Deconfining Phase Boundary of Rapidly Rotating Hot and Dense Matter and Analysis of Moment of Inertia

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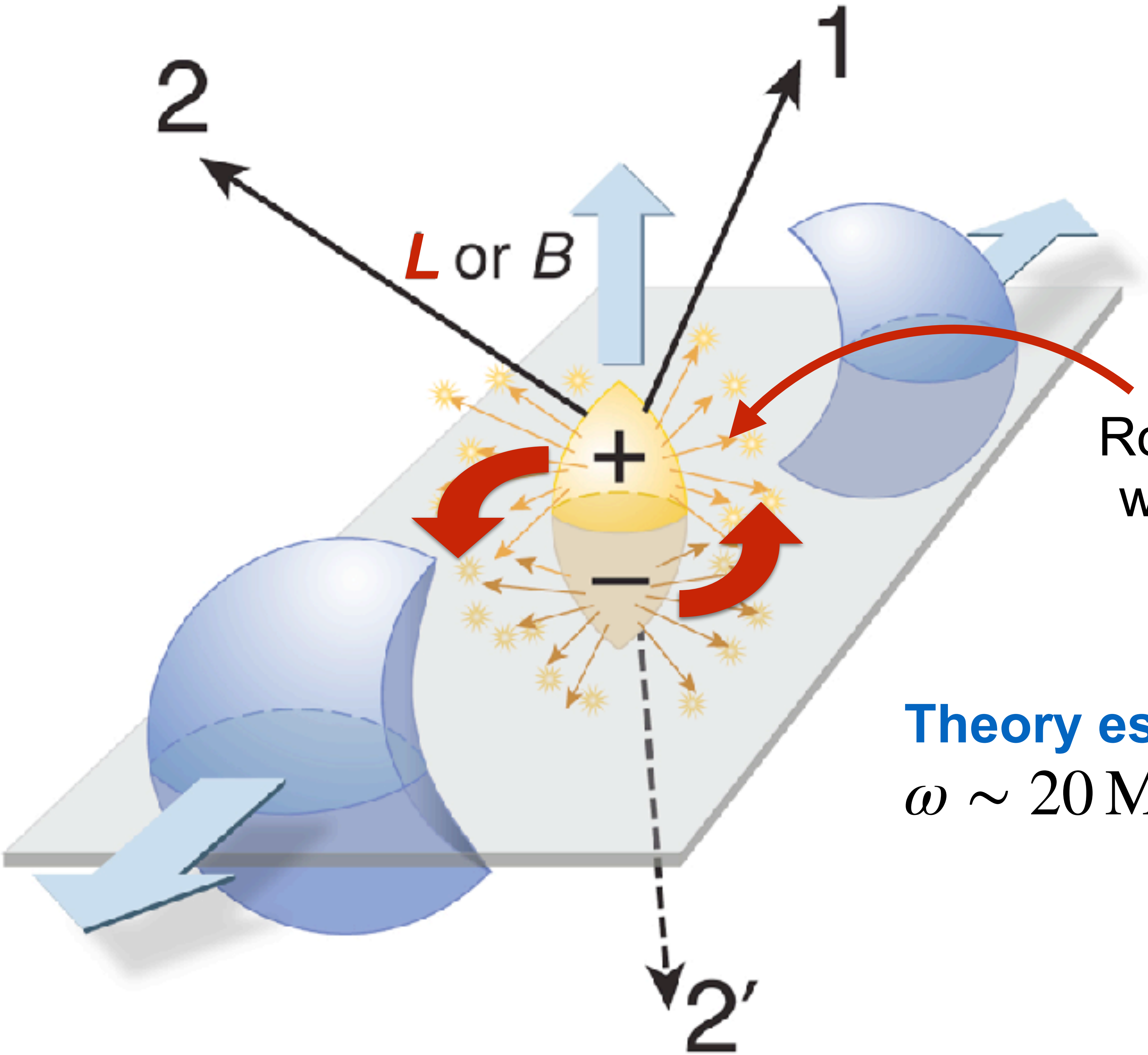
Reference:

Y. Fujimoto, K. Fukushima, Y. Hidaka,"Deconfining Phase Boundary of Rapidly Rotating Hot and Dense Matter and Analysis of Moment of Inertia," Phys. Lett. B 816 (2021) 136184, arXiv:[2101.09173 [hep-ph]](https://arxiv.org/abs/2101.09173).

12 November 2021, Hard Problems of Hadron Physics

Rotating quark-gluon matter

Non-central heavy-ion collisions:



Rotating matter is created with angular momentum

$$L \sim 10^6 \, \hbar$$

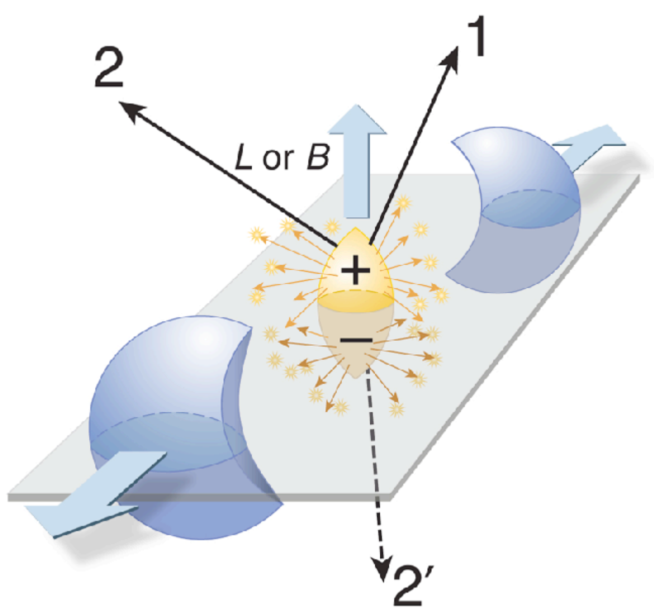
Theory estimate:

$\omega \sim 20\,\mathrm{MeV}$ Jiang,Lin,Liao (2016)

Global Λ polarization

STAR collaboration (2017)

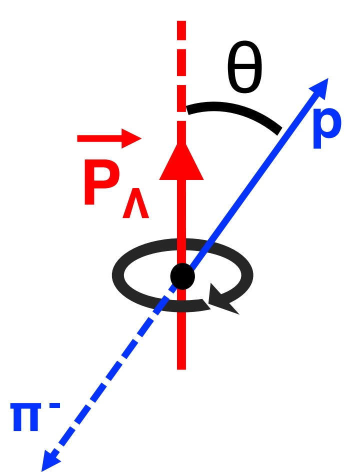
Experimental estimate (global Λ polarization):

