Three hard problems

Strong CP problem, nucleon electric dipole moment and the fate of axions

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Objective

- QCD allows for a CP-violating term, $S_\theta = i\theta Q$, in the action. Thus, there is the possibility of new sources of CP violation, which might shed light on the baryon asymmetry of the Universe. A nonvanishing value of θ would result in an electric dipole moment, d_n, of the neutron

- The current experimental upper limit is $|d_n| < 1.8 \times 10^{-13} e\,$fm, which suggests that θ is anomalously small. This feature is referred to as the strong CP problem

- The assumption here is that QCD is in a single (confinement) phase for $0 \leq |\theta| < \pi$. The popular Peccei-Quinn solution, e.g., is realized by the shift symmetry $\theta \to \theta + \delta$, trading the θ term for the hitherto undetected axion [$> 90,000$ publications on axions !]

- It is known from the massive Schwinger model that a θ term may change the phase of the system. Callan, Dashen and Gross claimed that a similar phenomenon occurs in QCD, in which the color fields produced by quarks and gluons are screened by instantons for $|\theta| > 0$. 't Hooft has shown that due to the joint presence of gluons and monopoles a rich phase structure may emerge as a function of θ

- In this talk I will show that CP is naturally conserved in the confining phase of QCD, consequently $d_n = 0$, while the axion extension of the Standard Model is inconsistent with confinement

QCD at Long Distances

To reveal the nonperturbative properties of the theory, we are faced with a multi-scale problem, involving the passage from the short-distance perturbative regime to the long-distance confining regime. The gradient flow provides a powerful framework for scale setting, and as such is a particular realization of the coarse-graining step of momentum space RG transformations Lüscher, Suzuki et al.

The gradient flow describes the evolution of fields as a function of flow time t. The flow of SU(3) gauge fields is defined by

$$\partial_t B_\mu(t,x) = D_\nu G_{\mu\nu}(t,x)\,, \quad G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu + [B_\mu, B_\nu]$$

where $B_\mu(t=0,x) = A_\mu(x)$ is the original gauge field of QCD. The renormalization scale μ is set by the flow time, $\mu = 1/\sqrt{8t}$ for $t \gg 0$.

The expectation value of the energy density $E(t,x) = \frac{1}{4}\, G^a_{\mu\nu}(t,x)\, G^a_{\mu\nu}(t,x)$ defines a renormalized coupling

$$g^2_{GF}(\mu) = \frac{16\pi^2}{3}\, t^2 \langle E(t)\rangle\,\Big|_{t=1/8\mu^2}$$

at flow time t in the gradient flow scheme Lüscher

For a start we may restrict our investigations to the Yang-Mills theory. If the strong CP problem is resolved in the Yang-Mills theory, then it is expected to be resolved in QCD as well. We use the plaquette action to generate representative ensembles of fundamental gauge fields on three different volumes

$$S = \beta \sum_{x,\,\mu<\nu} \Big(1 - \frac{1}{3}\,\mathrm{Re\,Tr}\; U_{\mu\nu}(x)\Big)$$

	16^4	24^4	32^4
#	4000	5000	5000

$\beta = 6.0 \quad a = 0.082\,\mathrm{fm}$

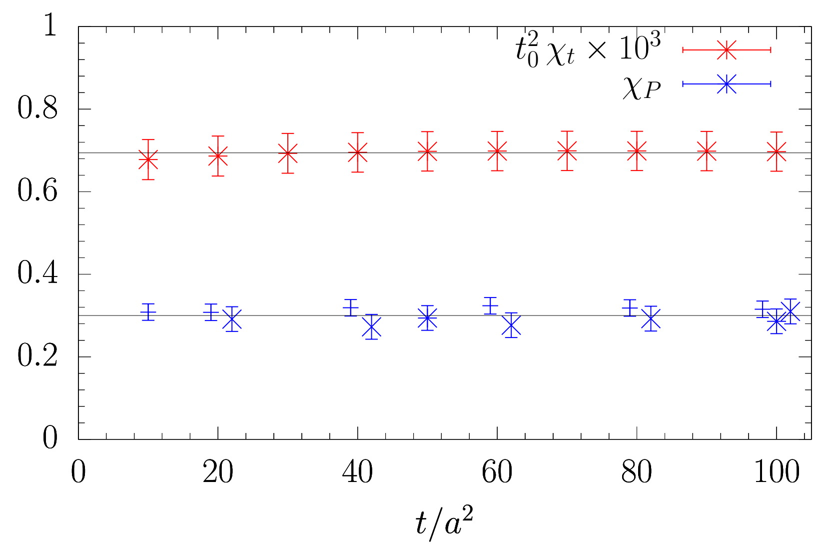
Physical quantities are independent of the RG scale. Two examples: [more to come]

