

# QCD phase transition for various number of flavours

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A.Yu. Kotov, M.P. Lombardo and A. Trunin, *Symmetry* 13 (2021) 10, 1833

# Outline

- QCD phase transition from  $N_f=2$  till the onset of conformal window
- $N_f=2+1$  with strange quark mass as an interpolator between  $N_f=2$  and  $N_f=3$
- $N_f=2+1+1$  (physical strange and charm, various  $m_\pi$ )
  - scaling behaviour
  - new order parameter
  - thresholds in QGP

# Symmetries of QCD

$$L_{\text{QCD}} = \bar{\psi}_L \not{D} \psi_L + \bar{\psi}_R \not{D} \psi_R + m(\bar{\psi}_L \psi_R + \bar{\psi}_R \psi_L) + \text{gauge part}$$

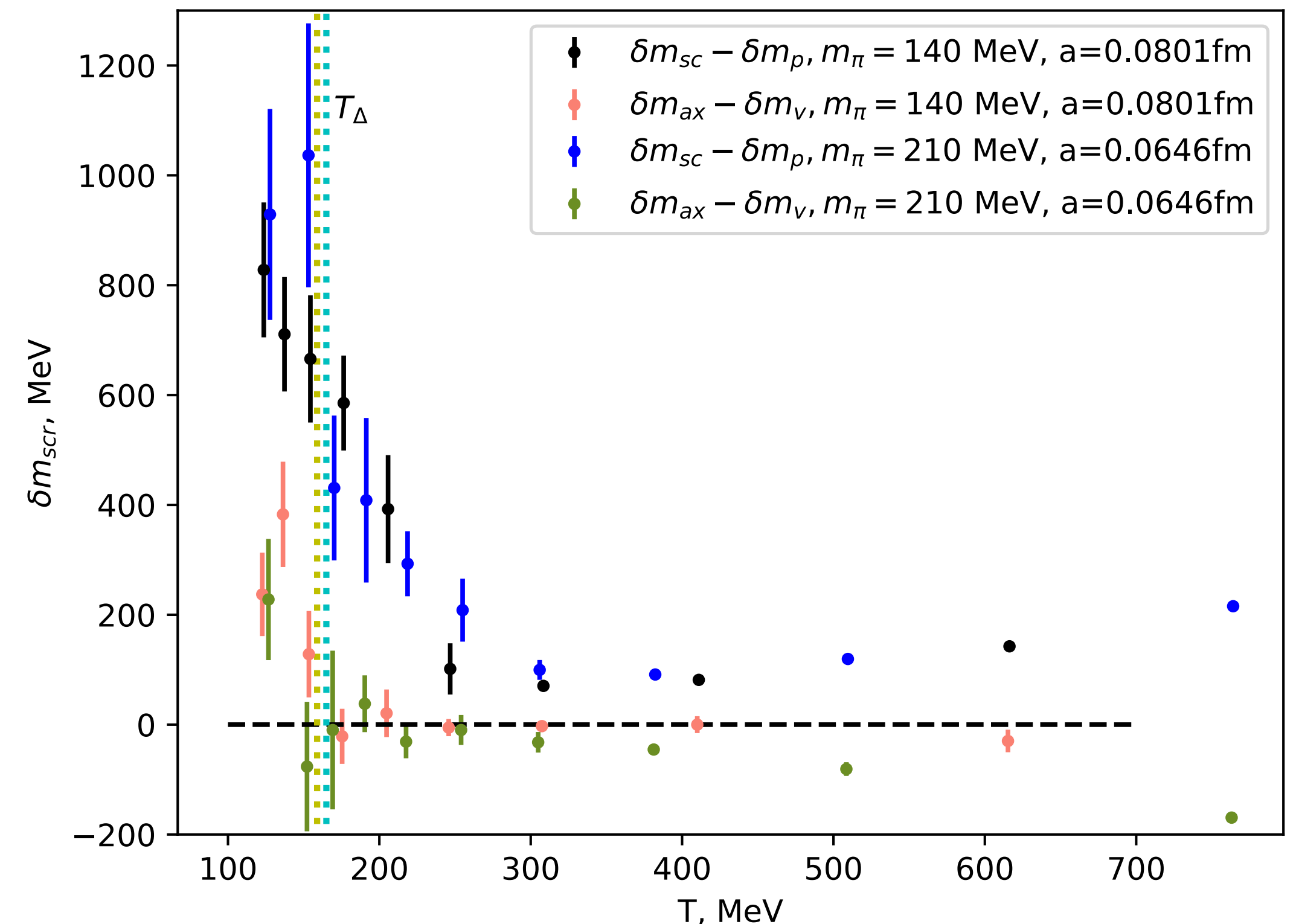
$$\text{Symmetry at } m=0: SU_L(N) \times SU_R(N) \times U_V(1) \times U_A(1)$$

