Excited States of Isolated Fermions in the Higgs phase of gauge Higgs theories

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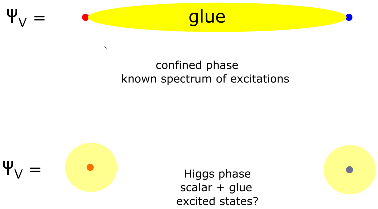
Motivations

Composite systems (molecules, atoms, nuclei, hadrons...) generally have a spectrum of excitations. What about the charged "elementary" particles like quarks and leptons?

By Gauss's Law, a charged particle is accompanied by a surrounding gauge (and possibly other) fields. If these surrounding fields interact with themselves, could they not also exhibit a spectrum of excitations? This would *look like a mass spectrum* of the isolated elementary particle.



This doesn't happen in pure QED. Any energy eigenstate containing a static $\pm$ charge pair is just the Coulomb field plus some number of photons. *Gauge Higgs theories could be different.*

Motivation: superconductivity, electroweak sector.

Are all physical states gauge invariant?

No, not quite. The Gauss law constraint only requires invariance under infinitesimal gauge transformations. In QED, in an infinite volume, a physical state containing a single static charge *transforms under a global subgroup of the gauge group.*

The ground state of pure QED containing a single static electric charge at point **x** is the Dirac state

$$|\Psi_{\mathbf{x}}\rangle = \overline{\psi}^{+}(\mathbf{x})\rho_C(\mathbf{x};A)|\Psi_0\rangle$$

where

$$\rho_C(\mathbf{x};A) \quad = \quad \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]$$

It is easy to check that $|\Psi_{\mathbf{x}}\rangle$ satisfies the Gauss Law. However, let $g(x) = e^{i\theta(x)}$ be an arbitrary U(1) gauge transformation, and we separate out the zero mode $\theta(x) = \theta_0 + \tilde{\theta}(x)$. Then

$$\psi(\mathbf{x}) \rightarrow e^{i\theta(\mathbf{x})}\psi(\mathbf{x})$$

but

$$\rho_C(\mathbf{x};A) \rightarrow e^{i\tilde{\theta}(x)}\rho_C(\mathbf{x};A)$$

Pseudomatter fields

It follows that

$$|\Psi_{\mathbf{x}}\rangle \to e^{-i\theta_0}|\Psi_{\mathbf{x}}\rangle$$

Local symmetries cannot break spontaneously, of course (Elitzur). But global symmetries can.

The operator

$$\rho_C(\mathbf{x};A) \quad = \quad \exp\left[-i\frac{e}{4\pi}\int d^3z\, A_i(\mathbf{z})\frac{\partial}{\partial z_i}\frac{1}{|\mathbf{x}-\mathbf{z}|}\right]$$

is a first example of a **pseudomatter field**.

Definition

A pseudomatter field $\rho(\mathbf{x};A)$ is a non-local functional of the gauge field which transforms like a matter field in the fundamental representation of the gauge group, *except* under the global center subgroup of the gauge group.

We combine the scalar field and pseudomatter fields with the static charge operator to create physical states in gauge Higgs theories.

1. Any SU(N) gauge transformation $g_F(\mathbf{x};A)$ to a physical gauge $F(A)=0$ can be decomposed into N pseudomatter fields $\{\rho_n\}$, and vice-versa:

$$\rho_n^a(\mathbf{x};A) = g_F^{\dagger an}(\mathbf{x};A)$$

In particular, the operator $\rho_C^*(\mathbf{x};A)$ defined earlier is precisely the gauge transformation to Coulomb gauge in an abelian theory. This operator dresses a static charge with a surrounding Coulomb field: $\overline{\psi}(\mathbf{x})\rho_C(\mathbf{x};A)\Psi_0$.

2. In an SU(N) lattice gauge theory, any eigenstate $\xi_n(\mathbf{x};U)$ of the covariant Laplacian operator,

$$-D^2\xi_n = \kappa_n\xi_n$$

where

$$(-D^2)^{ab}_{\mathbf{xy}} = \sum_{k=1}^{3}\left[2\delta^{ab}\delta_{\mathbf{xy}} - U_k^{ab}(\mathbf{x})\delta_{\mathbf{y},\mathbf{x}+\hat{k}} - U_k^{\dagger ab}(\mathbf{x}-\hat{k})\delta_{\mathbf{y},\mathbf{x}-\hat{k}}\right]$$

is a pseudomatter field

$$\rho^a(\mathbf{x};U) = \xi_n^a(\mathbf{x};U)$$

(This is the idea behind the Laplacian gauges of Vink and Wiese.)