- Topological susceptibility

$$\chi_t = \frac{\langle Q^2\rangle - \langle Q\rangle^2}{V}$$

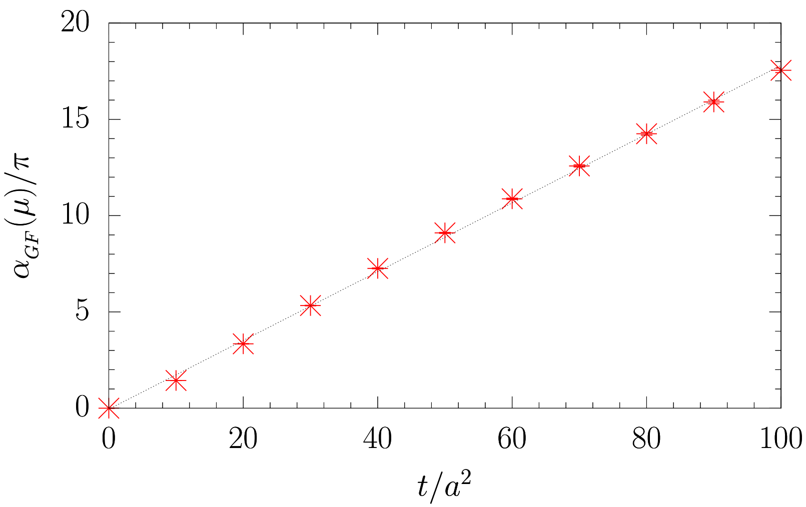
- Renormalized Polyakov susceptibility

$$\chi_P = \frac{\langle |P|^2\rangle - \langle |P|\rangle^2}{\langle |P|\rangle^2} \qquad P = \frac{1}{V_3}\sum_{\vec{x}} P(\vec{x})$$



At long distances the theory freezes practically to tree diagrams of quarks and gluons, which gives the strong coupling constant α_S a special meaning

The gradient flow running coupling



$$\frac{\partial\, \alpha_{GF}(\mu)}{\partial\, \ln\, \mu} \equiv \beta_{GF}(\alpha_{GF}) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -\,2\,\alpha_{GF}(\mu)$$

$$\frac{\Lambda_{GF}}{\mu} = (4\pi b_0 \alpha_{GF})^{-\frac{b_1}{2\,b_0^2}} \exp\left\{-\frac{1}{8\pi b_0 \alpha_{GF}} - \int_0^{\alpha_{GF}} d\alpha\, \frac{1}{\beta_{GF}(\alpha)} + \frac{1}{8\pi b_0 \alpha^2} + \frac{b_1}{2b_0^2 \alpha}\right\}$$

$$\alpha_{GF}(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_{GF}^2}{\mu^2}$$

To make contact with phenomenology, it is desirable to transform the gradient flow coupling α_{GF} to a common scheme. A preferred scheme in the Yang-Mills theory is the V scheme

$$\frac{\Lambda_{GF}}{\Lambda_V} = \exp\left\{-\int_0^{\alpha_{GF}} d\alpha \frac{1}{\beta_{GF}(\alpha)} + \int_0^{\alpha_V} d\alpha \frac{1}{\beta_V(\alpha)}\right\}$$

$$\beta_V(\alpha_V) \underset{\mu \ll 1\,\mathrm{GeV}}{=} -2\,\alpha_V(\mu)$$

$$\alpha_V(\mu) \underset{\mu \ll 1\,\mathrm{GeV}}{=} \frac{\Lambda_V^2}{\mu^2}$$

$$\frac{\Lambda_V}{\Lambda_{\overline{MS}}} = 1.60\,, \quad \frac{\Lambda_{\overline{MS}}}{\Lambda_{GF}} = 0.534$$

The linear growth of $\alpha_V(\mu)$ with $1/\mu^2$ is commonly dubbed infrared slavery. The static quark-antiquark potential can be described by the exchange of a single dressed gluon

$$V(r) = -\frac{1}{(2\pi)^3}\int d^3\mathbf{q}\; e^{i\,\mathbf{qr}}\; \frac{4}{3}\,\frac{\alpha_V(q)}{\mathbf{q}^2 + i0} \underset{r \gg 1/\Lambda_V}{=} \sigma\, r$$

where $\sigma = \frac{2}{3}\Lambda_V^2$, giving the string tension $\sqrt{\sigma} = 445(19)$ MeV

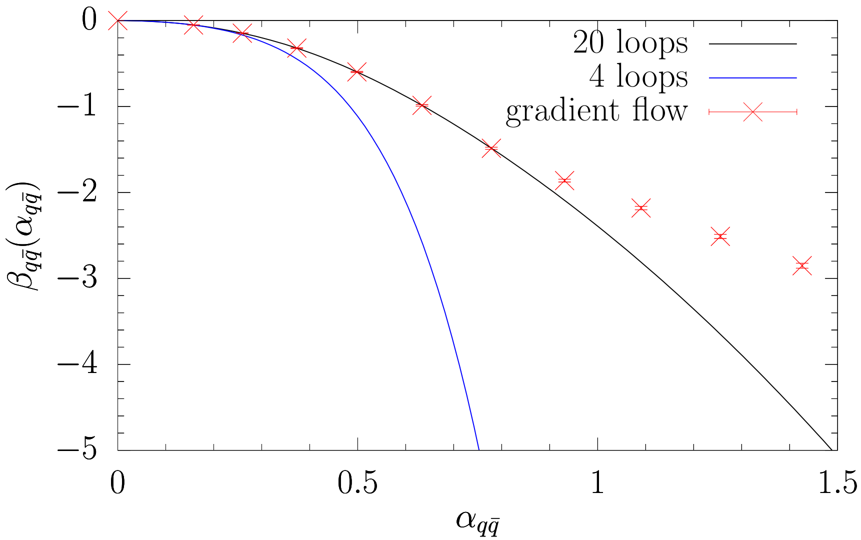
$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.217(7)$$

