# MOAT REGIMES & THEIR SIGNATURES IN HEAVY-ION COLLISIONS

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[Pisarski, FR, PRL 127 (2021)]

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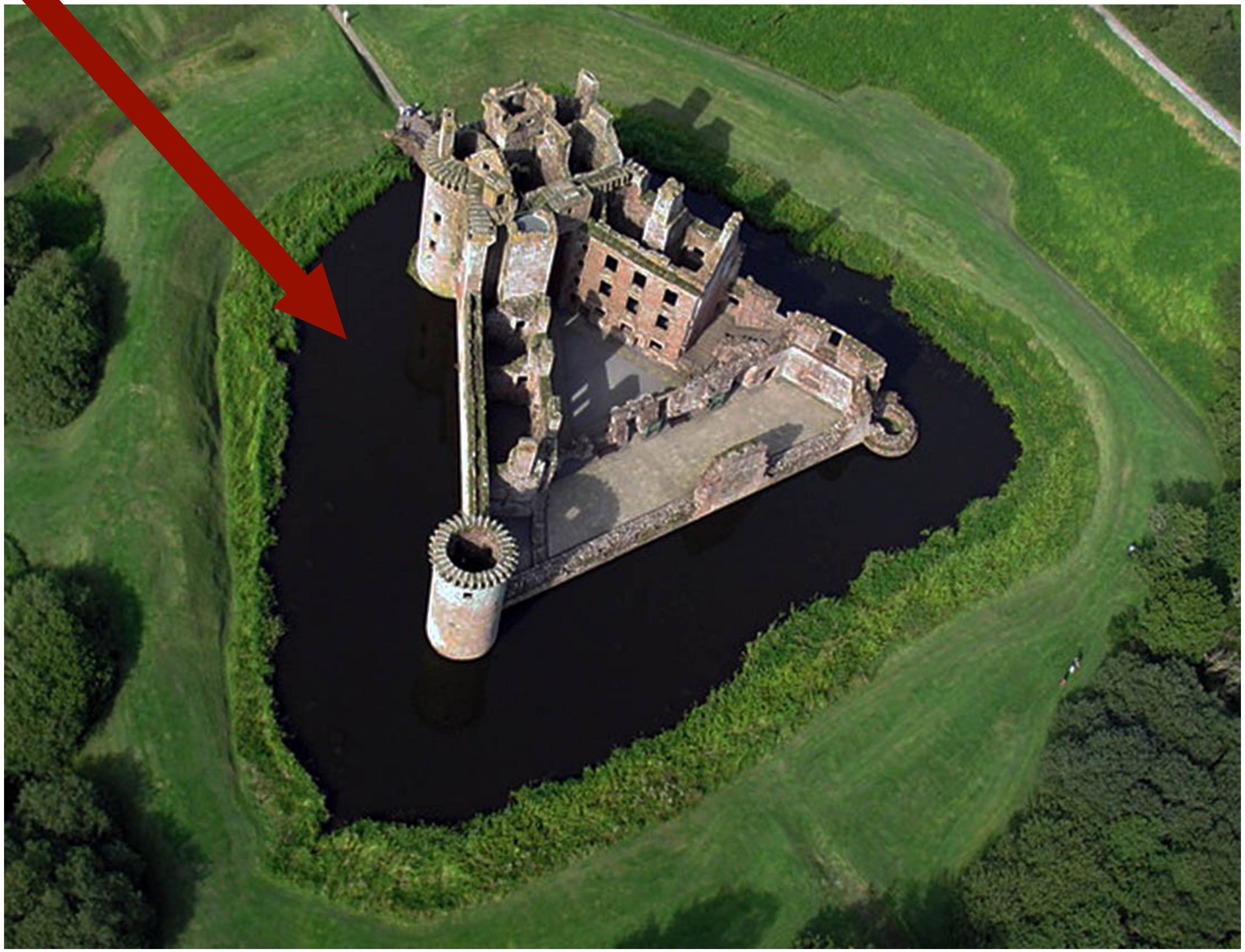
# MOATS IN THE QCD PHASE DIAGRAM

# A MOAT

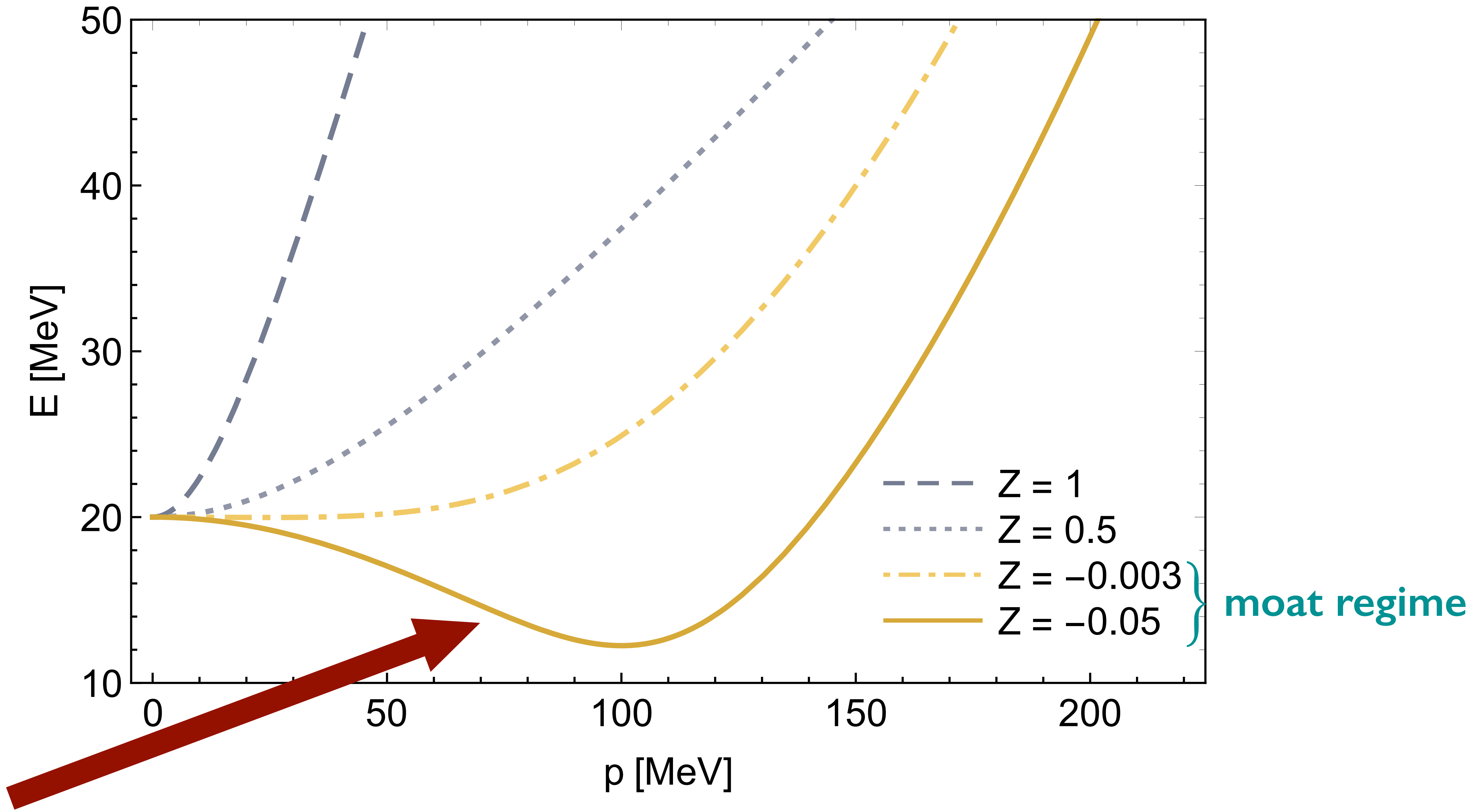


[Caerlaverock Castle, Scotland (source: Wikipedia)]

