Anomalies for anomalous symmetries

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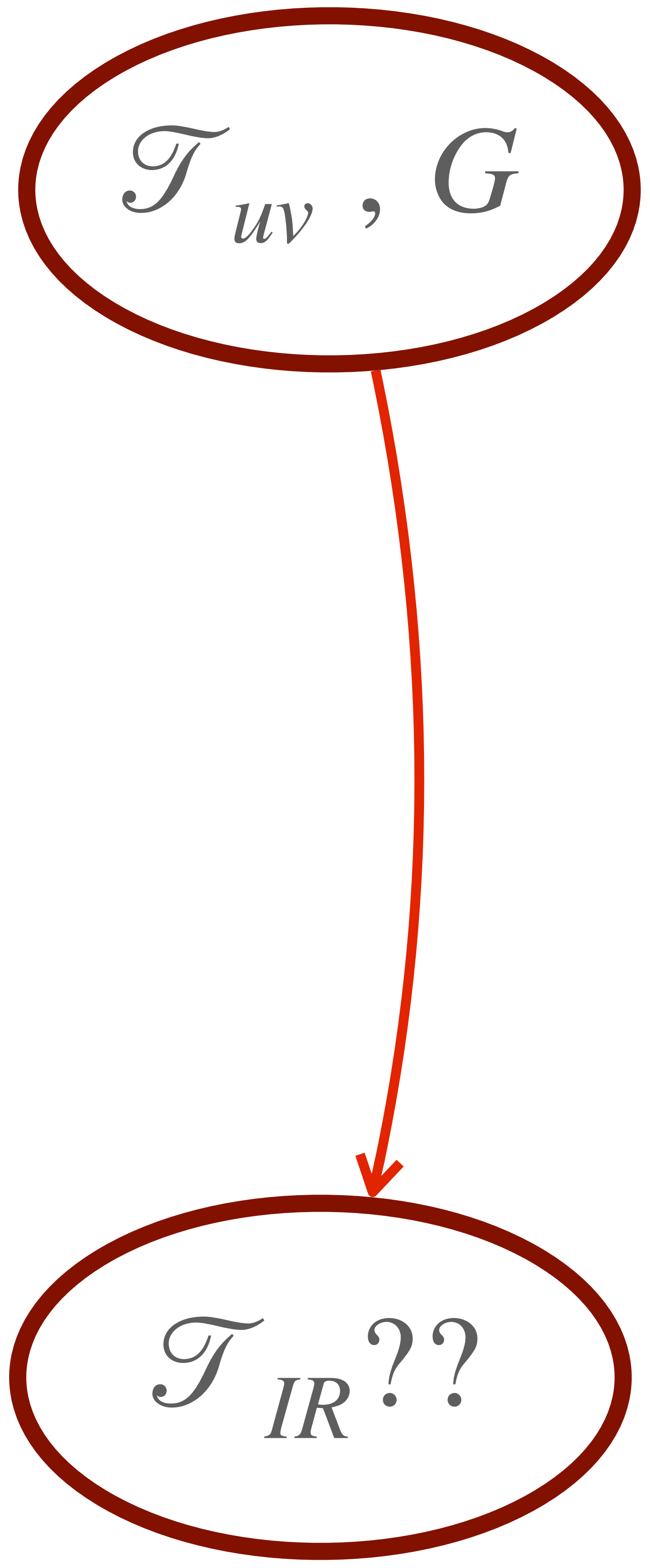
Hard problems of hadron physics: Non-perturbation QCD & related quests
Rigorous results in gauge QFT

Anomalies

- ‘t-Hooft anomaly matching conditions is one of the most important theoretical tools in analysing strongly interacting QFTs



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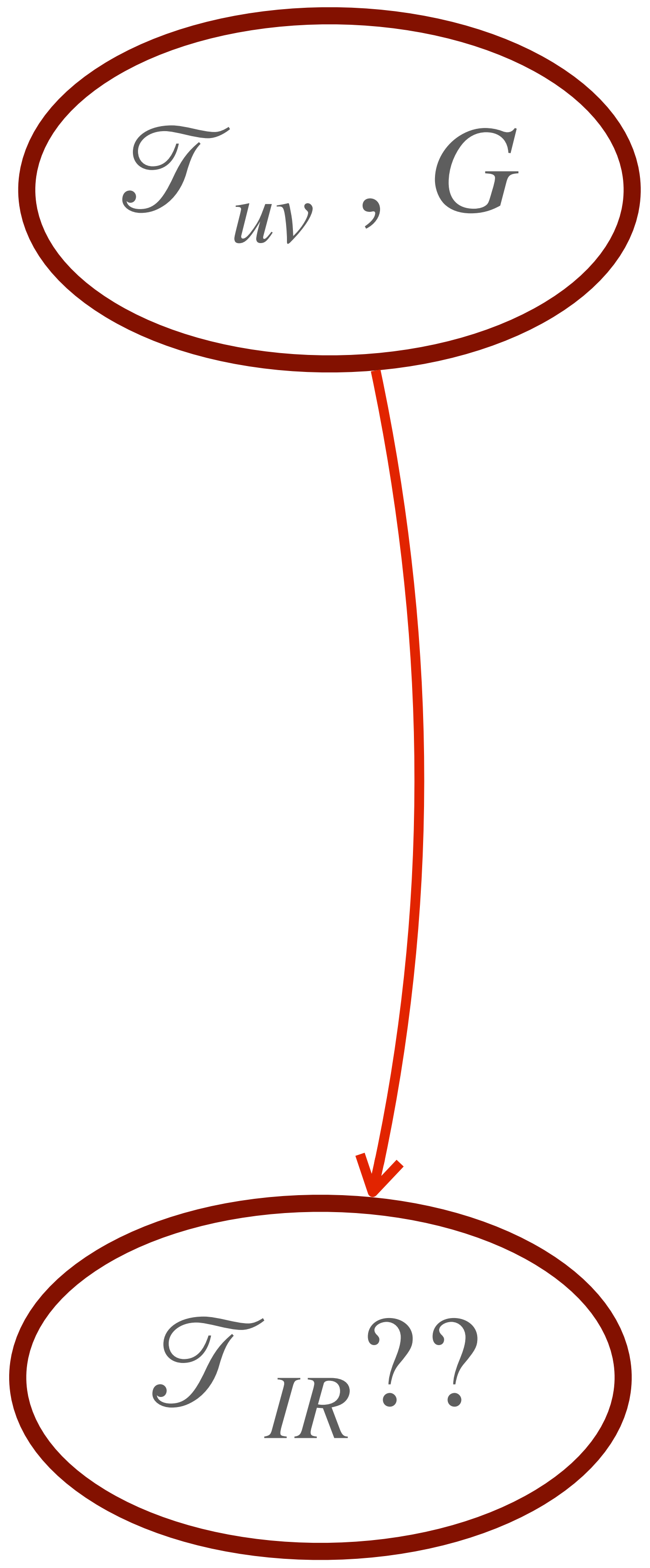
- Couple the symmetry G to background gauge fields A.

