A lattice QCD view of the hadronic contributions to the anomalous magnetic of the muon

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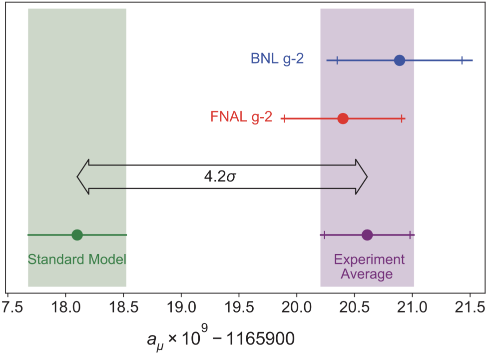
$(g-2)$ of the muon: current status

Current 4.2σ tension between the direct measurement (BNL + FNAL) and Standard Model prediction:



Standard Model prediction: White Paper 2006.04822 (Phys.Rept).

Figure from Muon (g-2) collaboration, 2104.03281 (PRL).

Outline

Reducing the hadronic uncertainties using lattice QCD:

- the hadronic vacuum polarisation (O(α^2), 0.2% precision desirable)
- the ‘hadronic light-by-light’ contribution (O(α^3), 10% precision desirable)

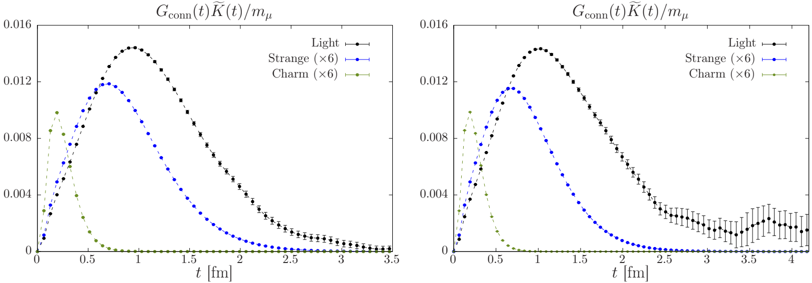
Useful resources:

- online workshop “Muon g-2 theory initiative workshop in memoriam Simon Eidelman”, KEK, Japan, 28 June - 3 July 2021.
- (online) Lattice conference 2021, MIT, July 26-30, 2021.
- (online) TAU2021 conference, Indiana University, Sep. 27 to Oct. 1, 2021.

Computing $a_\mu^{\rm hvp}$ from lattice-QCD current-current correlators $G(t)$

$$G(t) \equiv \int d^3x \langle j_3(t,\vec{x})\, j_3^\dagger(0)\rangle = \int_0^\infty d\omega\, \omega^2 \frac{R_{e^+e^-\to{\rm hadrons}}(\omega^2)}{12\pi^2}\, e^{-\omega t}.$$

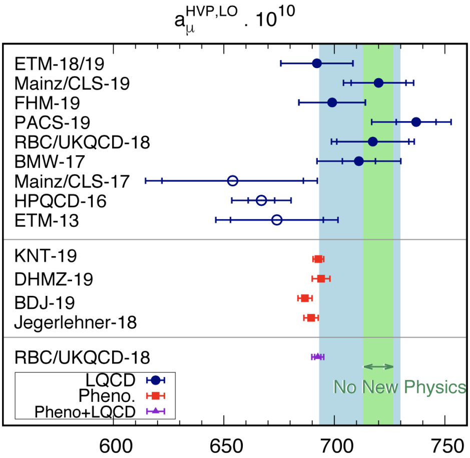
$a = 0.064{\rm fm}:$ $m_\pi = 200\,{\rm MeV}$ $m_\pi = 130\,{\rm MeV}$ Mainz/CLS 1904.03120.



$$a_\mu^{\rm hvp} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \widetilde{K}(t) G(t), \qquad \text{where} \qquad \widetilde{K}(t) \sim \begin{cases} \frac{\pi^2}{9} m_\mu^2 t^4 & t \ll m_\mu^{-1} \\ 2\pi^2 t^2 & t \gg m_\mu^{-1}. \end{cases}$$

- The tail of the integrand is affected by statistical fluctuations, by finite-size effects and depends strongly on the pion mass due to the $\pi\pi$ channel.

