The gauge group and flavor number dependence of m_V/f_{PS}

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1905.01909 – JHEP 1905 (2019) 197
1912.04114 – PoS LATTICE2019 (2019) 237
2107.05996 – JHEP 2107 (2021) 202

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Context

- Strongly interacting extensions of Standard Model
- Composite Higgs (f_0 or σ)
- Non-trivial new particle prediction: $V = \varrho$ vector
- How well can $m_V = m_\varrho$ differentiate between models?

Motivation

How well can m_ϱ differentiate between models?

- Gauge group G, representation R, flavor number N_f
- Lattice prediction for dimensionless ratios m_ϱ/f_π ($f_{PS} = f_\pi$)
- $f_\pi = 246\,GeV$, m_ϱ should be experimentally measured in GeV
- Main question: what is the model dependence of m_ϱ/f_π?

Motivation

Previous lattice results in various $SU(3)$ models:

$m_\varrho / f_\pi \sim 8$ not much N_f or R dependence

Jin/Mawhinney 0910.3216 1304.0312

LatKMI 1302.6859

LSD 1312.5298 1601.04027 1807.08411

LatHC 0907.4562 1209.0391 1605.08750 1601.03302

But no controlled infinite volume and continuum and chiral extrapolation. (Except QCD but $m \neq 0$)

Goal

Fully controlled results with $SU(3)$

$R = fund$ and $N_f = 2, 3, 4, 5, 6, 7, 8, 9, 10$

$L/a \to \infty$ $\quad m \to 0$ $\quad a \to 0$

Note: m_ϱ / f_π in chiral limit well-defined also inside conformal window

Outline

- Finite volume effects m_π, f_π, m_ϱ
- Chiral-continuum extrapolation, results
- KSRF relation, $g_{\varrho\pi\pi}$
- Speculation inside conformal window
- General $SU(N)$, N_f, R
- Conclusion

Lattice setup

$SU(3)$ gauge group, Symanzik tree level improved gauge action, staggered stout-improved fermion action

At each $N_f = 2, 3, 4, 5, 6$: 4 lattice spacing, 4 masses for each lattice spacing: 16 points

At each $N_f = 7, 8, 9, 10$: 3 lattice spacing, 4 masses for each lattice spacing: 12 points

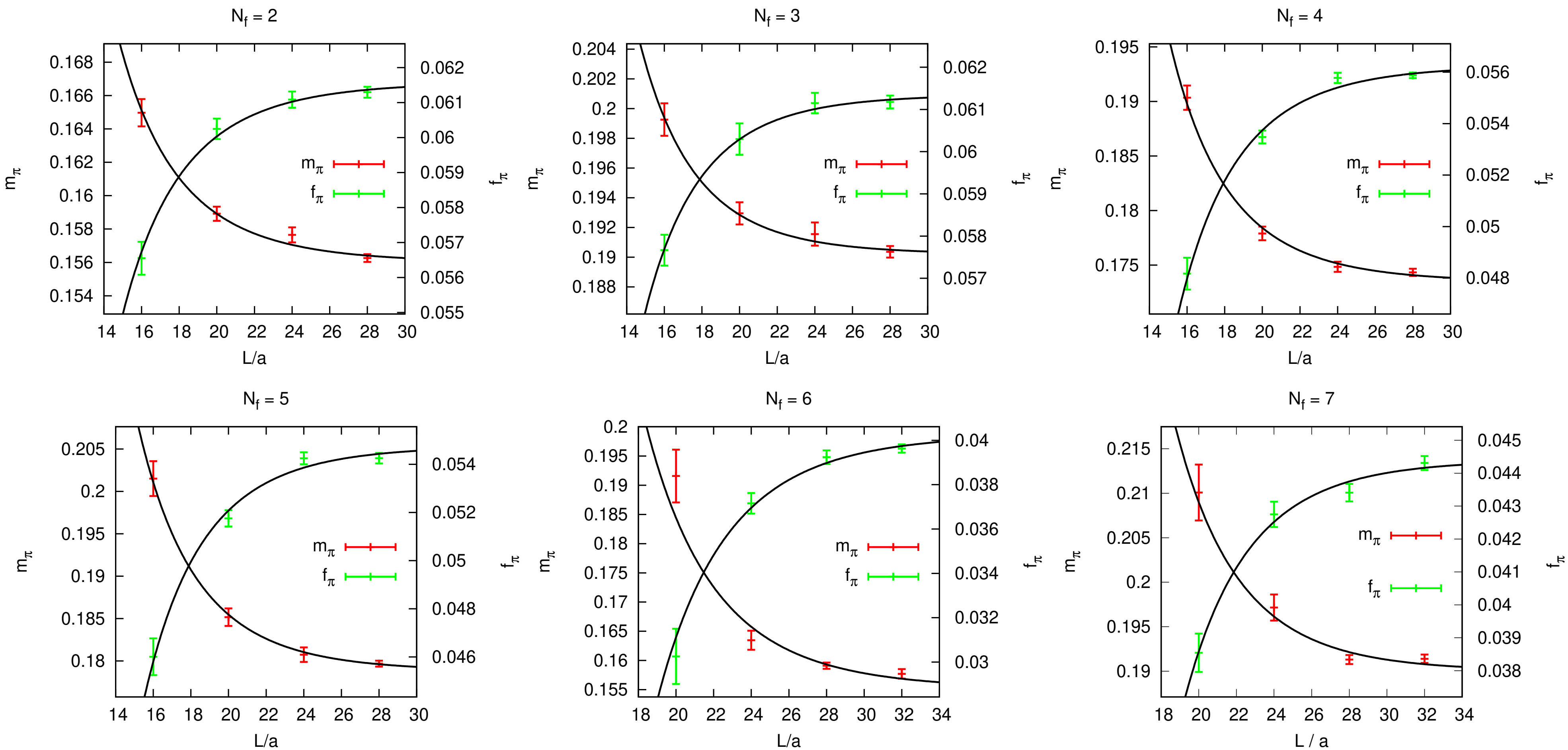
Finite volume effects - m_π, f_π

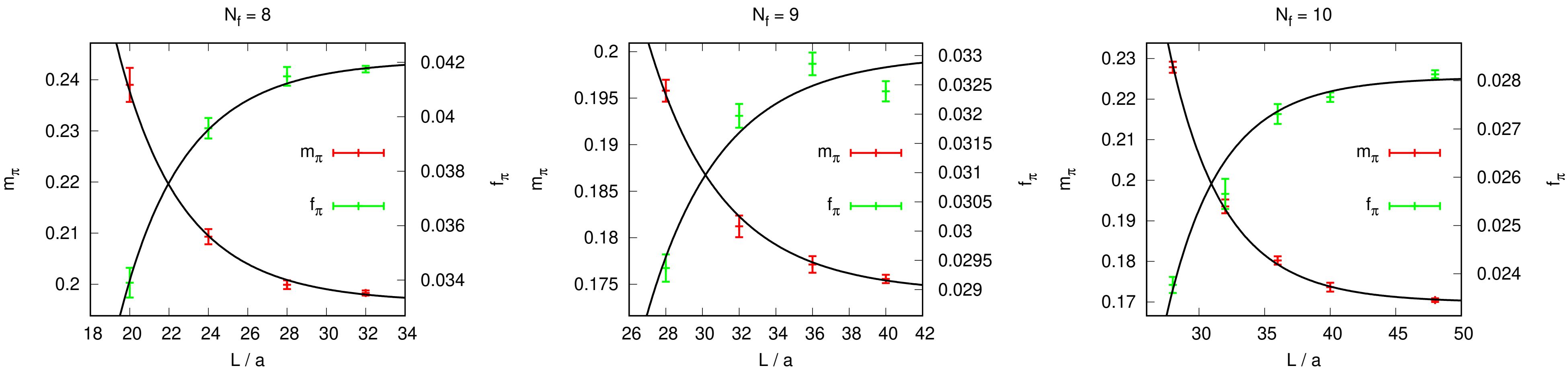
Exponential finite volume effects for m_π, f_π

How large does $m_\pi L$ needs to be to have less than 1% finite volume effect?

