QCD phase structure in Polyakov linear-sigma model with non-zero isospin density

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Agenda

- Short introduction to linear-Sigma Model,

  a) Mesonen, b) Quarks.
- Integrating in Polyakov-loop potential.
- PLSM at finite:
  - Temparatur and baryon density,
  - Isospin assymetry.
- Results and Conclusions.

Related publications

- 1908.05939 [hep-ph] → PRC
- Chin.Phys. C43 (2019) 034103
- Int.J.Mod.Phys. A34 (2019) 1950199
- J.Phys. G45 (2018) 055008
- J.Exp.Theor.Phys. 126 (2018) 620-632
- EPJ Web Conf. 177 (2018) 09005
- Indian J.Phys. 91 (2017) no.1, 93-99
- 1611.06926 [hep-lat] JINR-Konferenz
- J.Phys.Conf.Ser. 668 (2016) 012102
- Int.J.Adv.Res.Phys.Sci. 3 (2016) 4-14
- Int.J.Mod.Phys. A31 (2016) 1650175
- PoS ICHEP2016 (2016) 634
- Adv.High Energy Phys. 2016 (2016) 1381479
- Indian J. Phys. (2016)
- European Phys. J. A (2016)
- Adv.HighEnergy Phys. 2016 (2016) 1381479
- J.Phys.Conf.Ser. 668 (2016) 012082
- J.Phys. G42 (2015) 015004
- Adv.HighEnergy Phys. 2015 (2015) 563428
- Phys.Rev. C91 (2015) 015206
- Phys.Rev. C91 (2015) 015204
- J.Phys. G42 (2015) 0150041
- Phys.Rev. C90 (2014) 015204
- Phys.Rev. C89 (2014) 055210

Sigma Model

Long before the invension of QCD in 1960, sigma model was introduced by Gell-Mann und Levy


Il NuovoCimento 16, 705 (1960)

- The name σ-Modell is that of the field, which was presented by Julian Schwinger, a spinless mesonic scalar σ.

Ann. Phys. 2(407), 1957

In QFT, a non-linear σ-Model describes a scalar field Σ (Minkowski-space M differentiable Mapping), which takes values from a non-linear Manifold to a target manifold T. It is with Riemannschen Metrik g furnished.

- The Lagrangian density

$$\mathcal{L} = \frac{1}{2} g(\partial^\mu \Sigma, \partial_\mu \Sigma) - V(\Sigma)$$

in time-like chira form

$$\mathcal{L} = \frac{1}{2} g_{ab}(\Sigma)(\partial^\mu \Sigma^a)(\partial_\mu \Sigma^b) - V(\Sigma)$$

in coordination

Sigma Model

- Sigma model describes a physical system with the Langrangian

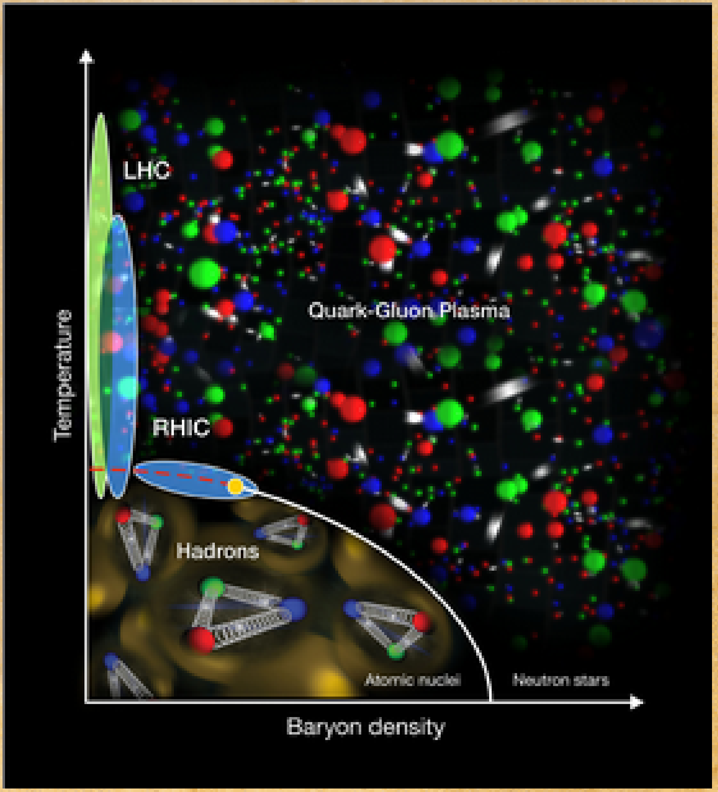
$$\mathcal{L}(\phi_1, \phi_2, \ldots, \phi_n) = \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} \, \mathrm{d}\phi_i \wedge *\mathrm{d}\phi_j$$

[Wedge Produkt]

- Mapping or Skalar g_{ij} defines whether this is linear or non-linear.
- The fields ϕ_i provide a general [Mapping] from a spacetime basic distributor, which is known as world sheet, to a Riemanian Manifold target distributor of scalar, which are distinguishable from each other of connected through internal symmetries.
- Sigma-Model with a Manifold, through which a real line exists, which parameterizes target and target space, gives a fundamental example on GFT in 1D.
- Sigma Model is a prototype of [spontaneous symmetry breaking], where the three [broken axial generators] for pions, which are the manifestations of the chiral symmetry breaking.

Why Linear-Sigma Model?

- LSM could be an economic alternative to QCD lattice simulations.
- No supercomputers are neeeed, conventional PCs and simple algorithms and programming techniques would be enough.
- Finite magnetic and electric fields and finite isospin asymmetry could be integrated in.
- Various symmetry breaking scenarios could be easily studied, for instance different properties of strong interactions chould be investigated
  - thermodynamics and EoS of Hadron and Parton,
  - chiral phase structure and meson masses (16 pseudoscalar, scalar, vector and axial vector)
  - transport and conductivity coefficients of QCD mater.



DoF in Linear Sigma Model

- This low-energy model has generators $T_a=\lambda_a/2$, where λ_a are Gell-Mann Matrices and real classical field; O(4) $\vec{\Phi} = T_a(\vec{\sigma}_a, i\vec{\pi}_a)$
- The chiral symmetry is explicitly broken through 3×3-Matrixfield $H=T_a h_a$, where h_a are external Fields.
- Under the chiral transformation $SU(2)_L \times SU(2)_R$, $\Phi \rightarrow L^+ \Phi R$, σ_a gets a finite vacuum expectation values, which also causes chiral symmetry breaking to $SU(2)_L \times SU(2)_R \rightarrow SU(2)_{L+R}$.
- The vacuum expectation values play the role of order parameters of phase transition.
- This leads to massive Sigma-particles und zu light oder even massless Goldstone-Bosons, the Pions.
- The original DoF are spissless scalar Sigma σ_a field and triplet-pseudoscalar Fields π_a (π+N interactions).
    - LSM breaks axiale anomaly U(1), at quantum level
    - LSM: explicit symmetry breaking for non-vanishing quartk masses
    - LSM: Spontaneous symmetry breaking through chiral condensate <qq>

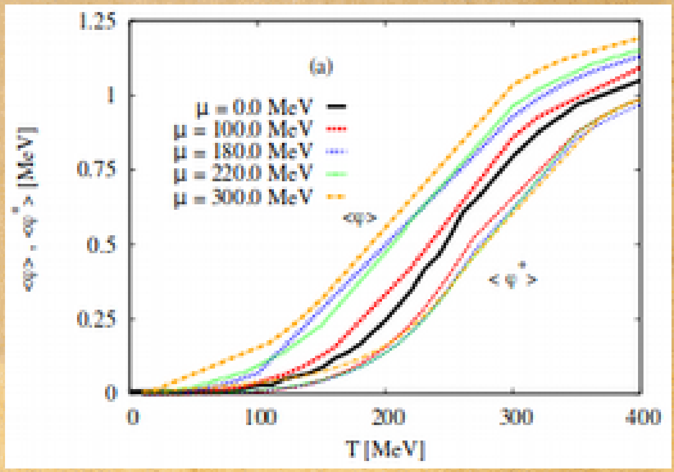
DoF: Mesons + Quarks

- QCD as theory of strong interactions has the following properties
  - (i) Asymptotic Freedom (ii) Confinement.
  - (iii) hidden spontaneous breaking of chiral Symmetry.
- Birse and Banerjee suggested in 1985 a model for nucleons and delta particles, which represents the strong QCD forces as chiral $SU(2)_L x SU(2)_R$-symmetry and assures a role separation between these forces.
- This symmetry is responsible for the binding of quarks in hadrons und for the forces, which cause an absolute limitation, i.e. short range.
- This leads to a linear sigma-model, which describes the interactions of quarks with sigma- and pion-mesons.
- The constituent quarks gain mass, for instance $m_q = g f_\pi$, where g and f_π are the coupling and die pion decay constants.
  - Accordingly, fermions could be integrated either as nucleons or as quarks.
  - It was shown, that the chiral field transformation shows the same behaviour as that of the quark condensates, namely the σ-Order parameter for chiral QCD phase-transition.

DoF: Mesons + Quarks + Gluons

- Similar to PNJL, Polyakov-Quark-Meson (PQM) was suggested
- B.-J. Schaefer, M. Wagner, J. Wambach, [Phys. Rev. D81 (2010) 074013]()
- → combination of chiral linear **σ**-model with Polyakov-loops
- The thermal expectation values of colored Wilson-loops in time dimension reads
$$\Phi(\vec{x}) = \frac{1}{N_c}\langle \mathrm{tr}_c\, \mathcal{P}(\vec{x})\rangle$$
- where P(x) are the matix-values of the Polyakov-loop parameter in the basic representation of SU(Nc)-group; similar to the temporal vectorfeeld A^0
$$\mathcal{P}(\vec{x}) = \mathrm{P}\exp\left(i\int_0^\beta d\tau A_0(\vec{x},\tau)\right)$$
- where P is order of the path integral und **β**=1/T is the inverse Temperatur.