Leading hadronic contribution to $(g-2)_\mu$ from lattice QCD



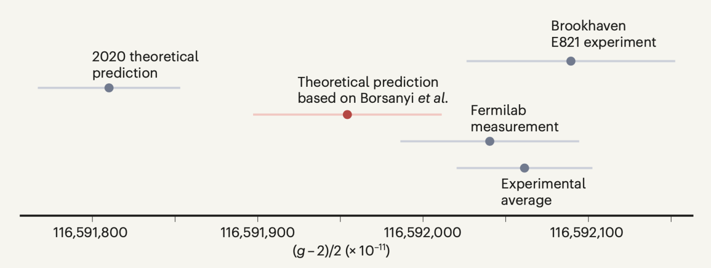
Summary plot from White Paper 2006.04822 (Phys.Rept).

Lattice average: $a_\mu^{\mathrm{HVP,LO}} = (711.6 \pm 18.4) \times 10^{-10}$

To be compared with $a_\mu^{\mathrm{hvp,pheno}} = (693.1 \pm 4.0) \times 10^{-10}$.

The Budapest-Marseille-Wuppertal lattice calculation

BMW result: $a_\mu^{\mathrm{hvp,BMW}} = 707.5(2.3)_{\mathrm{stat}}(5.0)_{\mathrm{syst}}(5.5)_{\mathrm{tot}} \times 10^{-10}$.



[BMW, 2002.12347 (Nature vol 593, 51 (2021))].

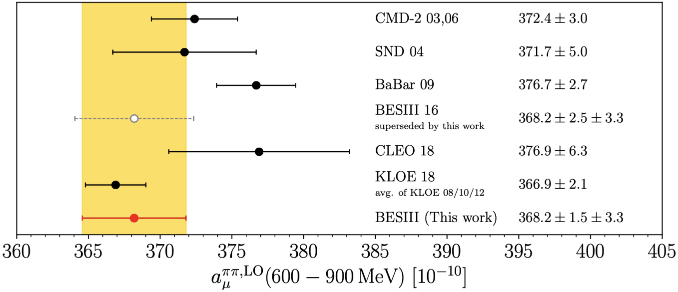
Figure from HM, *Thrill of the Magnetic Moment*, Nature Vol 593, 44 (2021).

Aspects of the dispersive approach: $a_\mu = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_0^\infty \frac{ds}{s^2}\, \hat{K}(s)\, R(s)$

Selected exclusive channels contributing to a_μ^{hvp}: From White Paper 2006.04822

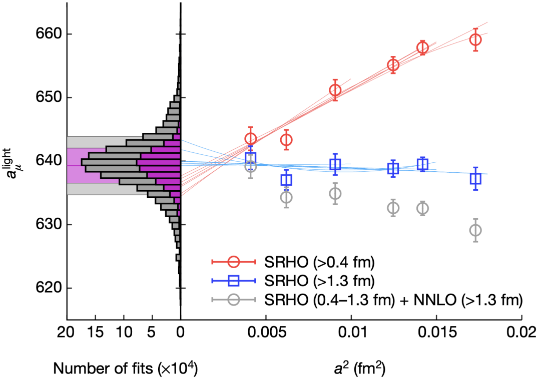
	DHMZ19	KNT19	Difference
$\pi^+\pi^-$	507.85(0.83)(3.23)(0.55)	504.23(1.90)	3.62
$\pi^+\pi^-\pi^0$	46.21(0.40)(1.10)(0.86)	46.63(94)	−0.42
$\pi^+\pi^-\pi^+\pi^-$	13.68(0.03)(0.27)(0.14)	13.99(19)	−0.31
$\pi^+\pi^-\pi^0\pi^0$	18.03(0.06)(0.48)(0.26)	18.15(74)	−0.12
K^+K^-	23.08(0.20)(0.33)(0.21)	23.00(22)	0.08
K_SK_L	12.82(0.06)(0.18)(0.15)	13.04(19)	−0.22
$\pi^0\gamma$	4.41(0.06)(0.04)(0.07)	4.58(10)	−0.17
Sum of the above	626.08(0.95)(3.48)(1.47)	623.62(2.27)	2.46
[1.8, 3.7] GeV (without $c\bar{c}$)	33.45(71)	34.45(56)	−1.00
J/ψ, $\psi(2S)$	7.76(12)	7.84(19)	−0.08
[3.7, ∞) GeV	17.15(31)	16.95(19)	0.20
Total $a_\mu^{\text{HVP, LO}}$	$694.0(1.0)(3.5)(1.6)(0.1)_\psi(0.7)_{\text{DV+QCD}}$	692.8(2.4)	1.2