Simulations at fixed β, m for each N_f with at least 4 L/a

Finite volume effects - m_π, f_π





- Functional form: Gasser-Leutwyler
- Good fits, read off fit parameters
- Determine $m_\pi L$ such that finite volume effects are at most 1%

Finite volume effects - m_π, f_π

$$m_\pi L > 3.46 + 0.12 N_f + 0.03 N_f^2$$

For 1% finite volume effects on m_π, f_π

For example $N_f = 2$: $m_\pi L > 3.82$

For example $N_f = 10$: $m_\pi L > 7.66$

Finite volume effects - m_ϱ

m_ϱ is different: $\varrho \to \pi\pi$ resonance, need in finite volume

$$\frac{m_\varrho}{2m_\pi} < \sqrt{1 + \left(\frac{2\pi}{m_\pi L}\right)^2}$$

Finite volume effects from full Luscher

Fits to finite volume energy levels $E(m_\varrho, g_{\varrho\pi\pi}, L)$ or $E(m_\varrho, \Gamma_\varrho, L)$

If just one volume L: obtain $g_{\varrho\pi\pi}$ a posteriori from KSRF relation (see later)

Check finite volume effect on m_ϱ a posteriori

Taste breaking

$O(a^2)$ scaling of taste broken Goldstones from $\chi = \frac{\langle Q^2 \rangle}{V}$

$\rightarrow$ backup slides if interested :)

Systematics

- Volumes large enough
- Lattice spacing small enough
- Mass small enough

$\rightarrow$ chiral - continuum extrapolation of $m_\varrho Y$ and $f_\pi Y$ with some physical length scale Y

$Y = w_0$ for $N_f = 2, \ldots, 6$

$Y = \sqrt{t_0}$ for $N_f = 7, \ldots, 10$

Chiral - continuum extrapolation

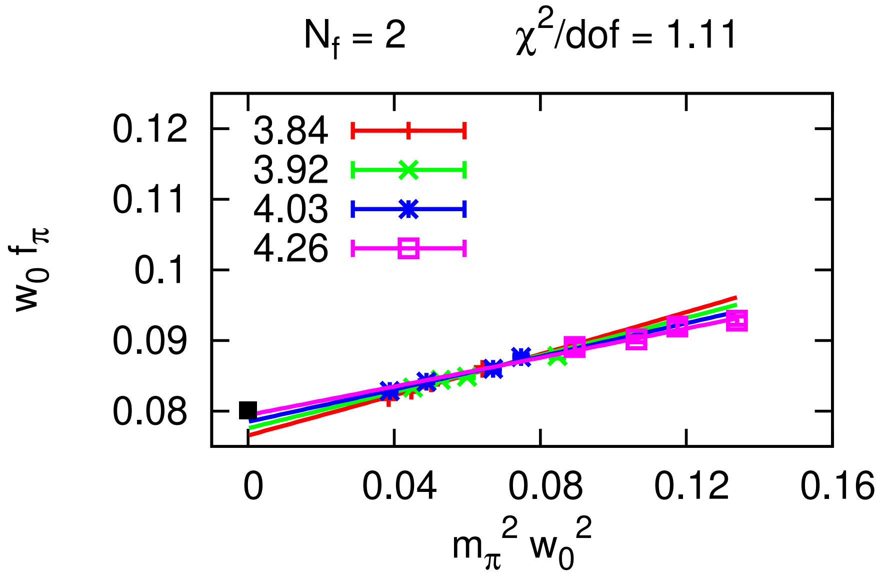
Global fit of $X = m_\varrho$ and $X = f_\pi$

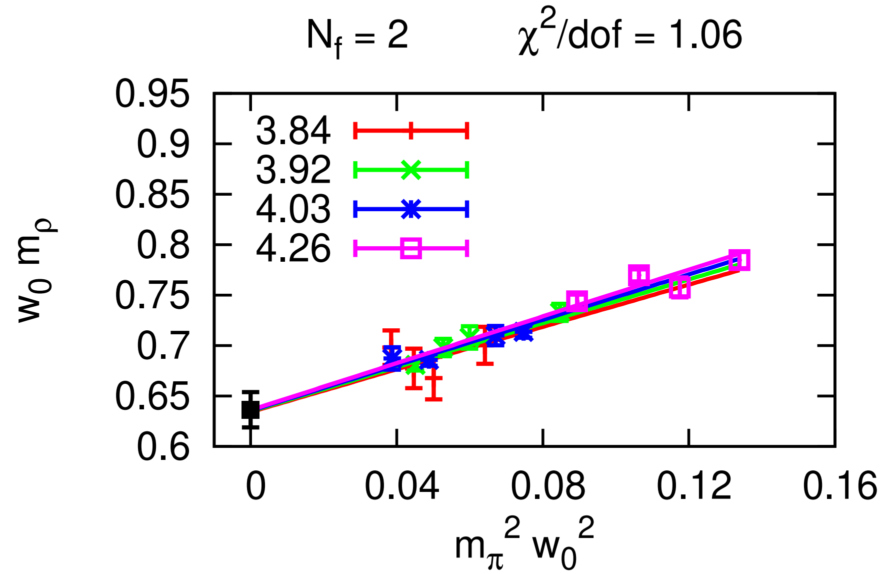
$$XY = C_0 + C_1 m_\pi^2 Y^2 + C_2 \frac{a^2}{Y^2} + C_3 \frac{a^2}{Y^2} m_\pi^2 Y^2$$

$Y = w_0$ for $N_f = 2, \ldots, 6,$ 16 points, dof = 14

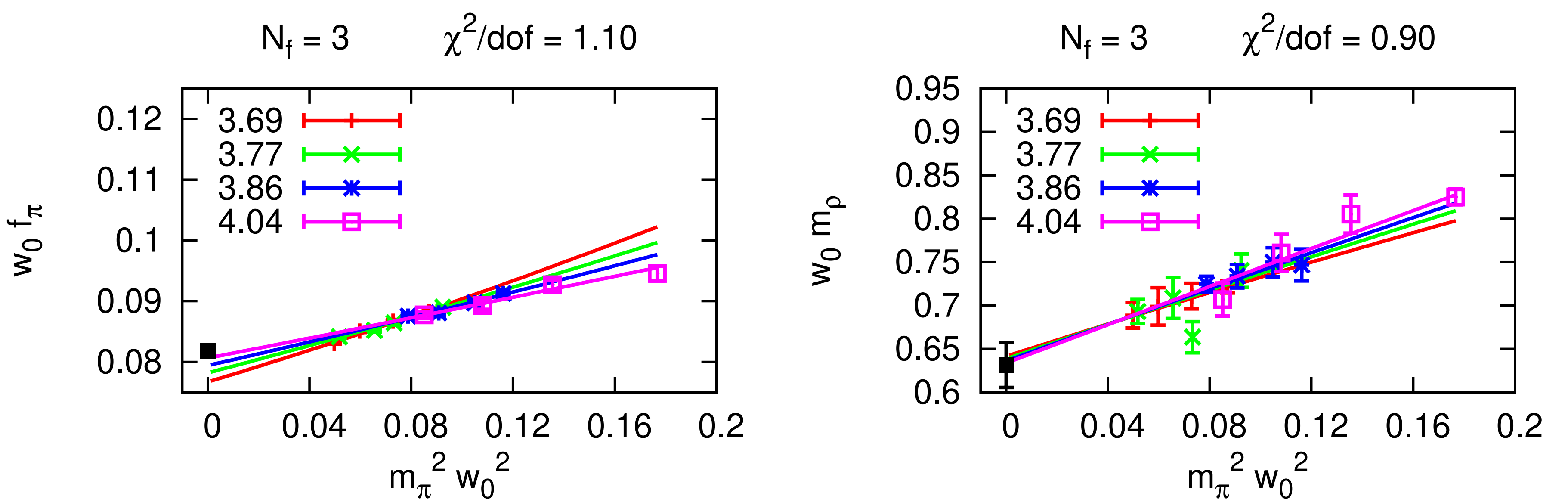
$Y = \sqrt{t_0}$ for $N_f = 7, \ldots, 10,$ 12 points, dof = 8

