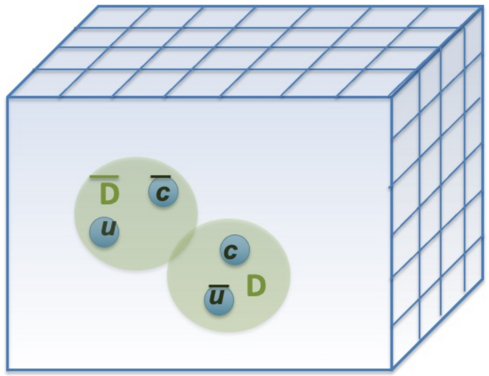
Lattice study of quarkonium-like states



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Workshop on High Energy Physics
Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quests

Protvino, Russia, online
9th November 2021

Outline

Lattice QCD study of

- charmonium-like resonances with I= 0 $\bar{c}\,c$ $\bar{c}c\bar{q}q$ $\bar{c}c\bar{s}s$

- bottomonium-like resonances with I=1 $\bar{b}\,b\,\bar{d}\,u$

Motivation to study charmonium resonances:

Experimentally discovered exotic hadrons

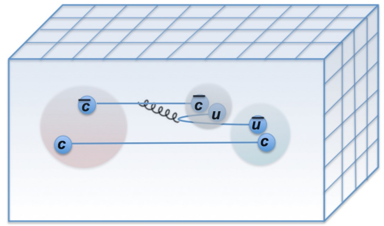
- Most of them contain $\underline{c}c$
- All of them are resonances (decay strongly)

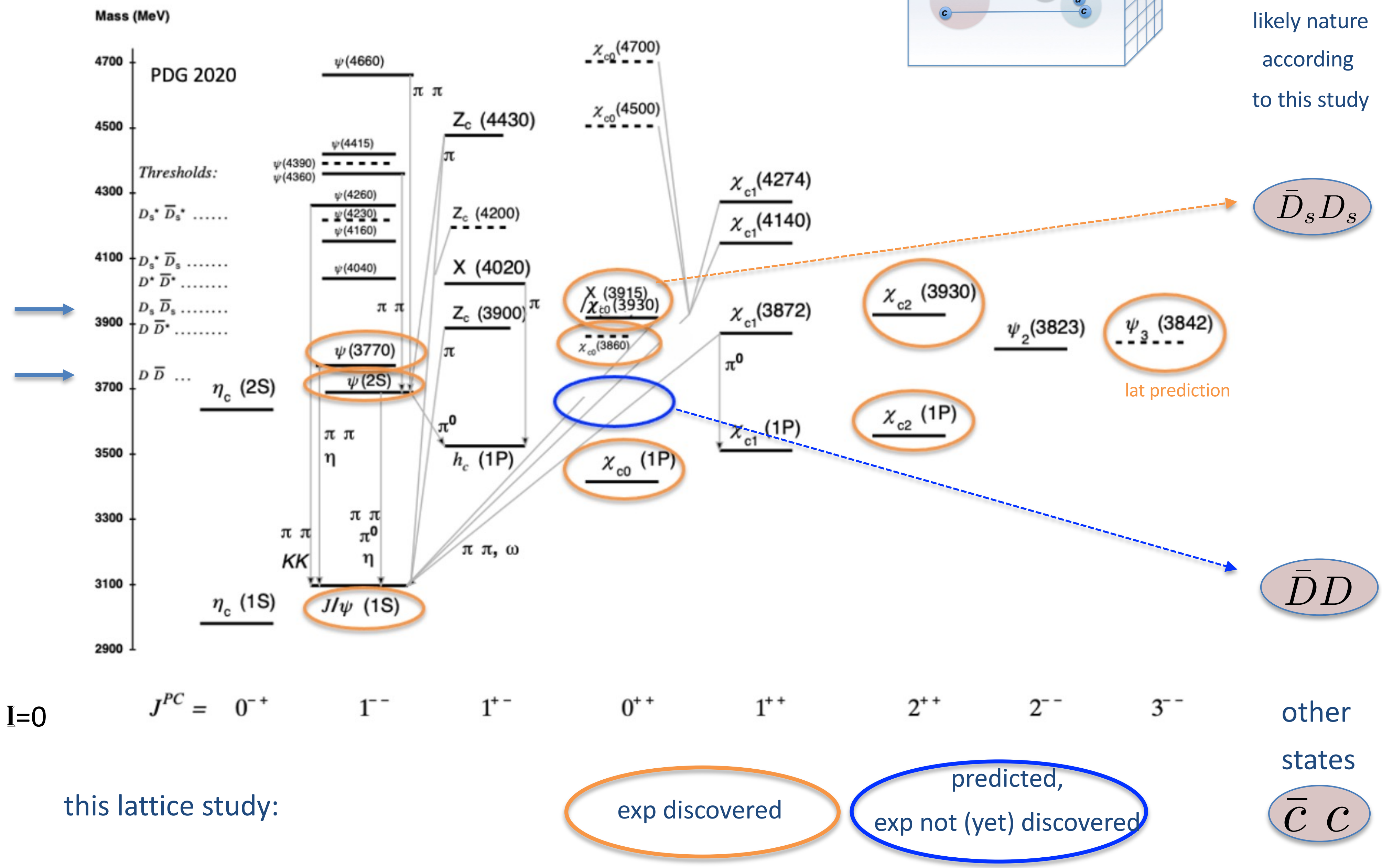
Charmonium-like resonances with I=0

S. P., Collins, Mohler, Padmanath and Piemonte
2011.02542, PRD 2021, $J^{PC}=0^{++}, 2^{++}$
1905.03506, PRD 2019, $J^{PC}=1^{--}, 3^{--}$
2111.02934 (proceedings for Lattice 2021)

PDG 2020

Charmonia with I=0 considered



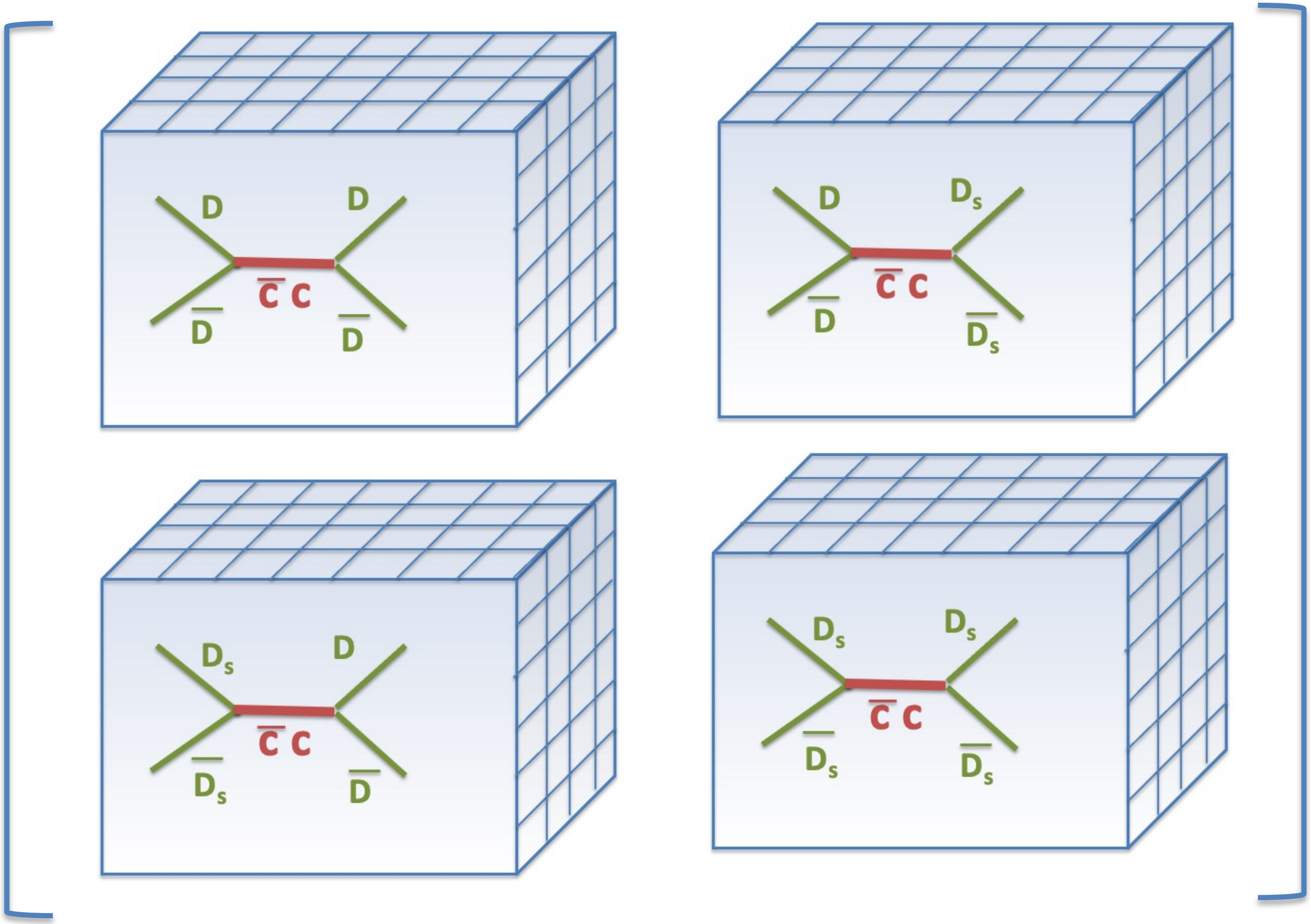


the first extraction of the scattering matrix for coupled channels in the charmonium sector

Charmonium resonances in coupled $D\bar{D} - D_s \bar{D}_s$ scattering

aim: extract scattering matrix $t_{ij}(E)$ illustrated below using Luscher's finite volume method

$t_l(E) =$



$l=0,1,2,3$

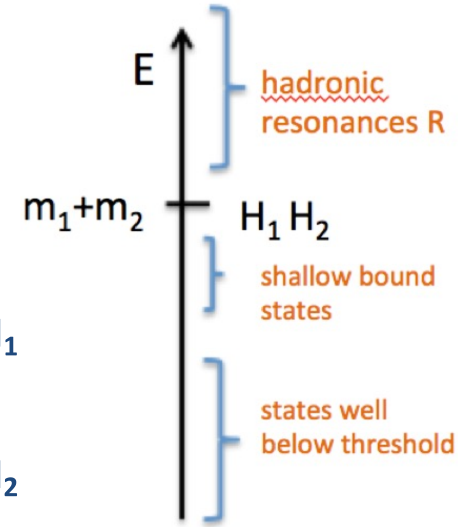
l=1,2,3

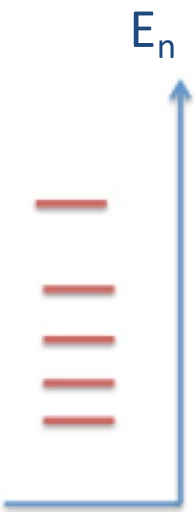
etracted from

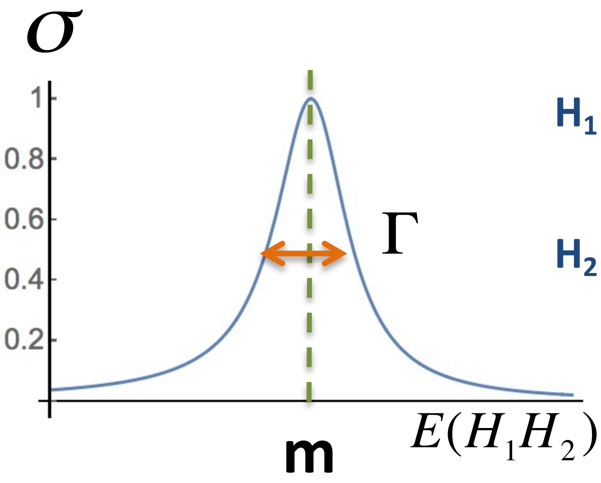
$D\bar{D}$ scattering

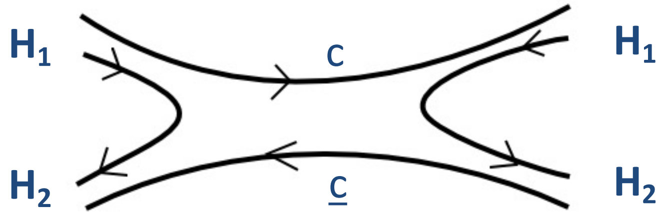
Hadronic resonances and shallow bound states from lattice