Tension in $\pi^+\pi^-$ channel data: Fig from BES III, 2009.05011 (PLB).



BMW calculation: continuum extrapolation of light-quark (connected) contribution

In this calculation using stout-smeared staggered fermions, pion taste-splittings are the source of the dominant cutoff effects:



Error estimated from the distribution of results obtained in the continuum, applying different a^2 correction schemes to the data.

[BMW, 2002.12347 (Nature vol 593, 51 (2021))].

Recent progress on understanding cutoff effects from short distances

At short distances in massless lattice QCD:

$$G(t,a) = G_{\rm cont}(t)(1 + {\rm O}((a/t)^2))$$

Therefore, since $G_{\rm cont}(t) \sim 1/t^3$,

$$\int_0^t dt'\, t'^4\, G(t',a) = \int_0^t dt'\, t'^4\, G_{\rm cont}(t') + {\rm const} \times a^2 \int_a^t dt'\, t'^4\, \frac{1}{t'^5} + {\rm O}(a^2)$$

one obtains a logarithmically enhanced cutoff effect from short distances.

In leading order of Wilson lattice perturbation theory, one finds

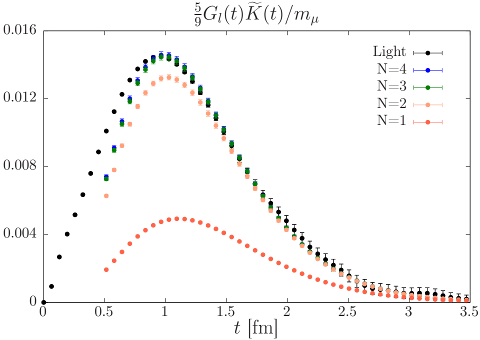
$$\int_0^t dt'\, t'^4\, G(t',a) = \int_0^t dt'\, t'^4\, G_{\rm cont}(t') + \frac{7N_c \sum_f \mathcal{Q}_f^2}{60\pi^2} a^2 \log(1/a) + {\rm O}(a^2).$$

Cè, Harris, HM, Toniato, Török [2106.15293].

See Husung, Marquard, Sommer 1912.08498 (EPJC) for recent results on logarithmic effects to be expect in on-shell quantities.

Possible strategies to improve control over the long-distance tail

1. Auxiliary calculation of the (discrete, finite-volume) spectrum of $\pi\pi$ states and their coupling to the e.m. current.
   The low-lying states saturate the correlator at long distances.



D200:
$a = 0.064\,\mathrm{fm}$
$m_\pi = 200\,\mathrm{MeV}$
Fig. from 1904.03120.

See also RBC/UKQCD 2019; Fermilab-HPQCD-MILC 2021.

2. 'all-to-all' propagators using the low eigenmodes of the Dirac operator
   RBC collaboration 1801.07224 (PRL); BMW collaboration 2002.12347 (Nature).
3. approximate factorization of the QCD path integral with bias correction.
   Dalla Brida et al., 2007.02973.