Excitations of elementary fermions in gauge Higgs theories

For static quarks in a pure gauge theory there is a tower of metastable states

$$\Psi_n(R) = \overline{q}(\mathbf{x}) V_n(\mathbf{x}, \mathbf{y}; U) q(\mathbf{y}) \Psi_0$$

corresponding to string excitations. This has been observed in computer simulations.
Juge, Kuti, and Morningstar, (2003), Brandt and Meineri (2016)

For light quarks, the excited states lie on Regge trajectories. A spectrum of excitations exists in the confinement region of a gauge Higgs theory.

In the Higgs phase, is there a similar tower of metastable states of the form

$$\Psi_n(R) = \overline{q}^a(\mathbf{x}) \left[\sum_m c_m^{(n)} \rho_m^a(\mathbf{x}) \rho_m^{\dagger b}(\mathbf{y}) \right] q^b(\mathbf{y}) \Psi_0$$

where the $\{\rho_m(\mathbf{x})\}$ are pseudo-matter fields?

The Models

We investigate:

1. SU(3) gauge Higgs theory. The Higgs scalar is in the fundamental representation.
   J.Greensite, PRD 102 (2020) 5, 054504 , arXiv: 2007.11616 [hep-lat]

2. The $q = 2$ abelian Higgs model. The Higgs scalar has charge $q = 2$.
   K.M., PRD 103 (2021) 7, 074508 , arXiv: 2012.13991 [hep-lat]

3. Landau-Ginzburg effective action for superconductivity.
   K.M. and J.Greensite, in progress

4. Chiral U(1) gauge Higgs theory (Smit-Swift formulation). The Higgs scalar has charge $q = 1$.
   J.Greensite, arXiv: 2104.12237 [hep-lat]

In each of these models we impose a unimodular constraint $|\phi| = 1$ for simplicity.

In the study of static fermion excitations, we find that each model has its own special features which must be taken into account.

Notation

A reminder: In a lattice Euclidean time field theory, with periodic boundary conditions and time extent N_t, there is an operator τ whose matrix elements in Schrodinger representation are known as the transfer matrix

$$Z = \text{Tr}\,\tau^{N_t} \quad , \quad \tau = e^{-Ha}$$

But when computing a Euclidean-time correllator of operators $A(t), B(0)$, the relevant operator is actually the rescaled transfer matrix $\mathcal{T}$

$$\langle A(t)B(0)\rangle = \langle \Psi_0 | A\mathcal{T}^t B | \Psi_0\rangle$$

where

$$\mathcal{T} = e^{-(H-\mathcal{E}_0)a}$$

and $\Psi_0, \mathcal{E}_0$ are the vacuum state and vacuum energy respectively.

With a slight abuse of language, I will refer to $\mathcal{T}$, rather than τ, as the transfer matrix.

How to find excitation enregy

If $|\Psi(R)\rangle$ is some arbitrary physical state containing a static fermion-antifermion pair with separation R, and $E_1(R)$ is the lowest energy of such states above the vacuum energy $\mathcal{E}_0$, then on general grounds

$$\langle\Psi(R)|\mathcal{T}^T|\Psi(R)\rangle \quad = \quad \sum_n c_n e^{-E_n(R)T} \to c_1 e^{-E_1(R)T} \quad \text{as} \;\; T \to \infty$$

Drawback: We get the ground state, not easy to find excited states.

Alternatively, let $\{|\Phi_\alpha(R)\rangle\}$ span a subspace of the full Hilbert space with the two static charges. Then one could get an approximate spectrum by diagonalizing $\mathcal{T}$ in this subspace. This approach is followed in some lattice QCD spectrum calculations.

