Nucleon axial form factors from lattice QCD

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The 23rd International Workshop on High Energy Physics
Hard problems of hadron physics: non-perturbative QCD and related quests
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Motivation

- Proton Electromagnetic form factors are well-measured
  ➜ can be used as bench mark quantities for lattice QCD

- Axial form factors are not well measured
  ➜ important for weak interactions, neutrino scattering, and parity violation experiments.
  ➜ check of PCAC and Goldberger-Treiman relations, and enables extraction axial and pseudo-scalar couplings

C. A. *et al.* (ETMC) Phys.Rev.D 103 (2021) 3, 034509, arXiv: 2011.13342
C. A., S. Bacchio, M. Constantinou, K. Hadjiyiannakou, K. Jansen G. Koutsou, *Phys.Rev.D* 104 (2021) 074503, arXiv: 2106.13468

Introduction

❋ Gauge ensembles generated by ETMC

These are now generated with 2+1+1 flavours
at physical values of the light, strange and charm quark masses

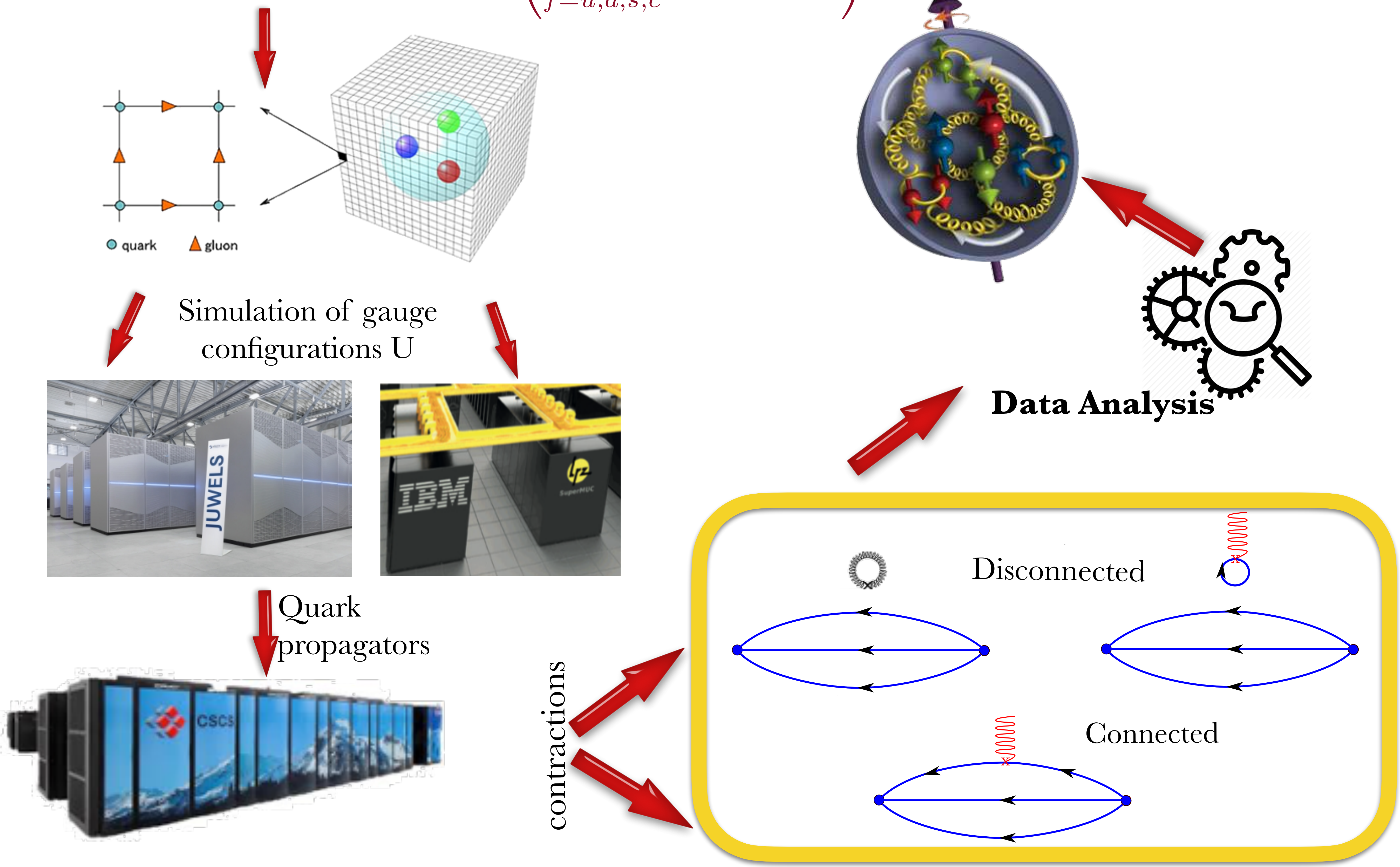
❋ Analysis of these ensembles for various observables

❋ Main collaborators for nucleon form factors

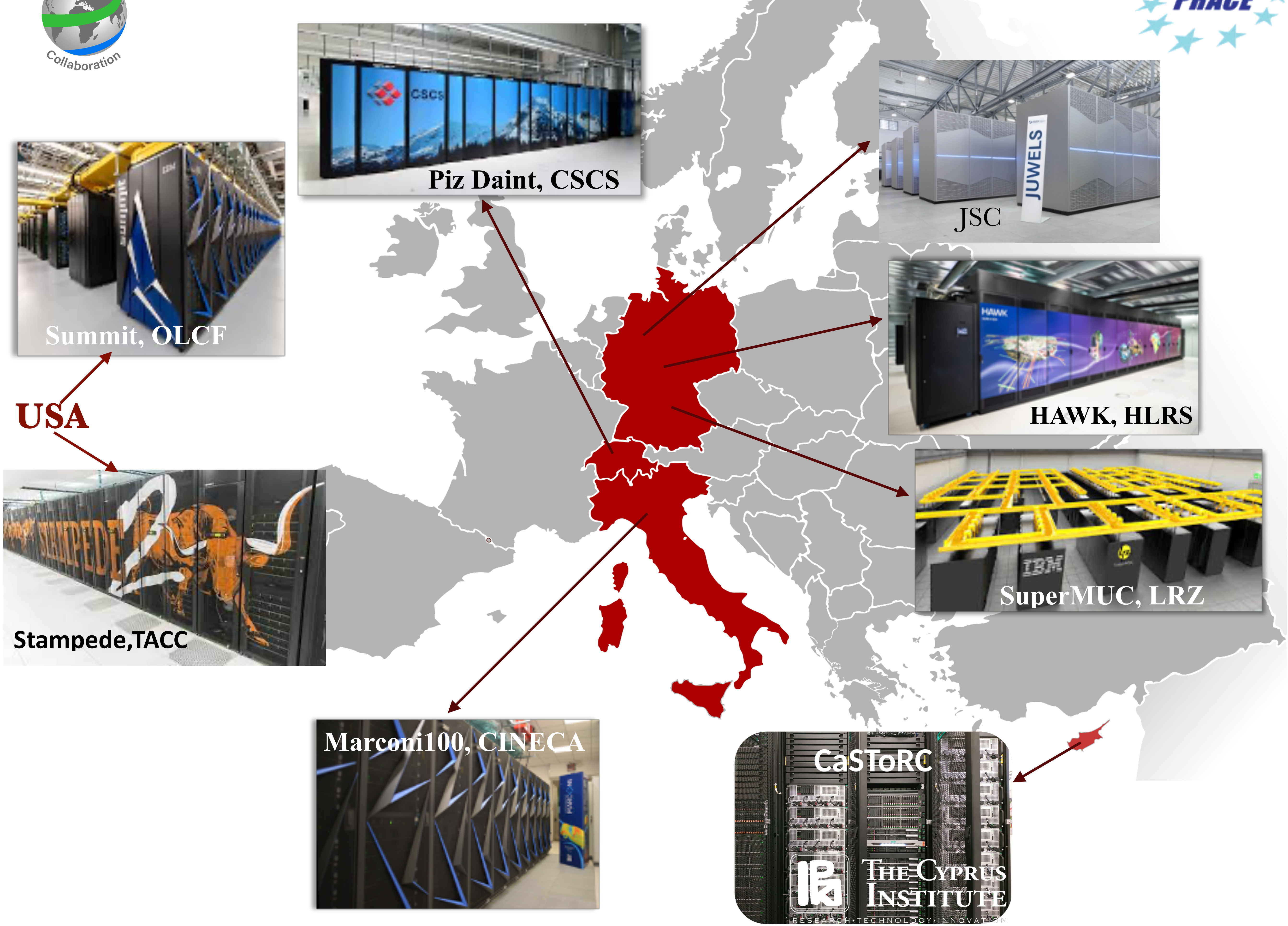
- *S. Bacchio, The Cyprus Institute*
- *M. Constantinou, Temple University*
- *J. Finkenrath, The Cyprus Institute*
- *K. Hadjiyiannakou, University of Cyprus & The Cyprus Institute*
- *K. Jansen, DESY-Zeuthen*
- *G. Koutsou, The Cyprus Institute*
- *T. Leontiou, Frederick University*

Extraction of matrix elements in lattice QCD

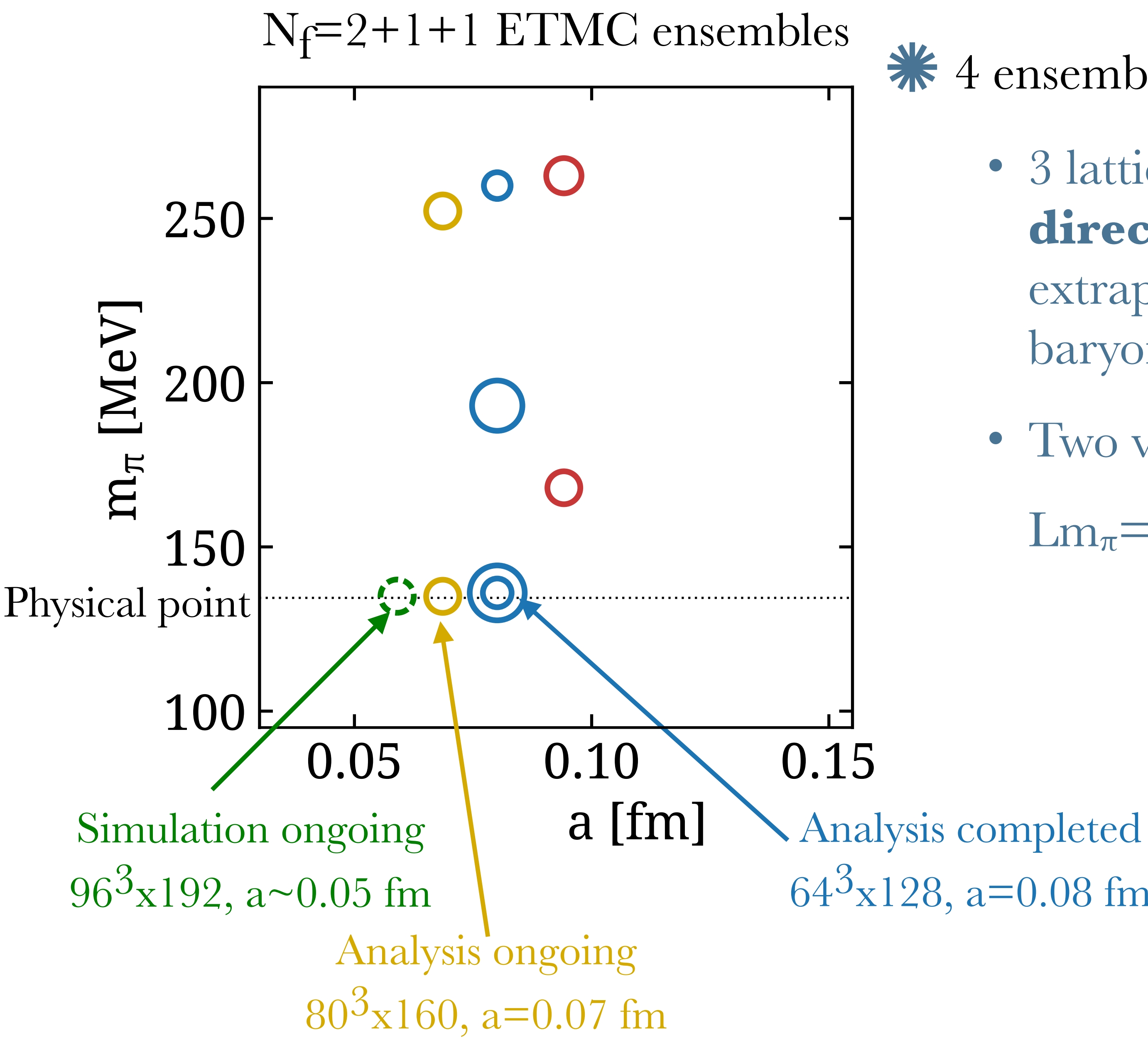
$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] \mathcal{O}(D^{-1}[U], U) \left(\prod_{f=u,d,s,c} \mathrm{Det}(D_f[U]) \right) e^{-S_{\mathrm{QCD}}[U]}$$



Computational resources



Gauge ensembles generated by ETMC

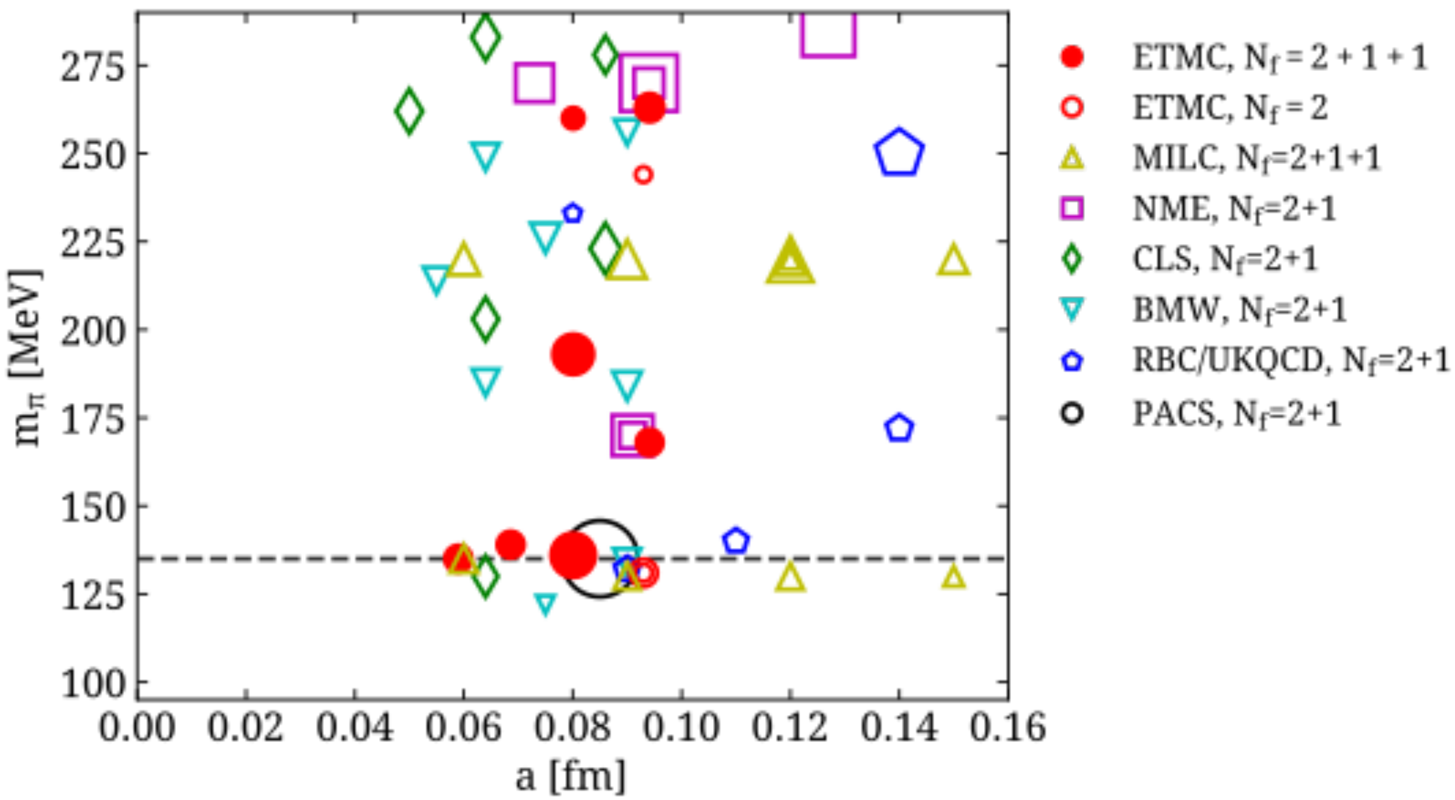


✳ 4 ensembles at physical pion mass

- 3 lattice spacings 0.05<a<0.1 fm —> take continuum limit **directly at the physical point** avoiding chiral extrapolation removing a major systematic error in the baryon sector
- Two volumes at a=0.08 fm of Lm_π=3.6 (5.1 fm) and Lm_π=5.4 (7.7 fm)

