

# Heavy quarkonium at finite temperature

(from lattice QCD)

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*“Hard Problems of Hadron Physics: Non-Perturbative QCD & Related Quests”*  
Logunov Institute, Protvino, Russia, 11/10/2021

Bottomonium ... using ... quantum trajectories ... :

JHEP 05 (2021) 136; arXiv:2107.06147[physics.comp-ph]; Phys.Rev.D ? (2021)

Bottomonium melting from screening correlators at high temperature,  
Phys.Rev.D 104 (2021) 5, 054511

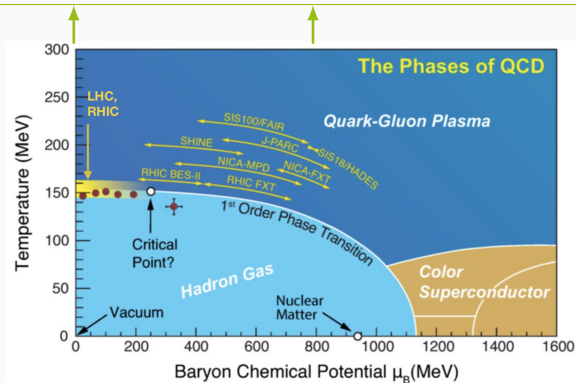
**Static quark anti-quark interactions at non-zero temperature from lattice QCD, arXiv:2110.11659[hep-lat]**

# Outline

- 1 **Appetizer:** link to heavy-ion phenomenology
  - Motivation
  - Historical perspective on “Heavy quarkonium at  $T > 0$ ”
- 2 **Main course:** modern perspective on “Heavy quarkonium at  $T > 0$ ”
  - In-medium quarkonium at weak coupling
  - Lattice QCD
  - Relativistic bottomonium
  - Nonrelativistic  $q\bar{q}$  pair on the lattice
  - In-medium static quarkonium
- 3 **Dessert:** bringing “Heavy quarkonium at  $T > 0$ ” full circle

# Phase diagram of QCD & heavy-ion collisions

Weak-coupling picture may eventually work at very high  $T$  or  $\mu_B$



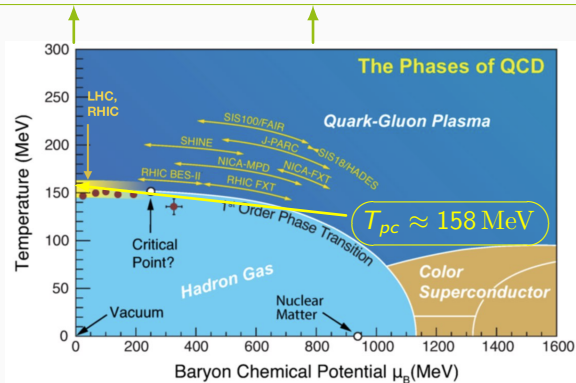
Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)

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# Phase diagram of QCD & heavy-ion collisions

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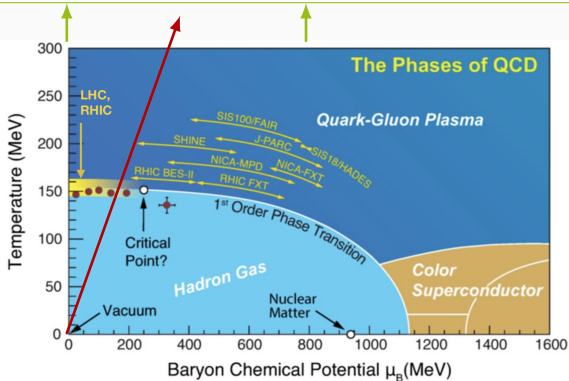
Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)
- Primordial state of nuclear matter before the hadronic freezeout
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# Phase diagram of QCD & heavy-ion collisions

Weak-coupling picture may eventually work at very high  $T$  or  $\mu_B$

Lattice Gauge Theory

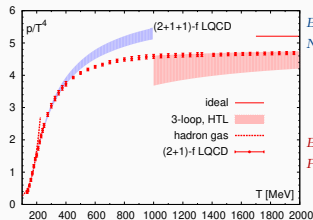


Neutron stars

- High temperature phase of nuclear matter: quark-gluon plasma (QGP)
- Primordial state of nuclear matter before the hadronic freezeout
- QGP can be produced in **heavy-ion collision (HIC) experiments**
- How to define thermodynamic state variables for such dynamic media?