Drawback: This requires a pretty big set $\sim$ hundreds of states. Not practical for our purposes, where it is expensive to generate the $|\Phi_\alpha(R)\rangle$.

general idea

Generate a small set of states $\{|\Phi_\alpha(R)\rangle\}$, and diagonalize either $\mathcal{T}$ or $\mathcal{T}^p$ in the small subspace spanned by these states. The hope is that one or more of the eigenstates $|\Psi_n\rangle$ in the subspace will be orthogonal (nearly) to the true ground state. If $|\Psi\rangle$ is such a state, then

$$\begin{aligned}\langle\Psi|\mathcal{T}^T|\Psi\rangle &= \sum_n c_n e^{-E_n(R)T} \\ &\to c_{ex} e^{-E_{ex}(R)T} \quad \text{at large } T\end{aligned}$$

There are no guarantees, it just has to be tried.

Let ξ_n denote the eigenstates of the lattice Laplacian operator (no time derivatives)

$$-D^2 \xi_n = \kappa_n \xi_n$$

Consider at each quark separation $R = |\mathbf{x} - \mathbf{y}|$, the 4-dimensional subspace spanned by three quark-pseudomatter states, and one quark-scalar state

$$\begin{aligned}
\Phi_n(R) &= [\overline{q}^a(\mathbf{x})\xi_n^a(\mathbf{x})] \times [\xi_n^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0 \quad (n = 1, 2, 3) \\
\Phi_4(R) &= [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0
\end{aligned}$$

We calculate numerically the matrix elements and overlaps in the non-orthogonal basis

$$\begin{aligned}
[\mathcal{T}]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \mathcal{T} | \Phi_\beta \rangle \\
[O]_{\alpha\beta}(R) &= \langle \Phi_\alpha | \Phi_\beta \rangle
\end{aligned}$$

The eigenvalues of $\mathcal{T}$ in the subspace are obtained by solving the generalized eigenvalue problem

$$[\mathcal{T}]\vec{v}_n = \lambda_n [O]\vec{v}^{(n)}$$

and we have eigenstates of $\mathcal{T}$ in the subspace

$$|\Psi_n(R)\rangle = \sum_{i=1}^{4} v_i^{(n)} |\Phi_i(R)\rangle$$

Then we consider evolving states for Euclidean time T, and compute

$$\begin{aligned} \mathcal{T}^T_{nn}(R) &= \langle \Psi_n | \mathcal{T}^T | \Psi_n \rangle \\ &= v_i^{(n)*} \langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle v_j^{(n)} \\ E_n(R,T) &= -\log \left[\frac{\mathcal{T}^T_{nn}(R)}{\mathcal{T}^{T-1}_{nn}(R)} \right] \end{aligned}$$

The second equation is the lattice version of a logarithmic time derivative.

$E_n(R,T)$ can be understood as the energy expectation value of the state

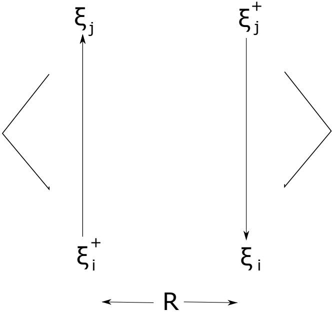
$$\Psi\left(R, \frac{1}{2}(T-1)\right) = \mathcal{T}^{(T-1)/2}\Psi(R)$$

which is obtained by evolving $\Psi(R)$ by $\frac{1}{2}(T-1)$ units of Euclidean time.

Integrating out the massive (i.e. static) fermion fields generates a pair of Wilson lines.

The numerical computation of $\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle$ involves expectation values of products of Wilson lines, terminated by matter or pseudomatter fields:

$$\langle \Phi_i | \mathcal{T}^T | \Phi_j \rangle \quad =$$



Three possibilities:

1. $\Psi_n(R)$ is an *eigenstate* in the full Hilbert space. Then $E_n(R) = E(R,T)$ is time independent.

2. $\Psi_n(R)$ evolves to the ground state. Then $E_n(R,T)$ drops steadily to the lowest energy with increasing T.

3. $\Psi_n(R)$ evolves in Euclidean time to a stable or metastable *excited* state. Then $E_n(R,T)$ converges to a value which is almost constant, over some range of Euclidean time. Analogous to string excitations in the confining phase.