Other aspects of lattice calculations of $a_\mu^{\rm hvp}$

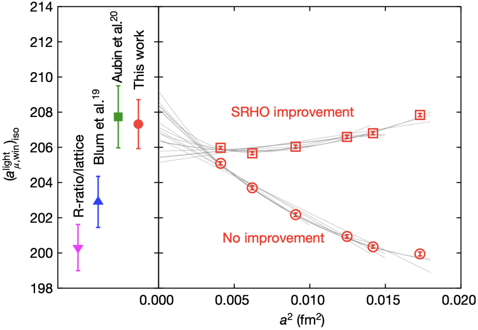
Beyond controlling the lattice-spacing dependence and the statistical precision of the tail, there are many aspects to the calculation:

- "scale setting": the lattice spacing must be calibrated to a precision of a few permille
- finite-size effects
- disconnected diagrams
- strange and charm contributions (contribute about 54 and 15 $\times 10^{-10}$)
- isospin-breaking effects: expanding around $m_\pi = m_{\pi^0}^{\rm phys}$, appear to be very small due to large cancellations between different contributions (BMW, 2002.12347). Several formalisms are available to handle QED in lattice QCD.

Contribution of an intermediate window of Euclidean time

- A full lattice calculation of a_μ^{hvp} at the subpercent level requires control over a vast number of contributions and effects.
- $\Rightarrow$ community strategy: perform comparisons of particularly precisely determined subcontributions or closely related quantities.

$$a_\mu^W = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt\, \widetilde{K}(t) \cdot \mathrm{window}_{[t_0,t_1]}(t) \cdot G(t), \qquad t_0 = 0.4\,\mathrm{fm},\ t_1 = 1.0\,\mathrm{fm}.$$



Expect updates in coming few months.

Figure from BMW, 2002.12347 (Nature).

Hadronic light-by-light contribution to $(g-2)_\mu$ from lattice QCD

Direct lattice calculation of HLbL in $(g-2)_\mu$

At first, this was thought of as a QED+QCD calculation
[pioneered in Hayakawa et al., hep-lat/0509016].

Today's viewpoint: the calculation is considered a QCD four-point Green's function, to be integrated over with a weighting kernel which contains all the QED parts.

RBC-UKQCD: calculation of $a_\mu^{\rm HLbL}$ using coordinate-space method in muon rest-frame; photon+muon propagators:

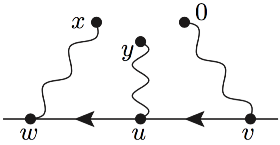
- either on the $L \times L \times L$ torus (QED$_L$) (1510.07100–present)
- or in infinite volume (QED$_\infty$) (1705.01067–present).

Mainz:

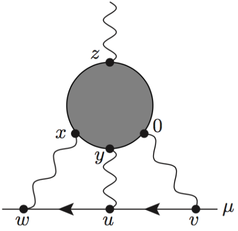
- manifestly covariant QED$_\infty$ coordinate-space approach, averaging over muon momentum using the Gegenbauer polynomial technique (1510.08384–present).

Coordinate-space approach to $a_\mu^{\rm HLbL}$, Mainz version

QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$



$\Rightarrow$



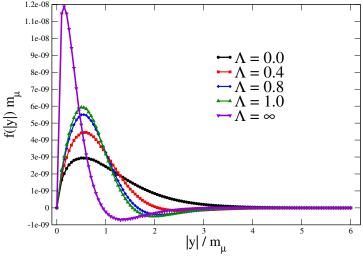
$$a_\mu^{\rm HLbL} = \frac{me^6}{3} \underbrace{\int d^4y}_{=2\pi^2|y|^3 d|y|} \Big[\int d^4x \, \underbrace{\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)}_{\rm QED} \; \underbrace{i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y)}_{\rm =QCD\ blob}\Big].$$

$$i\widehat{\Pi}_{\rho;\mu\nu\lambda\sigma}(x,y) = -\int d^4z \; z_\rho \, \Big\langle \, j_\mu(x)\, j_\nu(y)\, j_\sigma(z)\, j_\lambda(0) \Big\rangle.$$

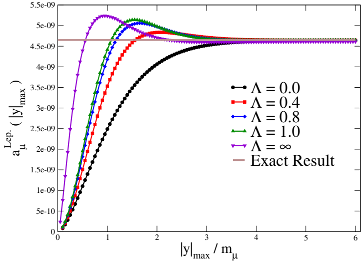
- $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ computed in the continuum & infinite-volume
- no power-law finite-volume effects & only a 1d integral to sample the integrand in $|y|$.

