Chiralspin symmetry and confinement in QCD

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Marco Catillo, November 8-12th, 2021

Introduction

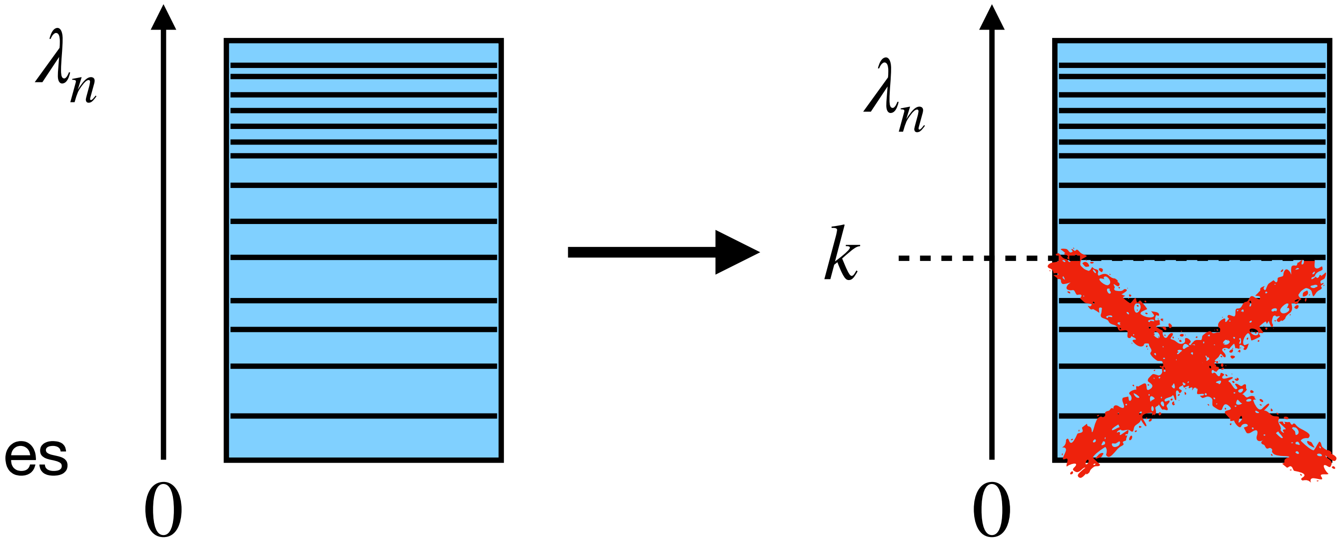
- The first study about the chiralspin group was made by Denissenya et al. (2015) in **truncated studies** in lattice QCD

- Suspect degeneracy has been observed in hadron masses.

$$D_{tr}^{-1} = D^{-1} - \sum_{i=1}^{k} \frac{1}{\lambda_k} |\lambda_k\rangle\langle\lambda_i|$$

$$D|\lambda_i\rangle = \lambda_i|\lambda_i\rangle$$

k= number of removed low-lying modes



Introduction

Plot from M. Denissenya et al. Phys. Rev. D91 (2015)

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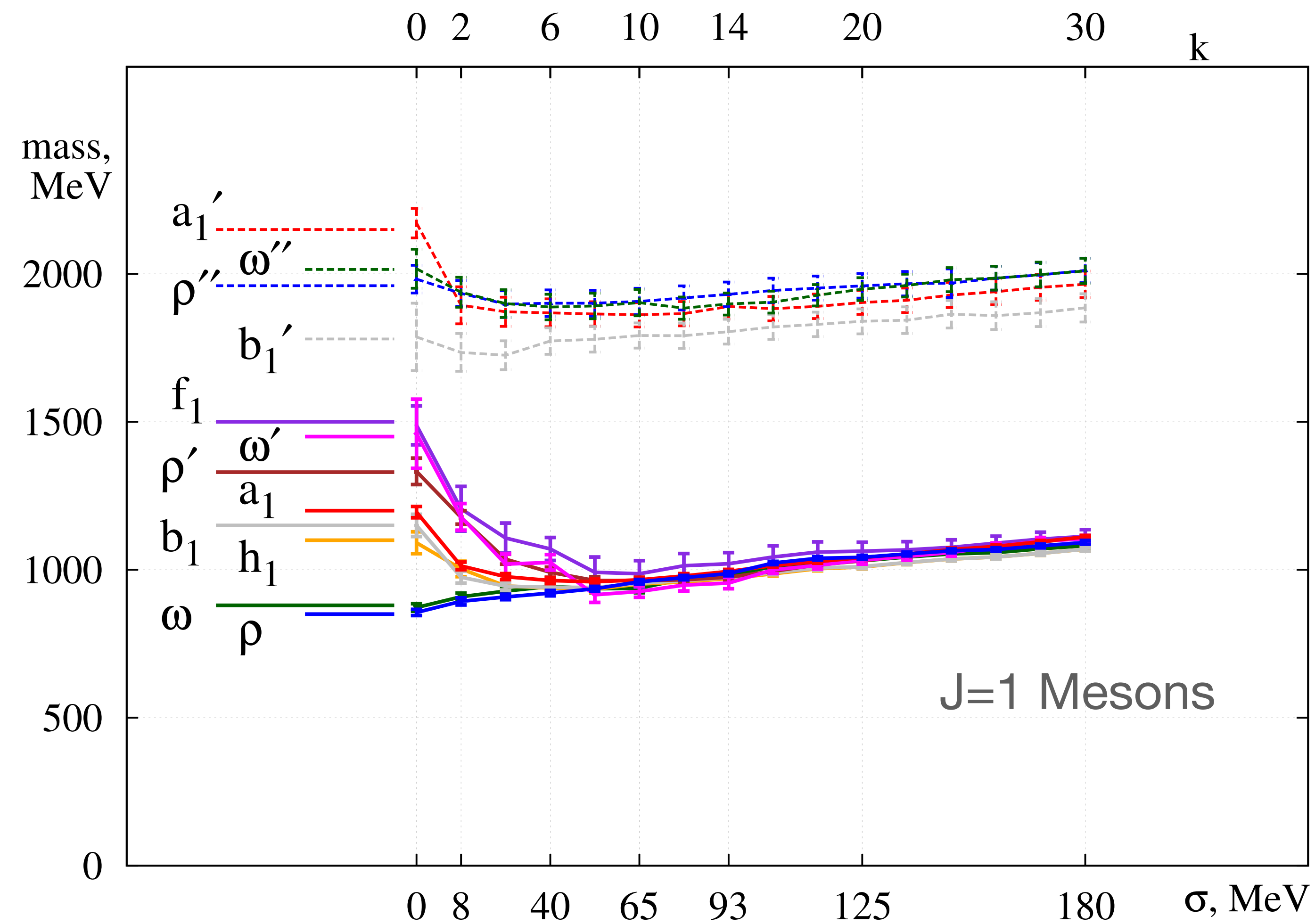


