Jet transport coefficient $\hat{q}$ in lattice QCD

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A. Majumder, Phys.Rev.C 87 (2013) 034905;

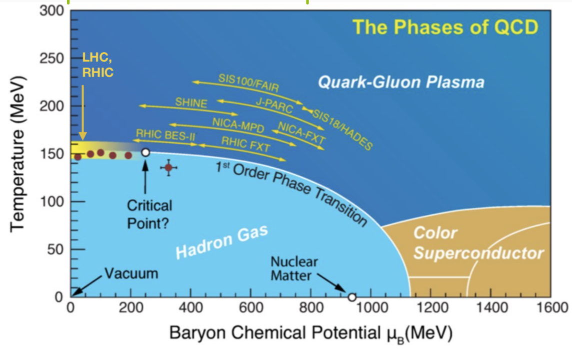
arXiv:2010.14463

Outline

1. Hot matter (QGP)
2. Hard probes (Jets)
3. Jets on QGP (continuum)
    - Weak-coupling picture
    - Higher twist approach
    - OPE
4. Jets on QGP (lattice)
    - OPE and lattice QCD
    - Renormalization in pure gauge theory
    - Beyond pure gauge theory
    - Quantitative comparison and summary

Phase diagram of QCD & heavy-ion collisions

Weak-coupling picture may eventually work at very high T or μ_B



Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)
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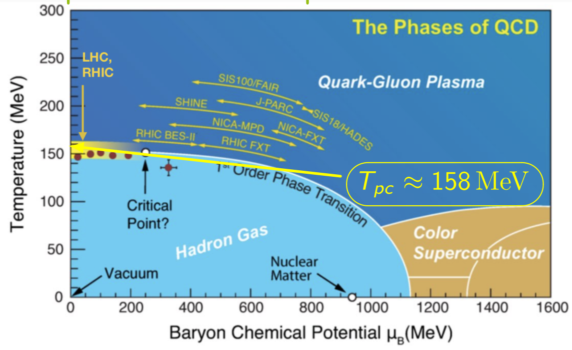

Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)
- Primordial state of nuclear matter before the hadronic freezeout
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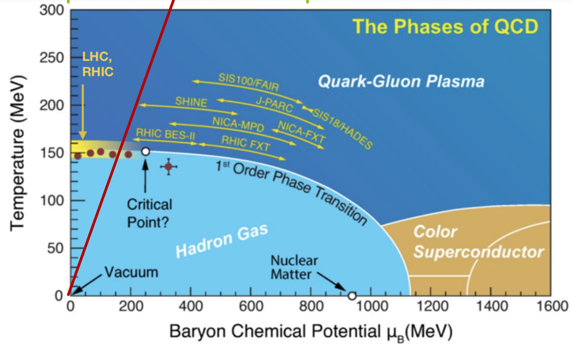
Lattice Gauge Theory

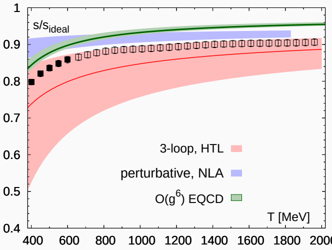


Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)
- Primordial state of nuclear matter before the hadronic freezeout
- QGP can be produced in heavy-ion collision (HIC) experiments
- How to define thermodynamic state variables for such dynamic media?

Interplay between lattice gauge theory and heavy-ion collisions

- Fruitful interplay of lattice gauge theory & heavy-ion collisions
- Search for the critical point and scan of the QCD phase diagram
- or **equation of state**, QNS, ...
- But: many heavy-ion collision experiments shift their focus

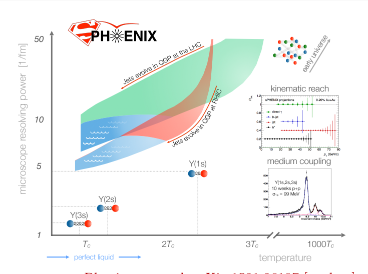


source" → *Weber, et al., PoS Conf. 2018*

Questions @ *ALICE*, *CMS*, *sPhenix*, ...

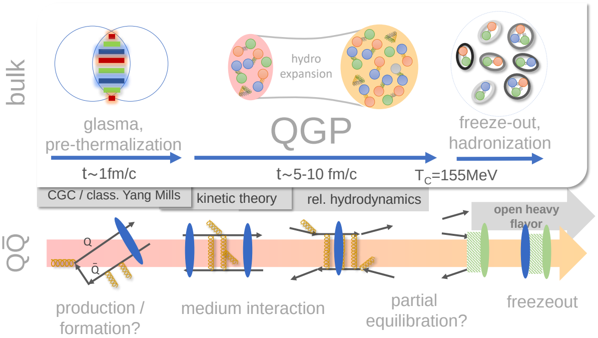
- How do **parton showers** develop and propagate in QGP?
- How to reconcile asymptotic freedom and observed strongly-coupled QGP
- Which **dynamical degrees of freedom** play a role in QGP?