    - **Φ** is finite for high temperatures; that of the deconfined phase, und vanishing for low temperatures in the confined and central symmetry phase



Polyakov Loops: Deconfinement & color DoF

- Gluonic DoF are depending on Polyakov-Variables phi und phi* and accordingly of the color DoF.
- All are based wilson-loops, for which the various parameters could be dermined from lattice QCD simulations.
- Polyakov-loop potential and its conjugate are

$$\phi = (\mathrm{Tr}_c\,\mathcal{P})/N_c,$$
$$\phi^* = (\mathrm{Tr}_c\,\mathcal{P}^\dagger)/N_c,$$

- The Polyakov loops could be represented as matrices in color space
- $\mathcal{P}(\vec{x}) = \mathrm{P}\exp\left(i\int_0^\beta d\tau A_0(\vec{x},\tau)\right)$ and $A_4 = iA^0$ Gauge-Polyakov
- Matrices of the Polyakov loops can be given as diagonal representation.
- Die coupling with the Qaurks is given by covariant derivative.
- where $A_\mu=\delta_{\mu 0}A_0$ are limited by the chiral Limits $D_\mu = \partial_\mu - iA_\mu$

PLSM: SU(3) Langrangian

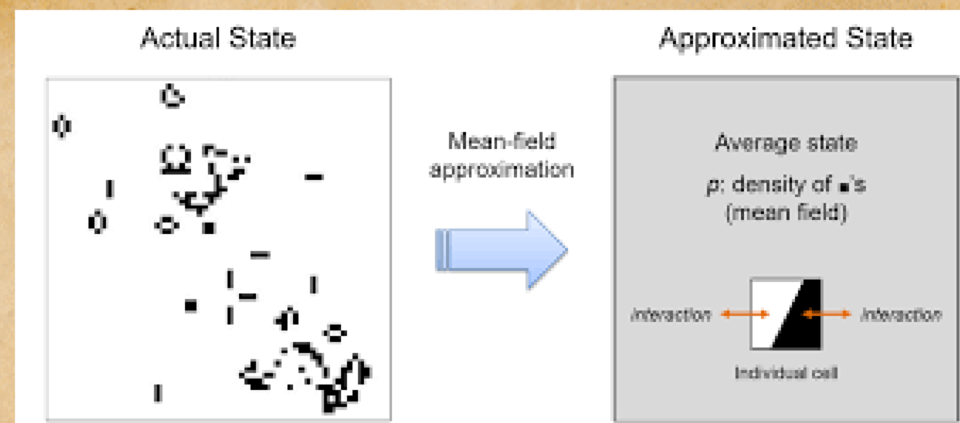
- For N_f=3 quark flavor and N_c=3 color DoF, where the suarks are coupled to the Polyakov-loop dynamics, $\mathcal{L} = \mathcal{L}_{chiral} - \mathcal{U}(\phi, \phi^*, T),$
- where $\mathcal{L}_{chiral} = \mathcal{L}_q + \mathcal{L}_m$ has $SU(3)_L \times SU(3)_R$ symmetry
- a) For Quarks: $\mathcal{L}_q = \sum_f \overline{\psi}_f (i\gamma^\mu D_\mu - g T_a(\sigma_a + i\gamma_5 \pi_a))\psi_f,$
- b) For Mesons: $$\mathcal{L}_m = \mathrm{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\mathrm{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \mathrm{Tr}(\Phi^\dagger \Phi)^2 + c[\mathrm{Det}(\Phi) + \mathrm{Det}(\Phi^\dagger)] + \mathrm{Tr}[H(\Phi + \Phi^\dagger)]$$
- where **ϕ** is 3x3-matrix including the nonet-Meson states $\Phi = \sum_{a=0}^{N_f^2-1} T_a(\sigma_a - i\pi_a)$
- c) Polyakov-loop dynamics: $$\frac{\mathcal{U}(\phi, \phi^*, T)}{T^4} = -\frac{b_2(T)}{2}|\phi|^2 - \frac{b_3}{6}(\phi^3 + \phi^{*3}) + \frac{b_4}{4}(|\phi|^2)^2,$$
- where $b_2(T) = a_0 + a_1 \left(\frac{T_0}{T}\right) + a_2 \left(\frac{T_0}{T}\right)^2 + a_3 \left(\frac{T_0}{T}\right)^3$ $\quad a_0 = 6.75, \quad a_1 = -1.95, \quad a_2 = 2.625, \quad a_3 = -7.44, \quad b_3 = 0.75, \quad b_4 = 7.5$
- D_μ, **μ**, **γ**, **μ** and g are covariant derivative, Lorentz-Index, chiral spinoren or Yukawa-coupling constants. **ψ** are Dirac-Spinor fields for quark flavors f=[u, d, s]

$$\Omega(T,\mu) = \Omega_{\bar{q}q}(T,\mu) + \mathscr{U}_{\mathrm{PolyLog}}(\varphi, \varphi^*, T) + U(\sigma_l, \sigma_s, \sigma_c) = -T \ln \mathscr{Z}/V$$

Phys. Rev. D 62, 085008 (2000).

PLSM: Approximation: MFA

- All fields are takes as constants in space and imaginary time directions.
- Averaged fields are defined as $\bar{\phi} = \frac{T}{V}\int_0^{\beta} d\,\tau \int d^3\,\phi(\nabla x)$
- MFA drastically reduces the higher to lower dimension, so that the dynamics of the system is adjusted according to the „averaged fields“.
- Then the dynamics is accordingly described, similar to single cells with an averaged status, which varies with the time.



It is assumed that every cell in the entire space selects its next state regardless of the probabilities that are determined by the average state of the system.

PLSM: Approximation: MFA

σ and π replaced by time-space independence averaged values, thus

$$\mathcal{Z} = e^{-\beta V U(\langle\sigma\rangle,\langle\vec{\pi}\rangle)} \int \mathcal{D}\bar{q}\mathcal{D}q \exp\left\{ - \int_x \bar{q}\left[\gamma^0\partial_\tau - \vec{\gamma}\cdot\nabla + g(\langle\sigma\rangle + i\gamma^5\vec{\tau}\cdot\langle\vec{\pi}\rangle) - \mu\gamma^0\right]q \right\}$$

Fourier transform fields ψ's and then calculation space-time integral, Helmholtz' free energy reads

$$\mathcal{F} = -T\sum \mathrm{tr}\log\left[\beta\left(i\gamma^0(\omega_n + i\mu) + \vec{\gamma}\cdot\mathrm{p} + g\sigma + ig\gamma^5\vec{\tau}\cdot\vec{\pi}\right)\right]$$

When expanding log around $g\sigma$ where all γ matrices are traceless

$$\mathcal{F} = -2TN_f\sum \log\left[\beta^2\left((\omega_n + i\mu)^2 + \mathrm{p}^2 + g^2(\sigma^2 + \vec{\pi}^2)\right)\right].$$

With dispersion relation $\omega^2 = \mathrm{p}^2 + m^2$ and $m^2 = g^2(\sigma^2 + \vec{\pi}^2)$

$$\mathcal{F} = -TN_f\sum \log\left[\beta^2\left(\omega_n^2 + (\omega+\mu)^2\right)\right] + \log\left[\beta^2\left(\omega_n^2 + (\omega-\mu)^2\right)\right]$$

$$\mathcal{F} = -2N_fV\int \frac{d\mathrm{p}}{(2\pi)^3}\left\{\omega + T\log\left[1 + e^{-(\omega+\mu)\beta}\right] + T\log\left[1 + e^{-(\omega-\mu)\beta}\right]\right\}$$

PLSM: Approximation: MFA

In thermal equilibrium, the grand-canonical distribution function can be defined by a path integral over the quark, antiquark and meson field, which contains the chemical potential

$$\Omega(T,\mu) = \Omega_{\bar{q}q}(T,\mu) + \mathscr{U}_{\text{PolyLog}}(\varphi,\varphi^*,T) + U(\sigma_l,\sigma_s,\sigma_c) = -T\ln \mathscr{Z}/V$$

$$\begin{aligned}\mathcal{Z} &= \mathrm{Tr}\exp[-(\hat{\mathcal{H}} - \sum_f \mu_f \hat{\mathcal{N}}_f)/T] \\ &= \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int_x (\mathcal{L} + \sum_f \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right],\end{aligned}$$

where μ_f is the chemical potential of f-ten Quarks, and

$$\int_x \equiv i\int_0^{1/T} dt \int_V d^3x$$

PLSM: Approximation: Optimum perturbation theory (OPT)

- The basic idea of OPT becomes clear when we expand the chiral Lagrangian from which we see that even analytical non-intrusive calculations become accessible that go beyond what MFA would achieve.

$$\mathcal{L}^{\delta} = (1-\delta)\,\mathcal{L}_0(\eta) + \delta\,\mathcal{L} = \mathcal{L}_0(\eta) + \delta\left[\mathcal{L} - \mathcal{L}_0(\eta)\right]$$