$N_F = 2$ Degenerate quark flavours

$Q_{top} = 0$ gauge configurations

$$\sum_{\mathbf{x},\mathbf{y}} \langle O(y)\bar{O}(x)\rangle \sim \langle Tr(\Gamma D^{-1}\Gamma D^{-1})\rangle = a_0 \exp(-mt)) + \ldots$$

$$\rightarrow \quad \sum_{\mathbf{x},\mathbf{y}} \langle O(y)\bar{O}(x)\rangle^{(k)} \sim \langle Tr(\Gamma D_{tr}^{-1}\Gamma D_{tr}^{-1})\rangle = a_k \exp(-m_{(k)}t)) + \ldots$$

Emergent Symmetry in QCD

The truncated studies reveal a restoration of:

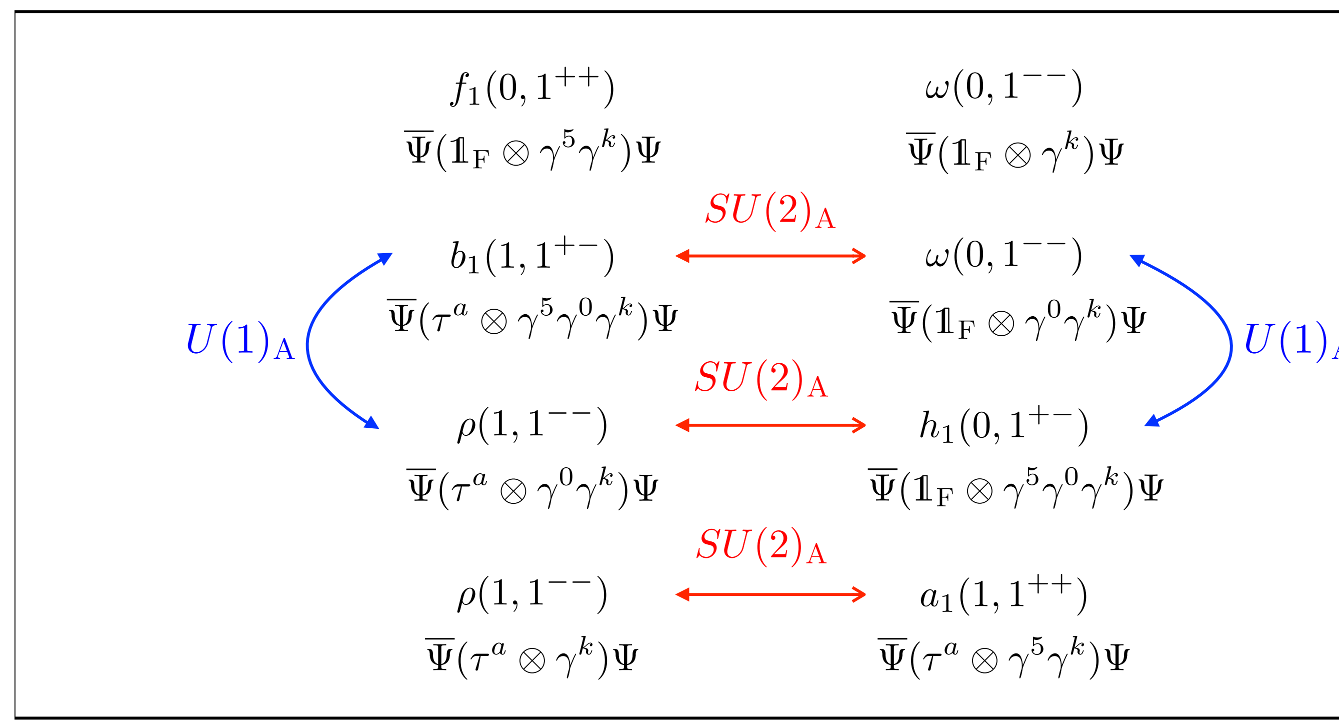
- Chiral symmetry $SU(N_F)_L \times SU(N_F)_R$
- Axial symmetry $U(1)_A$

} → Understood by Banks-Casher relation:

$$\Sigma = -\langle \bar{q}q \rangle = \pi\rho(0)$$

Atiyah-Singer theorem: $Q_{top} = n_- - n_+$

Denissenya et al. Phys. Rev. D91 (2015)



J=1 Mesons

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$\Big\} \longrightarrow$ Both understood by Banks-Casher relation:

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Emergence of a **new** symmetry group

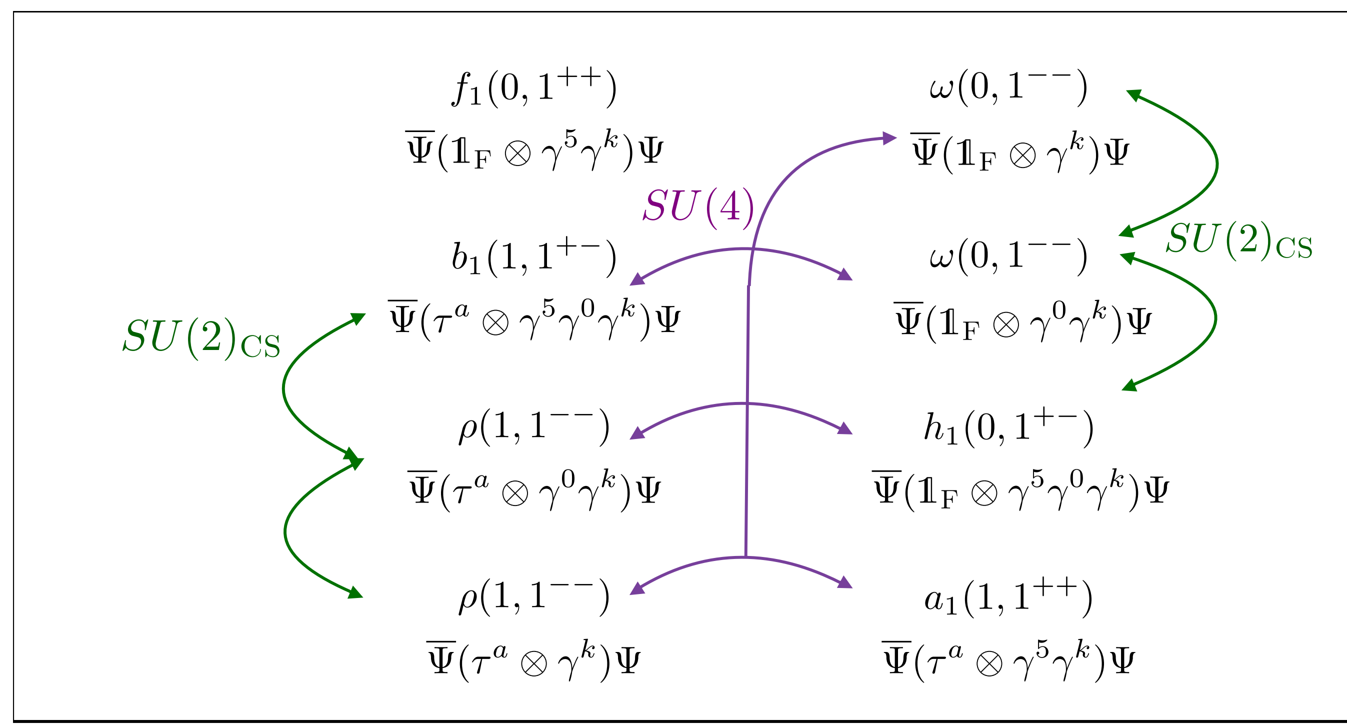
$$SU(2N_F) \supset SU(N_F)_L \times SU(N_F)_R \times U(1)_A$$