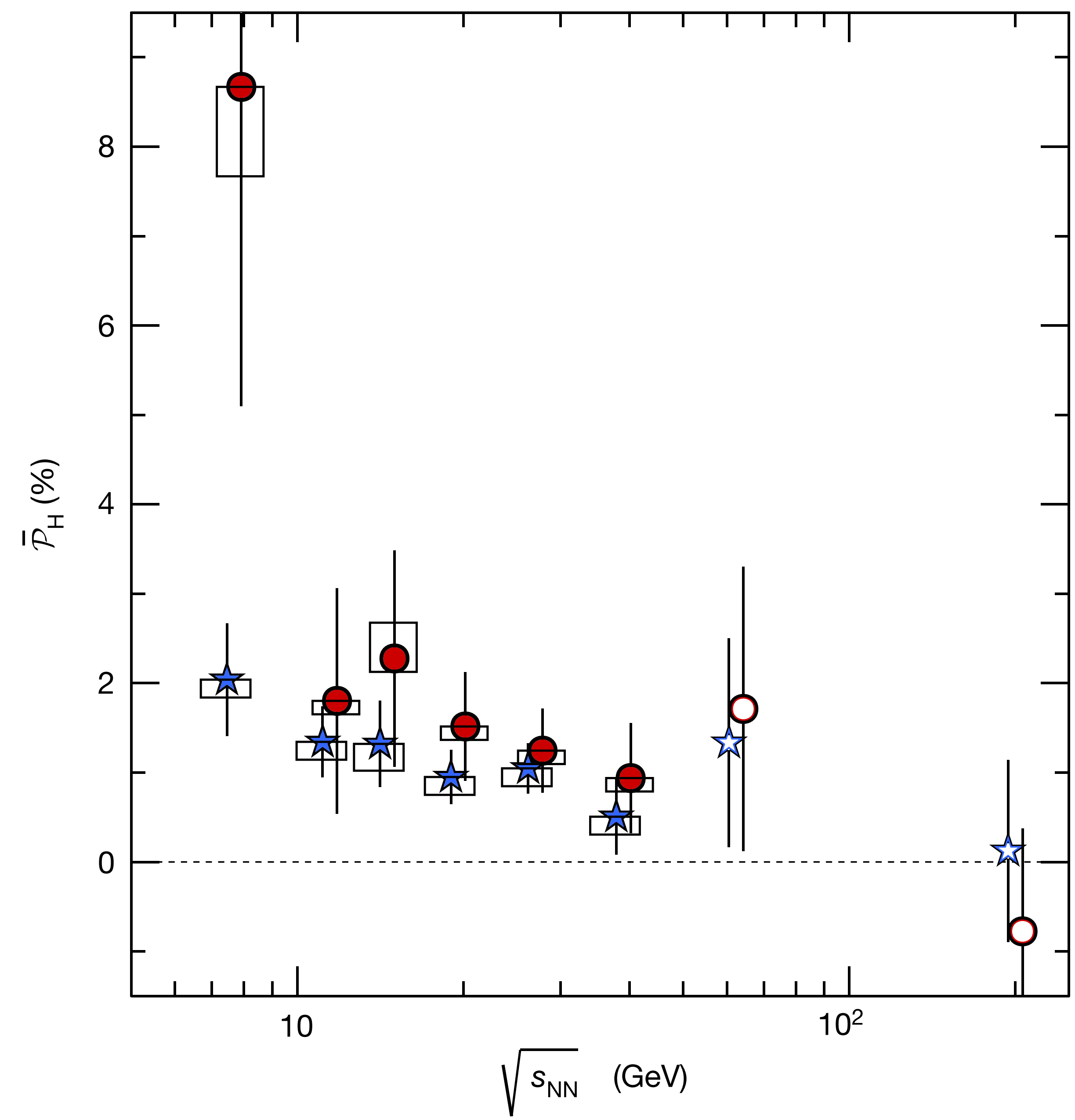


parity-violating weak decay of Λs

$$\Lambda \to p + \pi^-$$

(BR: 63.9%, cτ~7.9 cm)



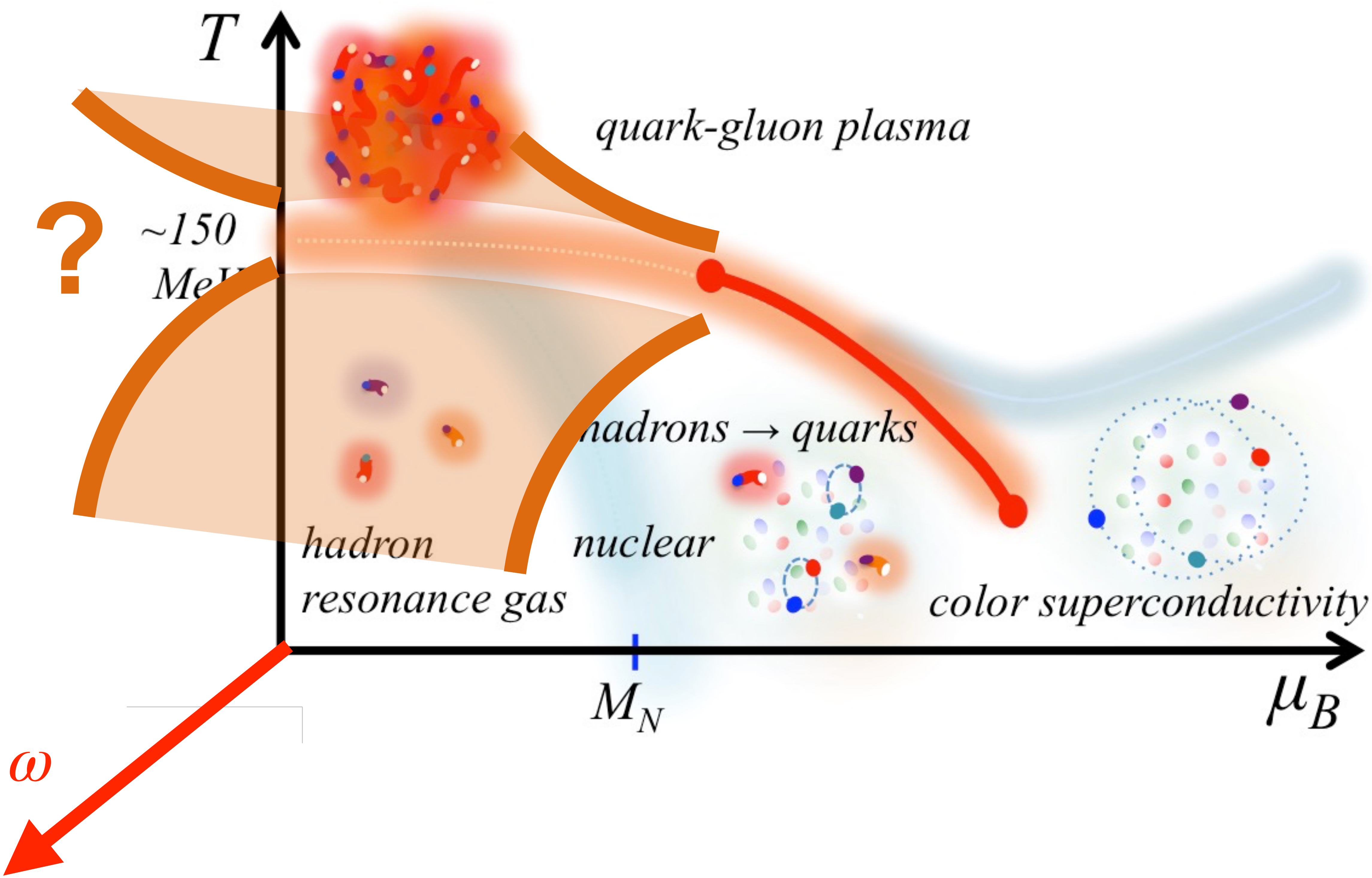


predicts roughly $\omega \sim 6\,\mathrm{MeV}$

Figure taken from T. Niida's slide

QCD phase diagram under rotation

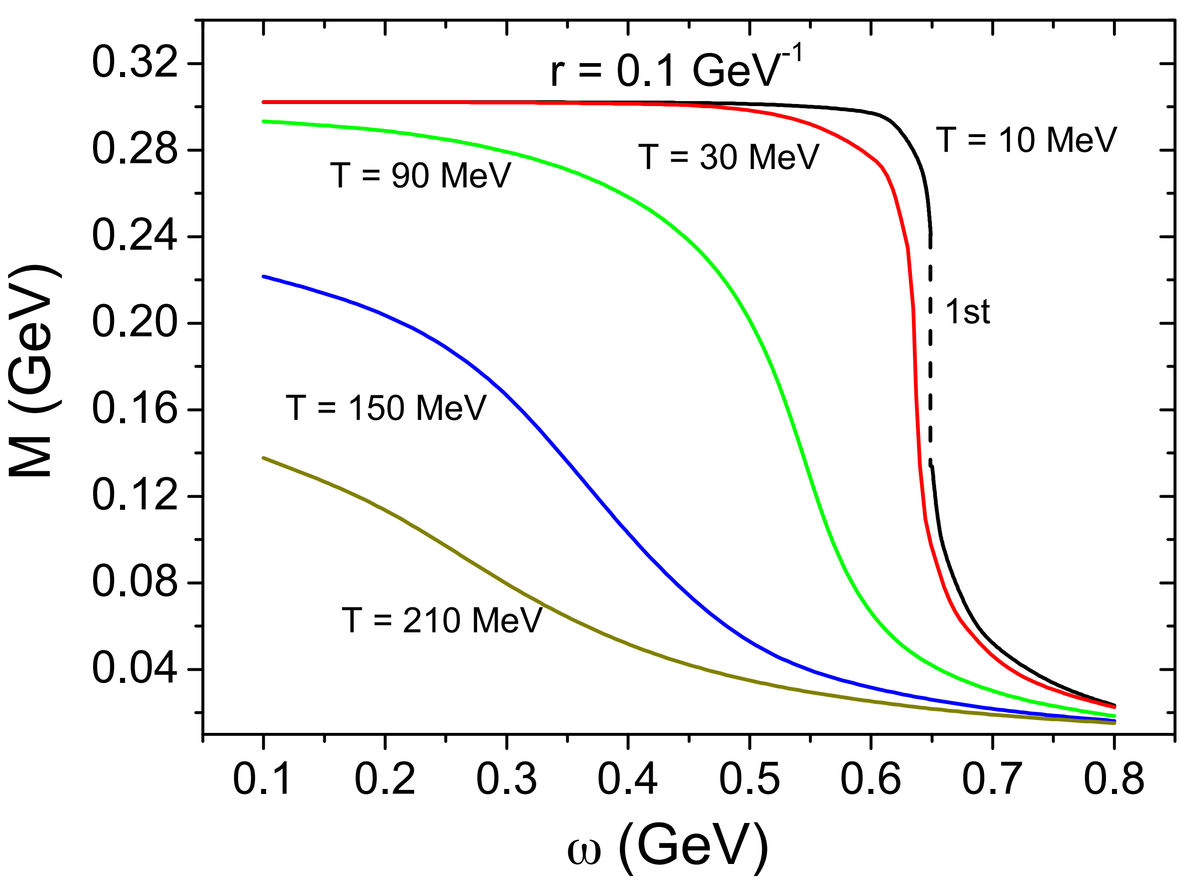
Taken from: Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