Chiral - continuum, $N_f = 2$ $\quad$ $24 \leq L/a \leq 36$

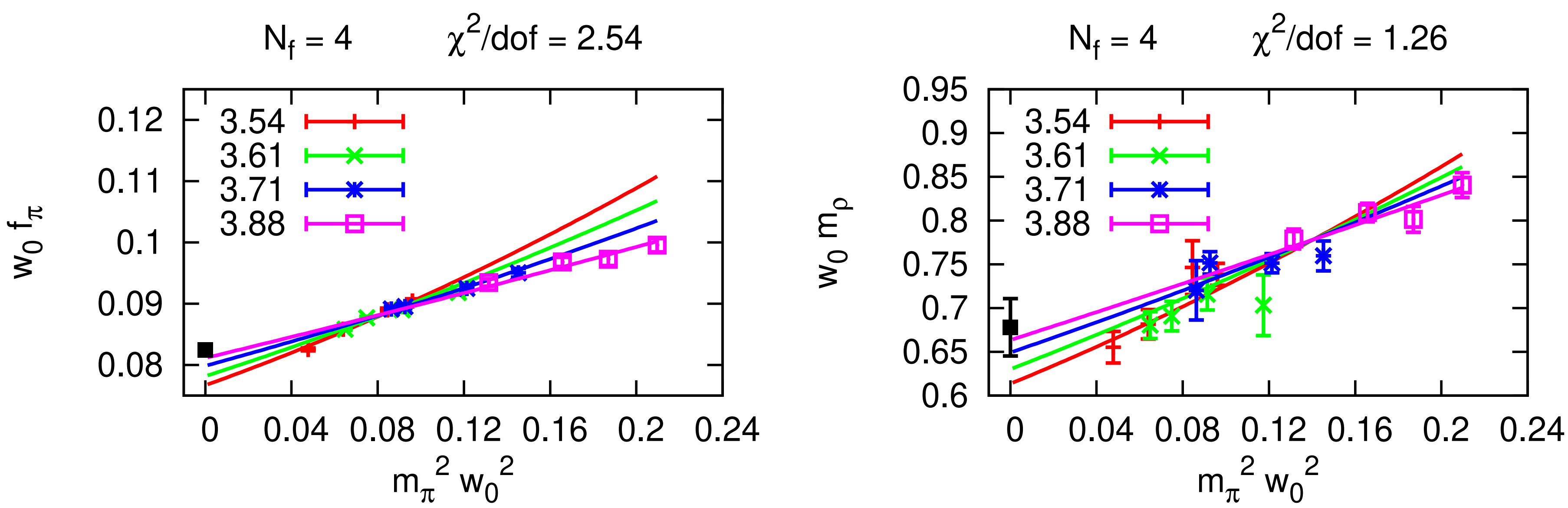




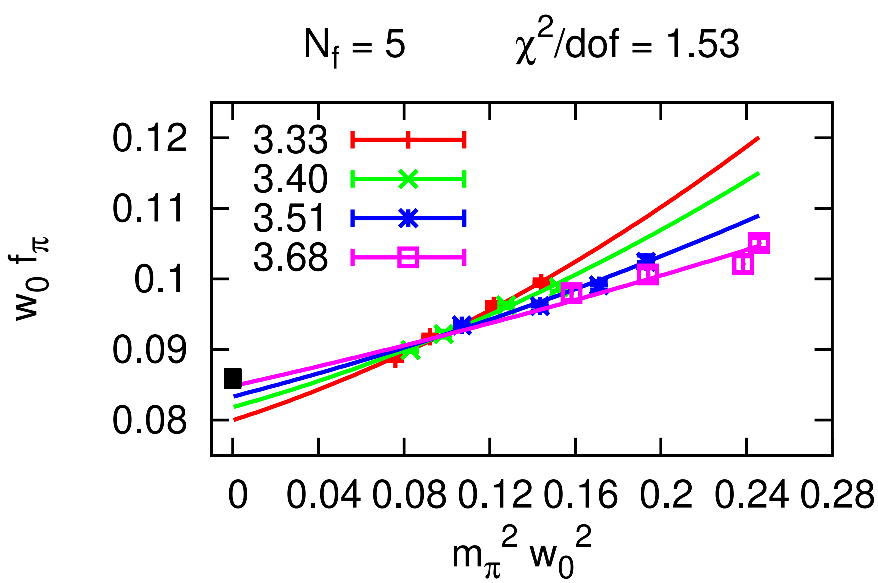
Chiral - continuum, $N_f = 3$ $\quad 20 \leq L/a \leq 36$