- The partition function satisfies in general

$$\mathcal{Z}(A) \rightarrow \mathcal{Z}(A') = e^{iS(A)}\mathcal{Z}(A)$$

- This equation is RG invariant. In particular must be satisfied by the low energy theory $\mathcal{T}_{IR}$



4d SU(N) gauge theories

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- Idea: define $\mathscr{T}_{x-ray}$: $SU(N-1) - \lambda_D - SU(N) - \psi_i$

- $\mathscr{T}_{x-ray}$ has an exact $\mathbb{Z}_N \subset U(1)_A$ axial symmetry with anomalies

- In the $\Lambda_{N-1} \gg \Lambda_N$ limit, can relate the flow of $\mathscr{T}_{x-ray}$ to the flow of $\mathscr{T}_{uv} \to \mathscr{T}_{IR}$

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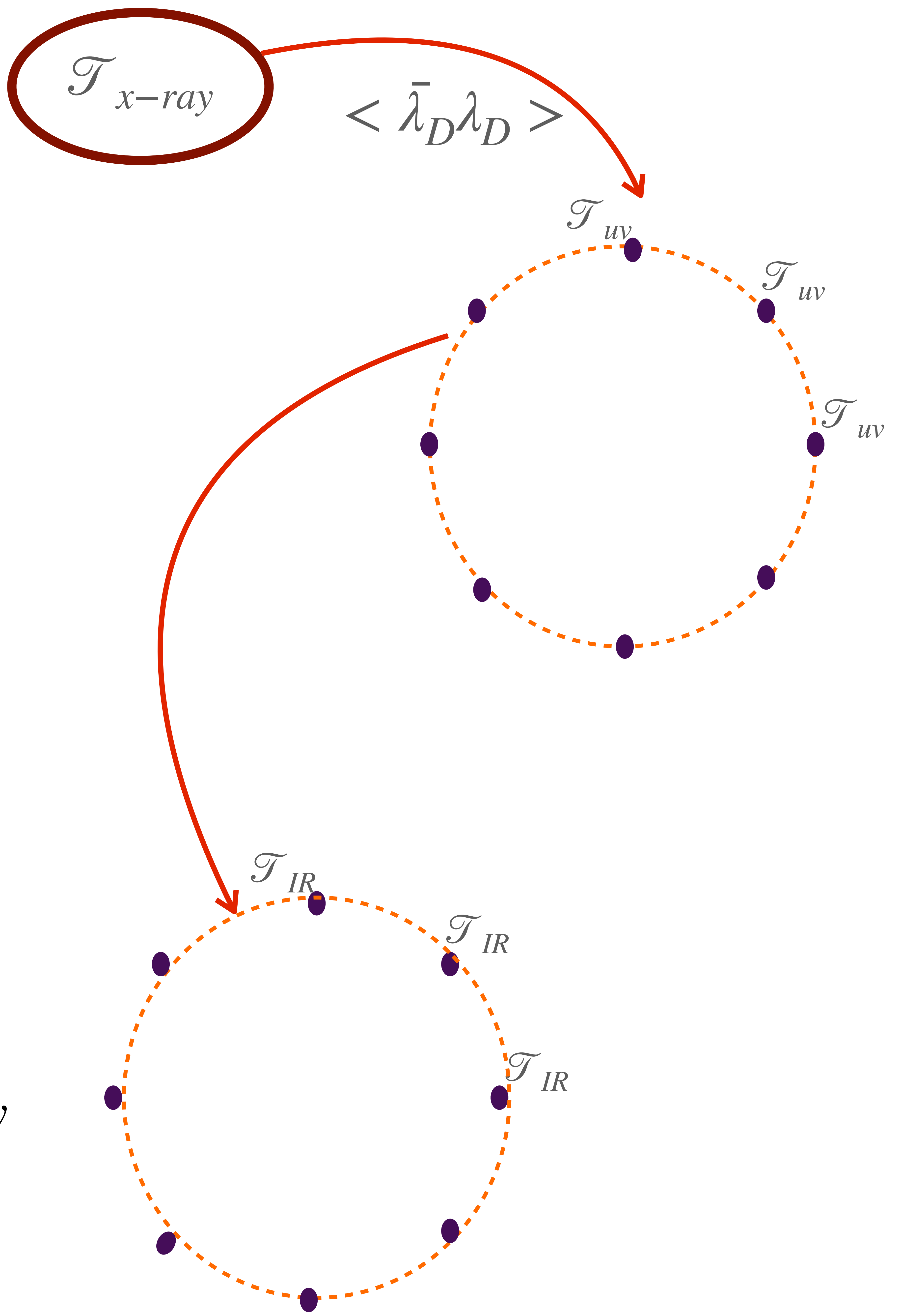

What can we learn about $\mathscr{T}_{IR}$?

- In the uv we are already in the $\mathbb{Z}_N$ broken phase.
- There are 4 types of $\mathbb{Z}_N$ anomalies in 4d:

1. $\mathbb{Z}_N \times G^2$, G^2- 3d anomaly
2. $\mathbb{Z}_N \times G^2$, G^2- 2d anomaly
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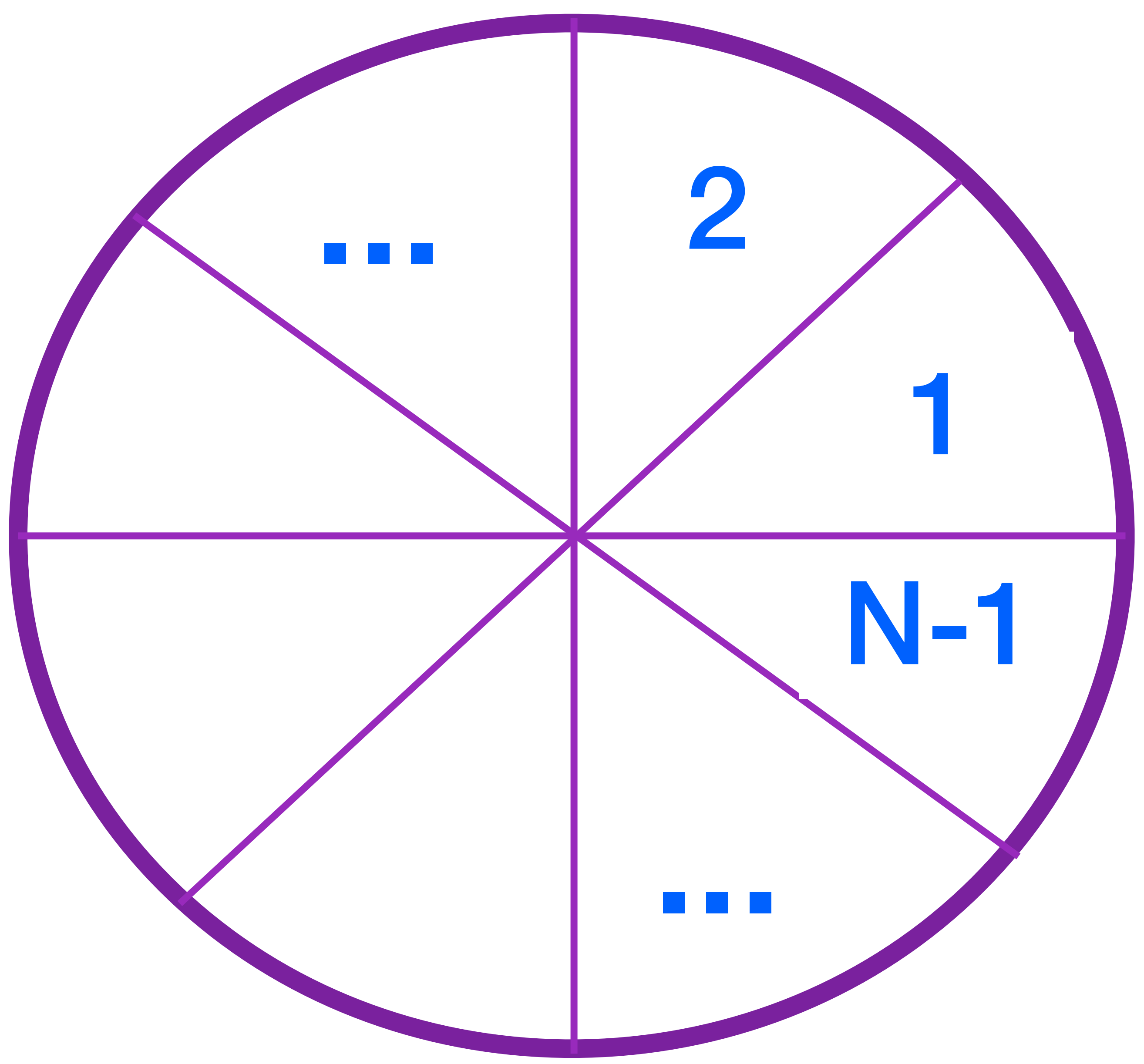
$\mathbb{Z}_N$ is not a symmetry of the DW. $\mathbb{Z}_N^2$, $\mathbb{Z}_N \times G$ cannot appear on the DW