Answers in terms of models or PQCD...



source: *sPhenix proposal, arXiv:1501.06197 [nucl-ex]*

Why focus on hard probes in heavy-ion collisions?



source: *Rothkopf, Phys.Rept. 858 (2020) 1-117*

- Hard probes are produced in a few **hard processes** in initial collision, neither created or destroyed afterwards, but can alter their nature
- Most important probes: **jets**, open heavy flavor & heavy quarkonium

heavy quarkonium: ⇒ check my talk on Wednesday, 11/10/2021, 12:30 [UTC+3]

- **What happens to jets if they impinge on QGP?**

Quark-hadron duality

- **Quark-hadron duality** explains hadrons in terms of their partons
- **Infinitely many partons** partake in the nonperturbative binding
- In particular, microscopic, **nonperturbative calculations** in terms of parton degrees of freedom describe all the known properties of hadrons

  Lattice gauge theory demonstrates this convincingly . . .
- The probability of finding a parton with specific properties in a hadron is given by its nonperturbative **parton distribution function** (PDF)

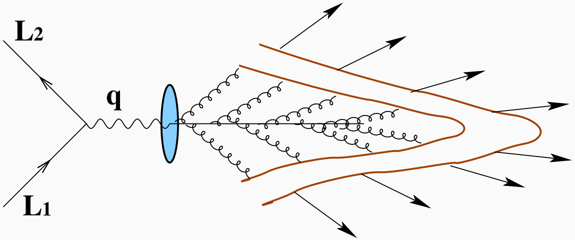
  PDFs are in principle accessible from lattice gauge theory. . .
- **Hard probes** ($Q^2 \gg \Lambda^2_{\mathrm{QCD}}$) resolve individual partons in hadrons

  Weak-coupling methods apply to hard processes . . .
- *Nearly instantaneous* **hard processes** on much shorter timescales ($\tau_Q \sim 1/Q$) than **rearrangements in hadronic states** ($\tau_\Lambda \sim 1/\Lambda_{\mathrm{QCD}}$)

  Ignore the rearrangements, only need to consider the PDF . . .

Jets

- **Hard probes** ($Q^2 \gg \Lambda^2_{\mathrm{QCD}}$) can knock out hard, off-shell partons that carry SU(3)-color charge, and thus must successively radiate hard gluons

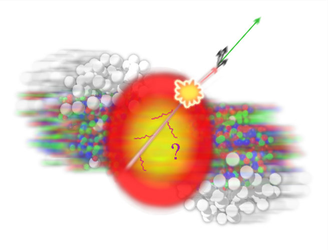


source: Majumder, van Leeuwen (2010)

- Radiation softens along path, eventually hadronizes as **collimated jet**
- The hard processes can be treated through **perturbative expansion**
- **Factorization** of **hard** and **nonperturbative** parts of the reaction is possible due to hierarchically ordered scales – $\mathcal{O}((\Lambda_{\mathrm{QCD}}/Q)^2)$ corrections

Jet modification by scattering on bulk nuclear matter

- Initial hard jet **traveling in $-z$ direction** with light-cone momentum
$$q_i \equiv [q^+, q^-, 0_\perp], \quad q^- = [q^0 - q^3]/\sqrt{2} \sim Q \gg q^+ = [q^0 + q^3]/\sqrt{2} \sim \lambda^2 Q$$
- Jet scatters on **bulk nuclear matter**, absorbing momentum k
$$q_f^2 = (q+k)^2 = 2\lambda^2 (q^-)^2 + 2\lambda^2 q^- k^- + 2k^+ (q^- + k^-) - k_\perp^2$$



- Jet retains off-shellness for $k^\pm \sim \lambda^2 Q$
- $k^+ \sim \lambda Q$ drives jet parton more off-shell, must radiate hard gluons
- $k^- \sim \lambda Q$: energy loss, drives bulk partons off-shell that may radiate
- Many interactions with bulk over length L: **Gaussian** approximation

The first two moments for energy loss and
the second moment for transversal momentum broadening
specify jet modification in Gaussian approximation:

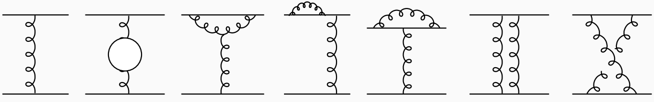
$$\hat{e} \equiv \frac{k^-}{L}, \quad \hat{e}_2 \equiv \frac{(\Delta k^-)^2}{L}; \quad \hat{q} \equiv \frac{|\boldsymbol{k}_\perp|_L^2}{L} = \frac{|k_{x\perp}|_L^2 + |k_{y\perp}|_L^2}{L}$$

Jet transport coefficient $\hat{q}$ and collision kernel in weak coupling (I)

- Jet transport coefficient $\hat{q}$ is truncated integral of collision kernel $C(k_\perp)$

$$\hat{q}(k_\perp^{\max}) = \hat{q}_{\rm soft}(k_\perp^*) + \hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\max}) \equiv \int_0^{k_\perp^*} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2 C_{\rm soft}(k_\perp) + \int_{k_\perp^*}^{k_\perp^{\max}} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2 C_{\rm hard}(k_\perp)$$