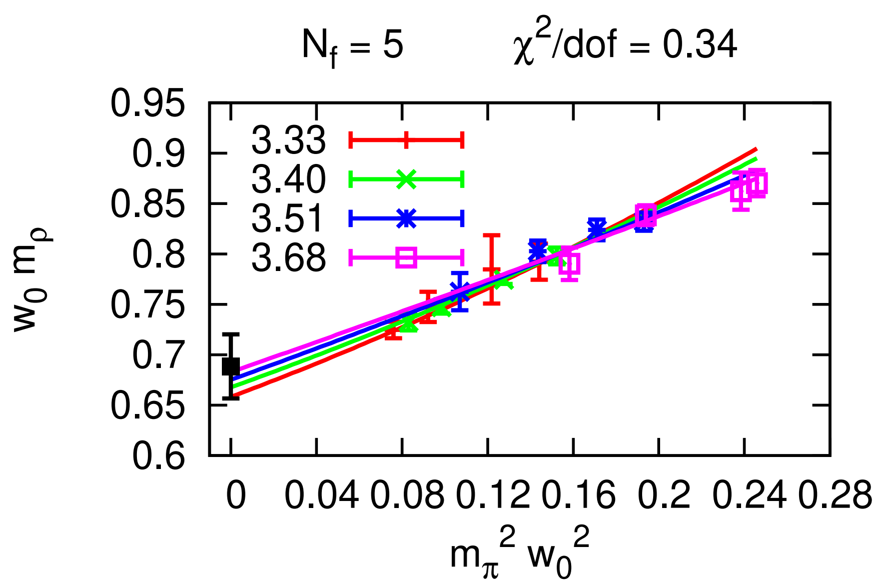


Chiral - continuum, $N_f = 4$ $20 \leq L/a \leq 36$

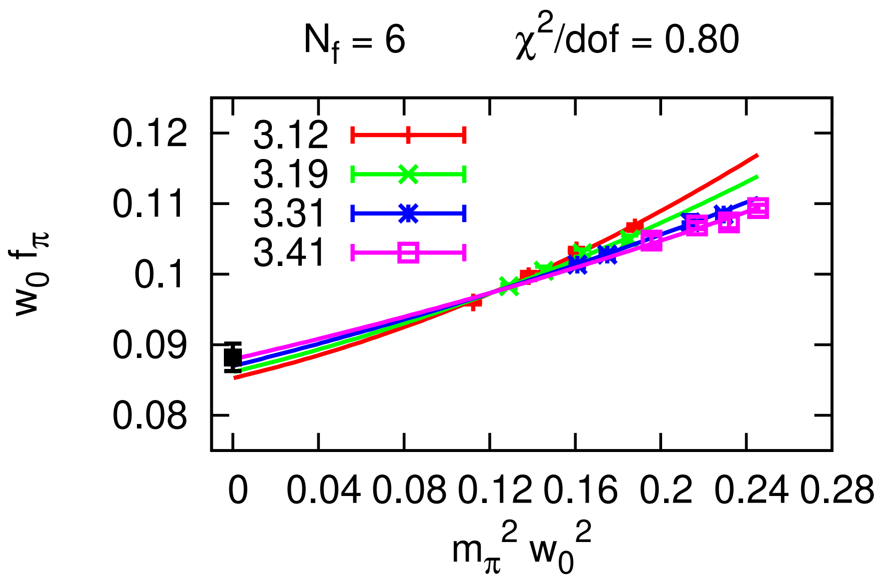


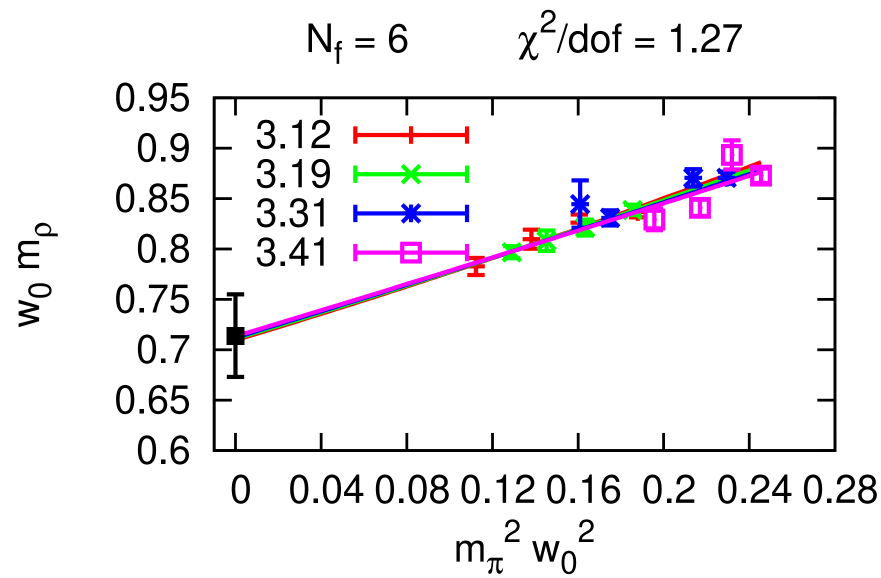
Chiral - continuum, $N_f = 5$ 20 $\leq L/a \leq$ 36



