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Precision nuclear physics with chiral EFT

- Determination of the πN coupling constants: ~1% accuracy

  Reinert, Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

- Calculation of the ^{2}H structure radius: ~1‰ accuracy

  Filin, Baru, EE, Krebs, Möller, Reinert, Phys. Rev. Lett. 124 (2020) 082501,
  Phys. Rev. C 103 (2021) 024313

Precision determination of pion-nucleon coupling constants

Patrick Reinert, Hermann Krebs, EE, Phys. Rev. Lett. 126 (2021) 9, 092501

- Fundamental observables that control the strength of the nuclear forces due to π-exchange
- Insights into isospin breaking at the hadronic/nuclear levels
- Gold-plated benchmarks for lattice-QCD + QED [talk by Constantia Alexandrou]

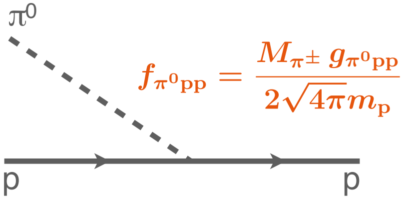
πN coupling constants & nucleon FFs

- Interaction of the nucleon with the isovector weak current $A_i^\mu(0)$ in the isospin limit:

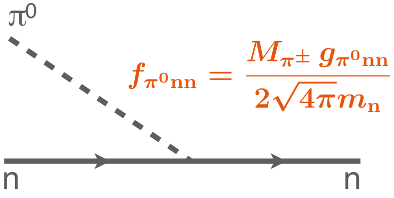
$$\langle \mathrm{N}(p')|A_i^\mu(0)|\mathrm{N}(p)\rangle = \bar{u}(p')\left[\gamma^\mu \underset{\substack{\uparrow \\ \text{axial form factor}}}{G_A(q^2)} + \frac{q^\mu}{2m_\mathrm{N}}\underset{\substack{\uparrow \\ \text{induced pseudoscalar form factor}}}{G_P(q^2)}\right]\gamma_5\frac{\tau_i}{2}u(p)$$

  - — axial FF: a smooth function near $q^2 = 0$; axial charge: $g_A \equiv G_A(0) = 1.2756(13)$ [PDG 2020]
  - — induced pseudoscalar FF: $G_P(q^2) = 4m_\mathrm{N}\dfrac{g_{\pi\mathrm{NN}}F_\pi}{(M_\pi^2 - q^2)} + \text{non-pole terms}$
  - — Goldberger-Treiman relation: $F_\pi g_{\pi\mathrm{NN}} = g_A m_\mathrm{N}(1 + \Delta_\mathrm{GT})$
  - — equivalently, the pseudovector coupling constant: $f_{\pi\mathrm{NN}} = M_{\pi^\pm}\, g_{\pi\mathrm{NN}}/(2\sqrt{4\pi}m_\mathrm{N})$

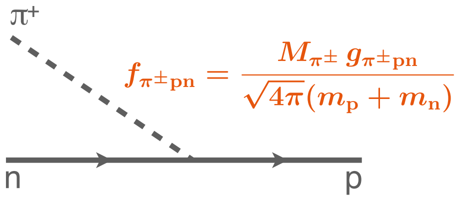
- Away from the isospin limit, one has to introduce 3 coupling constants:



$$f_{\pi^0\mathrm{pp}} = \frac{M_{\pi^\pm}\, g_{\pi^0\mathrm{pp}}}{2\sqrt{4\pi}m_\mathrm{p}}$$