Spontaneously broken to  $SU_V(N)$

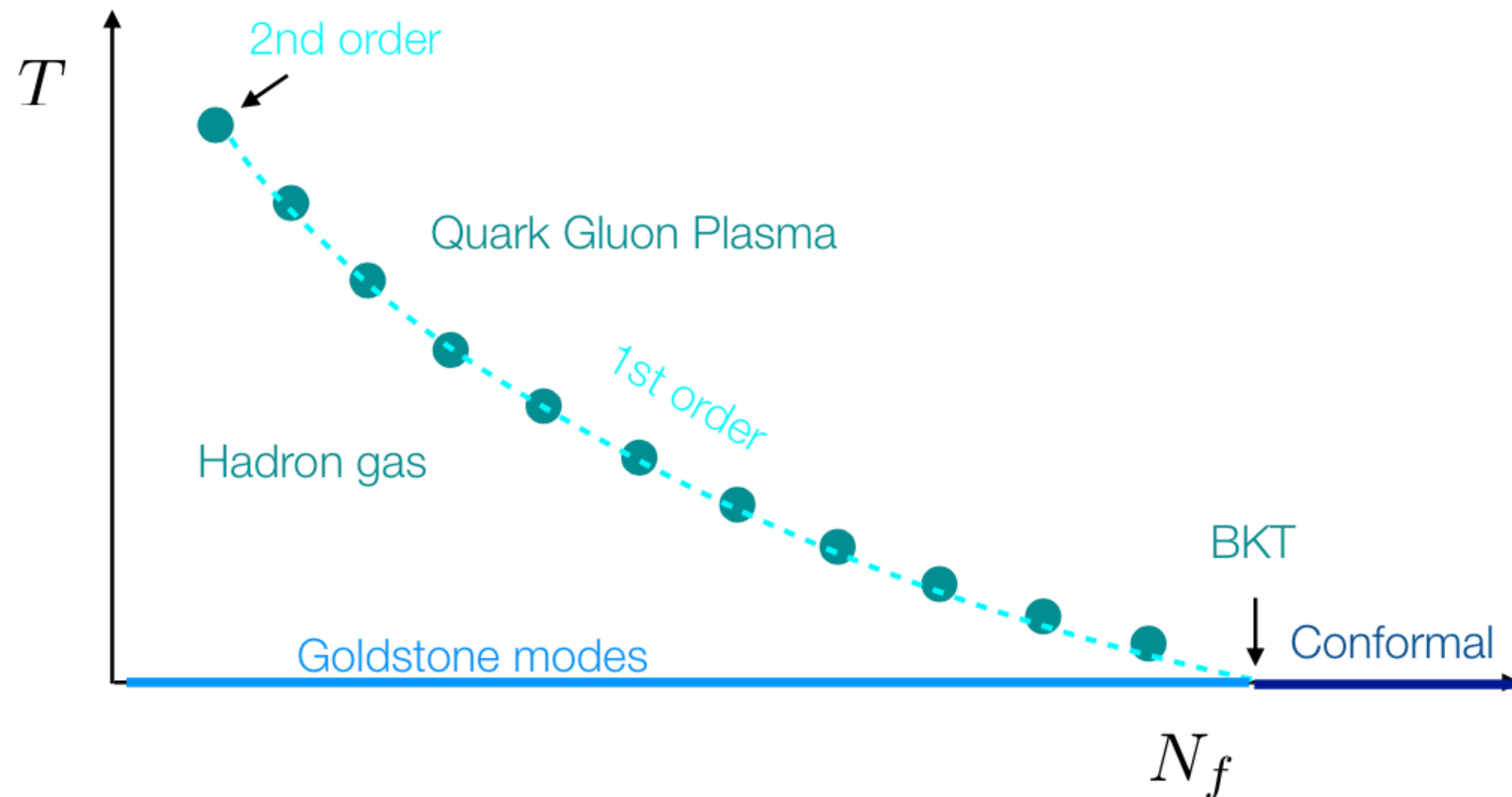
Baryon number

Broken by anomaly

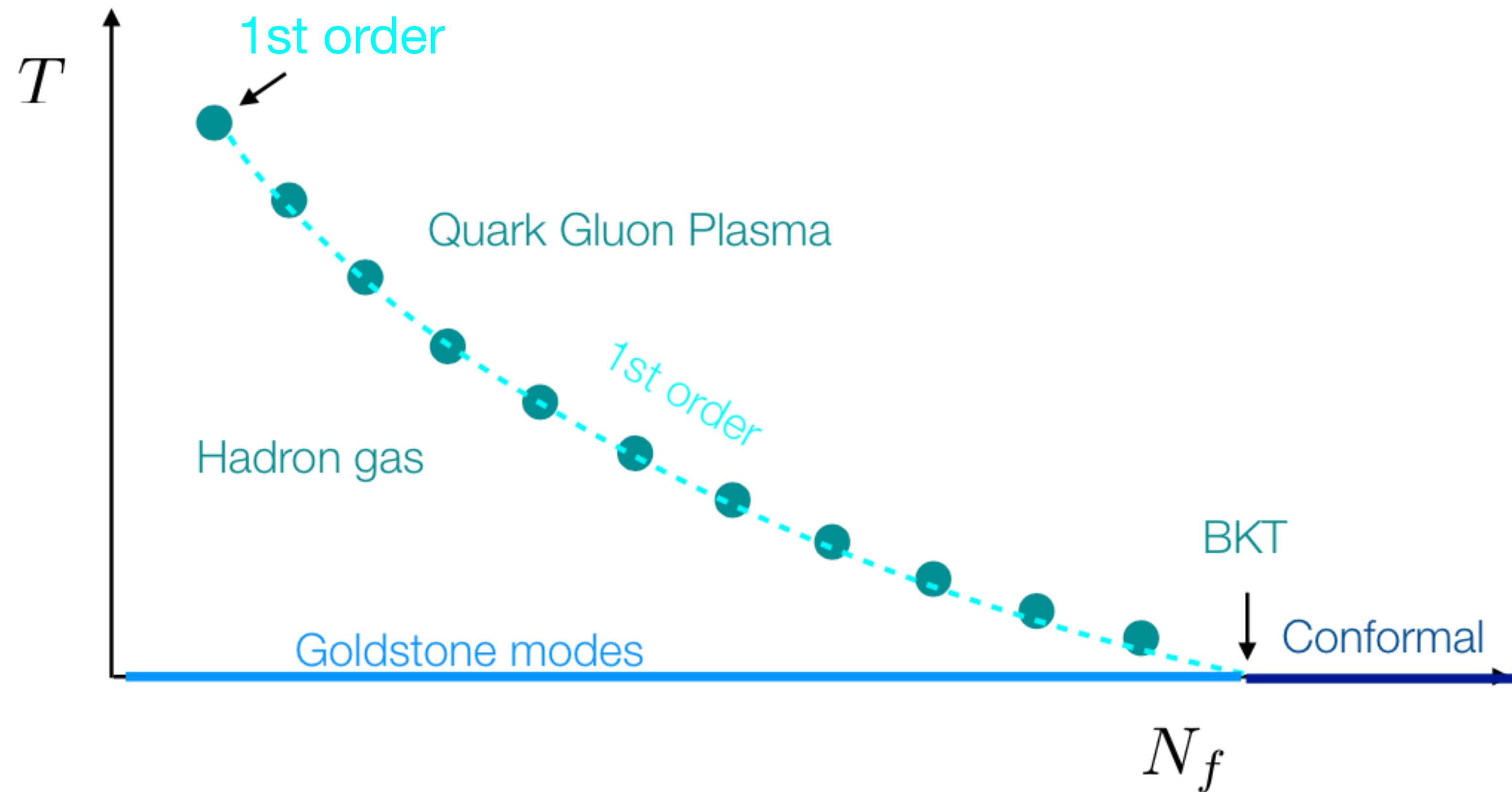
- Imprint on hadron spectrum
  - Goldstone bosons
  - Symmetry partners
- Restoration at finite temperature
- Nature of the QCD phase transition for various N



# Scenarios for thermal phase transition

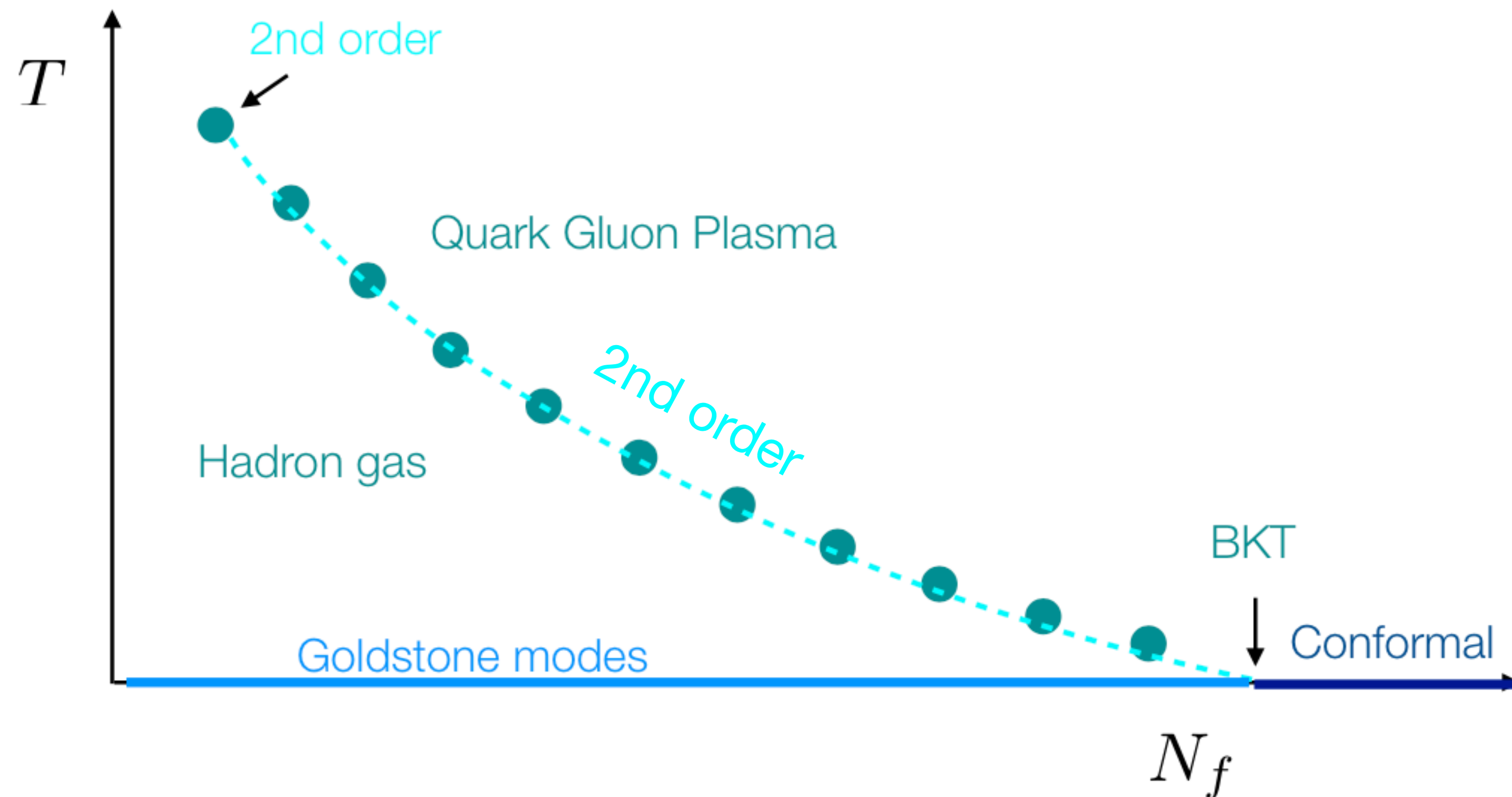


# Scenarios for thermal phase transition



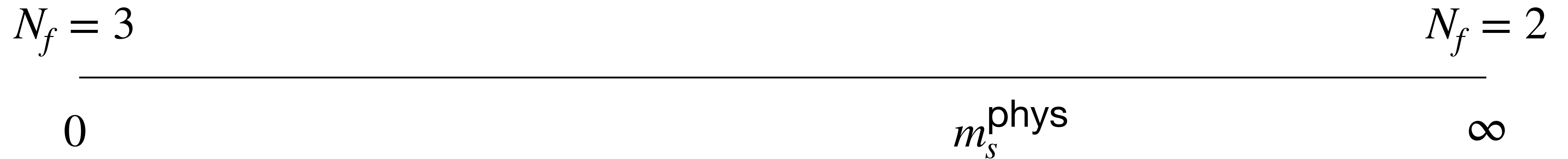
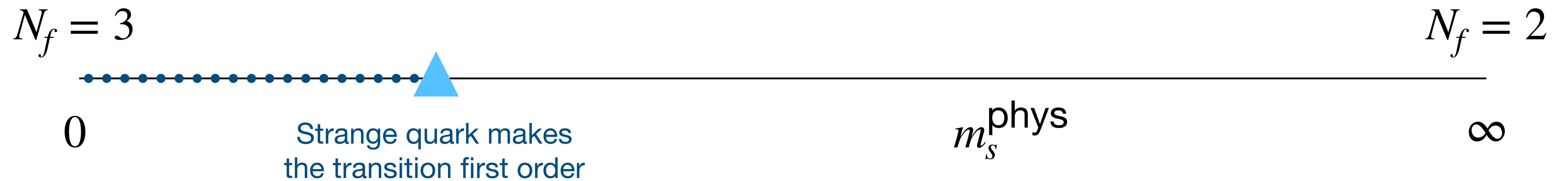
# Scenarios for thermal phase transition

[Cuteri, Philipsen, Sciarra, 2021]