The real novelty of such group is the

Chiralspin group:

$$SU(2)_{CS} \subset SU(2N_F)$$

Denissenya et al. Phys. Rev. D91 (2015)



J=1 Mesons

Chiralspin group

Truncated lattice studies have been performed in different channels

- J = 0, 1, 2 Mesons
- J = 1/2, 3/2 Baryons

Denissenya et al. Phys.Rev.D 91 (2015), Phys.Rev.D 92 (2015)

Glozman et al. Phys.Rev.D 92 (2015)

→ All of them show the same mass degeneration upon removing the low-lying eigenmodes, which can be explained by the chiralspin group.

Chiralspin transformations:

Generators

L. Ya. Glozman Eur. Phys. J. A (2015)

$$\psi(x) \to \exp(i\alpha_n \Sigma_n)\, \psi(x) \qquad \Sigma_n = \{\gamma_4, i\gamma_5\gamma_4, -\gamma_5\} \qquad [\Sigma_i, \Sigma_j] = 2i\epsilon_{ijk}\Sigma_k$$

→ $U(1)_A \subset SU(2)_{CS}$

It contains the axial group as subgroup

- Where chiralspin comes from?
- Has QCD a new hidden symmetry?

Spectrum Dirac operator

We recognise that the higher eigenmodes (eigenvalues or eigenvectors) should be chiralspin symmetric.

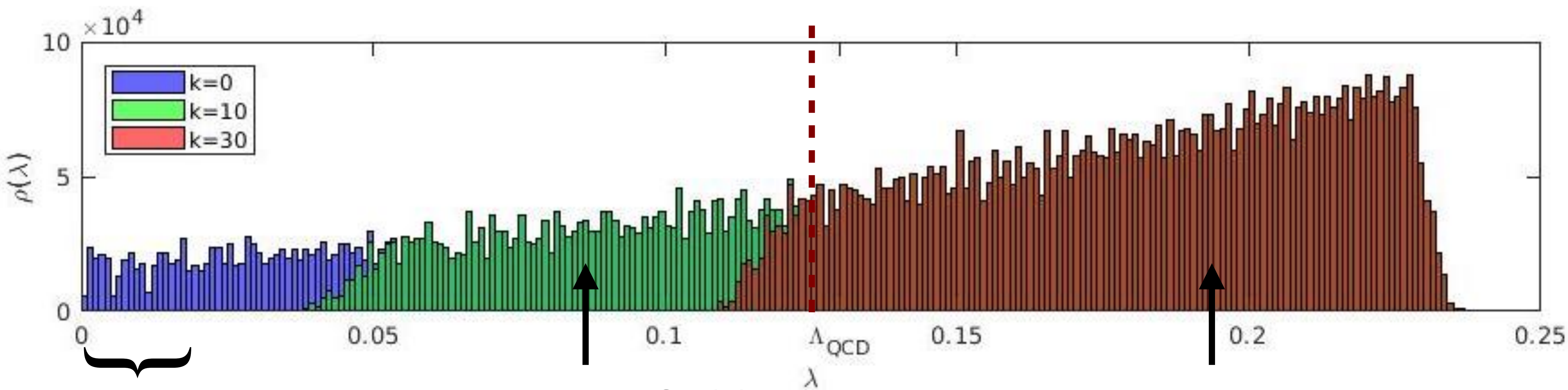
We have taken the same lattice setup on which $SU(2)_{CS}$ has been discovered.

m_π= 289(2) MeV

$\Sigma = (251 \pm 7 \pm 11\ \mathrm{MeV})^3$

$\Rightarrow V\Sigma m \simeq 4$ and $m_\pi L \simeq 3$

Outside ε-regime



χRMT

Emergence of $SU(2)_{cs}$ in mesons

Emergence of $SU(2)_{cs}$ in baryons

Spectrum Dirac operator

- The information on chiralspin symmetry seems to be hidden in the eigenvectors, as has been also given in the analysis of baryon and meson correlators M. Catillo et al. PRD 99 (2019), C.B. Lang PRD 97 (2018)

- The axial and chiral symmetry is encoded by the quark propagator: $\{D^{-1}, \gamma_5\} = 0$

- Two parts of the quark propagator:

$$D^{-1} = \sum_{\epsilon=\pm} \sum_{\lambda_n<\Lambda} \frac{1}{\lambda_n} v_n^{(\epsilon)} v_n^{(\epsilon)\dagger} + \sum_{\epsilon=\pm} \sum_{\lambda_n>\Lambda} \frac{1}{\lambda_n} v_n^{(\epsilon)} v_n^{(\epsilon)\dagger}$$

D_{CS}^{-1}

Chiralspin symmetric part of quark propagator

- Chiralspin symmetry constraint $\left[D_{CS}^{-1}, \gamma_4\right] = 0$

- Structure of higher eigenmodes in case of chiralspin symmetry:

$$v_n^{(\pm)} = \begin{pmatrix} \pm\chi_n \\ \tau_n \chi_n \end{pmatrix} \qquad \tau_n = \tau_n^*$$

Chiralspin group

We recognise some main characteristics of such truncated studies

- They have been performed with configurations with $Q_{top} = 0$
- The chiral condensate $\Sigma = 0$ since the eigenvalues distribution is forced to have $\lim_{\lambda \to 0} \rho(\lambda) = 0$
- Chiral symmetry and axial symmetry are present.

However truncated studies represent not the physical QCD.

Nevertheless, is there a physical regime of QCD with same characteristics?