Domain walls junction

- The anomalies constrain the effective 2d theory on the junction

- On the junction, the broken $\mathbb{Z}_N$ symmetry is restored (and embedded inside a U(1))



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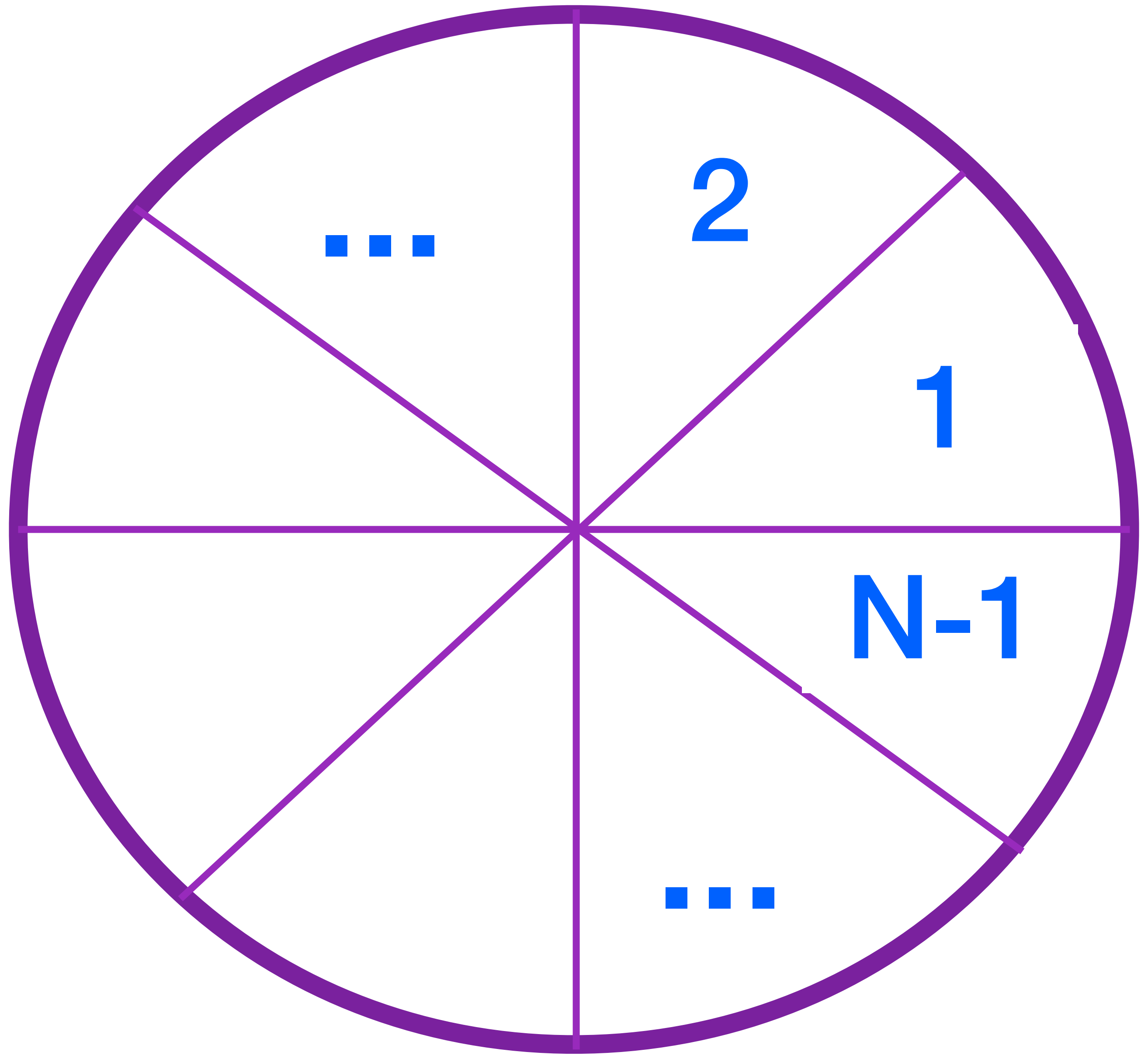
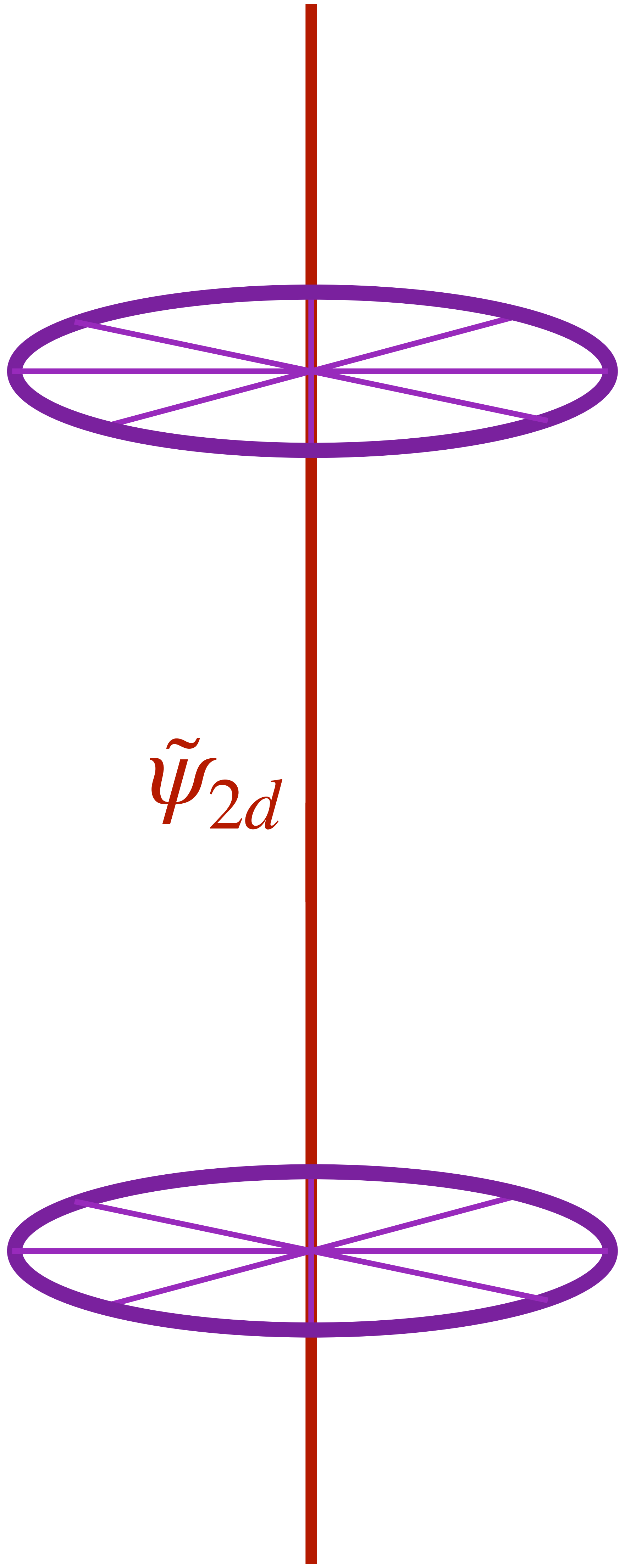