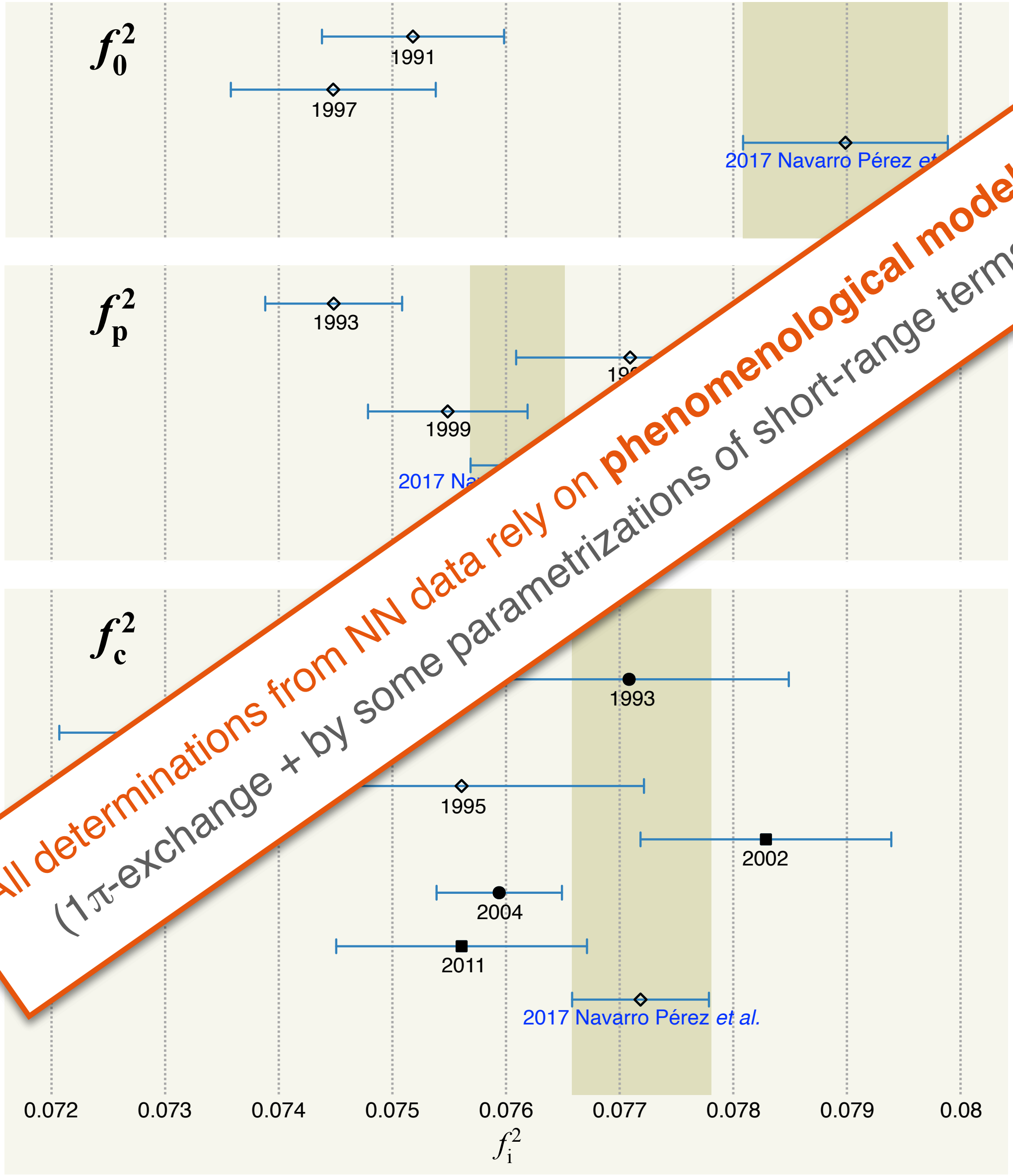
$$f_{\pi^0\mathrm{nn}} = \frac{M_{\pi^\pm}\, g_{\pi^0\mathrm{nn}}}{2\sqrt{4\pi}m_\mathrm{n}}$$



$$f_{\pi^\pm\mathrm{pn}} = \frac{M_{\pi^\pm}\, g_{\pi^\pm\mathrm{pn}}}{\sqrt{4\pi}(m_\mathrm{p} + m_\mathrm{n})}$$

→ fundamental observables that control the strength of nuclear forces

πN coupling constants: Some earlier determinations



All determinations from NN data rely on **phenomenological models**
(1π-exchange + by some parametrizations of short-range terms)

Standard notation:

$$\begin{aligned} f_0^2 &= -f_{\pi^0 \mathrm{nn}}\, f_{\pi^0 \mathrm{pp}} \\ f_\mathrm{p}^2 &= f_{\pi^0 \mathrm{pp}}\, f_{\pi^0 \mathrm{pp}} \\ 2f_\mathrm{c}^2 &= f_{\pi^\pm \mathrm{pn}}\, f_{\pi^\pm \mathrm{pn}} \end{aligned}$$

● — fixed-t dispersion relations of πN scattering
Markopoulou-Kalamara, Bugg '93; Arndt et al. '04

■ — πN scattering lengths + Goldberger-Miyazawa-Oehme sum rule
Ericson et al. '02; Baru et al. '11

▽ — proton-antiproton PWA
Timmermans et al. '94

◇ — neutron-proton (+ proton-proton) PWA
Klomp et al. '91; Stoks et al. '93; Bugg et al. '95; de Swart et al. '97; Rentmeester et al. '99

2017 Granada PWA: evidence for significant charge dependence of the coupling constants:

$$f_0^2 - f_\mathrm{p}^2 = 0.0029(10)$$

Navarro Perez et al., PRC 95 (2017) 6, 064001

Anatomy of the calculation

The goal: *Bayesian* determination of f_c^2, f_p^2 and f_0^2 by performing a full-fledged PWA of NN data up to pion-production threshold in the framework of chiral EFT

1. Experimental data:

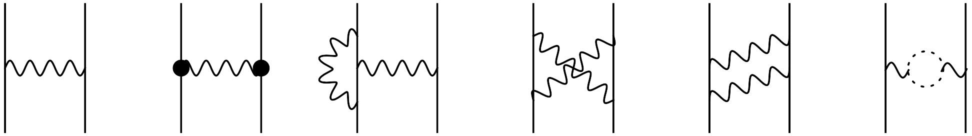
— About 8000 published np and pp scattering data below E_{lab} = 350 MeV. Not all data are mutually compatible…

— Selections of mutually compatible data: Nijmegen 1993, Granada 2013, 2017

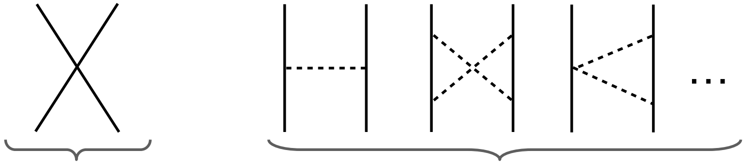
— Performed own selection of compatible data (found some differences to Granada…)

2. Interaction model (chiral EFT):

— Long-range EM interactions (included in all PWA…)