- where **η** is a mass parameter, that could be determined from a suitable variation process and L_0(**η**) is density of free Lagrangian, in which **η** is included.
- The implementation from OPT to PLSM thus apparently goes hand in hand with an expansion with regard to the arbitrary parameter **δ**.
- In PLSM:

$$\begin{aligned}
\mathcal{L}_{\bar{\psi}\psi} &= \sum_f \overline{\psi}_f\left[i\gamma^\mu D_\mu - \delta\, g\, T_a(\sigma_a + i\gamma_5\pi_a) - (1-\delta)\,\eta\right]\psi_f, \\
\mathcal{L}_m &= \mathrm{Tr}\left[\partial_\mu\Phi^\dagger\partial^\mu\Phi - \left(m^2 + (1-\delta)\ \eta^2\right)\Phi^\dagger\Phi\right] \\
&+ \delta\left\{c\left(\mathrm{Det}\left[\Phi\right] + \mathrm{Det}\left[\Phi^\dagger\right]\right) - \lambda_1\ \left(\mathrm{Tr}\left[\Phi^\dagger\Phi\right]\right)^2 - \lambda_2\,\mathrm{Tr}\left[\Phi^\dagger\Phi\right]^2 + \mathrm{Tr}\left[H\left(\Phi+\Phi^\dagger\right)\right]\right\}. \\
\mathcal{U}(\phi,\phi^*,T) &= -b\,T\left[54\,\phi\,\phi^*\,e^{-a/T} + \ln\left(1 - 6\phi\phi^* - 3\,(\phi\phi^*)^2 + 4\left(\phi^3+\phi^{*3}\right)\right)\right],
\end{aligned}$$

PLSM: Approximation: Optimum perturbation theory (OPT)

- When using OPT to evaluate the free energy F of the PLSM, non-disruptive analytical calculations are made possible by a rule known as the principle of minimum sensitivity (PMS).
- PMS dictates that F can be minimized to the variations of **η** at **δ = 1**.

$$\left.\frac{\partial \mathcal{F}_{\mathtt{OPT}}}{\partial \eta}\right|_{\bar{\eta},\delta=1} = 0$$

- The expected value of **η** is related to the sigma fields **σ**f and the color degrees of freedom Nc at **η~σ**.
- A global minimization should also be carried out for the order parameters

$$\left.\frac{\partial \mathcal{F}_{\mathtt{OPT}}}{\partial \bar{\sigma}_l}\right|_{\bar{\sigma}_l} = 0, \quad \left.\frac{\partial \mathcal{F}_{\mathtt{OPT}}}{\partial \bar{\sigma}_s}\right|_{\bar{\sigma}_s} = 0, \quad \left.\frac{\partial \mathcal{F}_{\mathtt{OPT}}}{\partial \bar{\phi}}\right|_{\bar{\phi}} = 0, \quad \left.\frac{\partial \mathcal{F}_{\mathtt{OPT}}}{\partial \bar{\phi}^*}\right|_{\bar{\phi}^*} = 0$$

$$\mathcal{Z} = \mathrm{Tr} \exp\left[\frac{\sum_{f=u,d,s} \mu_f \hat{\mathcal{N}}_f - \hat{\mathcal{H}}}{T}\right] = \int \prod_a \mathcal{D}\sigma_a \mathcal{D}\pi_a \int \mathcal{D}\psi \mathcal{D}\bar{\psi} \exp\left[\int d^4x (\mathcal{L} + \sum_{f=u,d,s} \mu_f \bar{\psi}_f \gamma^0 \psi_f)\right]$$

$$\mu_u = \frac{\mu_B}{3} + \frac{2\mu_Q}{3}, \qquad \mu_d = \frac{\mu_B}{3} - \frac{\mu_Q}{3}, \qquad \mu_s = \frac{\mu_B}{3} - \frac{\mu_Q}{3} - \mu_S$$

PLSM: SU(2) → SU(3) → SU(4)

- The thermodynamic Potential reads

$N_f = 2,\quad \Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l),$

- Polyakov Mesons Quarks

$N_f = 3,\quad \Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s)$

$N_f = 4,\quad \Omega = \mathcal{U}(\phi, \phi^*) + U(\sigma_l, \sigma_s, \sigma_c) + \Omega_{\bar{q}q}(\phi, \phi^*, \sigma_l, \sigma_s, \sigma_c)$

- For example, for SU(4):

$$
\begin{aligned}
U(\sigma_l, \sigma_s, \sigma_c) &= -h_l\sigma_l - h_s\sigma_s - h_c\sigma_c + \frac{m^2(\sigma_l^2 + \sigma_s^2 + \sigma_c^2)}{2} - \frac{c\,\sigma_l^2\sigma_s\sigma_c}{4} + \frac{\lambda_1\sigma_l^2\sigma_s^2}{2} \\
&+ \frac{\lambda_1\sigma_l^2\sigma_c^2}{2} + \frac{\lambda_1\sigma_s^2\sigma_c^2}{2} + \frac{(2\lambda_1 + \lambda_2)\sigma_l^4}{8} + \frac{(\lambda_1 + \lambda_2)\sigma_s^4}{4} + \frac{(\lambda_1 + \lambda_2)\sigma_c^4}{4}
\end{aligned}
$$

$$
\sigma_l = \frac{\sigma_0}{\sqrt{2}} + \frac{\sigma_8}{\sqrt{3}} + \frac{\sigma_{15}}{\sqrt{6}}, \qquad \sigma_s = \frac{\sigma_0}{2} - \sqrt{\frac{2}{3}}\sigma_8 + \frac{\sigma_{15}}{2\sqrt{3}}, \qquad \sigma_c = \frac{\sigma_0}{2} - \frac{\sqrt{3}}{2}\sigma_{15}
$$

$$
\begin{aligned}
\Omega_{\bar{q}q}(T, \mu) = -2T \sum_{f=l,s,c} \int_0^\infty \frac{d^3\vec{P}}{(2\pi)^3} \Bigg\{ &\ln\left[1 + 3\left(\varphi + \varphi^* e^{-\frac{E_f - \mu}{T}}\right) \times e^{-\frac{E_f - \mu}{T}} + e^{-3\frac{E_f - \mu}{T}}\right] \\
&+ \ln\left[1 + 3\left(\varphi^* + \varphi e^{-\frac{E_f + \mu}{T}}\right) \times e^{-\frac{E_f + \mu}{T}} + e^{-3\frac{E_f + \mu}{T}}\right]\Bigg\}
\end{aligned}
$$

PLSM: SU(2) → SU(3) → SU(4)

In $SU(4)_L \times SU(4)_R$ symmetries, the LSM Lagrangian can be constructed as $\mathscr{L}_{\text{chiral}} = \mathscr{L}_q + \mathscr{L}_m$, where $q \in (u,d,s,c)$,

$$\mathscr{L}_q = \sum_f \bar{q}_f \left[i\gamma^\zeta D_\zeta - gT_a(\sigma_a + i\gamma_5 \pi_a) \right] q_f,$$

and g is the flavor-blind Yukawa coupling . ζ is an additional Lorentz index. The pure mesonic part is given as

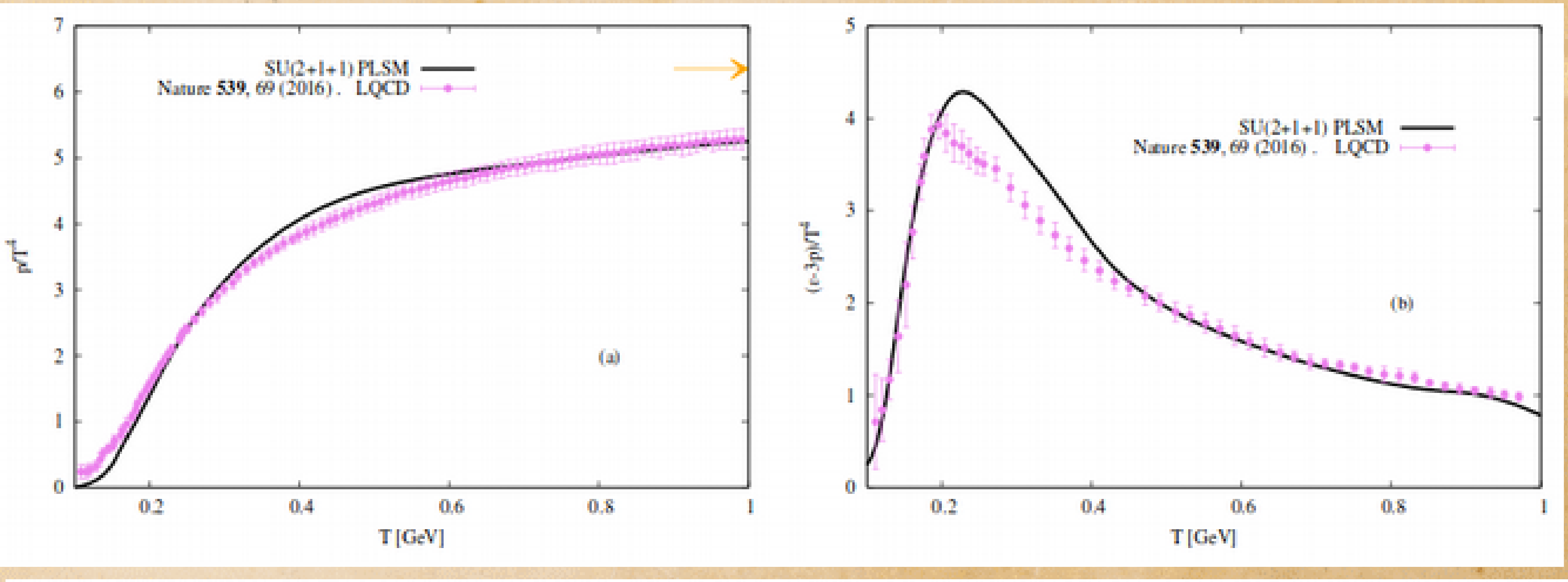
$$\mathscr{L}_m = \text{Tr}(\partial_\mu \Phi^\dagger \partial^\mu \Phi - m^2 \Phi^\dagger \Phi) - \lambda_1 [\text{Tr}(\Phi^\dagger \Phi)]^2 - \lambda_2 \text{Tr}(\Phi^\dagger \Phi)^2 + \text{Tr}[H(\Phi + \Phi^\dagger)],$$