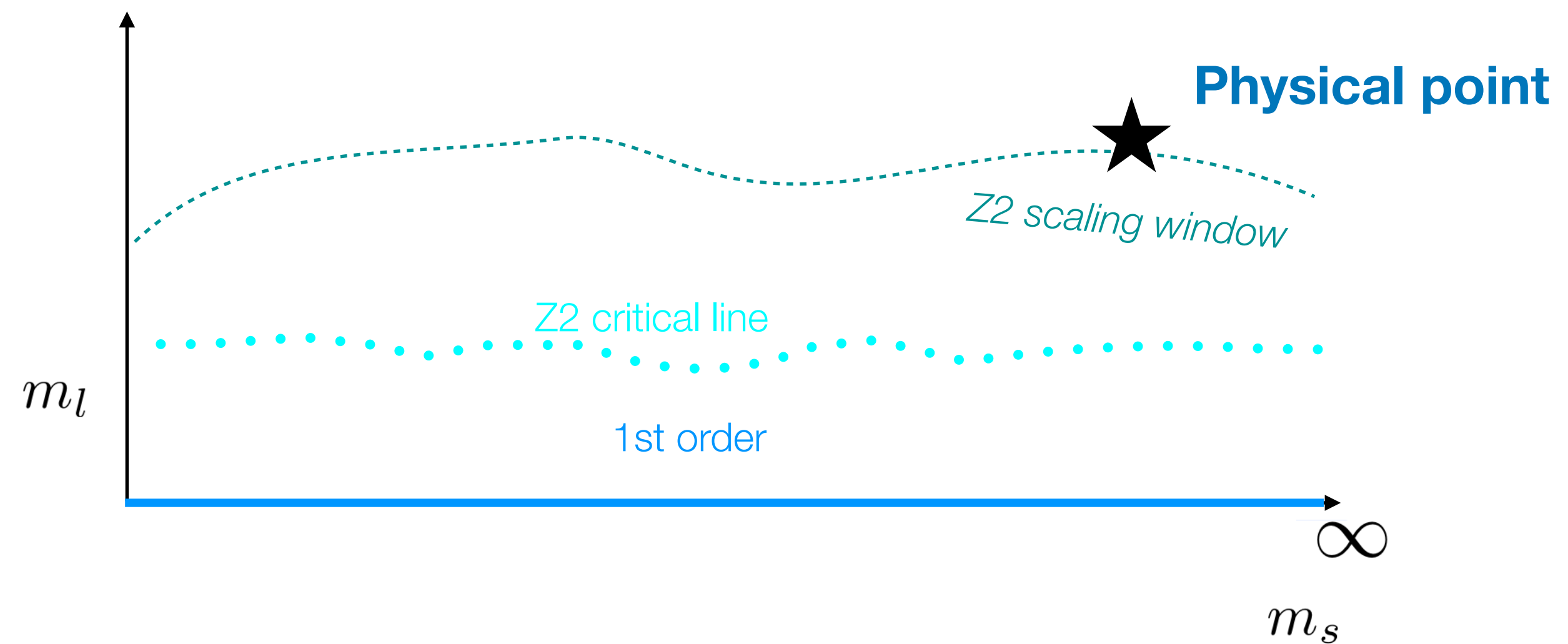
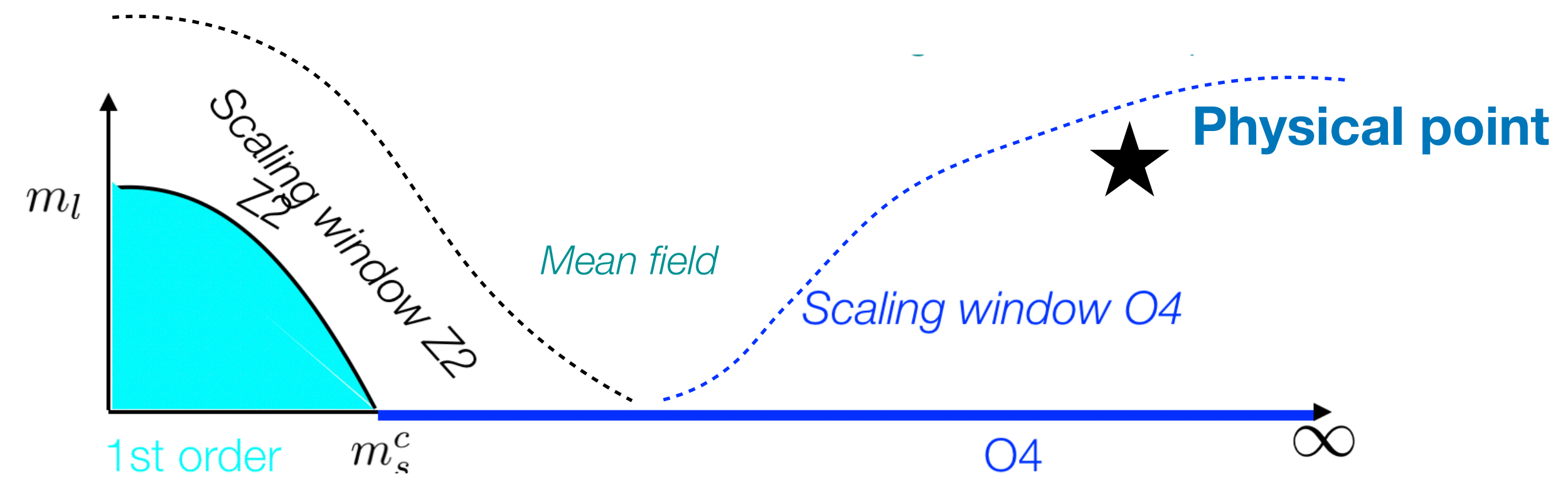


# Between $N_f=2$ and $N_f=3$

## Strange quark mass as an interpolator



# Scaling windows





# Our ensembles

$N_f = 2 + 1 + 1$  Wilson twisted mass fermions

$m_\pi^{\text{phys}} < m_\pi < 380 \text{ MeV}$

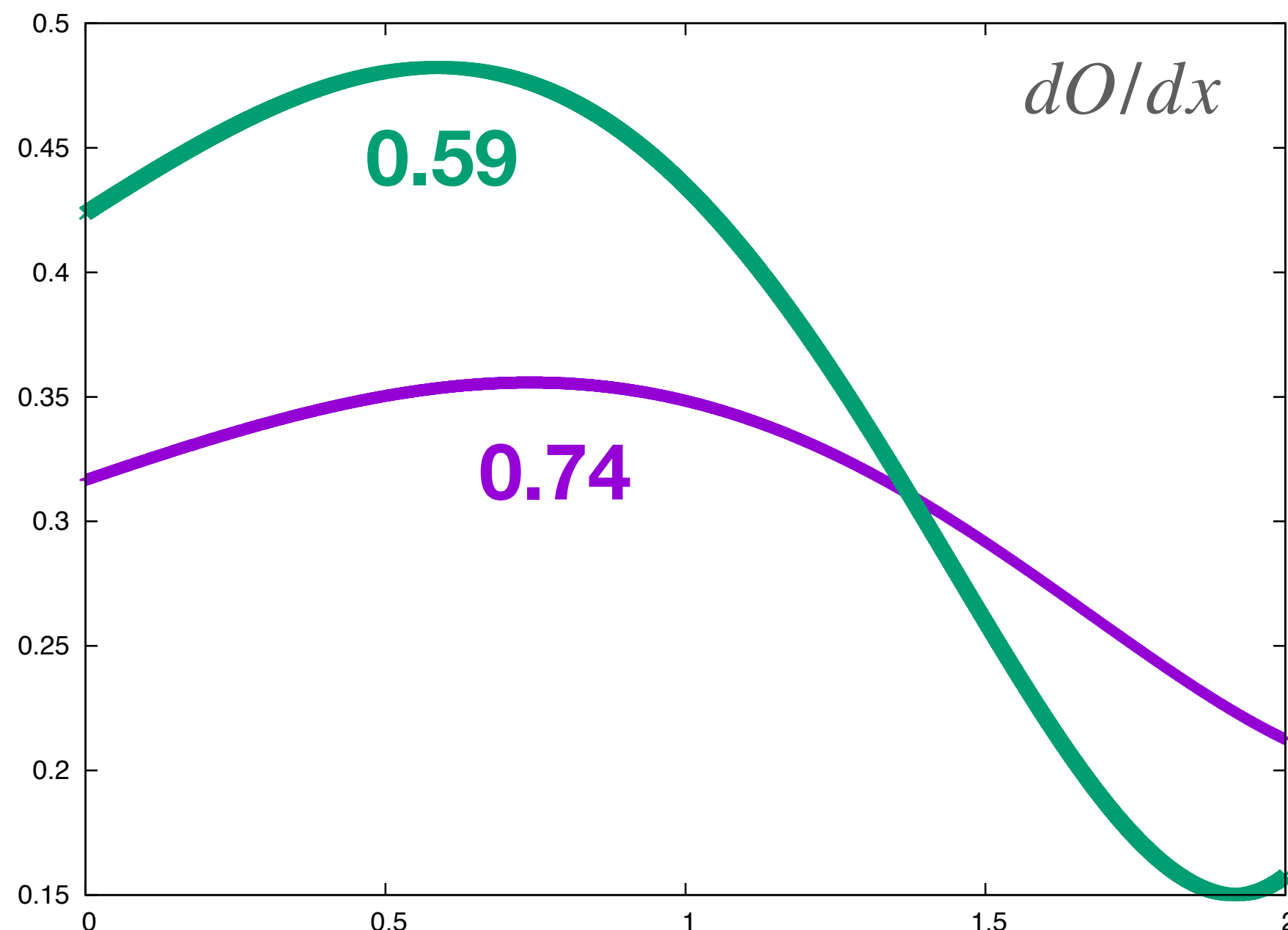
$0.06 \text{ fm} \lesssim a \lesssim 0.08 \text{ fm}$

Fixed scale approach  $a = \text{const}$

$m_\pi$ [MeV]	$a$ [fm]
139.7(3)	0.0801(4)
225(5)	0.0619(18)
383(11)	0.0619(18)
376(14)	0.0815(30)

# New order parameter

- Chiral condensate  $\langle \bar{\psi}\psi \rangle$  has large linear in  $m$  contributions (divergence, regular)
- $\langle \bar{\psi}\psi \rangle_3 = \langle \bar{\psi}\psi \rangle - m\chi$  is free from linear terms in mass [W. Unger, 2010]
- **EoS:**  $\langle \bar{\psi}\psi \rangle = Ah^{1/\delta}f(x) + \dots$ ,  $h \equiv m$ ,  $x = t/h^{1/\beta\delta}$