Literature:

$$\sqrt{t_0}\,\Lambda_{\overline{MS}} = 0.220(3)$$

arXiv:1905.05147

It is interesting to compare the nonperturbative gradient flow beta function with the perturbative beta function known up to twenty loops



20 loops arXiv:1309.4311

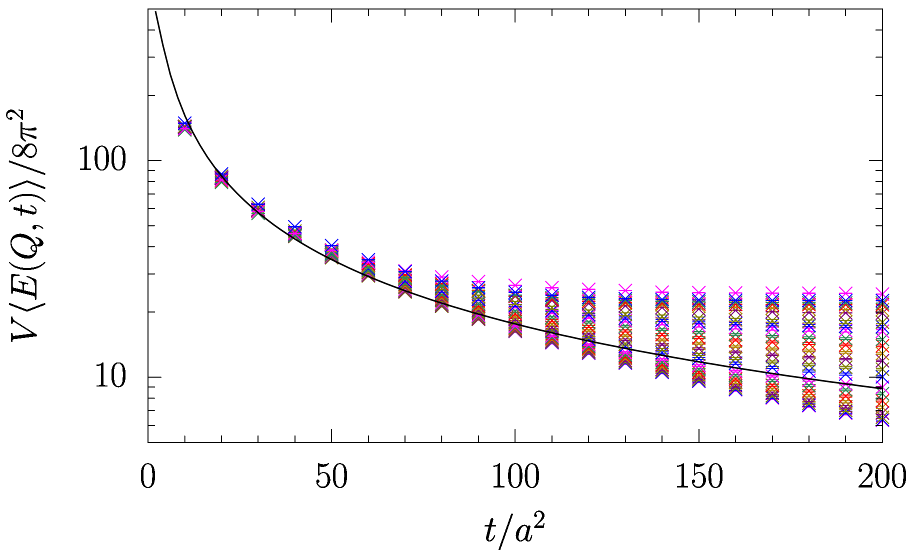
4 loops arXiv:1012.3037

$$\frac{\Lambda_{q\bar{q}}}{\Lambda_V} = 0.655$$

As was to be expected, the perturbative beta function gradually approaches the nonperturbative beta function with increasing order

Phase Structure

With increasing flow time the initial gauge field ensemble splits into effectively disconnected topological sectors of charge Q, at ever smaller flow time as β is increased Lefschetz thimble



$V\langle E(Q,t)\rangle/8\pi^2 \equiv S_Q \simeq |Q|$, while the ensemble average vanishes like $1/t$

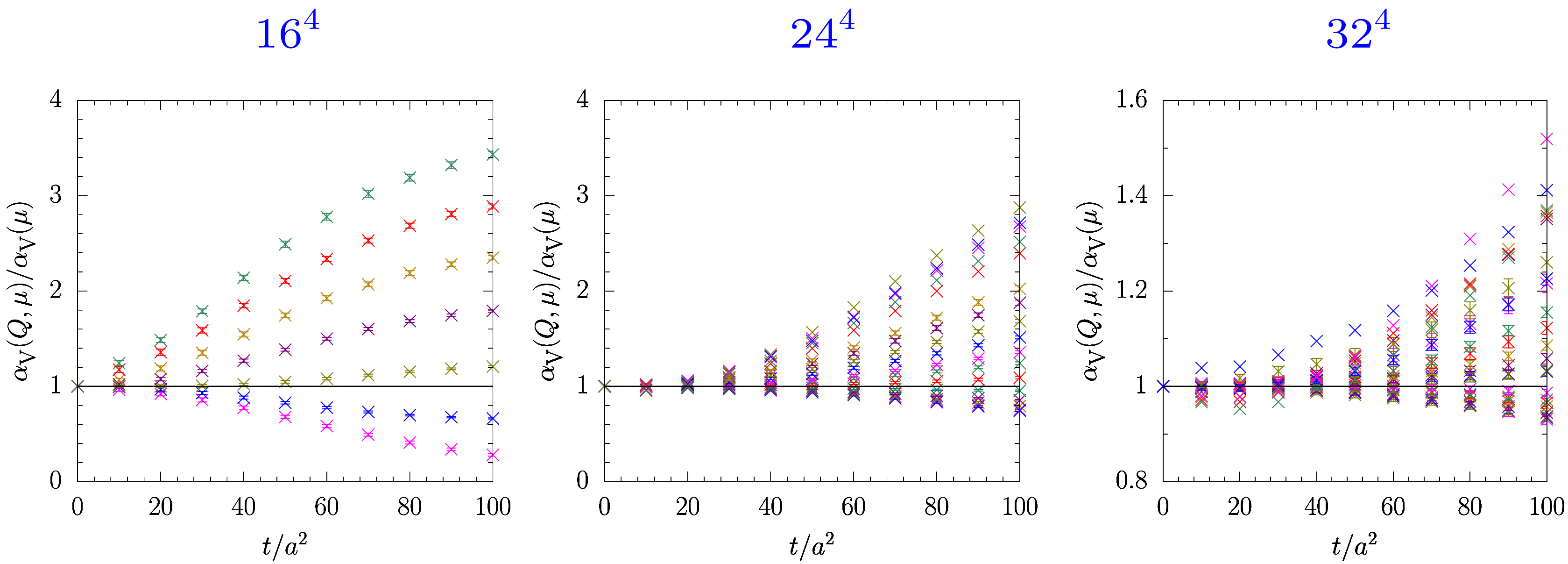
One is tempted to conclude that the vacuum is a dilute gas of instantons. However, this is not the case. We find a negative value for the 'kurtosis', $K = \langle Q^4\rangle_c/\langle Q^2\rangle_c^2$, on all lattice volumes, while $K > 0$ for a dilute instanton gas [more later]