— Semi-local chiral NN interaction at N^{4}LO$^+$ from P. Reinert, H. Krebs, EE, EPJA 54 (2018) 88



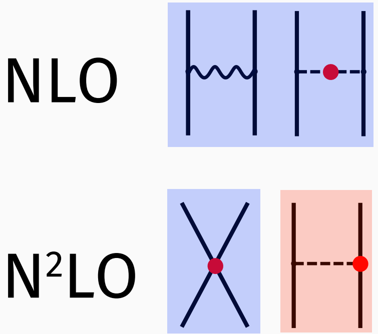
25 (+ 2 IB) contact terms

parameter-free: all πN LECs fixed from the πN-system, no IB for f_i^2 …

The treatment of isospin-breaking (IB) contributions was incomplete (limited to that of Nijmegen/Granada PWA)

We now include all charge-independence & charge-symmetry-breaking terms to N^{4}LO.

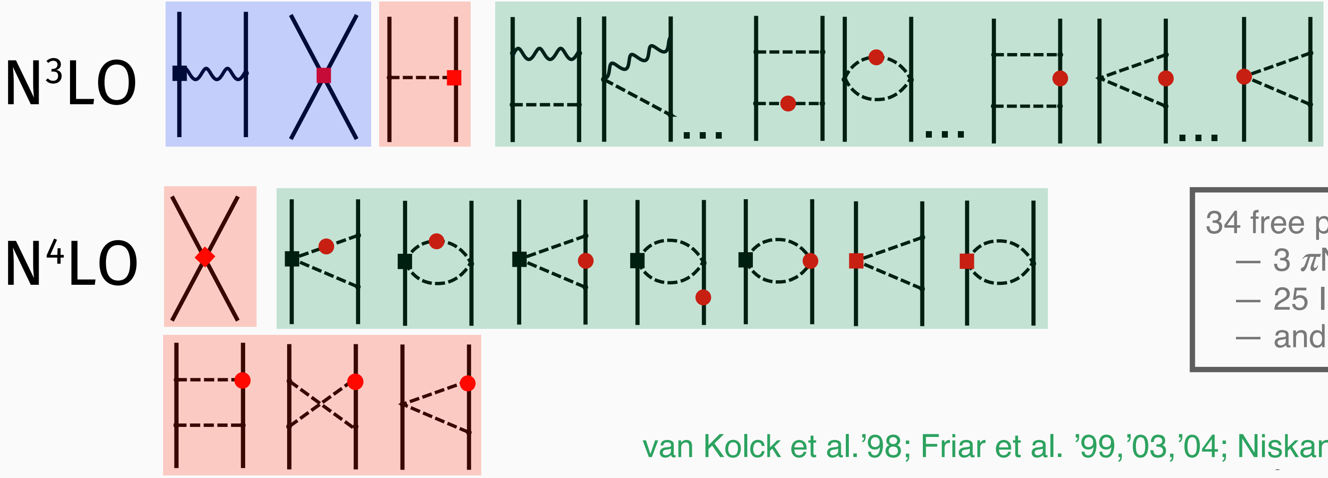
Isospin-breaking NN forces



Have been employed in **Reinert, Krebs, EE, EPJA 54 (2018) 88**

Parameter-free: depend on δM_π, $\delta m = 1.29$ MeV and $(\delta m)^{QCD} = 2.05(30)$ MeV **[Gasser, Leutwyler '75]**

Depend on 3 πN coupling constants + 3 IB contact terms in p-waves



34 free parameters comprising:
- 3 πN coupling constants,
- 25 IC + 5 IB contacts,
- and a cutoff Λ