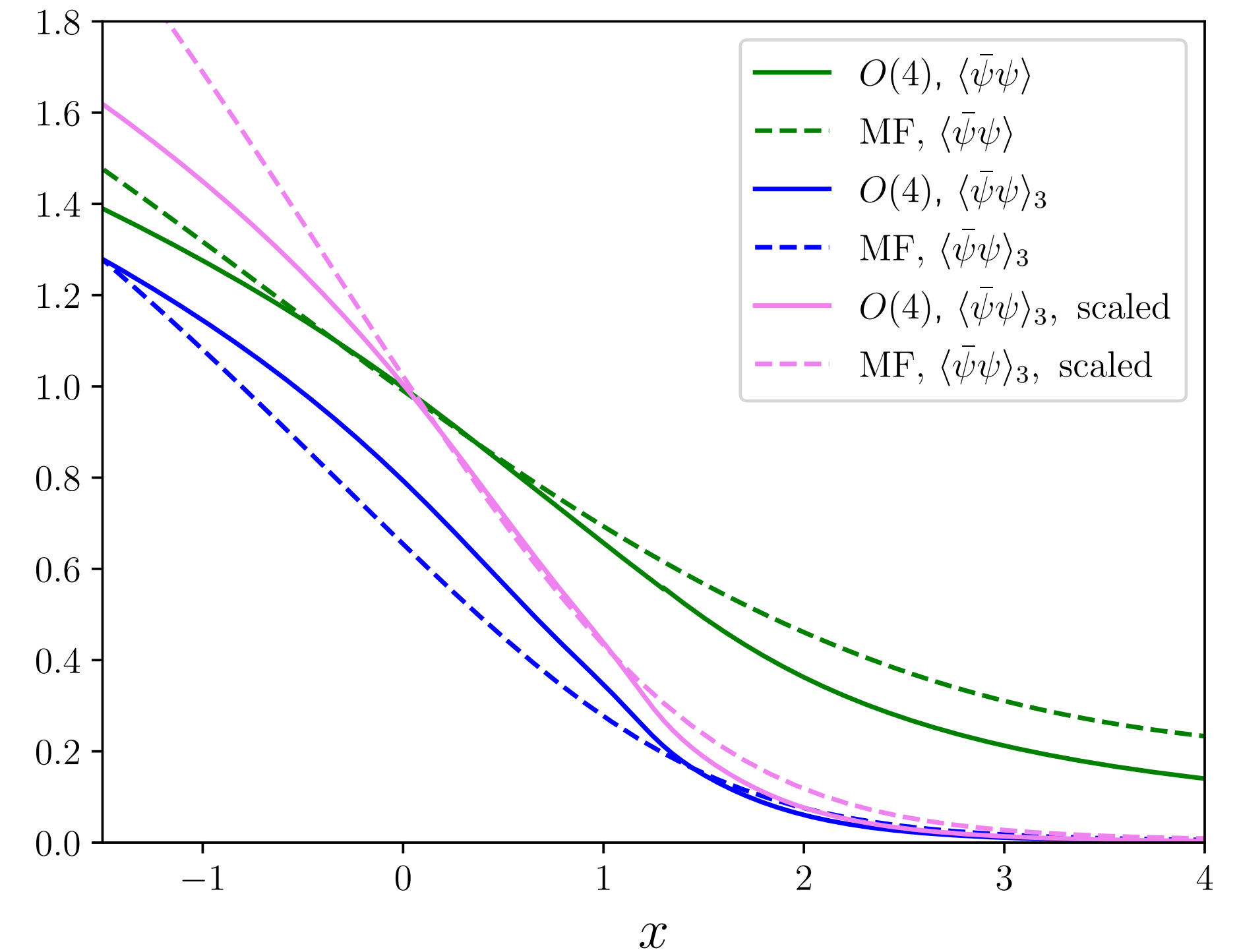
Observable	$\chi$	$\bar{\psi}\psi$	O
$k_s$	1.35(3)	0.74(4)	0.59(1)

$$T_c = T_0 + Ak_s m_\pi^{2/\beta\delta}$$

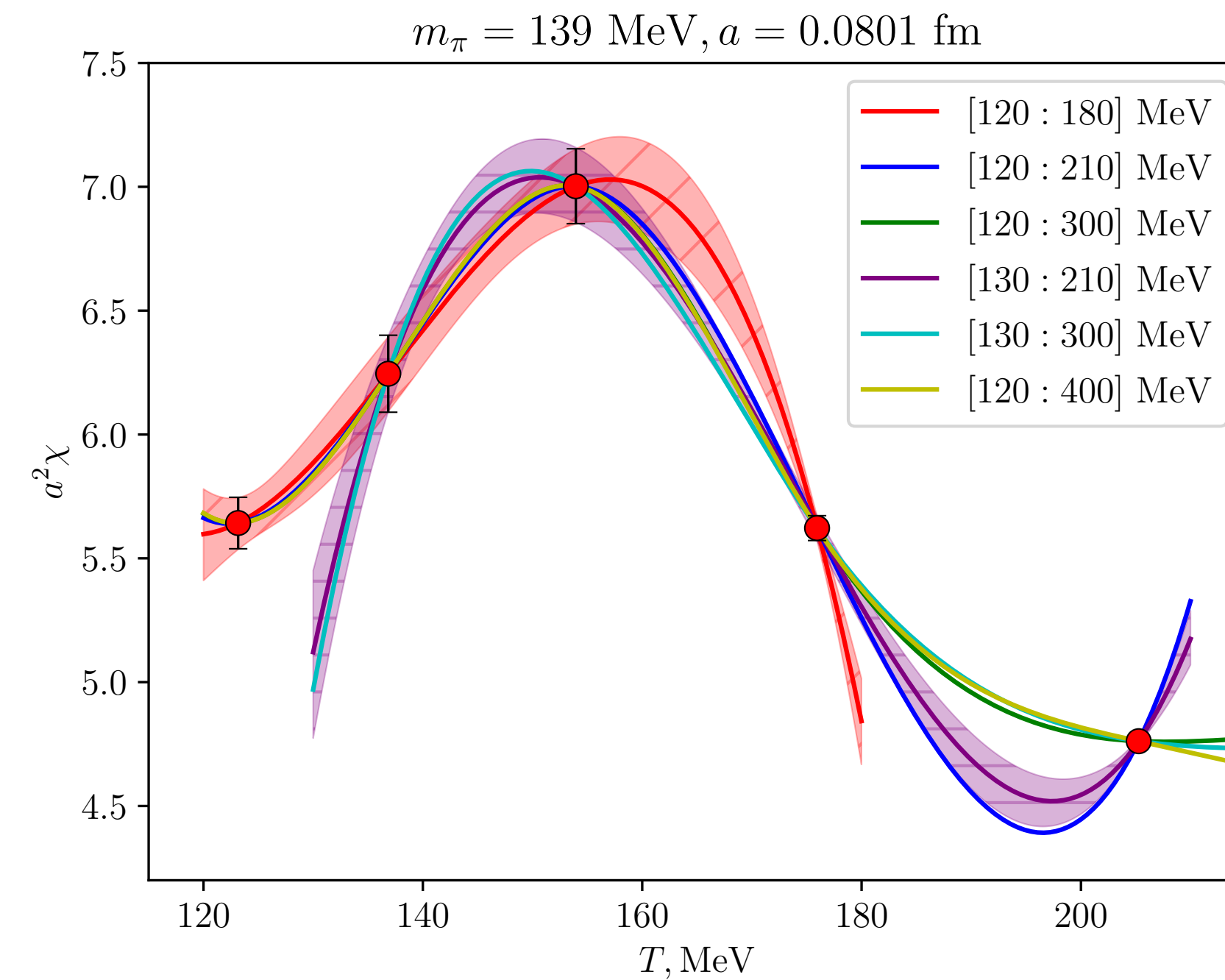
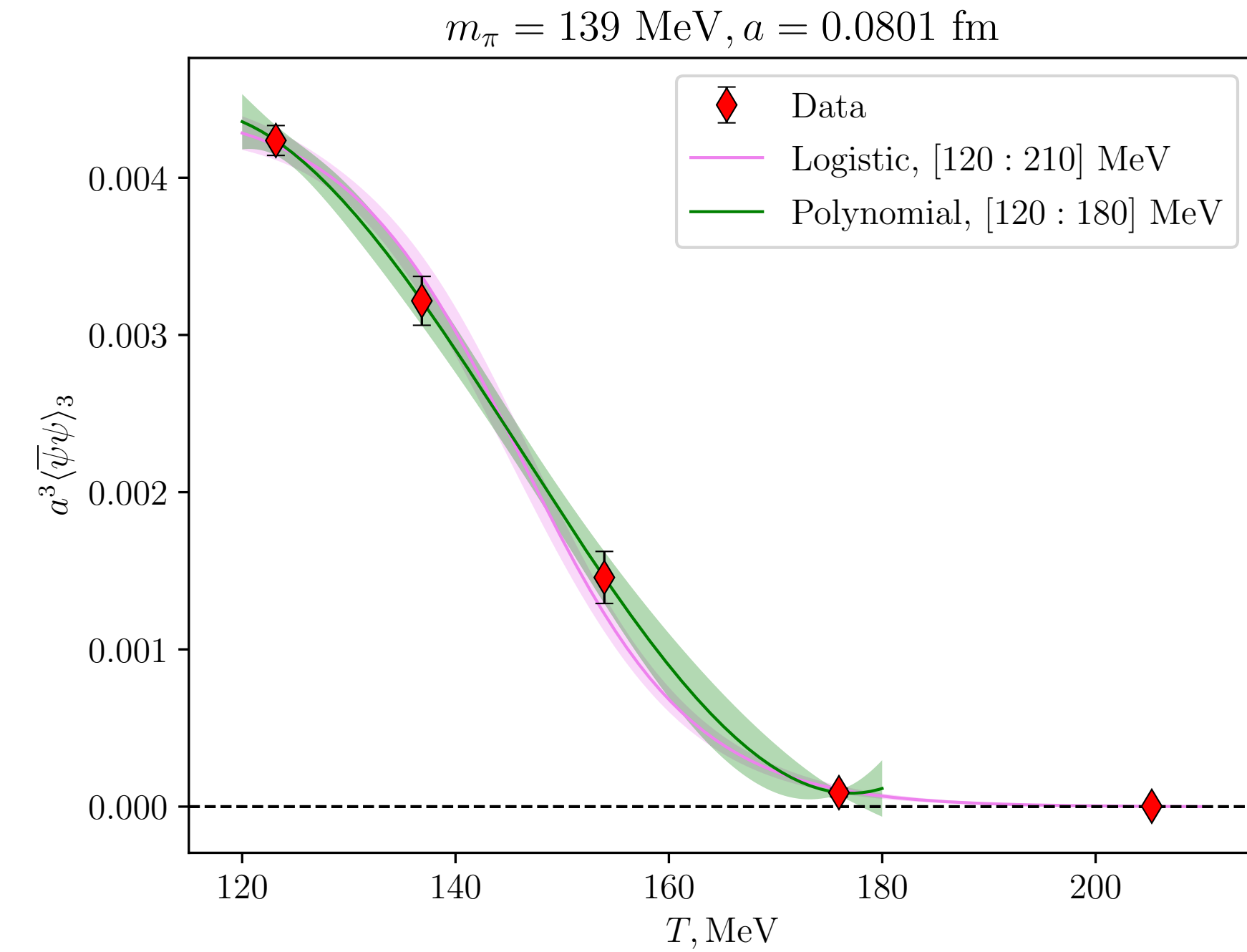
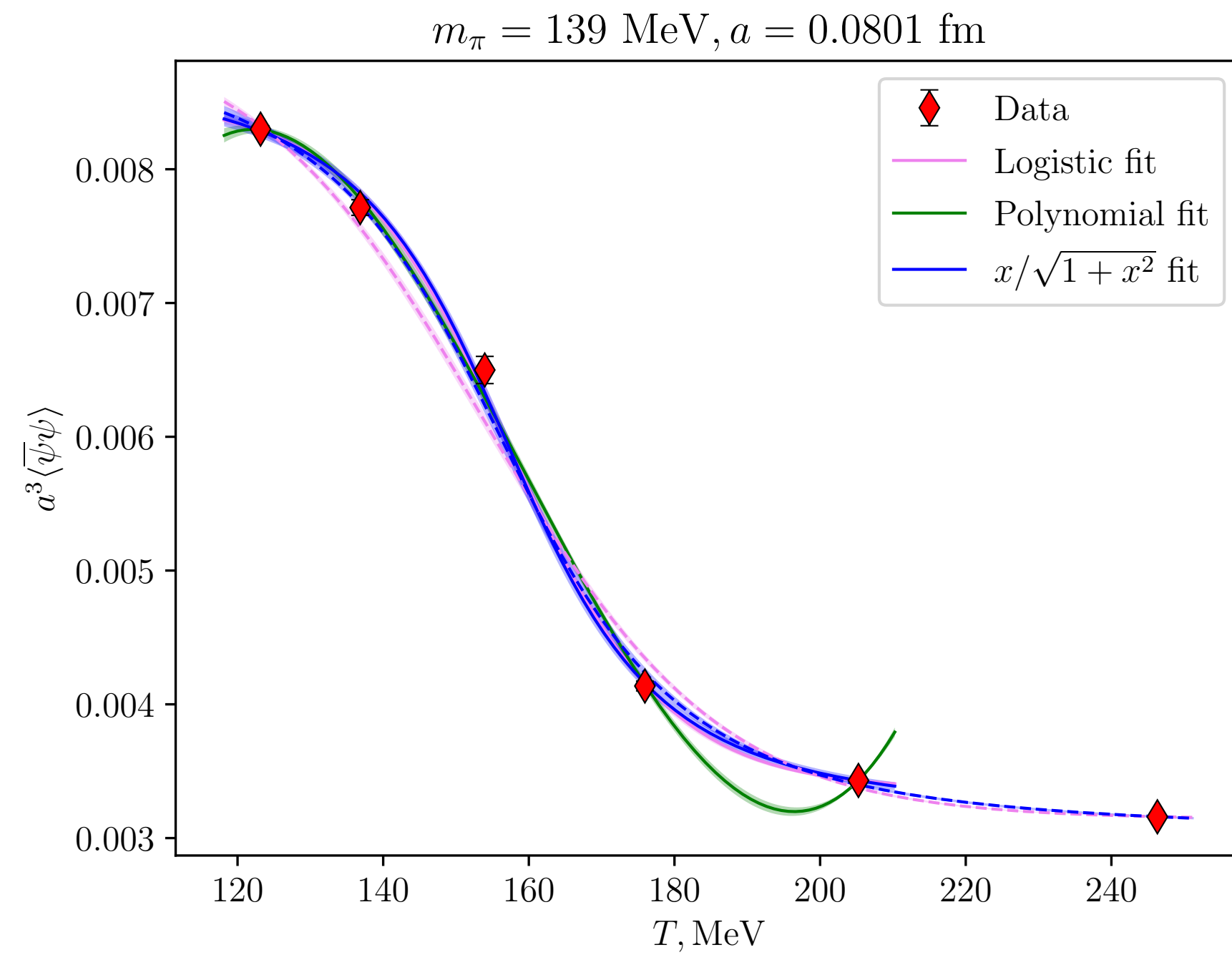
# New order parameter