- L. Glozman proposed to look at high temperature QCD above chiral phase transition

L. Ya. Glozman EPJ Web Conf. 164 (2017),
Acta Phys.Polon.Supp. 10 (2017)

Some issues

Searching chiralspin symmetry at high temperature presents some obstacles

- $SU(2)_{CS}$ is not a symmetry of free massless quark action:

$$S = \int d^4x\, \bar{\psi}(x)\gamma_\mu \partial_\mu \psi(x) = \underbrace{\int d^4x\, \bar{\psi}(x)\gamma_4 \partial_4 \psi(x)}_{\text{Chiralspin symmetric}} + \underbrace{\int d^4x\, \bar{\psi}(x)\gamma_k \partial_k \psi(x)}_{\text{Breaks explicitly Chiralspin}}$$

Only the chromo-electric part $\bar{\psi}(x)\gamma_4 A_4(x)\psi(x)$ is invariant under $SU(2)_{CS}$

- At high temperature quarks tend to be weakly interacting (quark-gluon plasma)

- If at $T = \infty$ QCD should become a free theory, how does this conciliate with the deconfinement regime of QCD at high T?

High temperature QCD

- As compromise between deconfinement and having the symmetry in the chirally symmetric phase, chiralspin symmetry is found in a region of temperatures between $1.2T_c - 2.8T_c$

C. Rohrhofer et al. PRD 100 (2019)

Studies of correlators in spatial direction z

$$\sum_{x,y,t} \langle O(x,y,z,t)\bar{O}(0,0,0,0)\rangle$$

- $\mathcal{O}(4)$ Lorentz invariance of the Euclidean action has been exploited

$$\psi(x) \to \exp(i\alpha_n\Sigma_n)\,\psi(x) \qquad \Sigma_n = \{\gamma_k, i\gamma_5\gamma_k, -\gamma_5\} \qquad [\Sigma_i, \Sigma_j] = 2i\epsilon_{ijk}\Sigma_k$$

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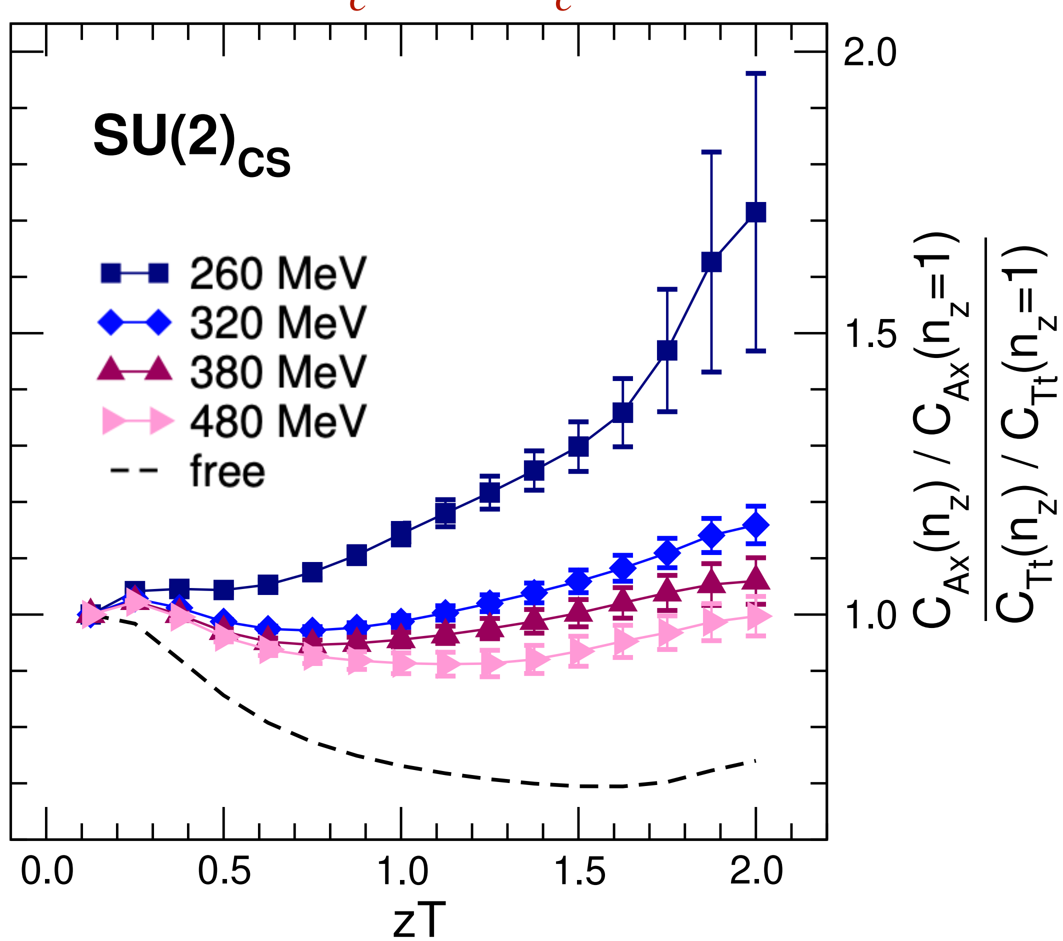
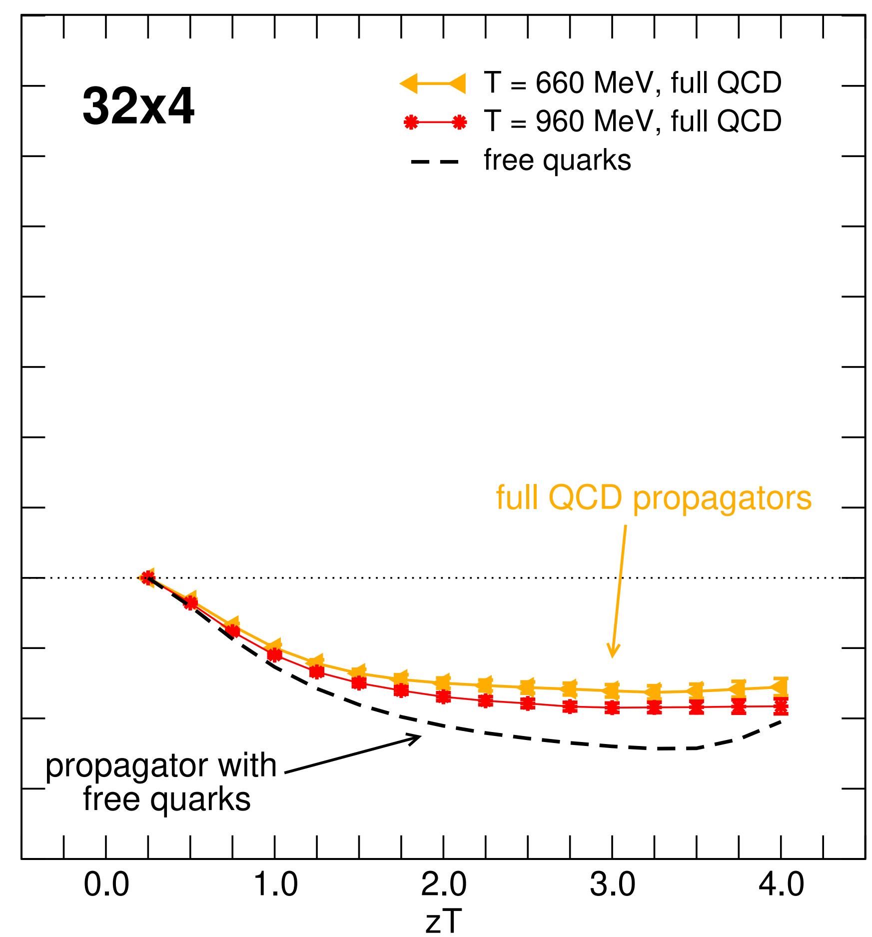
Studies of correlators in spatial direction z