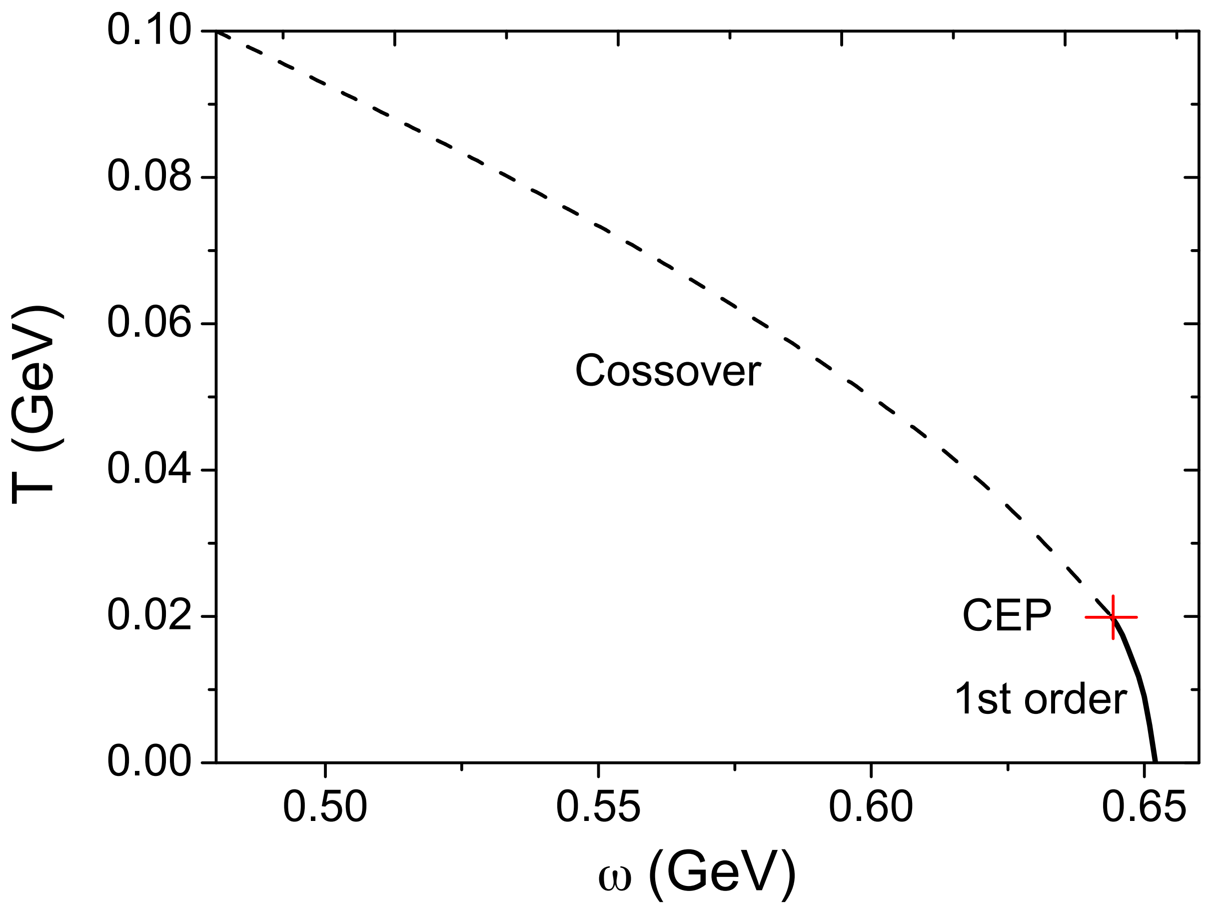


Chiral transition of rotating matter

Jiang,Liao (2016)

NJL model analysis shows…





↑ Rotation suppresses the chiral condensate

More or less accepted consensus:

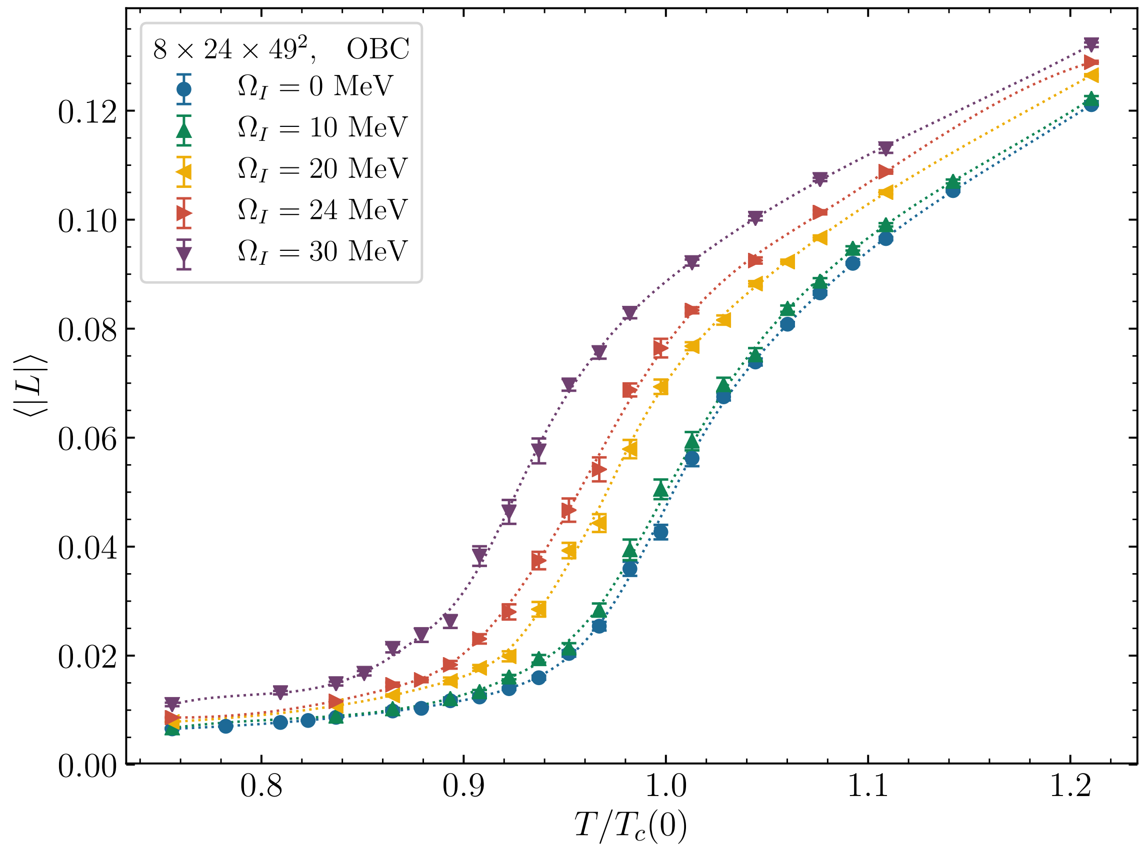
Critical temperature T_c **drops with increasing** ω