# A MOAT

**energy dispersion of particle $\phi$:**

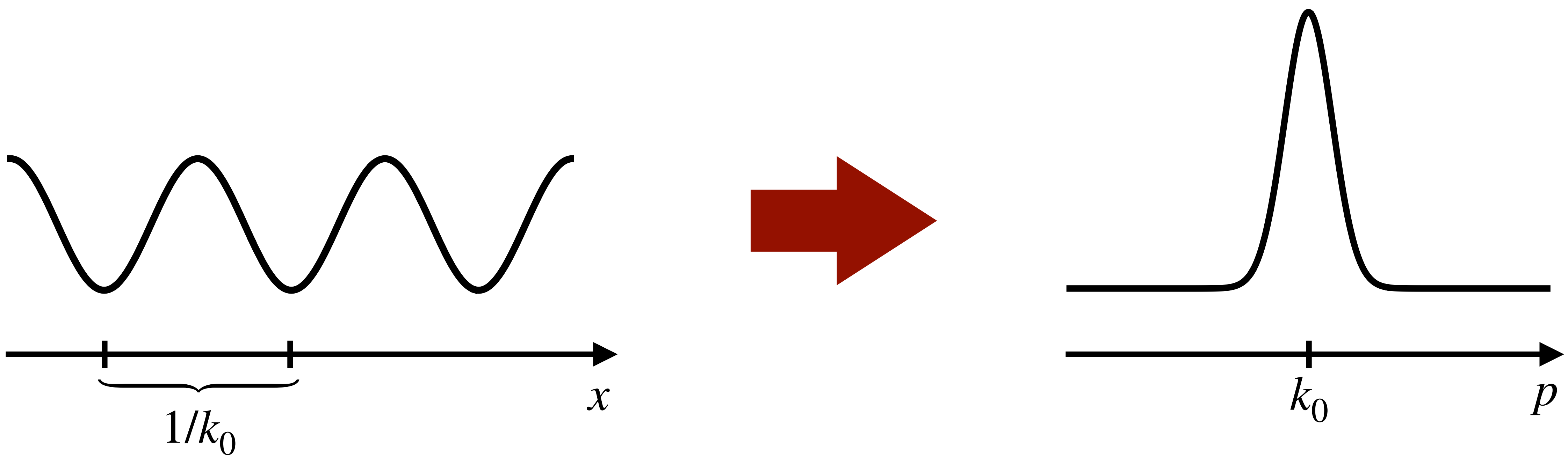
$$E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m_{\text{eff}}^2}$$



# WHERE DOES THE MOAT COME FROM?

spatial oscillation
$\cos(2\pi k_0 x)$

momentum space peak
$\delta(p - k_0)$




- particles subject to a spatial modulation are favored to have finite momentum $k_0$

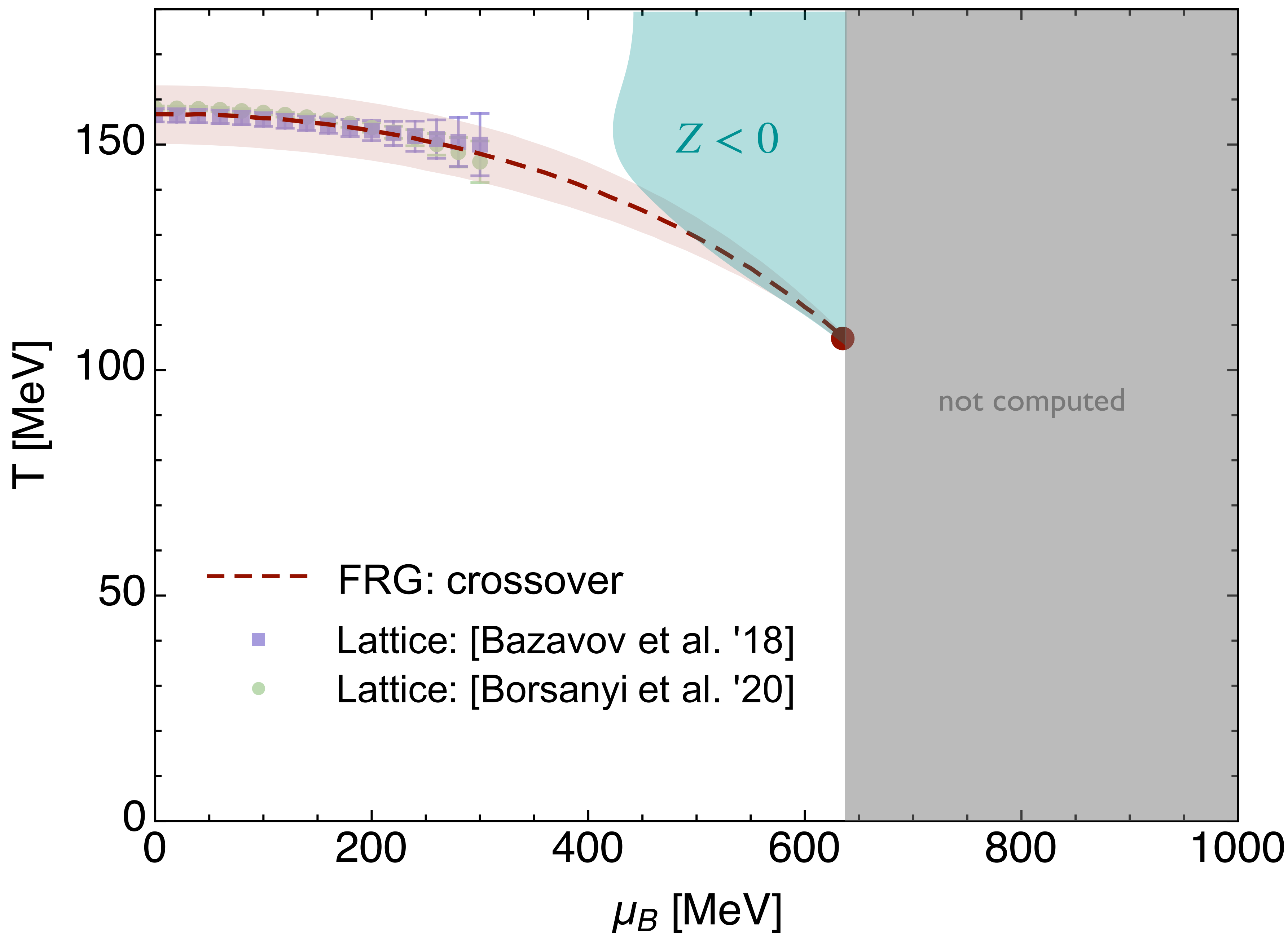
$\longrightarrow$ moat energy dispersion
(minimal energy at $k_0$)

- typical for inhomogeneous/crystalline phases or a quantum pion liquid (Q$\pi$L)

# WHERE CAN MOAT PHASES APPEAR?

**At large $\mu_B$ in the QCD phase diagram:**

[Fu, Pawlowski, FR (2019)]