# Interplay between lattice gauge theory and heavy-ion collisions

- Fruitful interplay of **lattice gauge theory** & **heavy-ion collisions**
- Search for the critical point and scan of the QCD phase diagram
- or **equation of state**, QNS, ...
- But: many **heavy-ion collision experiments** shift their focus



Borsanyi, et al.,  
Nature 539 (2016)

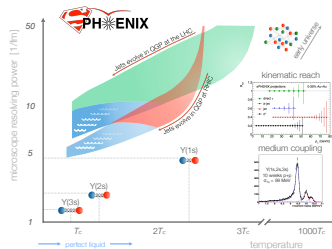
Bazavov, et al.,  
PRD 97 (2018)

Recent results  $\rightarrow$  Bazavov, et al., aXiv:2110.03606 [hep-lat]

Questions @ ALICE, CMS, sPhenix, ...

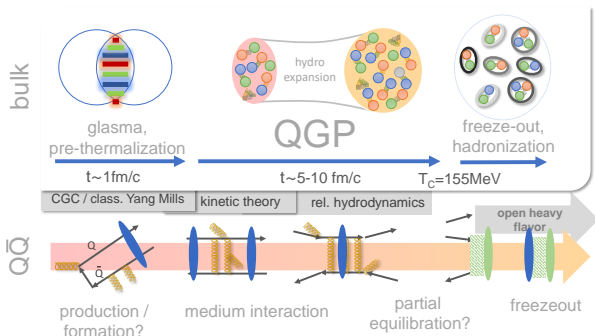
- How do **parton showers** develop and propagate in QGP?
- How to reconcile asymptotic freedom and observed strongly-coupled QGP
- Which **dynamical degrees of freedom** play a role in QGP?

Answers in terms of models or PQCD...



source: sPhenix proposal, arXiv:1501.06197 [nucl-ex]

# Why focus on hard probes in heavy-ion collisions?



source: Rothkopf, Phys.Rept. 858 (2020) 1-117

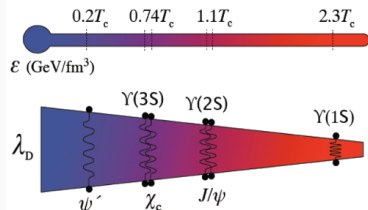
- Hard probes are produced in a few **hard processes** in initial collision, neither created or destroyed afterwards, but can alter their nature
- Most important probes: jets, open heavy flavor & **heavy quarkonium**
- **What happens to quarkonium if we increase the temperature?**

# Heavy quarkonium in the hot medium as a local thermometer

- Idea to look at **quarkonium** in the QGP is old and famous

*Matsui, Satz, PLB 178 (1986)*

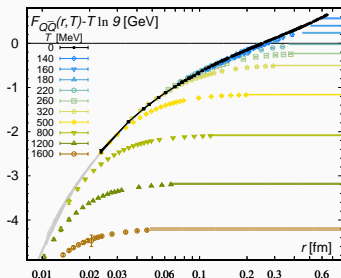
- Debye screening** of electric gluons ( $A_0$ ) dictates a limit of the radius of hadronic bound states
- Consequence: QGP formation  $\Leftrightarrow$  **quarkonium suppression**



source: *USQCD whitepaper 2018, EPJ A 55 (2019)*

- Color screening** usually studied via Polyakov loop correlator
 
$$C_P(r, T) = \langle P(0)P^\dagger(\mathbf{r}) \rangle_T^{\text{ren}} = e^{-F_{Q\bar{Q}}(r, T)/T}$$
- $rT \ll 1$ : **singlet/octet** decomposition
 
$$C_P(r, T) = 1/9e^{-F_S(r, T)/T} + 8/9e^{-F_O(r, T)/T}$$
- $rm_D \gtrsim 1$ : screening regime; decompose
 
$$C_P(r, T) = \langle \text{Re } P(0)\text{Re } P^\dagger(\mathbf{r}) \rangle_T^{\text{ren}} + \langle \text{Im } P(0)\text{Im } P^\dagger(\mathbf{r}) \rangle_T^{\text{ren}}$$
 into  $\mathcal{C}$  **even** or **odd** contributions