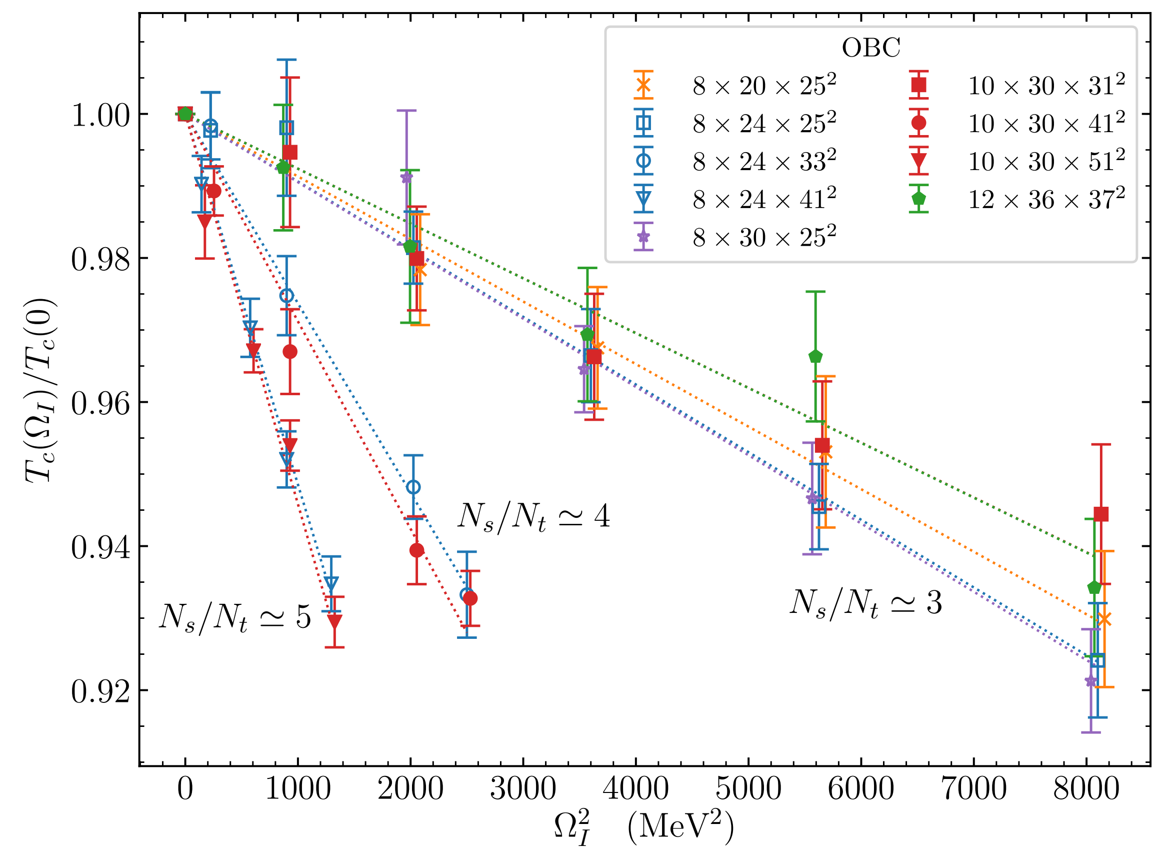
Other studies: Ebihara,Fukushima,Mameda (2016); Chernodub,Gongyo (2016); Wang,Wei,Li,Huang (2018); Zhang,Hou,Liao (2018); …

Deconfinement of rotating matter

Lattice formulation of imaginary rotation: Yamamoto,Hirono (2013)
Braguta,Kotov,Kuznedelev,Roenko (2020,21)

Lattice result of the Polyakov loop in pure QCD under imaginary rotation $\Omega_I = -i\omega$:





Deconfinement temperature T_c **rises** with increasing ω

$$\frac{T_c(\Omega_I)}{T_c(0)} = 1 - C\Omega_I^2$$

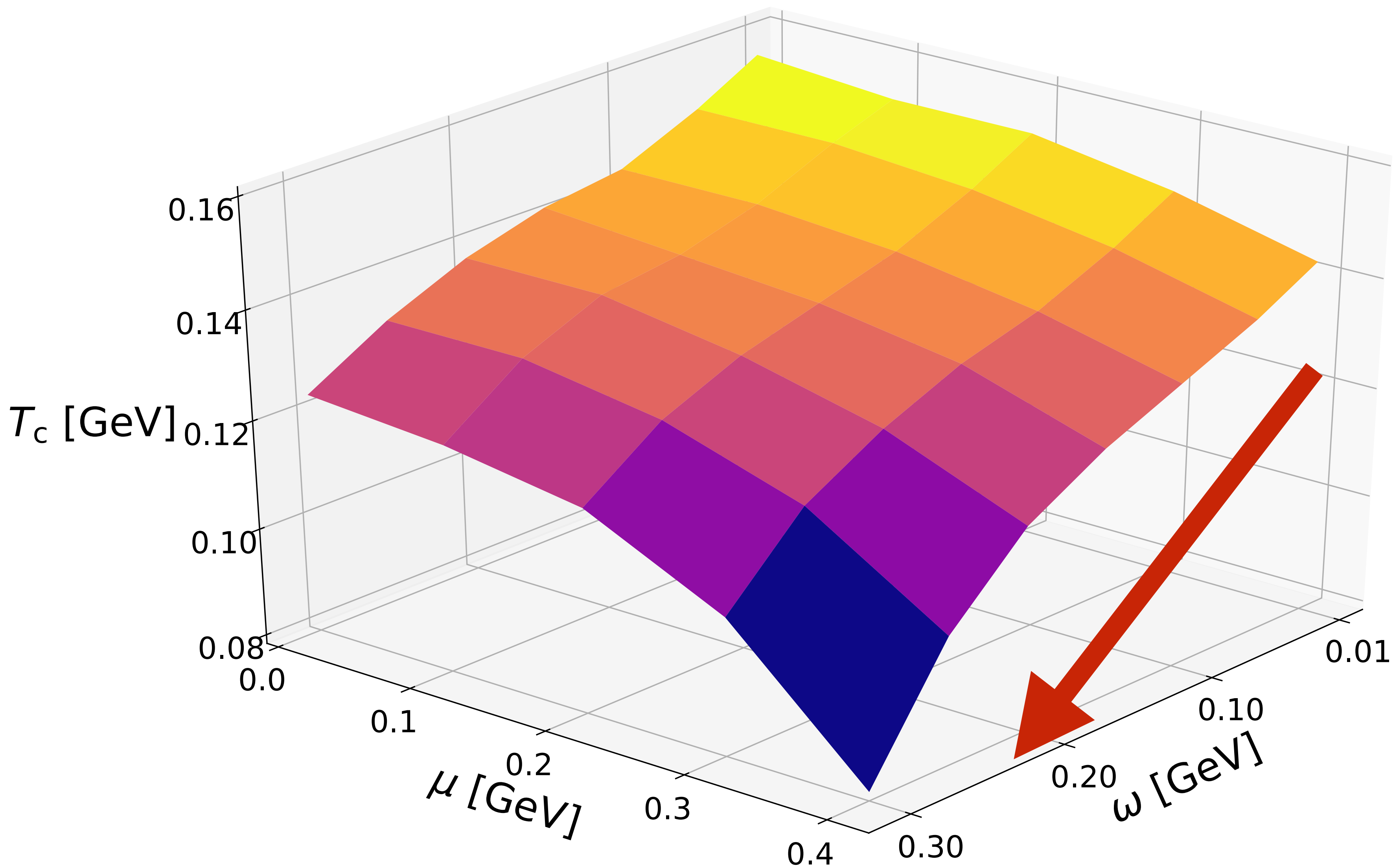
$$= 1 + C\omega^2$$

In tension with chiral transition!?