We have computed $E_n(R,T)$ for SU(3) gauge theory with a unimodular Higgs field on a $14^3 \times 32$ lattice volume, at $\beta = 5.5$ with $\gamma = 0.5$ and $\gamma = 3.5$, in the confinement and Higgs phases respectively. The action is

$$\begin{aligned} S &= -\frac{\beta}{3} \sum_{plaq} \mathrm{ReTr}[U_\mu(x) U_\nu(x+\hat{\mu}) U_\mu^\dagger(x+\hat{\nu}) U_\nu^\dagger(x)] \\ &\quad -\gamma \sum_{x,\mu} \mathrm{Re}[\phi^\dagger(x) U_\mu(x) \phi(x+\widehat{\mu})] \end{aligned}$$

confinement vs Higgs phase

Consider in particular

$$\Phi_1(R) = [\overline{q}^a(\mathbf{x})\xi_1^a(\mathbf{x})] \times [\xi_1^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0$$

$$\Phi_4(R) = [\overline{q}^a(\mathbf{x})\phi^a(\mathbf{x})] \times [\phi^{\dagger b}(\mathbf{y})q^b(\mathbf{y})]\ \Psi_0$$

Φ_4 is just a pair of color neutral objects, which can be separated to $R \to \infty$ with a finite cost in energy.

Φ_1 is different. The distinction between the Higgs and confinement phase is that in the confinement phase the energy of every pseudomatter state (such as Φ_1) diverges as $R \to \infty$, no matter which pseudomatter field is used.

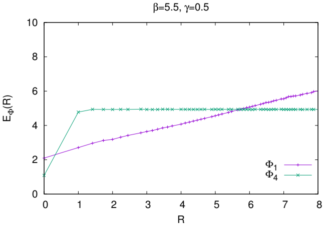
That is the definition of *separation-of-charge (S_c) confinement*, which is associated with metastable flux tubes and Regge trajectories. S_c confinement disappears in the Higgs phase, where the global center subgroup of the gauge group is spontaneously broken.

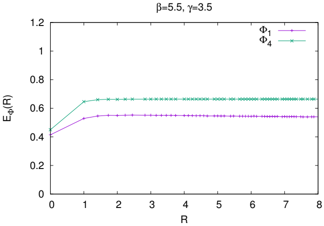
This can be proven, but we can also check it numerically.

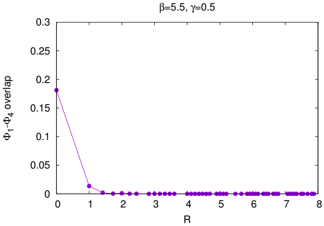
Energy expectation values & overlaps for the Higgs $\Phi_4(R)$ and pseudomatter $\Phi_1(R)$ states at $\beta = 5.5$ in the confinement phase ($\gamma = 0.5$) and Higgs phase ($\gamma = 3.5$) respectively.

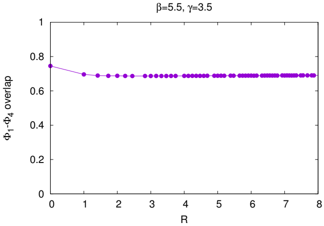
Confinement

Higgs



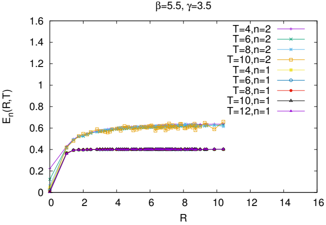




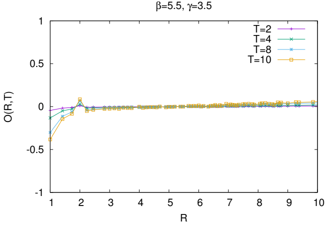


Excitation spectrums in SU(3) gauge Higgs theory

Now we show $E_n(R,T)$ and the overlap for $\Psi_1(R)$, $\Psi_2(R)$ and $T = 4 - 12$.



(a) Energies



(b) Overlap

There seems to be clear evidence of an stable excited state in the spectrum, orthogonal to the ground state.