- Soft contribution ($k_\perp \ll T$) to collision kernel known up to $\mathcal{O}(g^6)$ in HTL



$$C_{\rm soft}(k_\perp) = g^2 T C_R \left\{ \frac{m_D^2}{k_\perp^2 \left(k_\perp^2 + m_D^2\right)} \right\} + g^4 T^2 C_R N_c \times \left\{ \frac{7}{32 k_\perp^3} + \frac{m_D}{4\pi \left(k_\perp^2 + m_D^2\right)} \left[\frac{3}{k_\perp^2 + 4m_D^2} - \frac{2}{k_\perp^2 + m_D^2} - \frac{1}{k_\perp^2} \right] \right.$$

$$\left. - \frac{k_\perp m_D + 2\left(k_\perp^2 - m_D^2\right) \arctan(\frac{k_\perp}{m_D})}{4\pi k_\perp \left(k_\perp^2 + m_D^2\right)^2} + \frac{m_{\rm E}^2 \arctan(\frac{k_\perp}{2m_D})}{2\pi k_\perp^3 \left(k_\perp^2 + m_D^2\right)} + \frac{2k_\perp m_D - \left(k_\perp^2 + 4m_D^2\right) \arctan(\frac{k_\perp}{2m_D})}{16\pi k_\perp^5} \right\} + \mathcal{O}(g^6)$$

Caron-Huot, PRD 79 (2009)

$$\hat{q}_{\rm soft}(k_\perp^*) = \frac{g^2 T m_D^2 C_R}{2\pi} \ln \frac{k_\perp^*}{m_D} + \frac{g^4 T^2 m_D C_R N_c}{2\pi} \left\{ -\frac{k_\perp^*}{16 m_D} + \frac{3\pi^2 + 10 - 4\ln 2}{16\pi} + \mathcal{O}\left(\frac{m_D}{k_\perp^*}\right) \right\} + \mathcal{O}(g^6)$$

Jet transport coefficient $\hat{q}$ and collision kernel in weak coupling (II)

$$\hat{q}(k_\perp^{\rm max}) = \hat{q}_{\rm soft}(k_\perp^*) + \hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\rm max}) \equiv \int_0^{k_\perp^*} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2\, C_{\rm soft}(k_\perp) + \int_{k_\perp^*}^{k_\perp^{\rm max}} \frac{d^2\boldsymbol{k}_\perp}{(2\pi)^2} \boldsymbol{k}_\perp^2\, C_{\rm hard}(k_\perp)$$

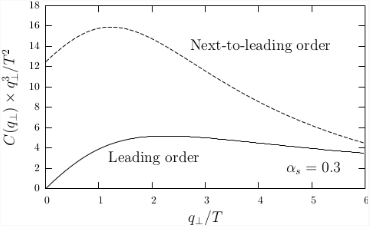
- Hard contribution ($k_\perp \gtrsim T$) to collision kernel known up to $\mathcal{O}(g^6)$ as well

$$\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\rm max}) = g^4 T^3 C_R \Big\{ \frac{N_c}{6\pi} \Big[\log\Big(\frac{T}{k_\perp^*}\Big) + \frac{\zeta(3)}{\zeta(2)} \log\Big(\frac{k_\perp^{\rm max}}{T}\Big) - 0.06885\ldots + \frac{3}{16}\frac{k_\perp^*}{T} + \ldots \Big] + \frac{N_f T_f}{6\pi} \Big[\log\Big(\frac{T}{k_\perp^*}\Big) + \frac{3}{2}\frac{\zeta(3)}{\zeta(2)} \log\Big(\frac{k_\perp^{\rm max}}{T}\Big) - 0.07286\ldots + \ldots \Big] \Big\}$$

Arnold, Xiao, PRD 78 (2008)

- $\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\rm max} \to \infty)$ is finite
- Cancellation of $k_\perp^*$ dependence between $\hat{q}_{\rm soft}(k_\perp^*)$, $\hat{q}_{\rm hard}(k_\perp^*, k_\perp^{\rm max})$
- Soft $\mathcal{O}(g^5)$ exceeds $\mathcal{O}(g^4)$ term $\Rightarrow$ expansion in g converges poorly
- Contributions at $\mathcal{O}(g^6)$ from magnetic scale ($g^2 T$) are small

Laine, EPJC 72 (2012)

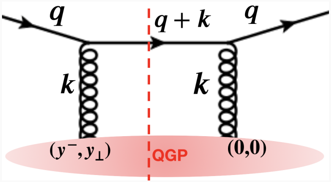


source: *Caron-Huot, PRD 79 (2009)*

LO is accidentally small, NLO is regularly large, and the non-perturbative magnetic contribution is hardly relevant – is this the **end of the story**???

Hard jet scattering a QGP brick

- Uniform **QGP brick** of length L, temperature T, Debye mass m_D
- A **jet** with energy E, virtuality Q traverses this QGP: $E \gg Q \gg T, m_D$
- On-medium scattering is dominated by **one-gluon exchange** (OGE)



source: *arXiv:2010.14463*

- Regularize integrals in finite box $V = L^3$, interaction time $T_I = {}^{L}/_{c}$
- The average momentum broadening among N_e OGE scattering events is

$$\hat{q} = \sum_i^{N_e} \frac{[k_\perp^i]^2}{N_e T_I}$$

- The tree-level matrix element (implies OGE approximation) for the scattering probability $W^{n,X}(k) = {}^{|\mathcal{M}|^2}/_{2N_c}$ is given by

$$\mathcal{M} = \langle q_f| \otimes \langle X| \int_0^{T_I} dt d^3x g\bar{\psi}(x)\gamma^\mu t^a A_\mu^a(x)\psi(x)\, |n\rangle \otimes |q_i\rangle$$

- $|n\rangle$ or $|X\rangle$ represent initial or final states of the nuclear medium
- $|q_i\rangle$ or $|q_f\rangle$ represent initial or final states of the hard parton