C. A. *et al.* (ETMC), "Simulating twisted mass fermions at physical light, strange and charm quark masses" Phys. Rev.,D98(5):054518, 2018

Status of current simulations



- Algorithmic improvements needed to go to a<0.05 fm due to critical slow down in HMC (long autocorrelations)

  —> new approaches e.g. Machine learning approaches using equivariant flows

  G. Kanwar, et al., Phys. Rev. Lett. 125 (2020) no.12, 121601, 2003.06413; D. Boyda, et al., 2008.05456

- A number of collaborations has physical point ensembles:

- Wilson-type: ETMC, BMW, CLS, PACS
  - Most have 1-2 lattice spacings 0.05<a<0.1 fm
  - PACS has a large volume ensemble
- Staggered at physical point: MILC with 3 lattice spacings 0.06<a<0.15 fm
- Domain wall at physical point RBC/UKQCD with 2 lattice spacings

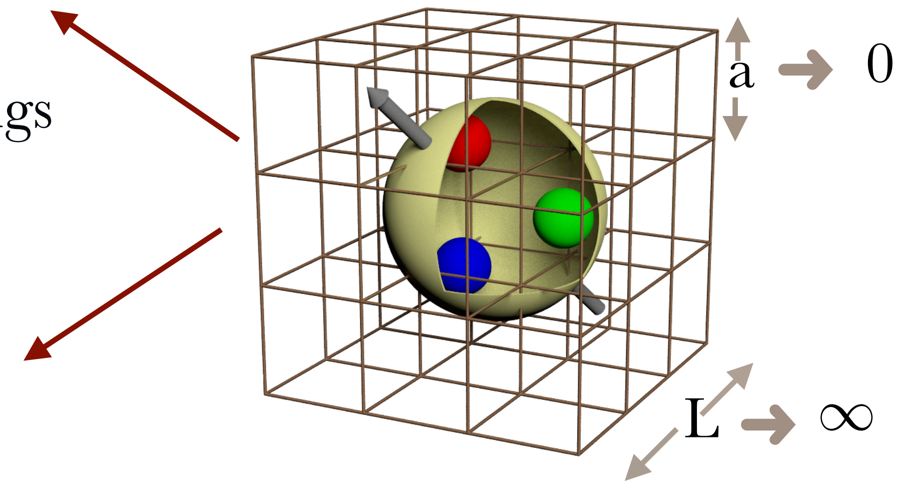
Systematics & Challenges

- **Discretisation effect:** Continuum limit

  —> need simulations for at least 3 lattice spacings

- **Finite volume effects:** Infinite volume limit

  —> need simulations for at least 3 volumes



Typically done using simulations for heavier than physical values of the pion mass

- **Simulations directly at the physical point** ✓

  Systematic effects from chiral extrapolation are eliminated

- **Ground-state identification**

  Cross-check (one-, two- and three-state fits, summation)

  Two-particle state contribution complicate the identification of the ground state

- **Renormalisation**

  Non-perturbatively with improvements e.g using perturbative subtraction of lattice artefacts

- In what follows we assume **isospin symmetry** i.e. up and down quarks have equal mass, and **neglect EM effects**

Axial and pseudo scalar form factors (isovector)

Extract from lattice QCD

$$\langle N(p',s')|A_\mu|N(p,s)\rangle = \bar{u}_N(p',s')\left[\gamma_\mu G_A(Q^2) - \frac{Q_\mu}{2m_N}G_P(Q^2)\right]\gamma_5 u_N(p,s)$$

$$\langle N(p',s')|P_5|N(p,s)\rangle = G_5(Q^2)\bar{u}_N(p',s')\gamma_5 u_N(p,s)$$

$q^2=-Q^2$

- Check the PCAC : $\partial^\mu A_\mu = 2m_q P, \;\; m_q = m_u = m_d$

$$G_A(Q^2) - \frac{Q^2}{4m_N^2}G_P(Q^2) = \frac{m_q}{m_N}G_5(Q^2)$$

Goldberger-Treiman relation

- Relate to pion field: $G_5(Q^2) = \frac{F_\pi m_\pi^2}{m_q}\frac{G_{\pi NN}(Q^2)}{m_\pi^2 + Q^2}$

$$\longrightarrow \quad G_P(Q^2) = \frac{4m_N^2}{Q^2+m_\pi^2}G_A(Q^2) \qquad m_N G_A(Q^2) = F_\pi G_{\pi NN}(Q^2)$$

- At the pion pole we get the pion nucleon coupling: $g_{\pi NN} \equiv G_{\pi NN}(Q^2 = -m_\pi^2)$

$$\lim_{Q^2\to -m_\pi^2}(Q^2+m_\pi^2)G_P(Q^2) = 4m_N F_\pi g_{\pi NN}$$

and $$g_{\pi NN} = m_N G_A(-m_\pi^2)/F_\pi \xrightarrow{m_\pi\to 0} \frac{m_N}{F_\pi}g_A$$

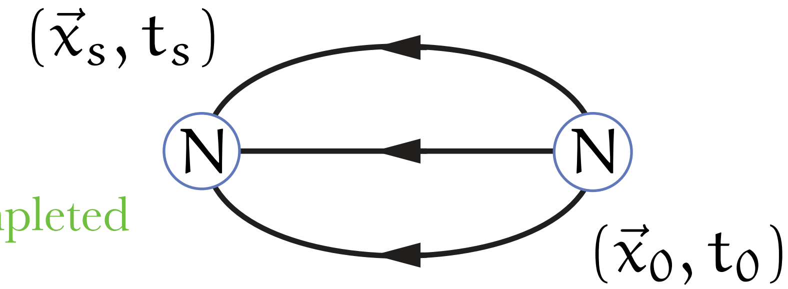
Extracting Nucleon Axial Form Factors

N_f=2=1+1 twisted mass fermion ensembles

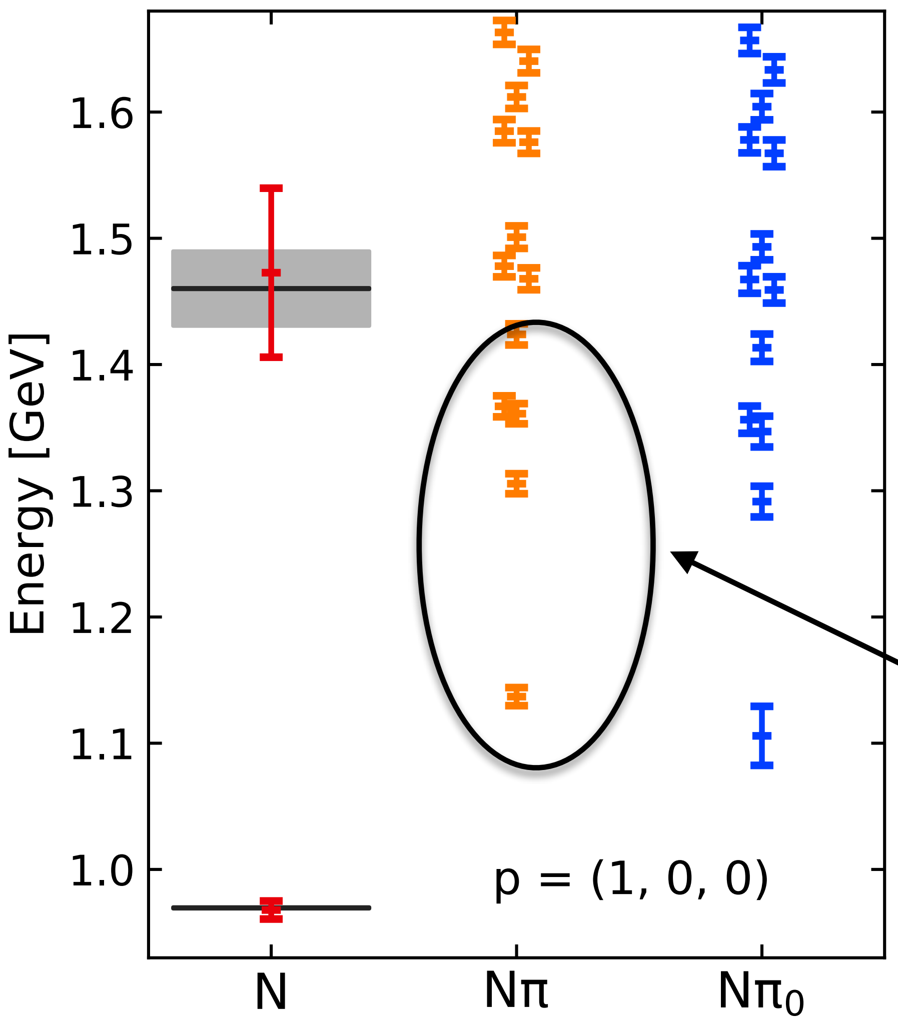
N_f	Ens. ID	Vol.	a [fm]
2+1+1	cB211.072.64	64×128	0.081
2+1+1	cB211.072.96	96×192	0.081
2+1+1	cC211.060.80	80×160	0.070

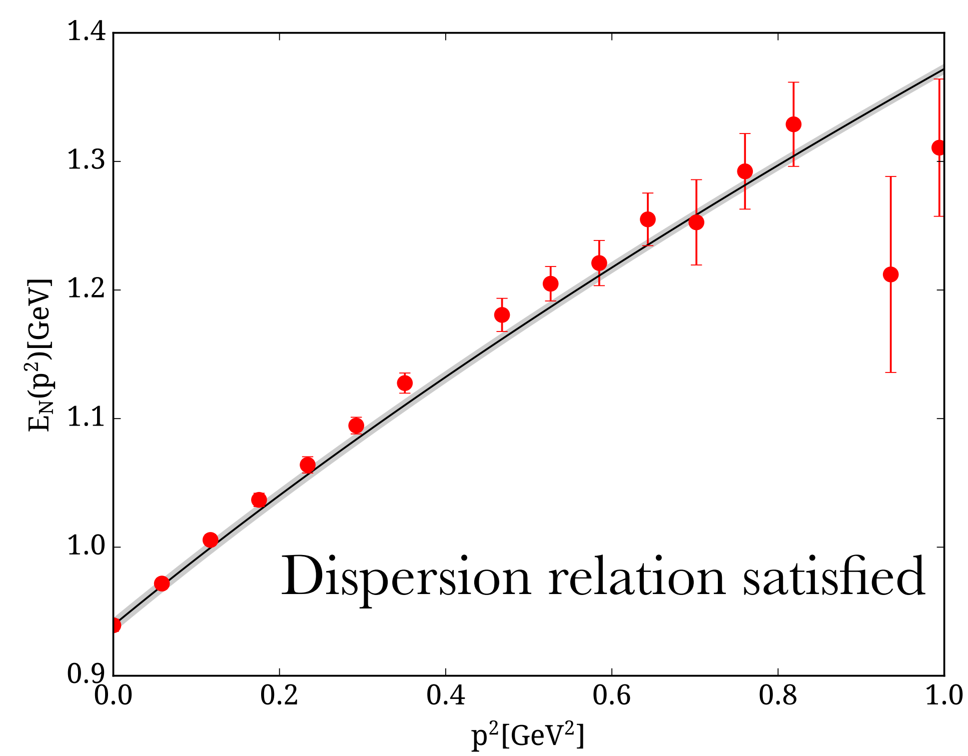
← Analysis completed

← ← Analysis ongoing



$$C_{2pt}(\Gamma_0;\vec{p}=\vec{0},t_s)=\sum_{\vec{x}_s}\mathrm{Tr}\left[\langle\Gamma_0 J_N(t_s,\vec{x}_s)\bar{J}_N(t_0,\vec{x}_0)\rangle\right]\xrightarrow{t_s\to\infty} A_0e^{-m_N t_s}+A_1e^{-E_1 t_s}+\cdots$$

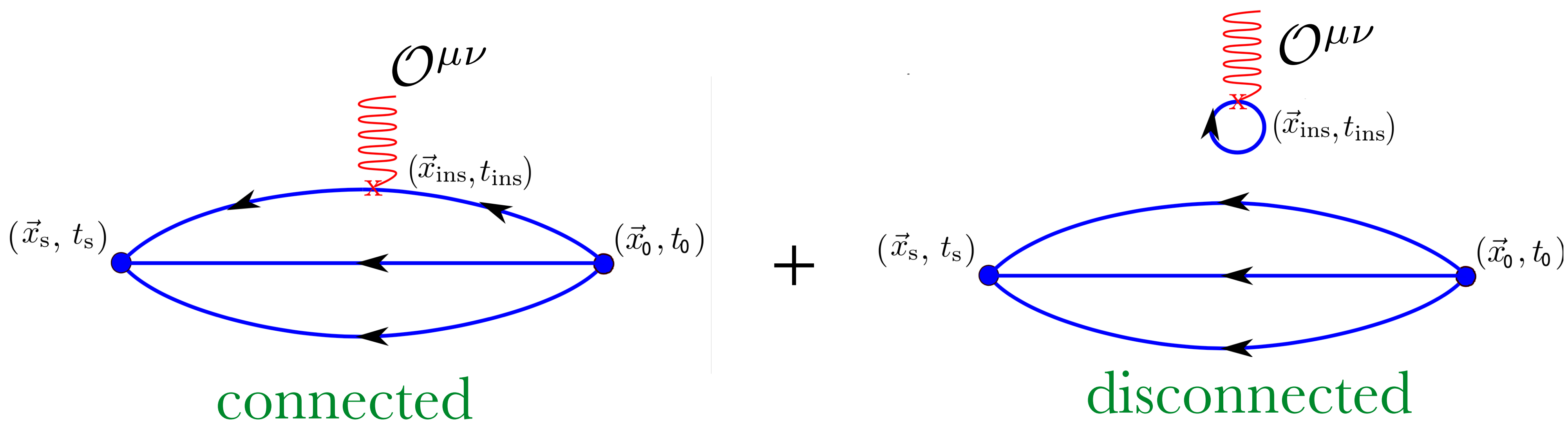




Two-particle states not seen in 2-point function

Nucleon matrix elements

$$C^{\mu\nu}_{3\text{pt}}(\Gamma;\vec{q}=0,t_s,t_{\text{ins}})=\sum_{\vec{x}_{\text{ins}},\vec{x}_s}\text{Tr}\left[\langle\Gamma J_N(t_s,\vec{x}_s)\mathcal{O}^{\mu\nu}(t_{\text{ins}},\vec{x}_{\text{ins}})\bar{J}_N(t_0,\vec{x}_0)\rangle\right]$$