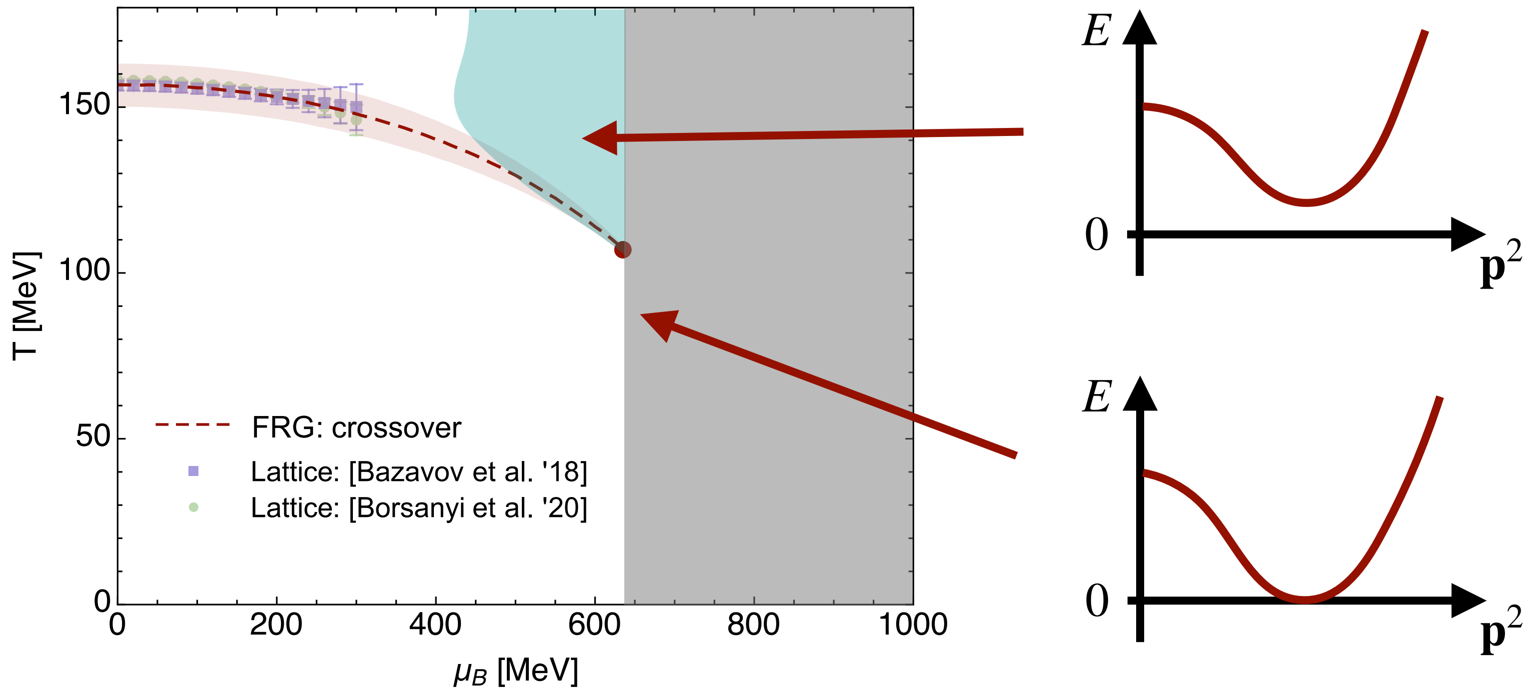

→ indication for extended region with $Z < 0$ in QCD: moat regime

$$E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\big(\mathbf{p}^2\big)^2 + m_{\text{eff}}^2}$$

# IMPLICATIONS OF THE MOAT

There are basically two possibilities:

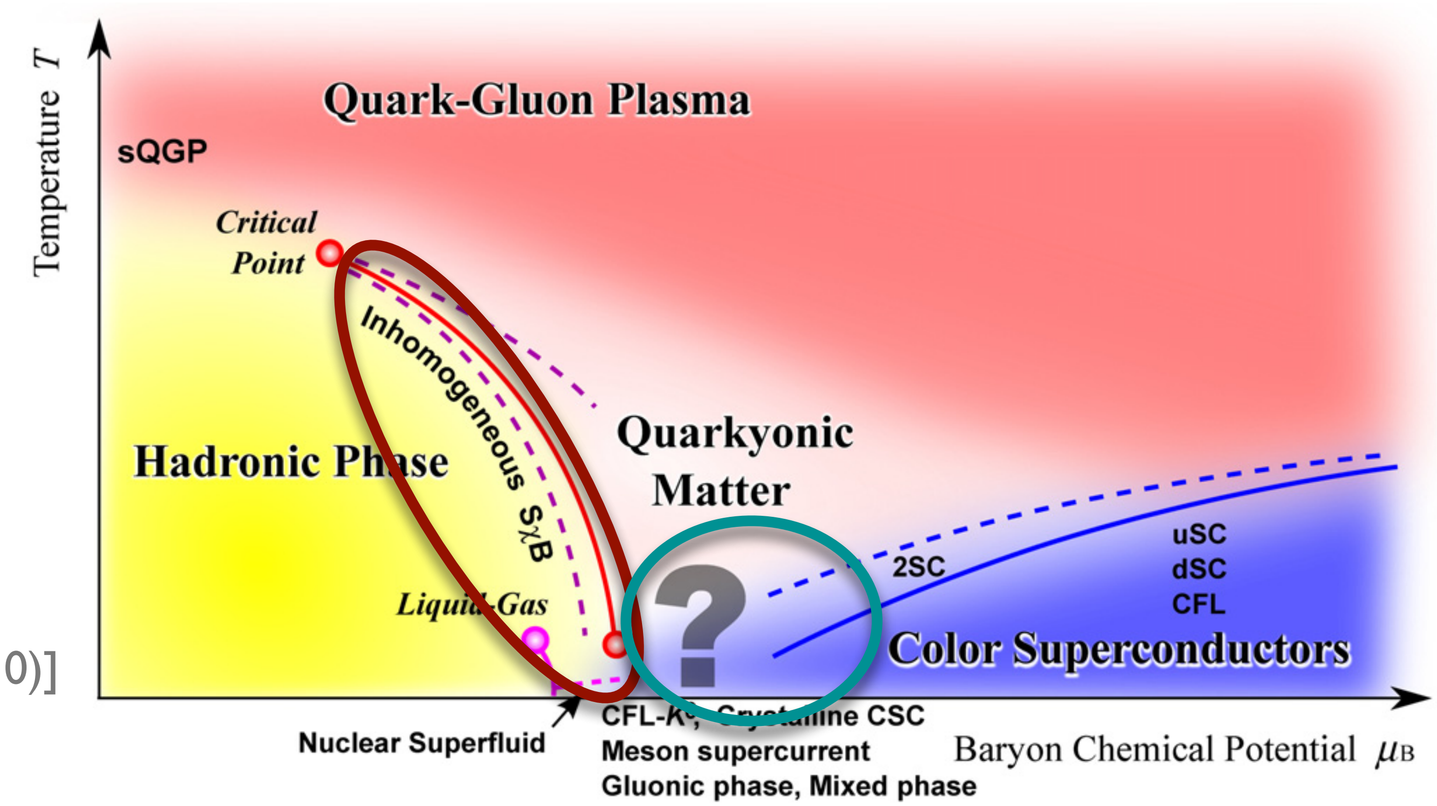
**Option 1: inhomogeneous phase at lower T/larger $\mu_B$**




$E > 0$ for all $\mathbf{p}^2$

$E = 0$ at $\mathbf{p}^2 > 0$:
instability towards formation of an inhomogeneous condensate

- moat is a **precursor** for an inhomogeneous phase




[Fukushima, Hatsuda (2010)]

# IMPLICATIONS OF THE MOAT

**But:** fundamental problem with inhomogeneous condensates with fluctuations:

- basic example: fluctuations around $O(N)$ chiral density wave

$$\phi = \Delta \begin{pmatrix} \cos(k_0\, z) \\ \sin(k_0\, z) \\ 0 \\ \vdots \\ 0 \end{pmatrix} + \begin{pmatrix} \delta\phi_{\parallel} \\ \delta\phi_{\perp} \end{pmatrix}$$

static (large $T$) propagator of transverse (Goldstone) modes

$$G_{\phi_\perp} = \frac{1}{W\,(\mathbf{p}^2 - k_0^2)^2}$$

$\longrightarrow$ double pole at nonzero momentum

- tadpole corrections in any dimension lead to linear IR divergences at finite T:

$$T \int \frac{d^d\mathbf{p}}{(2\pi)^d}\, G_{\phi_\perp} \sim \frac{T}{W}\, k_0^{d-3} \int_{|\mathbf{p}|\sim k_0} \frac{d\,|\mathbf{p}|}{(|\mathbf{p}| - k_0)^2}$$

transverse fluctuations $\delta\phi_\perp$ disorder the system:

$\longrightarrow$ **no inhomogeneous phase for $N > 2$**

(rigorous for $O(N)$ chiral density wave at $N \to \infty$)

[Pisarski, Tsvelik, Valgushev '20]

# IMPLICATIONS OF THE MOAT

There are basically two possibilities:

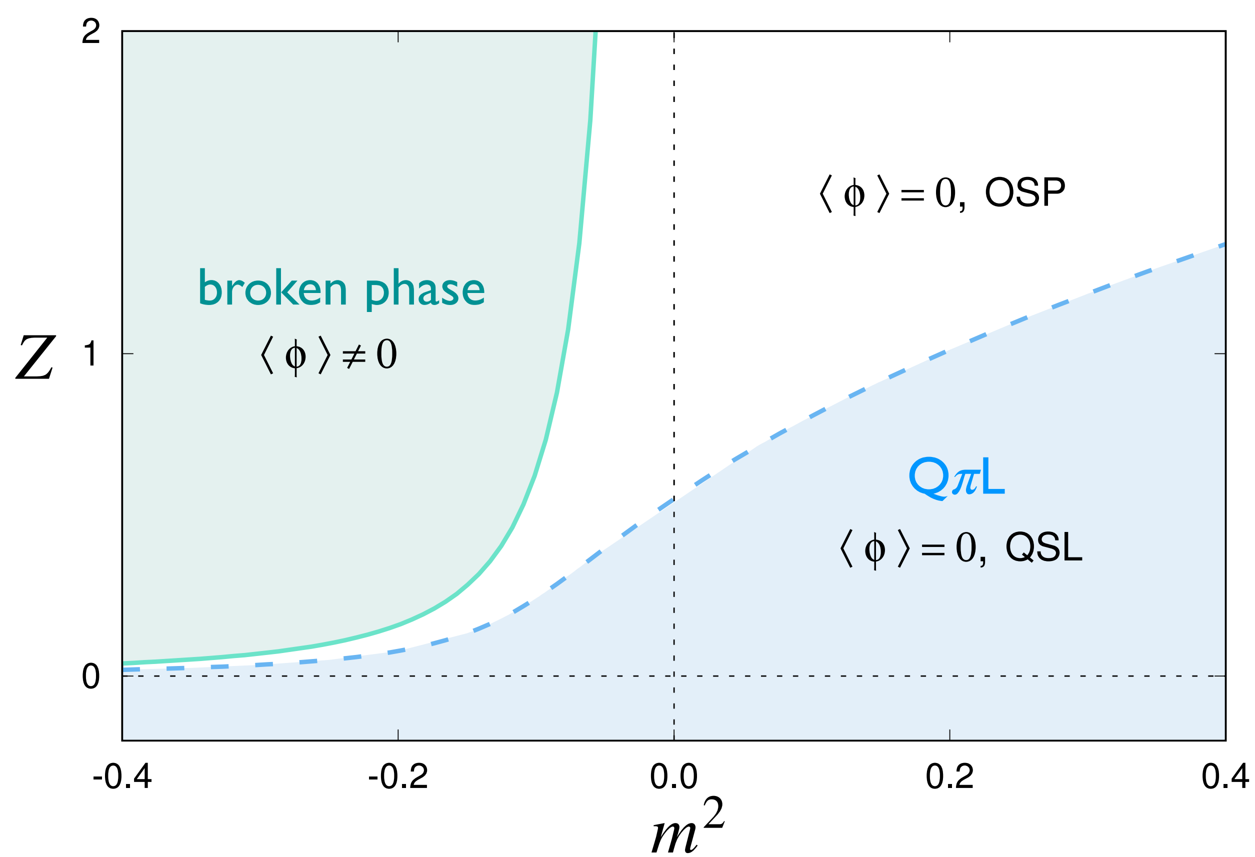
**Option 2: quantum pion liquid**

[Pisarski, Tsvelik, Valgushev '20]
[Pisarski '21]

**instead of inhomogeneous phase**

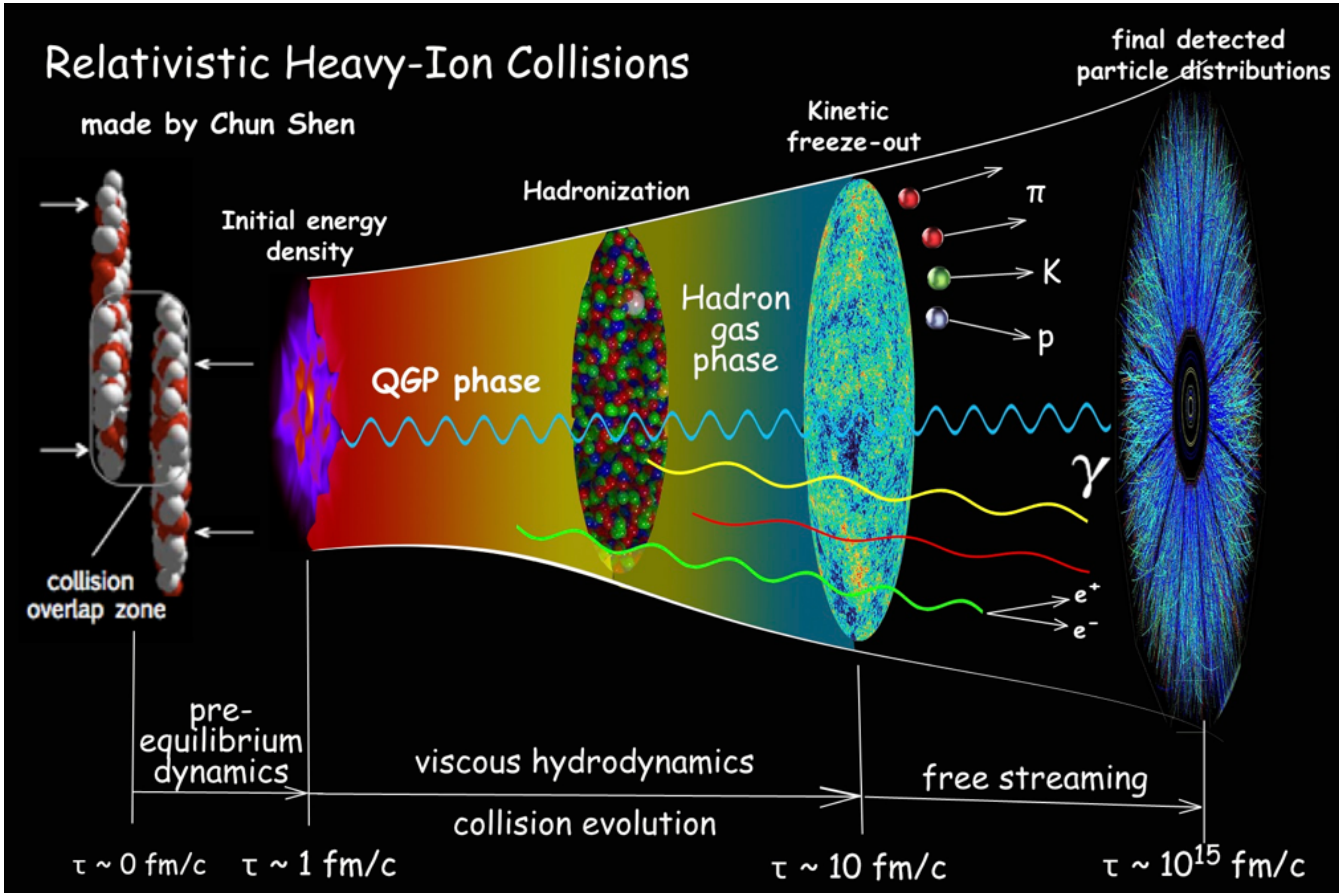
- disordered phase with a moat spectrum ($E > 0$ for all $\mathbf{p}^2$)
- spatial modulations: $\langle \phi(x)\phi(0)\rangle \sim e^{-m_r x} \cos(m_i x)$ for large $x$