- QCD**:  $F_{Q\bar{Q}, S, O}(r, T)$  **screened** @  $T = 0$



source: *Brambilla, et al., PRD 98 (2018)*

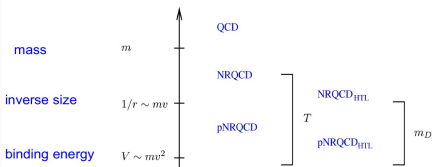


# Screening is not the whole story... (at weak coupling)

Matsui & Satz's idea of the **quarkonium suppression mechanism** was turned inside out by **weak-coupling EFT results** emerging 15 years ago

NR hierarchy:

$$V \sim \alpha_s$$



Thermal hierarchy:

$$m_D \sim gT$$

- For  $1/r \sim m_D \ll T$ :  $\text{Re}[V_s] = F_S + \mathcal{O}(g^4)$  and  $\text{Im}[V_s] \sim \mathcal{O}(g^2 T)$

$$V_s(T, r) = -C_F \alpha_s \left\{ \frac{e^{-rm_D}}{r} + m_D + iT \phi(rm_D) \right\}, \phi(x) = 2 \int_0^\infty \frac{dz z}{(z^2 + 1)^2} \left\{ 1 - \frac{\sin(zx)}{zx} \right\}$$

*Laine, et al., JHEP 03 (2007)*

- For  $\Delta V \ll 1/r \ll m_D \ll T$ :  $\text{Re}[V_s] = V_s + \mathcal{O}(g^4)$  and  $\text{Im}[V_s] \sim \mathcal{O}(g^4 r^2 T^3, g^6 T)$

$$V_s(T, r) = \frac{-C_F \alpha_s}{r} + r^2 T^3 \left\{ \mathcal{O}(g^4) + i \mathcal{O} \left( g^4, \frac{g^6}{(rT)^2} \right) \right\}$$

*Brambilla, et al., PRD 78 (2008)*

But an imaginary part leads to **dissociation** – is **screening** even relevant?

# In-medium quarkonium as an open quantum system

- Treat in-medium quarkonium in **OQS+pNRQCD** approach

*Brambilla, et al., PRD 96 (2017) + PRD 97 (2018)*

- Density matrix  $\rho \equiv \rho_{\text{QGP}} \otimes \rho_{q\bar{q}}$ : trace out environment  $\Rightarrow$  **OQS**

- Tower of EFTs: **QCD**  $\xrightarrow{\alpha_s M \ll M} M$  **NRQCD**  $\xrightarrow{\alpha_s/r \ll 1/r} 1/r$  **pNRQCD**

$$\begin{aligned} \mathcal{L}_{\text{pNRQCD}} = \int d^3\mathbf{r} \text{tr} \left\{ S^\dagger \left[ i\partial_0 - \left( \frac{D^2}{2M} + \sum_{j=0}^{\infty} \frac{V_s^{(j)}(r)}{M^{j-1}} \right) \right] S + O^\dagger \left[ i\partial_0 + \frac{D^2}{2M} + \sum_{j=0}^{\infty} \frac{V_o^{(j)}(r)}{M^{j-1}} \right] O \right\} \\ + V_A(r) \text{tr} \left\{ O^\dagger \mathbf{r}_g \mathbf{E} S + S^\dagger \mathbf{r}_g \mathbf{E} O \right\} + V_B(r) \text{tr} \left\{ O^\dagger \mathbf{r}_g \mathbf{E} O + O^\dagger O \mathbf{r}_g \mathbf{E} \right\} \\ + \mathcal{O} \left( r^2, \frac{1}{M^2} \right) + \mathcal{L}_{\text{light}} \end{aligned}$$

- **Coupled master equations** for the two-component density matrix

$$\dot{\rho}_s(t;t) = -i[h_s, \rho_s(t;t)] - \Sigma_s(t)\rho_s(t;t) + \rho_s(t;t)\Sigma_s^\dagger(t) + \Xi_{so}(\rho_o(t;t), t)$$

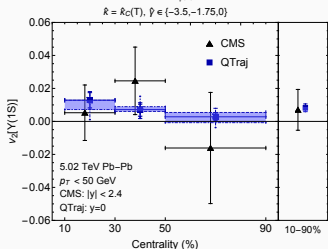
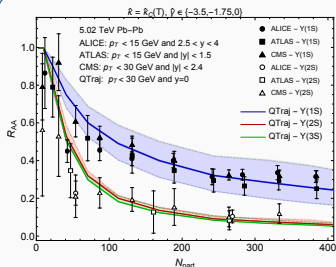
$$\dot{\rho}_o(t;t) = -i[h_o, \rho_o(t;t)] - \Sigma_o(t)\rho_o(t;t) + \rho_o(t;t)\Sigma_o^\dagger(t) + \Xi_{os}(\rho_s(t;t), t) + \Xi_{oo}(\rho_o(t;t), t)$$

thermal self-energies  $\Sigma_{..}$ , **dipole transitions** by QGP via  $\Xi_{..}(\rho_{..})$