one-channel example



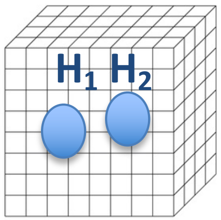






$$T_B(E) \propto \frac{1}{s-m_B^2} \qquad T_R(E) = \frac{-m_R\,\Gamma}{E^2 - m_R^2 + i\,m_R\,\Gamma}$$

$$T_B(E=m_B)=\infty$$



$$\sigma(E) \propto |T(E)|^2$$

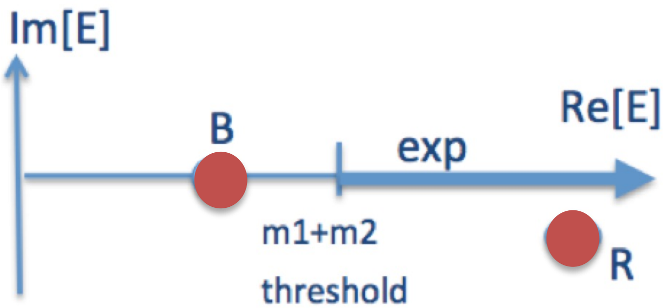
energy of eigenstate

scattering matrix for real E

E → $T(E)$

analytic relation: Luscher 1991

continuation to complex E



location of poles in complex E plane

Towards E_n for coupled-channel $D\bar{D} - D_s \bar{D}_s$ scattering

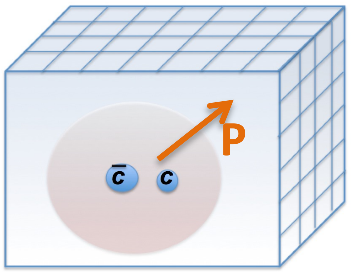
$$C_{ij}(t) = \langle 0 | \mathcal{O}_i(t)\ \mathcal{O}_j^+(0) | 0 \rangle = \sum_n Z_i^n\ Z_j^{n*}\ e^{-E_n t}$$

	lat	exp
m_π~280 MeV	$m_{u/d}$	$> m_{u/d}^{exp}$
	m_s	$< m_s^{exp}$
	$m_u+m_d+m_s = m_u^{exp} + m_d^{exp} + m_s^{exp}$	
	m_c	$\gtrsim m_c^{exp}$

CLS Nf=2+1 ensembles

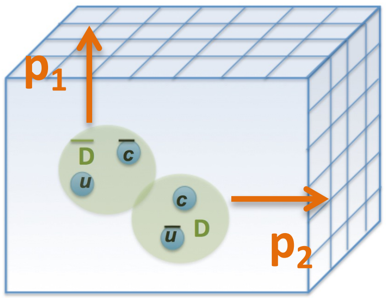
Implemented operators

$$O^{\bar{c}c} = (\bar{c}\Gamma c)_{\vec{P}}$$



$$O^{\bar{D}D} = (\bar{c}\Gamma_1 q)_{\vec{p}_1} (\bar{q}\Gamma_2 c)_{\vec{p}_2} = \bar{D}(\vec{p}_1)\ \ D(\vec{p}_2)$$

$$O^{\bar{D}_s D_s} = (\bar{c}\Gamma_1 s)_{\vec{p}_1} (\bar{s}\Gamma_2 c)_{\vec{p}_2} = \bar{D}_s(\vec{p}_1)\ \ D_s(\vec{p}_2)$$



$$\vec{P} = \vec{p}_1 + \vec{p}_2$$

P: 0
(0,0,1) $2\pi/N_L$
(1,1,0) $2\pi/N_L$

N_L=24, 32

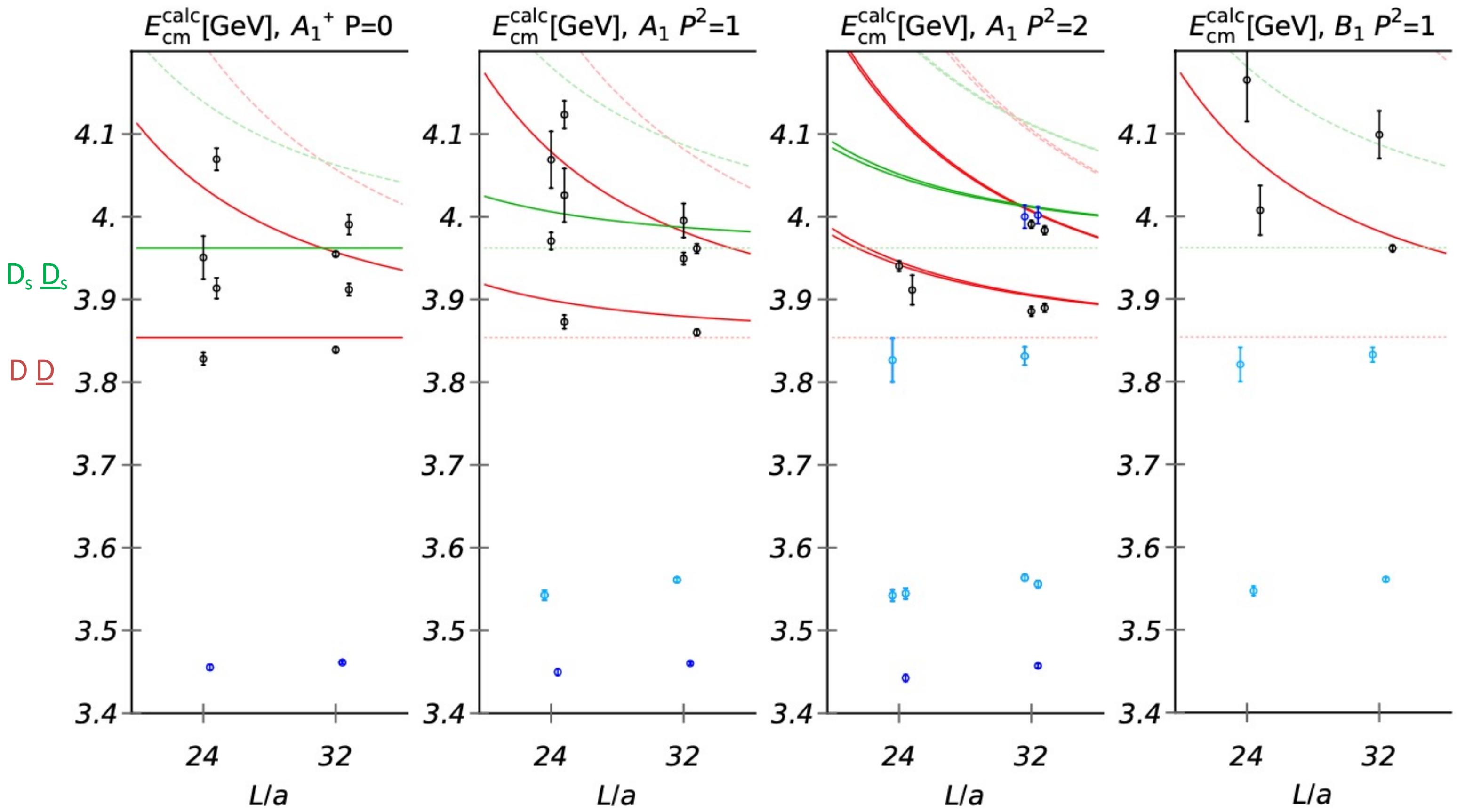
$$O^{J/\psi\ \omega} = J/\psi(\vec{p}_1)\ \ \omega(\vec{p}_2)$$

$$O^{\bar{D}^* D^*} = \bar{D}^*(\vec{p}_1)\ \ D^*(\vec{p}_2)$$

omission of channel $\eta_c\ \eta$ for 0++

Energies of eigen-states E_n in irreps that contain $J^{PC}=0^{++}, 2^{++}$

for m_D=1927 MeV



non-interacting energies denoted

D $\underline{D}$

D_s $\underline{D}_s$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{-+}$

black circles used for scattering analysis

Extraction of matrix t(E) :

$$S_{ij}(E_{cm}) = 1 + 2i\ \rho\ t_{ij}(E_{cm})$$

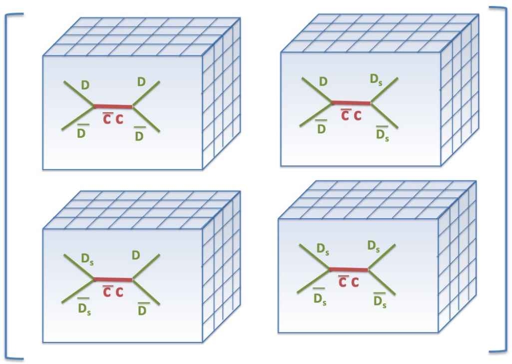
Luscher's equation for 2x2 coupled system

$$det[1 + i\ t(E_{cm})\ F(E_{cm})] = 0$$

known 2x2 matrix

$\rho_i \equiv 2p_i/E_{cm}$

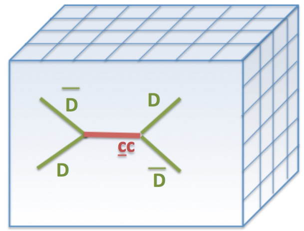
$t_1(E)$ =



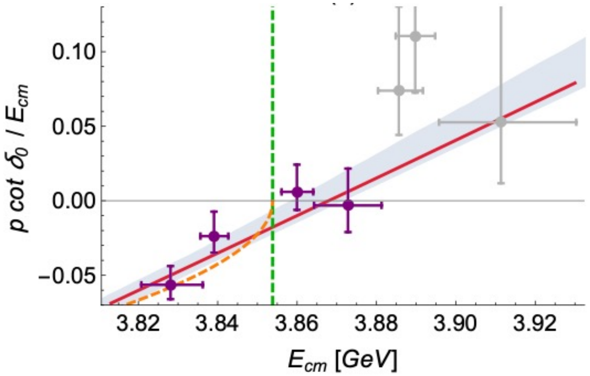
the need to parametrize t_{ij}(Ecm)

$J^{PC}=0^{++}$ (l=0) for low energy region :
unexpected shallow bound state slightly below $D\bar{D}$ threshold