$\longrightarrow$ **quantum pion liquid**
(in analogy to quantum spin liquids)



# SIGNATURES OF MOATS
# IN HEAVY-ION COLLISIONS

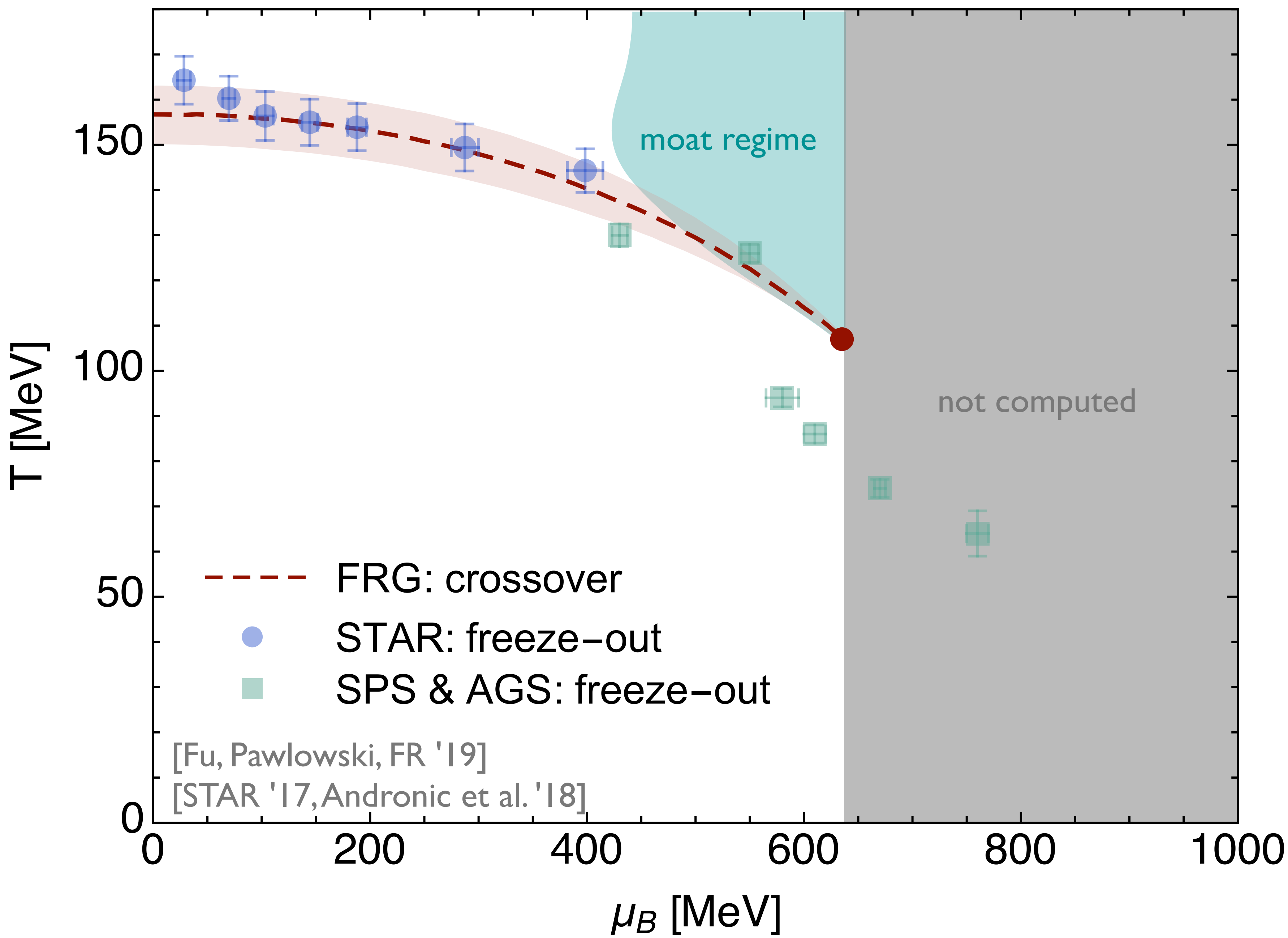
# PROBING THE PHASE DIAGRAM




→ imprints of the phase structure at freeze-out?

# PROBING THE PHASE DIAGRAM

Vary the beam energy to study the phase diagram different densities (smaller energy ↔ lager $\mu$)




<u>STAR @ RHIC</u>

$\sqrt{s} = 7.7 - 200\,\mathrm{GeV}$

$\mu_B \approx 400 - 30\,\mathrm{MeV}$

<u>HADES @ GSI</u>

$\sqrt{s} \approx 2.4\,\mathrm{GeV}$

$\mu_B \approx 770\,\mathrm{MeV}$

future experiments, e.g.,

<u>CBM @ FAIR</u>

$\sqrt{s} = 2.7 - 4.9\,\mathrm{GeV}$

$\mu_B \approx 730 - 540\,\mathrm{MeV}$

also: J-PARC, NICA, HIAF

**What are the signatures of the the phase diagram in heavy-ion collisions?**

# SEARCH FOR MOAT REGIMES [Pisarski, FR '21]

Moats arise in regimes with spatial modulations in the phase diagram at large $\mu_B$

Characteristic feature: minimal energy at nonzero momentum

$\Rightarrow$ **enhanced particle production at nonzero momentum**

$\rightarrow$ look for signatures in the **momentum dependence** of particle numbers and correlations

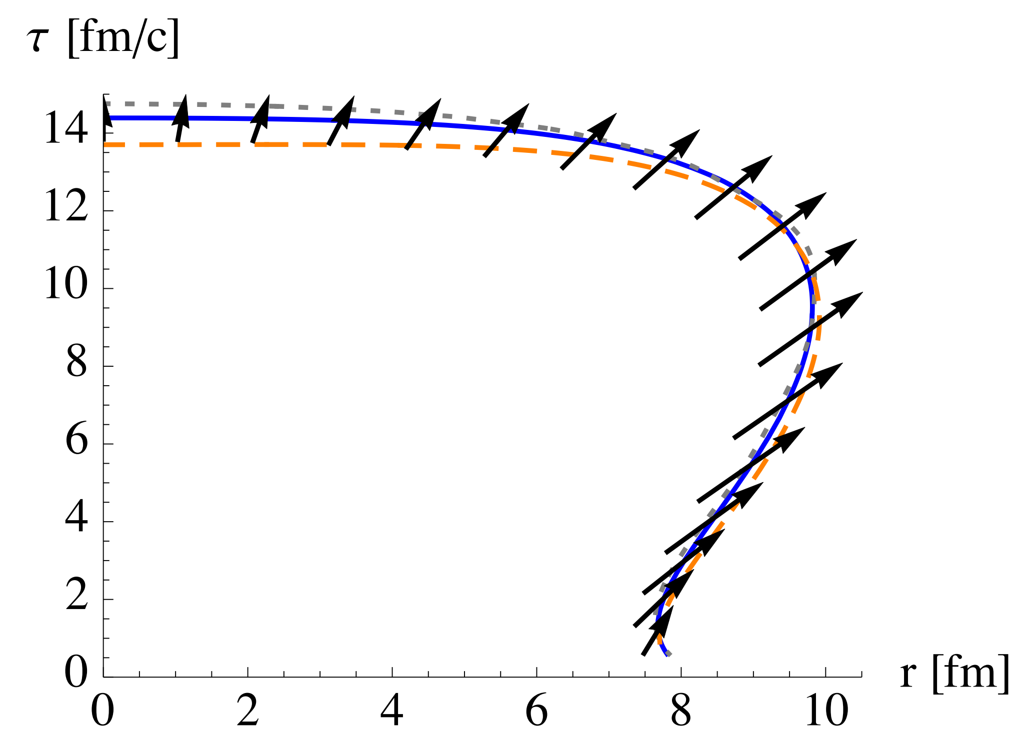
- Note: particle number correlations $\langle N_\phi^n \rangle$ are measured from event-by-event fluctuations!