Near light-cone separated gauge-invariant definition of $\hat{q}$

$$\hat{q} = c_0 \int \frac{dy^- d^2y_\perp d^2k_\perp}{(2\pi)^3} e^{i\frac{k_\perp^2 y^-}{2q^-} - ik_\perp \cdot y_\perp} \sum_n \frac{e^{-\beta E_n}}{Z}$$

$$\times \langle n| \ \mathrm{tr} \left[g_0^2 \ F^{+j}(y^-, y_\perp) U^\dagger(\infty^-, y_\perp; y^-, y_\perp) T^\dagger(\infty^-, \infty_\perp; \infty^-, y_\perp) \right.$$

$$\left. \times \ T(\infty^-, \infty_\perp; \infty^-, 0_\perp) U(\infty^-, 0_\perp; 0^-, 0_\perp) F_j^+(0) \right] |n\rangle$$

- $c_0 = 16\pi\alpha_s\sqrt{2}/(N_c^2-1)C_R$: quadratic Casimir $C_R = C_F = (N_c^2-1)/2N_c$ (hard quark)
- Bare field strength tensors F^{+j} are at ***near* light-cone separation**
- Specific path of infinite Wilson lines in adjoint representation

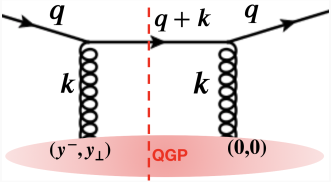
Garcia-Echevarria, et al., PRD84 (2011)

- Infinite transverse Wilson lines $T, T^\dagger$ (transverse to light cone)
- Infinite light-cone Wilson lines $U, U^\dagger$ along $(-)$ light-cone direction
- Covariant gauge to eliminate transverse Wilson lines
- Light-cone gauge to eliminate light-cone Wilson lines

Not straightforward to realize on the lattice ...

Near light-cone separated gauge-covariant definition of $\hat{q}$

$$\hat{q} = c_0 \int \frac{dy^- d^2 y_\perp}{(2\pi)^3} d^2 k_\perp e^{-i\frac{\boldsymbol{k}_\perp^2}{2q^-}y^- + i\boldsymbol{k}_\perp\cdot\boldsymbol{y}_\perp} \sum_n \frac{e^{-\beta E_n}}{Z}$$
$$\times \langle n|\ \mathrm{tr}\ \left[F^{+j}(0)F_j^{+}(y^-, y_\perp)\right] |n\rangle\,, \qquad j = \{1,2\} \text{ transverse directions}$$



source: *arXiv:2010.14463*

Make this amenable to lattice calculation

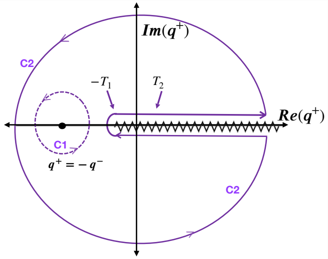
- Fix light-cone gauge: $A^- = 0$
- Omit remaining Wilson lines
- Rephrase as **gauge-invariant OPE**
- Refrain from k- and y- integration
- Study a generalized coefficient $\hat{Q}(q^+)$

$$\hat{Q}(q^+) = c_0 \int \frac{d^4 y d^4 k}{(2\pi)^4} \frac{2q^- e^{ik\cdot y}}{(q+k)^2 + i\epsilon} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n|\ \mathrm{tr}\left[g_0^2 F^{+j}(0) F_j^{+}(y)\right]|n\rangle$$

Generalized jet coefficient $\hat{Q}(q^+)$

$$\hat{Q}(q^+) = c_0 \int \frac{d^4y d^4k}{(2\pi)^4} \frac{2q^- e^{ik\cdot y}}{(q+k)^2 + i\epsilon} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \mathrm{tr}\left[g_0^2 F^{+j}(0) F_j^{+}(y)\right] |n\rangle$$

- $\hat{Q}(q^+)$ has a **branch cut** in the region $q^+ \sim T \ll q^-$, at which the internal quark propagator with $q+k$ **goes on the mass shell**
- Incoming hard quark is light-like $q_i^2 = 2q^+q^- \sim TQ \sim 0$ at branch cut
- **Thermal discontinuity** is related to $\hat{q}$: $Disc\left[\hat{Q}(q^+)\right]\Big|_{q^+\sim T} = 2\pi i\,\hat{q}$



source: dissertation A. Kumar

- **Vacuum discontinuity** of $\hat{Q}(q^+)$ at $q^+ \in (0,\infty)$: real hard gluon radition
- $\hat{Q}(q^+)$ for $q^+ \approx -q^-$ (deep space-like region) has no discontinuities nearby

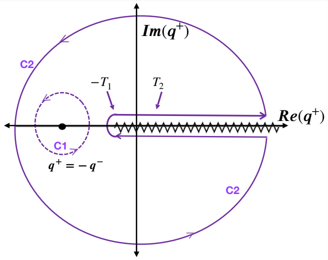
$\Rightarrow$ Turn $1/(q+k)^2$ into **geometric series**

$$\frac{1}{(q+k)^2} \simeq \frac{-\frac{1}{2(q^-)^2}}{1 - \frac{(k^+ - k^-)}{q^-}} = \frac{-1}{2(q^-)^2} \sum_{m=0}^{\infty} \left[\frac{\sqrt{2}k_3}{q^-}\right]^m$$