Observables consistently show a clear dependence on Q. This is the reason for a nontrivial θ dependence when Fourier transformed to the θ vacuum

Running coupling α_V

If the general expectation is correct and the color fields are screened for $|\theta| > 0$, we should, in the first place, find that the running coupling constant is screened in the infrared

From $\langle E(Q,t)\rangle$ we obtain $\alpha_V(Q,\mu)$ in the individual topological sectors $|Q|$ from bottom to top



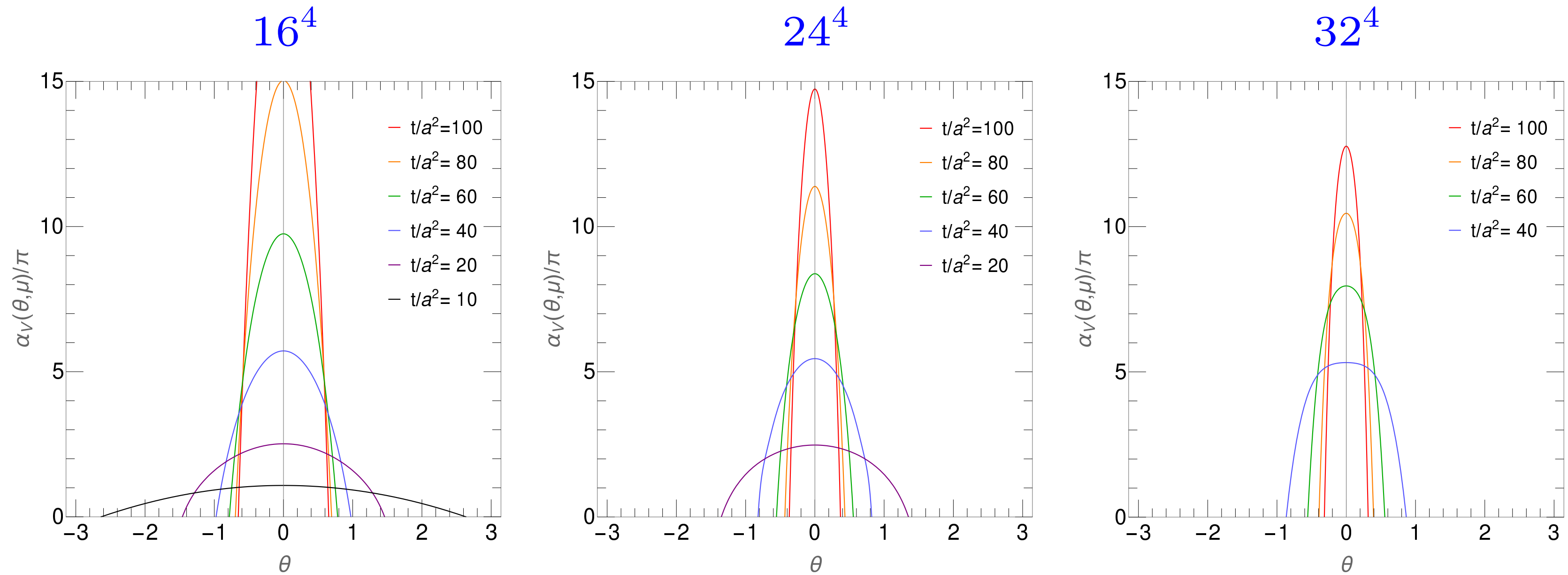
Interestingly, $\alpha_V(Q,\mu)$ vanishes in the infrared for $Q = 0$, while the ensemble average $\alpha_V(\mu)$ is represented by $|Q| \simeq \sqrt{2\langle Q^2\rangle/\pi}$

The transformation of $\alpha_V(Q,\mu)$ from Q to the θ vacuum is achieved by the discrete Fourier transform

$$\alpha_V(\theta,\mu) = \frac{1}{Z(\theta)} \sum_Q e^{i\,\theta\,Q} P(Q) \alpha_V(Q,\mu)$$

$$Z(\theta) = \sum_Q e^{i\,\theta\,Q} P(Q)$$

- $e^{i\theta Q} P_{\theta=0}(Q) = P_\theta(Q)$ 1502.02295
- Z_θ analytic at $\theta = 0$ Vafa-Witten
- Limits set by convergence of the Fourier sum