[Asmussen, Gérardin, Green, HM, Nyffeler 1510.08384, 1609.08454]

Tests of the framework and adjustments to the kernel



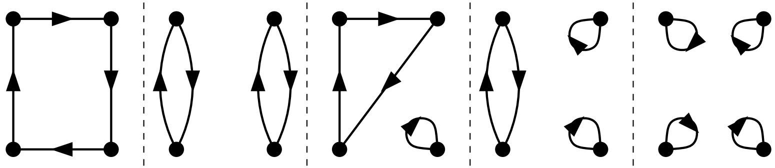
Integrands (Lepton loop, method 2)



Correspondings integrals

- The QED kernel $\bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y)$ is parametrized by six ‘weight’ functions of the variables $(x^2, x\cdot y, y^2)$.
-
$$\bar{\mathcal{L}}^{(\Lambda)}_{[\rho,\sigma];\mu\nu\lambda}(x,y) = \bar{\mathcal{L}}_{[\rho,\sigma];\mu\nu\lambda}(x,y) - \partial^{(x)}_{\mu}(x_\alpha e^{-\Lambda m_\mu^2 x^2/2})\bar{\mathcal{L}}_{[\rho,\sigma];\alpha\nu\lambda}(0,y) - \partial^{(y)}_{\nu}(y_\alpha e^{-\Lambda m_\mu^2 y^2/2})\bar{\mathcal{L}}_{[\rho,\sigma];\mu\alpha\lambda}(x,0),$$
- Using this kernel, we have reproduced (at the 1% level) known results for a range of masses for:
  1. the lepton loop (spinor QED, shown in the two plots);
  2. the charged pion loop (scalar QED);
  3. the π^0 exchange with a VMD-parametrized transition form factor.

Wick-contraction topologies in HLbL amplitude $\langle 0|T\{j_x^\mu j_y^\nu j_z^\lambda j_0^\sigma\}|0\rangle$



First two classes of diagrams turn out to be dominant, with a cancellation between them.

Example: $\Pi = \langle (j_u - j_d)(j_u - j_d)(j_u - j_d)(j_u - j_d)\rangle$ does not contain the π^0 pole (π^0 only couples to one isovector, one isoscalar current).

Write out the Wick contractions: $\Pi = 2\cdot\Pi^{(4)} + 4\cdot\Pi^{(2,2)}$

In kinematic regime where π^0 dominates: $|\Pi| \ll \Pi^{(4)} \Rightarrow \Pi^{(2,2)} \approx -\frac{1}{2}\Pi^{(4)}$.

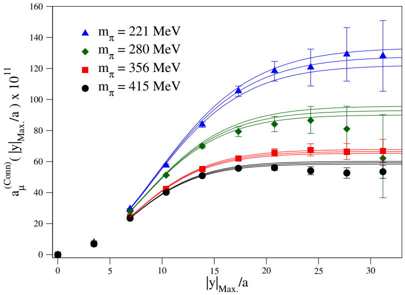
Including charge factors: $\left[(Q_u^2+Q_d^2)^2\Pi^{(2,2)}\right] = -\frac{25}{34}\left[(Q_u^4+Q_d^4)\Pi^{(4)}\right]$.

Large-N_c argument by J. Bijnens, 1608.01454; see also 1712.00421.