Plot from C. Rohrhofer et al. PRD 100 (2019)

$1.2T_c - 2.8T_c$



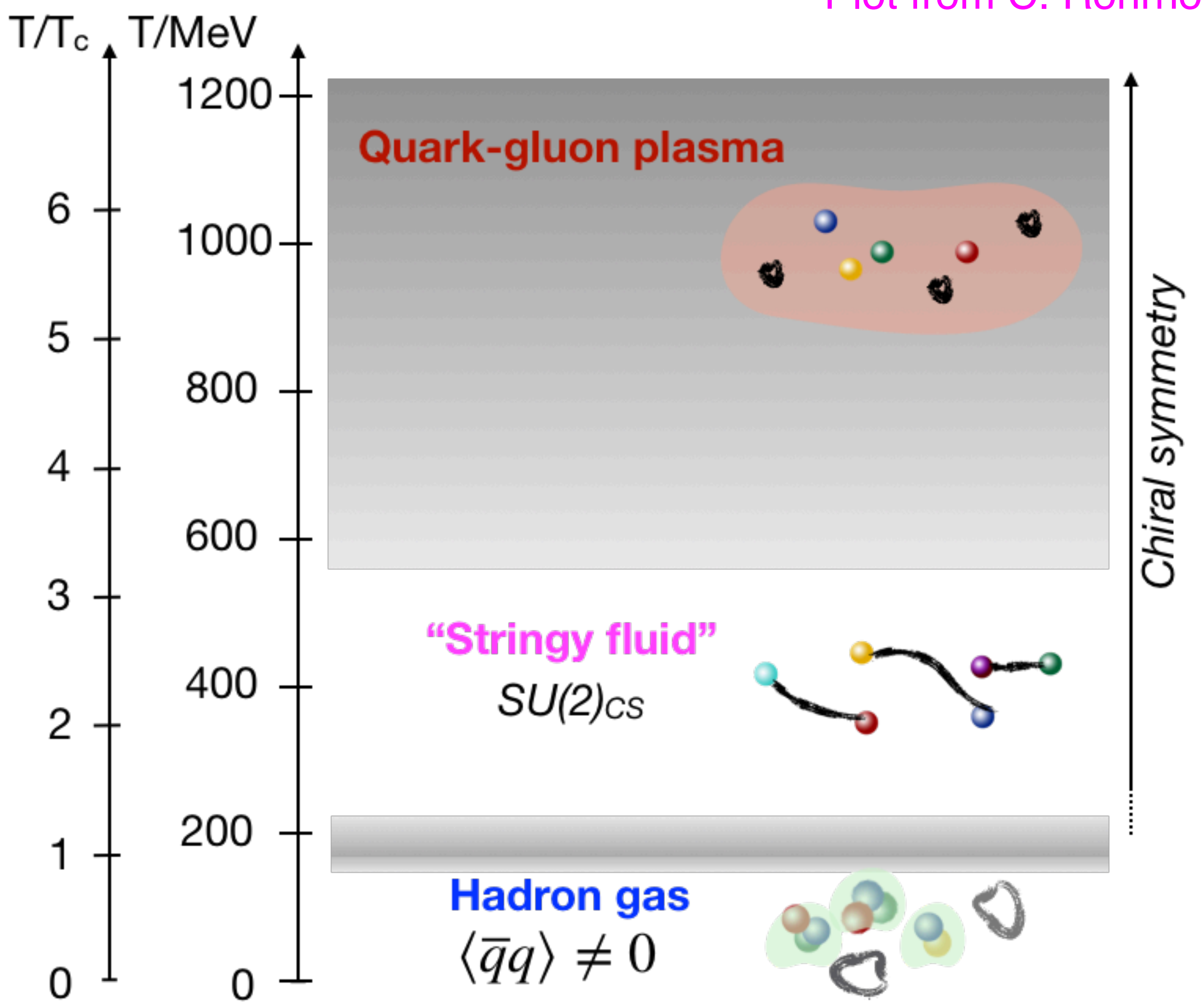
$T \gg 3T_c$



High temperature QCD

- Chiralspin symmetry disappears for $T \gg 3T_c$ because of deconfinement, hence there is only a narrow regime on which matter becomes chiralspin symmetric

Plot from C. Rohrhofer et al. PRD 100 (2019)



High temperature QCD

However there are still some doubts

- The correlators studied are averaged over space and time
- Which type of transition occurs after $\sim 3T_c$?
- The symmetry seems to be only approximate in the range $1.2T_c - 2.8T_c$
- Is the symmetry observed in the range $1.2T_c - 2.8T_c$, the same observed in truncated studies?

Let us see it is possible to explain the hadron mass degenarcy from the truncated studies using a different group and the relation with $SU(2)_{CS}$

Chiralspin group details

$SU(2)_{CS}$ group transformations are defined as

$$\psi(x) \to U^{\alpha}\,\psi(x) = \exp(i\alpha_n\Sigma_n)\,\psi(x) \qquad \Sigma_n = \{\gamma_4, i\gamma_5\gamma_4, -\gamma_5\}$$

It has 2 main $U(1)$ subgroups:

$$\psi(x) \to U_A^{\alpha_3}\,\psi(x) = \exp(-i\alpha_3\gamma_5)\,\psi(x) \quad \longleftarrow \quad U(1)_A$$

Axial group

$$\psi(x) \to U_4^{\alpha_1}\,\psi(x) = \exp(i\alpha_1\gamma_4)\,\psi(x) \quad \longleftarrow \quad U(1)_4$$
$$= \cos(\alpha)\psi(x) + i\sin(\alpha)\gamma_4\psi(x)$$

Group involving γ_4 as generator

Every element of $SU(2)_{CS}$ can be written as

$$U^{\alpha} = U_4^{\beta_1}U_A^{\beta_2}U_4^{\beta_3} \quad \longleftarrow$$

Euler angles

Therefore the real novelty in $SU(2)_{CS}$ is the subgroup $U(1)_4 \subset SU(2)_{CS}$.