If we think of the $\mathbb{Z}_N$ as coming from a broken U(1), the junction comes from a vortex-string. The anomalies are carried by it as in Callan-Harvey



Example: $N_f = 1$ QCD

- SU(N) + 1 massless fundamental Dirac fermion ψ
- The only symmetry is $U(1)_B = U(1)_V/\mathbb{Z}_N$ which is anomaly free
- The low energy theory is expected to be trivially gapped.
  But that doesn’t mean it is completely boring!
- Assume $<\bar{\psi}\psi> \neq 0$ and denote it’s phase by $\eta' = -ilog(\bar{\psi}\psi)$

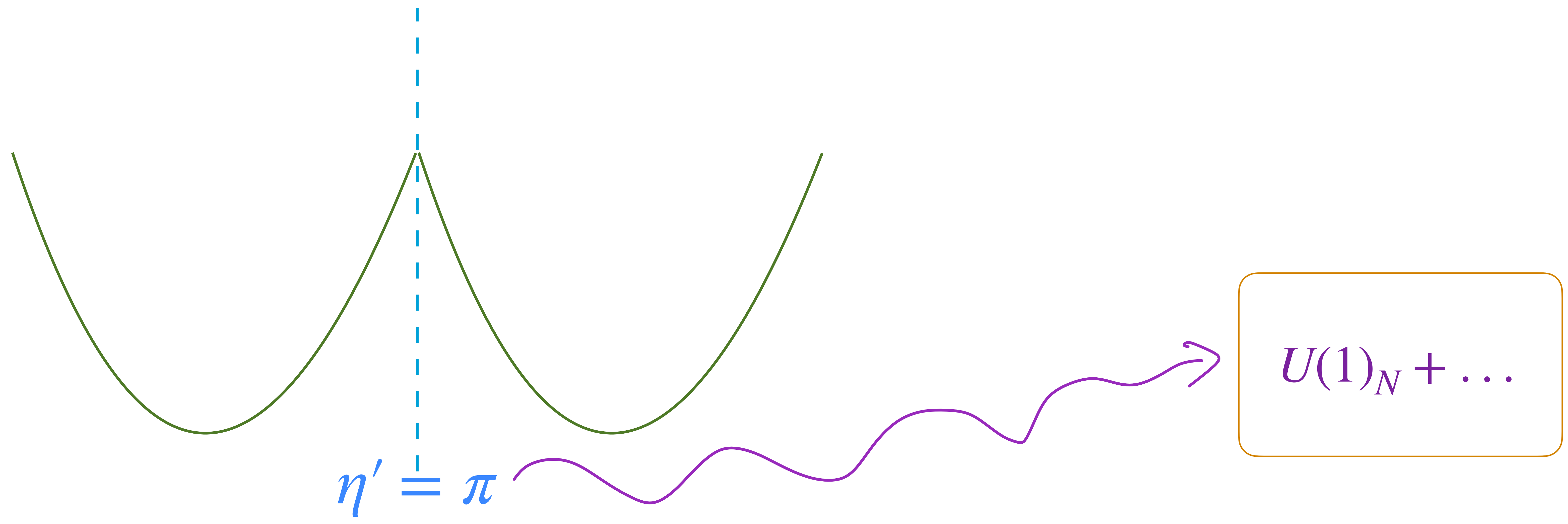
Cusp

- Large N arguments show that there is a cusp at $\eta' = \pi$ (Witten, Veneziano, Di-Vecchia)

- For $N_f > 1$, there must be a cusp due to anomalies involving $\mathbb{Z}_{N_f} \subset SU(N_f)_L$

- The anomalies constrain the effective 3d theory living on the cusp (related to the DW theory)

- There are conjectures about the $N_f = 1$ cusp and domain walls (Gaiotto, Komargodski, Seiberg)



Baryons

- In $N_f > 1$ QCD, there is a mixed $U(1)_B \times SU(N_f)_L^2$ anomaly
- Baryons must be part of the low energy effective theory!
- One consequence of the anomaly: A background with $\frac{1}{8\pi^2}\int_{space} tr\left(A_L \wedge dA_L - \frac{2i}{3}A_L^3\right) = 1$ has baryon charge 1
- To see it, plug in $dUU^\dagger = iA_L$ and get $\frac{1}{24\pi^2}\int tr\left(dUU^\dagger\right)^3 = 1$ which is exactly the skyrmion number
- There are conjectures about the $N_f = 1$ baryons (Komargodski; AK)

$(\mathbb{Z}_N)_L$ anomalies for $N_f = 1$ QCD

- Define $\mathcal{T}_{x-ray}$ as an $SU(N-1) \times SU(N)$ gauge theory with λ in the $(\square, \overline{\square})$ and ψ in the $(1, \square)$.

- There is a $\mathbb{Z}_N \times \frac{U(1)_\psi}{\mathbb{Z}_N} \times \frac{U(1)_\lambda}{\mathbb{Z}_{N-1}}$ anomaly

Symmetries:

$U(1)_\psi : \ \psi \to e^{i\alpha}\psi, \ \lambda \to e^{i\alpha}\lambda$

$U(1)_\lambda : \ \psi \to \psi, \ \lambda \to e^{i\beta}\lambda$

$(\mathbb{Z}_N)_L : \ \psi_L \to e^{\frac{2\pi i}{N}}\psi_L \ , \ \lambda_L \to e^{\frac{2\pi i}{N}}\lambda_L$

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- There is a $\mathbb{Z}_N \times \frac{U(1)_\psi}{\mathbb{Z}_N} \times \frac{U(1)_\lambda}{\mathbb{Z}_{N-1}}$ anomaly
- At energies $\Lambda_N \ll E \ll \Lambda_{N-1}$, there are k-Vacua: $< \bar{\lambda}\lambda > \sim e^{\frac{2\pi i k}{N}}$
- The choice of vacuum breaks $(\mathbb{Z}_N)_L$
- The effective theory at each vacuum is $N_f = 1$ QCD