❋ Identification of nucleon matrix element *M* $(t_0=0)$

Plateau and two-state fit:

$$R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\text{ins}})=\frac{C^{\mu\nu}_{3\text{pt}}(t_s,t_{\text{ins}})}{C_{2\text{pt}}(\Gamma_0,t_s)}\longrightarrow\boxed{\mathcal{M}}+\mathcal{O}(e^{-\Delta E(t_s-t_{\text{ins}})})+\mathcal{O}(e^{-\Delta E t_{\text{ins}}})$$

Included in the two-state fit

Summation:

$$\sum_{t_{\text{ins}}=a}^{t_s-a}R^{\mu\nu}(\Gamma;\vec{q}=\vec{0},t_s,t_{\text{ins}})\longrightarrow c+\boxed{\mathcal{M}}t_s+\mathcal{O}(e^{-\Delta E t_s})$$

L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys., B293: 420, 1987

Isovector matrix elements

N_f	Ens. ID	Vol.	a [fm]
2+1+1	cB211.072.64	64×128	0.081
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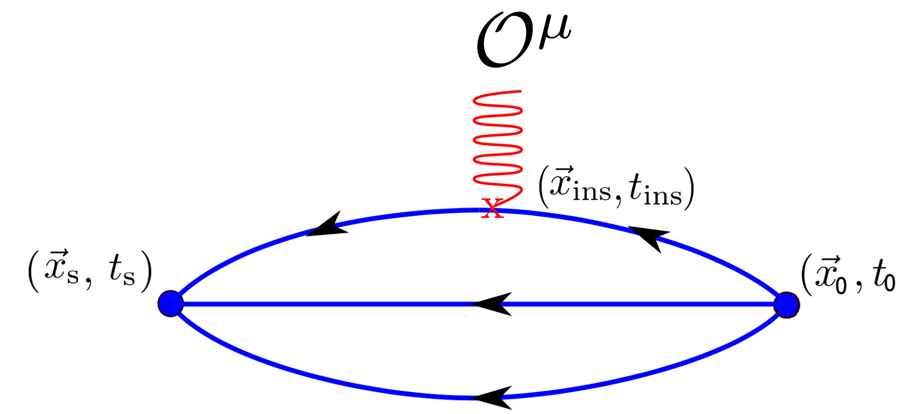


for isovector only connected

Statistics for connected contribution

t_s/a	N_{cnfs}	N_{srcs}	N_{meas}
8	750	1	750
10	750	2	1500
12	750	4	3000
14	750	6	4500
16	750	16	12000
18	750	48	36000
20	750	64	48000
2-pt	750	264	198000

0.64 fm

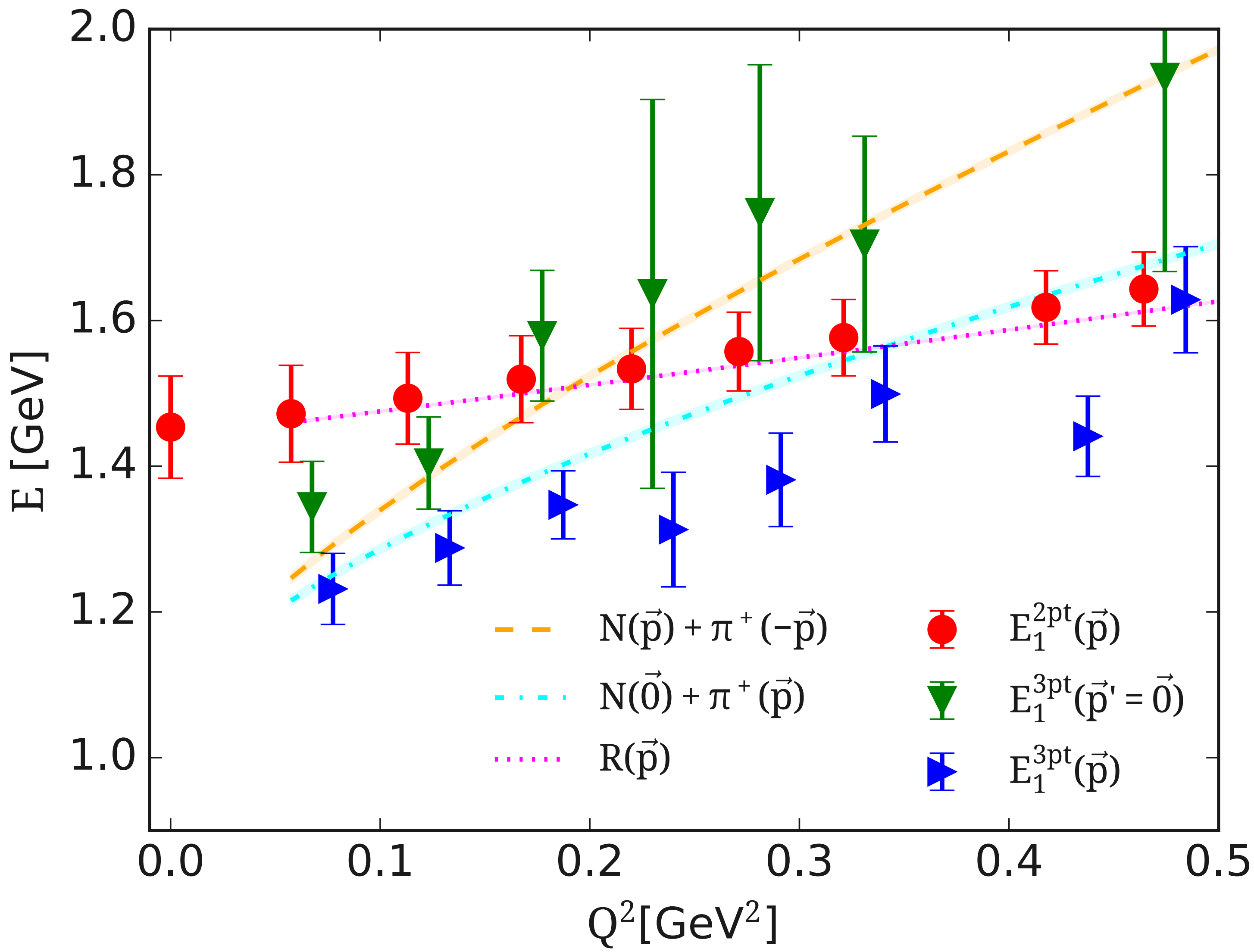
Needed for studying excited states

1.6 fm

Increase statistics to keep approx. constant error

Analysis of 3-pt functions

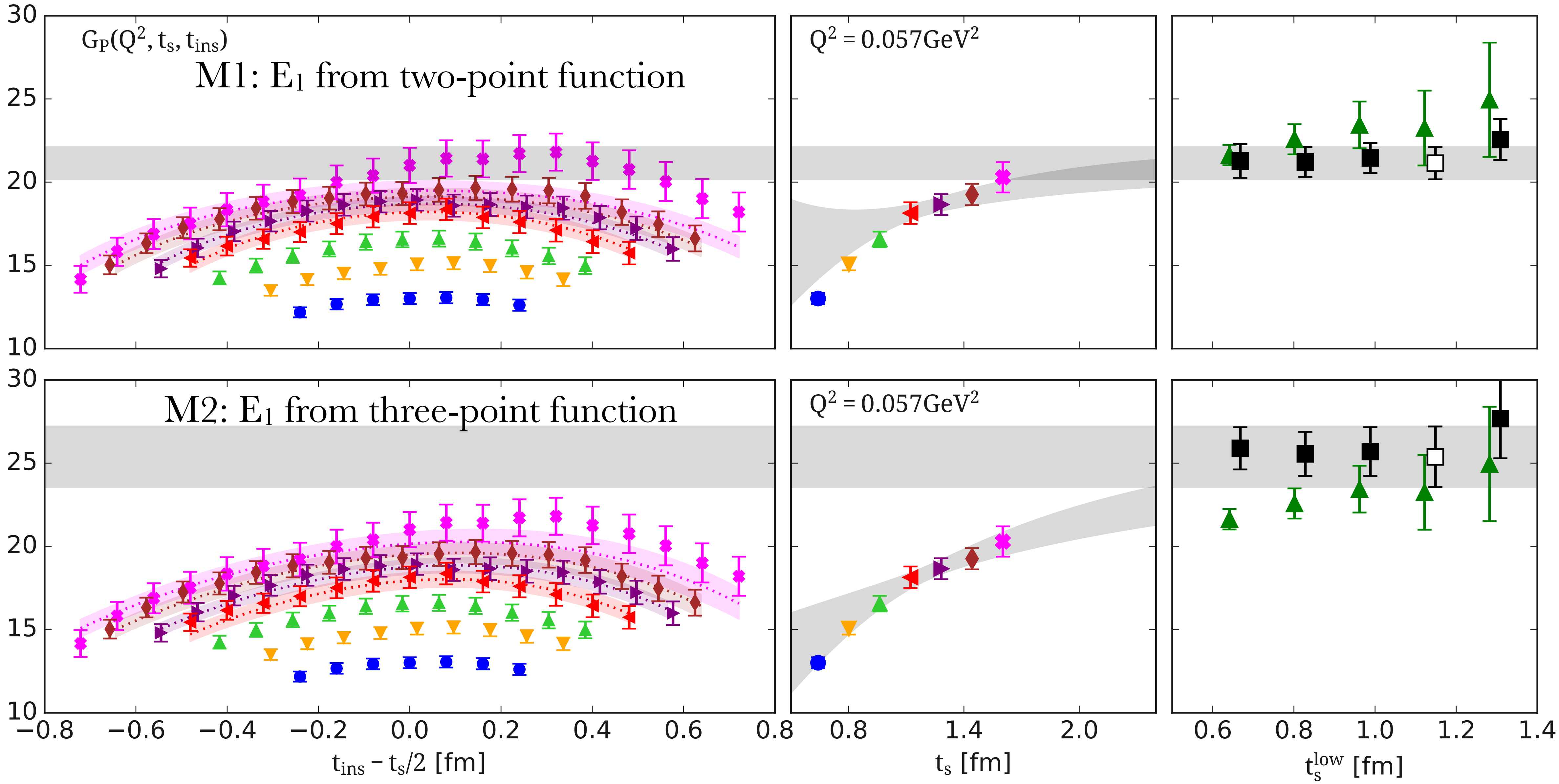
- We allow the first excited state to be different in the two- and three-point functions, motivated by chiral perturbation theory, O. Baer, Phys. Rev. D 99, 054506 (2019).



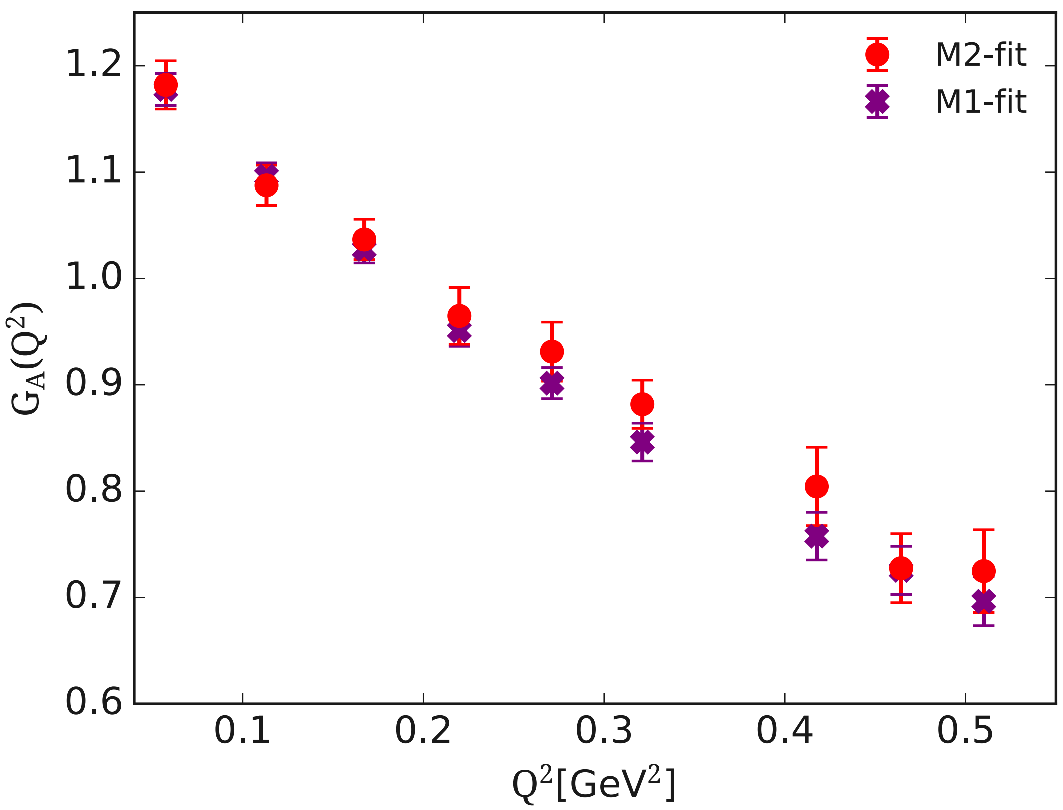
Analysis of 3-pt functions

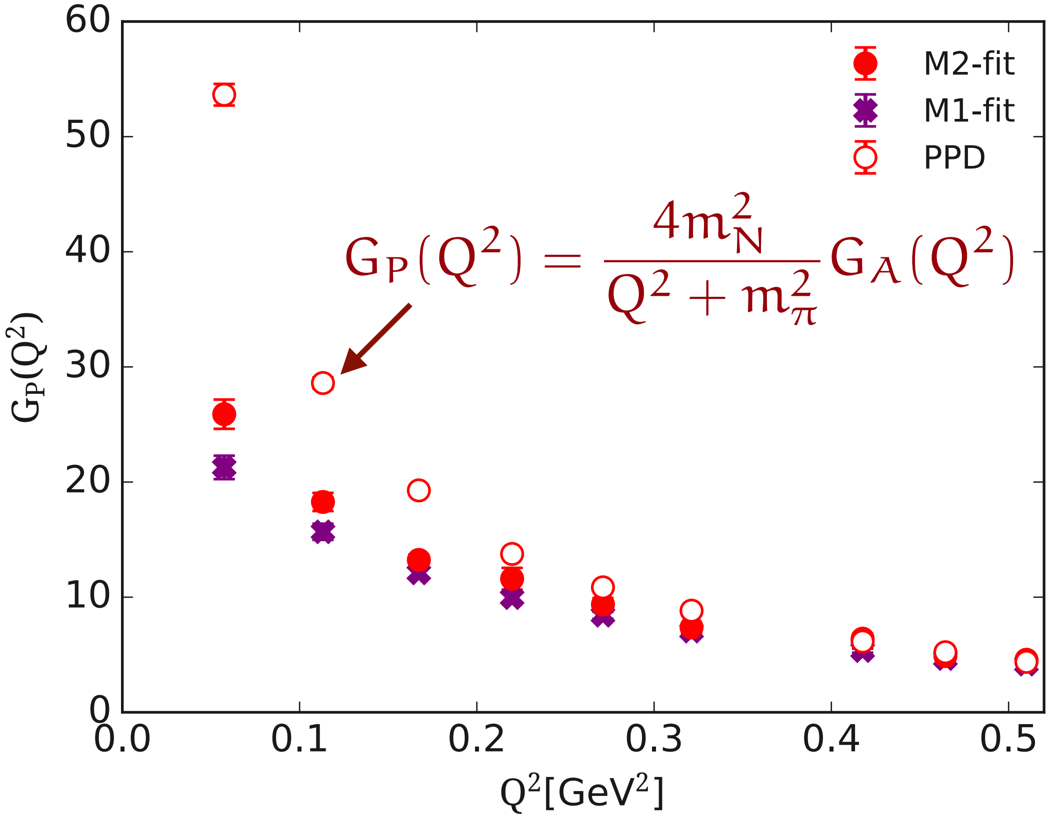
- We allow the first excited state to be different in the two- and three-point functions, motivated by chiral perturbation theory, O. Baer, Phys. Rev. D 99, 054506 (2019).