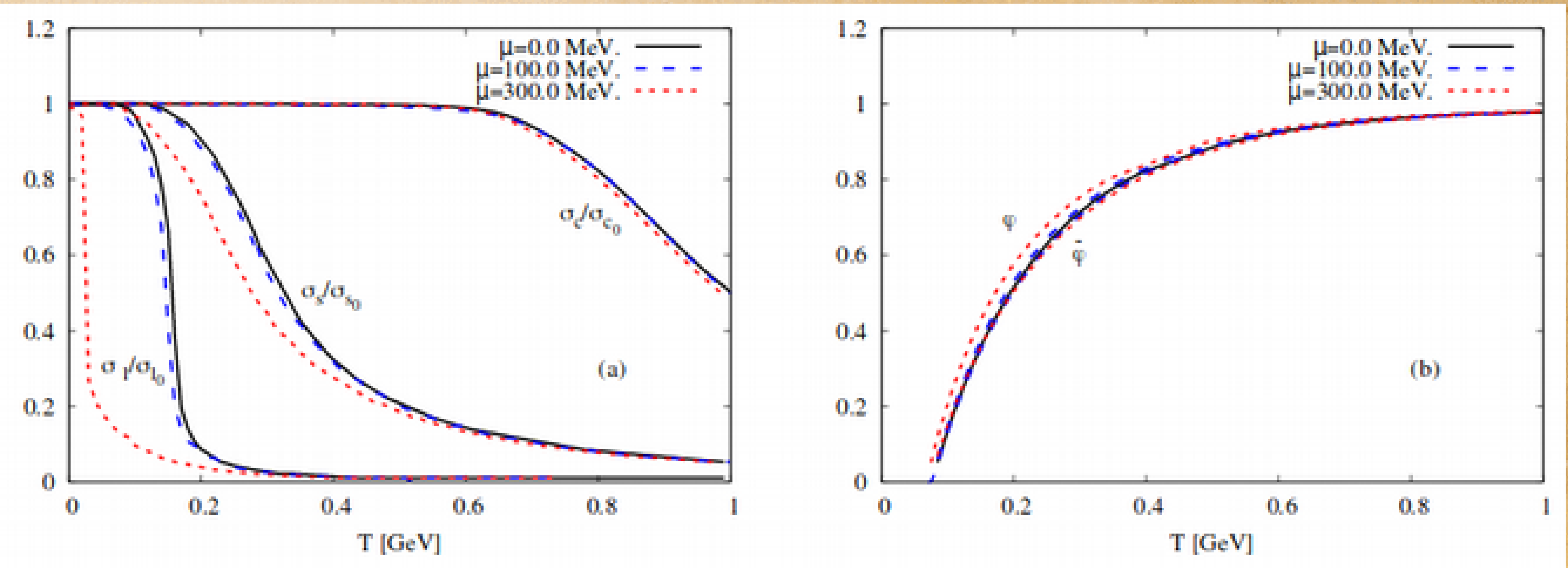
to which an c-term, $c[\text{Det}(\Phi) + \text{Det}(\Phi^\dagger)]$, is usually added, where Φ is a complex 4×4 matrix for scalar and pseudoscalar mesons σ_a and π_a, respectively, $\Phi = T_a(\sigma_a + i\pi_a)$, with $T_a = \lambda_a/2$ with $a = 0, \cdots, N_f^2 - 1$ are the generators of $U(4)$ symmetry group λ_a are Gell-Mann matrices, while $\lambda_0 = \hat{\mathbf{1}}/\sqrt{2}$

The values of the vacuum chiral-condensates are determined from pion, kaon and D-meson decay widths by means of the partially conserved axial-vector current relation (PCAC) [9, 13]. At $T = 0$, the quark condensates reads $\sigma_{l_0} = f_\pi = 92.4$ MeV, $\sigma_{s_0} = (2f_K - f_\pi)/\sqrt{2} = 94.5$ MeV and $\sigma_{c_0} = (2f_D - f_\pi)/\sqrt{2} = 293.87$ MeV.

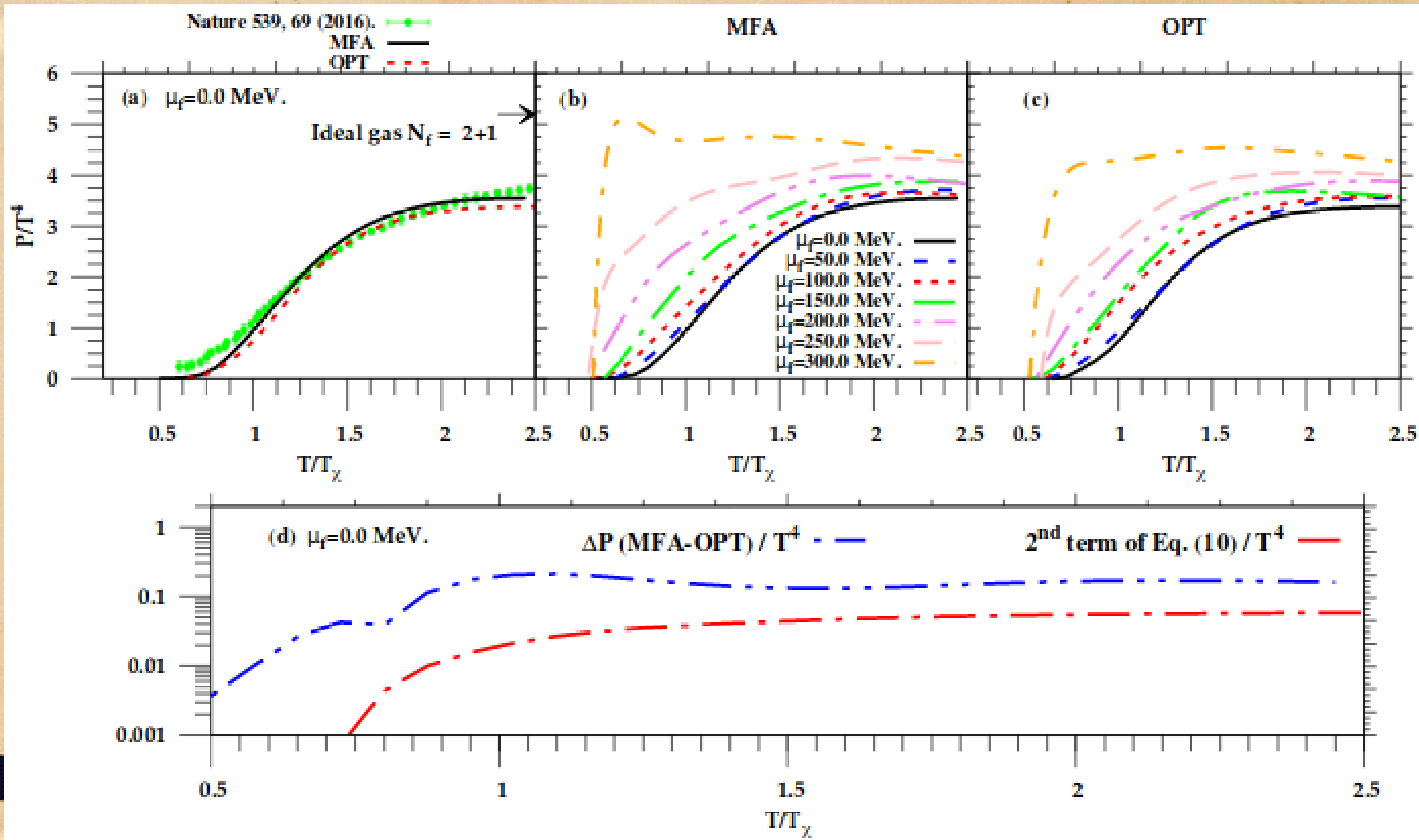
$$\Omega(T, \mu) = \Omega_{\bar{q}q}(T, \mu) + \mathscr{U}_{\text{PolyLog}}(\varphi, \varphi^*, T) + U(\sigma_l, \sigma_s, \sigma_c) = -T \ln \mathscr{Z}/V$$