New order parameter:  $\langle \bar{\psi}\psi \rangle_3 = \langle \bar{\psi}\psi \rangle - m\chi$

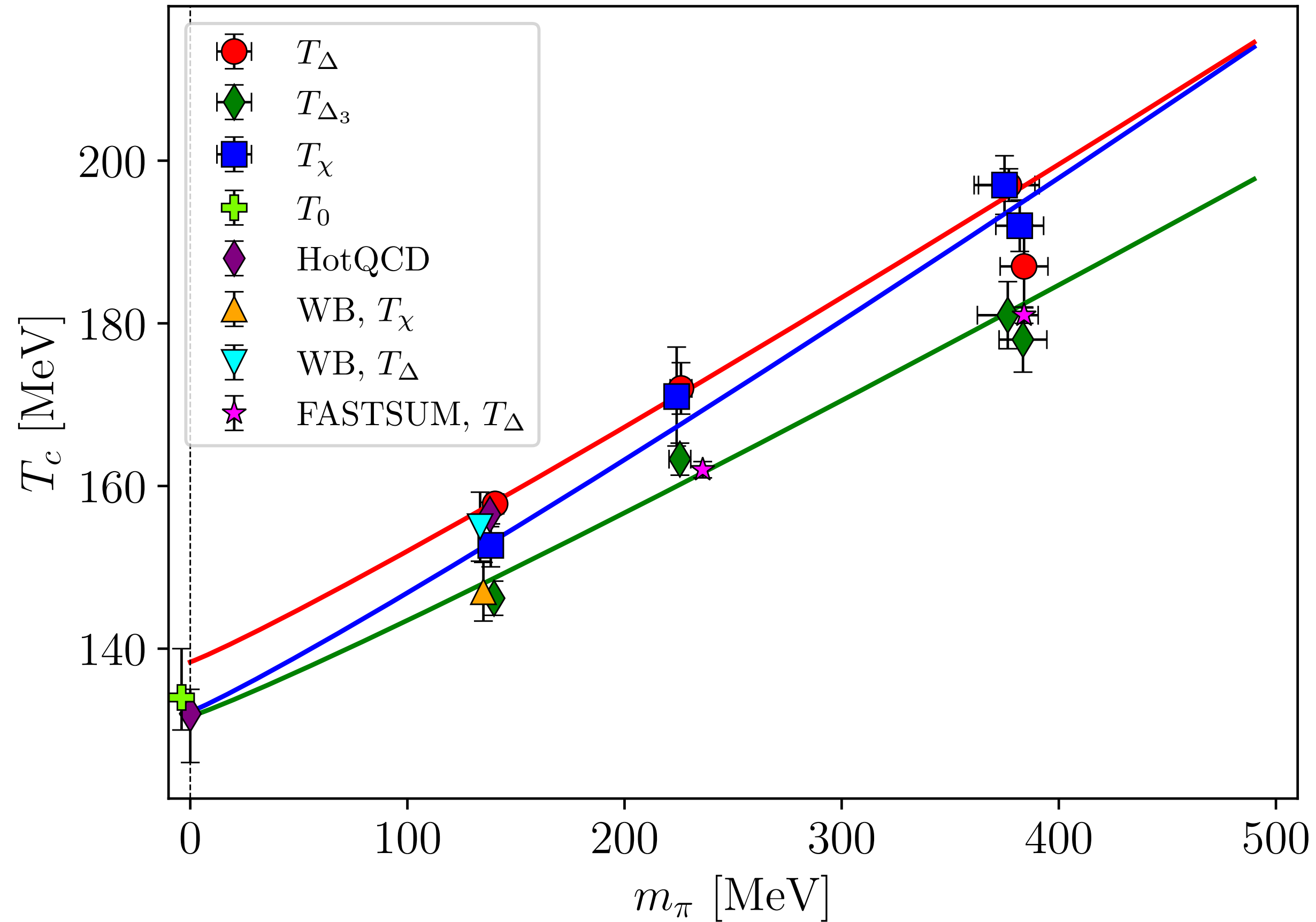
- terms  $\sim m$  cancel: divergences/regular
- $\sim m^3$  (symmetric phase)
- $\langle \bar{\psi}\psi \rangle_3 \sim t^{-\gamma-2\beta\delta}$  vs  $\langle \bar{\psi}\psi \rangle \sim t^{-\gamma}$  as  $t \rightarrow \infty$