—>The induced pesudoscalar increases



Axial and induced pseudoscalar form factors

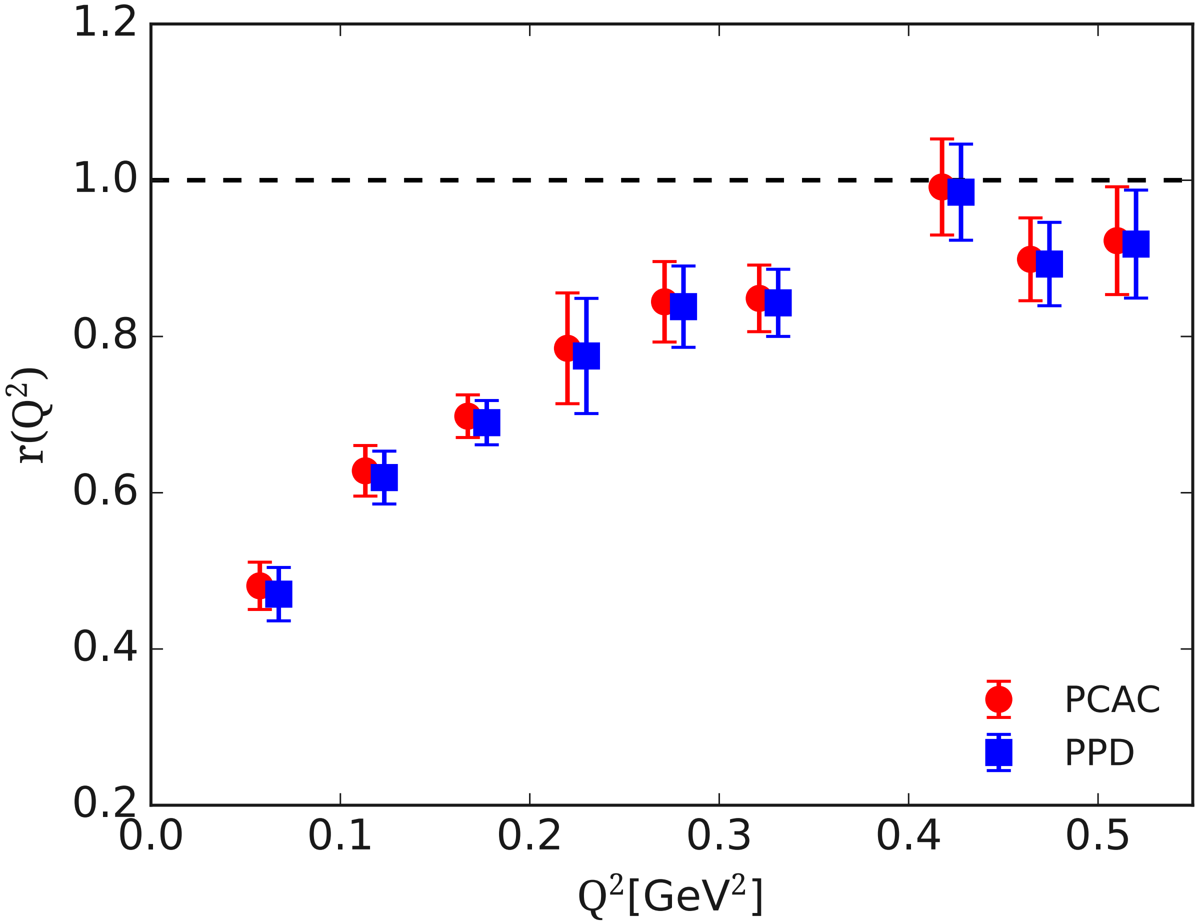




- Negligible effect on G_A; larger effect on G_P but not large enough to fulfil pion pole dominance
- For G_5 we find a similar similar behaviour to G_P
- Other lattice QCD collaborations find bigger effects when not constraining the first edited state in the 3-point function

Y.-C. Jang, R. Gupta, B. Yoon, and T. Bhattacharya, Phys. Rev. Lett. 124, 072002 (2020)
G. S. Bali, et al. (RQCD Collaboration), J. High Energy Phys. 05 (2020) 126

PCAC and pion pole dominance

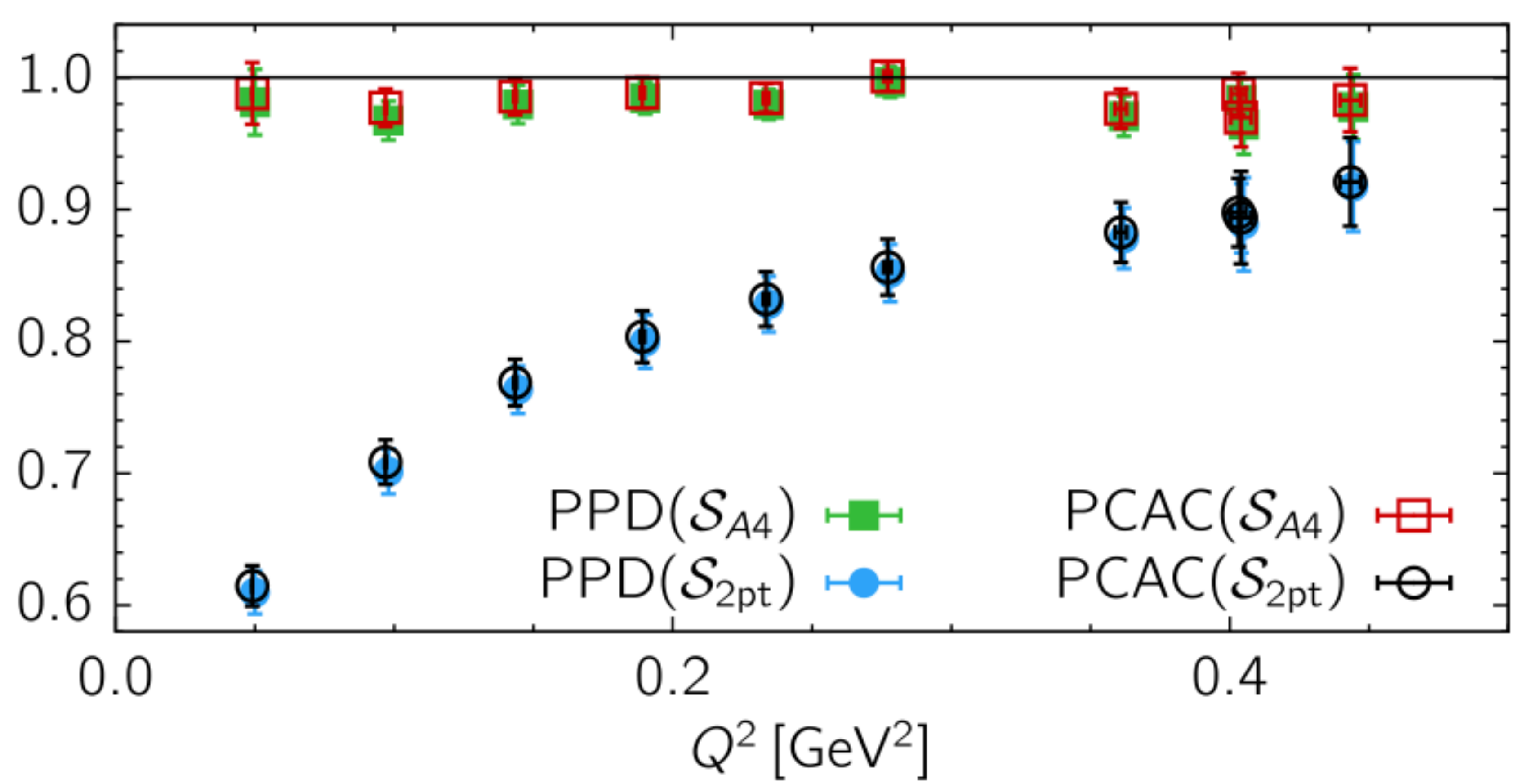


$$r_{\mathrm{PCAC}} = \frac{\frac{m_q}{m_N} G_5(Q^2) + \frac{Q^2}{4m_N^2} G_P(Q^2)}{G_A(Q^2)}$$

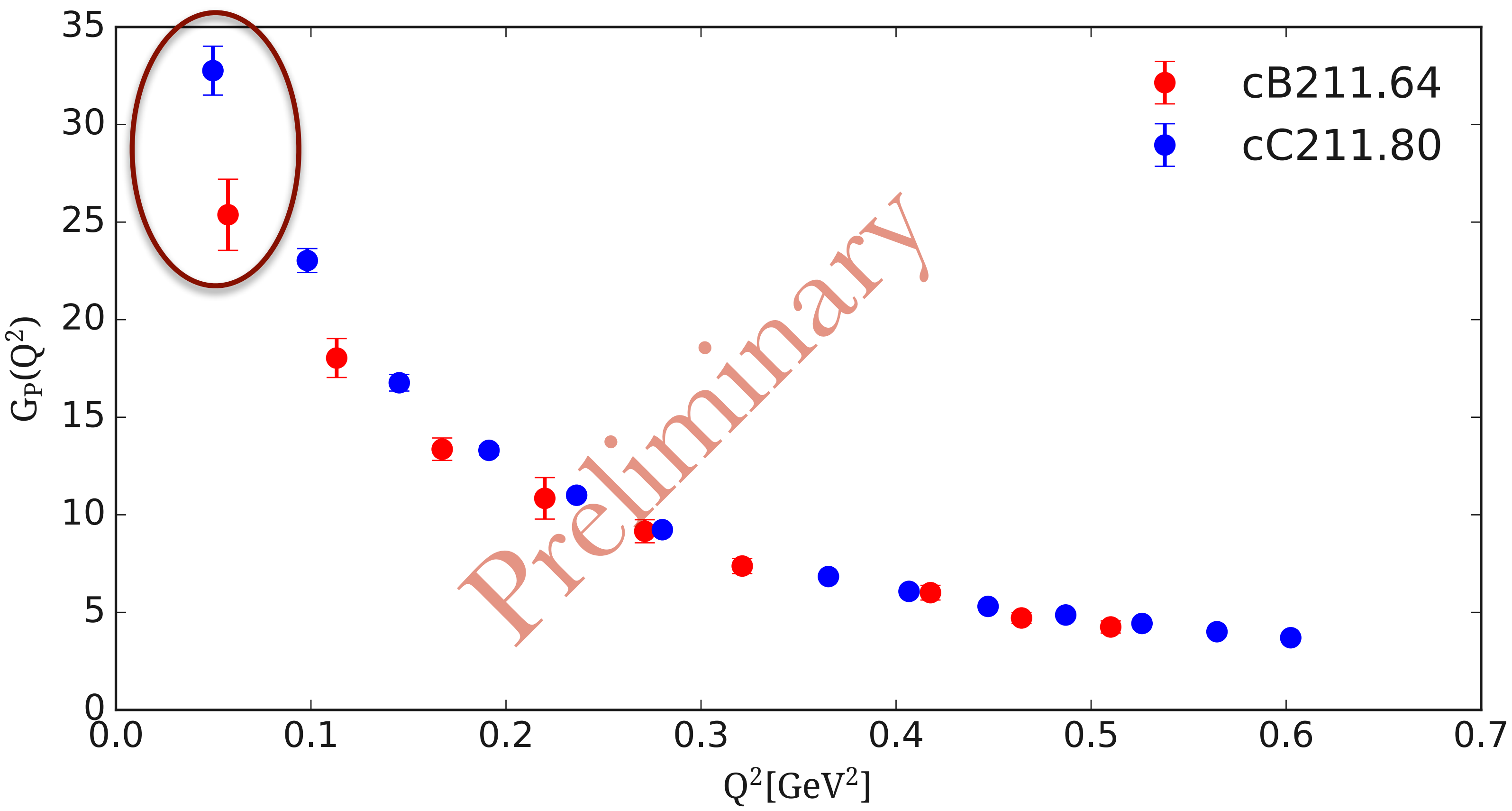
$$r_{\mathrm{PPD}} = \frac{G_P(Q^2)}{\frac{4m_N^2}{m_\pi^2 + Q^2} G_A(Q^2)}$$

✵ Both PCAC and PPD not satisfied for small Q^2

✵ Unlike in Y.-C. Jang, R. Gupta, B. Yoon, and T. Bhattacharya, Phys. Rev. Lett. 124, 072002 (2020)