- The **dilute limit** (bottomonium) –  $\Xi_{..}(\rho_{..})$  is **linear** in  $\rho_{..}$ .
- If Markovian + quantum Brownian motion  $\Rightarrow$  **Lindblad equation**

*Lindblad, CMP 48 (1976); V. Gorini et al., JMP 17 (1976)*

# Modern understanding of quarkonium melting from lattice+EFT



source: Brambilla, et al., arXiv:2107.06222

If  $M \gtrsim 1/a_0 \gg \pi T \sim m_D \gg E$   
 $\Rightarrow$  master equation has a Lindblad form, is discretized and solved stochastically<sup>a</sup>  $\rightarrow$  QTraj

- Temperature dependence from hydrodynamics evolution using lattice QCD equation of state
- For strongly-coupled plasma:  $T$  dependence via heavy-quark transport coefficients  $\kappa, \gamma$
- Lattice transport coefficients & EoS in OQS+pNRQCD approach: quarkonium suppression

<sup>a</sup> Brambilla, et al., JHEP 05 (2021) + arXiv:2107.06222 [hep-ph];

Ba Omar, et al., arXiv:2107.06147 [physics.comp-ph]

## QCD on a lattice

$$S_{\text{QCD}}[U, \bar{\psi}, \psi] = a^4 \sum_x \sum_{f=1}^{N_f} \bar{\psi}^f(x) \left( \not{D}[U(x)] + m_f \right) \psi^f(x) - a^4 \sum_x \sum_{\mu < \nu} \frac{2}{g_0^2} \text{Re tr} \left\{ 1 - U_{\mu, \nu}(x) + \mathcal{O}(a^2) \right\}$$

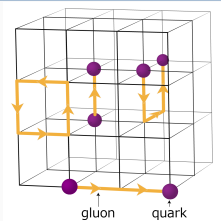
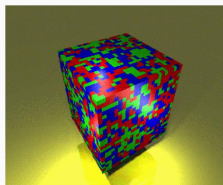
$$D_\mu[U_\mu(x)]\psi^f(x) = \frac{U_\mu(x)\psi^f(x + a\hat{\mu}) - U_\mu^\dagger(x - a\hat{\mu})\psi^f(x - a\hat{\mu})}{2a} + \mathcal{O}(a^2)$$

$$U_\mu(x) = \exp[iag_0 A_\mu(x)]$$

gauge link

$$U_{\mu, \nu}(x) = U_\mu(x)U_\nu(x + a\hat{\mu})U_\mu^\dagger(x + a\hat{\nu})U_\nu^\dagger(x)$$

plaquette


 $\xrightarrow{\text{HPC}}$ 


# Lattice QCD simulations in a box on a computer

Stochastically sample the (Euclidean) QCD path integral

$$\langle O \rangle_{\text{QCD}} = \frac{1}{Z} \frac{1}{N} \sum_{\{U\}}^N O[U] \prod_{f=1}^{N_f} \det(\not{D}[U] + m_f) \exp(-S_g[U]) + \mathcal{O}\left(\frac{1}{\sqrt{N}}\right)$$

using MCMC algorithm with importance sampling

QCD on a lattice with spacing  $a$  in a box of  $N_s^3 \times N_\tau$  points

- scale setting: lattice spacing  $a$  is determined a posteriori  
**control the approach to the continuum limit  $a \rightarrow 0$**
- time (Euclidean): periodic for gluons, antiperiodic for quarks
- space: periodic for gluons and quarks  
**always at finite temperature and in finite volume**  
 $aN_\tau = 1/T$  (**volumes only must be large enough**)
- quark masses: light quarks at the physical point are expensive  
**control the quark mass dependence through  $\chi$ PT**
- quark flavors: usually  $N_f = 2 + 1$  or  $N_f = 2 + 1 + 1$ , or  $N_f = 0$

# At which $T$ are there either bound states or melted $q\bar{q}$ pairs?

- **Euclidean Correlators** are towers of exponential decays  $G(\tau) = \sum_i A_i e^{-E_i \cdot \tau}$