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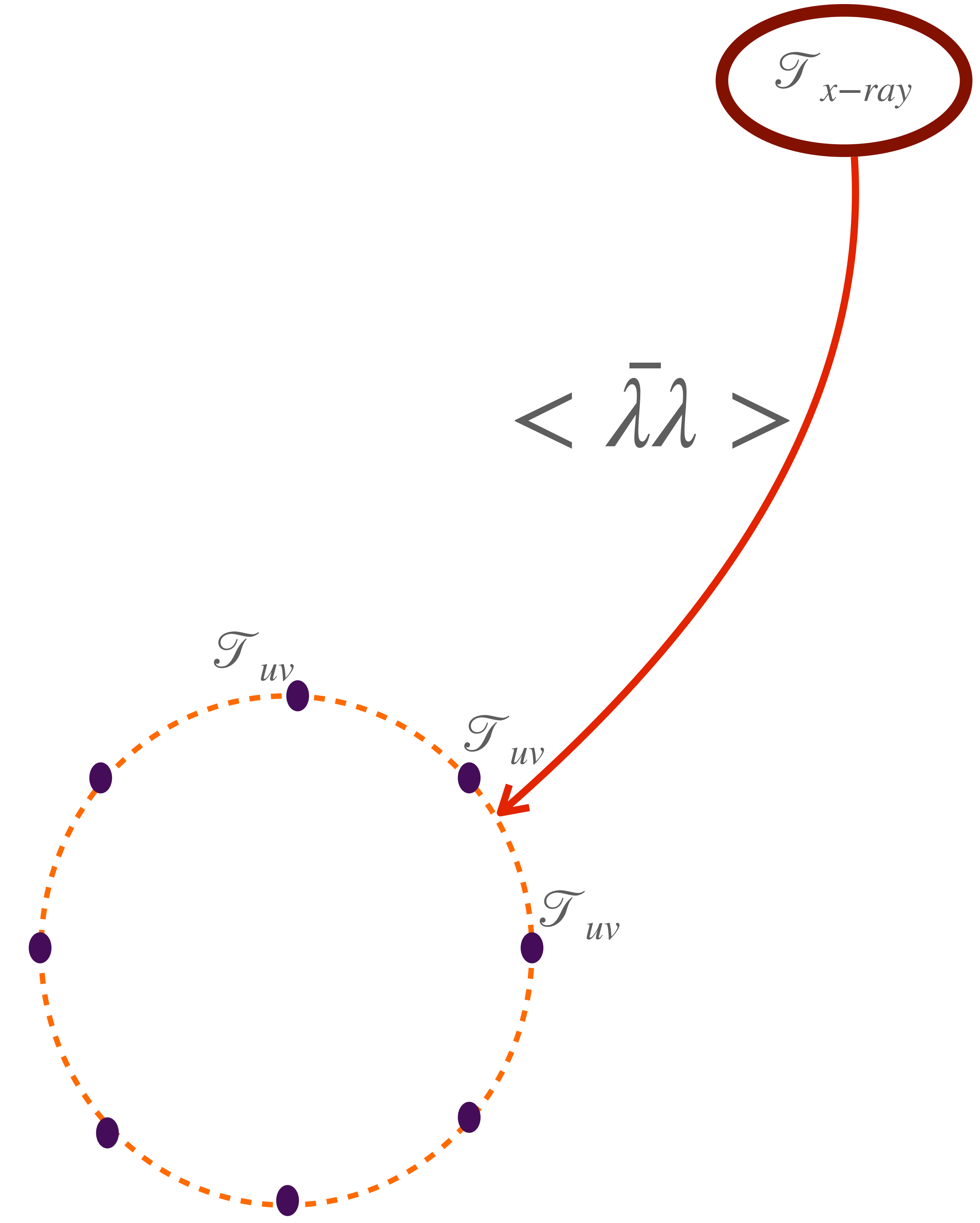
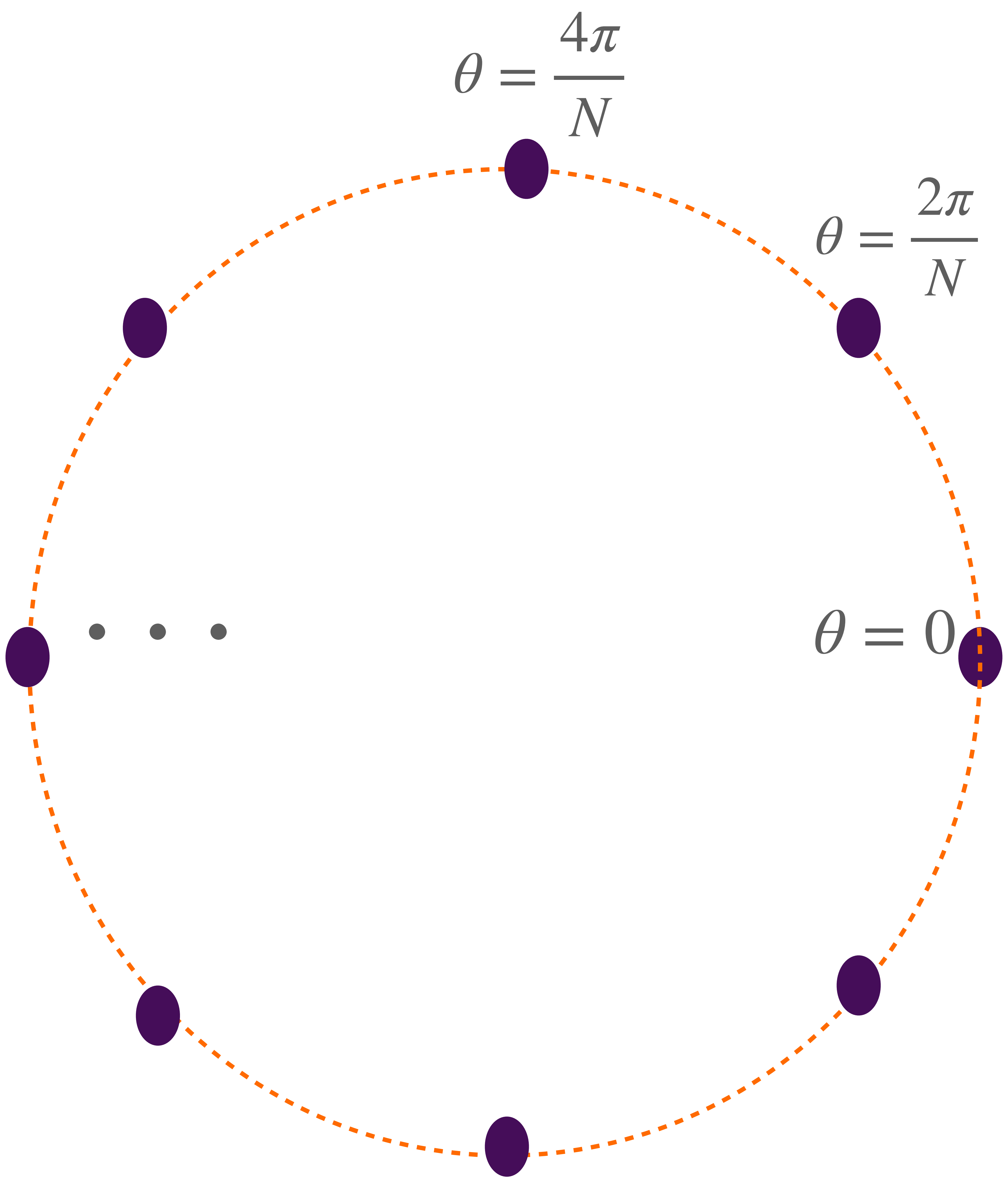