Deconfinement of rotating matter

Fujimoto,Fukushima,Hidaka (2021)

Our result based on the hadron resonance gas model:



Deconfinement temperature T_c **drops** with increasing ω

See also: Holography approach: Chen,Zhang,Li,Hou,Huang (2020)
Compact QED approach: Cherdodub (2020)
these works also give the same behavior as ours

Our phenomenological approach

Hadron resonance gas (HRG) model

total pressure: $p(T, \mu) = \sum_i p_i^{\text{ideal}}$

Only control parameter (T, μ);
Parameter free (fixed by experiments)

Each particle's contribution is very small,
but in total, it becomes big

$$p_i^{\text{ideal}} = \pm \frac{T}{8\pi^2} \int dk_r^2 \int dk_z \, (2S_i + 1) \times \log\left\{ 1 \pm \exp\left[-\frac{E_{k,i} - \mu_i}{T} \right] \right\}$$

i : particle specie (e.g., π, K, p, n, …); $E_{k,i} = \sqrt{k^2 + m_i^2}$

Rotating reference frame

General coordinate transformation:

$\bar{x}^\mu$: **non-rotating** $\quad\rightarrow\quad$ x^μ: **rotating**

$$\begin{matrix}\bar{x} \\ \bar{y}\end{matrix} \rightarrow \begin{aligned} x &= +\bar{x}\cos\omega t + \bar{y}\sin\omega t \\ y &= -\bar{x}\sin\omega t + \bar{y}\cos\omega t \end{aligned}$$

$$g_{\mu\nu} = \eta_{ab}\frac{\partial \bar{x}^a}{\partial x^\mu}\frac{\partial \bar{x}^b}{\partial x^\nu} = \begin{pmatrix} 1-(x^2+y^2)\omega^2 & y\omega & -x\omega & 0 \\ y\omega & -1 & 0 & 0 \\ -x\omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

Energy spectrum: $\varepsilon \rightarrow \varepsilon - (\ell + s)\omega$

$(s = -S, -S+1, \cdots, S-1, S$ for spin-S particles$)$

Rotating hadron resonance gas model

$$p(T,\mu,\omega) = \sum_i p_i^{\rm rot}$$

$$p_i^{\rm rot} = \pm\frac{T}{8\pi^2}\int_{\boxed{(\Lambda_\ell^{\rm IR})^2}} dk_r^2 \int dk_z \boxed{\sum_{\ell=-\infty}^{\infty}\sum_{\nu=\ell}^{\ell+2S_i} J_\nu^2(k_r r)}$$
$$\times \log\left\{1 \pm \exp\left[-\frac{E_{k,i} \boxed{-(\ell+S_i)\omega} - \mu_i}{T}\right]\right\}$$

momentum discretization

$$k_r \geq \frac{\xi_{\ell,1}}{R}$$

& causality

$$R\omega \leq 1$$

$$\Lambda_\ell^{\rm IR} = \xi_{\ell,1}\omega$$

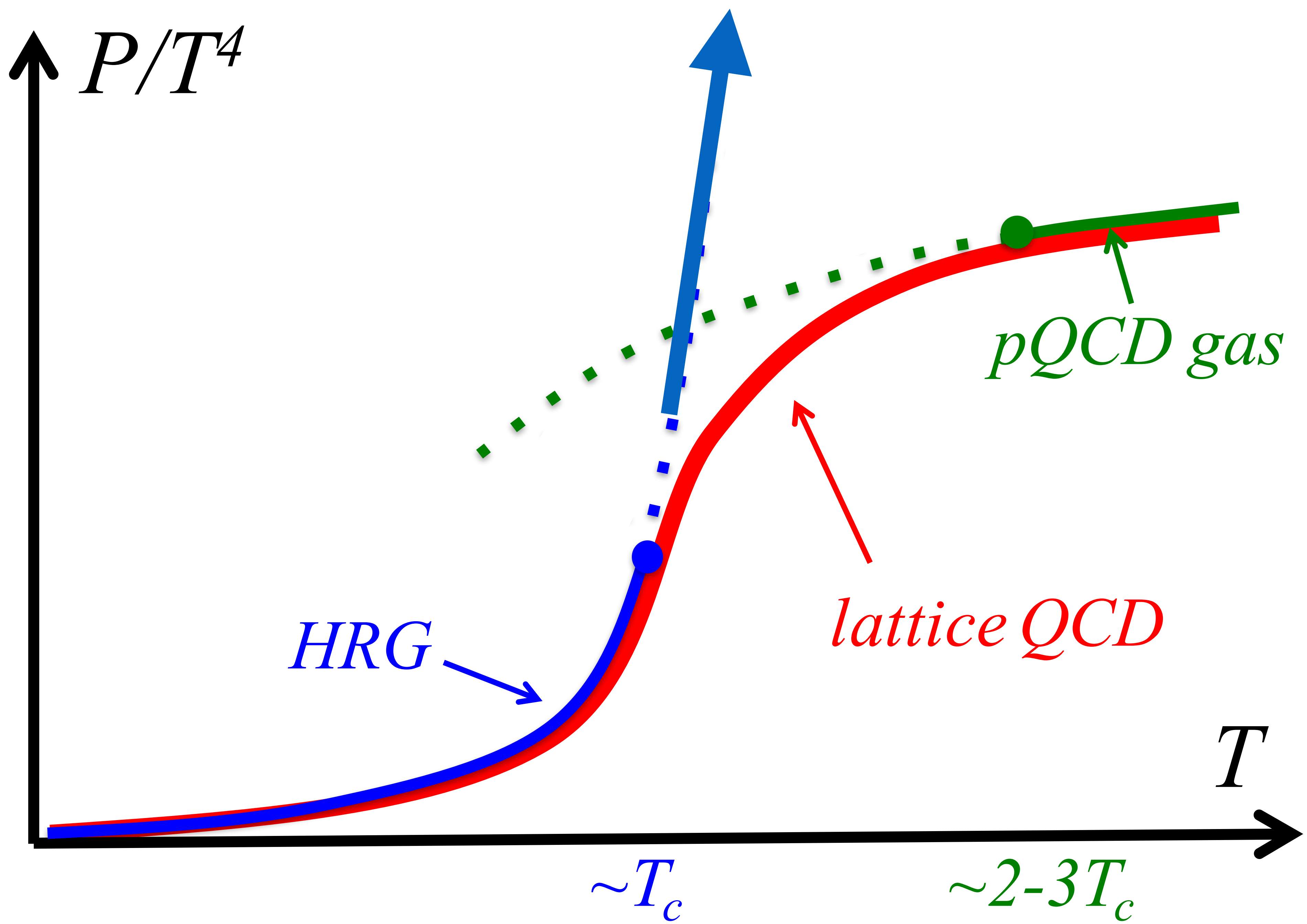
Compare with non rotating expression:

$$p_i^{\rm ideal} = \pm\frac{T}{8\pi^2}\int dk_r^2 \int dk_z\,(2S_i+1)$$
$$\times \log\left\{1 \pm \exp\left[-\frac{E_{k,i}-\mu_i}{T}\right]\right\}$$

HRG model is purely hadronic model,
but how can it capture the deconfinement of quarks?

Deconfinement in hadron resonance gas

HRG blow up → Signal for deconfinement



Taken from: Baym,Hatsuda,Kojo,Powell,Song,Takatsuka (2017)