The energy gap is far smaller than the threshold for vector boson creation.

q=2 Abelian Gauge-Higgs theory,

$$S = -\beta \sum_{plaq} \text{Re}[U_\mu(x)U_\nu(x+\hat{\mu})U^*_\mu(x+\hat{\nu})U^*_\nu(x)] - \gamma \sum_{x,\mu} \text{Re}[\phi^*(x)U^2_\mu(x)\phi(x+\widehat{\mu})] \; .$$

- the scalar field has charge $q = 2$ (as do Cooper pairs)
- impose a unimodular constraint $\phi^*(x)\phi(x) = 1$ (for simplicity)
- a relativistic generalization of the Landau-Ginzburg effective model of superconductivity

Four lowest-lying Laplacian eigenstates + Higgs field

In our calculation we make use of the four lowest-lying Laplacian eigenstates and the Higgs field to construct the Φ_α,

the four lowest lying Laplacian eigenstates,

$$\zeta_i(x) = \begin{cases} \xi_i(x) & i = 1,2,3,4 \\ \phi(x) & i = 5 \end{cases}$$

In general the five states $\Phi_\alpha(R)$ are non-orthogonal at finite R. Of course $\phi(x)$ is a $q = 2$ matter field, rather than pseudomatter field. We express the operator Q_α in terms of a non-local operator $V_\alpha(\mathbf{x}, \mathbf{y}; U)$

$$\begin{aligned} Q_\alpha(R) &= \overline{\psi}(\mathbf{x}) V_\alpha(\mathbf{x}, \mathbf{y}; U) \psi(\mathbf{y}) \\ V_\alpha(\mathbf{x}, \mathbf{y}; U) &= \zeta_\alpha(\mathbf{x}; U) \zeta^*_\alpha(\mathbf{y}; U) , \end{aligned}$$

and define the Euclidean time evolution operator of the lattice abelian Higgs model, $\mathcal{T} = e^{-(H-\mathcal{E}_0)}$, which is the transfer matrix multiplied by a constant $e^{\mathcal{E}_0}$ where $\mathcal{E}_0$ is the vacuum energy, evolving states for one unit of discretized time.

Calculations of the Transfer Matrix

$[\mathcal{T}]$ is the matrix element in the five non-orthogonal states Φ_α, with the matrix of overlaps, $[\mathcal{O}]$, of such states.

$$
\begin{aligned}
[\mathcal{T}]_{\alpha\beta} &= \langle \Phi_\alpha | e^{-(H-\mathcal{E}_0)} | \Phi_\beta \rangle = \langle Q_\alpha^\dagger(R,1) Q_\beta(R,0) \rangle \\
[O]_{\alpha\beta} &= \langle \Phi_\alpha | \Phi_\beta \rangle = \langle Q_\alpha^\dagger(R,0) Q_\beta(R,0) \rangle
\end{aligned}
$$

We obtain the five orthogonal eigenstates of $[\mathcal{T}]_{\alpha\beta}$ in the subspace of Hilbert space spanned by the Φ_α by solving the generalized eigenvalue problem.

$$
[T]_{\alpha\beta} \upsilon_\beta^{(n)} = \lambda_n [O]_{\alpha\beta} \upsilon_\beta^{(n)} ,
$$

with eigenstates,

$$
\Psi_n(R) = \sum_{\alpha=1}^{5} \upsilon_\alpha^{(n)} \Phi_\alpha(R)
$$

and ordered such that λ_n decreases with n.

Euclidean time evolution

Consider evolving the states Ψ_n in Euclidean time,

$$\begin{aligned}\mathcal{T}_{nn}(R,T) &= \langle \Psi_n | e^{-(H-\mathcal{E}_0)T} | \Psi_n \rangle \\ &= \upsilon_\alpha^{*(n)} \langle \Phi_\alpha | e^{-(H-\mathcal{E}_0)T} | \Phi_\beta \rangle \upsilon_\beta^{(n)} \\ &= \upsilon_\alpha^{*(n)} \langle Q_\alpha^\dagger(R,T) Q_\beta(R,0) \rangle \upsilon_\beta^{(n)} \ ,\end{aligned}$$

where Latin indices indicate matrix elements with respect to the Ψ_n rather than the Φ_α, and there is a sum over repeated Greek indices.