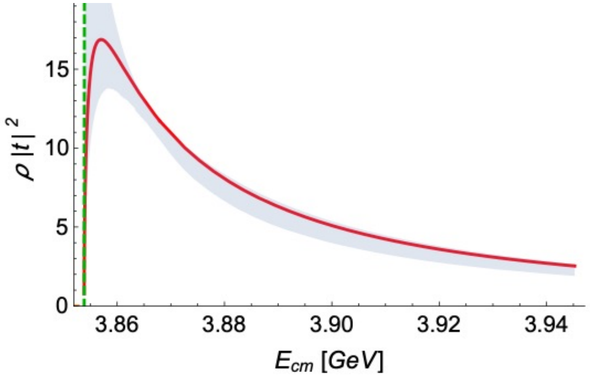
$\bar{D}D$



$D\bar{D}$



$D\bar{D}$



$$m - 2\, m_D = -4.0\,^{+3.7}_{-5.0}\,\mathrm{MeV}$$

$$D\bar{D} \to D\bar{D}$$

one-channel $D\bar{D}$ and two-channel analysis give consistent results

exp: not claimed yet

- possible hint from Belle [0708.3812] and BaBar [1002.0281] ?
- look for peak above $D\bar{D}$ threshold
- see strategies [Oset et al 1512.04048, 2004.05204, 2010.15431, 1211.1862]

lattice : a virtual bound state pole is present but not mentioned in

[Lang, Leskovec, Mohler, SP, 1503.05363]

pheno: predicted by effective models with vector meson exchange

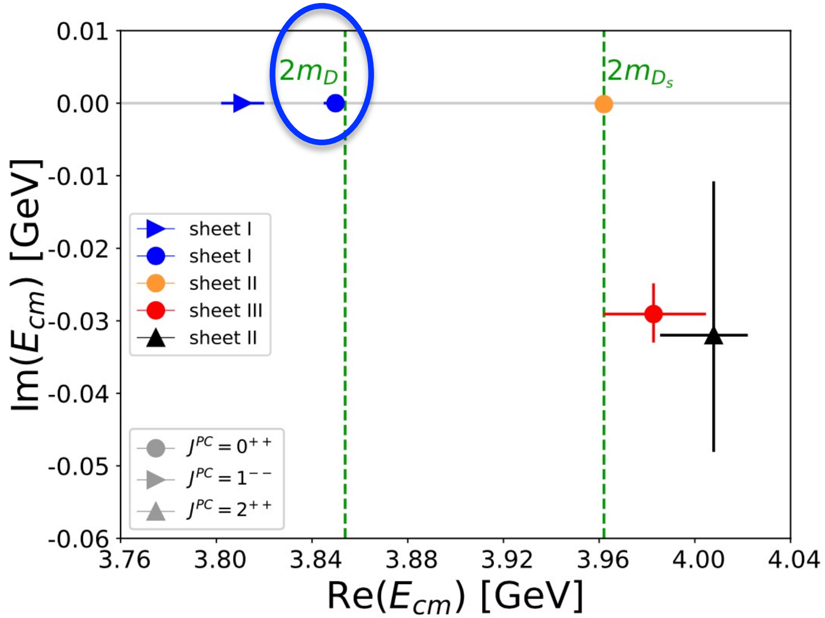
"X(3720)" [Oset et al, many works, eg. state at ~1720 MeV in Table 4 of 0612179]

predicted as spin partner of X(3872)

[Hildago Duque et al 1305.4487, Baru et al 1605.09649]

molecular models F.-K. Guo 2101.01021

poles in complex E plane



$D\bar{D}$ $\qquad$ $D_s\bar{D}_s$

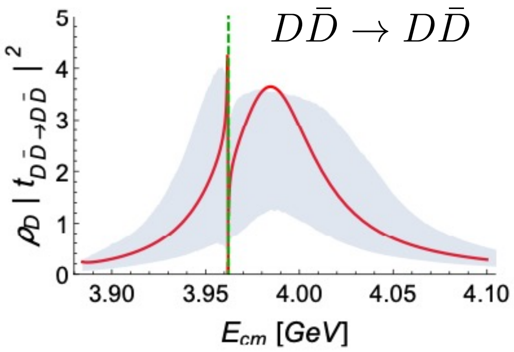
$J^{PC}=0^{++}$: higher energy region around $D_s\bar{D}_s$ threshold

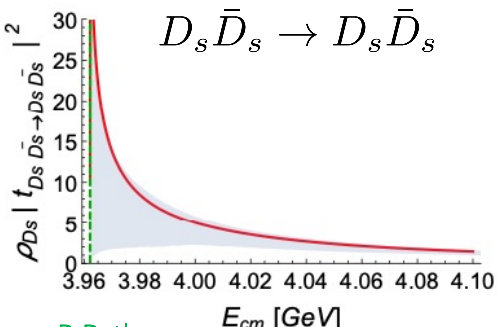
$D\bar{D} - D_s\bar{D}_s$

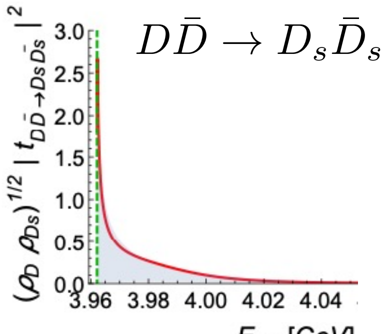
near pole

$$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$







- conventional broad resonance coupling mostly to $D\bar{D}$ $\bar{c}\ c$

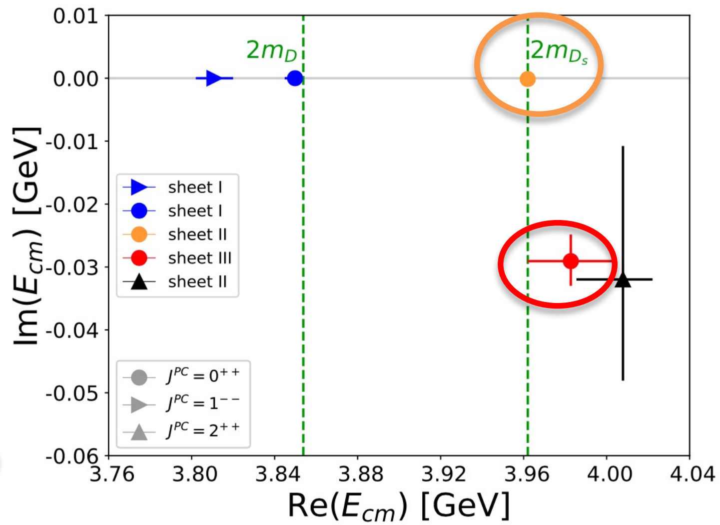
lat $\quad m - M_{av} = 880^{+28}_{-20}\,\text{MeV}\,, \quad g = 1.35\ ^{+0.04}_{-0.08}\,\text{GeV}$

exp $\chi_{c0}(3860)$: $m - M_{av} = 793\ ^{+48}_{-35}\,\text{MeV}\,, \quad g = 2.5^{+1.2}_{-0.9}\,\text{GeV}$

Belle 2017 $\quad M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

- state near $D_s\bar{D}_s$ threshold coupling mostly to $D_s\bar{D}_s$ $\quad \bar{D}_s D_s \quad \bar{c}s\bar{s}c$

lat $\quad m - 2m_{D_s} = -0.2\ ^{+0.16}_{-4.9}\,\text{MeV}\,, \quad g = 0.10\ ^{+0.21}_{-0.03}\,\text{GeV}$



LHCb 2009.00026

exp $\chi_{c0}(3930)$: $m - 2m_{D_s} = -12.9 \pm 1.6\,\text{MeV}\,, \quad \Gamma = 17 \pm 5\,\text{MeV}\,, \quad g = 0.67 \pm 0.10\,\text{GeV}$

exp $X(3915)$: $m - 2m_{D_s} = -18.3 \pm 1.9\,\text{MeV}\,, \quad \Gamma = 20 \pm 5\,\text{MeV}\,, \quad g = 0.72 \pm 0.10\,\text{GeV}$

Babar (those two are likely the same exp state: listed as one state in PDG)

$D_s\bar{D}_s$ nature explains why width to $D\bar{D}$ is so small

Pheno predictions of $c\bar{s}s\bar{c}$ state:

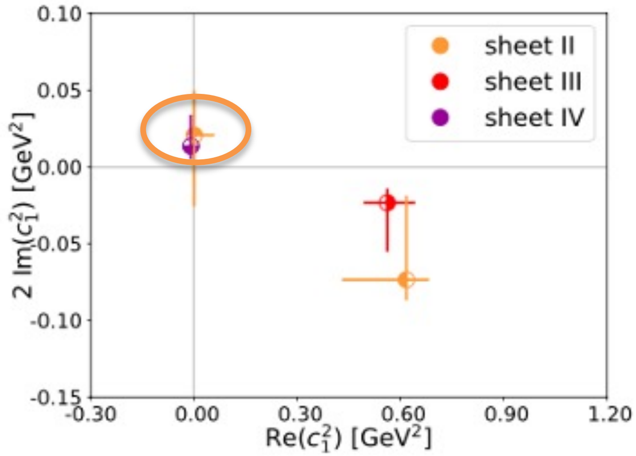
Lebed and Polosa: 1602.08421

2103.12425

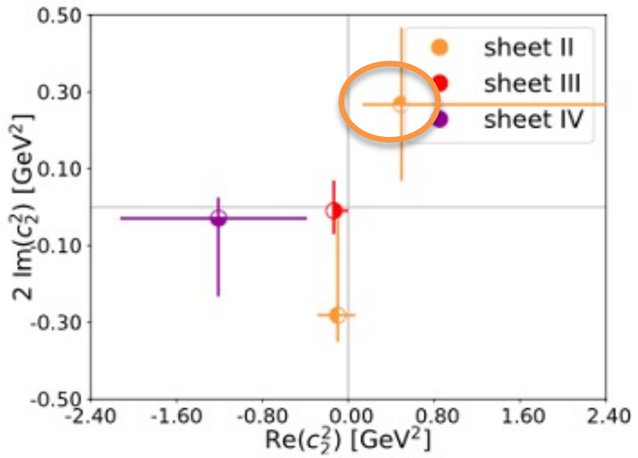
Guo et al, virtual state 2101.01021

couplings of state near $D_s\bar{D}_s$ threshold to both channels

$$t_{ij} \sim \frac{c_i\ c_j}{(E^p_{cm})^2 - E^2_{cm}}$$

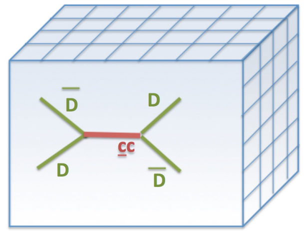


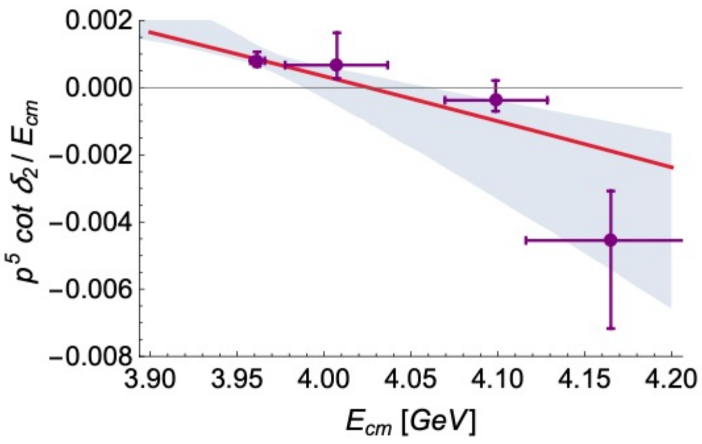
$D\bar{D}$

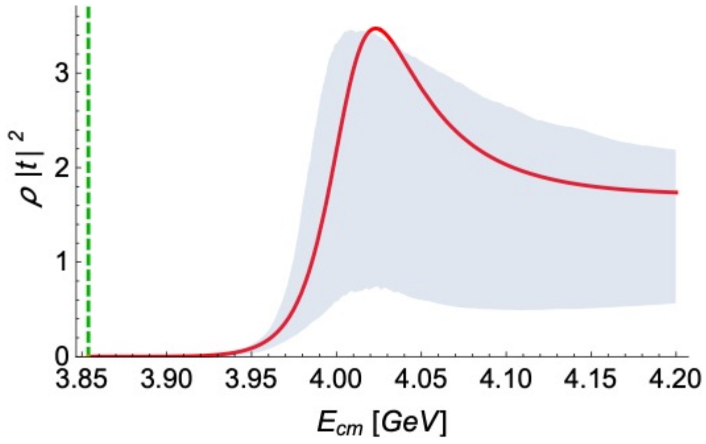


$D_s\bar{D}_s$

$J^{PC}=2^{++}$ (l=2) : conventional resonance $\bar{c}\ c$







$$D\bar{D} \to D\bar{D}$$

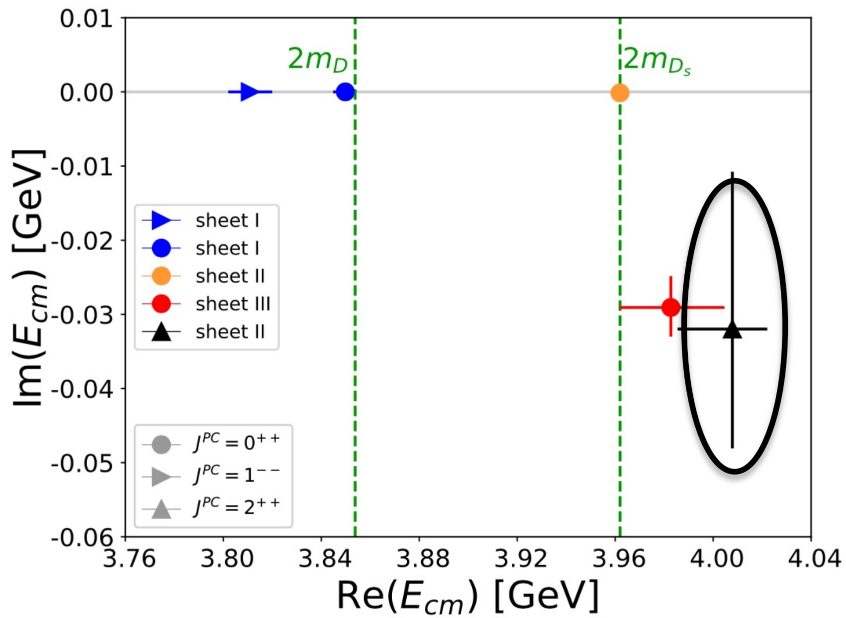
- 2++ resonance $\Gamma \equiv g^2 p_D^{2l+1}/m^2$

lat $\chi_{c2}(3930):\ m - M_{av} = 904\ ^{+14}_{-22}\,\text{MeV}, \quad g = 4.5^{+0.7}_{-1.5}\,\text{GeV}^{-1}$

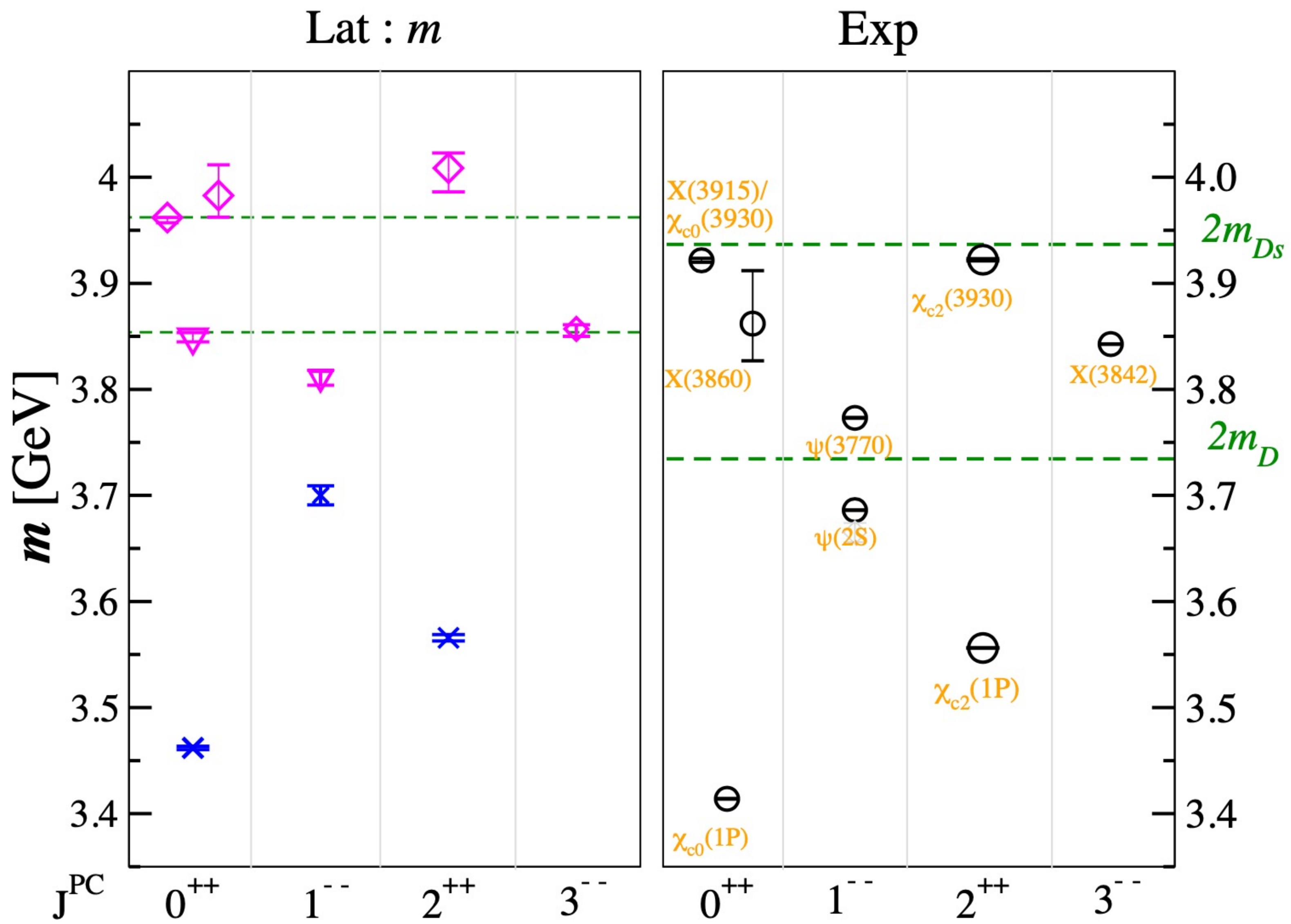
exp $\chi_{c2}(3930):\ m - M_{av} = 854 \pm 1\,\text{MeV}, \quad g = 2.65 \pm 0.12\,\text{GeV}^{-1}$

PDG

$$M_{av} = \tfrac{1}{4}(3m_{J/\psi} + m_{\eta_c})$$



summary of masses for charmonium-like states



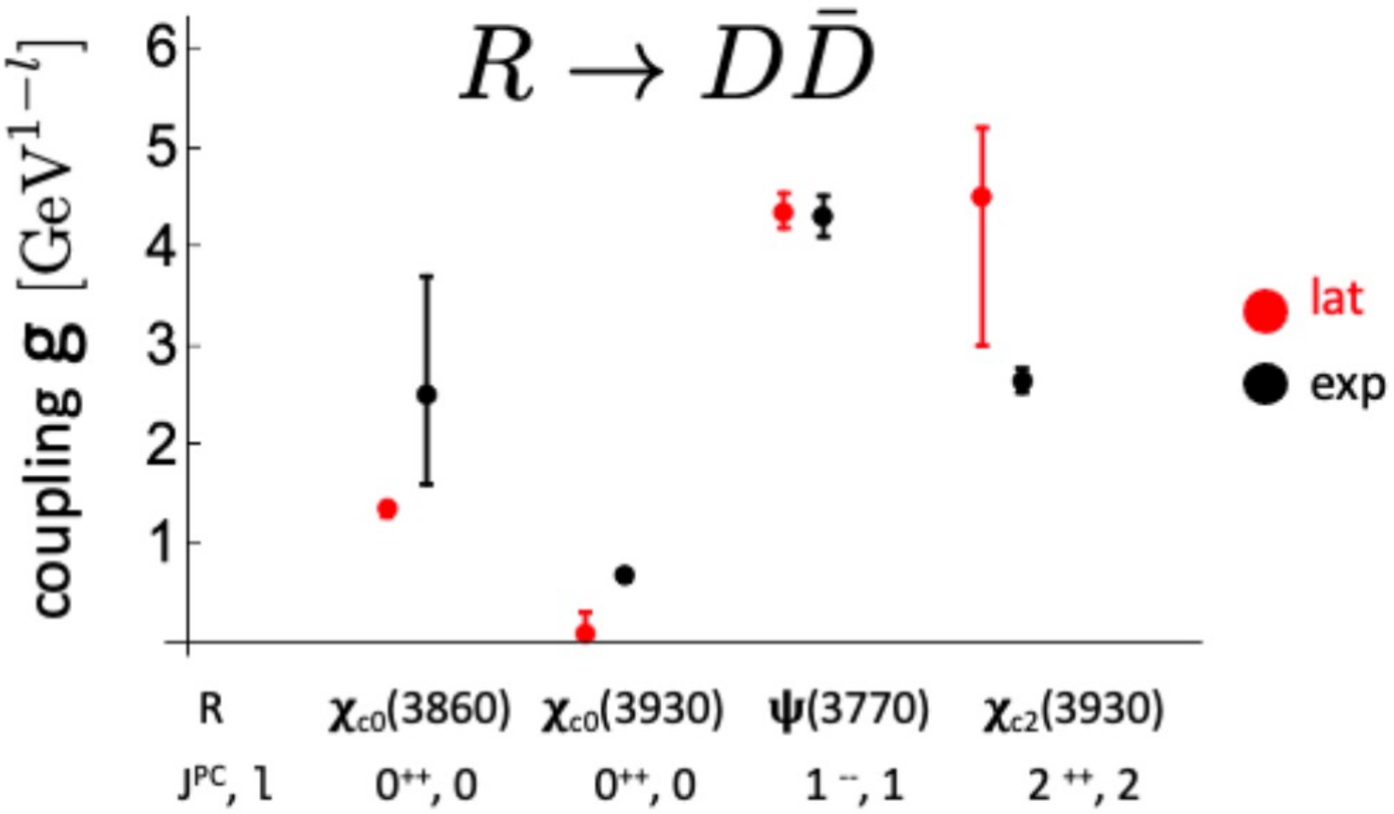
m_π~ 280 MeV $\quad m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