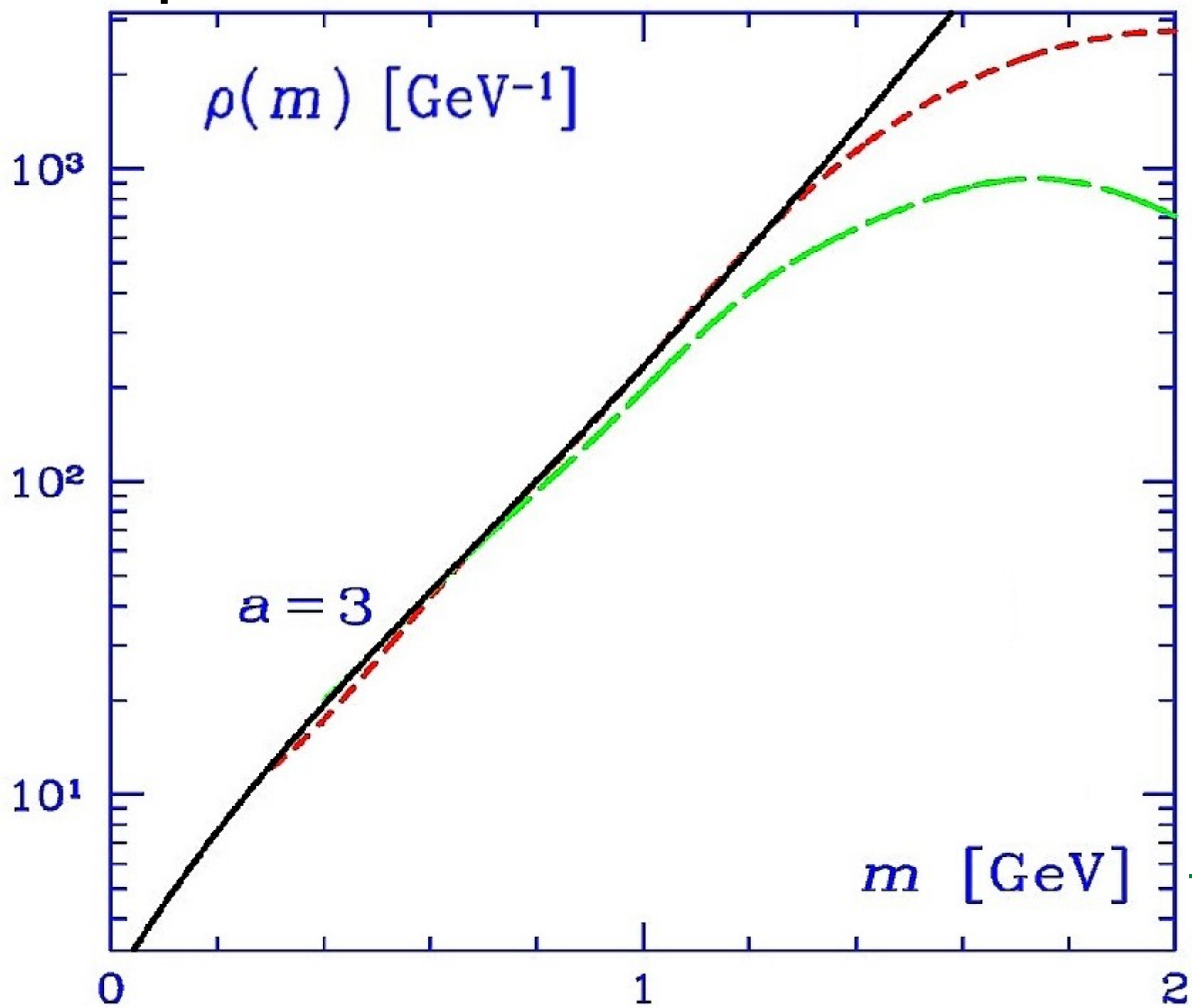
Deconfinement in hadron resonance gas

Hagedorn (1965)

$$Z = N \int dm\, \rho(m) e^{-m/T}, \qquad \rho(m) \propto e^{m/T_{\mathrm{H}}}$$

T_H: Hagedorn's limiting temeperature

hadron mass spectrum:



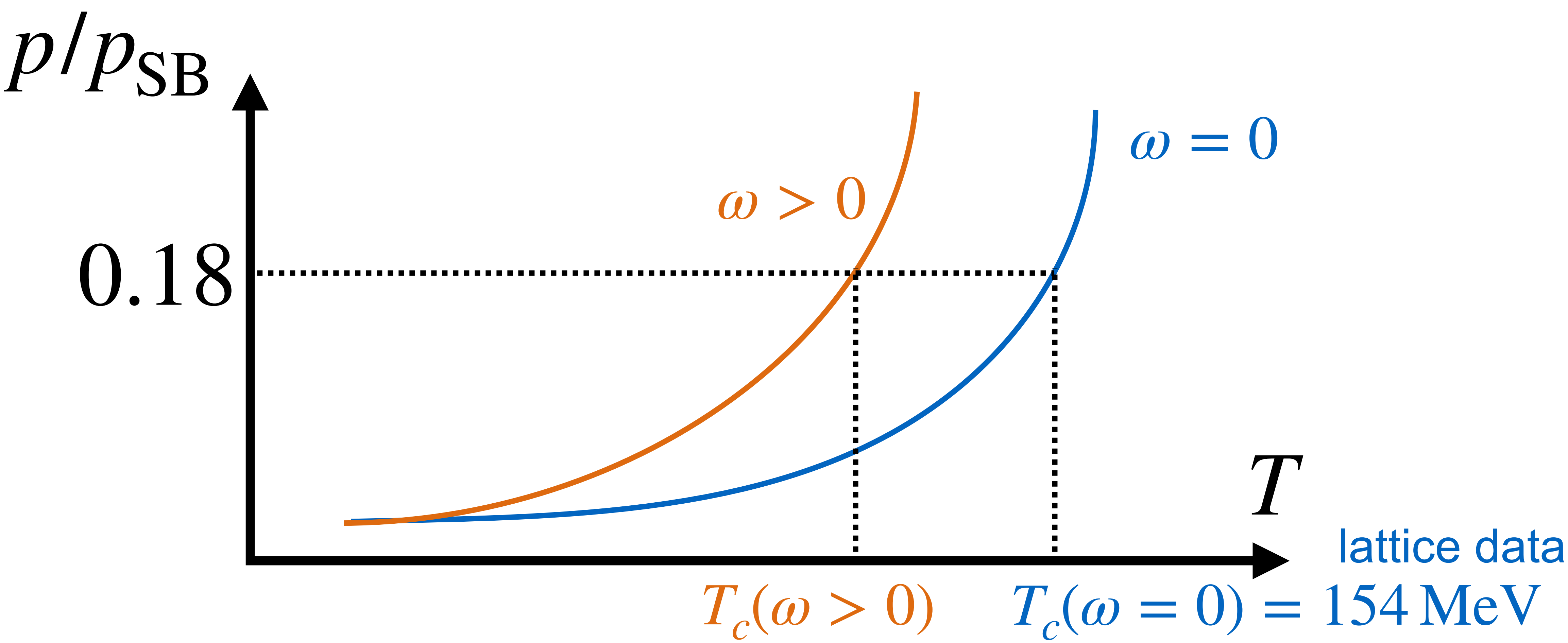
Taken from Rafelski (2016)

Our criterion of deconfinement

For each given (μ, ω), we identify T that satisfies the following condition as T_c:

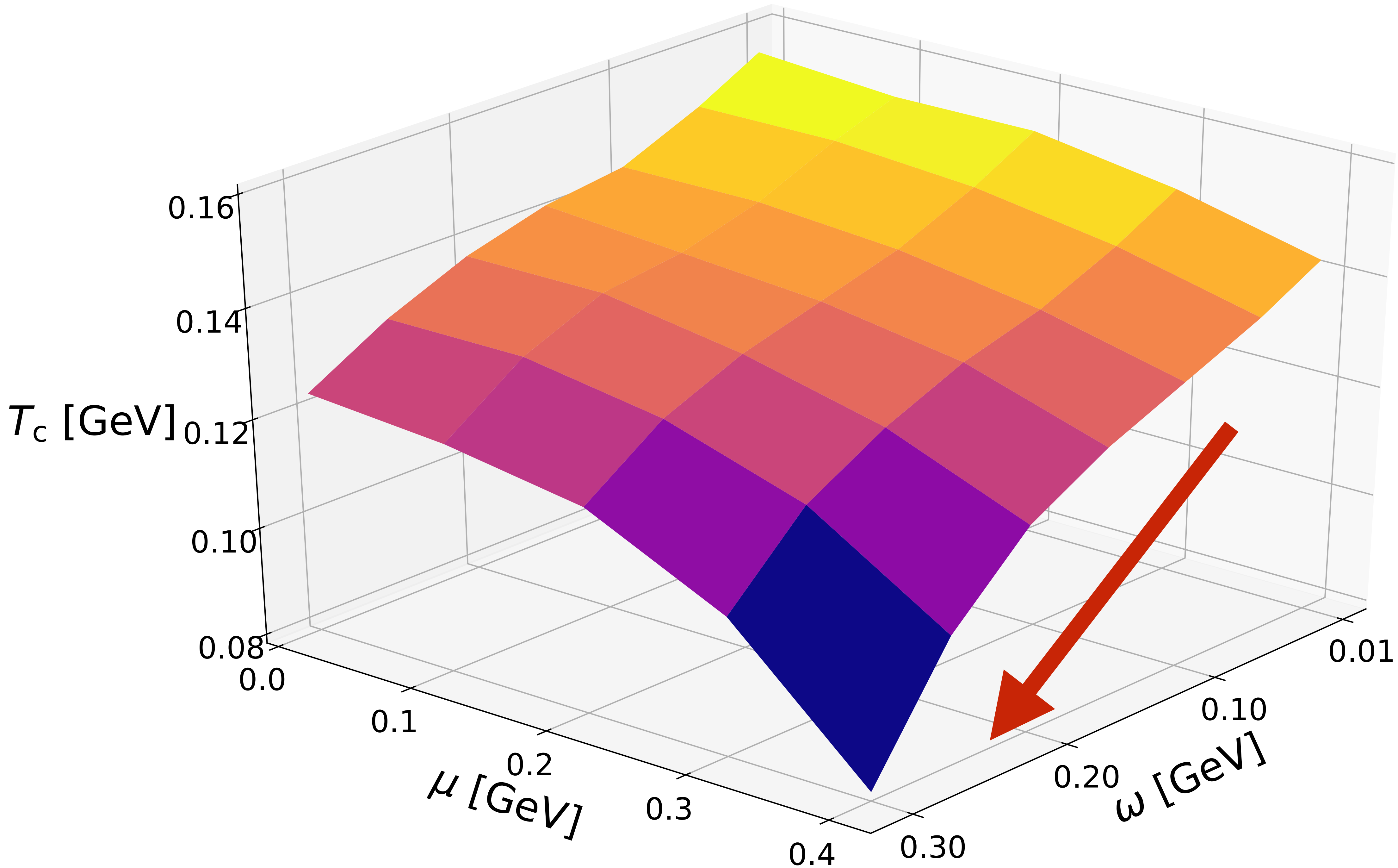
$$\frac{p}{p_{\mathrm{SB}}}(T = T_c, \mu, \omega) = 0.18$$