Integration through contour deformation

- Roll over $k_3^m F_j^+(y)$ as partial derivative, then promote to $(iD_{y_3})^m F_j^+(y)$
- No k-dependence besides $e^{ik\cdot y}$, k integration yields $\delta^4(y)$, then $y \to 0$

$$\hat{Q}(q^+ = -q^-) = \frac{1}{q^-} c_0 \sum_{m=0}^{\infty} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \,\mathrm{tr}\left[g_0^2 F^{+j}(0) \left[\frac{i\sqrt{2}D_3}{q^-} \right]^m F_j^+(0) \right] | n \rangle$$



source: dissertation A. Kumar

$$I = \oint_{C_1} \frac{dq^+}{2\pi i} \frac{\hat{Q}(q^+)}{q^+ + q^-} = \hat{Q}(q^+ = -q^-)$$

$$= \int_{-T_1}^{T_2} \frac{dq^+}{2\pi i} \frac{Disc\left[\hat{Q}(q^+)\right]}{q^+ + q^-}$$

$$+ \int_0^{\infty} \frac{dq^+}{2\pi i} \frac{Disc\left[\hat{Q}(q^+)\right]}{q^+ + q^-}$$

- $-T_1, T_2$ bound the thermal discontinuity of $q^+ = k^+ + k_\perp^2/2(q^- + k^-)$
- T-independent **vacuum discontinuity**: subtract vacuum analog

Gauge-invariant, local OPE for $\hat{q}$

$$\frac{\hat{q}}{T^3} = \sum_{m=0}^{\infty} \left[\frac{T}{q^-}\right]^{2m} c_0 \frac{T}{T_1+T_2} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{+j}(0)\Delta^{2m}F_j^+(0)\right]}{T^4} |n\rangle_{T-V} \quad \text{with} \quad \Delta \equiv \mathrm{i}\sqrt{2}D_3/T$$

- Width of thermal discontinuity $T_1+T_2 \simeq 2\sqrt{2}T$
- Terms odd in $\Delta \propto D_3$ are odd under either **parity or time reflection**

$\Rightarrow$ vanish for a medium at rest that satisfies these invariances

- Manifestly **gauge-invariant** & local; $[T/q^-]^{2m}$ suppresses **higher twist**
- Our result equally applies for **both pure gluon plasma or full QGP**

Evaluate the **gauge-invariant, local operators** on the **lattice**

$$O_m \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{+j}(0)\Delta^{2m}F_j^+(0)\right]}{T^4} |n\rangle_{T-V}$$

What could **go wrong**?

Gauge-invariant, local OPE for $\hat{q}$

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What could **go wrong**? Much more than we initially anticipated...

Gauge-invariant, local, gluonic operators in imaginary time

$$O_m \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{+j}(0)\Delta^{2m}F_j^{+}(0)\right]}{T^4} |n\rangle_{T-V}$$

- To use MCMC in lattice QCD: **Wick rotation** to imaginary time
$$x^0 \to ix^4, \quad A^0 \to iA^4 \Rightarrow F^{0j} \to iF^{4j}, \quad F^{+j} \to iF^{4j} - F^{3j}$$
$$\Rightarrow F^{+j}\Delta^{2m}F^{+j} \to \left[F^{3j}\Delta^{2m}F^{3j} - F^{4j}\Delta^{2m}F^{4j}\right] + i\left[F^{3j}\Delta^{2m}F^{4j} + F^{4j}\Delta^{2m}F^{3j}\right]$$
$\mathrm{tr}\left[F^{3j}\Delta^{2m}F^{4j} + F^{4j}\Delta^{2m}F^{3j}\right]$ not invariant under parity or time reflection, contributes only in a moving frame; vanishes for QGP in its rest frame
- Leading-twist operator O_0 is (up to $T_F = 1/2$) just gluonic contribution to the non-singlet component of **energy-momentum-tensor** (EMT) $T_G^{(9)}$
- In QCD, $T_G^{(9)}$ mixes with contribution from sea quark flavors $T_Q^{(9)}$
- Total EMT is conserved, i.e. $T_G^{(9)}$ in pure gauge, or $T_G^{(9)} + T_Q^{(9)}$ in QCD

$\Rightarrow$ QCD result for O_0 (and thus $\hat{q}$) is necessarily scheme dependent

- Consider pure gauge plasma first (using Wilson plaquette action)
- Later: (2+1)-flavor QCD plasma (using Symanzik + HISQ action)

Renormalization and mixing on quenched lattices

$$O_m \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}(0)\Delta^{2m}F^{3j}(0) - F^{4j}(0)\Delta^{2m}F^{4j}(0)\right]}{T^4} |n\rangle_{T-V}$$

- **Reduced symmetry group** in the lattice formulation: $SO(4) \overset{\text{broken}}{\rightarrow} SW_4$

$\Rightarrow$ Analyze operators in terms of irreps of SW_4 of given mass dimension

Gluonic contributions to EMT	dim.	irrep.	–
$\mathbf{1}\ \delta_{\mu\nu}$	**0**	**singlet**	cancels against $T = 0$
$T_F {}^1\!/\!_4\ \mathrm{tr}\ [F^{\mu\rho}F^{\mu\rho}]\ \delta_{\mu\nu}$	**4**	**singlet**	mixes with dim $= 0$
$T_F \mathrm{tr}\ [F^{\mu\rho}F^{\mu\rho} - F^{\nu\rho}F^{\nu\rho}]\ [1-\delta_{\mu\nu}]$	**4**	**triplet**	$O_0 \equiv T_F\, T_G^{(3)}$
$T_F \mathrm{tr}\ [F^{\mu\rho}F^{\nu\rho} + F^{\nu\rho}F^{\mu\rho}]\ [1-\delta_{\mu\nu}]$	**4**	**sextet**	vanishes at rest