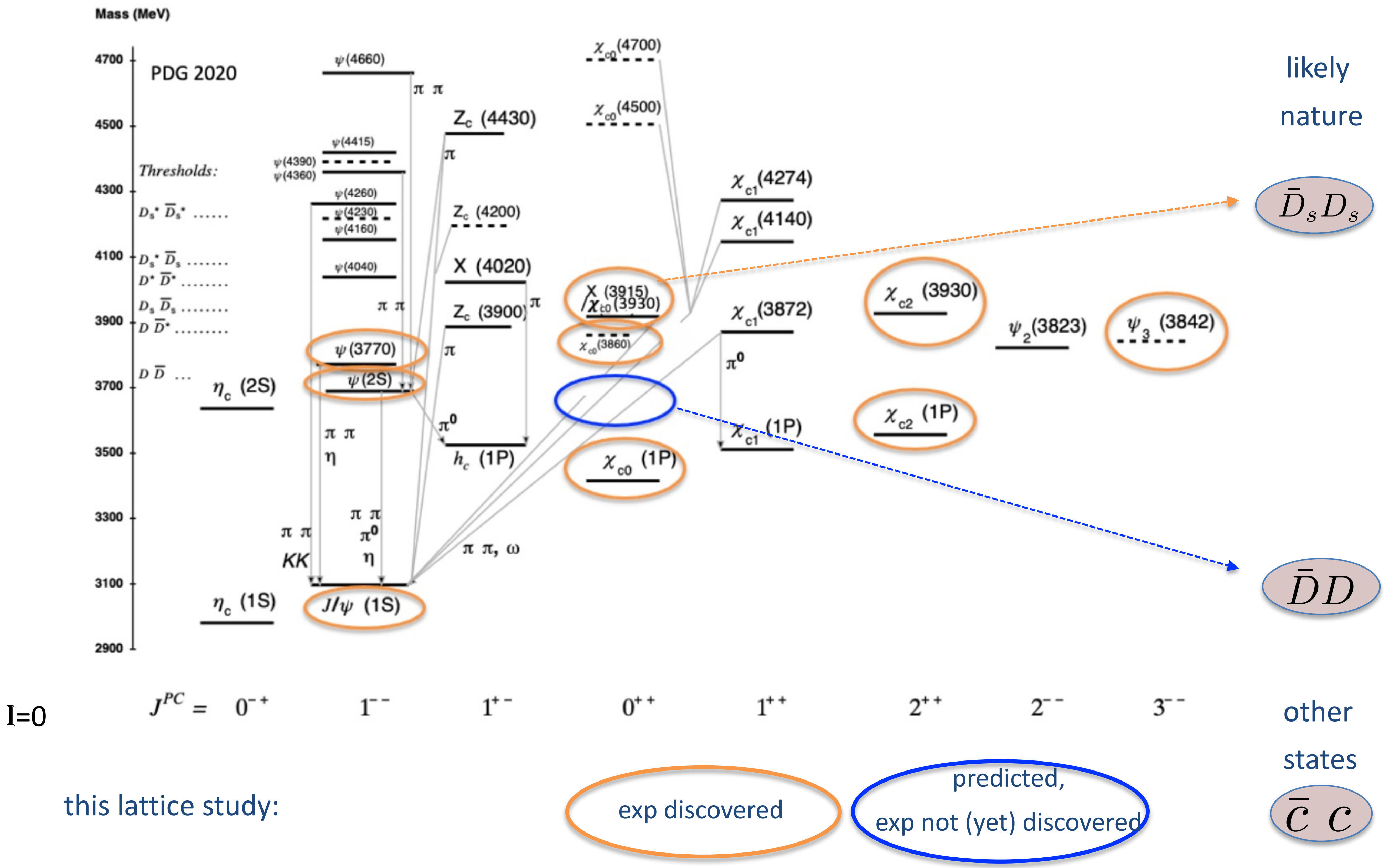
summary of couplings that parametrize the width

$$\Gamma \equiv g^2 p_D^{2l+1}/m^2$$



PDG 2020

summary on charmonium-like states



$\bar{b} b \bar{d} u$

Bottomonium-like resonances with I=1

M. Sadl, S. P.: 2109.08560, accpeted to PRD
S.P., Bahtyar, Petkovic: 1912.02656, PLB

$\bar{b}b\bar{d}u$ with Lattice QCD, non-static b quarks and Luscher's method : **to challenging !**

$Z_b^+(10610)$, $Z_b^+(10650)$ $\quad$ I=1, $J^{PC}=1^{+-}$

observed decays

$\Upsilon(1S)\,\pi$, $\Upsilon(2S)\,\pi$, $\Upsilon(3S)\,\pi$

$h_b(1S)\,\pi$, $h_b(2S)\,\pi$

$$\bar{b}b\bar{d}u \rightarrow \bar{b}b + \bar{d}u$$

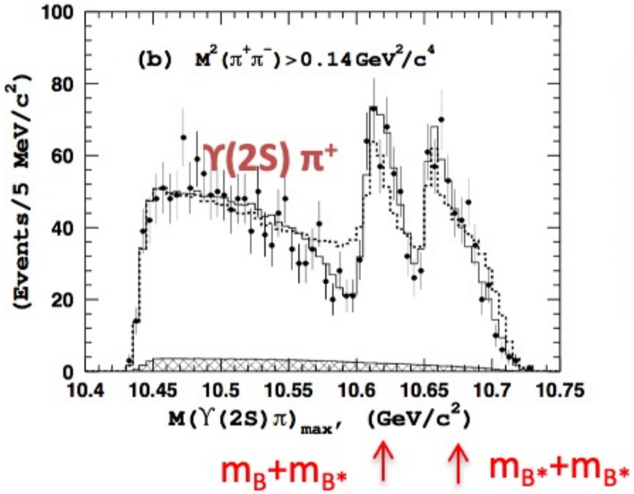
dominant Br: B $\underline{B}$* , B* $\underline{B}$*

$$\bar{b}b\bar{d}u \rightarrow \bar{b}u + \bar{d}b$$

allowed (unobserved) $\quad \eta_b\,\rho$

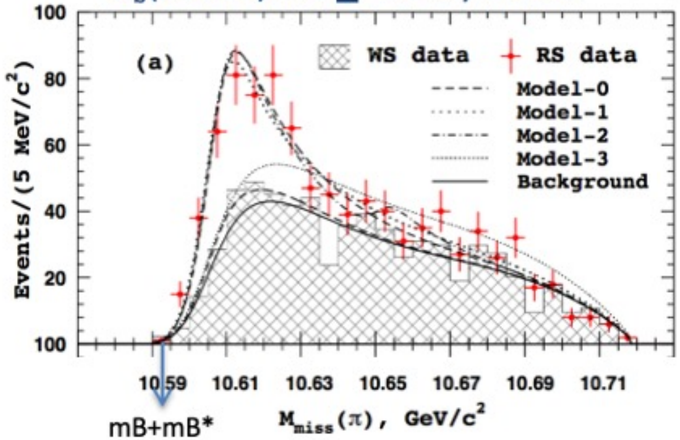
discovered by Belle in 2011

[PRL 108 (2012) 122001]



[Belle, 1512.07419, PRL 2016]

$Z_b(10610)$ in B$\underline{B}$* decay mode $\quad$ Br≈85 %



Rigorous treatment to challenging:

- at least 7 two-particle channels coupled
- very dense B$\underline{B}$* and B*$\underline{B}$* energy levels

Z_b with static b and $\underline{b}$

general idea: talk by Marc Wagner

Idea and the only previous lat study

Bicudo, Cichy, Peters, Wagner [proceedings : Lat16: 1602.07621]

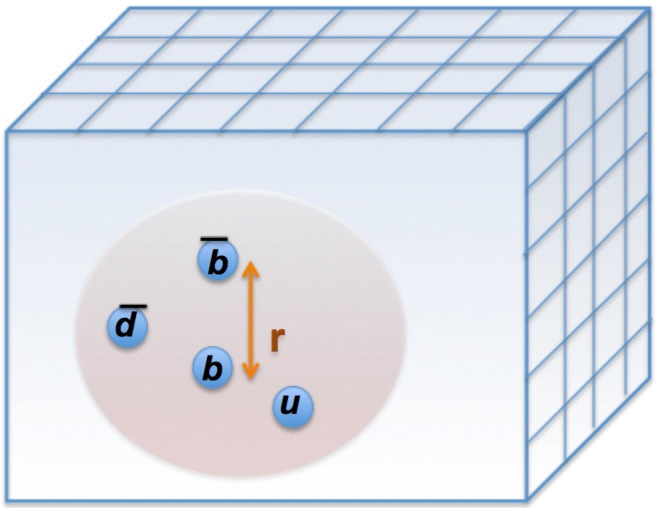
Born-Oppenheimer approach

h = heavy: b, $\underline{b}$ l=light:u,d,gluons

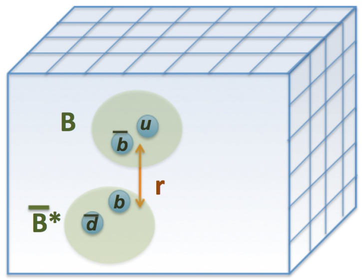
Step 1: fix static b and $\underline{b}$ at distance r: determine $E_n(r)$ for light d.o.f.: lattice QCD

Step 2: consider motion of heavy d.o.f. in the potential determined in step 1 with non-relativistic Schrodinger equation

[Braaten et al PRD 1402.0438 , Brambilla et al PRD 1707.09647, Bali et al. hep-lat/0505012 PRD, Bicudo & Wagner 1209.6274 + many others ..]

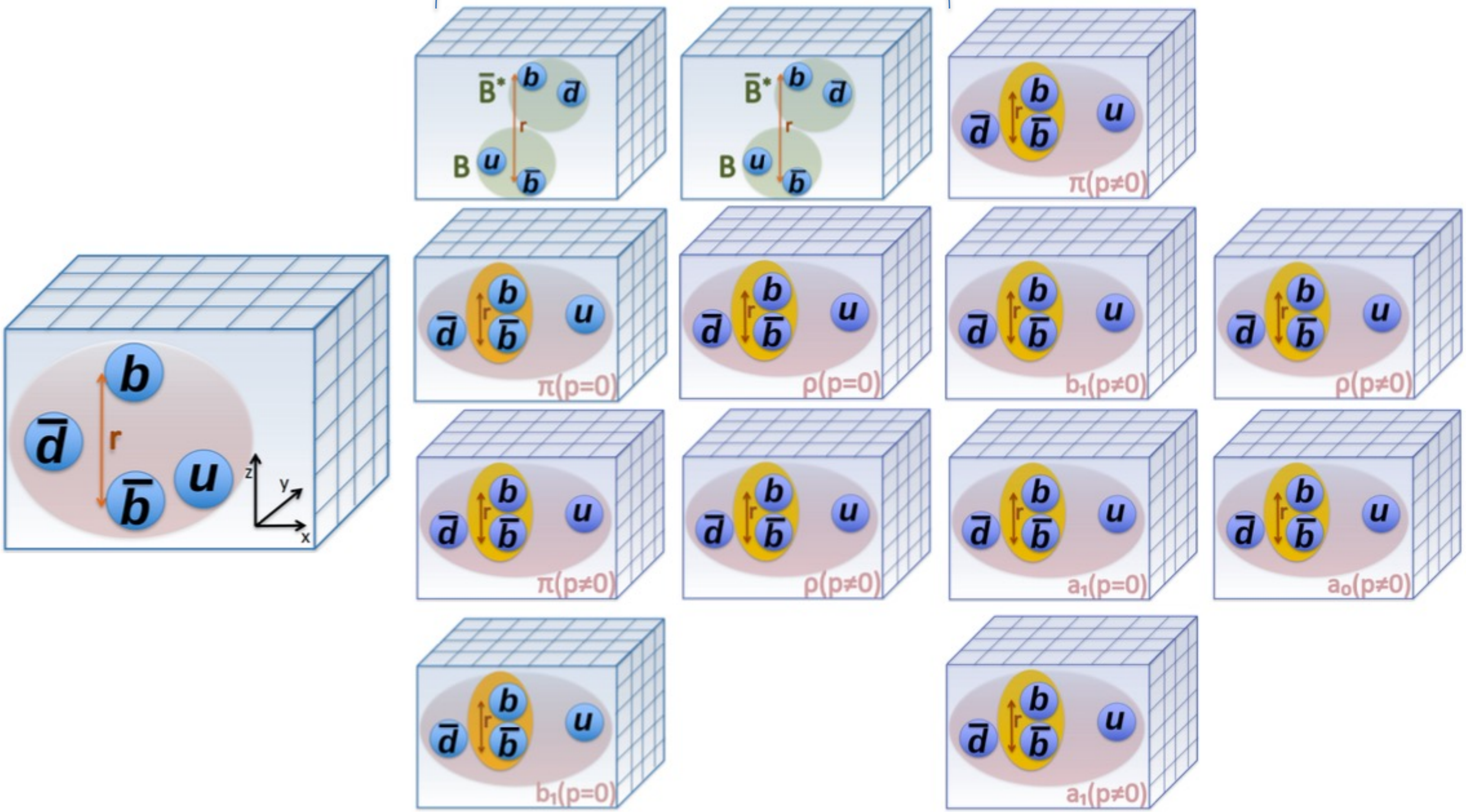


aim:



Four different sets of quantum numbers considered

couple to $J^{PC}=1^{+-}$ (Zb)