To calculate this expression, we define timelike $q = 2$ Wilson lines of length T,

$$P(\mathbf{x},t,T) = U_0^2(\mathbf{x},t) U_0^2(\mathbf{x},t+1) \ldots U_0^2(\mathbf{x},t+T-1) \ .$$

After integrating out the massive fermions, whose worldlines lie along timelike Wilson lines, we have

$$\langle Q_\alpha^\dagger(R,T) Q_\beta(R,0) \rangle = \langle \mathrm{Tr}[V_\alpha^\dagger(\mathbf{x},\mathbf{y};U(t+T)) P^\dagger(\mathbf{x},t,T) V_\beta(\mathbf{x},\mathbf{y};U(t)) P(\mathbf{y},t,T)] \rangle \ .$$

Transfer matrix, a sum of exponentials

On general grounds, $\mathcal{T}_{nn}(R,T)$ is a sum of exponentials

$$\begin{aligned} \mathcal{T}_{nn}(R,T) &= \langle \Psi_n(R) | e^{-(H-\mathcal{E}_0)T} | \Psi_n(R) \rangle \\ &= \sum_j |c_j^{(n)}(R)|^2 e^{-E_j(R)T} \ , \end{aligned}$$

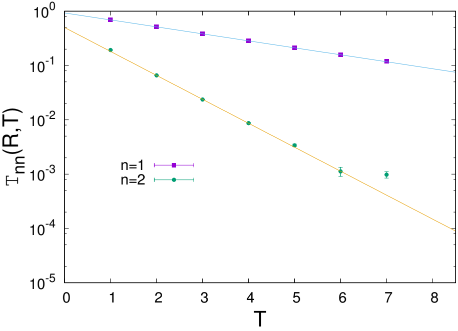
where $c_j^{(n)}(R)$ is the overlap of state $\Psi_n(R)$ with the j-th energy eigenstate of the abelian Higgs theory containing a static fermion-antifermion pair at separation R, and $E_j(R)$ is the corresponding energy eigenvalue minus the vacuum energy.

Numerics

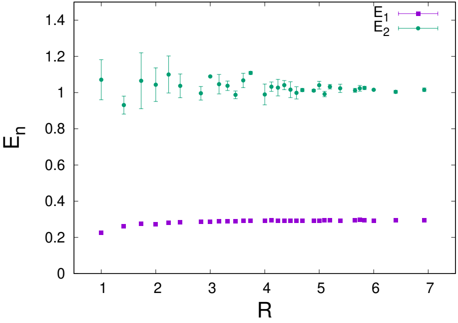
I work in the Higgs region at β=3 and γ=0.5, the photon mass is determined from the plaquette-plaquette correlator to be 1.57 in lattice units.

The energies $E_n(R)$ for $n = 1, 2$

The energies $E_n(R)$ for $n = 1, 2$ are also obtained by fitting the data for $\mathcal{T}_{nn}(R, T)$ vs. T, at each R, to an exponential falloff.

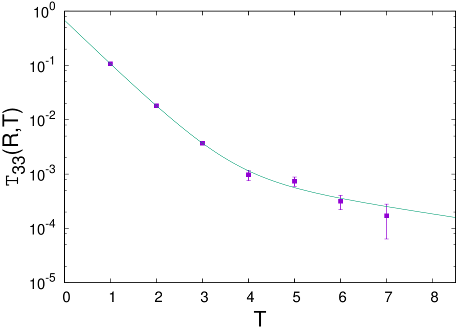


An example of these fits at $R = 6.93$ on a 16^4 lattice with couplings $\beta = 3, \gamma = 0.5$. Fitting through the points at $T = 2 - 5$, with $E_1 = 0.2929(6)$ and $E_2(R) = 1.01(1)$



Energy expectation values $E_n(R)$ vs. R for $n = 1$ and $n = 2$, obtained from a fit to a single exponential. The data and errors were obtained from ten independent runs, each of 77,000 sweeps after thermalization, with data taken every 100 sweeps, computing $\mathcal{T}_{nn}$ from each independent run.