$(\mathbb{Z}_N)_L$ anomalies for $N_f = 1$ QCD

- Some of the anomaly is carried by the $<\bar{\lambda}\lambda>$ condensate, some is carried by ψ:
  $$S_{anomaly} = S_\lambda + S_\psi$$
- S_ψ should be carried by the $<\bar{\psi}\psi>$ condensate in the IR.
- S_ψ includes $(\mathbb{Z}_N)_L \times \left(\frac{U(1)_\psi}{\mathbb{Z}_N}\right)^2$ and $(\mathbb{Z}_N)_L^2 \times \frac{U(1)_\psi}{\mathbb{Z}_N}$



The cusp and the $(\mathbb{Z}_N)_L \times \left(U(1)_\psi/\mathbb{Z}_N\right)^2$ anomaly

- There is no such anomaly in the "x-ray".
  In the uv, the contribution from $<\bar{\lambda}\lambda>$ is cancelled by an opposite contribution from ψ

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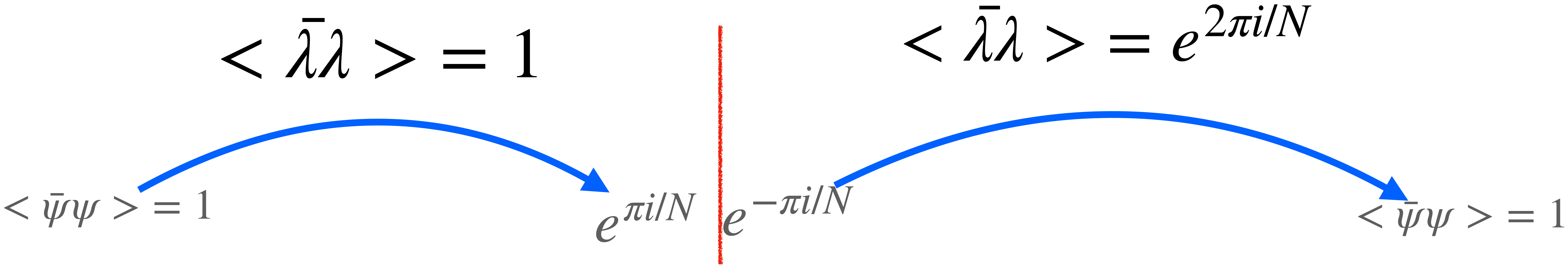

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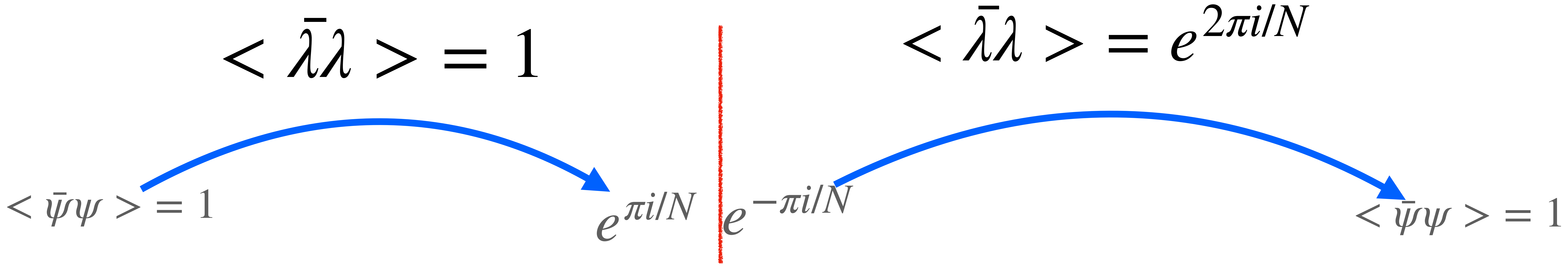
- In the IR, the contribution from $<\bar{\lambda}\lambda>$ must be cancelled by $<\bar{\psi}\psi>$.

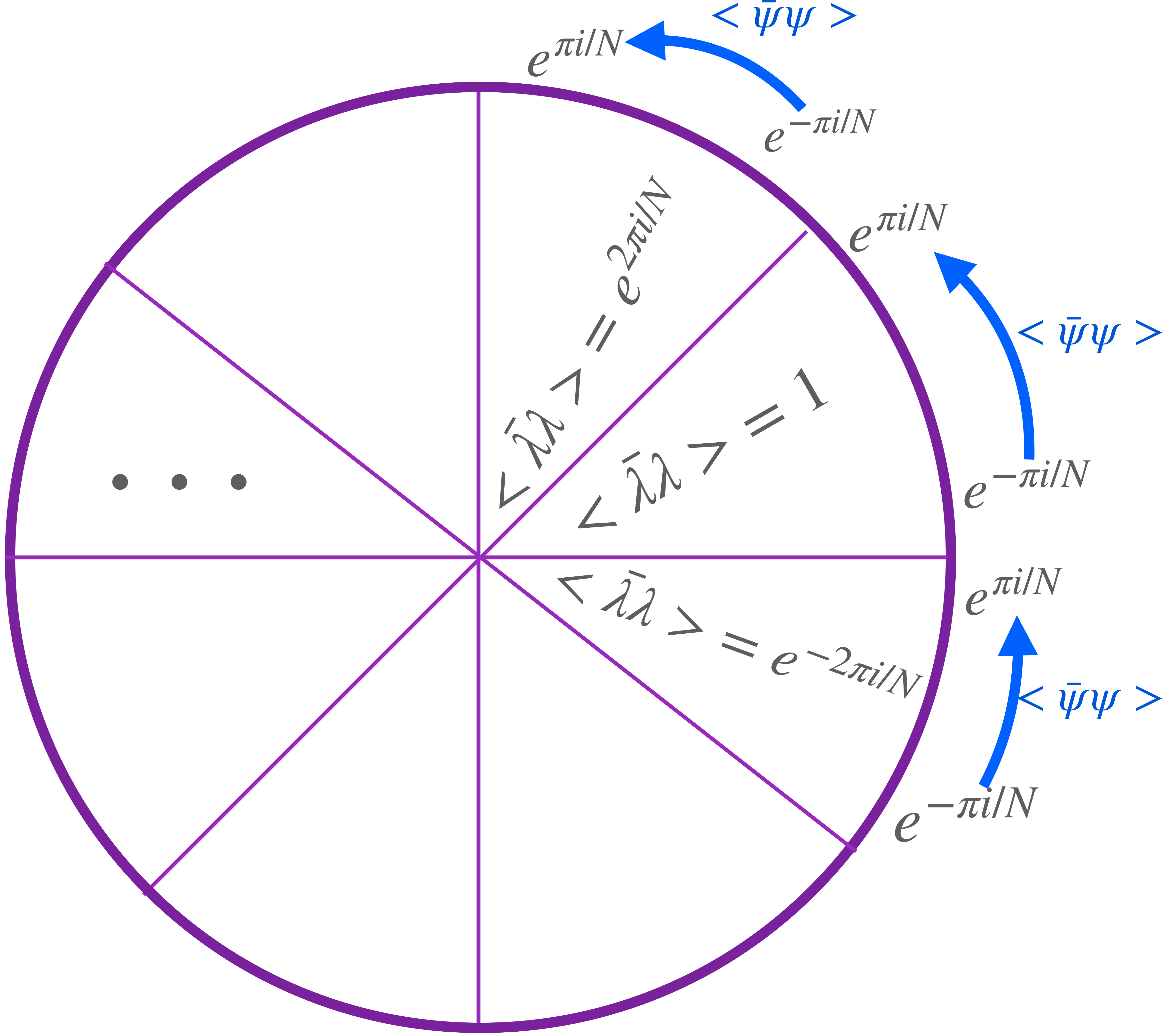
- This fixes the coupling of η' to $U(1)_\psi$ gauge field $-\frac{1}{8\pi^2 N}\eta' dA_\psi \wedge dA_\psi$