$$p_{\mathrm{SB}} \equiv (N_c^2 - 1)p_{\mathrm{gluon}} + N_c N_f (p_{\mathrm{quark}} + p_{\mathrm{antiquark}})$$



Deconfining boundary

Fujimoto,Fukushima,Hidaka (2021)

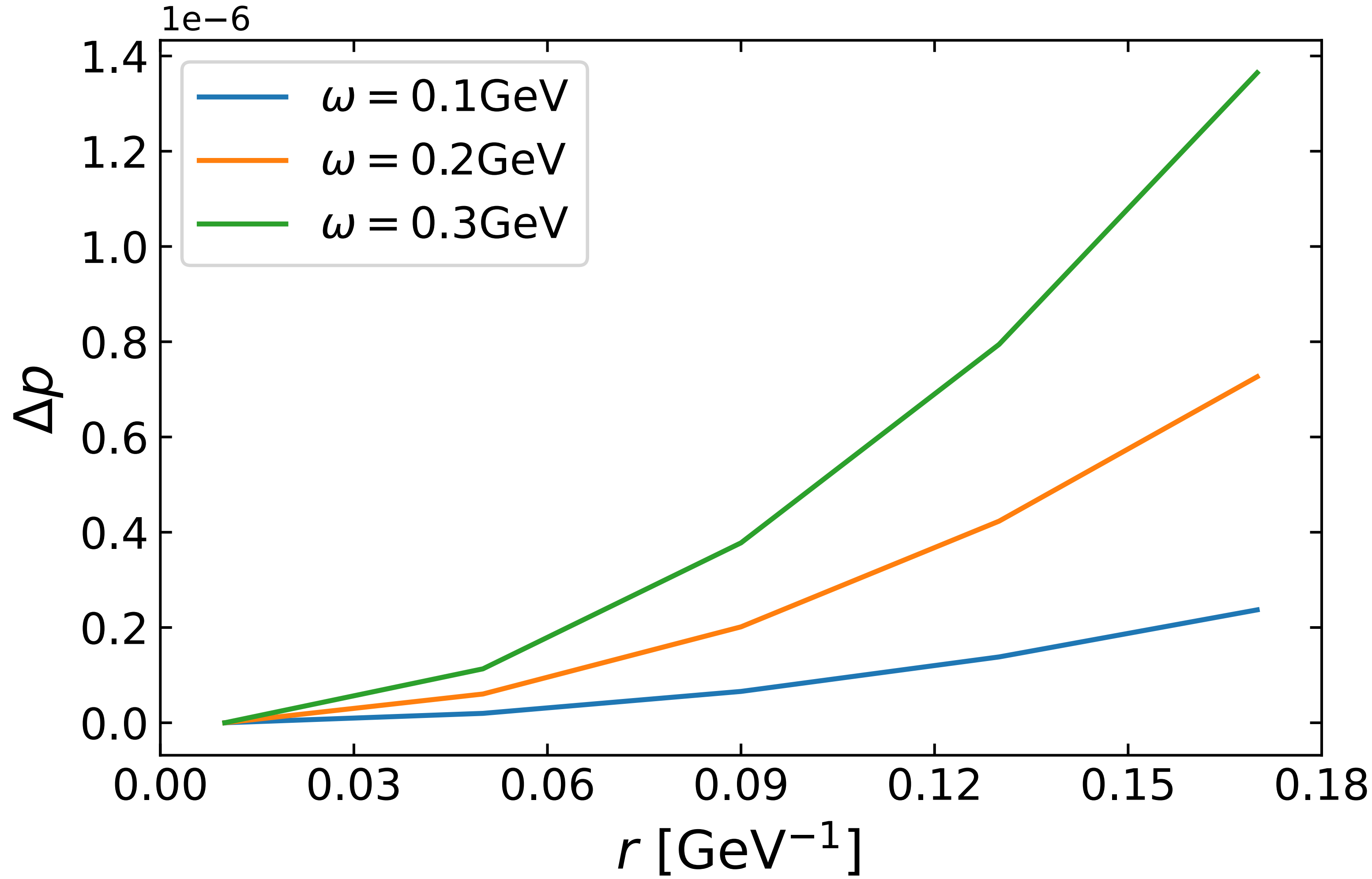


Deconfinement temperature T_c **drops**

with increasing ω

Radial dependence of pressure

radial distance dependent part of pressure

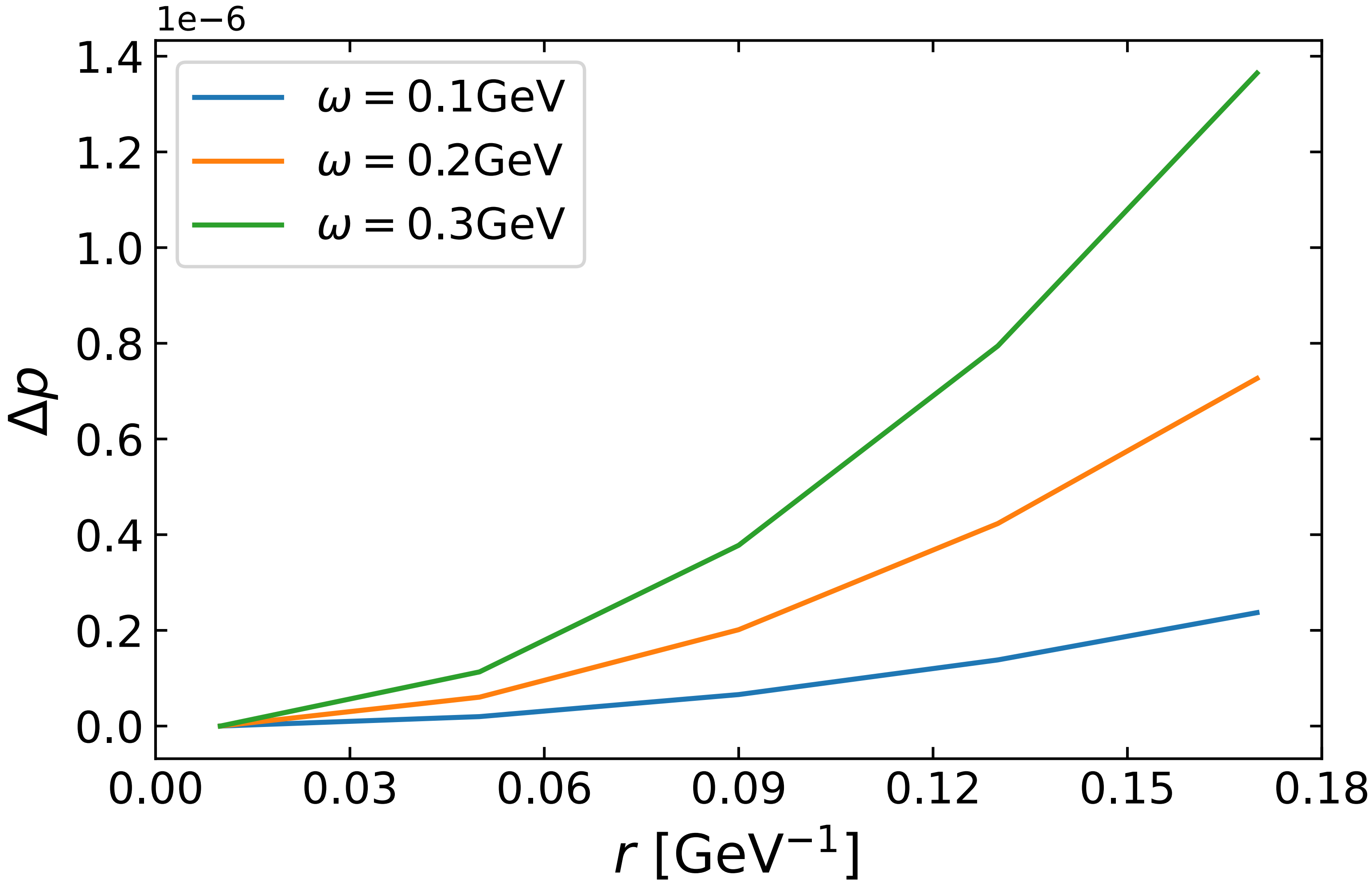


$$p_i^{\mathrm{rot}} = \pm \frac{T}{8\pi^2} \int dk_r^2 \int dk_z \sum_{\ell=-\infty}^{\infty} \sum_{\nu=\ell}^{\ell+2S_i} \boxed{J_\nu^2(k_r r)}$$

$$\times \log\left\{ 1 \pm \exp\left[-\frac{E_{k,i} - (\ell + S_i)\omega - \mu_i}{T} \right] \right\}$$

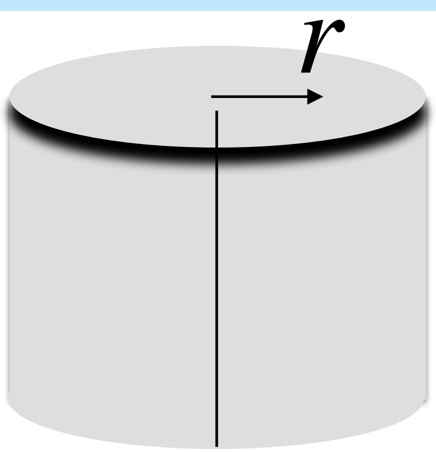
this r-dependence leads to r-dependent pressure

Radial dependence of pressure



$$\langle j \rangle (r) = \frac{\partial p(r)}{\partial \omega}$$

(cf: $n = \partial p / \partial \mu$)



$$\langle j \rangle (r)\, dV \simeq \boxed{dI(r)}\,\omega$$

moment of inertia $\propto r^2$

$$p(r) = p(0) + \Delta p(r)$$