- EMT components on lattice need **renormalization** $T_G^{(3)R} = Z_T^{(3)} T_G^{(3)B}$
- $Z_T^{(3)} \equiv z_T Z_T^{(6)}$: non-perturbative finite momentum **Ward Identities** (WI)
  $z_T, Z_T^{(6)}$ *in* $\overline{\mathrm{MS}}$ *for plaquette action from Giusti, Pepe, PRD 91 (2015); PLB 769 (2017)*
- **Higher-twist**: $O_m,\ m \geq 1$ mix with lower-twist, T-dependent operators
  $\Rightarrow$ **no cancellation** vs $T = 0$! mixing not studied systematically yet

$\Rightarrow$ Consider $q^- \to \infty$ first: higher-twist operators do **NOT** contribute!

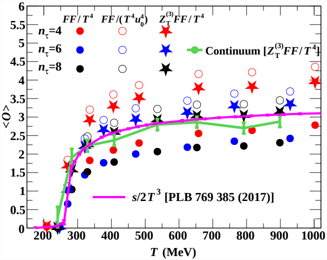
Leading-twist operator for a pure gauge plasma

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} | n \rangle_{T-V}$$

- We use Wilson plaquette action in pure gauge theory
- The renormalized lattice result

$$O_0^R[a(g_0^2), T = 1/N_\tau a(g_0^2)] = Z^{(3)}(g_0^2) O_0^B[a(g_0^2), T = 1/N_\tau a(g_0^2)]$$

  is extrapolated at fixed T to the continuum via $1/N_\tau^2 = (aT)^2 \to 0$
- For $N_\tau > 4$: cutoff effects $\sim 10\%$



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Renormalized triplet comp. in rest frame ⇒ entropy density $T^{(3)R} = Ts = 2O_0^R$

Common approximation in lattice gauge theory: use tadpole factors

$$u_0(g_0^2) = \sqrt[4]{\langle \mathrm{tr}\ [U_{\mu,\nu}] \rangle / N_c}$$

in place of renormalization factor: $1/u_0^4(g_0^2) \approx Z_T^{(3)}(g_0^2)$: overestimation $\sim 10\%$

QCD: explicit sea quarks as uninvited guests

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}$$

- Symmetry breaking $SO(4) \overset{\text{broken}}{\rightarrow} SW_4$ in QCD as in pure gauge theory

$\Rightarrow$ Quark contributions in same irreps of SW_4: $T_Q^{(1)} \rightarrow T_Q^{(1)}$, $T_Q^{(9)} \rightarrow \{T_Q^{(3)}, T_Q^{(6)}\}$

Fermionic contribution to EMT	dim.	irrep.	–
$\mathbf{1}\ \delta_{\mu\nu}$	**0**	**singlet**	cancels against $T = 0$
$m\ \bar\psi\psi\ \delta_{\mu\nu}$	4	**singlet**	mixes with dim $= 0$
$\bar\psi\left[\gamma_\mu D_\mu - \gamma_\nu D_\nu\right]\psi\ [1-\delta_{\mu\nu}]$	**4**	**triplet**	mixes with $T_G^{(3)}$
$\bar\psi\left[\gamma_\mu D_\nu + \gamma_\nu D_\mu\right]\psi\ [1-\delta_{\mu\nu}]$	4	**sextet**	vanishes at rest

- Renormalization of EMT in QCD requires complete mixing matrix

$$\begin{pmatrix} T_G^{(3)R} \\ T_Q^{(3)R} \end{pmatrix} = \mathcal{Z} \begin{pmatrix} T_G^{(3)B} \\ T_Q^{(3)B} \end{pmatrix}, \quad \mathcal{Z} \equiv \begin{pmatrix} \mathcal{Z}_{GG}^{(3)} & \mathcal{Z}_{GQ}^{(3)} \\ \mathcal{Z}_{QG}^{(3)} & \mathcal{Z}_{QQ}^{(3)} \end{pmatrix}$$

1. Still missing the bare quark contribution $\Rightarrow$ straightforward to compute
2. All four renormalization factors unknown for choice of action $\Rightarrow$ could obtain 2 out of 4 via finite momentum WI via QCD in a moving frame

Dalla Brida, et al., JHEP 04 (2020)

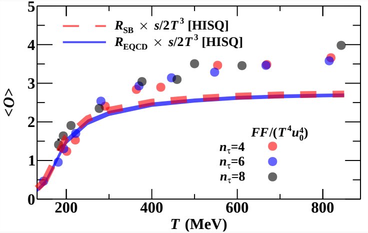
QCD: estimating the influence of the sea quarks

$$O_0 \equiv \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} | n \rangle_{T-V}$$

- We use Symanzik + HISQ action in (2+1)-flavor QCD
- Mixing between $T_G^{(i)}$ and $T_Q^{(i)}$: N_f-dependent coefficients smaller than N_f independent ones $Z_T^{(i)}$ at 1-loop level (plaquette+Wilson)

  Dalla Brida, et al., JHEP 04 (2020)