- The potential for η' cannot be smooth. A cusp with $U(1)_N$ CS can do the job



$N_f = 1$ baryons and the $(\mathbb{Z}_N)^2_L \times U(1)_\psi/\mathbb{Z}_N$ anomaly

On the junction, $(\mathbb{Z}_N)_L$ is restored and enhanced to a $U(1)$



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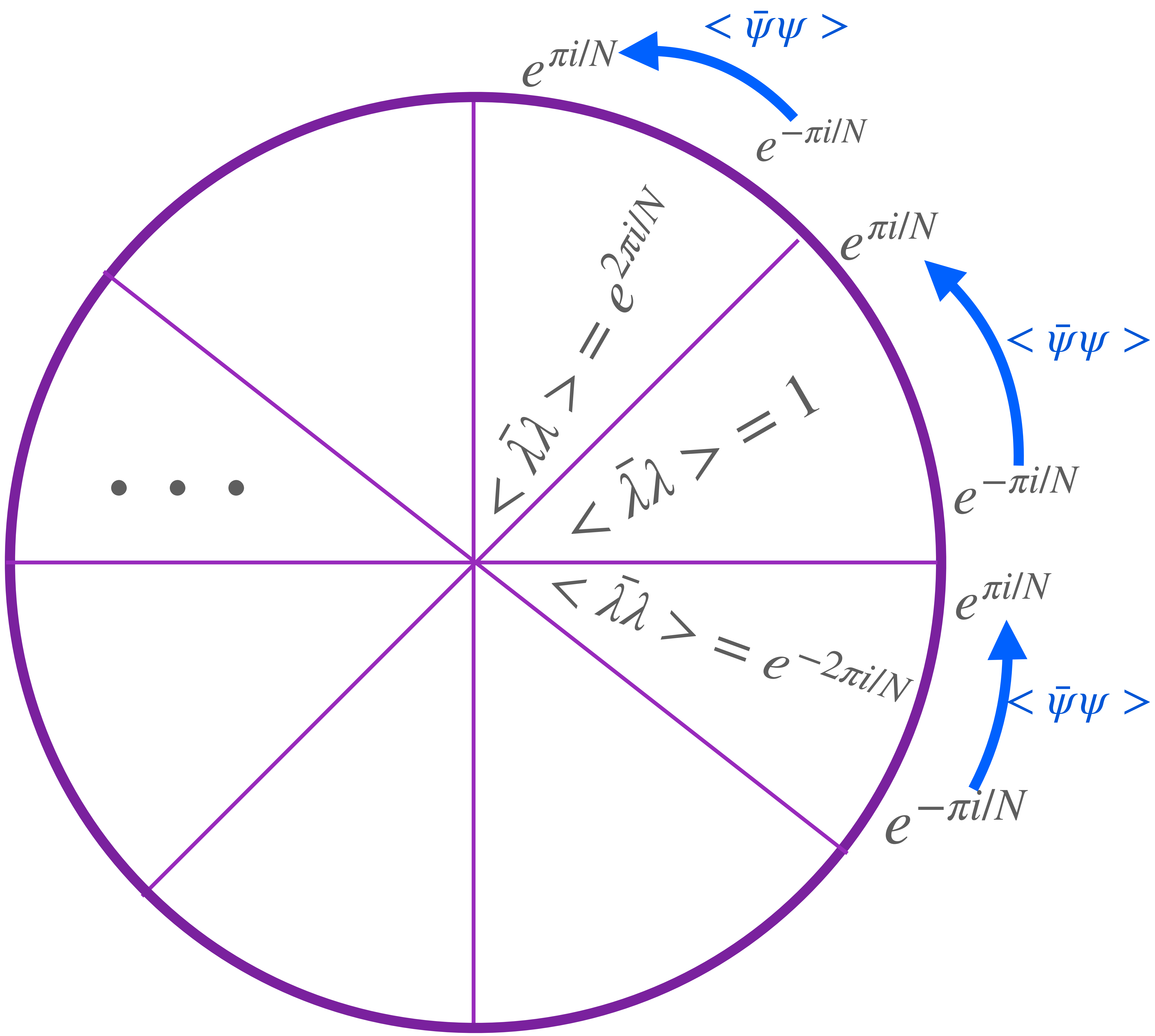
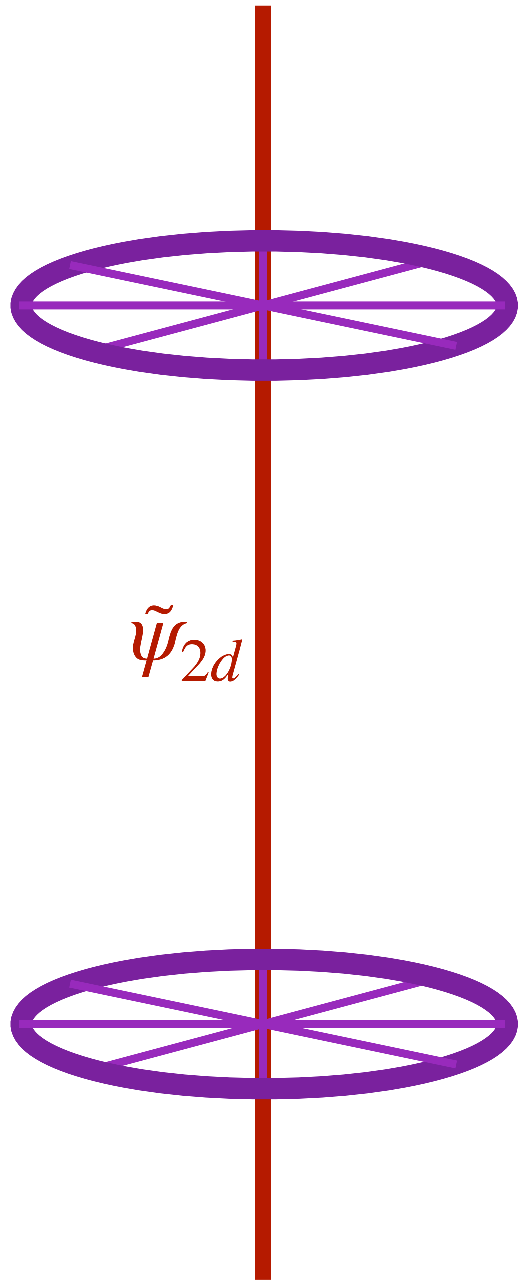
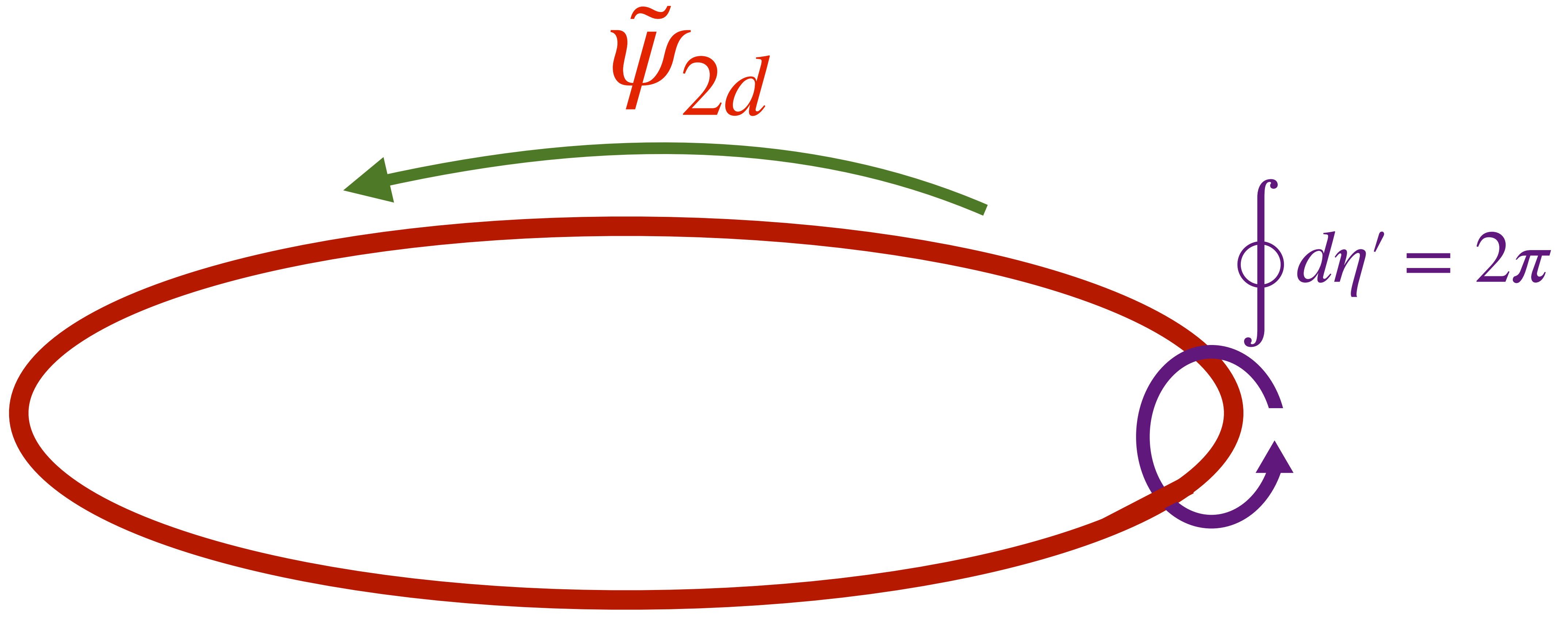