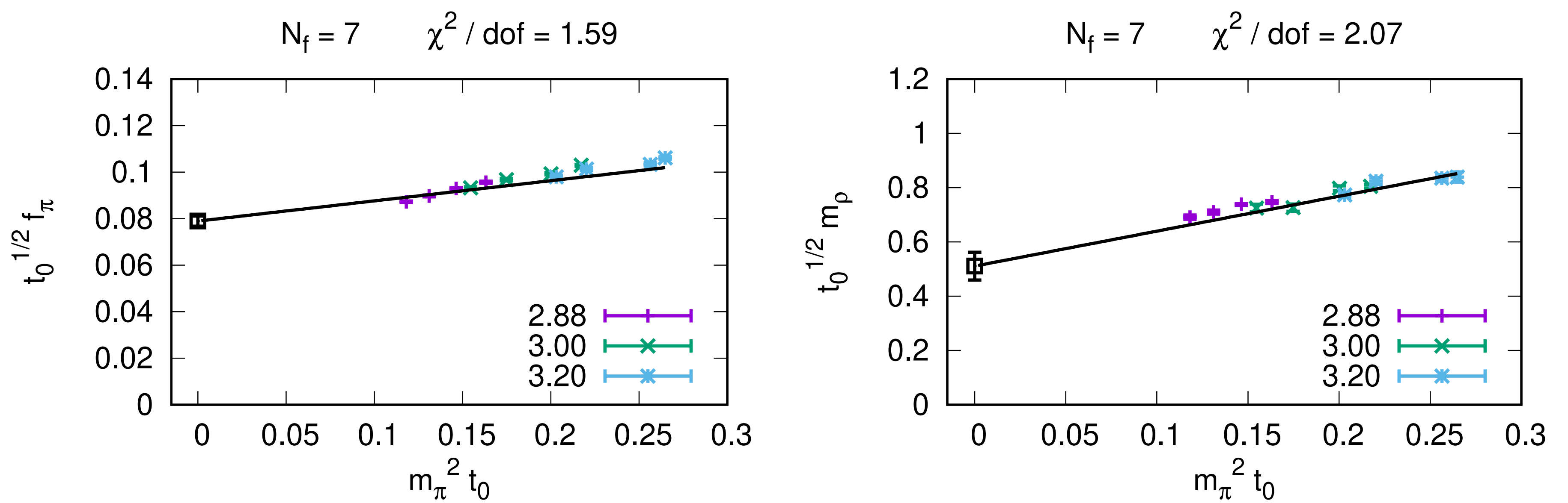


Chiral - continuum, $N_f = 6$ $\quad 20 \leq L/a \leq 36$

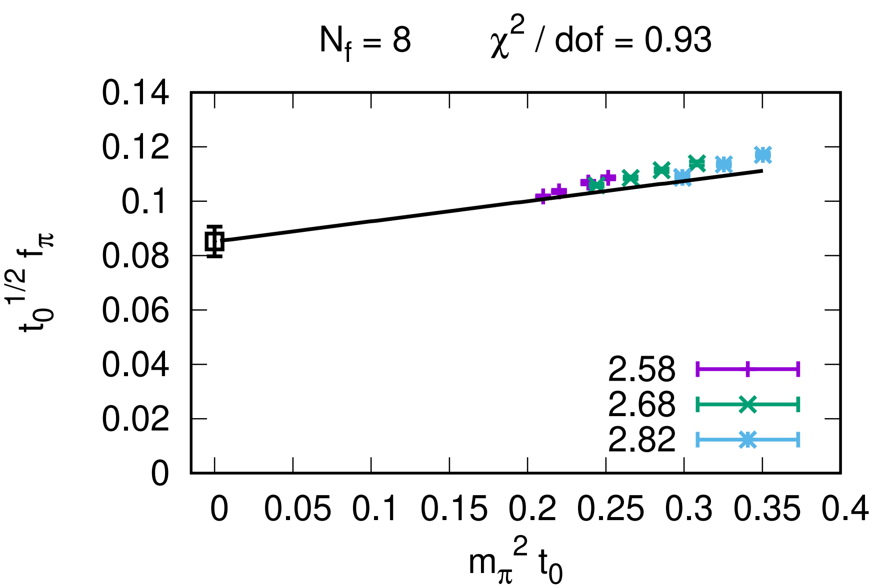


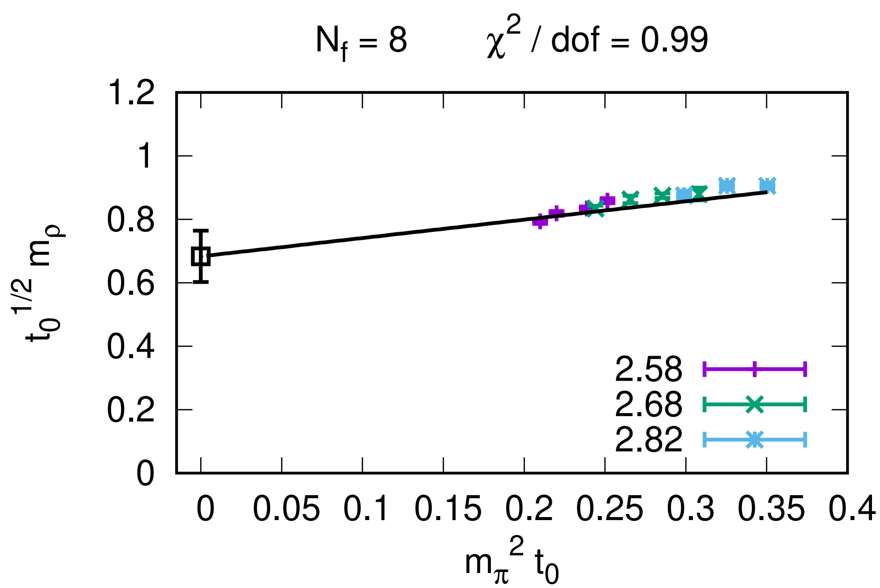


Chiral - continuum, $N_f = 7$ $24 \le L/a \le 40$

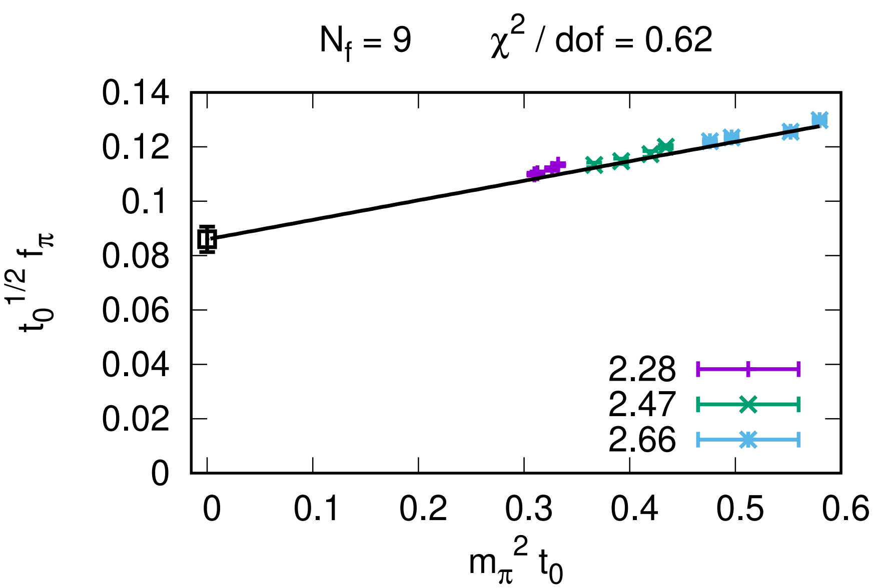


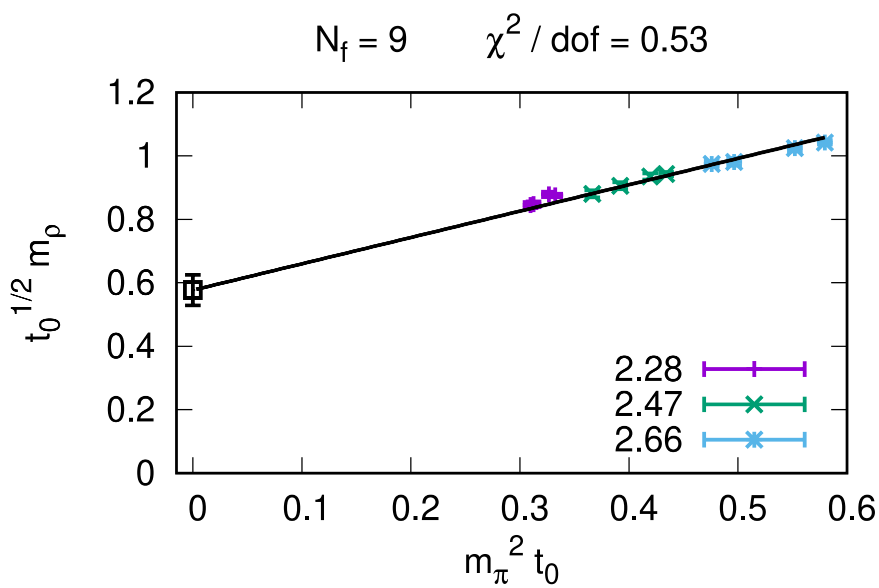
Chiral - continuum, $N_f = 8$ $\quad 24 \le L/a \le 40$



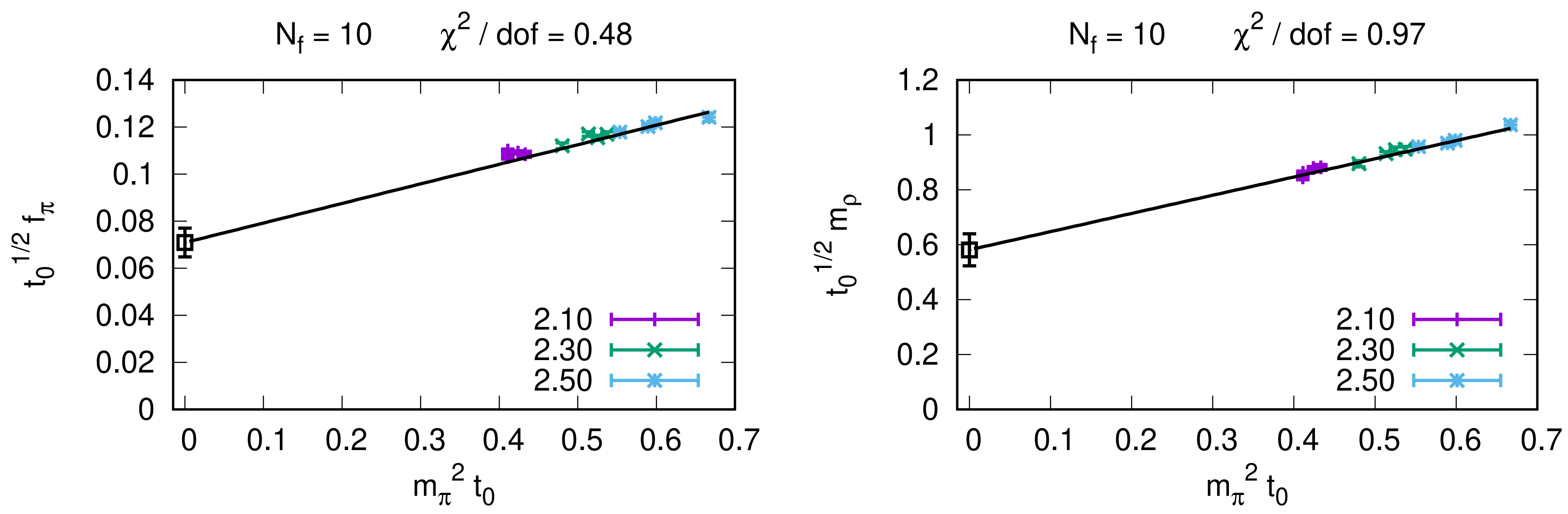


Chiral - continuum, $N_f = 9$ $\quad 28 \leq L/a \leq 48$

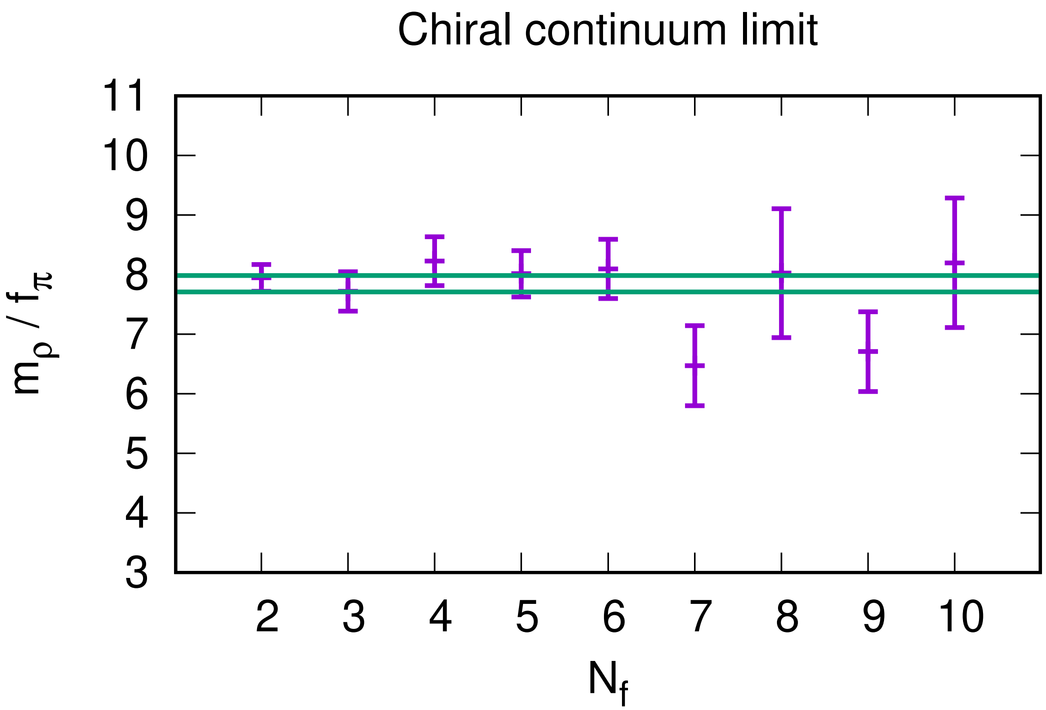




Chiral - continuum, $N_f = 10$ $\quad 32 \leq L/a \leq 48$



Chiral-continuum for ratio m_ϱ/f_π



Constant fit

$$\frac{m_\varrho}{f_\pi} = 7.85(14) \text{ with } \chi^2/dof = 1.1$$

Conclusion

No statistically significant N_f-dependence!