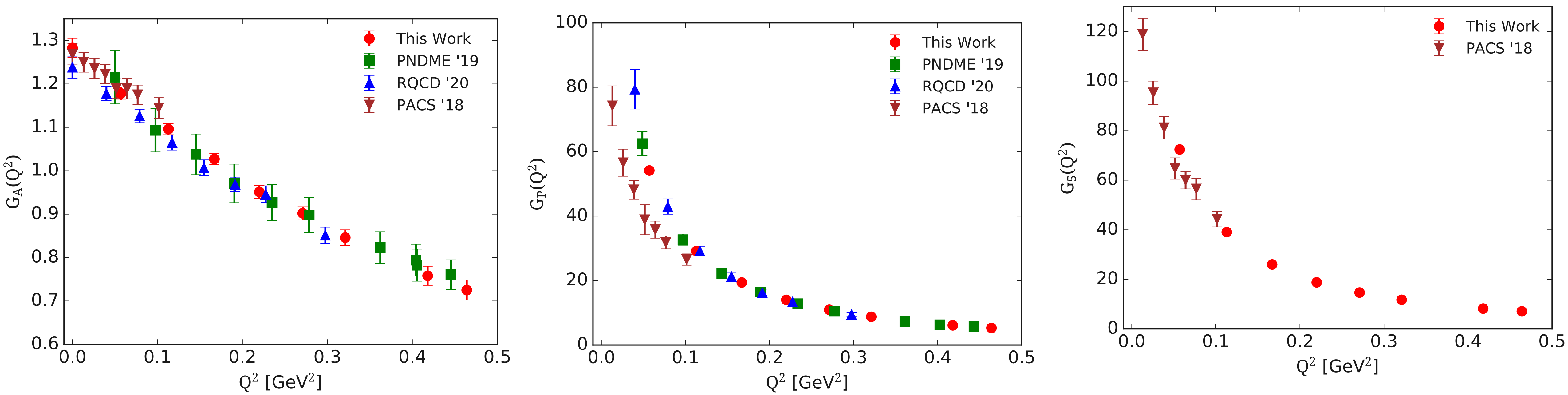


Lattice cut-off effects

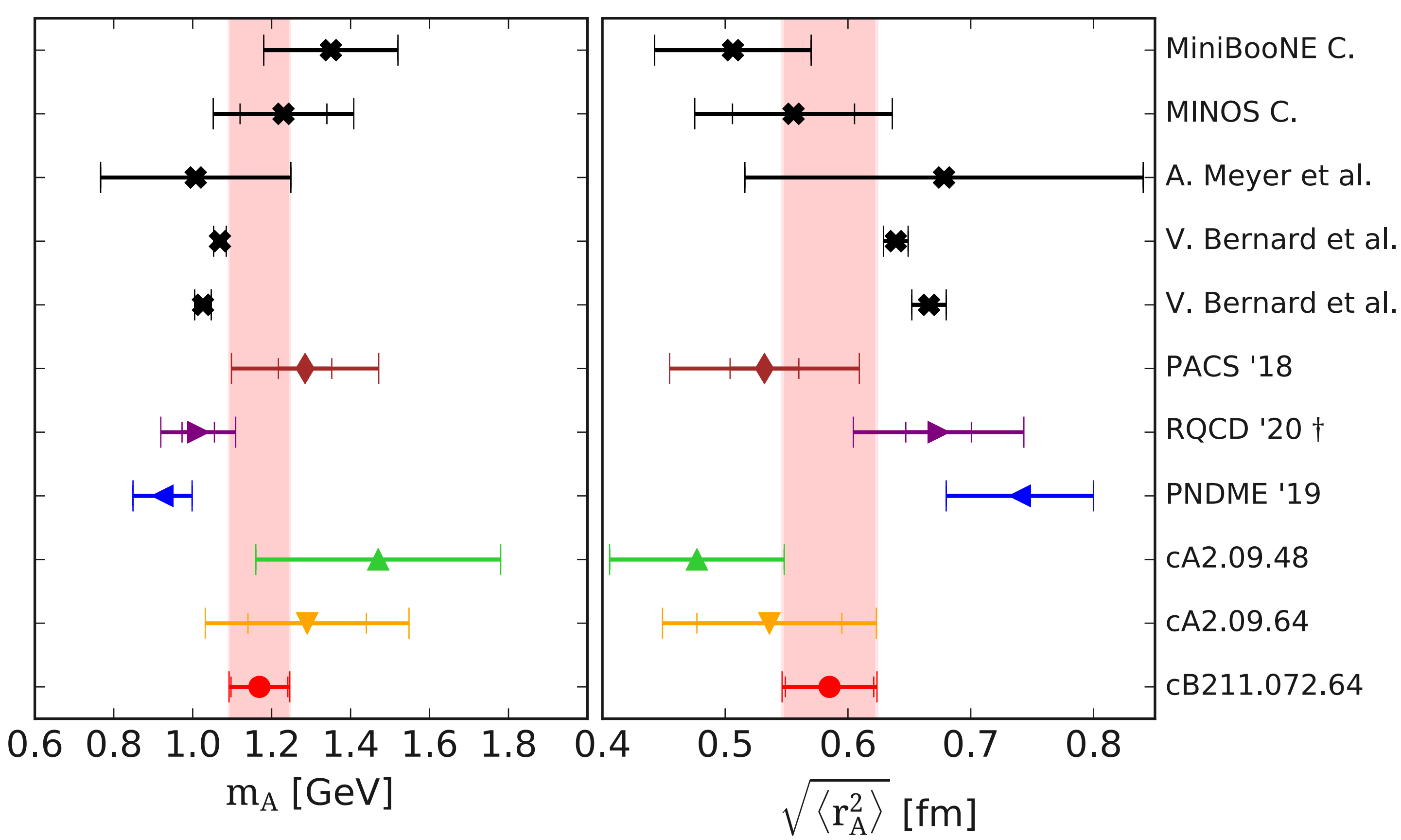


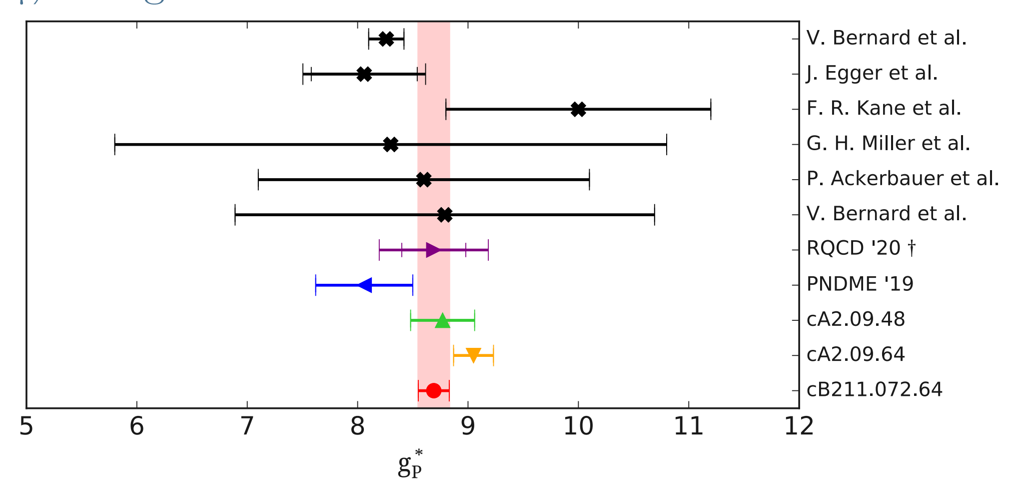
- Large effects on $G_P(Q^2)$ at low Q^2 —> may (partly) explain the discrepancy from pion dominance
- Cut-off effects small for G_A —> for the rest of this talk we will use lattice results on G_A to extract G_P
- G_5 shows the same behaviour as observed for G_P

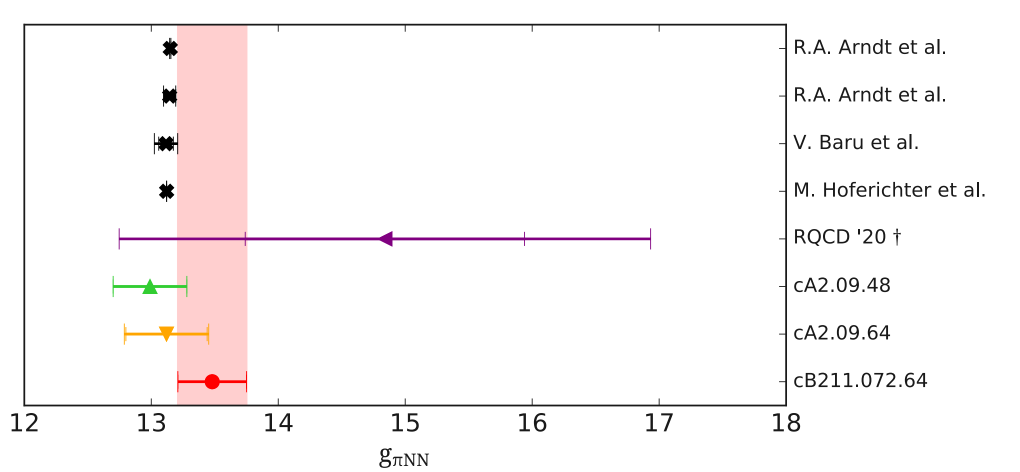
Results and comparisons



- Good agreement for G_A
- G_P and G_5 still needs investigation - our data are extracted from G_A, RQCD data from M2 fits and PACS from plateau fits
- Fitting the Q^2-dependence we extract the axial mass and radius, $g_P^*=G_P(0.88m_\mu)$ and $g_{\pi NN}$

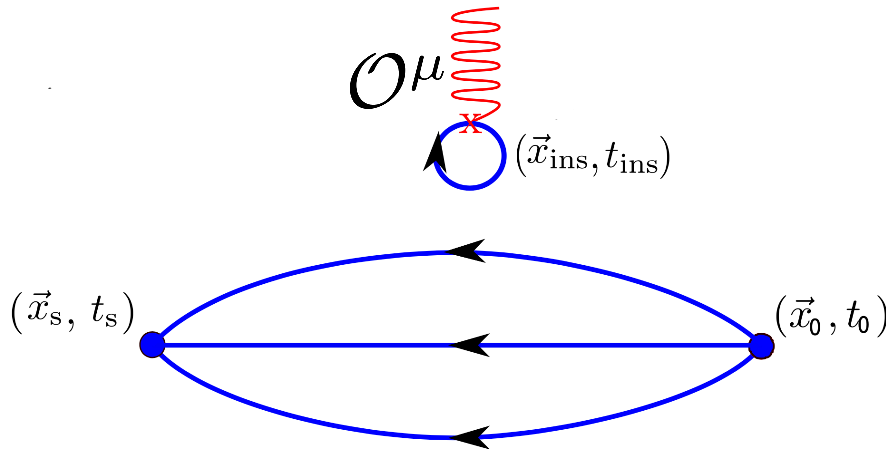






Beyond isovector - each flavour contribution

- Need the complete non-valence contributions
- An order of magnitude more computational resources needed
- Non-singlet renormalisation needed

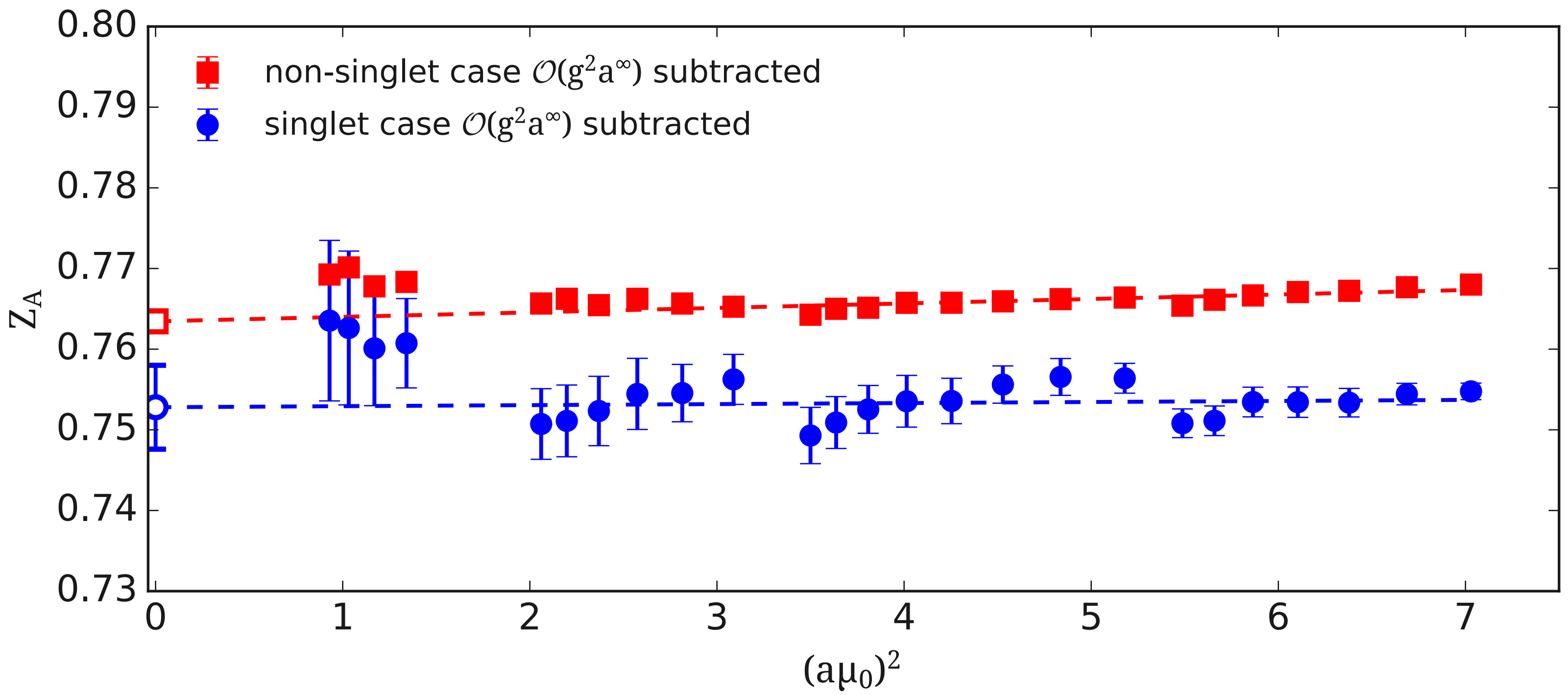


Use hierarchical probing
no. of Hadamard vectors

Statistics for disconnected contribution

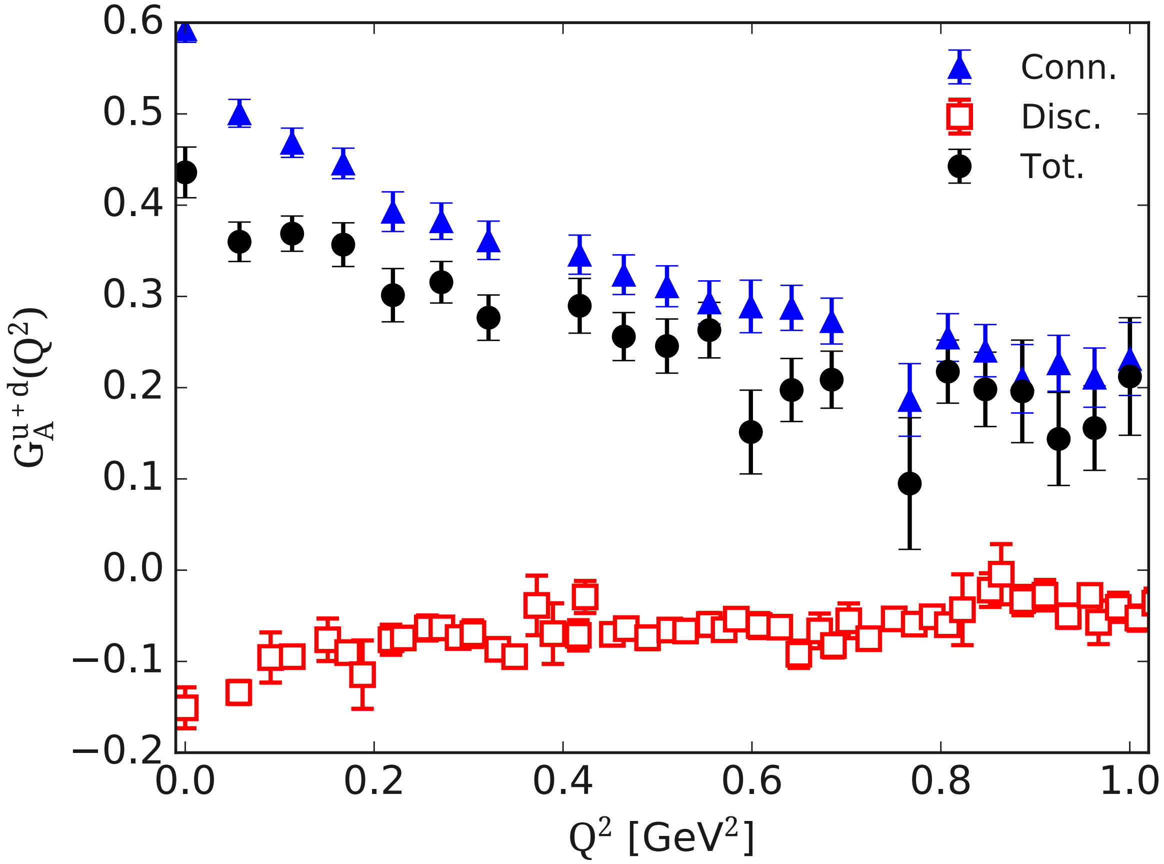
2pt	(u+d)-quark loop	s-quark loop	c-quark loop
600000	750×512 + deflation of 200 modes	750×512	9000×32