$\Rightarrow$ mixing $\sim 10\%$ correction to $T_G^{(i)R}$



source: *arXiv:2010.14463*

Compare O_0^B/u_0^4 to entropy density $T^{(3)R} = Ts$ rescaled by weak-coupling ratio

$$R\left(\frac{T}{T_c}\right) = \frac{[s^{N_f=0}/T^3]\,(T/T_c)}{[s^{N_f=3}/T^3]\,(T/T_c)} \quad \text{using} \quad T_c \approx \begin{cases} 270\ \text{MeV} & N_f = 0 \\ 155\ \text{MeV} & N_f = 3 \end{cases}$$

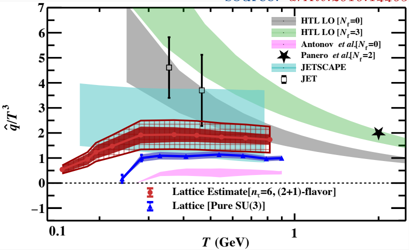
N_f-dependent "critical" temperatures and $T_F = 1/2$; deviation $\lesssim 30\%$

We employ $N_\tau = 6$ assuming $\mathcal{Z}_{GG}^{(3)} \approx 1/u_0^4$, $\mathcal{Z}_{GQ}^{(3)} \approx 0$, 30% systematic error

Quantitative comparison: HTL, lattice, models, phenomenology

$$\frac{\hat{q}_\infty}{T^3} \equiv \frac{\hat{q}}{T^3}\left(q^- \to \infty\right) = \underbrace{c_0 \frac{T}{T_1+T_2}}_{=\pi\ C_R\ g_R^2} \underbrace{\sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \frac{\mathrm{tr}\left[F^{3j}F^{3j} - F^{4j}F^{4j}\right]}{T^4} |n\rangle_{T-V}}_{=O_0}$$

source: *arXiv:2010.14463*



- 1-loop, $N_f = \{0, 3\}$, $\overline{\mathrm{MS}}$ running coupling $g_R^2(\mu)$, scale $\mu = (2\ldots4)\pi T$
- Volume $V \cdot T^3 = N_\sigma/N_\tau = 4$ for $T > 0$
- $T = 0$ ensembles with $N_\tau = N_\sigma$
- **Pure gauge**: $0.2\,\mathrm{GeV} \lesssim T \lesssim 1\,\mathrm{GeV}$
- **QCD** $N_\tau = 6$: $0.15\,\mathrm{GeV} \lesssim T \lesssim 0.8\,\mathrm{GeV}$
- $\hat{q}_\infty/T^3$ is nearly **flat** at $T > 0.3\,\mathrm{GeV}$
- **Phenomenological** results by **JET** & **JETSCAPE** collaborations

  Burke, et al. (JET), PRC 90 (2014); Soltz (JETSCAPE), PoS HardProbes 2018
- **Stochastic vacuum model** $N_f = 0$ (lattice input): Landau damping

  Antonov, Pirner, EPJC55 (2008)
- **Soft contribution in** $N_f = 2$ **EQCD**: $\hat{q}_{\mathrm{soft}} \approx \hat{q}_{\mathrm{NLO}}\big|_{m_D^{\mathrm{LO}} \to m_D^{\mathrm{NP}}}$

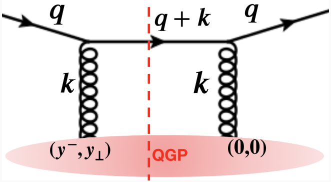
  Panero, et al., PRL 112 (2014)
- **LO HTL** with $q^- = 100\,\mathrm{GeV}$ for $N_f = 0$ or $N_f = 3$

Thank you for your attention!

Explict derivation of $\hat{q}$

- **Ergodicity** implies that we can replace an average over N_e events by an average over **Boltzmann-weighted initial medium states**

$$\hat{q} = \sum_{n,X} \frac{e^{-\beta E_n}}{Z T_l} \int d^2 k_\perp k_\perp^2 \frac{d^2 W^{n,X}(k_\perp)}{d^2 k_\perp}$$



source: *arXiv:2010.14463*

- SCET power counting of A_μ fields:
  $$A_\mu^+ \sim \lambda^2 Q, \quad A_\mu^\perp \sim \lambda^3 Q$$
- Roll over $k_\perp^2$ to derivatives $\partial_\perp$ on A_μ
- Prop up to field strength tensors F_j^+
  from here: transverse comp. $j \in \{1, 2\}$

- Use finite volume wave-functions for the spinors
- Shift one vertex to origin, eliminate $\int d^4x$ against prefactor $1/VT_l$
- Eliminate final medium state $|X\rangle$ via completeness $1 = \sum_X |X\rangle \langle X|$
- k^+-integral is eliminated by on-shell delta function $\delta[k^+ - k_\perp^2/(2q^-)]$
- Integrate over $k^- \ll q^- \Rightarrow$ delta function $2\pi\delta[y^+]$, integrate $y^+ \to 0$

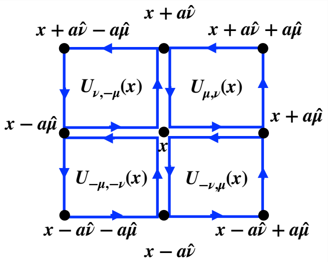
Sufficiently-improved field-strength operator on the lattice