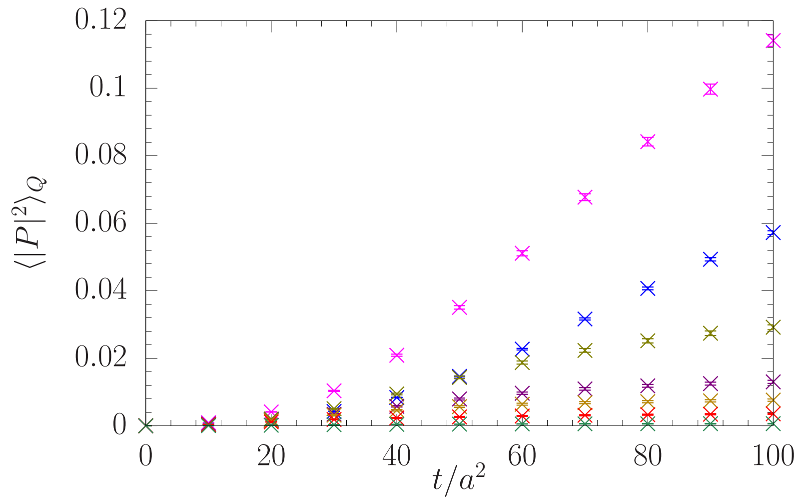
The color charge is totally screened for $|\theta| \gtrsim 0$ in the infrared, while it becomes gradually independent of θ as we approach the perturbative regime

Precision test by comparing different volumes

Polyakov loop

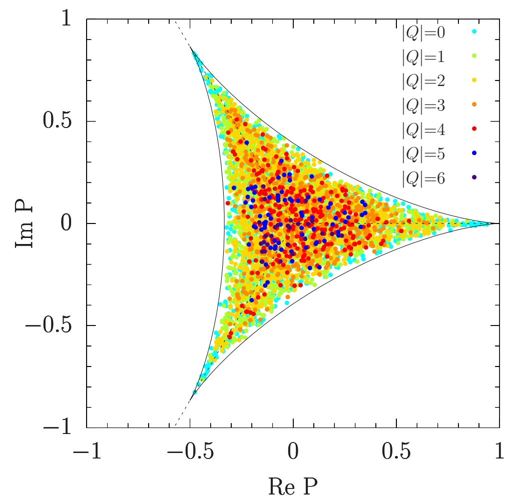
The Polyakov loop P describes the propagation of a single static quark travelling around the periodic lattice

16^4



From $Q = 0$ (top) to 6 (bottom)

$t/a^2 = 60$



$\langle P \rangle = 0$ in each sector. That implies center symmetry throughout. P rapidly populates the entire theoretically allowed region for small values of $|Q|$, while it stays small for larger values of $|Q|$

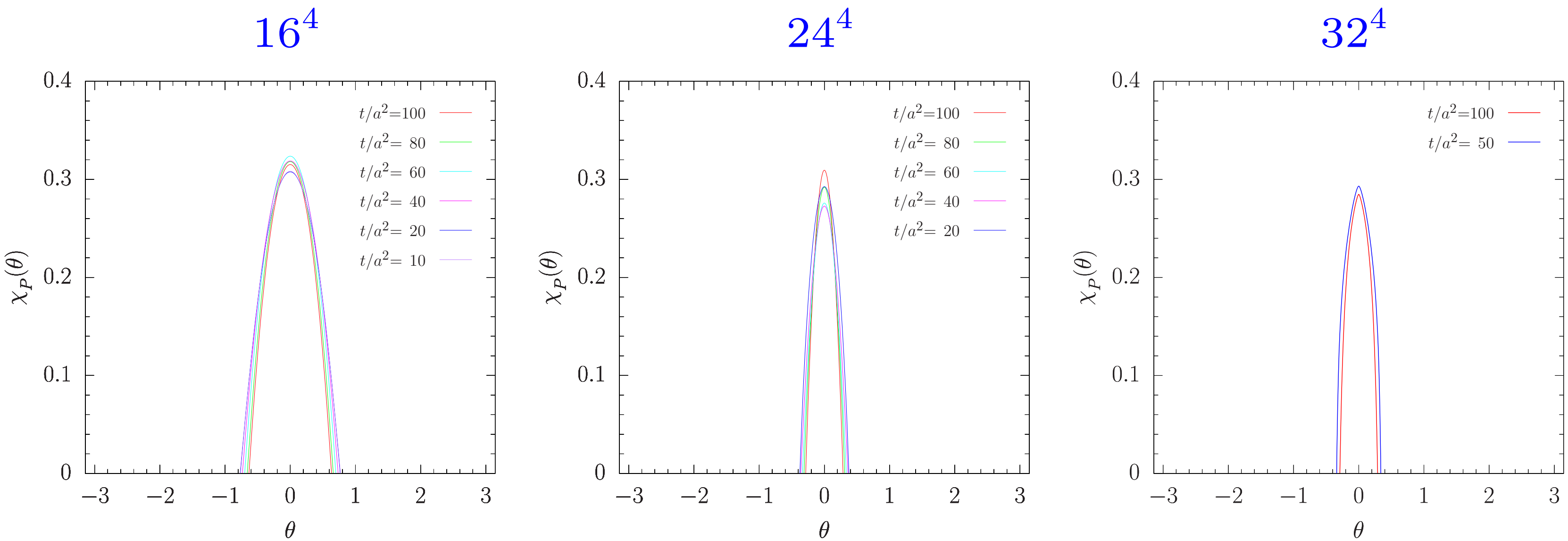
The transformation of the Polyakov loop expectation values to the θ vacuum is again achieved by the discrete Fourier transform