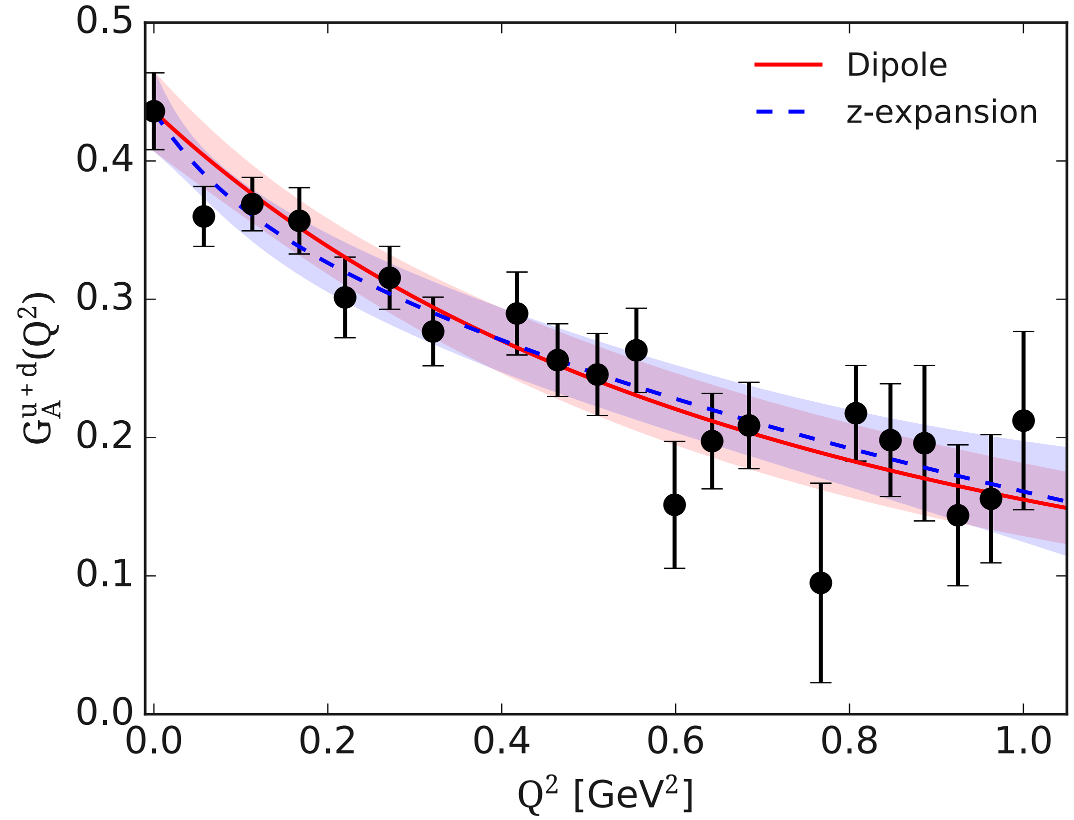
no. of stochastic vectors



Clear difference between singlet and non-singlet

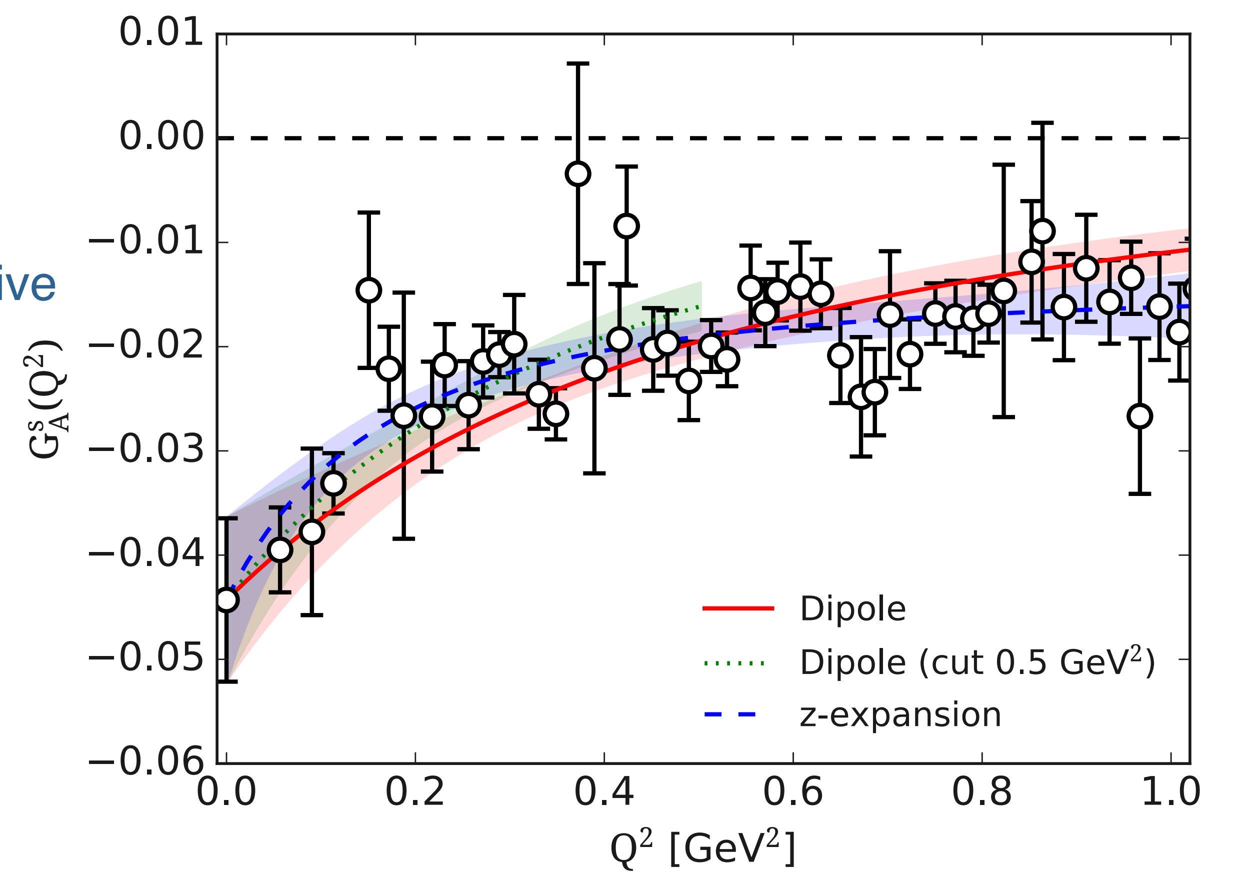
Quark flavour decomposition of G_A



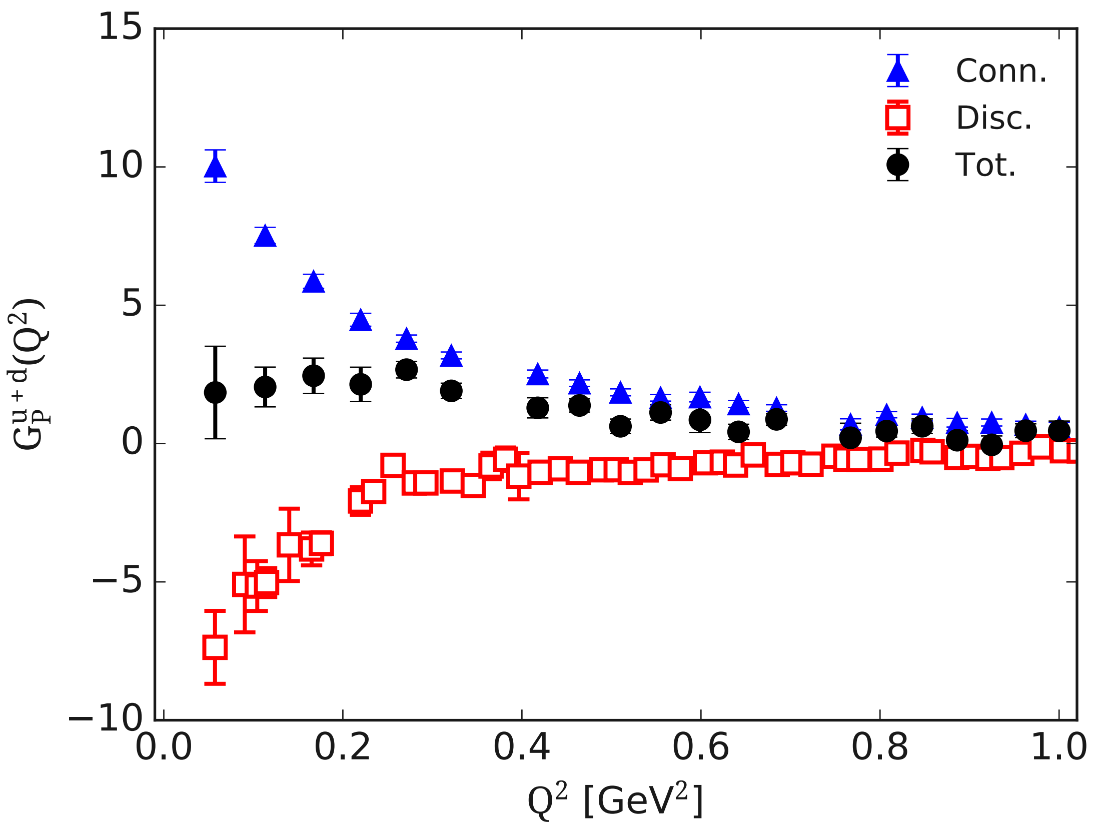


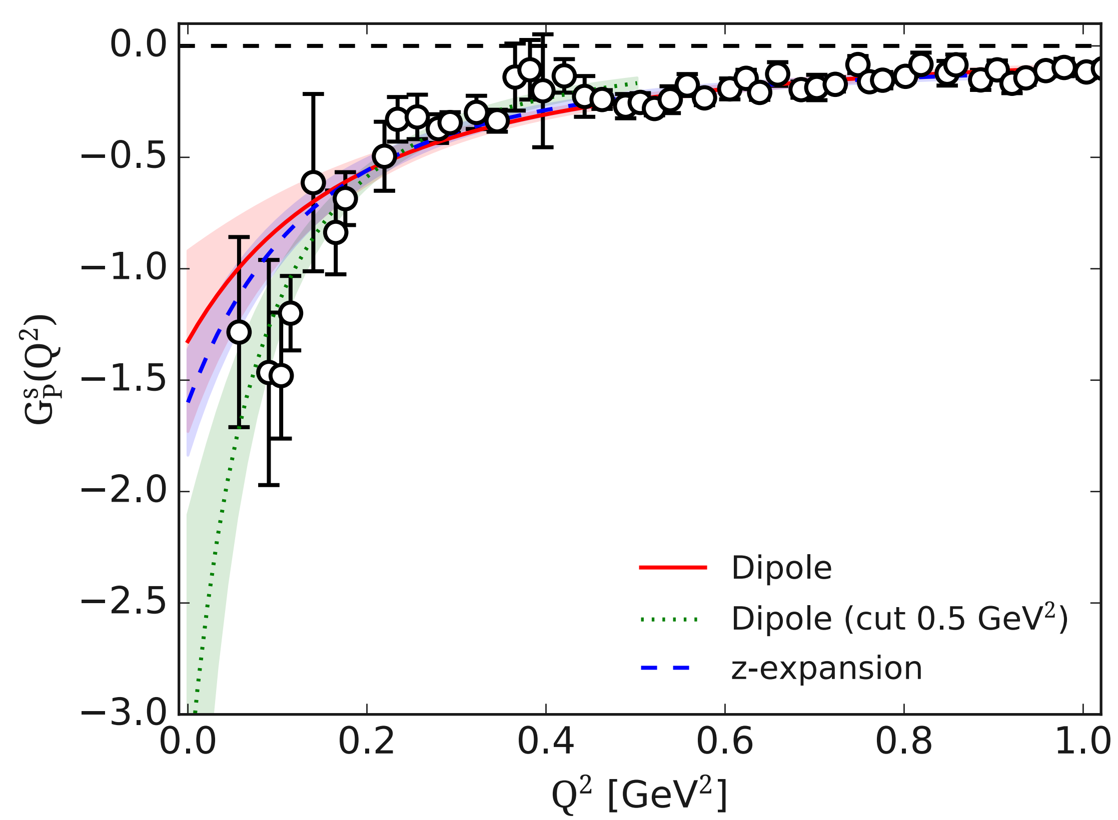
Contribution of disconnected diagrams

- Significant contribution of disconnected to u+d combination
- Negative disconnected contribution: subtracts from connected
- Good signal for strange contribution: clearly non-zero and negative



Quark flavour decomposition of G_P

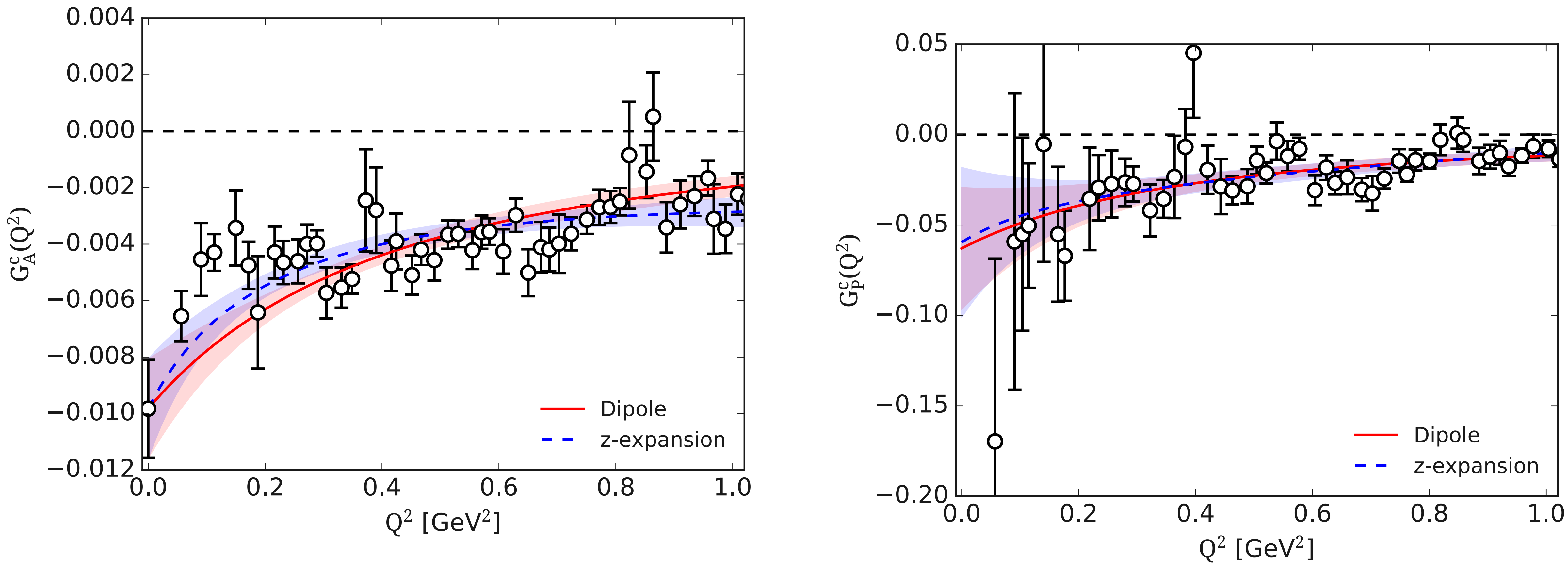




Contribution of disconnected diagrams

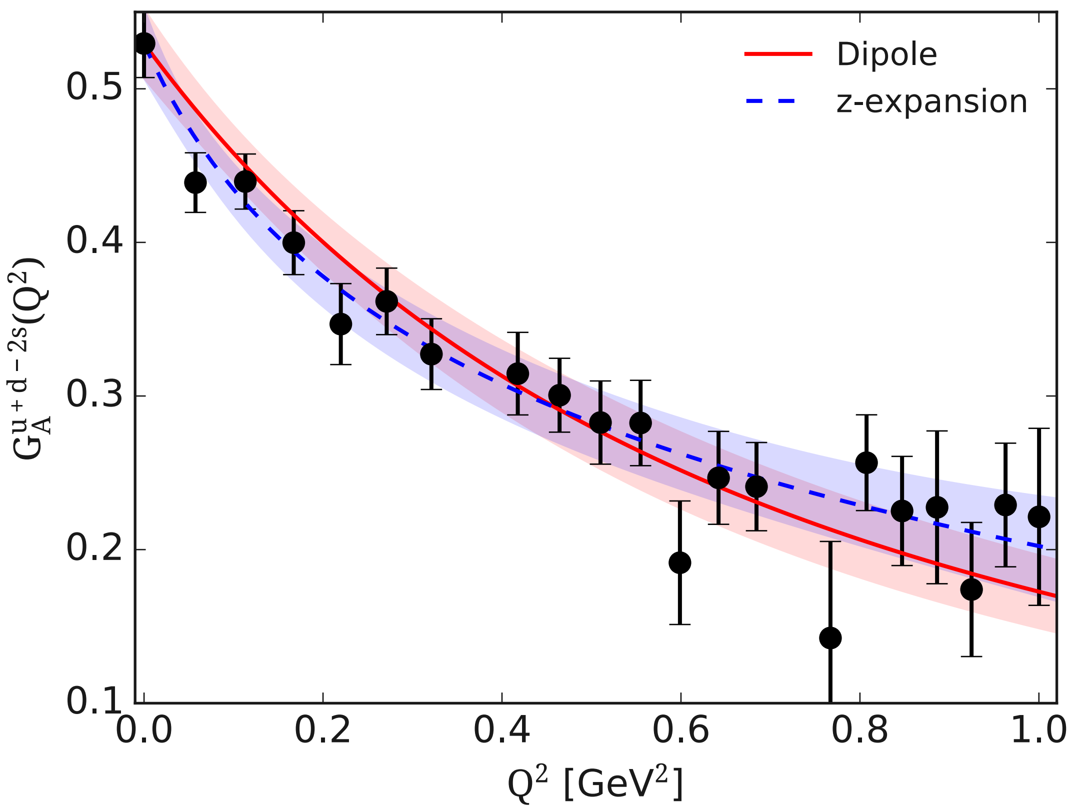
- Significant contribution of disconnected to u+d combination
- Negative large disconnected contribution: subtracts from connected
- Large negative strange quark contribution