For mesons: same  $E_0$  in temporal or spatial directions

- **Spatial  $q\bar{q}$  pair correlators** are a model-independent analysis tool

*Bazavov, et al., PRD 91 (2015)*

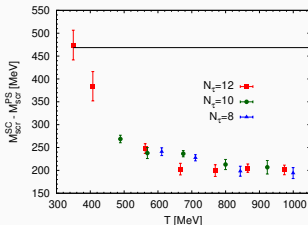
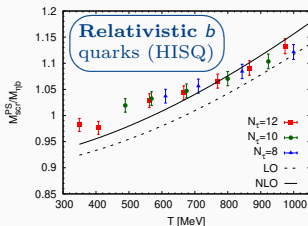
$$G(z, T) = \int_0^{1/T} d\tau \int d^2x_\perp \langle \mathcal{J}(\tau, \mathbf{x}_\perp, z) \mathcal{J}^\dagger(0) \rangle$$

$$= \int_0^\infty \frac{2d\omega}{\omega} \int_{-\infty}^\infty dp_z e^{ip_z z} \rho(\omega, p_z, T)$$

with spectral function  $\rho(\omega, p_z, T)$

$$\sim \begin{cases} \delta[\omega^2 - p_z^2 - M_0^2] & \text{mesons} \\ \delta\left[\omega - \sum_{q_i} \sqrt{m_{q_i}^2 + [\pi T]^2}\right] & \text{free quarks} \end{cases}$$

- Lowest pseudoscalar & vector are hardly modified at  $T \lesssim 450$  MeV
- Lowest scalar & axialvector are hardly modified at  $T \lesssim 350$  MeV
- How can we understand the **melting mechanism** at work?



source: *Petreczky, et al., PRD 104 (2021)*

# Nonrelativistic bottomonium with extended sources (HotQCD)

- NRQCD correlator study on  $T = 0$  or  $N_\tau = 12$  lattices: **extended sources**

*Larsen, et al., ...*

- Lattice NRQCD: no continuum limit  $\Rightarrow$  upper limit on  $T$  for fixed  $N_\tau$

- Point source vs Gaussian smearing, new scheme for removing UV part: **thermal width**, small mass shift

*PRD 100 (2019)*

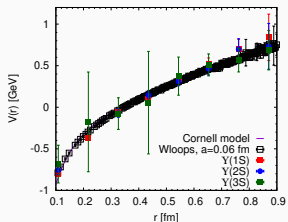
- Cornell pot. eigenstates  $\rightarrow$  GEVP  $\Rightarrow$  obtain lattice **BS amplitudes**
- BS amp. allow recovering potential

$$\left(\frac{-\Delta}{m_b} + V(r)\right) \phi_\alpha = E_\alpha \phi_\alpha$$

*PLB 800 (2020) + PRD 102 (2020)*

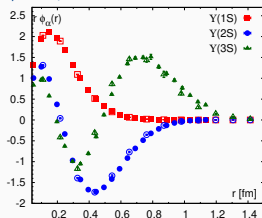
- Small  $\tau$ : BS amplitudes at  $T = 0$  and  $T > 0$  almost indistinguishable
- Large  $\tau$ : BS amplitudes at  $T > 0$  visibly modified  $\Leftrightarrow$  thermal width

$T = 0, a = 0.06$  fm: NRQCD vs static  $q\bar{q}$



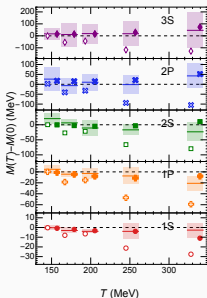
source: *Larsen, et al., PRD 102, (2020)*

$N_\tau = 12, T = 334$  MeV vs  $T = 151$  MeV



# Machine learning the potential from NRQCD amplitudes

Machine learning (DNN) applied to lattice BS amp.:  $T > 0$  potential

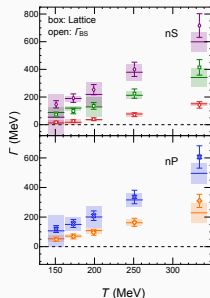


BS amp. fed into DNN  
 $\Rightarrow$  reconstruct potential

Unscreened real part of potential, but its imaginary part implies a large thermal width

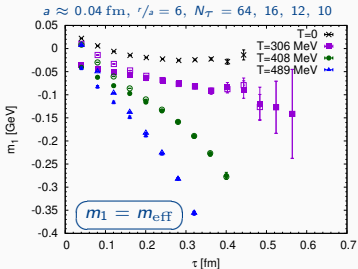
Clearly smaller thermal mass shift and larger width than in **Hard Thermal Loop (HTL)** perturbation theory