  prominent application: CEP search

- particles freeze out at certain temperature $T_f$

  $\longrightarrow$ defines 3d hypersurface: freeze-out surface $\Sigma$

**How does the moat regime affect particles on $\Sigma$?**




[Floerchinger, Wiedemann '13]

# GENERALIZED COOPER-FRYE FORMULA

## compute particle numbers on the freeze-out surface

- probability distribution of finding a particle $\phi$ with momentum $p$ in thermal equilibrium: Wigner function

$$F_\phi(p) = 2\pi\, \underbrace{\rho_\phi}_{\text{spectral function}}(p_0, \mathbf{p})\, f(p_0)$$

- particles on $\Sigma$ boosted with fluid velocity $u^\mu(x)$:

energy: $\breve{p}_0 = u^\mu p_\mu$

spatial momentum: $\breve{\mathbf{p}}^2 = \left(u^\mu u^\nu - g^{\mu\nu}\right) p_\mu p_\nu$

- particle spectrum from integrating particle number current over freeze-out surface:

$$\frac{d^3 N_\phi}{d\mathbf{p}^3} = \frac{2}{(2\pi)^3} \int_\Sigma d\Sigma_\mu \underbrace{\int \frac{dp_0}{2\pi}\, p^\mu\, \Theta(\breve{p}_0)\, F_\phi(\breve{p})}_{\sim \text{ particle number current density}}$$

- reduces to Cooper-Frye formula for free vacuum spectral function: $\rho_\phi(p) = \mathrm{sign}(\mathrm{p}_0)\, \delta\left[\mathrm{p}_0^2 - \left(\mathbf{p}^2 + \mathrm{m}^2\right)\right]$

# PARTICLE SPECTRUM IN A MOAT PHASE

**use simple models to show general structure**

## Particle in a moat regime:

- **low-energy model** of free bosons in a moat regime $(Z < 0, W > 0)$:

$$\mathscr{L}_0 = \frac{1}{2}\left(\partial_0\phi\right)^2 + \frac{Z}{2}\left(\partial_i\phi\right)^2 + \frac{W}{2}\left(\partial_i^2\phi\right)^2 + \frac{m_{\text{eff}}^2}{2}\phi^2$$

- gives simple in-medium spectral function

$$\rho_\phi(p_0, \mathbf{p}^2) = \mathrm{sign}(\mathrm{p}_0)\,\delta\big[\mathrm{p}_0^2 - \mathrm{E}_\phi^2(\mathbf{p}^2)\big] \quad \text{with} \quad E_\phi(\mathbf{p}^2) = \sqrt{Z\,\mathbf{p}^2 + W\left(\mathbf{p}^2\right)^2 + m_{\text{eff}}^2}$$

- boost symmetry broken! (but spatial rotation symmetry still intact)

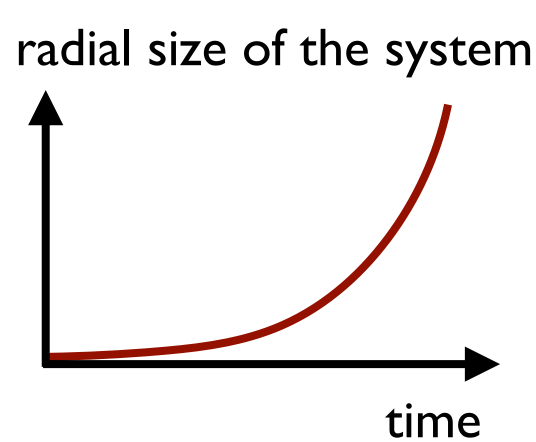
## Fluid velocity and freeze-out surface from hydro evolution

- boost invariant freeze-out at fixed temperature $T_f$ and fixed proper time $\tau_f$ $\left(=\sqrt{t^2 - z^2}\right)$

- blast wave approximation for the fluid velocity:

$$u^r = \bar{u}\,\frac{r}{\bar{R}}\,\theta(\bar{R} - r)$$

[Schnedermann, Sollfrank, Heinz (1993)]
[Teaney (2003)]



# PARTICLE SPECTRUM IN A MOAT PHASE

**use simple models to show general structure**

model parameters:

- pick a beam energy of $\sqrt{s} = 5\,\mathrm{GeV}$ and read off thermodynamic and blast wave parameters:

$$T_f = 115\,\mathrm{MeV}$$
$$\mu_{B,f} = 536\,\mathrm{MeV}$$

[Andronic, Braun-Munzinger, Redlich, Stachel (2018)]

$$\bar{u} = 0.3$$
$$\bar{R} = 8\,\mathrm{fm}$$
$$\tau_f = 5\,\mathrm{fm/c}$$

[Zhang, Ma, Chen, Zhong (2016)]

- thermodynamics (used later) from a hadron resonance gas [Braun-Munzinger, Redlich, Stachel (2003)]

- moat parameters: purely illustrative

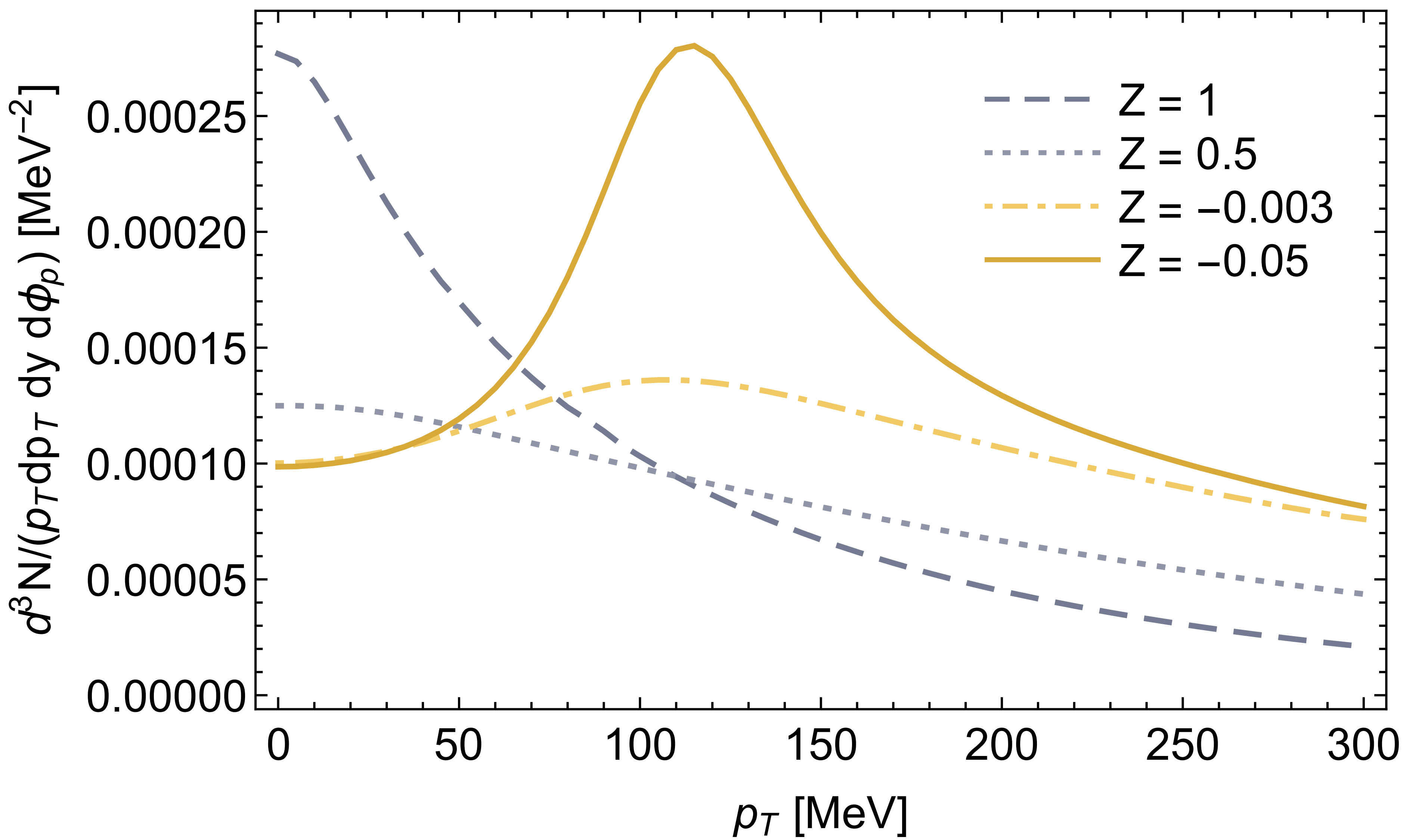
$$\text{if } Z < 0: \quad W = 2.5\,\mathrm{GeV}^{-2}$$