$$\langle |P|^2 \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{\,i\,\theta\,Q}\, P(Q) \langle |P|^2 \rangle_Q$$

$$\langle |P| \rangle_\theta = \frac{1}{Z(\theta)} \sum_Q e^{\,i\,\theta\,Q}\, P(Q) \langle |P| \rangle_Q$$

The connected part of $\langle |P|^2 \rangle_\theta$ is described by the renormalized Polyakov loop susceptibility

$$\chi_P(\theta) = \frac{\langle |P|^2 \rangle_\theta - \langle |P| \rangle_\theta^2}{\langle |P| \rangle_\theta^2}$$



The Polyakov loop gets totally screened for $|\theta| \gtrsim 0$. The renormalized Polyakov loop susceptibility is independent of flow time t (even for $\theta \neq 0$!)

Mass gap

$$\langle E^2\rangle = \frac{1}{T}\sum_t \langle E(0)E(t)\rangle$$

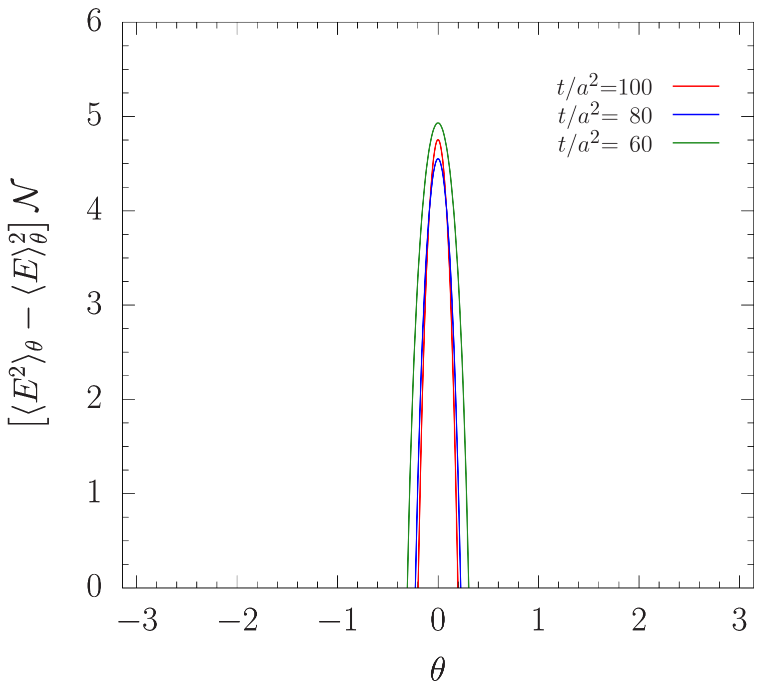
$$E(t) = \frac{1}{V_3}\sum_{\vec{x}} E(\vec{x},t)$$

$$[\langle E^2\rangle - \langle E\rangle^2]\,\mathcal{N} = \sum_{n>0,t} \frac{1}{2m_n}\,|\langle 0|E|n\rangle|^2\, e^{-m_n t}$$

$$\simeq \frac{1}{m^2_{0^{++}}}\,|\langle 0|E|0^{++}\rangle|^2 \;\propto\; \xi^2$$

Correlation length

24^4



$\langle E^2\rangle_\theta - \langle E\rangle_\theta^2$ Independent of flow time t

$\xi \simeq 0$ for $|\theta| \gtrsim 0$

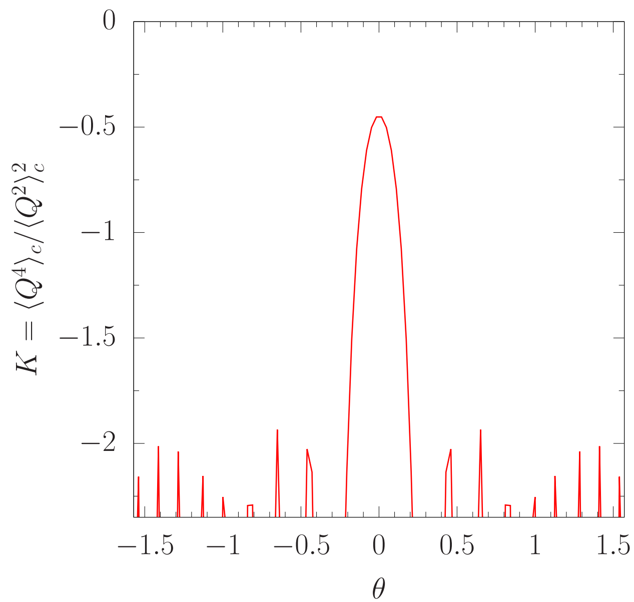
No mass gap

Kurtosis

The key to understanding the transition to nonvanishing θ lies in the topological structure of the vacuum. Our global analysis limits us to the investigation of moments of topological charge Q. A quantity of particular interest is the kurtosis K. In the θ vacuum