Towards a new chiralspin group

In order to define a new chiralspin group which contains $U(1)_A$ as subgroup, we redefine $U(1)_4$.

Parity: $\psi(x) \to \psi(x)^{\mathscr{P}} = \gamma_4 \psi(\mathscr{P}x)$ $\qquad \mathscr{P} = diag(-1,-1,-1,1)$

Exponentiation: $\psi(x) \to \psi(x)^{U_P^\alpha} = \sum_n \frac{(i\alpha)^n}{n!} \psi(x)^{\mathscr{P}^n} = \cos(\alpha)\psi(x) + i\sin(\alpha)\gamma_4\psi(\mathscr{P}x)$

New group transformation, we name it $U(1)_P$ which is different from $U(1)_4$

$U(1)_P$ and $U(1)_4$ coincide in the point $x = x^{(t)} = (\mathbf{0}, \mathbf{x_4})$

Now I introduce two parity partners $\psi_\pm(x) = \frac{1}{2}(\psi(x) \pm \psi(\mathscr{P}x))$, so that

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \to \Psi^{U_P^\alpha} = \begin{pmatrix} \psi_+^{U_P^\alpha} \\ \psi_-^{U_P^\alpha} \end{pmatrix} = \begin{pmatrix} e^{i\alpha\gamma_4}\psi_+ \\ e^{-i\alpha\gamma_4}\psi_- \end{pmatrix} = \exp(i\alpha(\sigma^3 \otimes \gamma_4))\Psi = U_P^\alpha \Psi$$

Towards a new chiralspin group

- We can apply the $U(1)_A$ transformation on Ψ as

$$\Psi = \begin{pmatrix} \psi_+ \\ \psi_- \end{pmatrix} \to \Psi^{U_A^\alpha} = \begin{pmatrix} \psi_+^{U_A^\alpha} \\ \psi_-^{U_A^\alpha} \end{pmatrix} = \begin{pmatrix} e^{i\alpha\gamma_5}\psi_+ \\ e^{i\alpha\gamma_5}\psi_- \end{pmatrix} = \exp(i\alpha(1 \otimes \gamma_5))\Psi = U_A^\alpha \Psi$$

- The set of all elements $U^\alpha = U_P^{\beta_1} U_A^{\beta_2} U_P^{\beta_3}$ form an $SU(2)$ group, which we call $SU(2)_{CS}^{\mathscr{P}}$

- $SU(2)_{CS}^{\mathscr{P}}$ and $SU(2)_{CS}$ are different but they are the same in the point $x = x^{(t)} = (\mathbf{0}, \mathbf{x_4})$.

Main result

M. Catillo arXiv:2109.03532

- Action of free massless fermions is $U(1)_P$ invariant

$$S = \int d^4x \, \bar{\psi}(x)\gamma_\mu \partial_\mu \psi(x)$$

→ $SU(2)_{CS}^{\mathscr{P}}$ invariant

- We have constructed a chiralspin group compatible with deconfinement in QCD

Gauge interaction

A gauge interaction term in the action breaks explicitly $U(1)_P$, here is why

$$S_I(\psi, \bar{\psi}, A) = i \int d^4\, \bar{\psi}(x)\gamma_\mu A_\mu(x)\psi(x) \longrightarrow S_I(\psi^{U_P^\alpha}, \bar{\psi}^{U_P^\alpha}, A) = i \int d^4\, \bar{\psi}(x)^{U_P^\alpha}\gamma_\mu A_\mu(x)\psi(x)^{U_P^\alpha}$$

$$= i \int d^4x \left[\bar{\psi}(x)\gamma_\mu(\cos(\alpha)^2 A_\mu(x) + \sin(\alpha)^2 A_\mu(x)^{\mathscr{P}})\psi(x) + i\sin(\alpha)\cos(\alpha)\, \bar{\psi}(x)\gamma_\mu(A_\mu(x) - A_\mu(x)^{\mathscr{P}})\gamma_4\psi(\mathscr{P}x)\right]$$

where $A_\mu(x)^{\mathscr{P}} = \mathscr{P}_{\mu\nu}A_\nu(\mathscr{P}x)$, which is mixed with $A_\mu(x)$.

It implies that also $SU(2)_{CS}^{\mathscr{P}}$ is broken explicitly

- A sufficient condition for having the invariance is that $A_\mu(x)^{\mathscr{P}} = A_\mu(x)$
- However $Q_{top}(A^{\mathscr{P}}) = -Q_{top}(A)$
- It means to restrict to gauge fields in the zero topological sector.

Gauge interaction

- Example with $SU(2)$ gauge theory:

- Instanton solution $$A^I_\mu(x;\rho,\bar{x}) = \frac{\eta^a_{\mu\nu}(x-\bar{x})_\nu \sigma^a}{[(x-\bar{x})^2+\rho^2]} \longrightarrow Q_{top}(A^I) = 1$$

- A parity transformation gives an anti-instanton solution

$$A^I_\mu(x;\rho,\bar{x})^{\mathscr{P}} = \mathscr{P}_{\mu\nu} A^I_\nu(\mathscr{P}x;\rho,\bar{x}) \longrightarrow A^I_\mu(x;\rho,\bar{x})^{\mathscr{P}} = \frac{\bar{\eta}^a_{\mu\nu}(x-\mathscr{P}\bar{x})_\nu \sigma^a}{[(x-\mathscr{P}\bar{x})^2+\rho^2]} = A^{\bar{I}}_\mu(x;\rho,\mathscr{P}\bar{x})$$

$$\uparrow \quad Q_{top}(A^{\bar{I}}) = -1$$

- Possible gauge structure compatible with $SU(2)^{\mathscr{P}}_{CS}$ are molecules of instanton and anti-instanton, like
$$A^{I\bar{I}}_\mu(x;\rho) = A^I_\mu(x;\rho,\bar{x}) + A^{\bar{I}}_\mu(x;\rho,\mathscr{P}\bar{x})$$

Ilgenfritz and Shuryak NPB 319 (1989)

Ilgenfritz and Shuryak NLB 325 (1994)

Schaefer et al. PRD 51 (1995)

- Which are present at high temperature QCD

Observables

- At $x = x^{(t)} = (\mathbf{0}, \mathbf{x_4})$, $U(1)_P$ and $U(1)_4$ group transformations coincide, i.e.