PLSM: SU(4) Thermodynamics

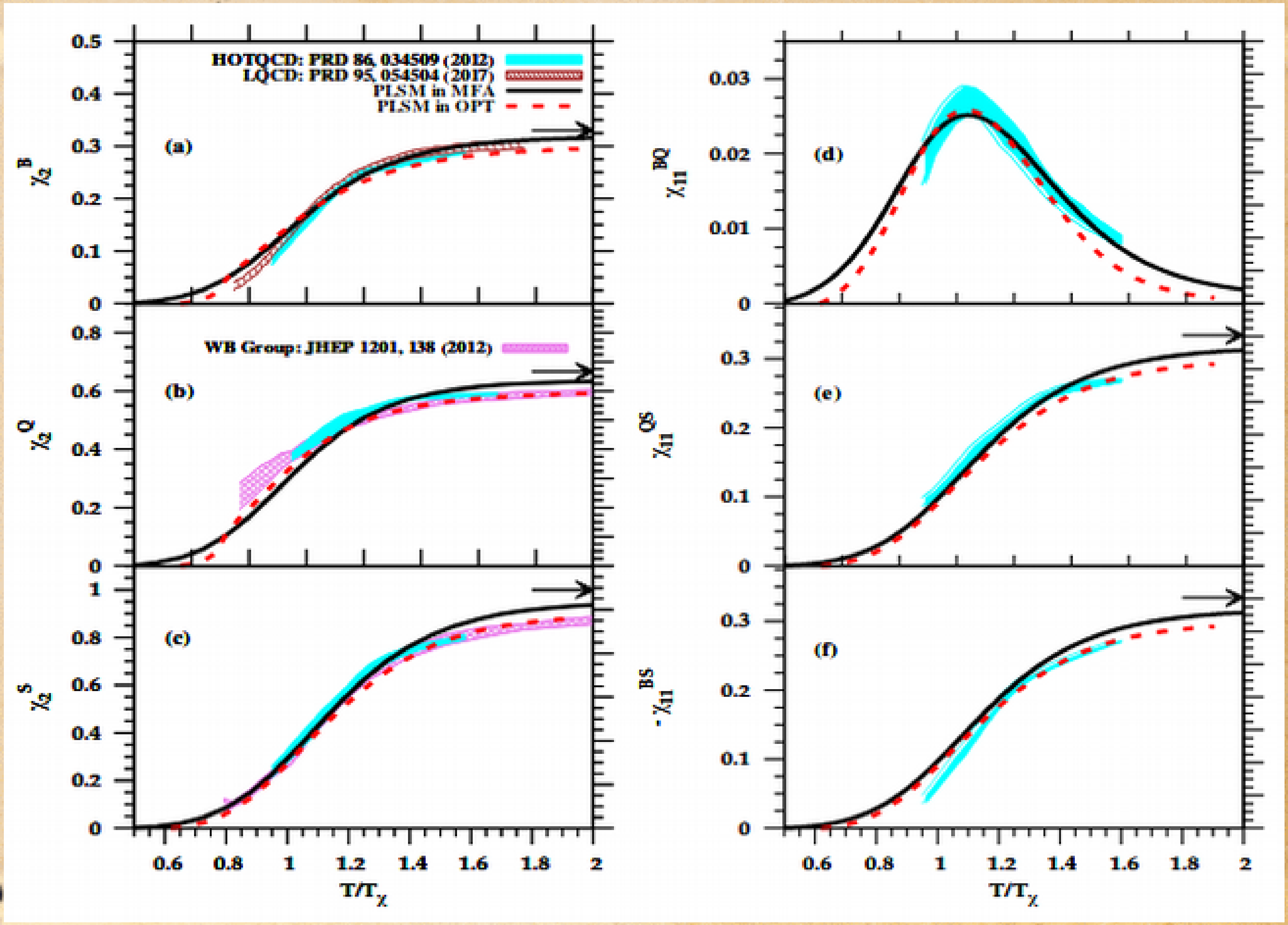




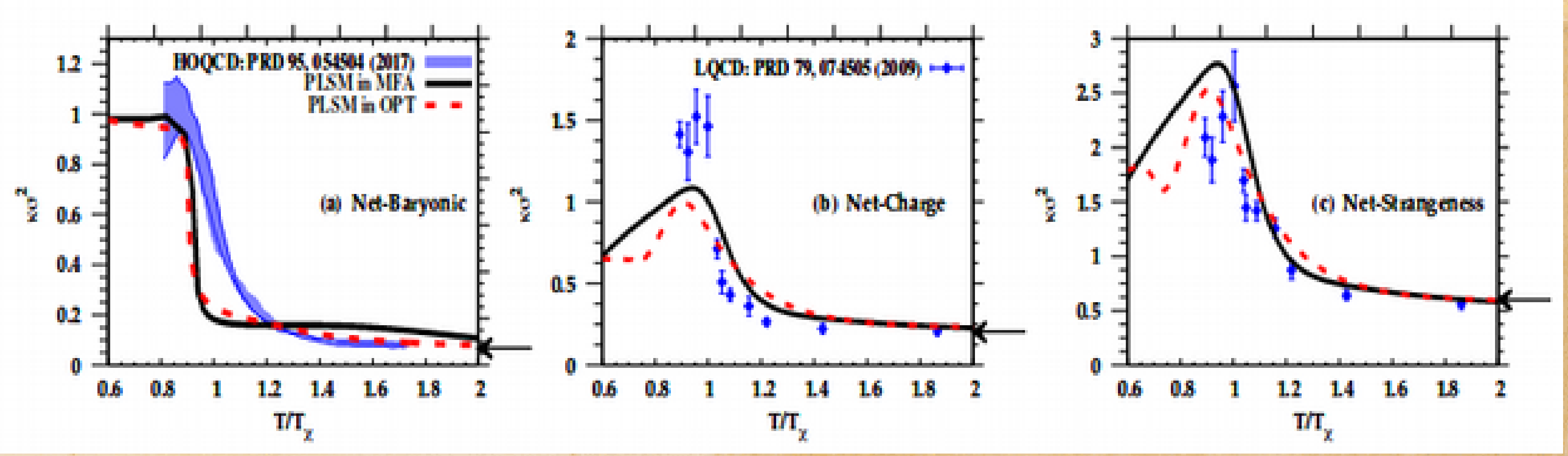
PLSM: SU(3) Thermodynamics

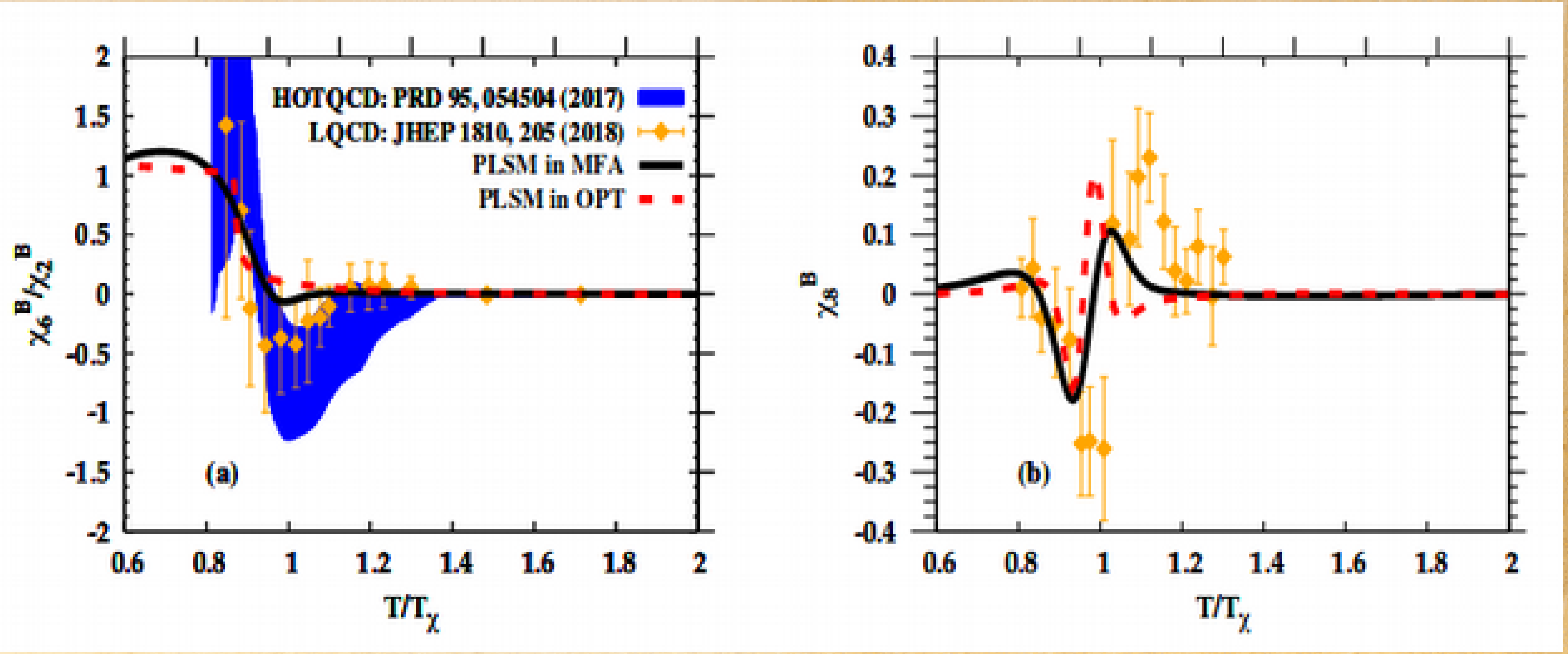


PLSM: SU(3) Thermodynamics



PLSM: SU(3) Thermodynamics





PLSM: SU(3) at finite Isospin-Asymmetry

- The explicit symmetry breaking, H = Ta ha, where ha are nine parameters of the explicit symmetry breaking in SU (3).
- As a result, the diagonal components of the symmetry generators h0, h3, h8 are finite.
- In addition, the mesonic field Φ is a (3 × 3) matrix for meson states.

$$\bar{\Phi} = \sum_{a=0}^{N_f^2-1} T_a(\bar{\sigma_a} + i\bar{\pi}_a)$$

- where σ_a and π_a are scalar and pseudo-scalar fields, respectively.
- In the vacuum state with U (1) anomaly and due to the spontaneous symmetry breaking, the expected values of the mesonic fields <Φ> and their conjugates with the quantum numbers of the vacuum are generated.