Inside conformal window

But N_f-dependence *must* enter at some N_f

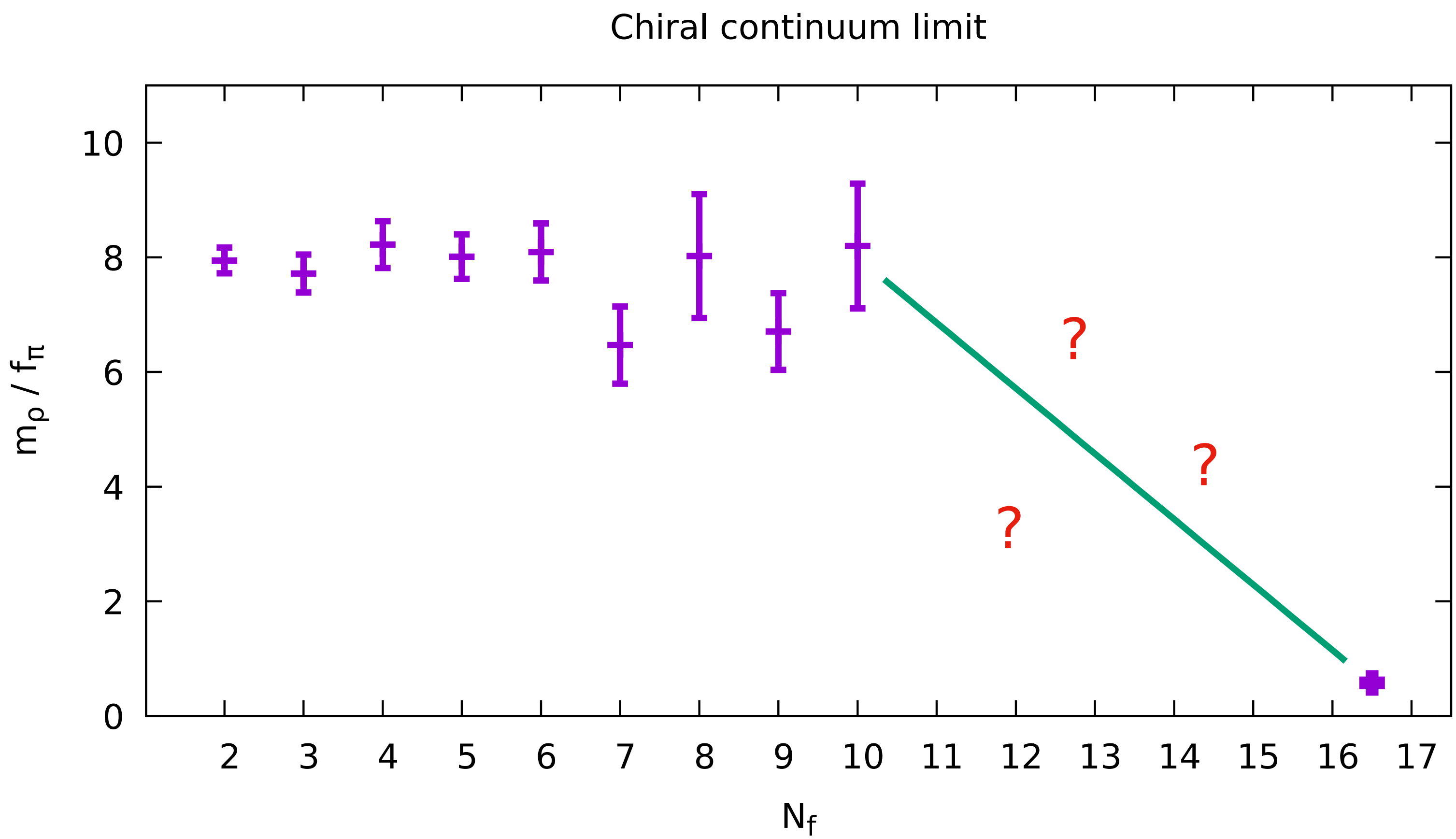
Below conformal window m_ϱ/f_π finite in chiral limit (of course)

Inside conformal window $m_\varrho \sim m^\alpha$, $f_\pi \sim m^\alpha$ with $\alpha = \frac{1}{1+\gamma}$

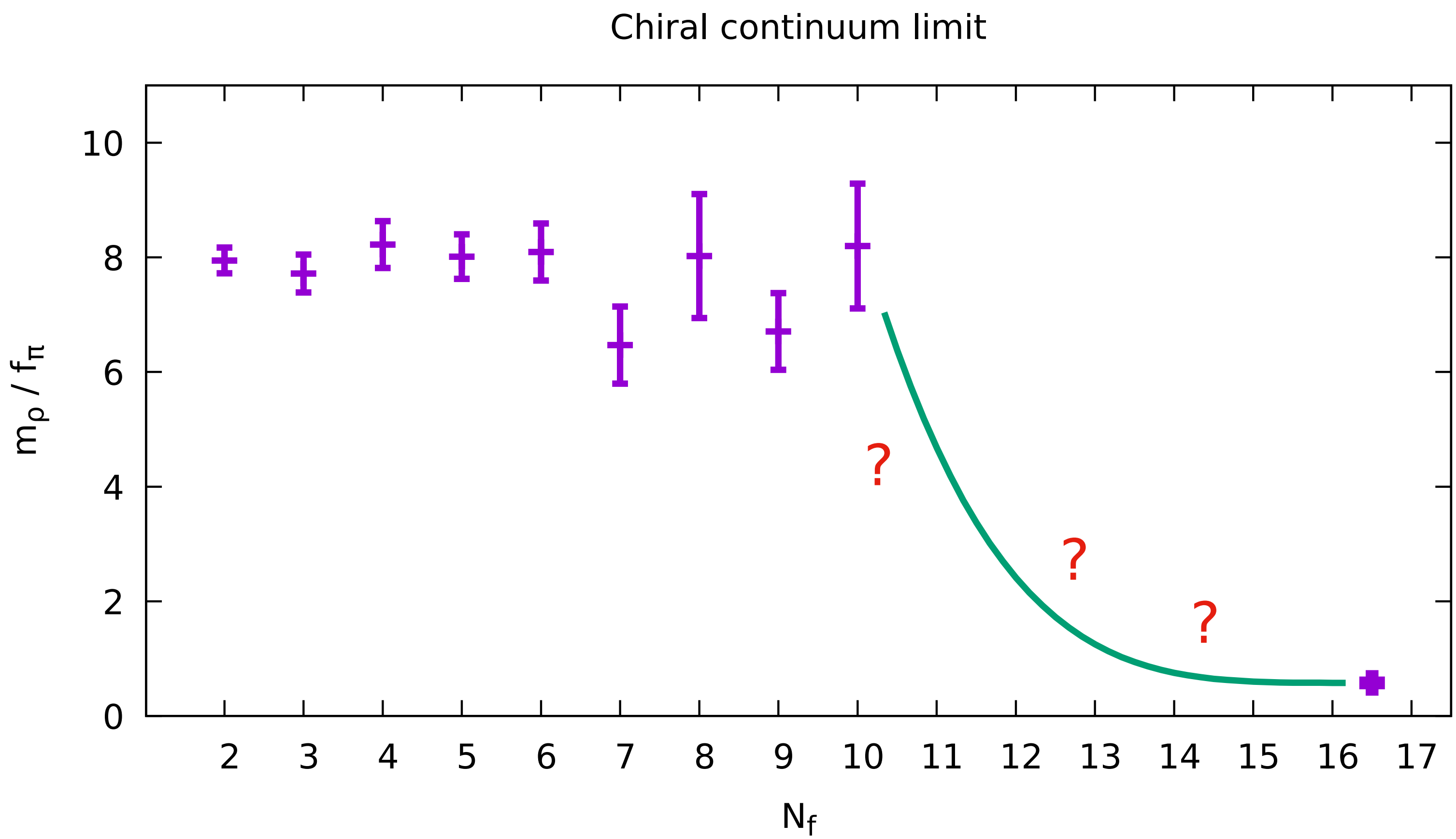
Inside conformal window m_ϱ/f_π finite in chiral limit also