The 2d theory on the junction should carry the $(\mathbb{Z}_N)_L \times U(1)_\psi/\mathbb{Z}_N$ anomaly



The $N_f = 1$ baryon



- Similar to $N_f > 1$ QCD, the anomaly tells you how to construct a baryon.
- $\frac{1}{8\pi^2}\int A_L \wedge dA_L = 1$ requires the product of two orthogonal windings:

1. The winding of η' around the singularity ⟷ Construction of the junction
2. The winding of the 2d mode along the singularity

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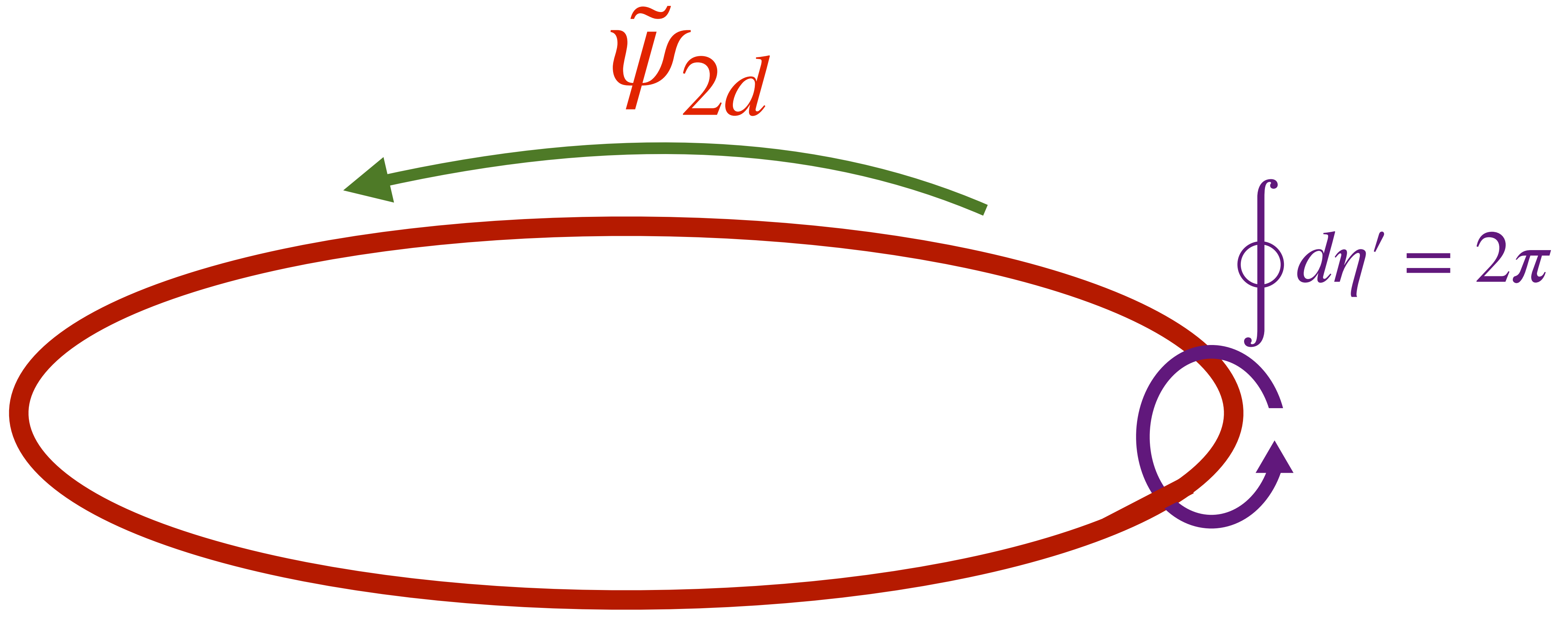

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This is equivalent to the proposal for baryons made by Komargodski!

Summary

- Many known/conjectured results about $N_f = 1$ QCD can be made rigorous and get a kinematic explanation.
- The same method can be used to many other theories.
- Can be used to predict IR phases
- "X-ray" embedding for other non-symmetry transformations? Other space time dimensions?
- Thank you very much!