# Physical pion mass



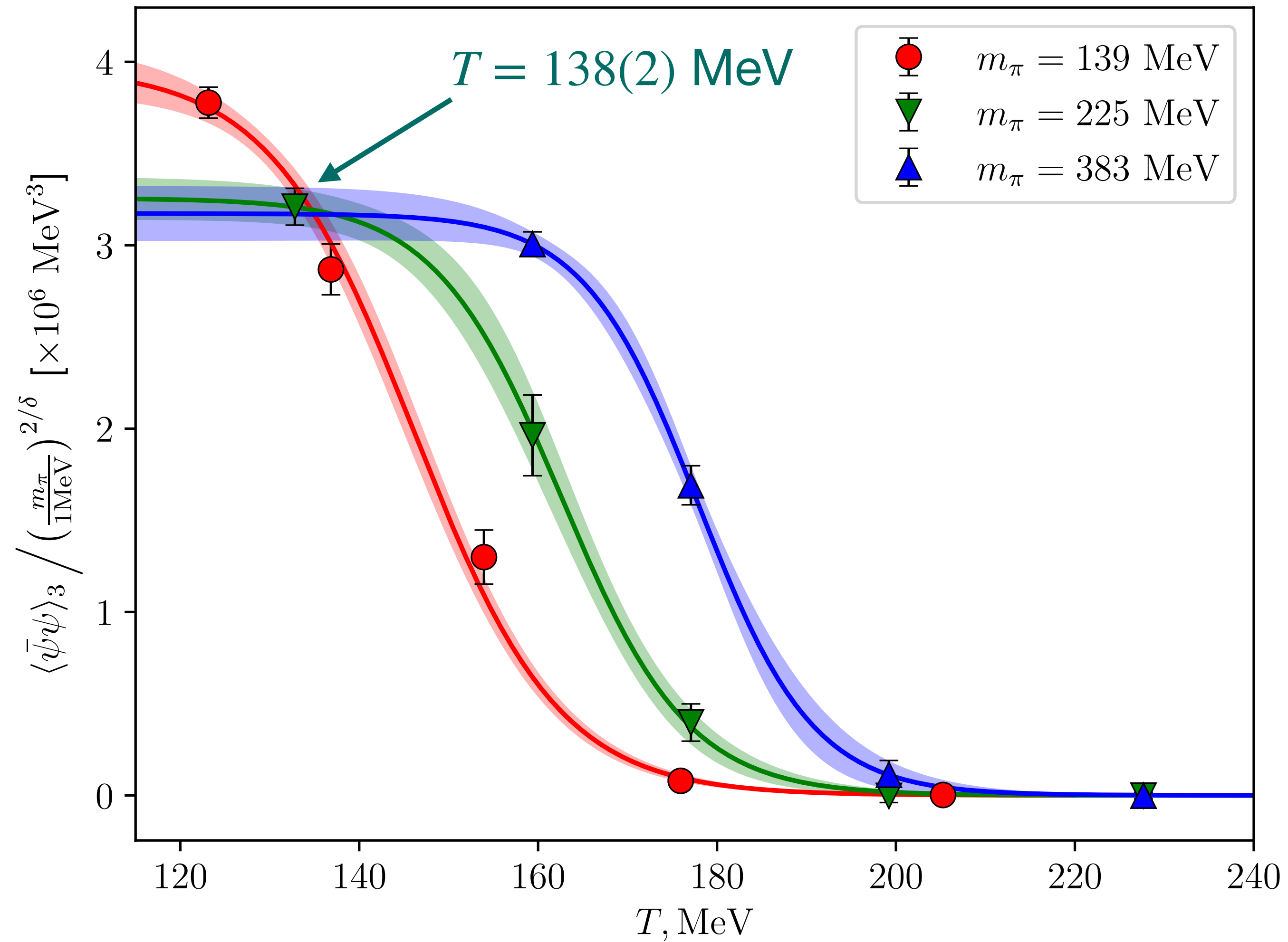
# Critical temperature and chiral extrapolation



	$T(m_{\pi} = 139 \text{ MeV})$ [MeV]	$T(m_{\pi} = 0)$ [MeV]
$\langle \bar{\psi}\psi \rangle$	157.8(12)	138(2)
$\chi$	153(3)	132(4)
$\langle \bar{\psi}\psi \rangle_3$	146(2)	132(3)

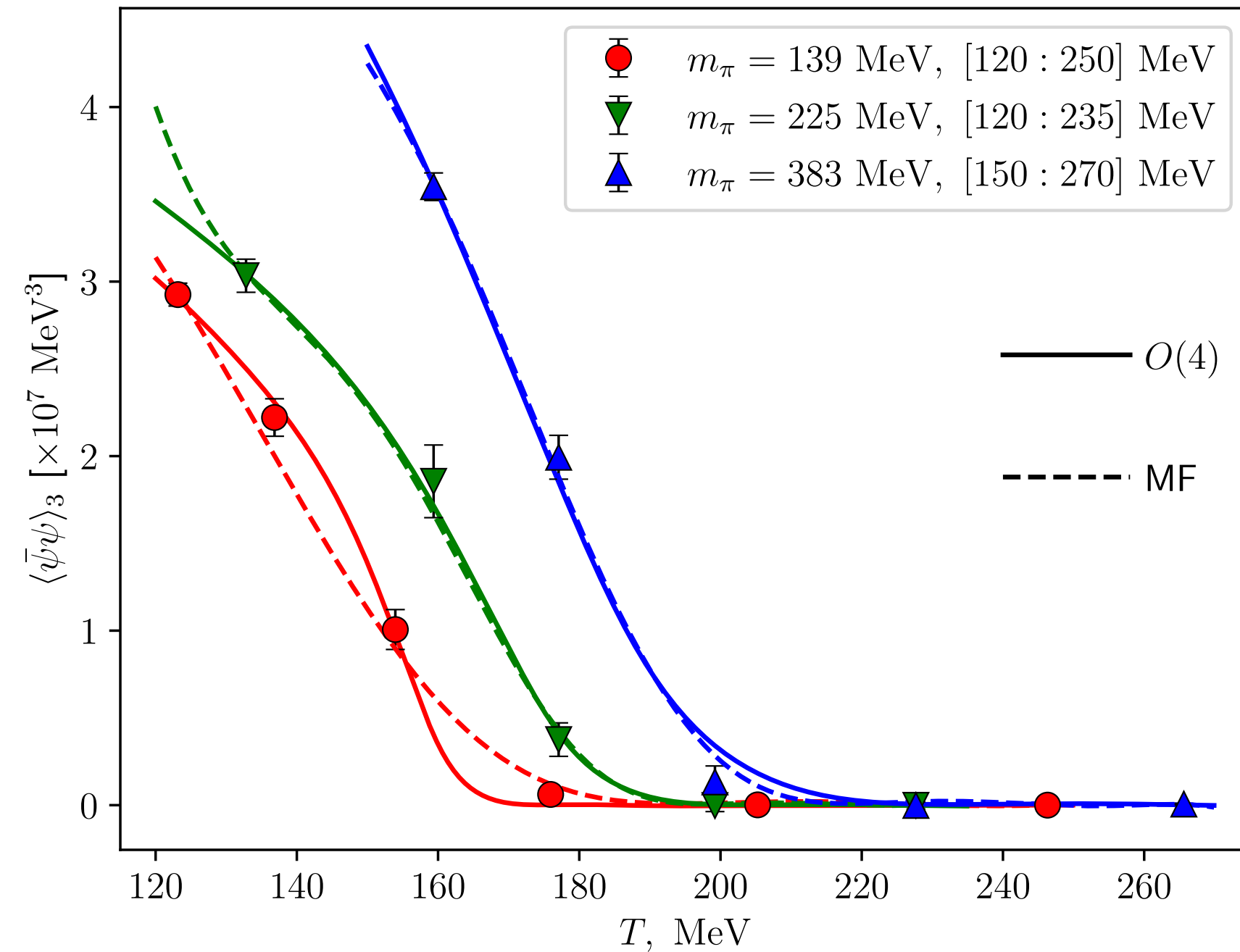
$$T_0 = 134^{+6}_{-4} \text{ MeV}$$

# Scaling of $\langle \bar{\psi}\psi \rangle_3$

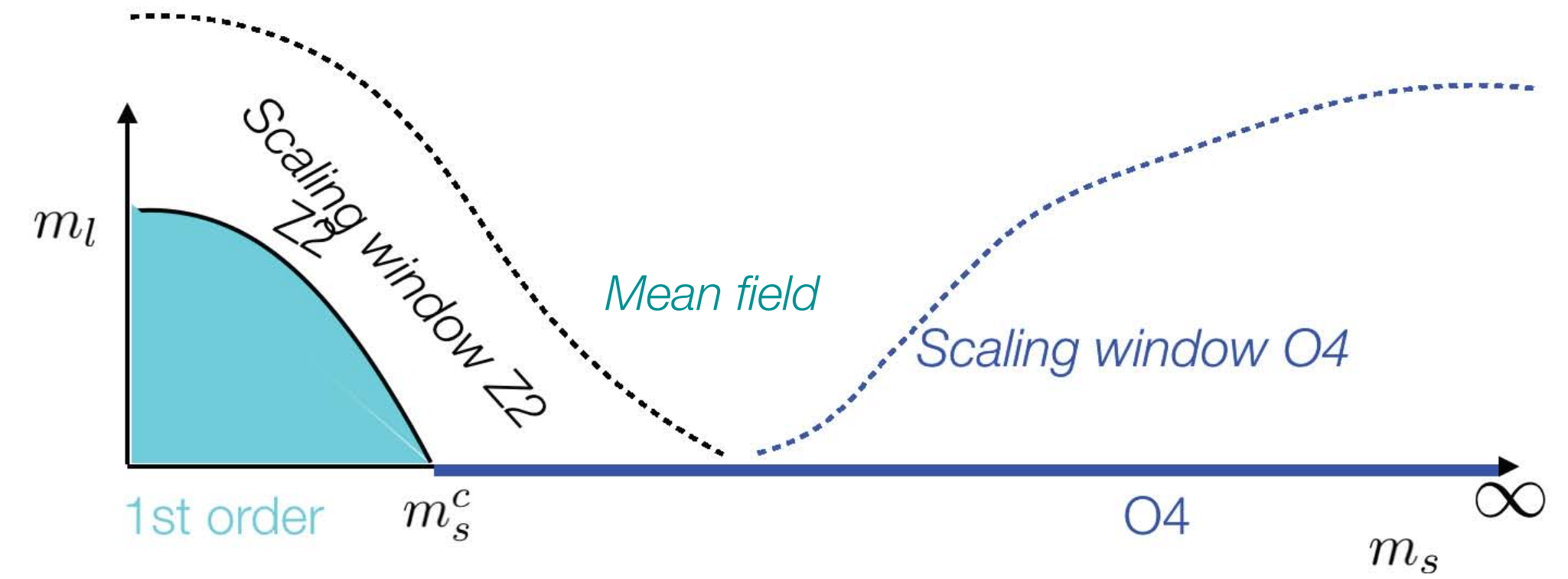


$$\frac{\langle \bar{\psi}\psi \rangle_3}{m^{1/\delta}} \sim \frac{\langle \bar{\psi}\psi \rangle_3}{m_\pi^{2/\delta}} = \text{const at } t=0$$