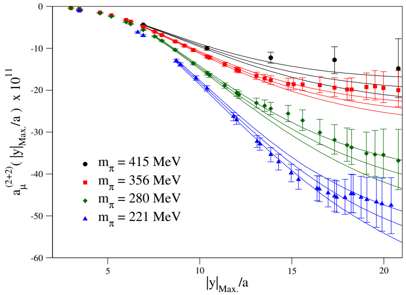
The connected and leading disconnected contribution

Cumulated $a_\mu^{\mathrm{HLbL}} = \int_0^{|y|_{\max}} d|y|\, f(|y|)$

Connected

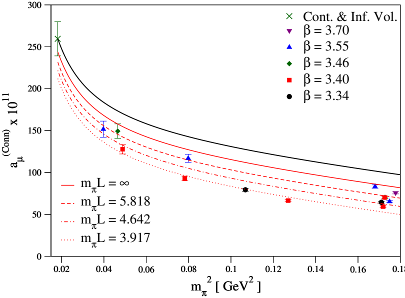


Leading disconnected

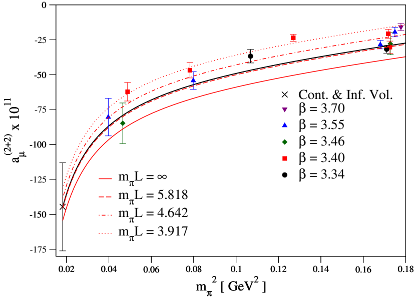


Chiral, continuum, volume extrapolation

Connected contribution



disconnected contribution

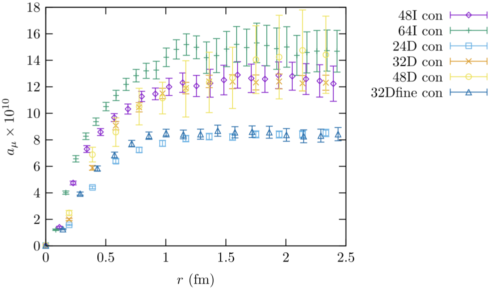


Contribution	Value$\times 10^{11}$
Light-quark fully-connected and $(2+2)$	107.4(11.3)(9.2)(6.0)
Strange-quark fully-connected and $(2+2)$	$-0.6(2.0)$
$(3+1)$	0.0(0.6)
$(2+1+1)$	0.0(0.3)
$(1+1+1+1)$	0.0(0.1)
Total	106.8(15.9)

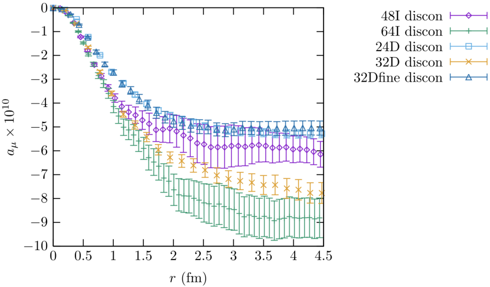
[Chao et al, 2104.02632 (EPJC)]

RBC/UKQCD (QED$_L$): cumulative contributions to a_μ^{HLbL}

Connected ⟶

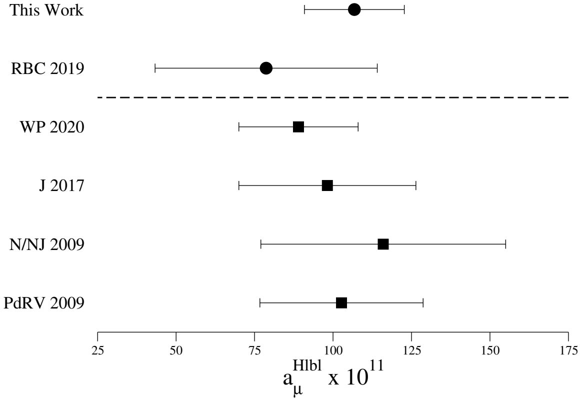


Disconnected ⟶



[Blum et al. 1911.08123 (PRL)]

Compilation of a_μ^{HLbL} determinations



Good consistency of different determinations.

Fig from Chao et al, 2104.02632 (EPJC).

Conclusion

- $(g-2)_\mu$ remains a hot topic in precision tests of the SM!
- HVP: consistency checks expected soon between different lattice collaborations
- the HLbL contribution shows good agreement between the dispersive approach and two independent lattice calculations.