*Shi, et al., arXiv:2105.07862*





# Static $q\bar{q}$ pair at $T > 0$ on the lattice



source: [Bala, et al., arXiv:2110.11659](#)

- Static  $q\bar{q}$  interaction is encoded in (real-time) **Wilson loops**<sup>a</sup>

$$W_{[r, \tau]}(t) = \left\langle e^{ig \oint_{r \times t} dz^\mu A_\mu} \right\rangle_{\text{QCD}, T}$$

- Stable (ground) state  $\Omega_r$  exists if

$$\Omega_{[r, \tau]} \equiv -i \lim_{t \rightarrow \infty} \partial_t W_{[r, \tau]}(t)$$

<sup>a</sup>We use Wilson line correlators in Coulomb gauge.

- Same spectral functions yield real- or imaginary-time correlators

$$W_{[r, \tau]} \left( \frac{t}{\tau} \right) = \int d\omega \begin{pmatrix} e^{+i\omega\tau} \\ e^{-\omega\tau} \end{pmatrix} \rho_{[r, \tau]}(\omega)$$

- Motivates generic decomposition

$$\rho_{[r, \tau]}(\omega) = \rho_{[r, \tau]}^{\{\Omega; \mathcal{O}(T)\}}(\omega) + \rho_{[r, \tau]}^{\text{tail}}(\omega) + \rho_{[r, \tau]}^{\text{UV}}(\omega)$$

- UV continuum  $\rho_{[r, \tau]}^{\text{UV}}(\omega)$  is far above **lowest feature  $\Omega$  + effects of  $\mathcal{O}(T)$**

$\Rightarrow$  Guess  $\rho_{[r, \tau]}^{\text{UV}}(\omega)$  via  $\rho_{[r, 0]}^{\text{UV}}(\omega) \Rightarrow$  subtract

**Note:** “tail” due to backward propagating UV physics (vacuum excited states) at  $\tau \lesssim 1/T$ .

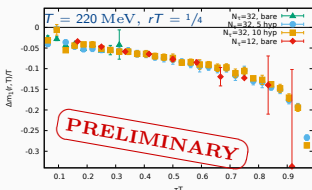
# Cumulants of spectral functions – what can we expect?

- Access **cumulants** of  $\rho_{[r, T]}(\omega)e^{-\omega\tau}$  via  $\tau$  (**log**) **derivatives** of  $W_{[r, T]}(\tau)$ 

$$m_1^{[r, T]}(\tau) = -\partial_\tau \ln W_{[r, T]}(\tau) \quad [\equiv m_{\text{eff}}^{[r, T]}(\tau)],$$

$$m_n^{[r, T]}(\tau) = -\partial_\tau m_{n-1}^{[r, T]}(\tau), \quad n > 1$$
- For  $N_\tau \leq 16$  obtain up to  $m_3^{[r, T]}(\tau)$ : supports  $\leq 5$  parameters for  $\rho_{[r, T]}(\omega)$
- Higher cumulants at small  $\tau$  need at least  $N_\tau > 16$ : bad signal-to-noise

Fully vacuum-subtracted result

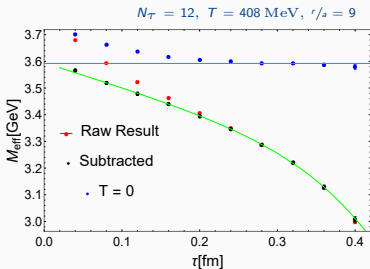


see: [Hoying, et al., arXiv:2110.00565 \[hep-lat\]](#)

Feasibility study with  $N_\tau = 32$ :  $m_n^{[r, T]}$ ,  $n > 2$  ?