$N_f = 16.5$ is free, $m_\varrho = 2m$ and $f_\pi = \sqrt{4N_c}\, m$ so $m_\varrho/f_\pi = 1/\sqrt{3}$

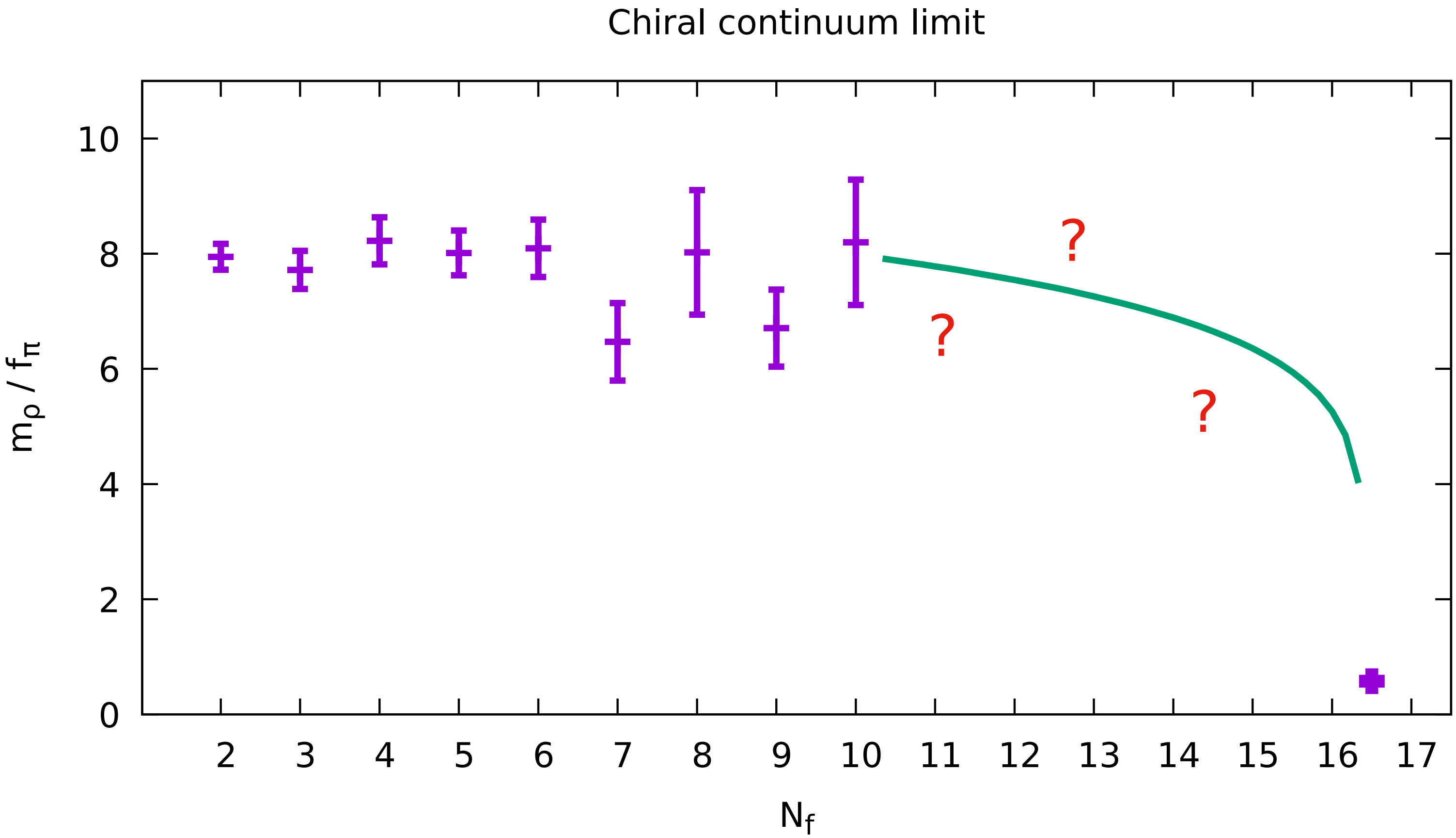
Inside conformal window - speculation



Inside conformal window - speculation



Inside conformal window - speculation



KSRF-relations

$$g_{\varrho\pi\pi} = \frac{m_\varrho}{f_\varrho} = \sqrt{48\pi\frac{\Gamma_\varrho}{m_\varrho}} = \frac{1}{\sqrt{2}}\frac{m_\varrho}{f_\pi}$$

Quite precise in QCD, should be more precise in $m \to 0$ limit

We have m_ϱ/f_π, assume KSRF $\to$ we have $g_{\varrho\pi\pi}$

$g_{\varrho\pi\pi}$ also N_f-independent ~ 5.55

Go back to $E(m_\varrho, g_{\varrho\pi\pi}, L)$ full Luscher $\to$ finite volume effects on m_ϱ small a posteriori

KSRF-relations

Assuming KSRF many ϱ related quantities are N_f-independent:

- $g_{\varrho\pi\pi}$
- Γ_ϱ/m_ϱ
- m_ϱ/f_ϱ

Looks like vector meson doesn't know anything about N_f

Gauge group dependence

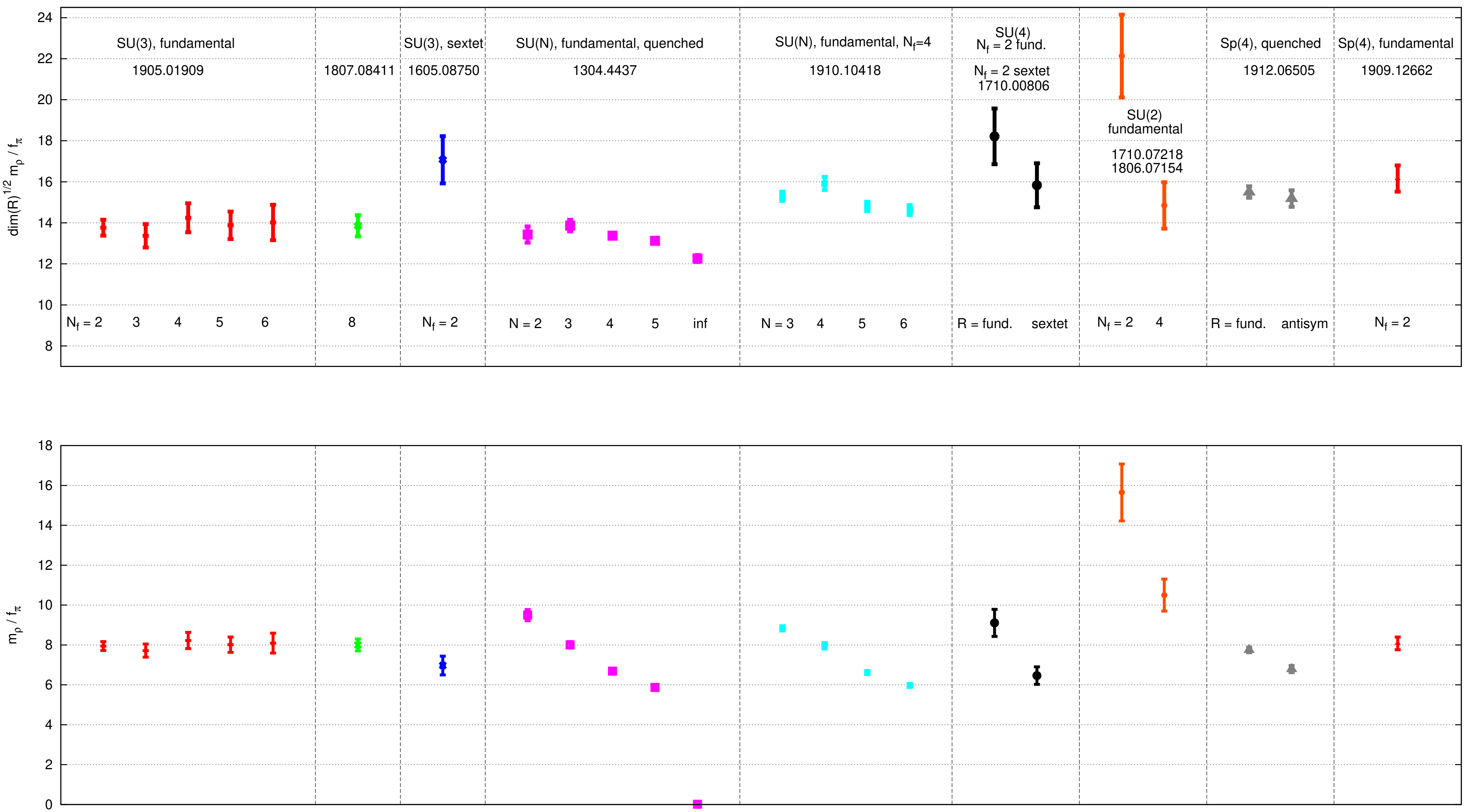
Large-N: $m_\varrho / f_\pi \sim 1/\sqrt{N}$