van Kolck et al.'98; Friar et al. '99,'03,'04; Niskanen '02; EE, Meißner '05

3. Bayesian determination of the πN coupling constants $f^2 \equiv \{f_c^2, f_p^2, f_0^2\}$:

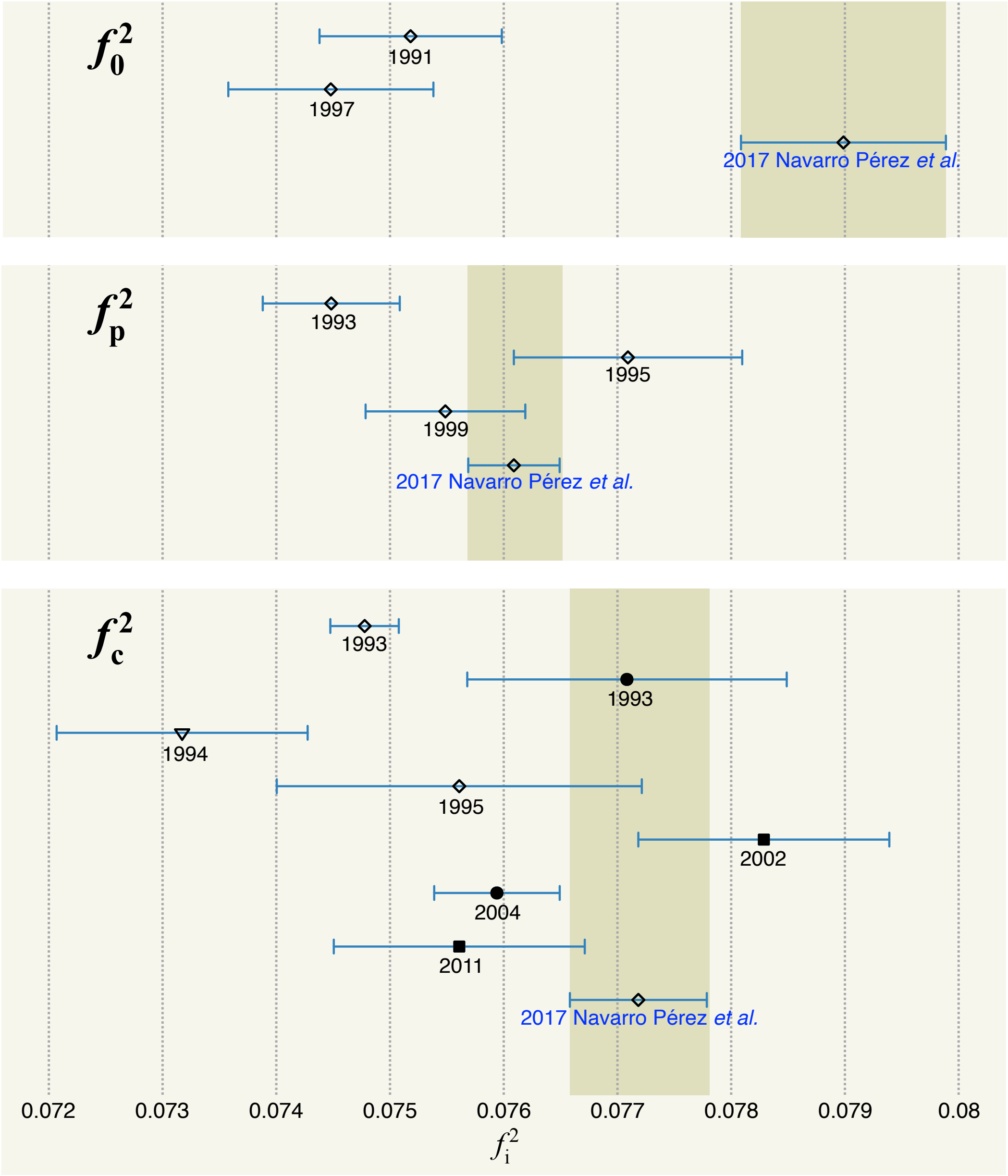
$$p(f^2|D) = \int d\Lambda\, dC\; p(f^2 C\Lambda|D) = \int d\Lambda\, dC\; \frac{p(D|f^2 C\Lambda)\; p(f^2 C\Lambda)}{p(D)}$$

↑ NN data

↑ use Bayes' theorem

↑ just a normalization constant

Determination of the πN constants



Determination of the πN constants

Our $g_{\pi\mathrm{NN}}$ value corresponding to f_c^2 reads:

$$g_{\pi\mathrm{NN}} = 13.23 \pm 0.04$$

Pionic hydrogen exp. at PSI (GMO sum rule)

[Hirtl et al., Eur. Phys. J. A57 (2021) 2, 70]

$$\epsilon_{1s}^{\pi\mathrm{H}} + \epsilon_{1s}^{\pi\mathrm{D}}: \quad g_{\pi\mathrm{NN}} = 13.10 \pm 0.10$$

$$\Gamma_{1s}^{\pi\mathrm{H}}: \quad g_{\pi\mathrm{NN}} = 13.24 \pm 0.10$$

Summary of the first part:

- extracted πN couplings from NN data in χEFT
- employed a Bayesian approach to account for statistical and systematic uncertainties
- achieved a statistically perfect description of NN data ($\chi^2/\mathrm{dat} = 1.005$ for ~ 5000 data in the energy range of $E_{\mathrm{lab}} = 0 - 280$ MeV)

Our result:

$$\begin{aligned} f_0^2 &= 0.0779(9)(1.3) \\ f_p^2 &= 0.0770(5)(0.8) \\ f_c^2 &= 0.0769(5)(0.9) \end{aligned}$$

statistical **and systematic** errors due to the EFT truncation, choice of E$_{\mathrm{max}}$ and data selection

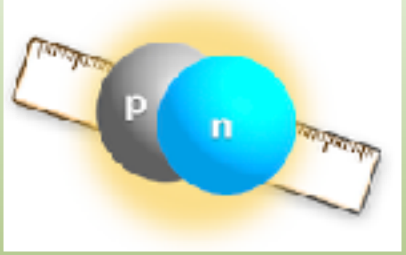
uncertainty in the subleading πN LECs

Our f_c^2 value is consistent with the extractions from the πN system

Contrary to the Granada group, we see no evidence for charge dependence of the πN coupling constants

Using the PDG-2020 value for the axial charge $g_A = 1.2756(13)$, the GT discrepancy amounts to (f_c):

$$\Delta_{\mathrm{GT}} \sim 1.7\%$$



High-accuracy calculation of the deuteron charge and quadrupole FFs

Arseniy Filin, Vadim Baru, EE, Hermann Krebs, Daniel Möller, Patrick Reinert, Phys. Rev. Lett. 124 (2020) 082501;

Phys. Rev. C103 (2021) 024313

- provides a nontrivial test the new chiral NN interactions
- can shed new light on the long-standing issue with underpredicted radii of medium-mass and heavy nuclei
- opens a way to extract the neutron radius from few-N data

How big is a neutron?