Charm quark contributions

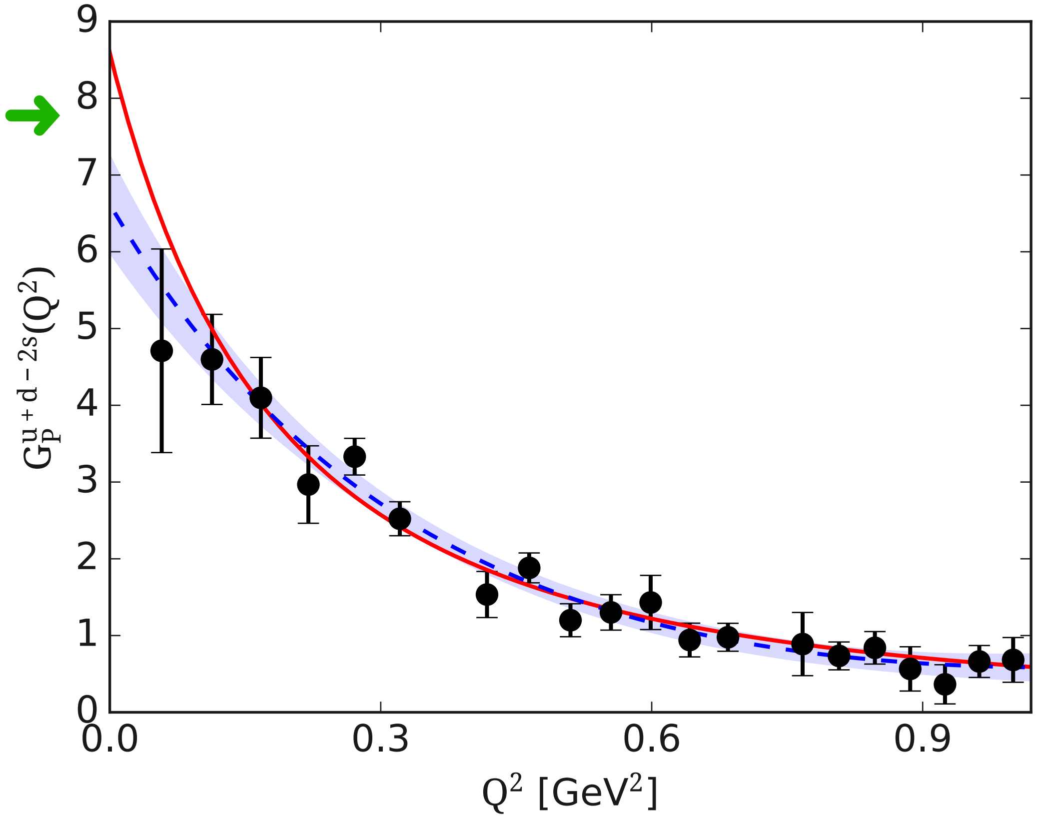


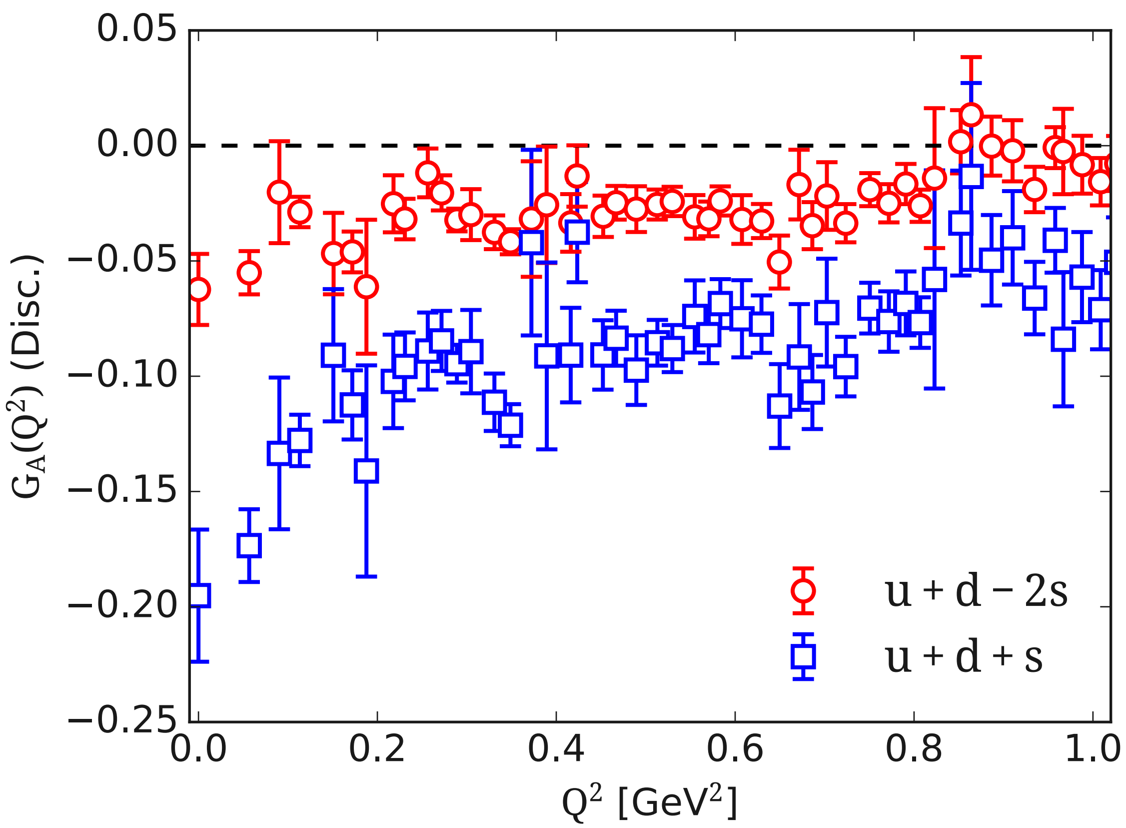
- Clearly non-zero negative contributions for both axial form factors

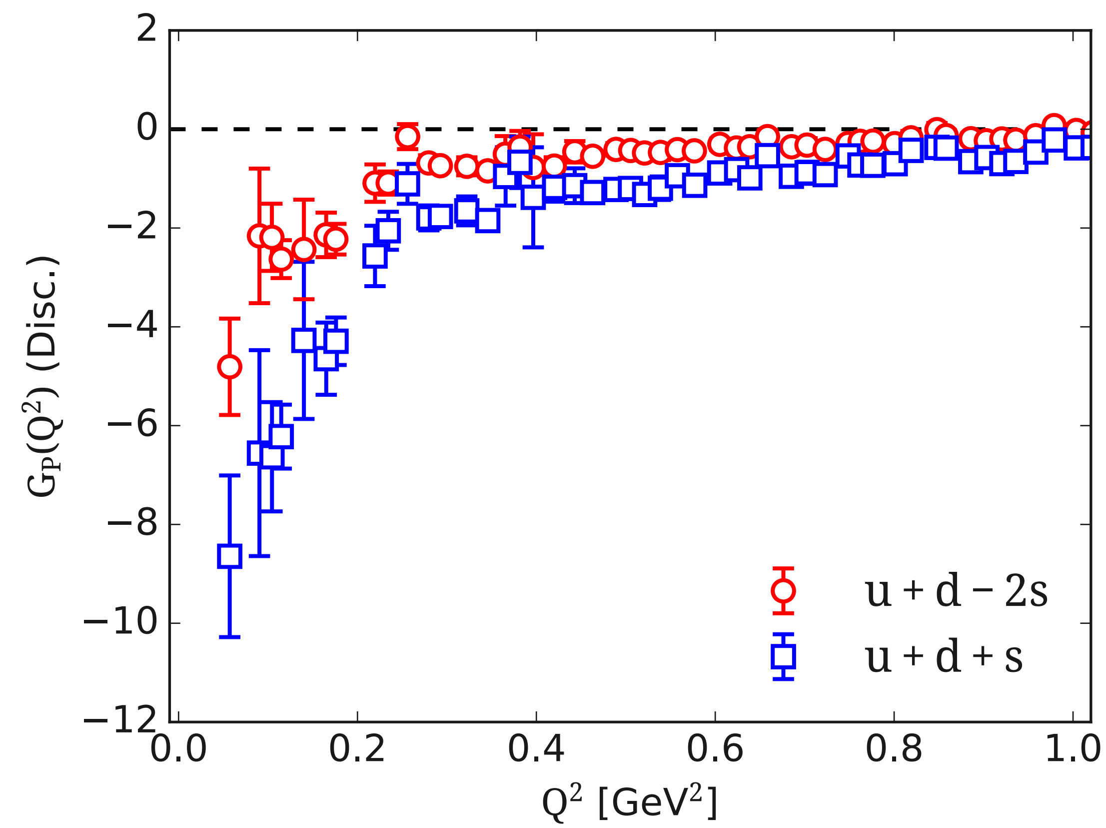
Check SU(3) symmetry



← No SU(3) symmetry assumed →







- In the SU(3) limit disconnected contributions should vanish in the octet combination u+d-2s
- Deviations of up to 10% are seen in $G_A^{u+d-2s}(0)$ and up tp 50% in $G_P^{u+d-2s}(0)$

Conclusions

- Axial form factors including contributions form non-valence quarks can be extracted precisely (precision era of lattice QCD) - we can extract a lot of interesting physics and make predictions

- The calculation of sea quark contributions is feasible providing valuable input e.g. for the determination of strange and charm form factors and for checking SU(3) symmetry

- Further study of PCAC and GT relations is required

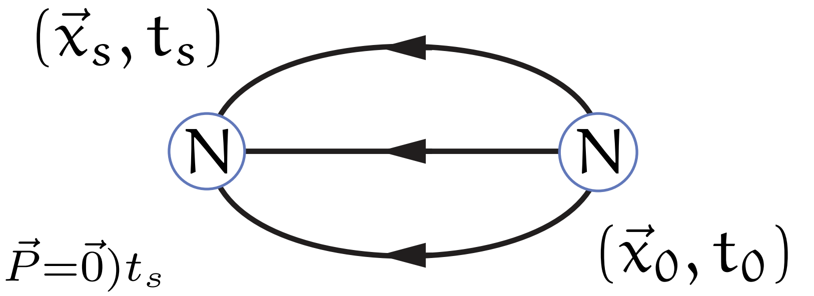
- Way forward:
    - continuum limit, study of volume effects
    - other hadrons, higher Mellin moment, direct computation of PDFs and GDPS, …

Very much progress over the last five years!!

Extended Twisted Mass Collaboration

Backup slides

Nucleon propagator



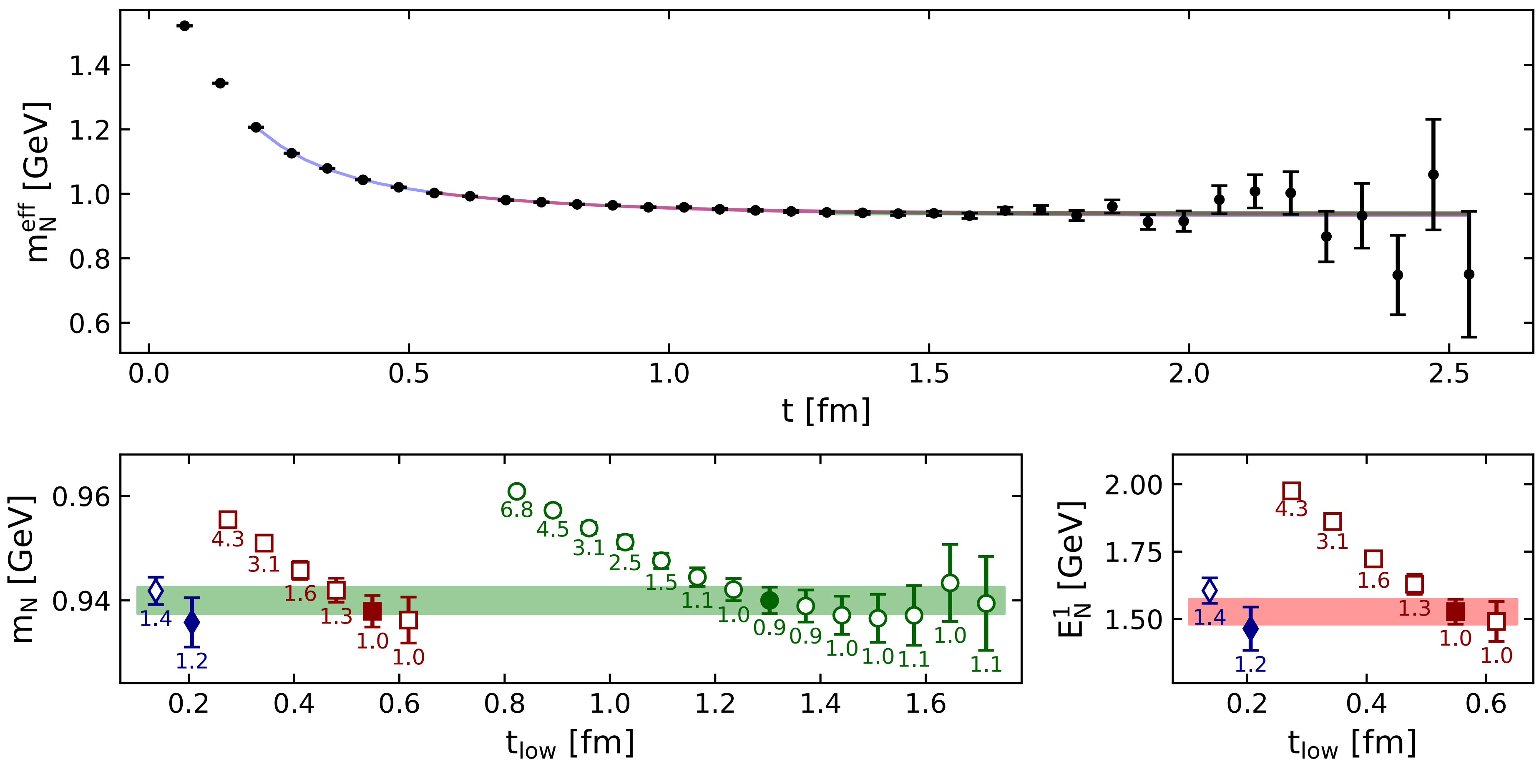
Analysis of two- and three-point functions C_{2pt} and C_{3pt}

$$C_{2\text{pt}}(\Gamma_0; \vec{P}=\vec{0}, t_s) = \sum_{\vec{x}_s} \text{Tr}\left[\langle \Gamma_0 J_N(t_s,\vec{x}_s)\bar{J}_N(t_0,\vec{x}_0)\rangle\right] = \sum_{n=0}^{\infty} a_n e^{-E_n(\vec{P}=\vec{0})t_s}$$

$$\xrightarrow{t_s\to\infty} a_0 e^{-m_N t_s} + \mathcal{O}\left(e^{-E_1(\vec{P}=\vec{0})t_s}\right)$$

Fit the nucleon two-point function or effective mass keeping up to two excited states

$$am_N^{\text{eff}}(t) = \log\left(\frac{C_{2\text{pt}}(t)}{C_{2\text{pt}}(t+a)}\right) \approx am_N + \log\left(\frac{1+\sum_{j=1}^{K} c_j e^{-\Delta_j t}}{1+\sum_{j=1}^{K} c_j e^{-\Delta_j (t+a)}}\right)$$



Renormalisation

- Non-perturbative renormalisation employing the RI' -MOM scheme:

the forward amputated Green function computed in the chiral limit and at a given (large Euclidean) scale $p^2 = \mu^2$ is set equal to its tree-level value.