$$K = \frac{\langle (Q - \langle Q \rangle_\theta)^4 \rangle_\theta}{\langle (Q - \langle Q \rangle_\theta)^2 \rangle_\theta^2} - 3 \equiv -V \frac{\partial^4 F(\theta)/\partial\theta^4}{(\partial^2 F(\theta)/\partial\theta^2)^2}\,, \qquad F(\theta) = -\frac{1}{V} \log Z(\theta)$$

32^4



The lowest value the kurtosis can take is $K = -2$, that is when

$$\langle (Q - \langle Q \rangle_\theta)^4 \rangle_\theta = \langle (Q - \langle Q \rangle_\theta)^2 \rangle_\theta^2$$

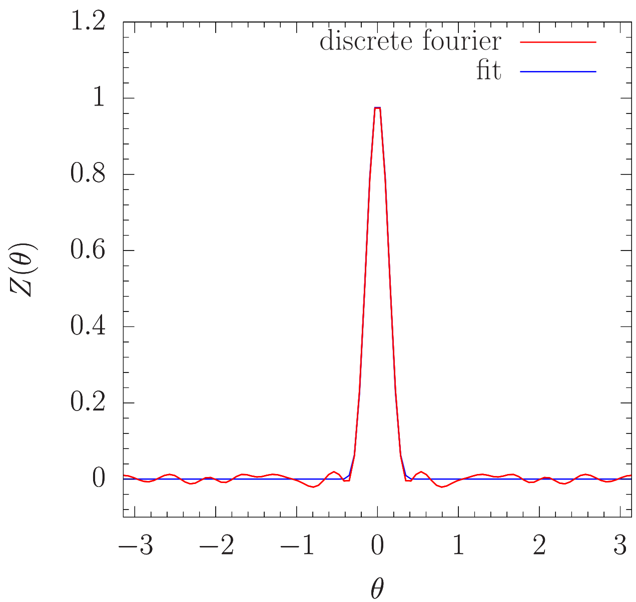
which means that the topological charge Q is screened on its own:

$$\chi_{t,\theta} = \frac{\partial^2 F(\theta)}{\partial\theta^2} = \frac{1}{V}\,\wp(\theta; 0, g_3),\ |\theta| \gtrsim 0$$

Errors

Source of errors

- Convergence of the (discrete) Fourier series $\sum_Q \exp\{i\theta Q\}\, P(Q) \cdots$
- Statistics
- Topological charge generally limited to $|Q| \leq |Q|_{\max}$, $|Q|_{\max} \propto \sqrt{V}$



$Z(\theta), \alpha_V(\theta), \chi_P(\theta), \cdots$ are positive functions of θ

After the quantities I showed have dropped to 'zero' at $|\theta| \gtrsim 0$, they start to oscillate around zero with frequency $\nu \approx |Q|_{\max}$ due to the truncated Fourier series

Various techniques to filter unphysical high-frequency modes are discussed in the literature. We fit the tail of the distributions to a smooth function. Alternatively, one can employ a low-pass filter, which practically gives the same result

Electric Dipole Moment

arXiv:1502.02295

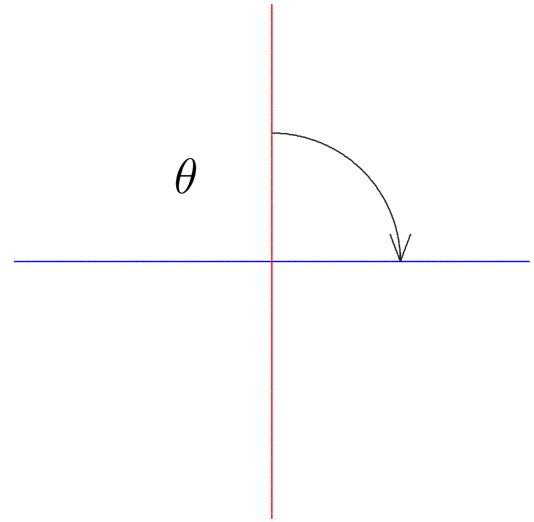
Aoki

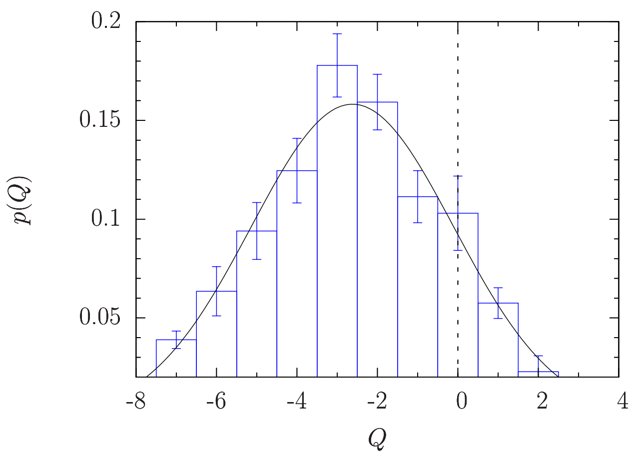
~~Aoki~~

According to Vafa-Witten the theory is analytic at $\theta = 0$. Hence, we may continue θ to imaginary $\bar{\theta} = -i\,\theta$. This leads to the action

$$S_\theta = \frac{1}{3}\bar{\theta}\hat{m}\sum_x(\bar{u}\gamma_5 u + \bar{d}\gamma_5 d + \bar{s}\gamma_5 s)$$

which is amenable to numerical simulations. At the end of the calculation the results are rotated back to real θ