# O(4) vs mean field



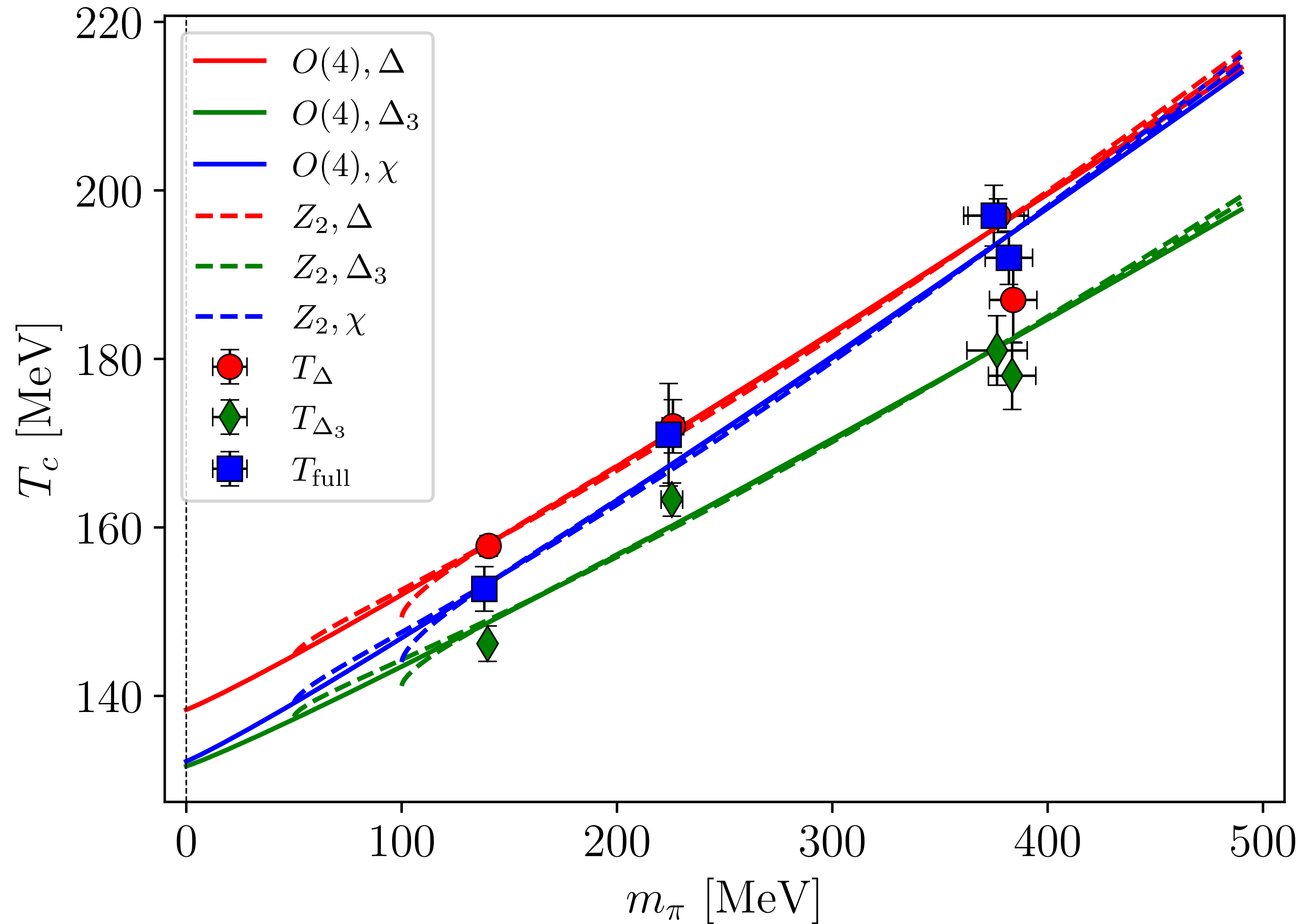
Mild tension between data and MF for  $m_\pi = 139 \text{ MeV}$



$m_\pi$ [MeV]	$T_0$ [MeV]
139	142(2)
225	159(3)
383	174(2)