This leads to an exact vanishing mean value of π_a and to ensure the finite mean value of σ_a, which corresponds to the diagonal generators U (3) as, $\bar{\sigma_0} \neq \bar{\sigma_3} \neq \bar{\sigma_8} \neq 0$ **where** $\langle \Phi \rangle = T_0\bar{\sigma_0} + T_3\bar{\sigma_3} + T_8\bar{\sigma_8}$

PLSM: SU(3) at finite Isospin-Asymmetry

- **On the other hand, σ_3 breaks the isospin asymmetry SU (2) and the potential for purely mesonic contributions in SU (Nf) can be written as**

J. T. Lenaghan, D. H. Rischke, and J. Schaffner-Bielich, Phys.Rev.D62, 085008 (2000).

$$U(\bar{\sigma}) = \left(\frac{m^2}{2} - h_a\right)\bar{\sigma}_a - 3\mathcal{G}_{abc}\bar{\sigma}_b\,\bar{\sigma}_c - \frac{4}{3}\mathcal{F}_{abcd}\,\bar{\sigma}_b\,\bar{\sigma}_c\bar{\sigma}_d,$$

- **where the coefficients are given as**

$$\mathcal{G}_{abc} = \frac{c}{6}\left[d_{abc} - \frac{3}{2}\left(d_{0bc}\delta_{a0} + d_{a0c}\delta_{b0} + d_{ab0}\delta_{c0}\right) + \frac{9}{2}d_{000}\delta_{a0}\delta_{b0}\delta_{c0}\right],$$

$$\mathcal{F}_{abcd} = \frac{\lambda_1}{4}\left[\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{cd} + \delta_{ac}\delta_{bd}\right] + \frac{\lambda_2}{8}\left[d_{abn}d_{ncd} + d_{adn}d_{nbc} + d_{acn}d_{nbd}\right].$$

- **The explicit symmetry breaking terms h_0, h_3 und h_8 can be determined by minimizing the potential to tree level.** $\partial U(\bar{\sigma})/\partial\bar{\sigma}_a = 0$

- **For example: h_0 and h_8 can be determined from the partially conserved axial current relationships (PCAC)**

$$h_0 = \frac{1}{\sqrt{6}}\left(m_\pi^2 f_\pi + 2m_K^2 f_K\right),$$

$$h_8 = \frac{2}{\sqrt{3}}\left(m_\pi^2 f_\pi - m_K^2 f_K\right).$$

PLSM:

The generator operator $\hat{T}_a = \hat{\lambda}_a/2$ in U(3) is a obtained from Gell-Mann matrices $\hat{\lambda}_a$ with the indices running as $a = 0, \cdots, 8$. From U(3) algebra, we have

$$\begin{aligned}\left[\hat{T}_a, \hat{T}_b\right] &= i f_{abc} \hat{T}_c, \\ \left\{\hat{T}_a, \hat{T}_b\right\} &= i d_{abc} \hat{T}_c,\end{aligned}$$

where f_{abc} and d_{abc} are the standard antisymmetric and symmetric structure constants of SU(3), respectively. The symmetric structure constant d_{abc} can be defined as

$$\begin{aligned}d_{abc} &= \frac{1}{4} Tr\left[\left\{\hat{\lambda}_a, \hat{\lambda}_b\right\} \hat{\lambda}_c\right], \\ d_{ab0} &= \sqrt{\frac{2}{3}}\, \delta_{ab}.\end{aligned}$$

In PCAC relation, the decay constant f_a is related to the symmetric structure constant as

$$f_a = d_{aab} \bar{\sigma}_a.$$

For h_0 and h_8

Accordingly, the decay constants of the charged and neutral pion mesons ($f_{\pi^\pm} = f_1$, $f_{\pi^0} = f_3$) and kaon meson ($f_{K^\pm} = f_4$, $f_{K^0} = f_6$) are given as

$$\begin{aligned}f_{\pi^0} &= f_{\pi^\pm} = \sqrt{\frac{2}{3}} \bar{\sigma}_0 + \frac{1}{\sqrt{3}} \bar{\sigma}_8, \\ f_{K^\pm} &= \sqrt{\frac{2}{3}} \bar{\sigma}_0 + \frac{1}{2} \bar{\sigma}_3 - \frac{1}{2\sqrt{3}} \bar{\sigma}_8, \\ f_{K^0} &= \sqrt{\frac{2}{3}} \bar{\sigma}_0 - \frac{1}{2} \bar{\sigma}_3 - \frac{1}{2\sqrt{3}} \bar{\sigma}_8,\end{aligned}$$

where the isospin sigma field, $\bar{\sigma}_3$, is the difference between the decay constants of neutral and charged kaon mesons as,

$$\bar{\sigma}_3 = f_{K^\pm} - f_{K^0}.$$

From the experimental and recent lattice review on physical constants $f_{\pi^\pm} = f_{\pi^0} = 92.4$ MeV and $f_{K^\pm} = 113$ MeV, $f_{K^0} = 113.453$ MeV.

PLSM: SU(3) at finite Isospin-Asymmetry

From this, the explicit symmetry breaking expression h_3 can be derived from $\partial U(\bar{\sigma})/\partial \bar{\sigma}_a = 0$

Phys.Rev.D62, 085008 (2000).

$$h_3 = \left[m^2 + \frac{c}{\sqrt{6}}\bar{\sigma}_0 - \frac{c}{\sqrt{3}}\bar{\sigma}_8 + \lambda_1\left(\bar{\sigma}_0{}^2 + \bar{\sigma}_3{}^2 + \bar{\sigma}_8{}^2\right) + \lambda_2\left(\bar{\sigma}_0{}^2 + \frac{\bar{\sigma}_3{}^2}{2} + \frac{\bar{\sigma}_8{}^2}{2} + \sqrt{2}\bar{\sigma}_0\bar{\sigma}_8\right)\right]\bar{\sigma}_3,$$

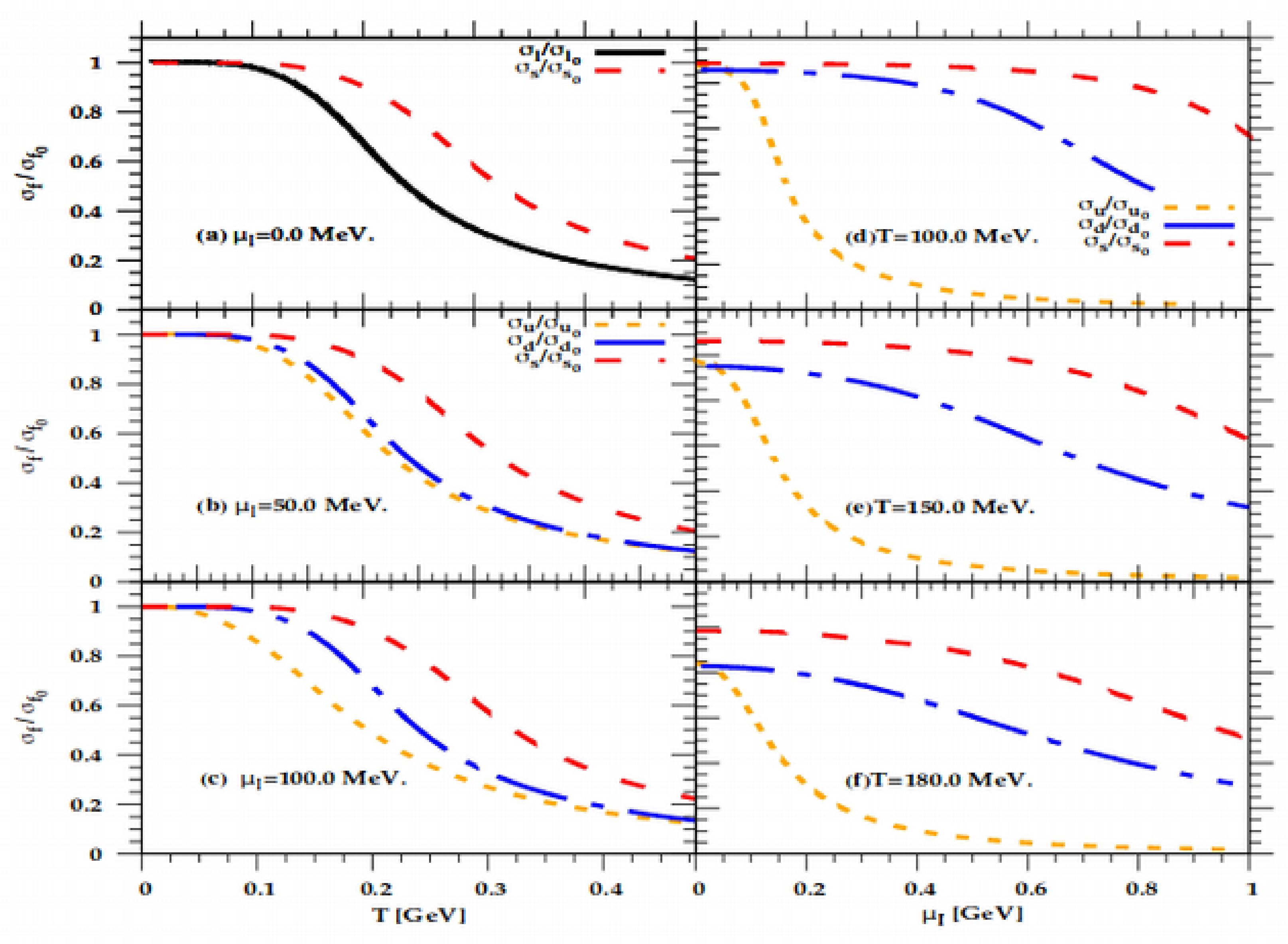
- **where** $\bar{\sigma}_3 = (f_{K^\pm} - f_{K^0})$ **and $[\ldots]^{1/2} = m_{a0}$**
- **We get** $h_3 = m_{a_0}^2 (f_{K^\pm} - f_{K^0})$
- **With orthogonal base transformation, the condensates can be converted from the original base σ_0, σ_3 and σ_8 into pure Up- (σ_u), Down- (σ_d) or strange (σ_s) quark aroma bases**

$$\begin{bmatrix}\bar{\sigma}_u\\ \bar{\sigma}_d\\ \bar{\sigma}_s\end{bmatrix} = \frac{1}{\sqrt{3}}\begin{bmatrix}\sqrt{2} & 1 & 1\\ \sqrt{2} & -1 & 1\\ 1 & 0 & -\sqrt{2}\end{bmatrix}\begin{bmatrix}\bar{\sigma}_0\\ \bar{\sigma}_3\\ \bar{\sigma}_8\end{bmatrix}$$

$$m_u = \frac{g}{2}\sigma_u, \qquad m_d = \frac{g}{2}\sigma_d, \qquad m_s = \frac{g}{\sqrt{2}}\sigma_s$$

$$\frac{\partial\Omega}{\partial\bar{\sigma}_u} = \frac{\partial\Omega}{\partial\bar{\sigma}_d} = \frac{\partial\Omega}{\partial\bar{\sigma}_s} = \frac{\partial\Omega}{\partial\bar{\phi}} = \frac{\partial\Omega}{\partial\bar{\bar{\phi}}}\bigg|_{min} = 0.$$