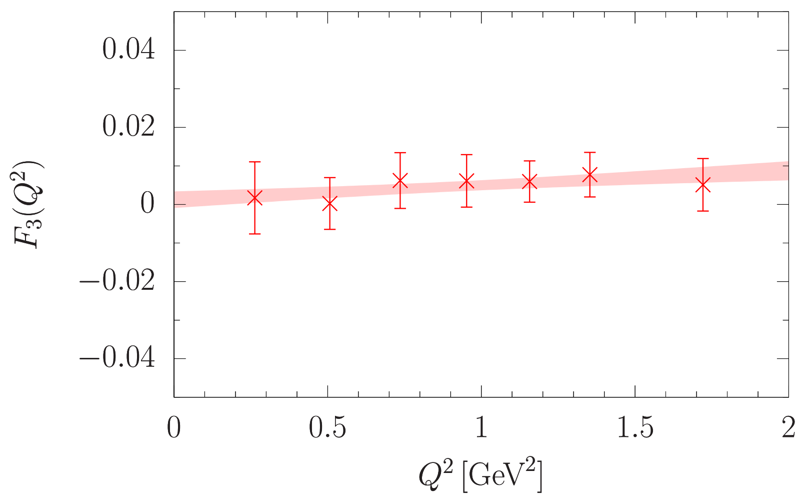
- $e^{i\theta Q}\,P_{\theta=0}(Q) = P_\theta(Q)$, which is reassuring
- The unique feature is that the calculation is done on a non-trivial topological background
- How can one find a nucleon in the screened phase? If the hadron radius is significantly smaller than the screening length

We expect the electric dipole moment of the neutron to be largest for heavy quarks, as it will vanish trivially in the chiral limit, $d_n \propto m_u m_d/(m_u + m_d)$

At the SU(3) flavor symmetric point, $m_\pi = m_K = 410$ MeV arXiv:1102.5200

$$(m_u = m_d = m_s)$$

$32^3 \times 64$



$|\theta| \approx 0.4$

This leads to

$$d_n = \frac{e\,F_3(0)}{2m_N} = 0.00028(30)\ [\text{e fm}\ \theta]$$

which is compatible with zero, as expected

Similar results have been reported for using reweighting

arXiv:1701.07792
arXiv:2011.01084
arXiv:2101.07230

Axion

In the Peccei–Quinn theory the CP violating action $S_\theta = i\theta Q$ is augmented by the axion interaction

$$S_\theta \to S_\theta + S_{\rm Axion} = \int d^4x \left[\frac{1}{2}(\partial_\mu \phi_a(x))^2 + i\left(\theta - \frac{\phi_a(x)}{f_a}\right) q(x)\right], \quad \int d^4x\, q(x) = Q$$

with

$$U_{\rm PQ}(1)\,|\theta\rangle \qquad \longrightarrow \qquad |\theta + \delta\rangle$$

It is then expected that QCD induces an effective potential $U_{\rm eff}(\theta - \phi_a/f_a)$, having a stationary point at $\theta - \phi_a/f_a = 0$

$$\theta \qquad \longrightarrow \qquad \frac{\phi_a(x)}{f_a}$$

CP violating   CP conserving

thus effectively eliminating CP violation in the strong interaction

However, QCD vacuum unstable under $U_{\rm PQ}(1)$

Conclusions

★ The numerical work is characterized by high statistics on three different volumes. A key point is that the path integral splits into disconnected topological sectors for $t \gtrsim 0$, which is expected to occur at ever smaller flow times with decreasing lattice spacing. Comparing results on different volumes enabled us to control the accuracy of the calculation

★ The gradient flow proved a powerful tool for tracing the gauge field over successive length scales and showed its potential for extracting low-energy quantities. The novel result is that color charges are screened for $|\theta| > 0$ by nonperturbative effects, limiting the vacuum angle to $\theta = 0$ at macroscopic distances, which rules out any strong CP violation at the hadronic level

★ The electric dipole moment of the neutron was found to be zero within the errorbars, as expected. In absence of a nonvanishing dipole moment no upper limit of θ can be drawn from the experimental bound

★ The nontrivial phase structure of QCD has far-reaching consequences for anomalous chiral transformations. In particular, the confining QCD vacuum will be unstable under the Peccei-Quinn chiral $U_{PQ}(1)$ transformation, realized by the shift symmetry $\theta \to \theta + \delta$, which thwarts the axion conjecture