- Statistical precision top priority for DW, TM, Wilson (in progress).  
Improved bounding method, low-lying states for long distance tail.
- Physical masses  
(most groups already)
- More, more precise disconnected and IB calculations needed.
- Continuum limit and scale setting (per-mil) are crucial.

Looking to the future:

- Careful, step-by-step study of differences between various lattice calculations now underway, data driven comparison to follow.
- Continuum limit is main focus now, expect it will shift.
- Lattice needs to build consensus, c.f. data driven approach. Happening within Muon  $g-2$  Theory Initiative.
- New results with errors comparable to BMW 2020 soon.





# Conclusions

- Fermilab's Muon  $g-2$  Experiment has confirmed BNL's result: the discrepancy between experiment and SM increases to  $4.2\sigma$ .
- All SM contributions other than HVP, including HLbL, now fully cross checked and understood to be under control.
- Data-driven HVP dominates theory uncertainty with 0.6% error.
- The BMW lattice QCD result weakens the exp-SM discrepancy. It must be confirmed or refuted by cross checks and other lattice calculations.
- Improvements to come:
  - Updated HVP evaluation with new measurements of hadronic cross section data.
  - HVP comparisons for BMW result and between lattice groups/R-ratio as part of theory initiative.
  - HLbL uncertainty to reach  $\sim 10\%$  .
  - New, full SM update from theory initiative before Fermilab's next result.