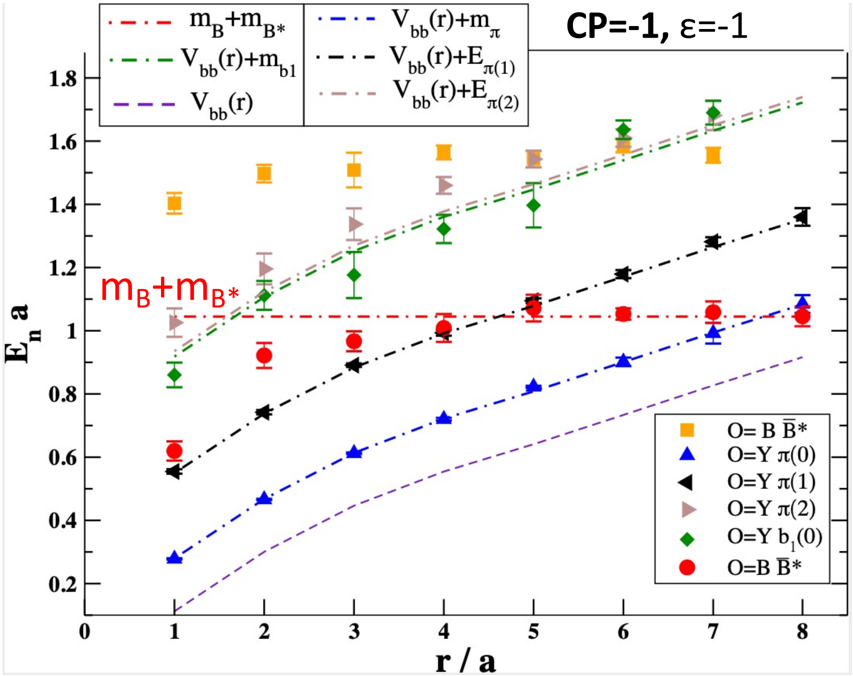
(a)

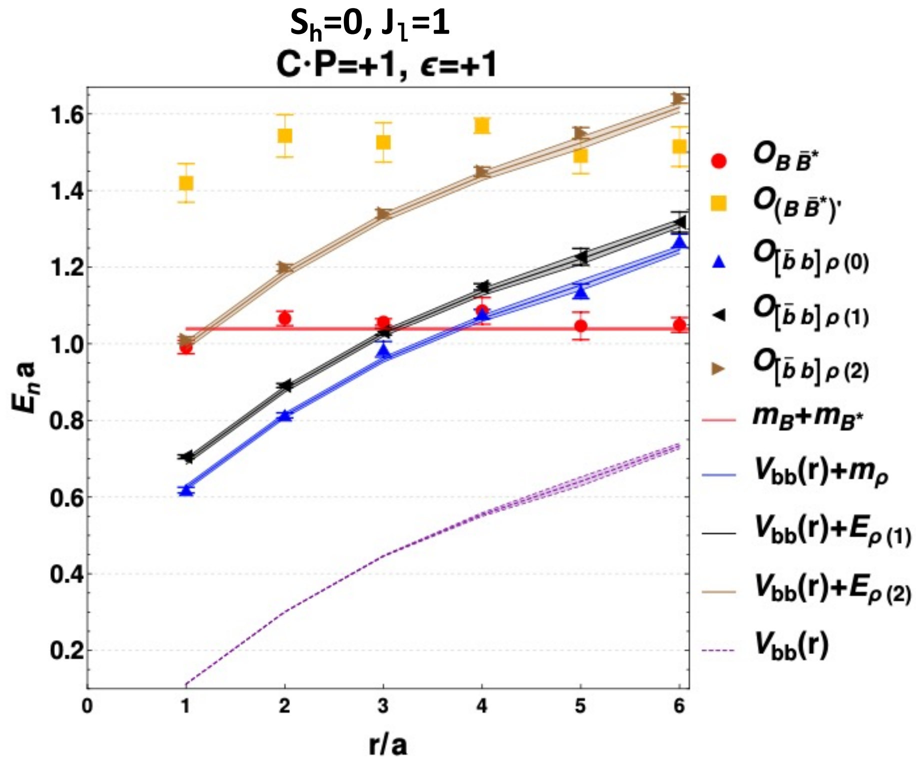
(b) $J_z^l = 0,\ C{\cdot}P = -1,\ \epsilon = -1$ (c) $J_z^l = 0,\ C{\cdot}P = +1,\ \epsilon = +1$ (d) $J_z^l = 0,\ C{\cdot}P = +1,\ \epsilon = -1$ (e) $J_z^l = 0,\ C{\cdot}P = -1,\ \epsilon = +1$

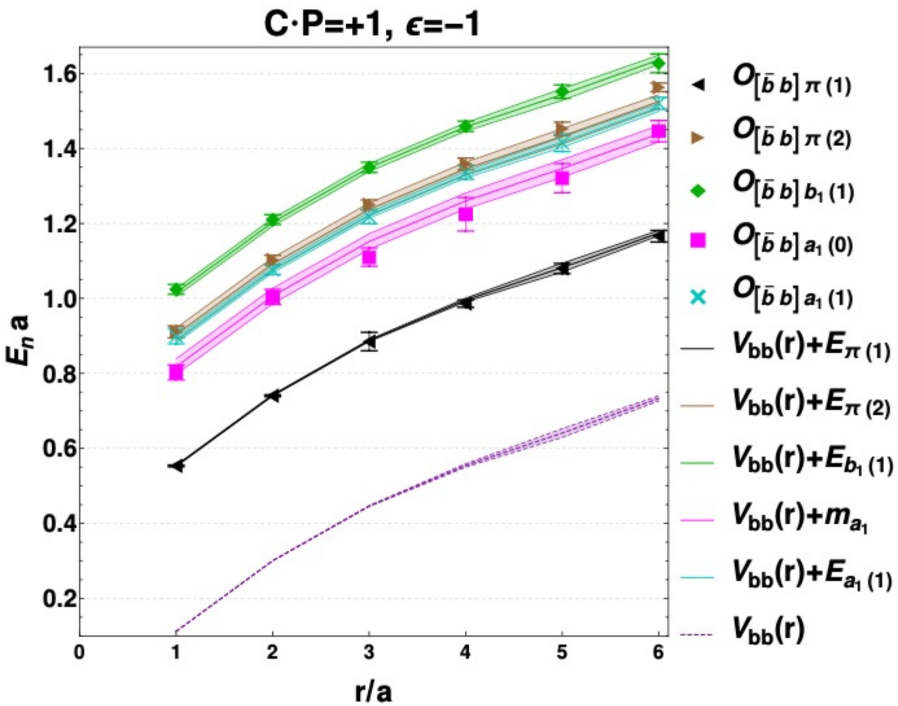
$S_h = 1\ \&\ J_l = 0$ $S_h = 0\ \&\ J_l = 1$

$\Upsilon\pi$ $\eta_c\rho$

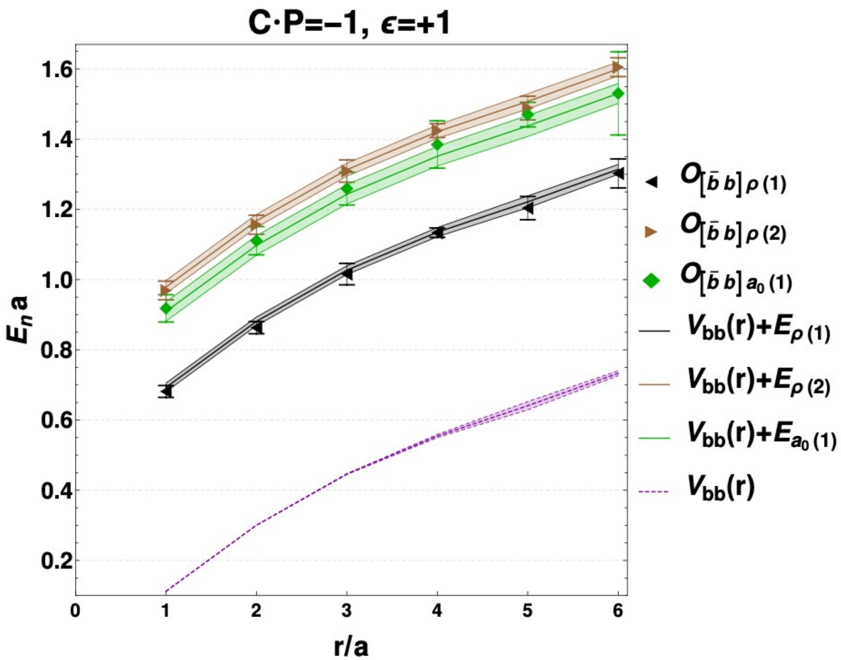
Eigen-energies $E_n(r)$



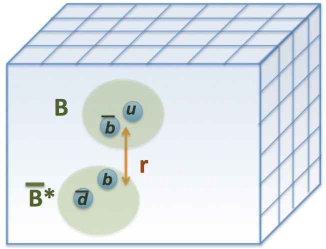


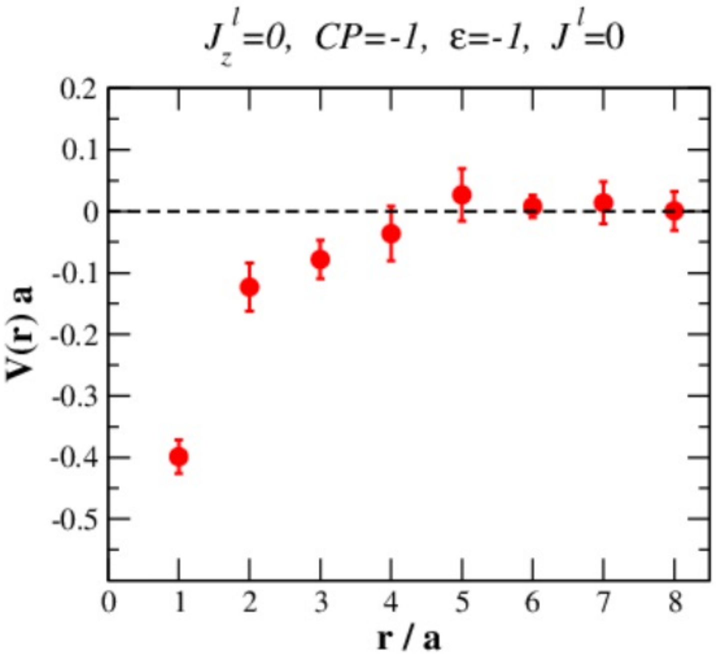


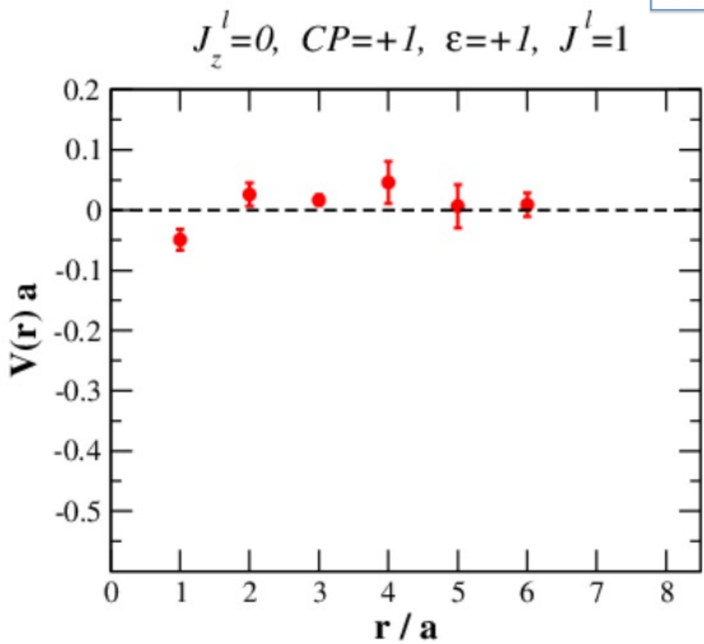
tudy



Potential V(r) between B and B̄*







$$\left[-\frac{\hbar^2}{2\mu}\frac{d^2}{dr^2}+\frac{\hbar^2 L(L+1)}{2\mu r^2}+V(r)\right]u(r)=Eu(r)$$

$$V(r) = -A\ e^{-(r/d)^F}$$

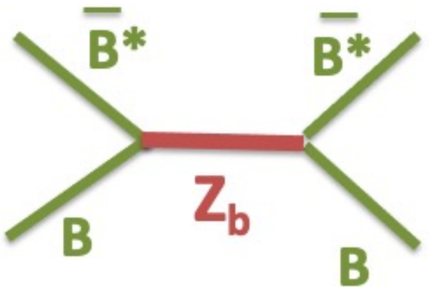
$$M - m_B - m_{B^*} = -48\ ^{+41}_{-108}\ \mathrm{MeV}$$

in agreement with only previous lattice study

Bicudo, Cichy, Peters, Wagner [proceedings : Lat16: 1602.07621]