Famous proton radius puzzle: pre-2010 electron-based experiments give the radius $> 7\sigma$ larger than muon-based experiments.

CODATA-2018 recommended value: $r_p = 0.8414 \pm 0.0019$ fm

What do we know about the neutron radius?

— no neutron targets; extrapolations of $G_C^n(Q^2)$ extracted from ^{2}H not reliable…

— the only information comes from (old) n-scattering experiments on Pb, Bi, …

$\longrightarrow$ PDG recommended value: $r_n^2 = -0.1161 \pm 0.0022$ fm^2

Idea: accurate calculation of the ^{2}H structure radius

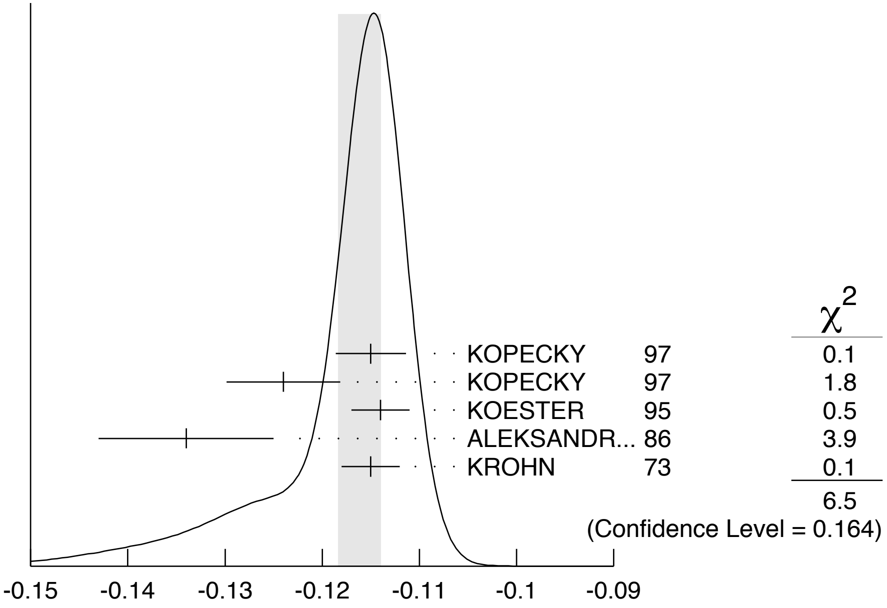
$$r_{\rm str}^2 = r_d^2 - r_p^2 - r_n^2 - \frac{3}{4m_p^2}$$

along with ^{1}H-^{2}H isotope shifts data

$$r_d^2 - r_p^2 = 3.82007(65)\ \text{fm}^2$$

Jentschura et al., PRA 83 (2011)

can be used to extract r_n^2 !



		χ^2
KOPECKY	97	0.1
KOPECKY	97	1.8
KOESTER	95	0.5
ALEKSANDR...	86	3.9
KROHN	73	0.1
		6.5

(Confidence Level = 0.164)

Outline of the calculation

Calculation of the deuteron charge radius:

The deuteron charge radius is defined in terms of the charge form factor G_C

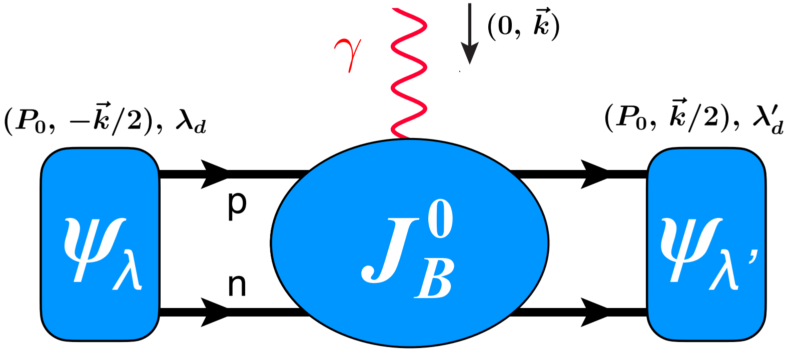
$$r_C^2 = (-6)\frac{\partial G_C(Q^2)}{\partial Q^2}\bigg|_{Q^2=0}$$

which can be computed as (in the Breit frame):

$$G_C(Q^2) = \frac{1}{3e}\frac{1}{2P_0}\sum_{\lambda}\langle P', \lambda | J_B^0 | P, \lambda\rangle$$

The matrix element is given by:

$$\frac{1}{2P_0}\langle P', \lambda' | J_B^\mu | P, \lambda\rangle = \int \frac{d^3 l_1}{(2\pi)^3}\frac{d^3 l_2}{(2\pi)^3}\,\psi_{\lambda'}^{\dagger}\left(\vec{l}_2 + \frac{\vec{k}}{4}, \vec{v}_B\right) J_B^\mu\, \psi_\lambda\left(\vec{l}_1 - \frac{\vec{k}}{4}, -\vec{v}_B\right)$$



calculated **consistently** in chiral EFT

Precision calculation of the deuteron charge radius in chiral EFT relies upon:

— accurate, high-precision two-nucleon interactions

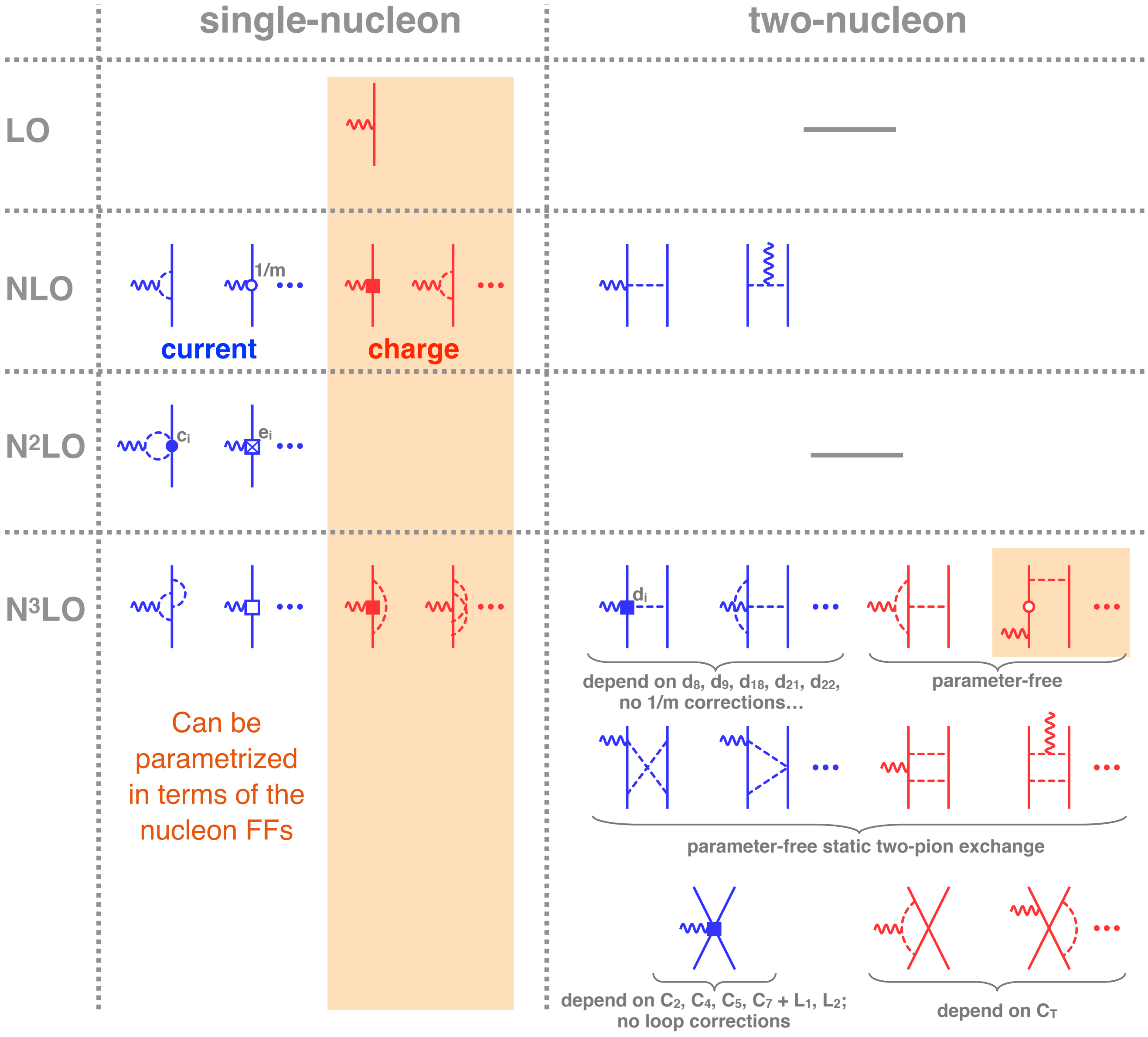
— **consistent** charge density operator

— careful error analysis

Nuclear electromagnetic currents

Kölling, EE, Krebs, Meißner, PRC 80 (09) 045502; PRC 86 (12) 047001; Krebs, EE, Meißner, FBS 60 (2019) 31

For review see:
H. Krebs, EPJA 56 (2020) 240



Can be parametrized in terms of the nucleon FFs

depend on d_8, d_9, d_{18}, d_{21}, d_{22},
no 1/m corrections…

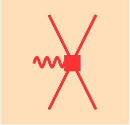
parameter-free

parameter-free static two-pion exchange

depend on C_2, C_4, C_5, C_7 + L_1, L_2;
no loop corrections

depend on C_T

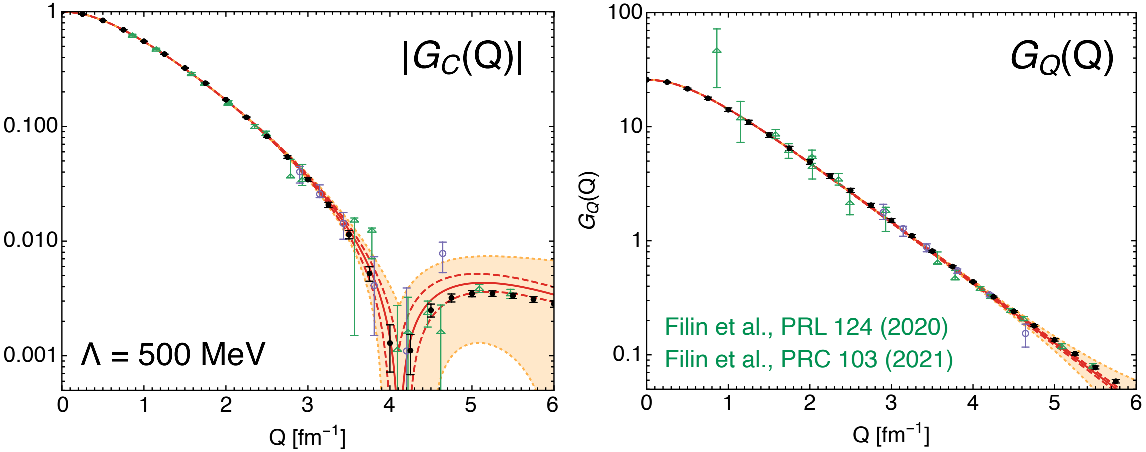
+ N^{4}LO
[3 LECs]