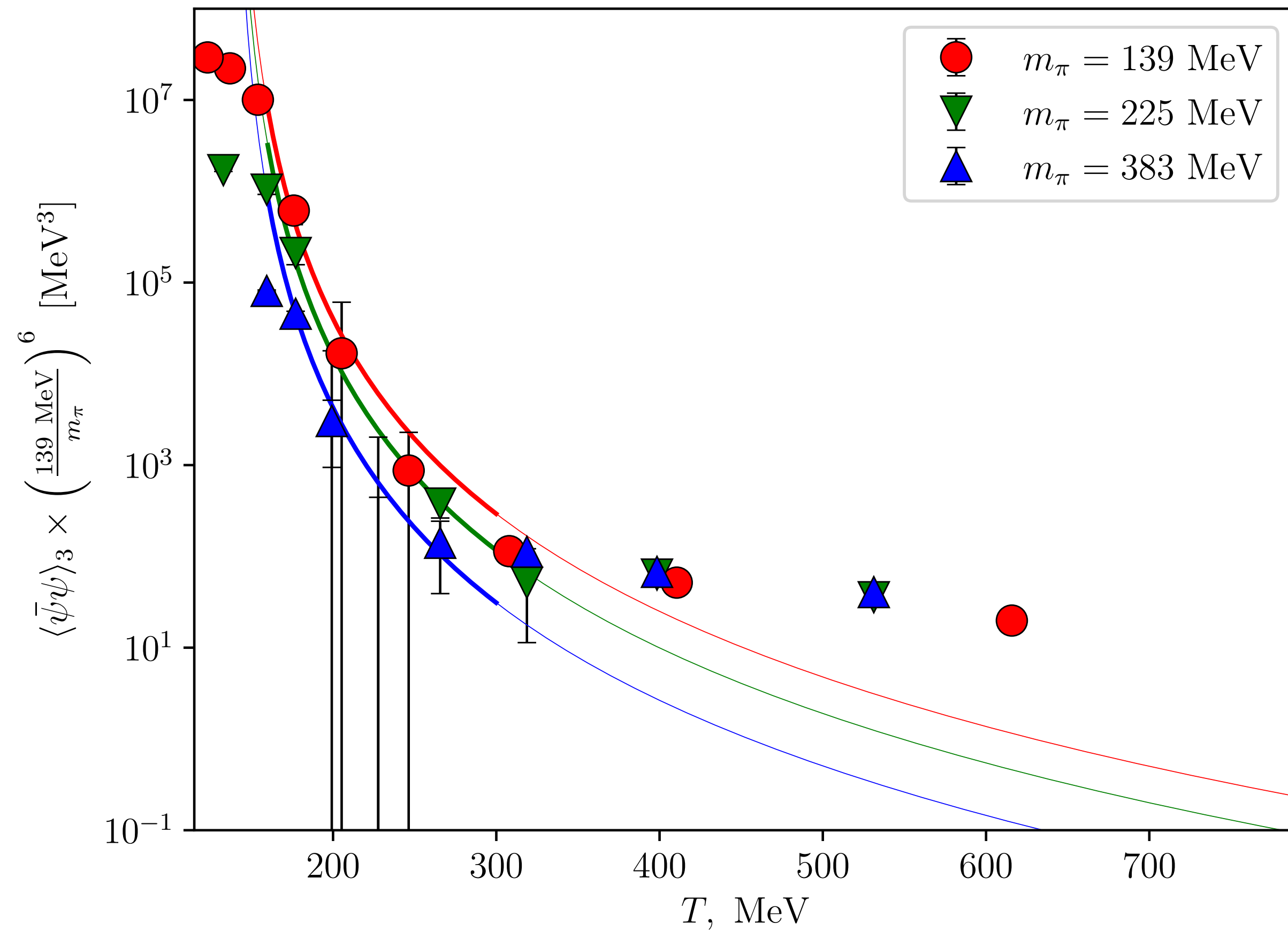
# Z<sub>2</sub> scaling



Can also describe data  
for  $m_\pi^c \in [0, m_{\text{phys}}]$

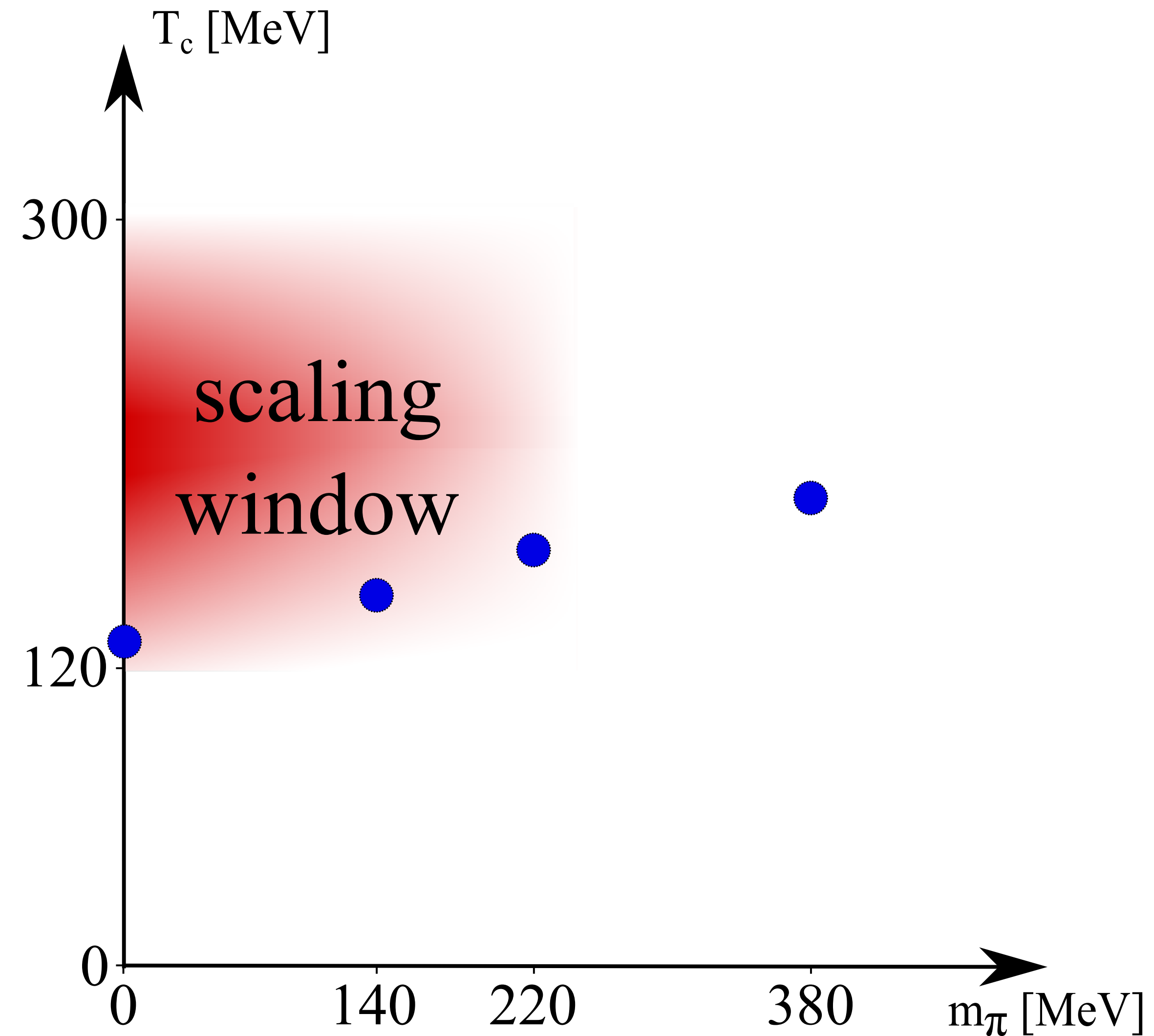


# Large temperature behaviour

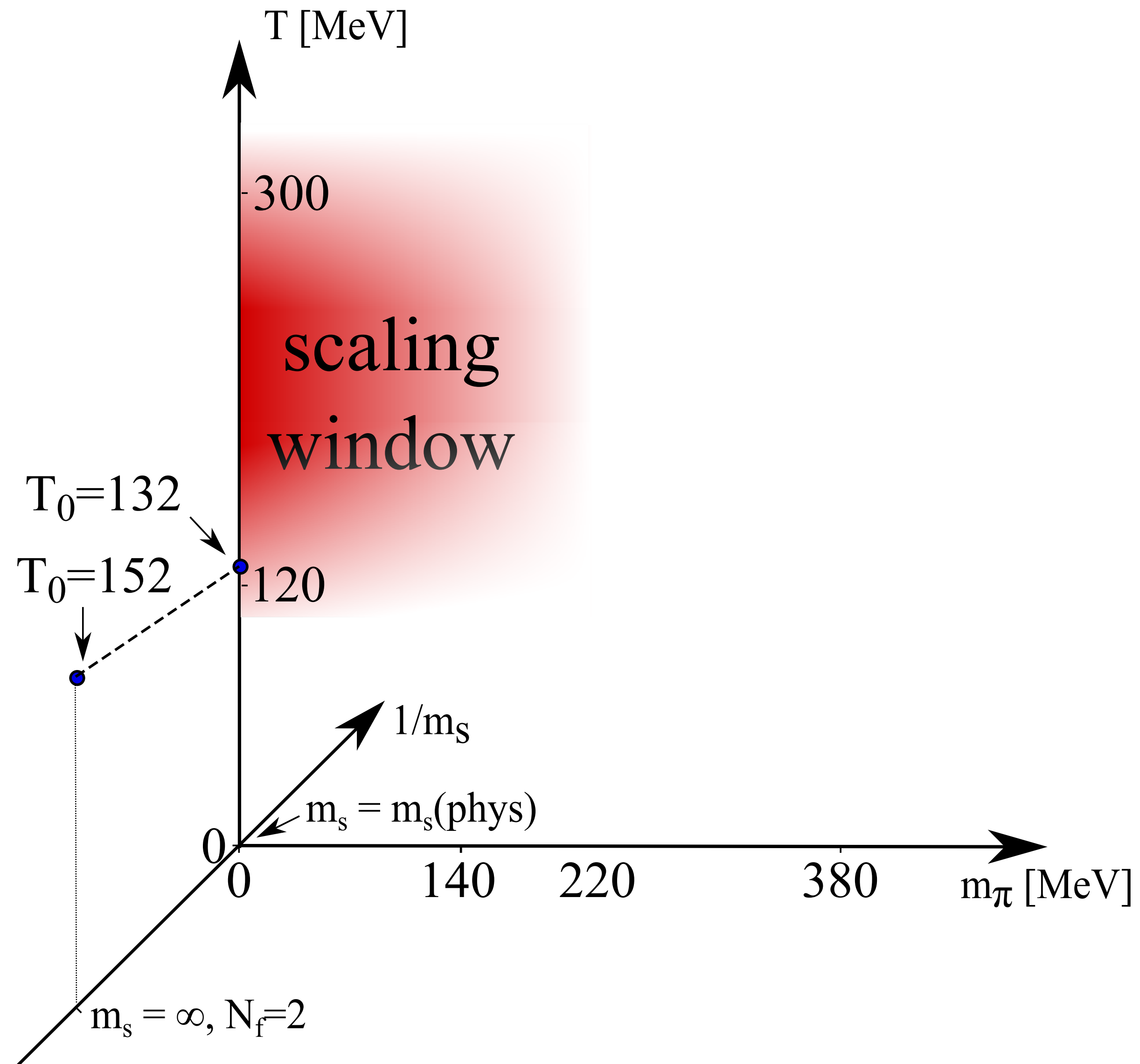


- O(4):  $\langle \bar{\psi}\psi \rangle_3 \sim t^{-\gamma-2\beta\delta}$
- Griffith analyticity:  
 $\langle \bar{\psi}\psi \rangle_3 \sim m^3 \sim m_\pi^6$
- $T \sim 300$  MeV

# Sketch of possible phase diagram, 2D

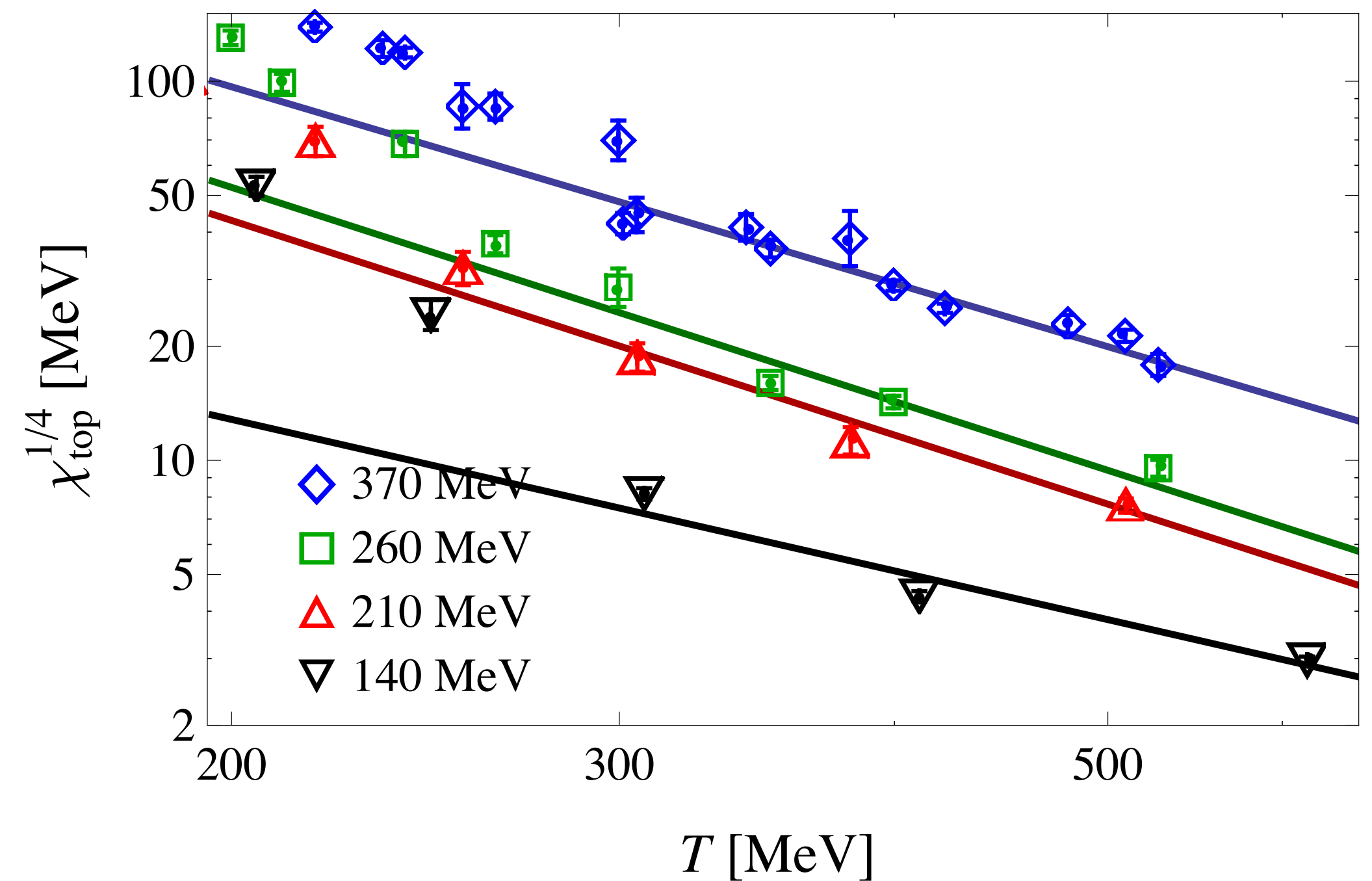


# Sketch of possible phase diagram, 3D



# Beyond the scaling behaviour

- $T \sim 300$  MeV: scaling behaviour  $\Leftrightarrow$  leading order Griffith analyticity
- Coincides with the onset of DIGA behaviour [Burger et al., 2018]
- Thresholds in QGP:  
[Glozman, 2019][Alexandru, Horvath, 2019]  
[Cardinali, D'Elia, Pasqui, 2021]



# Conclusions

Consistency with O(4) scaling for light pions  $m_\pi \lesssim m_\pi^{\text{phys}}$  and temperatures  $T < 300$  MeV

Order parameter  $\langle \bar{\psi}\psi \rangle_3 = \langle \bar{\psi}\psi \rangle - m\chi$

$T = 134_{-4}^{+6}$  MeV in the chiral limit

From

- Scaling of pseudocritical temperatures
- Scaling of  $\langle \bar{\psi}\psi \rangle_3$
- Fit to the Equation of State

$T=300$  MeV is close to the onset of DIGA behaviour, thresholds in QGP