- $SU(2)$: $m_\varrho \sim 2.4\ TeV$
- $SU(3)$: $m_\varrho \sim 2.0\ TeV$
- $SU(4)$: $m_\varrho \sim 1.7\ TeV$
- $SU(5)$: $m_\varrho \sim 1.5\ TeV$
- $SU(6)$: $m_\varrho \sim 1.4\ TeV$

Other gauge groups and representations in literature

- $SU(2) \quad N_f = 2, 4 \quad R = fund$
- $SU(3) \quad N_f = 2 \quad R = sextet$
- $SU(N) \quad N_f = 0 \quad R = fund$
- $SU(N) \quad N_f = 4 \quad R = fund$
- $SU(4) \quad N_f = 2 \quad R = fund, sextet$
- $Sp(4) \quad N_f = 0 \quad R = fund, anti - symm$
- $Sp(4) \quad N_f = 2 \quad R = fund$

Comparison



Top: $\sqrt{dim(R)}\, m_\varrho / f_\pi$ where $dim(R) = N$ for $R = fund$

Conclusions

- Dynamics very N_f-dependent
- But: $m_\varrho/f_\pi = 7.85(14)$ for $SU(3)$ and $2 \le N_f \le 10$
- KSRF: $g_{\varrho\pi\pi}$ also N_f-independent
- BSM

  experimental result for m_ϱ: conclude about $SU(N)$ not R, N_f

Outlook

- Theoretical understanding of KSRF?
  Proven in SQCD — Komargodski 1010.4105

- Theoretical understanding of N_f-independence?

- N_f-dependence of m_ϱ/f_π inside the conformal window?

- Should reach $m_\varrho/f_\pi = 1/\sqrt{3} = 0.577$ at $N_f = 16.5$

Thank you for your attention!

Backup slides

Taste breaking

One could measure all taste broken Goldstones directly

Instead: look for a quantity with the most N_f-dependence and see if it is reproduced or not

Good candidate: topological susceptibility $\chi = \frac{\langle Q^2 \rangle}{V}$, very sensitive to light degrees of freedom

Continuum: $\chi = \frac{m_\pi^2 f_\pi^2}{2N_f}$

Taste breaking

Chiral - continuum extrapolate χ, see N_f-dependence

Extrapolation in w_0 units (dof=13 with $N_f = 2, \ldots, 6$)

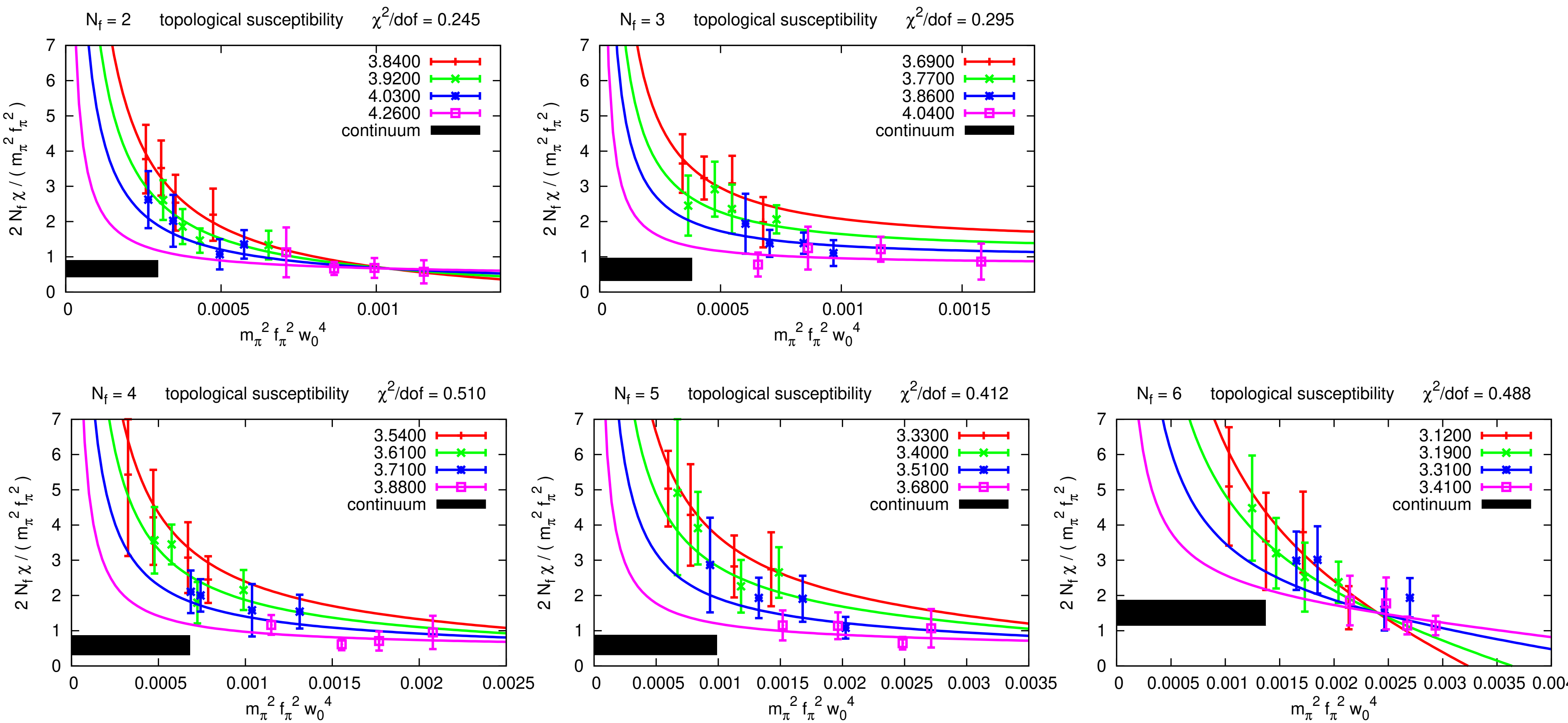
$$\chi w_0^4 = C_0 m_\pi^2 f_\pi^2 w_0^4 + C_1 \frac{a^2}{w_0^2} + C_2 \frac{a^2}{w_0^2}(m_\pi^2 f_\pi^2 w_0^4)$$

Continuum expectation: $C_0 = \frac{1}{2N_f}$

But taste broken Goldstones also enter χ

Billeter/Detar/Osborn hep-lat/0406032

Taste breaking - topological susceptibility



$\frac{2N_f\chi}{m_\pi^2 f_\pi^2}$ as a function of $m_\pi^2 f_\pi^2 w_0^4$ (should be 1)

Shift symmetry broken phase