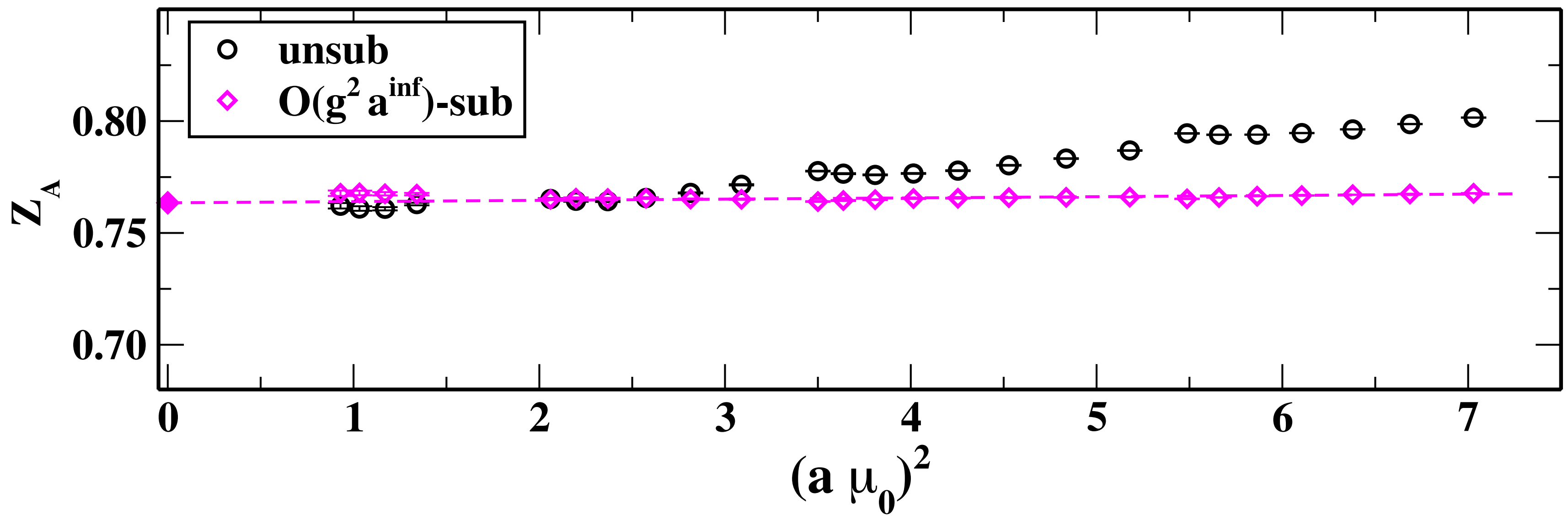
G. Martinelli, C. Pittori, C. T. Sachrajda, M. Testa and A. Vladikas, Nucl. Phys. B 445 (1995) 81, hep-lat/9411010

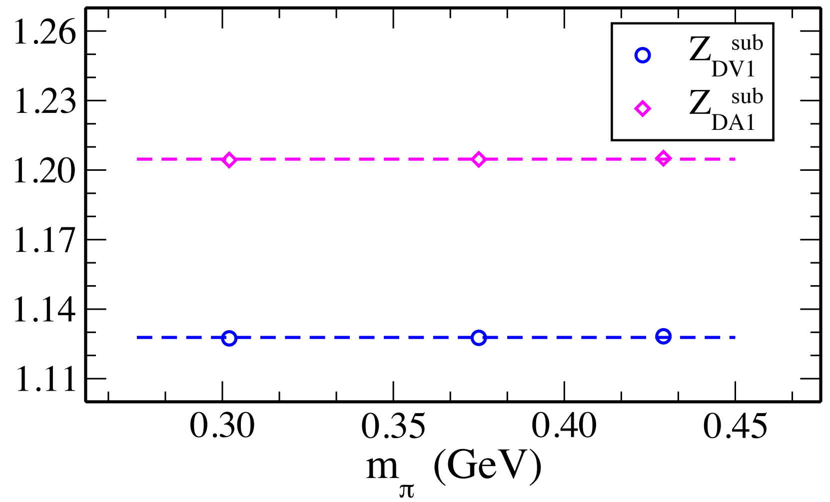
- Use N_f=4 ensembles to take chiral limit - very mild dependence
- Subtract lattice artefacts to $\mathcal{O}(g^2 a^\infty)$ perturbatively
- For scheme dependent operators translate them to the $\overline{\text{MS}}$ scheme at μ =2 GeV using a conversion factor computed in perturbation theory to three-loops

C.A, M. Constantinou, H. Panagopoulos, Phys. Rev. D95, 034505 (2017), 1509.00213

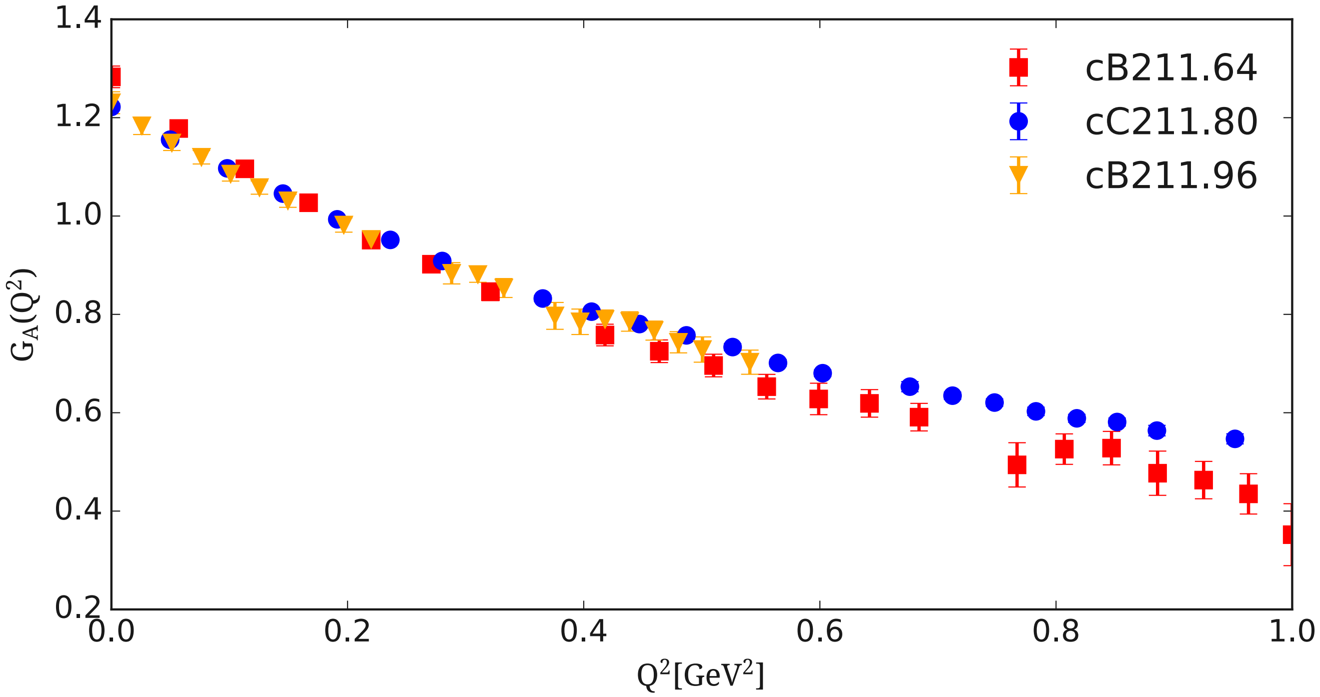
- Momentum source method leads to small statistical errors

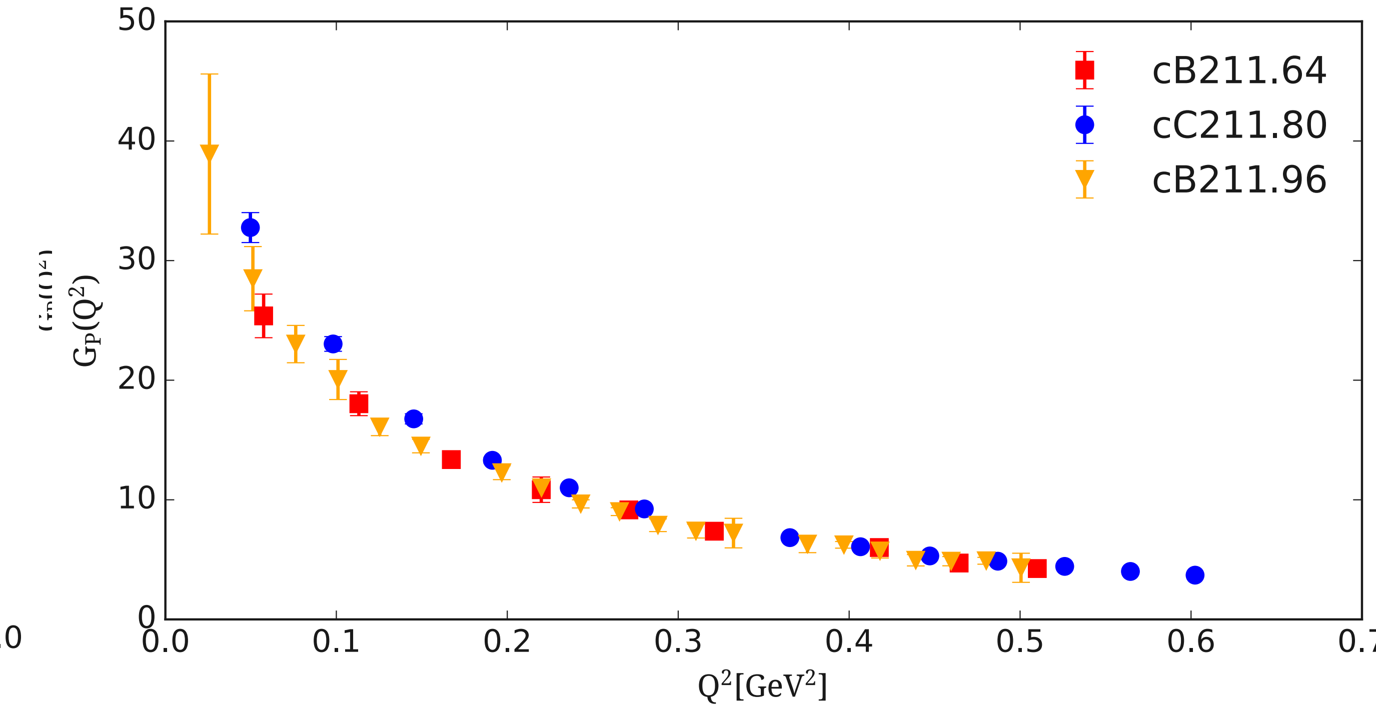
M. Gockeler *et al.* (QCDSF) Nucl. Phys. B544, 699 (1999), hep-lat/9807044; Phys. Rev. D 82 (2010) 114511, 1003.5756

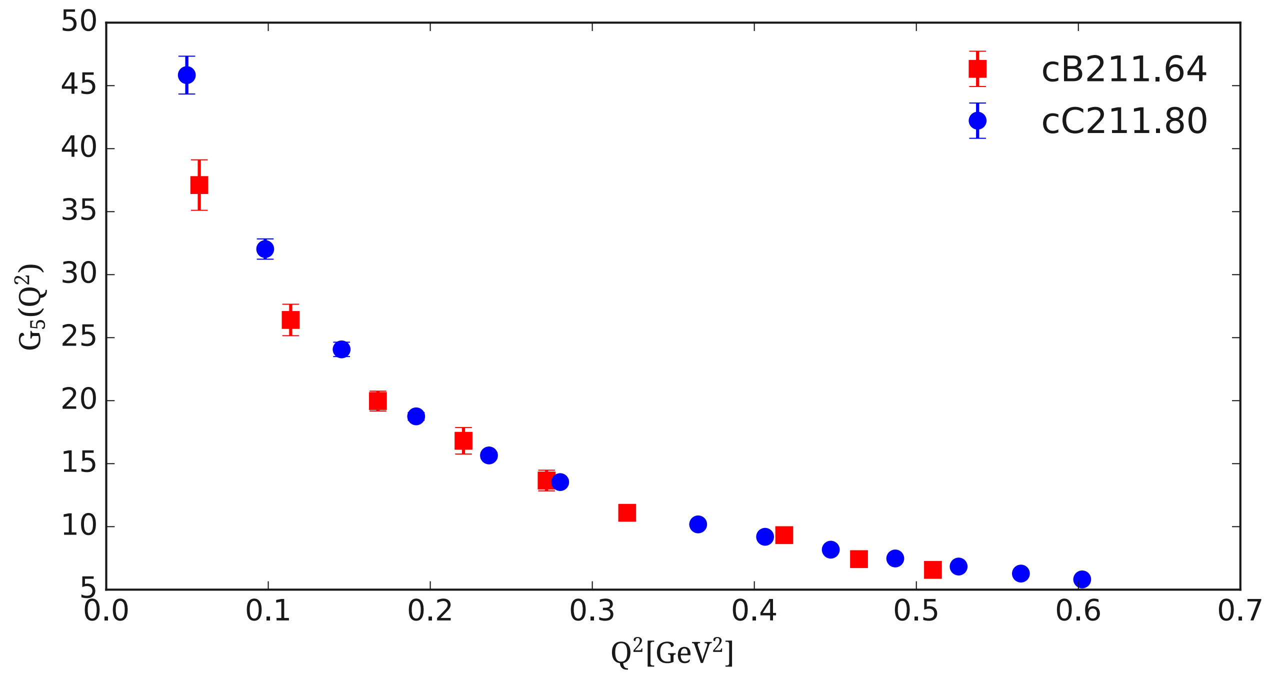




Axial and pseudoscalar form factors for two lattice spacings







- Smaller cut-off effects on G_A; Larger for G_P and G_5
- Crucial to take the continuum limit. Analysis of the third ensemble is ongoing.
- Volume effects small but we statistical errors still large; increased of statistics is ongoing