...see if there is any indication of a second stable excited state



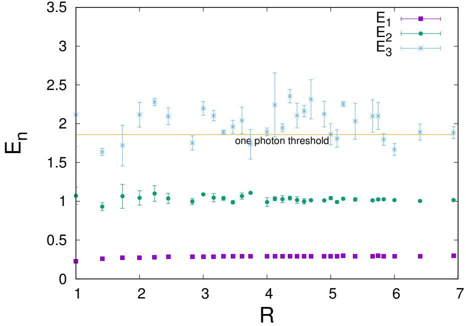
$\mathcal{T}_{33}(R,T)$ vs. T at fixed $R = 6.93$. The fit shown is to the sum of exponentials

$$\mathcal{T}_{33}(R,T) \approx a_1(R)e^{-E_1 T} + a_2(R)e^{-E_2 T} + a_3(R)e^{-E_3 T} ,$$

where $E_1 = 0.29, E_2 = 1.02$ are taken from the previous fits. A sample fit, again at $R = 6.93$, is shown. Obviously one cannot be very impressed by a four parameter fit through a handful of data points. A sample fit, again at $R = 6.93$ is shown.

Excitation spectrums

the values of E_1, E_2, E_3, together with the one photon threshold



The one photon threshold is simply $E_1 + m_{photon} = 0.29 + 1.57(1) = 1.86(1)$ in lattice units. The important observation is that $E_2(R)$ lies well below this threshold, which implies that the first excited state of the static fermion-antifermion pair is stable. The second point to note is that $E_3(R)$ seems to lie above or near the one photon threshold. The indications are that there is no second stable excited state. States above the first excited state most likely lie above the threshold, and are probably combinations of the ground state plus a massive photon.

Excitations in Effective Landau-Ginzburg model

This is work in progress (*K.M. and J. Greensite.*)

The effective Landau-Ginzburg model for ordinary superconductivity is a non-relativistic $q = 2$ abelian Higgs model of this form:

$$\begin{aligned} S &= -\beta \sum_{plaq} \mathrm{Re}[UUU^* U^*] - \gamma \sum_x \sum_{k=1^3} \phi^*(x) U_k^2(x) \phi(x + \hat{k}) \\ &\quad - \frac{\gamma}{\upsilon^2} \sum_x \phi^*(x) U_0^2(x) \phi(x + \hat{t}) \end{aligned}$$

where $\upsilon \sim 10^{-2}$ in natural units, is on the order of the Fermi velocity in a metal, and $\beta = 1/e^2 = 10.9$ for ordinary electrodynamics. Go to unitary gauge, so that $U_0(x) \approx \pm 1$. We then compute the excitations around a pair of static $q = \pm 1$ (e) charges, having electrons and holes in mind.

γ, β determine the photon mass (inverse to the penetration depth) in lattice units, so for a given γ the penetration depth fixes the lattice spacing in physical units.

But this time things are not so simple, and diagonalizing $\mathcal{T}$ in a small subspace doesn't work. Eigenstates in the subspace flow in Euclidean time to the ground state.

Landau-Ginzburg I

Let us instead (at each separation R) diagonalize $\mathcal{T}^{2t_0}$ in the basis Φ_α, so that

$$\langle\Psi_m|\mathcal{T}^{2t_0}|\Psi_n\rangle = \lambda_n(t_0)\delta_{mn}$$

and define

$$\Psi_n(t) = \mathcal{T}^t\Psi_n$$

Suppose, after evolving Ψ_1 by t_0 units of Euclidean time, that $\Psi_1(t_0)$ is approximately the true ground state in the full Hilbert space. It follows that $\Psi_{n>1}(t_0)$ is orthogonal to the ground state, because

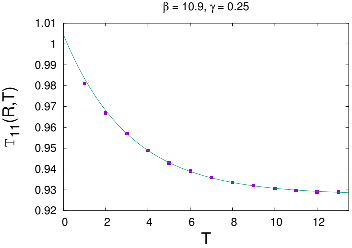
$$\langle\Psi_m(t_0)|\Psi_n(t_0)\rangle \propto \delta_{mn}$$

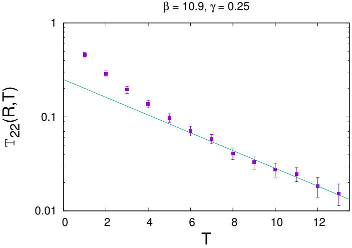
and therefore, at large $T > 2t_0$

$$\begin{aligned}
\mathcal{T}_{22}(R,T) &= \langle\Psi_2|\mathcal{T}^T|\Psi_2\rangle \\
&= \langle\Psi_2(t_0)|\mathcal{T}^{T-2t_0}|\Psi_2(t_0)\rangle \\
&\to \text{const} \times e^{-E_{ex}T} \quad \text{where} \quad E_{ex} > E_1
\end{aligned}$$

So we try that.