$$U_4^\alpha \psi(x^{(t)}) = \psi(x^{(t)})^{U_P^\alpha}$$

- Which implies that since $SU(2)_{CS}^{\mathscr{P}}$ share with $SU(2)_{CS}$ the axial group

$$U_{CS}^\alpha\, \psi(x^{(t)}) = \psi(x^{(t)})^{U_{CS\mathscr{P}}^\alpha} \longleftarrow$$

$SU(2)_{CS}$ and $SU(2)_{CS}^{\mathscr{P}}$ coincide

- This means that taking e.g. a meson observable $O_\Gamma(x) = \bar{\psi}(x)\Gamma\psi(x)$

$$O_{\Gamma'}(x) = \bar{\psi}(x)\Gamma'\psi(x) \equiv \bar{\psi}(x)\underbrace{\gamma_4 U_{CS}^{\alpha\dagger}\gamma_4}\Gamma U_{CS}^\alpha \psi(x) \longrightarrow O_{\Gamma'}(x^{(t)}) = \bar{\psi}(x^{(t)})^{U_{CS\mathscr{P}}^\alpha}\Gamma\psi(x^{(t)})^{U_{CS\mathscr{P}}^\alpha}$$

As in Minkowskian

In $x^{(t)}$ the two chiralspin groups act in the same way

Correlators

- If $SU(2)_{CS}$ is a symmetry in the theory (in truncated lattice studies), then a degeneration appears

$$\left.\begin{aligned} &\sum_{\mathbf{x}} \langle O_\Gamma(x + t\hat{4})\bar{O}_\Gamma(x)\rangle \sim \exp(-m_\Gamma t) \\ &\sum_{\mathbf{x}} \langle O_{\Gamma'}(x + t\hat{4})\bar{O}_{\Gamma'}(x)\rangle \sim \exp(-m_{\Gamma'} t) \end{aligned}\right\} \Rightarrow \qquad m_\Gamma = m_{\Gamma'}$$

- But the same mass mass degeneration can be obtained considering

$$\langle O_\Gamma(x^{(t)} + t\hat{4})\bar{O}_\Gamma(x^{(t)})\rangle \sim \exp(-m_\Gamma t)$$
$$\langle O_{\Gamma'}(x^{(t)} + t\hat{4})\bar{O}_{\Gamma'}(x^{(t)})\rangle \sim \exp(-m_{\Gamma'} t)$$

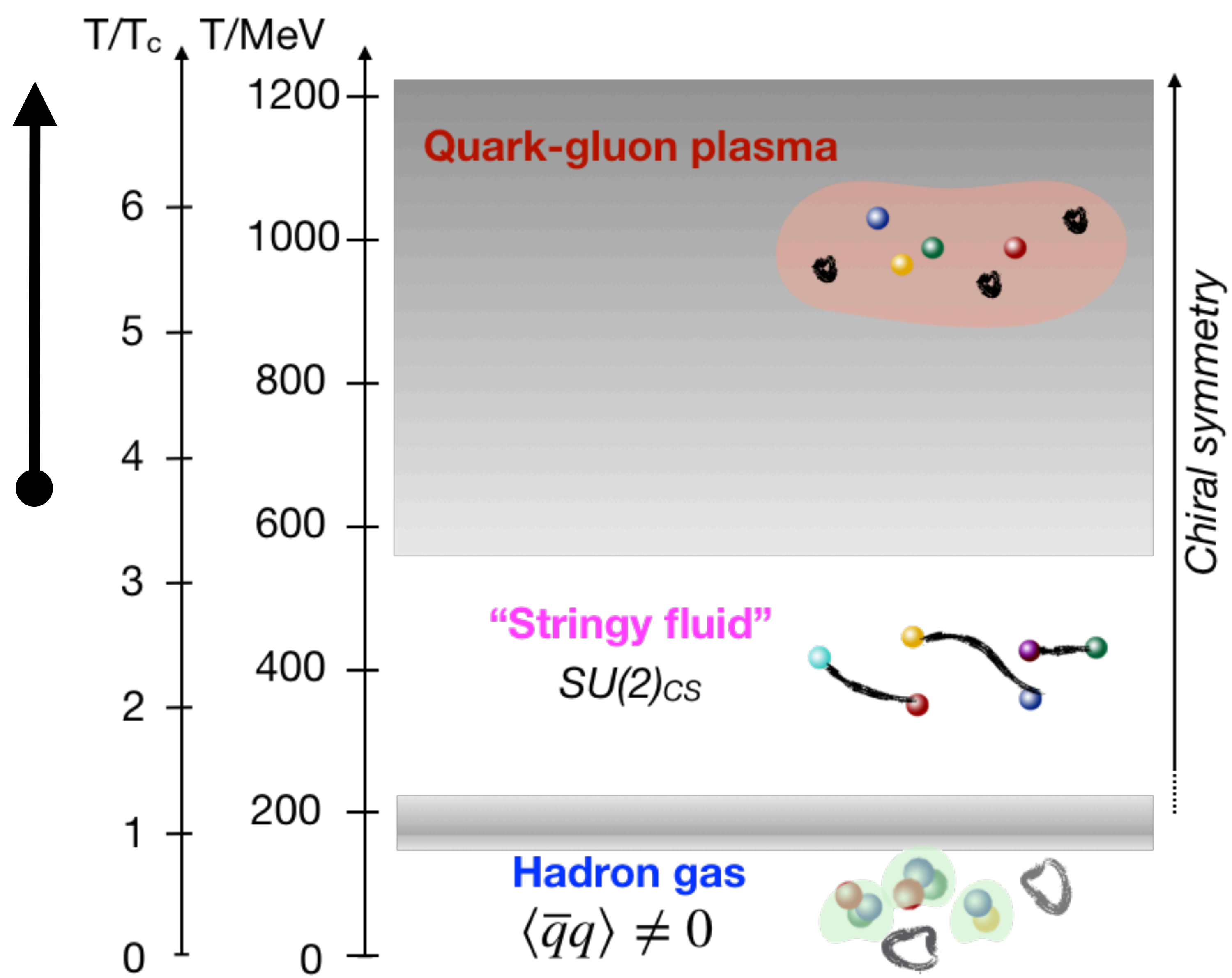
We have fixed some reference frame

- Nevertheless $O_{\Gamma'}(x^{(t)})$ can be also obtained by $SU(2)_{CS}^{\mathscr{P}}$ transformation.

- The same mass degeneration $m_\Gamma = m_{\Gamma'}$ can be explained by $SU(2)_{CS}$ and $SU(2)_{CS}^{\mathscr{P}}$ $\Rightarrow$ We can't distinguish them.

Phase diagram

Are there other interesting symmetries to consider?