Conclusions on S_h=1, J_l=0: peak above B$\underline{B}$* for shallow bound state Z_b

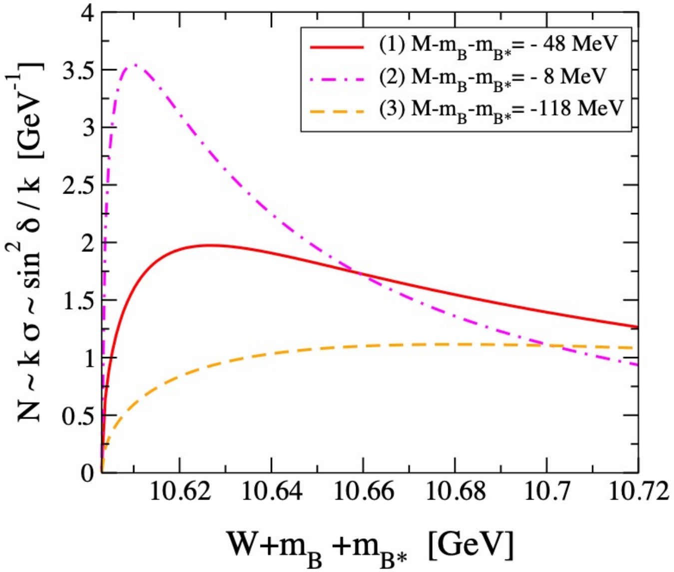
Schrodinger equation for B$\underline{B}$* motion -> scattering phase shift $\boldsymbol{\delta}$ -> cross section $\boldsymbol{\sigma}$



$$M - m_B - m_{B^*} = -48\,^{+41}_{-108}\ \text{MeV}$$

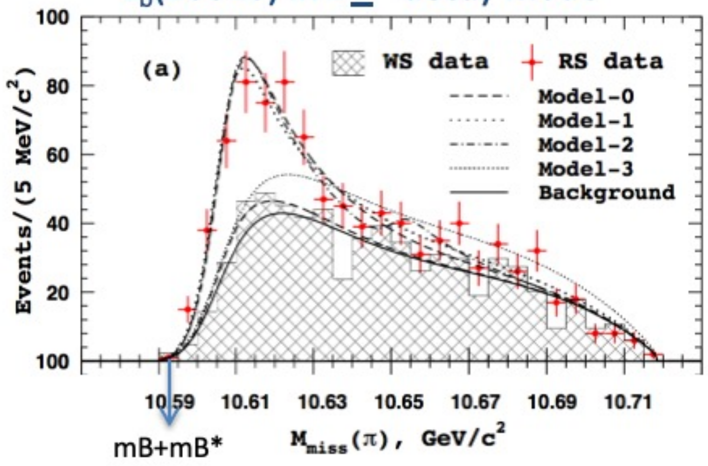
$$N_{B\bar{B}^*} \propto k\ \sigma$$

for three parametrizations consistent with lattice V(r)



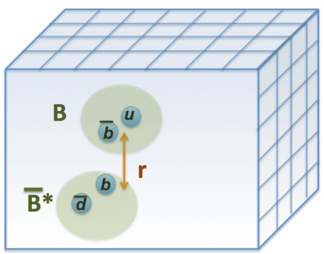
[Belle, 1512.07419, PRL 2016]

Z_b(10610) in B$\underline{B}$* decay mode



Conclusion from our lattice study [in agreement with Wagner & Bicudo & Peters]

- attraction between B and $\underline{B}^*$ renders bound state Zb
- for certain parametrizations bound state is close below threshold and renders peak in B$\underline{B}$* cross-section above threshold



Re-analysis of exp data

[Wang, Baru, Filin, Hanhart, Nefediev, Wynen, 1805.07453, PRD 2018]:

- Zb is virtual bound state few MeV below B$\underline{B}$*
  [when coupling to ($\underline{b}$b)($\underline{d}$u) omitted]
- renders peak above threshold

Conclusions

Results of these lattice QCD studies:

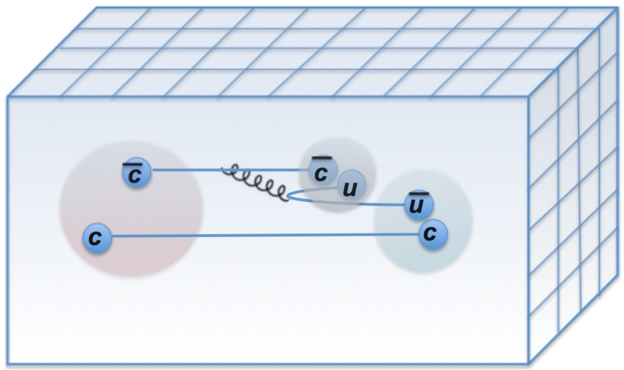
- many conventional charmonium resonances and bound states with I=0 confirmed $\bar{c}\,c$

- two unconventional charmonium-like states with I=0 identified $D\bar{D},\, D_s\bar{D}_s$

- Zb resonances likely related to significant attraction between B and B* $\bar{b}\,b\,\bar{d}\,u$

Backup

Lattice details for charmonium-like studies



CLS ensembles with u/d, s dynamical quarks

a≃0.086 fm

N_L=24, 32

lat exp

$m_{u/d} > m_{u/d}^{exp}$

$m_s < m_s^{exp}$

$m_u+m_d+m_s = m_u^{exp}+m_d^{exp}+m_s^{exp}$

$m_c \gtrsim m_c^{exp}$

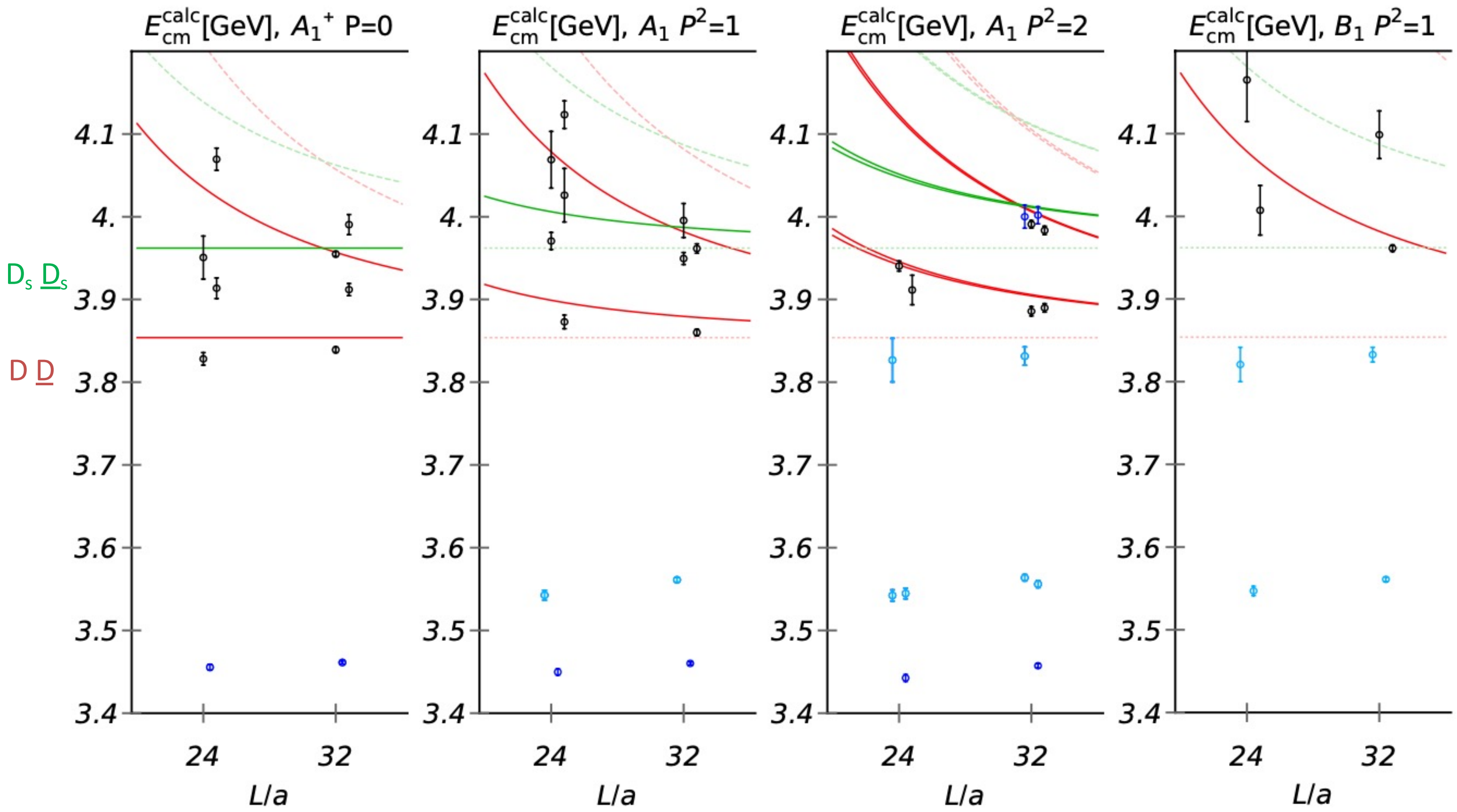
m [MeV]	lat	exp
m_π	280(3)	137
m_D	1927(2)	1867
m_{Ds}	1981(1)	1968
M_{av}	3103(3)	3068

$M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$

separation between DD and DsDs threshols smaller than in exp

Energies of eigen-states E_n in irreps that contain $J^{PC}=0^{++}, 2^{++}$

for m_D=1927 MeV



non-interacting energies denoted

D $\underline{D}$

D_s $\underline{D}_s$

$J^{PC}=0^{++}$

$J^{PC}=2^{++}, 2^{-+}$

black circles used for scattering analysis

Extraction of matrix t(E) : i,j=1,2 1: $D\underline{D}$, 2: $D_s\underline{D}_s$, l=0,2

$$(t^{-1})_{ij} = \frac{2}{E_{cm}\ p_i^l p_j^l}\ (\tilde{K}^{-1})_{ij} - i\ \rho_i\ \delta_{ij}$$

$$\det[\tilde{K}^{-1}_{l;ij}(E_{cm})\ \delta_{ll'} - B^{\vec{P},\Lambda}_{ll';i}(E_{cm})\ \delta_{ij}] = 0$$