- Fine lattices:  $a^{-1} \approx 7 \text{ GeV}$   $m_\pi \approx 0.3 \text{ GeV}$
- UV filtering (HYP) for **noise reduction**
- distortions cancel in vacuum subtraction
- Definitely still work in progress

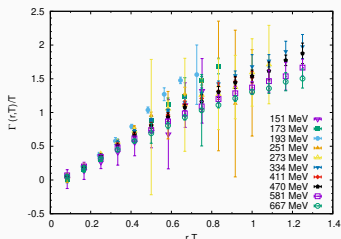
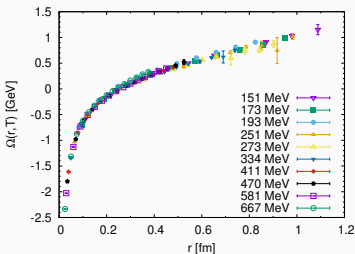
# Lowest spectral feature from fits using Gaussian approximation



- Quasiparticles are represented as **Breit-Wigner** in  $\rho_{[r,T]}(\omega)$
- Ansatz: approximate **BW** of  $\rho_r^{\{\Omega; \mathcal{O}(T)\}}(\omega)$  locally as **Gaussian**, include **delta function** for  $\rho_r^{\text{tail}}(\omega)$

$$W_{[r,T]}(\tau) = A_{[r,T]}^{\{\Omega; \mathcal{O}(T)\}} e^{-\Omega_{[r,T]}\tau + (\Gamma_{[r,T]}^G)^2 \tau^2 / 2} + A_{[r,T]}^{\text{tail}} e^{-\omega_{[r,T]}^{\text{tail}} \tau}, \quad \omega_{[r,T]}^{\text{tail}} \ll \Omega_{[r,T]}$$

$N_T = 12, \Omega(r, T) \equiv \Omega_{[r,T]}, \Gamma(r, T) \equiv \sqrt{2 \ln 2} \Gamma_{[r,T]}^G$ , subtracted correlators

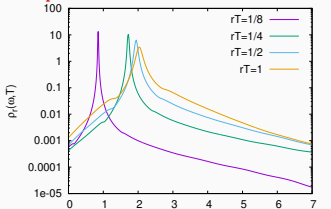


source: Bala, et al., arXiv:2110.11659

- Almost no  $T$  dependence in  $\Omega_{[r,T]}$  (naive correspondence:  $\text{Re } V_s(r, T)$ )
- Naively expected scaling of  $\Gamma(r, T)/T \approx \Gamma(rT)/T$  down to  $T \approx T_{pc}$

# Comparison: lattice QCD vs HTL

HTL spectral function for  $T = 667$  MeV



[NLO, 2-loop  $\alpha_s(2\pi T)$ ,  $\Lambda_{\overline{MS}}^{N_f=3} = 332$  MeV]

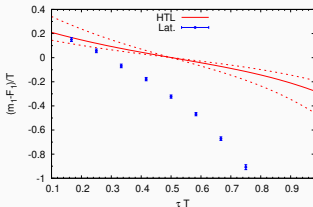
source: *Bala, et al., arXiv:2110.11659*

- **HTL** is an attractive proposition: **motivated & regularized BW**
- **HTL** result is **antisymmetric** around the midpoint  $\tau = 1/2T$ :

$$\log W_{[r, T]}(\tau) = -\text{Re } V_s(r, T) \times \tau + \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \left\{ e^{-\omega\tau} + e^{-\omega(1/\tau - \tau)} \right\} \times \{1 + n_B(\omega)\} \sigma_{[r, T]}(\omega)$$

- Leading **singularity** of  $\sigma_{[r, T]}(\omega)$  (transv. gluon spec. fun.) fixes  $\text{Im } V_s(r, T)$

$N_T = 12$ ,  $r/s = 12$ , subtracted correlator  
 $T=667$  MeV,  $rT=1$

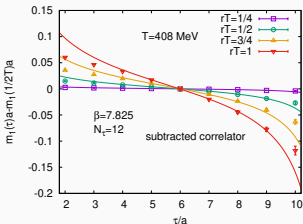


source: *Bala, et al., arXiv:2110.11659*

- **HTL** should work at  $r \sim 1/m_D$
- *Subtleties* due to renormalons and regulators: consider  $(m_1 - F_5)/T$   
Reminder:  $\text{Re}[V_s] = F_5 + \mathcal{O}(g^4)$  in **HTL**
- **No large UV component** in HTL, compare UV-subtracted result
- $m_1$  at midpoint lower than **HTL**, and  $m_2$  is much more negative

# Lowest spectral feature from fits using HTL-motivated Ansatz

$N_\tau = 12$ ,  $T = 408$  MeV,  $r/a = 9$



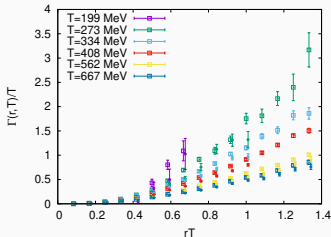
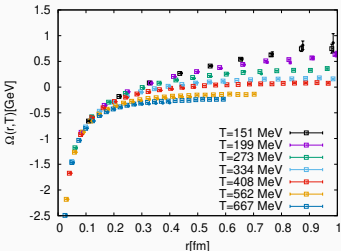
- Fit via HTL-motivated Ansatz