Conclusions

- The mass degeneracy from truncated studies has been explained by chiralspin group
- Chiralspin symmetry gives some constraints on higher eigenmodes
- $SU(2)_{CS}$ should be present in the regime where chiral symmetry is restored, and where gauge configurations with $Q_{top} = 0$ are predominant
- It is present when quarks are still strongly interacting particles (before quark-gluon plasma)
- Therefore $SU(2)_{CS}$ has been found in a range of temperature $\sim T_c - 3T_c$
- Supposing that the **truncated lattice studies** are explained by **another group $SU(2)^{\mathscr{P}}_{CS}$ compatible with deconfinement**, can we find such symmetry also at temperature higher than $3T_c$?

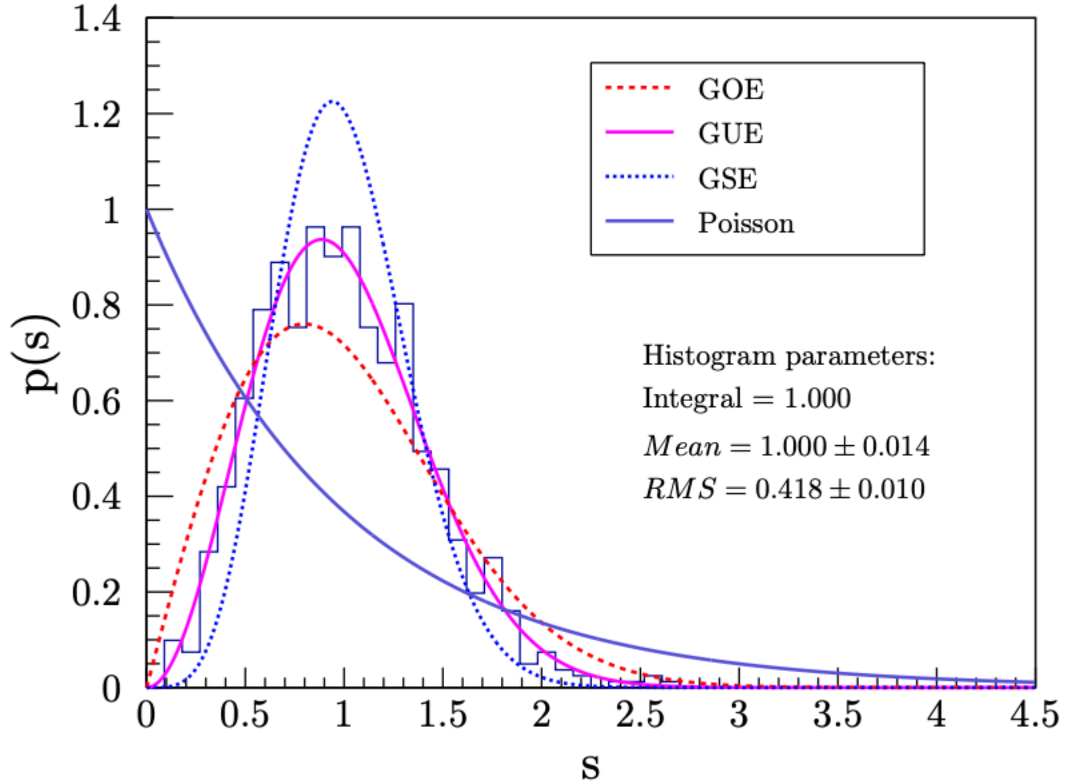
Backup

Different scales are involved in the Dirac spectrum and they are connected with emergence of symmetries in QCD

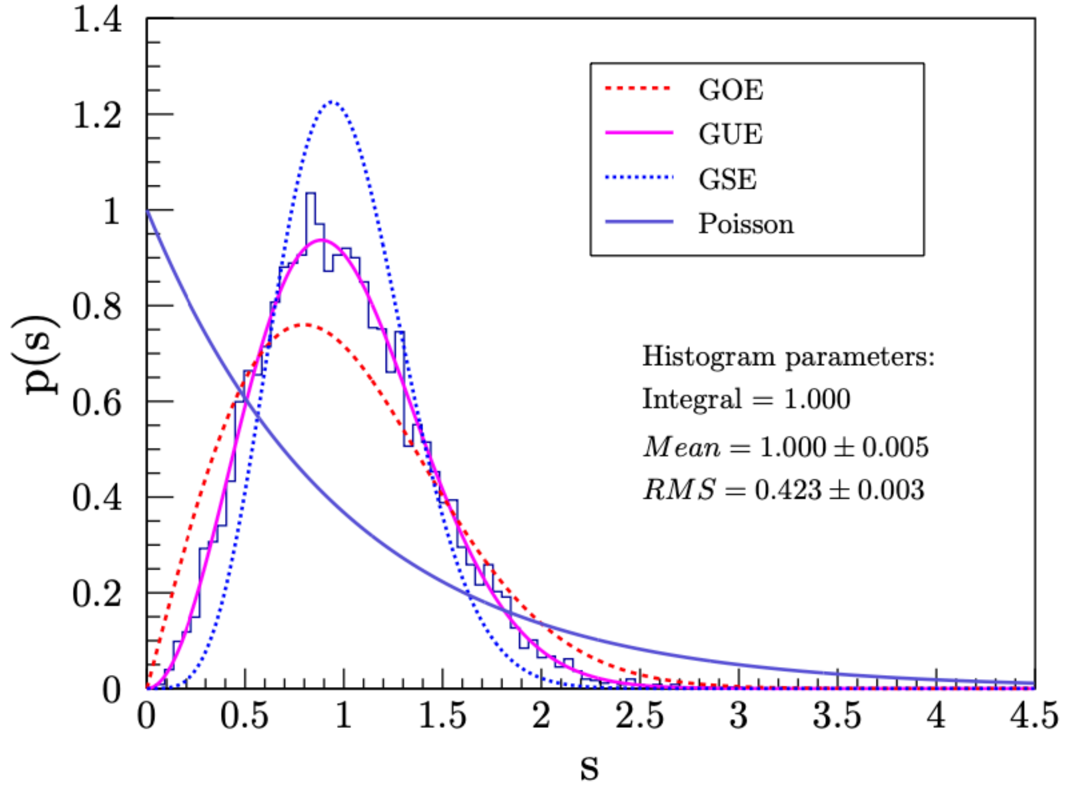
Unfolding procedure $$\xi_n \equiv \xi(\lambda_n) = \int_0^{\lambda_n} \rho(\lambda) d\lambda \Rightarrow s_n = \xi_{n+1} - \xi_n$$

Nearest neighbor spacing (NNS) distribution: $$p(s)_{GUE} = \frac{32}{\pi^2} e^{-\frac{4}{\pi}s^2}$$

Range eigenvalues: 1-10



Range eigenvalues: 81-100



They are identical: no so much information from the eigenvalue distribution