Luscher's equation

known matrix (we take into account that it is not diagonal in l=0,2)

package TwoHadronsInBox by C. Morningstar et al employed [1707.05817]

$$\rho_i \equiv 2p_i/E_{cm}$$

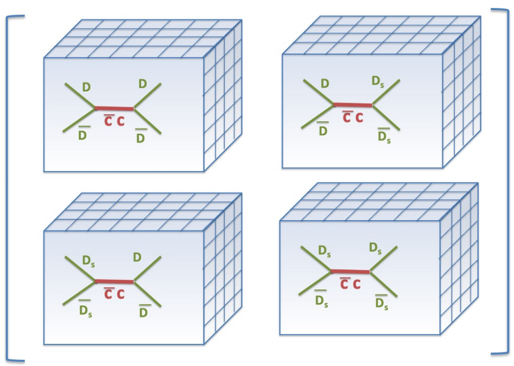
Parametrization for K(s) matrix in each of two energy regions

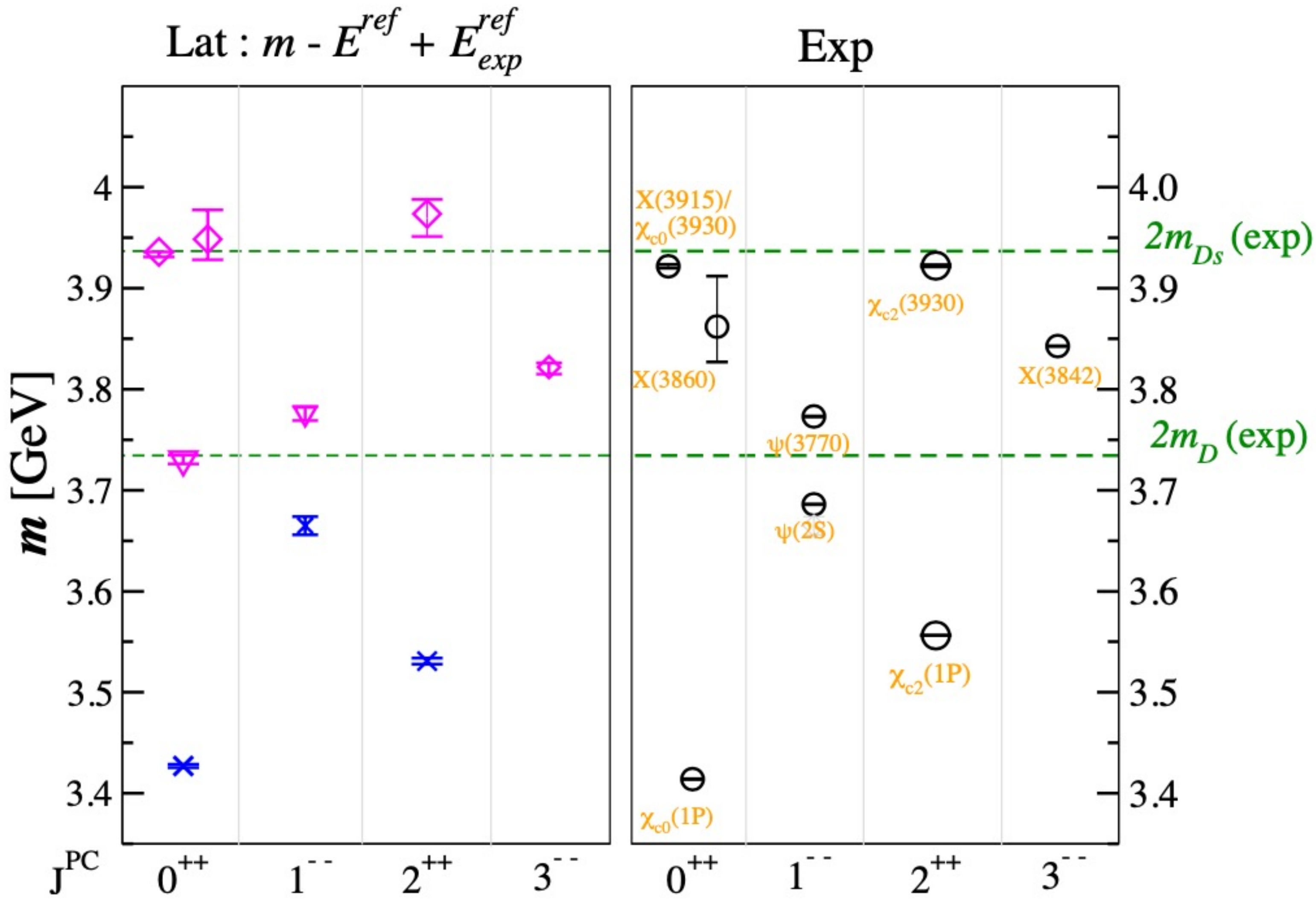
$$\frac{\tilde{K}^{-1}_{ij}(s)}{\sqrt{s}} = a_{ij} + b_{ij}s$$

$$s = E_{cm}^2$$

we verified aposteriory that both regions can be smoothly connected

t_1(E) =





(b) Left pane: The same masses m as above, but shifted to $m - E^{ref} + E^{ref}_{exp}$ in order to account for the dominant effect of unphysical quark masses in the simulation. The reference energy is $E^{ref} = 2m_D$ ($2m_{D_s}$) for the state closest to the $D\bar{D}$ ($D_s\bar{D}_s$) threshold, while $E^{ref} = M_{av} = \frac{1}{4}(3m_{J/\psi} + m_{\eta_c})$ for the remaining four states. The green lines denote experimental thresholds.

Quantum numbers relevant for Zb

h=heavy=b,$\underline{b}$ $\vec{S}_h = \vec{S}_b + \vec{S}_{\bar{b}}$

l=light=u,d,gluons

exp+pheno

continuum

$J^{PC}=1^{+-}$

$Z_b(10610)$ as B$\underline{B}$* molecule

$$\bar{b}\gamma_5 q\ \bar{q}\gamma_z b + \bar{b}\gamma_z q\ \bar{q}\gamma_5 b \propto (S_h\!=\!1)(J_l\!=\!0) + (S_h\!=\!0)(J_l\!=\!1)$$

B $\underline{B}$* B* $\underline{B}$

Bondar, Garmash, Milstein, Mizuk, Voloshin PRD84 054010, Voloshin PRD84 (2011) 031502

Wang, Baru, Filin, Hanhart, Nefediev, Wynen, 1805.07453, PRD 2018

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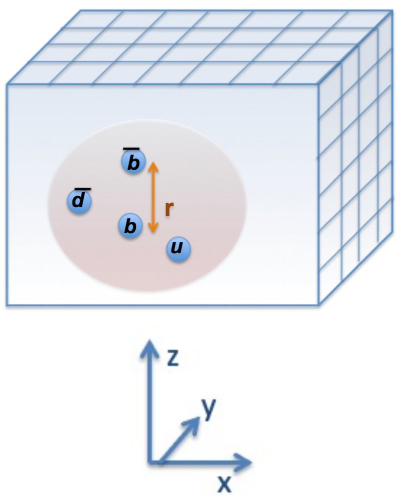
lattice with static b

$m_b = \infty$



static b → b quark can not flip spin via gluon exchange

S_h is conserved

quantum numbers of light degrees of freedom in static limit

$$S_h = 1\ \&\ J_l = 0\ (J_l^z = 0, CP = -1, \epsilon = -1\ \ : \Sigma_u^-)$$

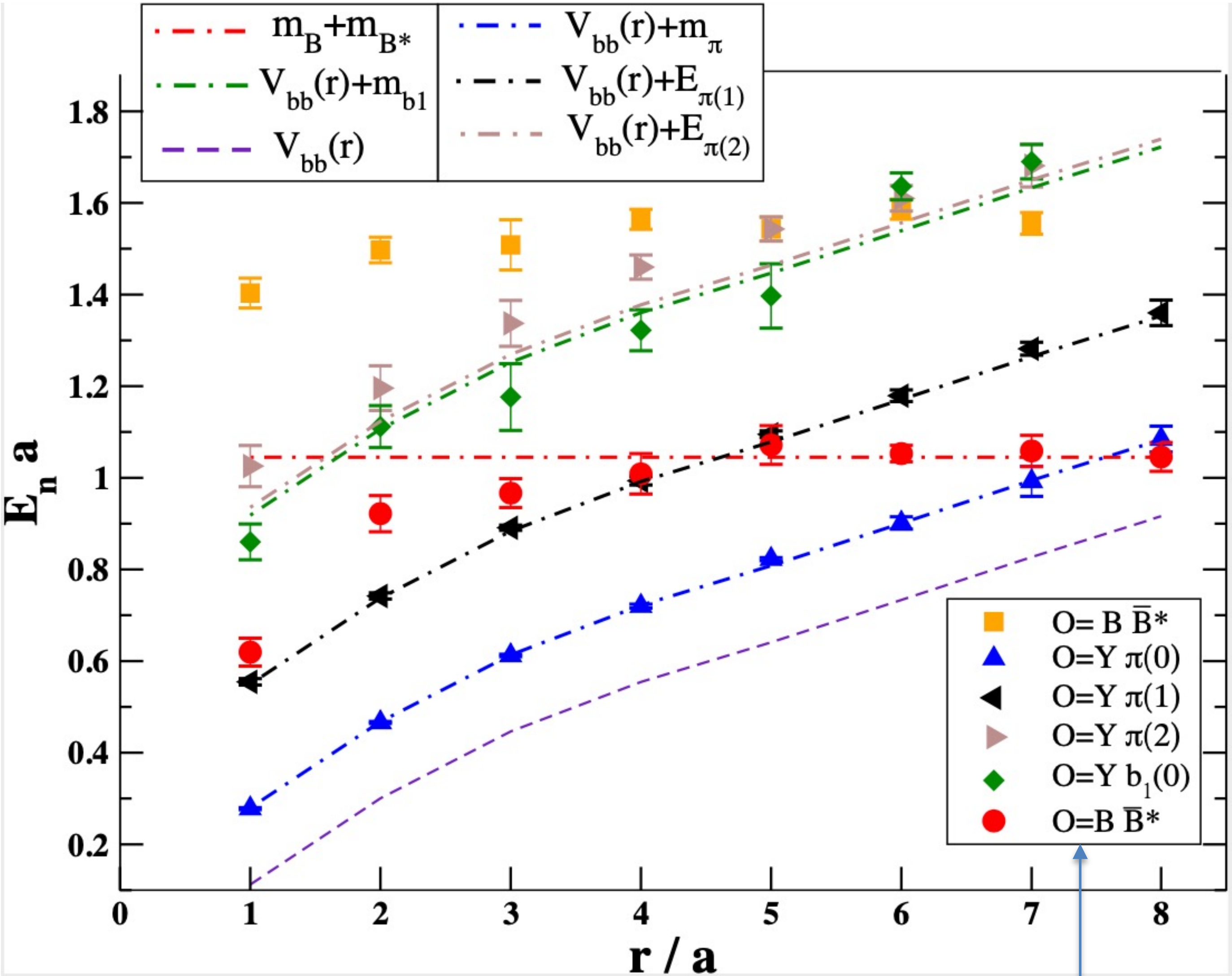
$$S_h = 0\ \&\ J_l = 1\ (J_l^z = 0, CP = +1, \epsilon = +1\ \ : \Sigma_g^+)$$

seperate channels, considered seperately

J^x, J^y not goot q.n.

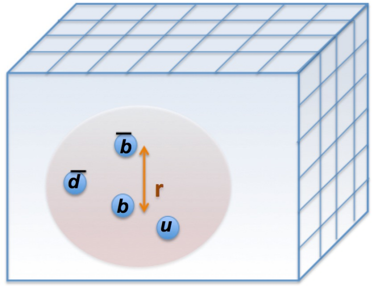
reflection over yz plane

Eigen-energies $E_n(r)$: channel S_h=1, J_l=0 (CP=-1, ε=-1)



m_B+m_{B*}

dominant operator
in each |n>
according to $\langle O_i | n \rangle$



dot-dashed-lines:
$E_n^{non\text{-}int}$