*Bala, Datta, PRD 101 (2020)*

$$W_{[r,T]}(\tau) = A_{[r,T]}^{BD} e^{-\Omega_{[r,T]}^{BD} \tau - i \frac{\Gamma_{[r,T]}^{BD}}{\pi} \log \sin(\pi \tau T)}$$

- Note: similar result via Gaussian around midpoint  $\tau = 1/2T$

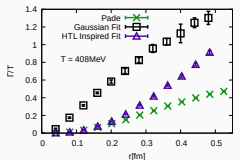
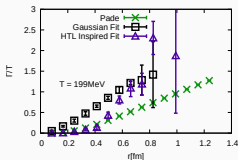
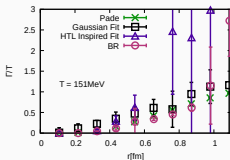
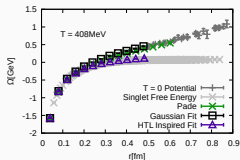
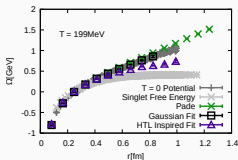
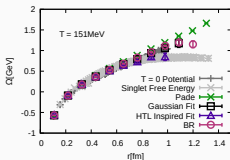
$N_\tau = 12$ ,  $\Omega(r, T) \equiv \Omega_{[r,T]}^{BD}$ ,  $\Gamma^{BD}(r, T) \equiv \Gamma_{[r,T]}^{BD}$ , (un-)subtracted correlators



source: *Bala, et al., arXiv:2110.11659*

- Significant  $T$  dependence in  $\Omega_{[r,T]}$  (naive correspondence:  $\text{Re } V_s(r, T)$ )
- Weaker than naive scaling of  $\Gamma(r, T)/T \approx \Gamma(rT)/T$

# Comparison: lowest spectral feature from four different methods



source: *Bala, et al., arXiv:2110.11659*

- Applied two further, independent methods (Padé rational approximation, Bayesian reconstruction) not discussed in detail
- $T \approx 150 \text{ MeV}$  conclusive:  $\Omega_{[r,T]} \approx F_S(r, T) \approx V_s(r)$  for  $r \lesssim 0.8 \text{ fm}$
- $T \lesssim 250 \text{ MeV}$ : all three methods yield  $\Omega_{[r,T]} \gg F_S(r, T)$
- $T \approx 400 \text{ MeV}$  inconclusive:  $\Omega_{[r,T]}^{BD} \approx F_S(r, T)$  vs  $\Omega_{[r,T]}^G \approx \Omega_{[r,T]}^P \approx V_s(r)$
- All methods find for all  $T$  nontrivial  $\Gamma_{[r,T]}$  that increases with  $r$  or  $T$

# Heavy quarkonium at finite temperature

Modern picture: quarkonium suppression is **NOT** due to screening.

- **OQS+pNRQCD** ( $\rightarrow$  QTraj): first-principles, non-Abelian evolution
- Use of gradient flow for **heavy-quark transport coefficients**
- Spatial correlation functions of **relativistic bottomonium**
  - Model-independent study of quarkonium melting in LQCD
  - $\eta_b$  or  $\Upsilon(1S)$  largely unmodified at  $T \approx 400$  MeV;  $\chi_{b0}$  or  $h_b$  already gone
- **Nonrelativistic bottomonium**
  - Extended sources or BS wave functions boost resolving power of LQCD
  - Spectral features are fully **consistent with static  $q\bar{q}$  pair**
- **Static quarkonium ( $q\bar{q}$  pair)**
  - Lowest spectral feature  $\{\Omega; \mathcal{O}(T)\}$  + tail + UV continuum
  - Model-independent cumulant analysis  $\rightarrow$  clear evidence for a large **thermal width being the main cause of quarkonium melting**
  - Consistent with minimal (Gaussian fit) or major (HTL-motivated fit) medium modification of real part  $\rightarrow$  insufficient resolution with  $N_\tau \leq 16$

The lattice + EFT is in good shape to deliver more accurate and more realistic results needed for HIC phenomenology in the coming years.

Thank you for your attention!