At $R = 5.385, \ \gamma = 0.25$





Choose $2t_0 = 9$. We fit $\mathcal{T}_{11}$ to

$$f_1(T) = a_1 \exp(-b_1 T) + c_1$$

The fact that $c_1 \neq 0$ means that the ground state energy $E_1 \approx 0$.
b_1 gives an excited state energy.

Then we fit T_{22} in the range $T > 6$ to a single exponential

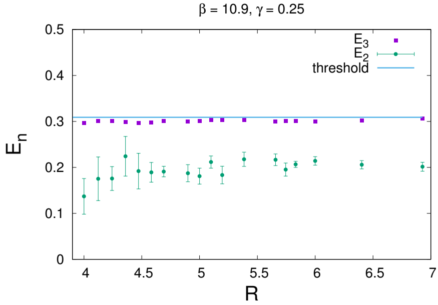
$$f_2(T) = a_2 \exp(-b_2 T)$$

The coefficient $b_2 < b_1$ gives another excitation energy.

Excitation spectrums in effective Landau-Ginzburg theory

The data at $R < 4.0$ are rather noisy, with large χ^2. Here are the results for $R \geq 4.0$.

At these couplings, $E_1(R) \approx 0$.



Once again, the first excited state is stable. The next excited state, which is right on the threshold, is presumably the ground state plus a massive photon.

Can such excitations be detected experimentally? E.g. by ARPES (angle-resolved photoemission spectroscopy) data? We don't yet know...

Excitations in chiral gauge theories

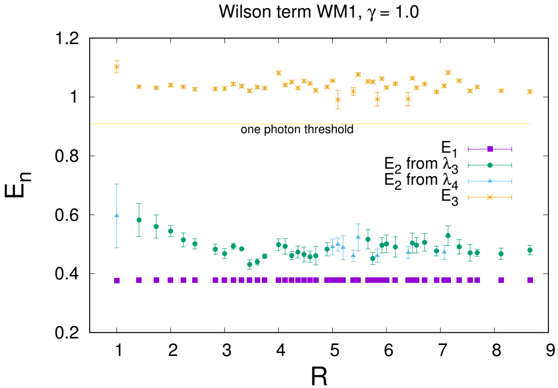
No known lattice formulation of chiral non-abelian gauge theories with a continuum limit. There is a formulation for U(1) gauge theories due to Lüscher, involving overlap fermions. Difficult to implement numerically.

In this exploratory work, Jeff Greensite chose a simpler option.

For *static* fermions, work instead with a quenched version, at fixed lattice spacing, of the Smit-Swift lattice action, U(1) gauge group, with oppositely charged right and left-handed fermions.

There are doublers, even with quenched fermions. The idea was to use a Wilson-style non-local mass term to take the mass of the doublers to infinity in the continuum.

The continuum limit doesn't work...Smit-Swift is not a true chiral gauge theory. Moreover, the positivity of the transfer matrix is unproven. But at least the non-local mass term breaks the mass degeneracy with the doublers. We can try it.



Energies E_1, E_2, E_3 vs. R at $\beta = 3, \gamma = 1$, shown together with the one photon threshold.

Conclusions

- The gauge+Higgs fields surrounding a charged static fermion have a spectrum of localized excitations, which cannot be interpreted as simply the ground state plus some massive bosons.
- This means that charged "elementary" particles can have a mass spectrum in gauge Higgs theories.
- This conclusion seems robust. We see it in SU(3), q=2 Abelian Higgs, Landau-Ginzburg, and chiral U(1) models.
- Observable? Maybe in ARPES studies of conventional superconductors? Core electron spectra above and below the transition temperature?
- Electroweak theory? Excitations of quarks and leptons?? W and Z bosons?? (we'll see...)