PLSM: SU(3) at finite Isospin-Asymmetry

- 

PLSM: SU(3) at finite Isospin-Asymmetry

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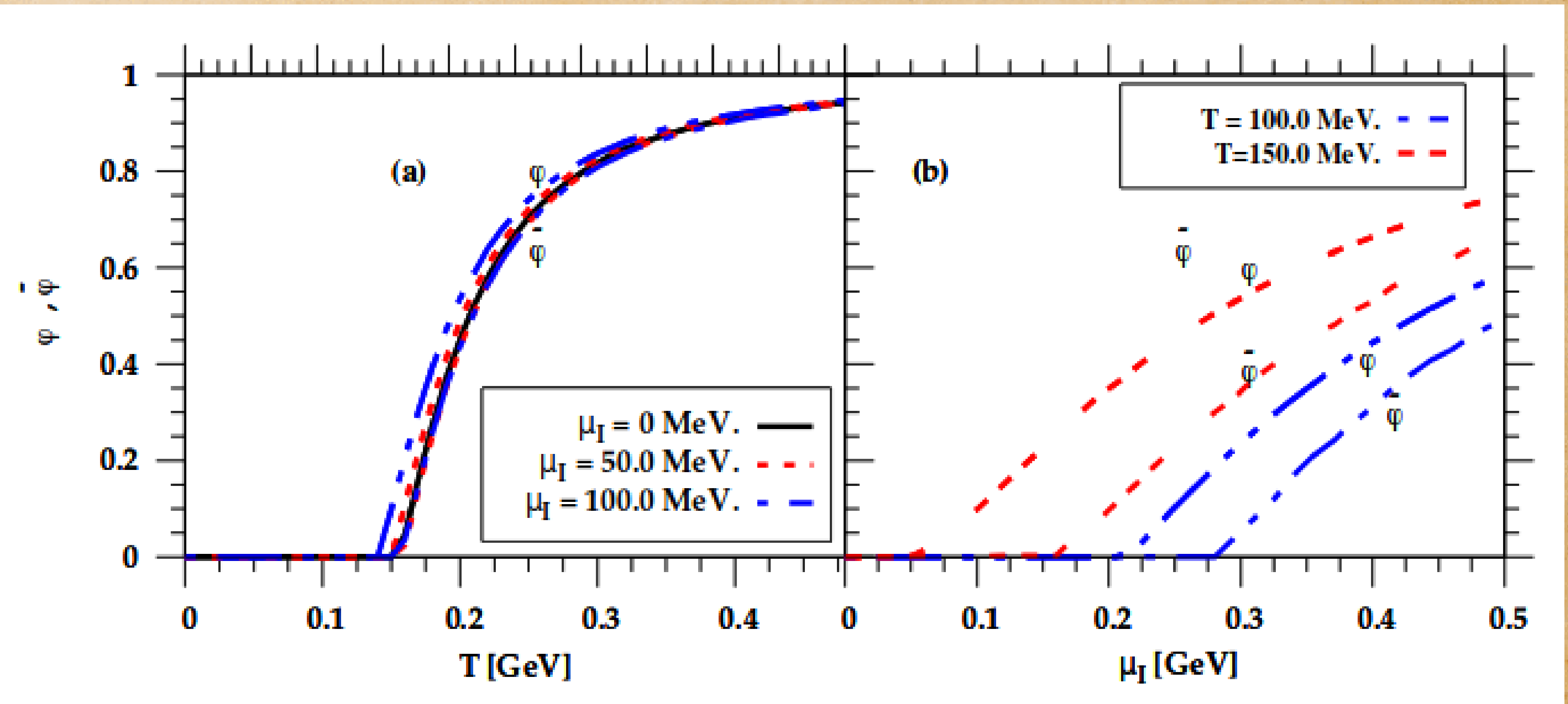

PLSM: SU(3) at finite Isospin-Asymmetry

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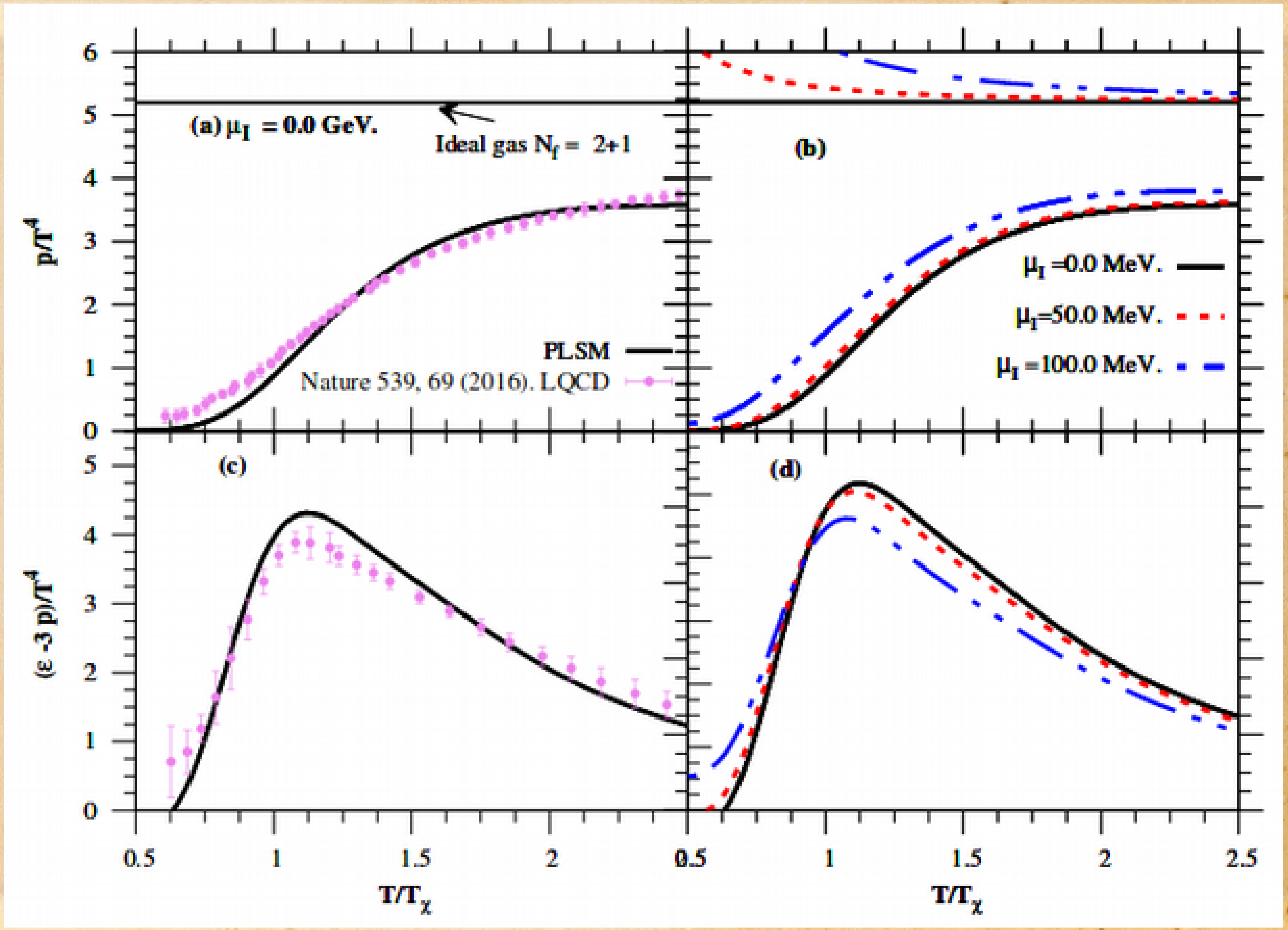