Deuteron charge and quadrupole FFs

Filin, Möller, Baru, EE, Krebs, Reinert, PRL 124 (2020) 082501; PRC 103 (2021) 024313

The charge and quadrupole form factors of the deuteron at N⁴LO



The extracted structure radius and quadrupole moment:

$$r_{\rm str} = 1.9729^{+0.0015}_{-0.0012}\ \text{fm}$$

$$Q_{\rm d} = 0.2854^{+0.0038}_{-0.0017}\ \text{fm}^2$$

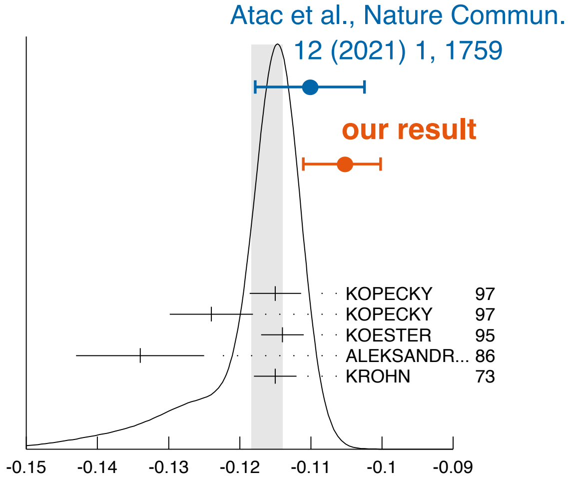
statistical and systematic errors due to the EFT truncation, choice of fitting range and πN LECs

IB effects are significant for $Q_{\rm d}$: using the 2018 N⁴LO⁺ potential would lead to $Q_{\rm d} = 0.2803$ fm^2

The value of $Q_{\rm d}$ is to be compared with $Q_{\rm d}^{\rm exp} = 0.285\,699(15)(18)$ fm^2 Puchalski et al., PRL 125 (2020)

Combining our result for $r_{\rm str}^2 = r_d^2 - r_p^2 - r_n^2 - \dfrac{3}{4m_p^2}$ with the ¹H-²H isotope shift datum $r_d^2 - r_p^2 = 3.82007(65)$ fm^2 leads to the prediction for the neutron radius:

$$\boxed{r_n^2 = -0.105^{+0.005}_{-0.006}\ \text{fm}^2}$$



Preliminary results for the charge radius of A = 3,4 nuclei

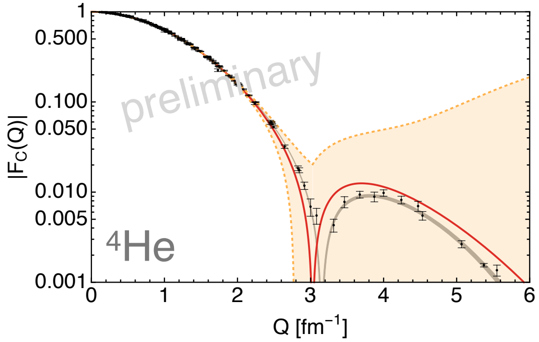
Arseniy Filin, Vadim Baru, EE, Christopher Körber, Hermann Krebs, Daniel Möller, Patrick Reinert,
in preparation

- precision test of the theory for ^{4}He
- theoretical prediction for the isoscalar ^{3}H-^{3}He charge radius 10 times more accurate than the current exp value — to be tested by CREMA soon!
- consistent regularization of isovector currents (π-loops) is still in progress $\Rightarrow$ limit ourselves to $T = 0$ nucleus (^{4}He) + isoscalar ^{3}H-^{3}He combination

Charge radii of light nuclei

Filin, Baru, EE, Körber, Krebs, Möller, Reinert, in preparation

2 out of 3 LECs in the short-range 2N charge density already fixed from the ^{2}H FFs; the remaining one is determined from the ^{4}He FF (lots of low-energy data…) ⟶