σ can be read off from the plot

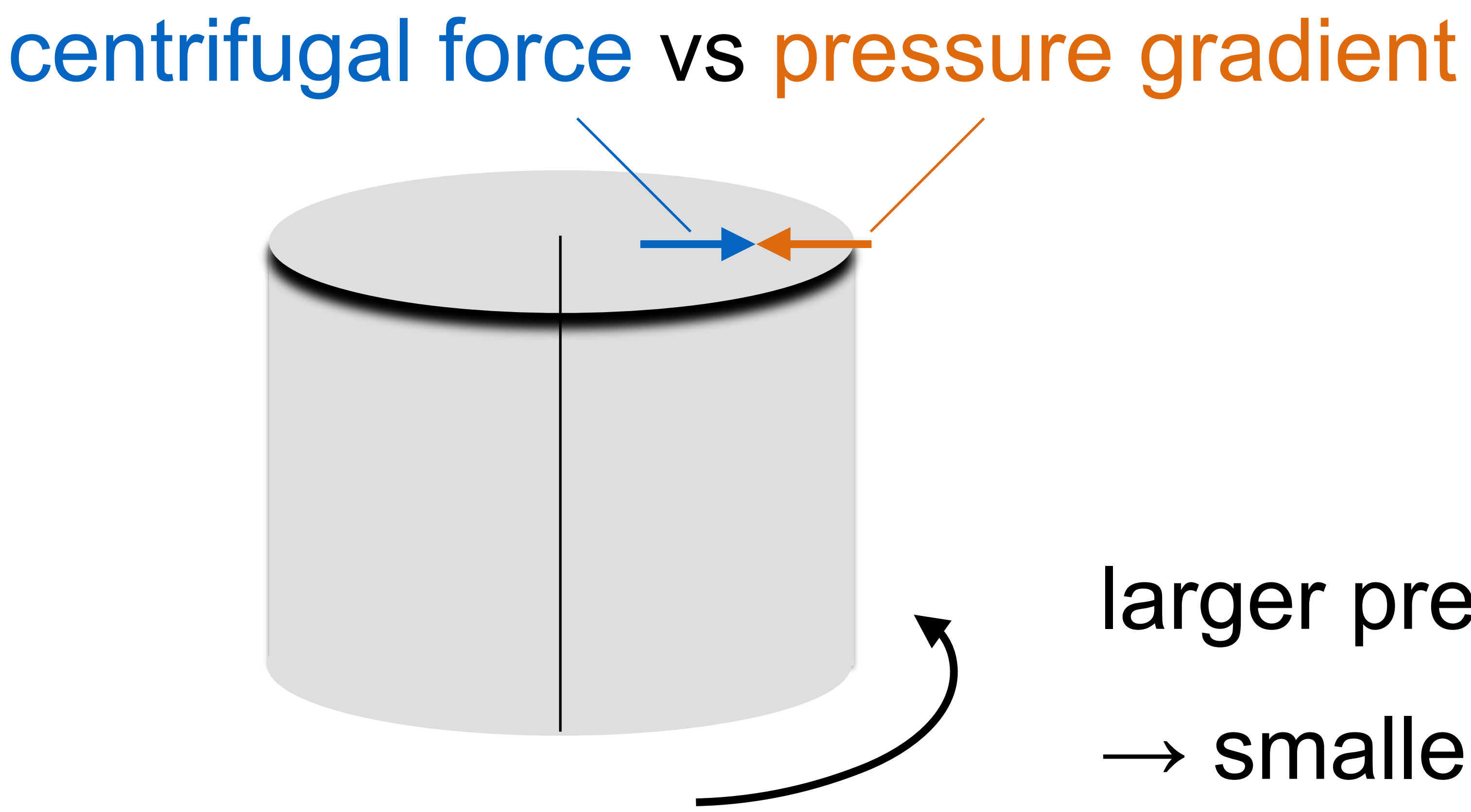
$$\sigma = 3.21 \longleftarrow \Delta p(r) \simeq \frac{\sigma}{2} T^4 r^2 \omega^2$$

Thermal d.o.f.:
$\nu \simeq 7$

$$\left(\sigma = \frac{2\pi^2 \nu}{45}\right)$$

larger than pion gas, but smaller than QGP → crossover region

Radial dependence of pressure



larger pressure outside

→ smaller free energy outside

→ Poyakov loop becomes

larger outside

$$e^{-f_q/T} = \langle \mathscr{P} e^{ig\int dx_4 A_4} \rangle$$

Summary

- Estimated the rotation effect on the deconfinement transition in QCD:
  the critical temperature T_c **drops** with increasing rotation

- We used the **Hadron Resonance Gas model**: a phenomenological and parameter-free approach

- Still there is a tension between our and the lattice result;
  We are looking for the thermodynamics at finite rotation on lattice.

- Radial dependent pressure may be interesting to see in the future analysis. Also, HRG only for gluonic sector?