# PARTICLE SPECTRUM IN A MOAT PHASE

**transverse momentum spectrum**

- compare normal phase (gray, $W = 0$) to moat phase (yellow, $W = 2.5\,\mathrm{GeV}^{-2}$)

$$\frac{d^3N}{p_T\,dp_T\,dy\,d\phi_p}$$




**enhanced particle production at nonzero momentum!**

**maximum related to the wavenumber of the spatial modulation**

# PARTICLE NUMBER CORRELATIONS

- correlations sensitive to in-medium modifications
- difficult to compute in systems with long-range order due to multi-particle correlations
- Q$\pi$L is disordered: single particle correlations can capture relevant features

⟶ **correlations on $\Sigma$ from (generalized) Cooper-Frye formula** [Pisarski, FR (2021)] [Floerchinger (unpublished)]

$n$-particle correlation:

$$\left\langle \prod_{i=1}^{n} \frac{d^3 N_\phi}{d\mathbf{p}_i^3} \right\rangle = \left[ \prod_{i=1}^{n} \frac{2}{(2\pi)^3} \int d\Sigma_i^\mu \int \frac{dp_i^0}{2\pi} (p_i)_\mu \, \Theta(\check{p}_i^0) \right] \left\langle \prod_{i=1}^{n} F_\phi(\check{p}_i) \right\rangle$$

thermodynamic average

- fluctuations, e.g., of thermodynamic quantities lead to fluctuations of $F_\phi$
- consider small fluctuations $T, \mu_B, u$ with $\kappa_i^\mu(x) = \big(T(x), \mu_B(x), u^\mu(x)\big)_i$ :

$$\left\langle F_\phi F_\phi \right\rangle_c = \frac{\partial F_\phi}{\partial \kappa_i^\mu} \frac{\partial F_\phi}{\partial \kappa_j^\nu} \bigg|_{\bar{\kappa}} \left\langle \delta\kappa_i^\mu \delta\kappa_j^\nu \right\rangle + \mathcal{O}\big(\delta\kappa^3\big)$$

connected correlator

fluctuations of $T, \mu_B, u$

# THERMODYNAMIC CORRELATIONS

- correlations $\langle \ldots \rangle$ from thermodynamic average
- weight configurations with the change in entropy due to fluctuations, $\Delta s^\mu$ [Landau, Lifshitz (vol. 5)]

⟶ generating functional of (connected) thermodynamic correlations

$$W[J] = \ln \int \mathscr{D}\kappa(x)\, \exp \int d\Sigma_\mu \left[\Delta s^\mu(x) + J(x)_{i\nu}\, \hat{v}^\mu\, \delta\kappa_i^\nu(x)\right]$$

normal to $\Sigma$

- connected n-particle correlations $\langle \delta\kappa^n \rangle_c$ from $\left.\dfrac{\delta^n W[J]}{\delta J^n}\right|_{J=0}$
- change of entropy in an ideal fluid $(T^{\mu\nu} = \epsilon\, u^\mu u^\nu + p\Delta^{\mu\nu})$ with Gaussian fluctuations:

$$\hat{v}_\mu \Delta s^\mu = -\frac{1}{2}\delta\kappa_{i\mu}(x)\, \mathscr{F}^{\mu\nu}_{ij}(x)\, \delta\kappa_{j\nu}(x)$$

local fluctuations!

fluctuation matrix $(\hat{u} = \hat{v}^\mu\, u_\mu)$

$$\mathscr{F}^{\mu\nu}_{ij} = \frac{1}{T}\begin{pmatrix} \hat{u}\,\dfrac{\partial s}{\partial T} & \hat{u}\,\dfrac{\partial s}{\partial \mu_B} & s\hat{v}^\nu \\ \hat{u}\,\dfrac{\partial s}{\partial \mu_B} & \hat{u}\,\dfrac{\partial n}{\partial \mu_B} & n\hat{v}^\nu \\ s\hat{v}^\mu & n\hat{v}^\mu & -\hat{u}\,(Ts + \mu_B n) g^{\mu\nu} \end{pmatrix}_{ij}$$