- Plaquette operator is most simple, but has real and imaginary parts

$$U_{\mu,\nu}(x) = \exp\left[a^2 i g_0 F_{\mu\nu}(x)\right] + \mathcal{O}(a^3)$$

- Clover operator is more symmetric, suppresses lattice artifacts

$$Q_{\mu\nu} = \frac{U_{\mu,\nu} + U_{\nu,-\mu} + U_{-\mu,-\nu} + U_{-\nu,\mu}}{4} = \exp\left[a^2 i g_0 F_{\mu\nu}\right] + \mathcal{O}(a^4)$$



source: dissertation A. Kumar

- Traceless-antihermitean **projection**

$$[Q]_{\mathrm{TA}} = \frac{Q - Q^\dagger - \frac{\mathrm{tr}\,[Q - Q^\dagger]}{N_c}}{2}$$

- Weak-coupling picture dysfunctional in practice – $[Q_{\mu,\nu}]_{\mathrm{TA}} \not\approx a^2 i g_0 F_{\mu\nu}$
- **Tadpole improvement** with factor $u_0 = \sqrt[4]{\langle \mathrm{Tr}[U_{\mu,\nu}]\rangle / N_c} \qquad \left(\lim_{a, g_0 \to 0} u_0 = 1\right)$

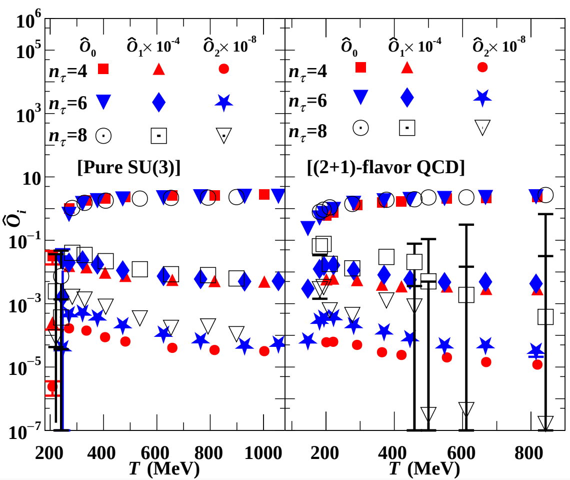
- Traceless-antihermitian projected, tadpole-improved clover operator

$$i g_0 \mathcal{F}_{\mu\nu}(x) = \frac{[Q_{\mu\nu}(x)]_{\mathrm{TA}}}{a^2 u_0^4} = i g_0 F_{\mu\nu} + \mathcal{O}(a^2)$$

Higher twist on the lattice (I)

$$O_m = \frac{1}{q^-} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n | \, \mathrm{tr} \left[F^{3j}(0) \Delta^{2m} F^{3j}(0) - F^{4j}(0) \Delta^{2m} F^{4j}(0) \right] | n \rangle \quad \text{suppressed by} \quad \left[\frac{T}{q^-} \right]^{2m}$$

assuming hard parton of $q^- = 100\,\mathrm{GeV}$



source: *arXiv:2010.14463*

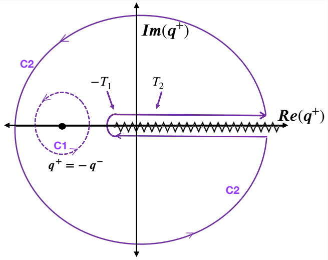
Higher twist on the lattice (II)

$$O_m = \frac{1}{q^-} \sum_n \frac{e^{-\beta E_n}}{Z} \langle n| \,\mathrm{tr}\, \left[F^{3j}(0)\Delta^{2m}F^{3j}(0) - F^{4j}(0)\Delta^{2m}F^{4j}(0)\right] |n\rangle$$

- Dimension six operators have many more SW_4 irreps – not fully sorted
- Covariant derivatives via finite differences on the lattice

⇒ Gauge-field dependence implies different renormalization of hopping and on-site terms, giving rise to obvious power-law divergences of form

$$O_1^{\mathrm{B}} \equiv X_6^{\mathrm{B}} = \#_{6,i} Z_{6,i}^{-1} X_{6,i}^{\mathrm{R}}(T) + \#_{4,i} Z_{4,i}^{-1} X_{4,i}^{\mathrm{R}}(T)\, a^{-2} + \#_{1,i} \mathbf{1}\, a^{-6}$$



source: dissertation A. Kumar

- Instead could use $q^+ = -xq^-$, $x \sim 1$

$$\frac{1}{(q+k)^2} \simeq \frac{-1}{2x(q^-)^2} \sum_{m=0}^{\infty} \left[\frac{\frac{1-x}{x}k_0 + \frac{1+x}{x}k_3}{\sqrt{2}q^-}\right]^m$$

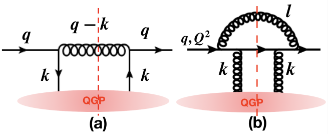
- Linear combination of different integrals could alleviate mixing

Missing contributions

- Hard parton at NLO interacting with non-perturbative medium

$\Rightarrow$ Quite possible that NLO contribution is larger than the tree-level one

- **Flavor-changing scattering** diagrams generate corrections, suppressed by odd powers $[T/q^-]^{2m+1}$ $\Rightarrow$ more relevant than O_m, $m > 0$.
- **not even considered yet** in published results using strict HTL perturbation theory or phenomenology
- Contribute both in QCD and in pure gauge theory: a presence of parton's flavor in the quark sea not necessary (but certainly important)



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- Emissions within the medium yield more complicated lattice operators

$\Rightarrow$ similar issues as higher-twist ops.?

All approaches so far consider only **independent scattering** events.