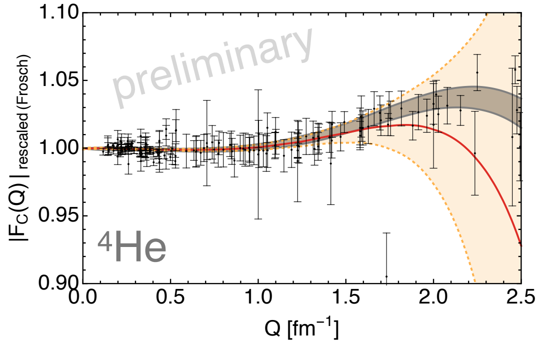


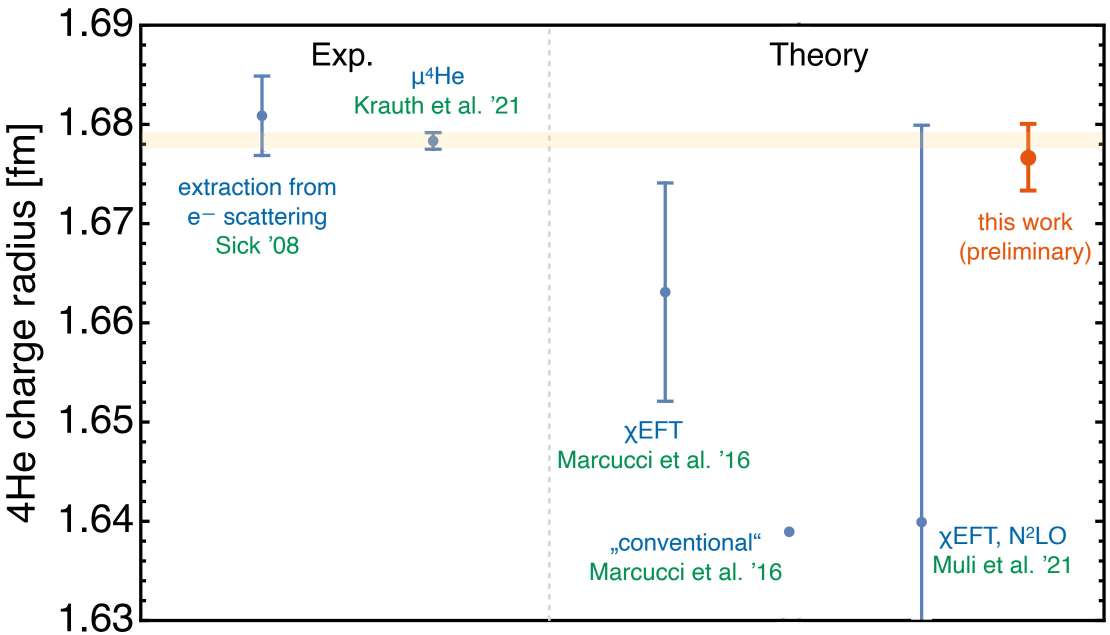
$$\boxed{r_{str}(^4He) = (1.4748 \pm 0.0030_{trunc} \pm 0.0013_{stat})\, fm}$$

preliminary, further uncertainty sources under investigation…

$$\Rightarrow \boxed{r_C(^4He) = (1.6766 \pm 0.0034)\, fm}$$

using CODATA r_p and own determination of r_n





The μ 4He exp. value is:

$$r_C^{exp}(^4He) = (1.67824 \pm 0.00083)\, fm$$

Krauth et al., Nature 589 (2021) 7843, 527-531

Charge radii of light nuclei

Filin, Baru, EE, Körber, Krebs, Möller, Reinert, in preparation

With all LECs being fixed, we can predict the isoscalar 3N charge radius $\sqrt{\frac{1}{3}r_C^2(^3\mathrm{H}) + \frac{2}{3}r_C^2(^3\mathrm{He})}$

$$\boxed{r_C(3N_{\mathrm{isoscalar}}) \;=\; (1.9065 \pm 0.0026)\ \mathrm{fm}}$$

preliminary, using CODATA-2018 r_p and own determination of r_n

On the experimental side:

- the ^{3}H radius poorly known (5%) from e⁻ scattering exp.: $r_C^{^3\mathrm{H}} = (1.755 \pm 0.086)\, fm$
  Amroun et al. '94 (world average)
- more (and more precise) measurements for ^{3}He
  - e⁻ scattering experiments: $r_C^{^3\mathrm{He}} = (1.959 \pm 0.030)\, fm$ Amroun et al. '94 (world average)
    $r_C^{^3\mathrm{He}} = (1.973 \pm 0.016)\, fm$ Sick '15 (world average)
  - muonic ^{3}He (preliminary): $r_C^{^3\mathrm{He}} = (1.9687 \pm 0.0013)\, fm$ Pohl '20

⇒ the current exp. value for the isoscalar radius: $\boxed{r_C^{\mathrm{exp}}(3N_{\mathrm{isoscalar}}) \;=\; (1.903 \pm 0.029)\ \mathrm{fm}}$

The ongoing T-REX experiment in Mainz [Pohl et al.] aims at measuring $r_C^{^3\mathrm{H}}$ with $\pm 0.0002\, fm$, which would determine the isoscalar radius with $\pm 0.0009\, fm$ ⇒ precision tests of nuclear chiral EFT!

Summary of the second part

- Calculated the charge and quadrupole FFs of light nuclei at N^{4}LO in chiral EFT

- Deuteron:
  - — determined r_{str} (0.1% accuracy) and Q_d (1.4% accuracy)
  - — combined with isotope-shift data, extracted the neutron charge radius (2σ tension with PDG…)

- ^{4}He: the extracted r_C (0.2% accuracy) agrees with the new μ^{4}He measurement

- ^{3}H-^{3}He: predicted the isoscalar r_C (0.1% accuracy) in agreement with the current exp. value (10 times bigger errors). The ongoing T-REX (μ^{3}H) exp. in Mainz will allow for a precision test of nuclear chiral EFT