PLSM: SU(3) at finite Isospin-Asymmetry

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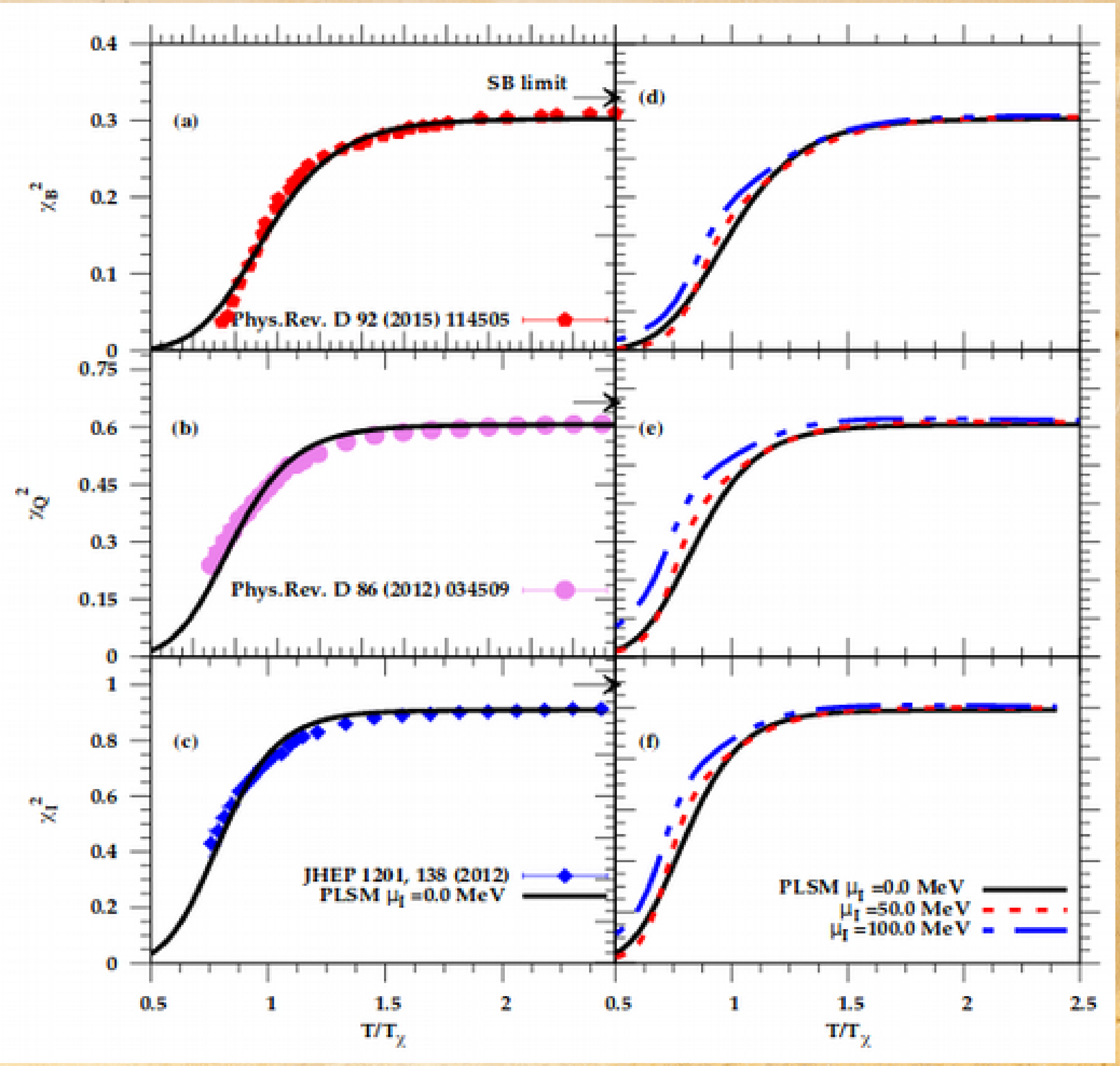

PLSM: SU(3) at finite Isospin-Asymmetry

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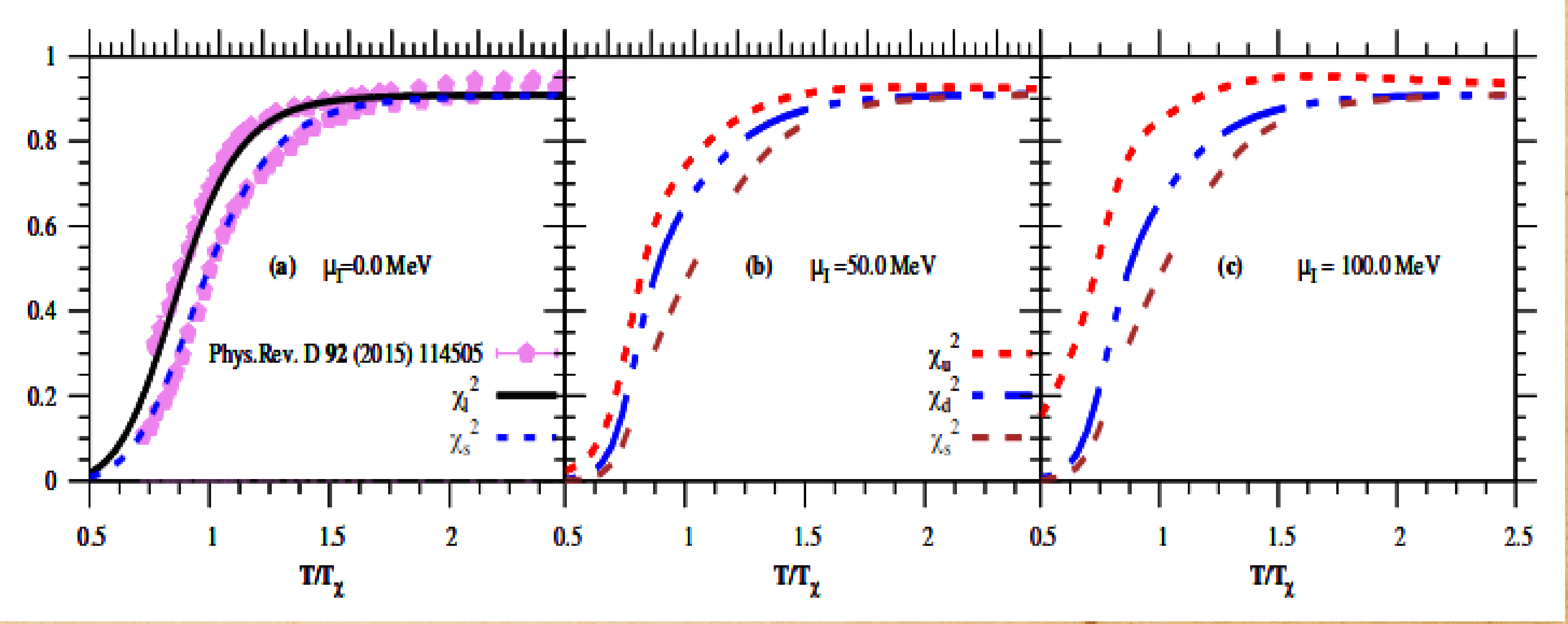
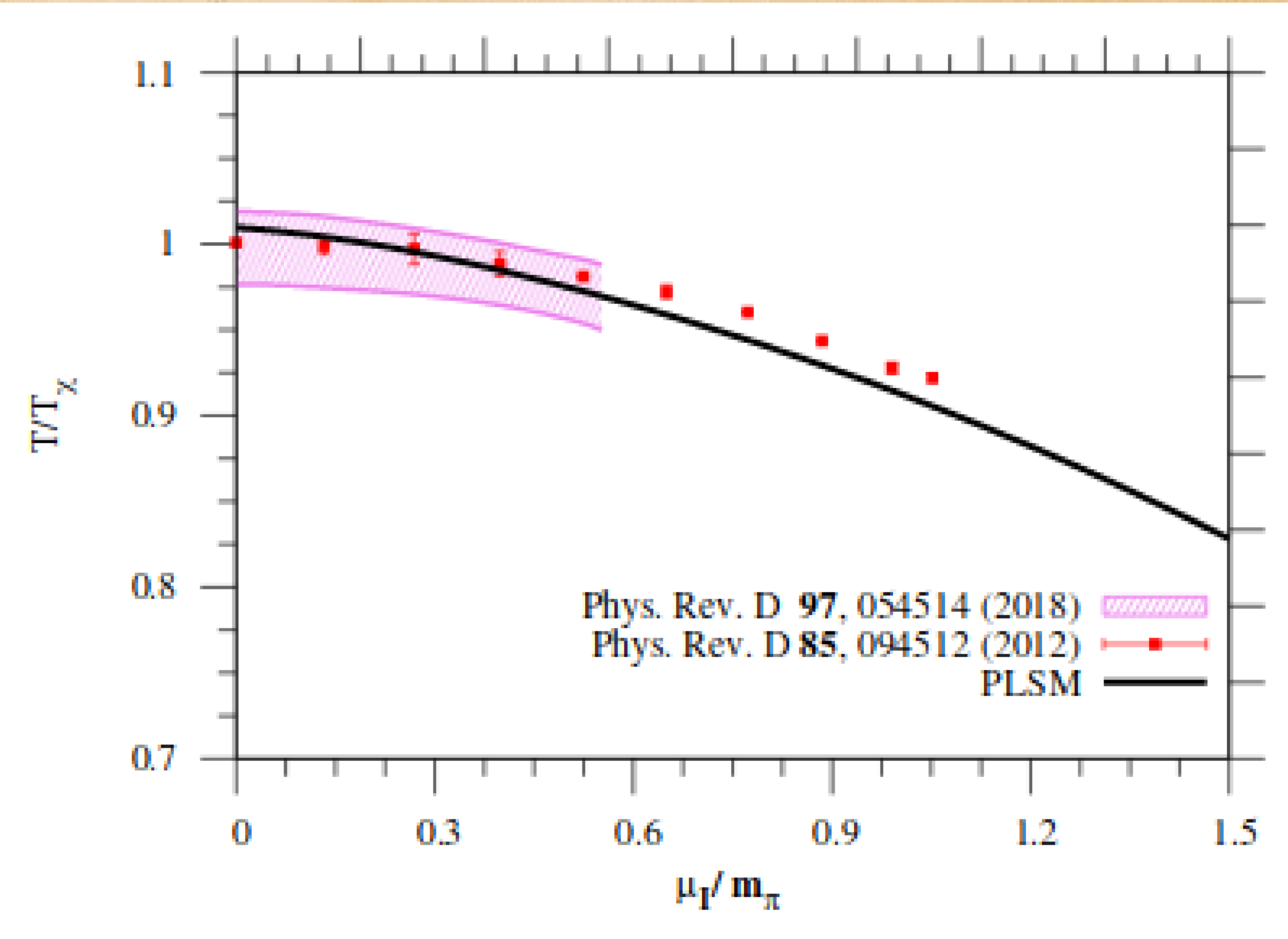


PLSM: SU(3) at finite Isospin-Asymmetry

- 

Results and Conclusions

- LSM: Mesons → Mesons + Quarks + Ployakov-loops
- PLSM: SU(2) → SU(3) → SU(4)
- PLSM: finite T, μ_b, eB and I_3
- PLSM: 16 Meson states at finite T, μ_b, eB
- PLSM: Viscosity und conductivity at finite T, μ_b, eB
- PLSM: QCD Thermodynamics and higher fluctuations
- PLSM: QCD Phase-diagram at finite μ_b, eB and I_3

Thank You!