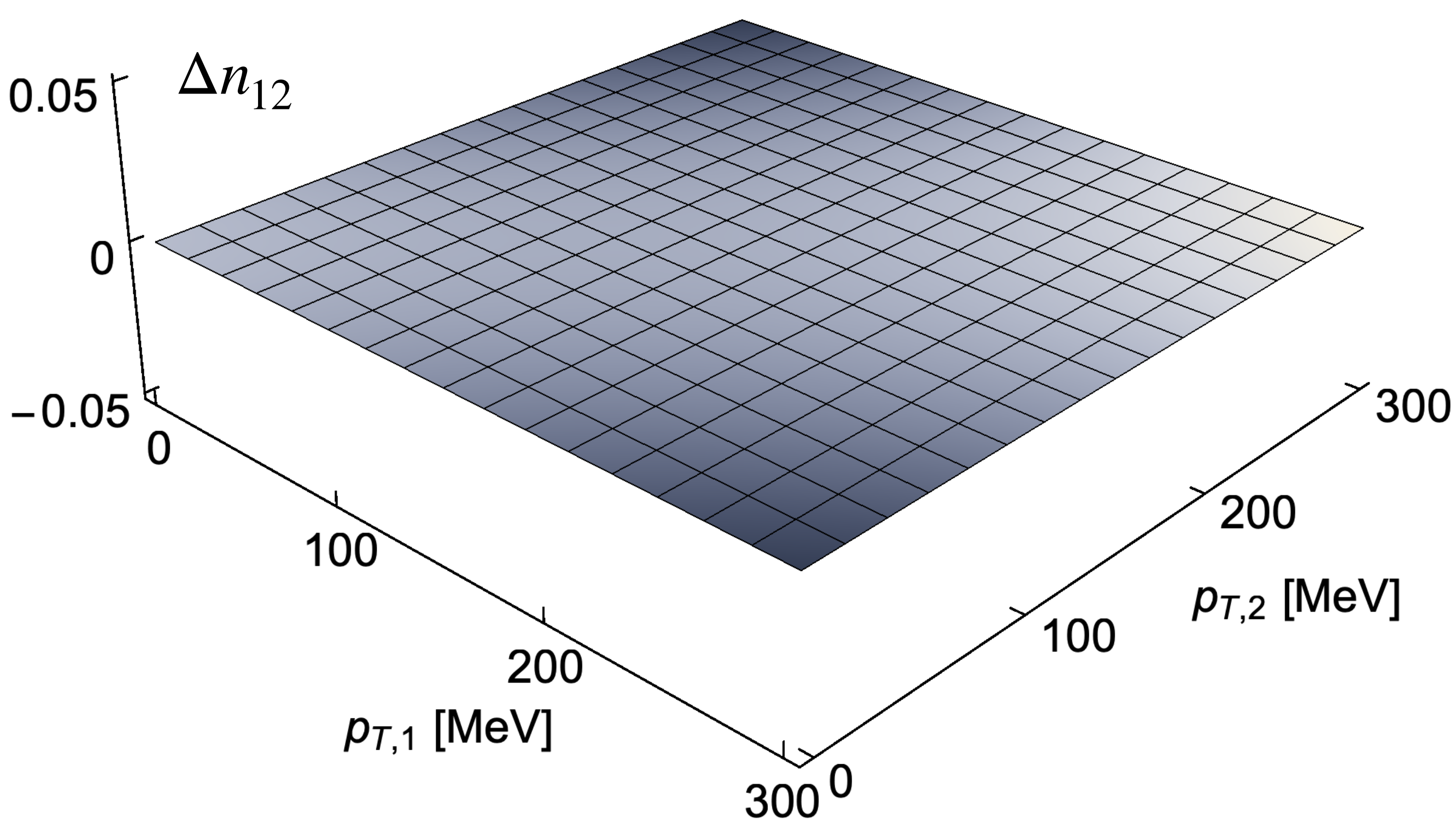
# PARTICLE NUMBER CORRELATIONS

- Gaussian fluctuations: only nontrivial correlation is two-point function (all others are products thereof (Wick's theorem))

$$\left\langle \frac{d^3N_\phi}{d\mathbf{p}_1^3}\frac{d^3N_\phi}{d\mathbf{p}_2^3} \right\rangle_c = \frac{4}{(2\pi)^6}\int d\Sigma^\mu \int \frac{dp_1^0}{2\pi}\frac{dp_2^0}{2\pi}\,(p_1)_\mu\,(\hat{v}\cdot p_2)\,\Theta(\breve{p}_1^0)\,\Theta(\breve{p}_2^0)\left(\frac{\partial F_\phi(\breve{p}_1)}{\partial\kappa_i^\rho}\frac{\partial F_\phi(\breve{p}_2)}{\partial\kappa_j^\sigma}\right)\Bigg|_{\bar{\kappa}}\left(\mathscr{F}_{ij}^{\rho\sigma}(w)\right)^{-1}$$

- look at normalized two-particle correlation $\Delta n_{12} = \left\langle \left(\frac{d^3N}{d\mathbf{p}^3}\right)^2 \right\rangle_c \Big/ \left\langle \frac{d^3N}{d\mathbf{p}^3} \right\rangle^2$

**normal phase**




(relatively) flat two-particle $p_T$ correlation in the normal phase

# PARTICLE NUMBER CORRELATIONS

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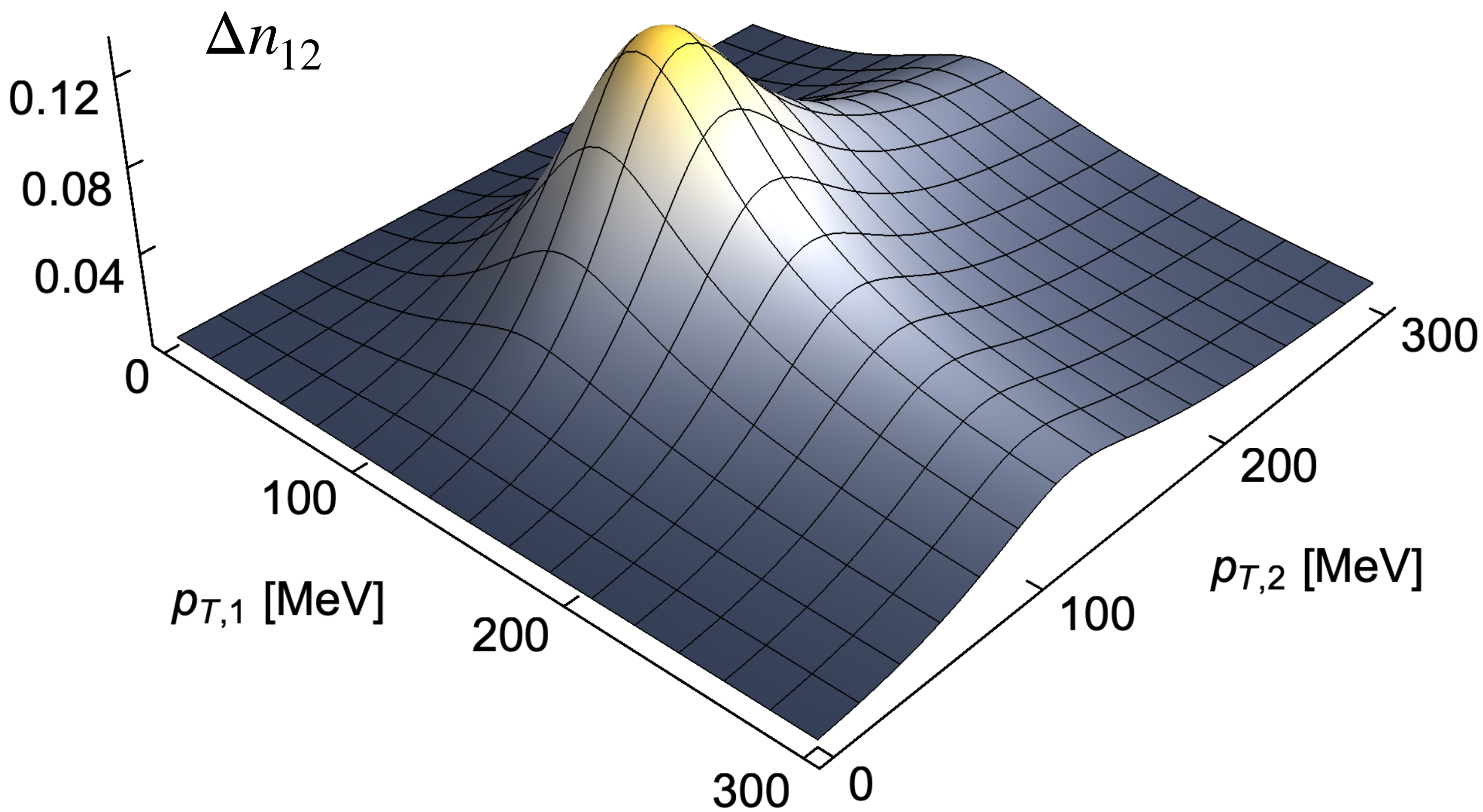
**moat phase**




**pronounced peak and ridges at nonzero $p_T$ related to wavenumber of spatial modulation!**

# SUMMARY

**Moats arise in regimes with spatial modulations**

- expected to occur at large $\mu_B$

**Enhanced production of moat particles at nonzero momentum**

- characteristic peaks (and ridges) in particle spectra and correlations at nonzero $p_T$

**Opportunity to discover novel phases with low-energy heavy-ion collisions through differential measurements of particles and their correlations at small momenta**

So far: basic description of qualitative effects
To do: quantitative description of moat regimes