CRC-TR 211
Strong-interaction matter under extreme conditions

UNIVERSITÄT BIELEFELD

QCD Phase diagram at non-zero real and imaginary chemical potential

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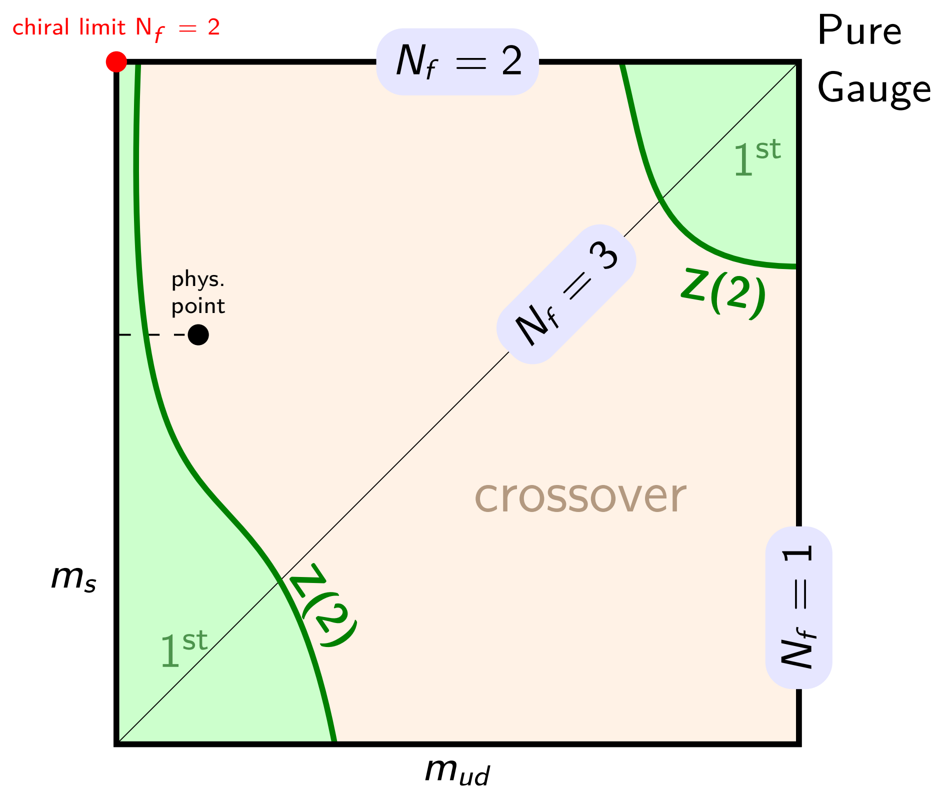
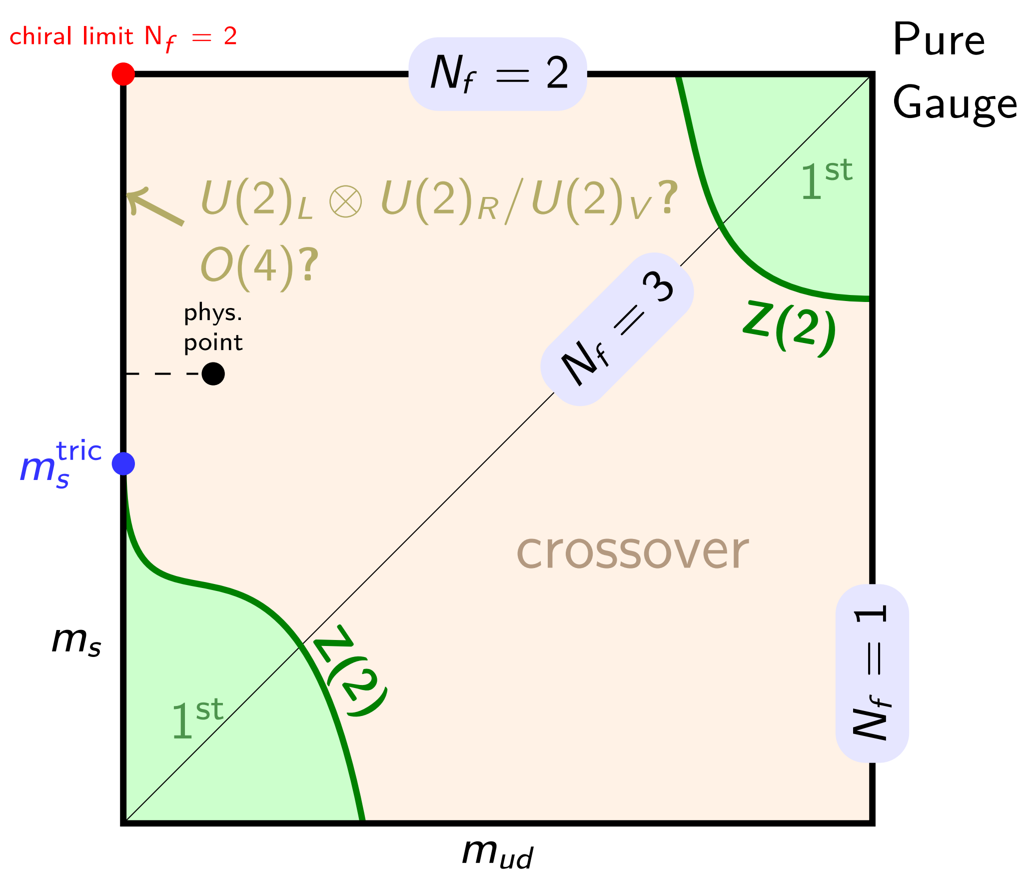
HotQCD Collaboration

XXXII International (ONLINE) Workshop on High Energy Physics
“Hot problems of Strong Interactions”, November 9-13, 2020

Transition in the chiral limit($m_u = m_d = 0$)?



Philipsen, Pinke, PRD 93 (2016)



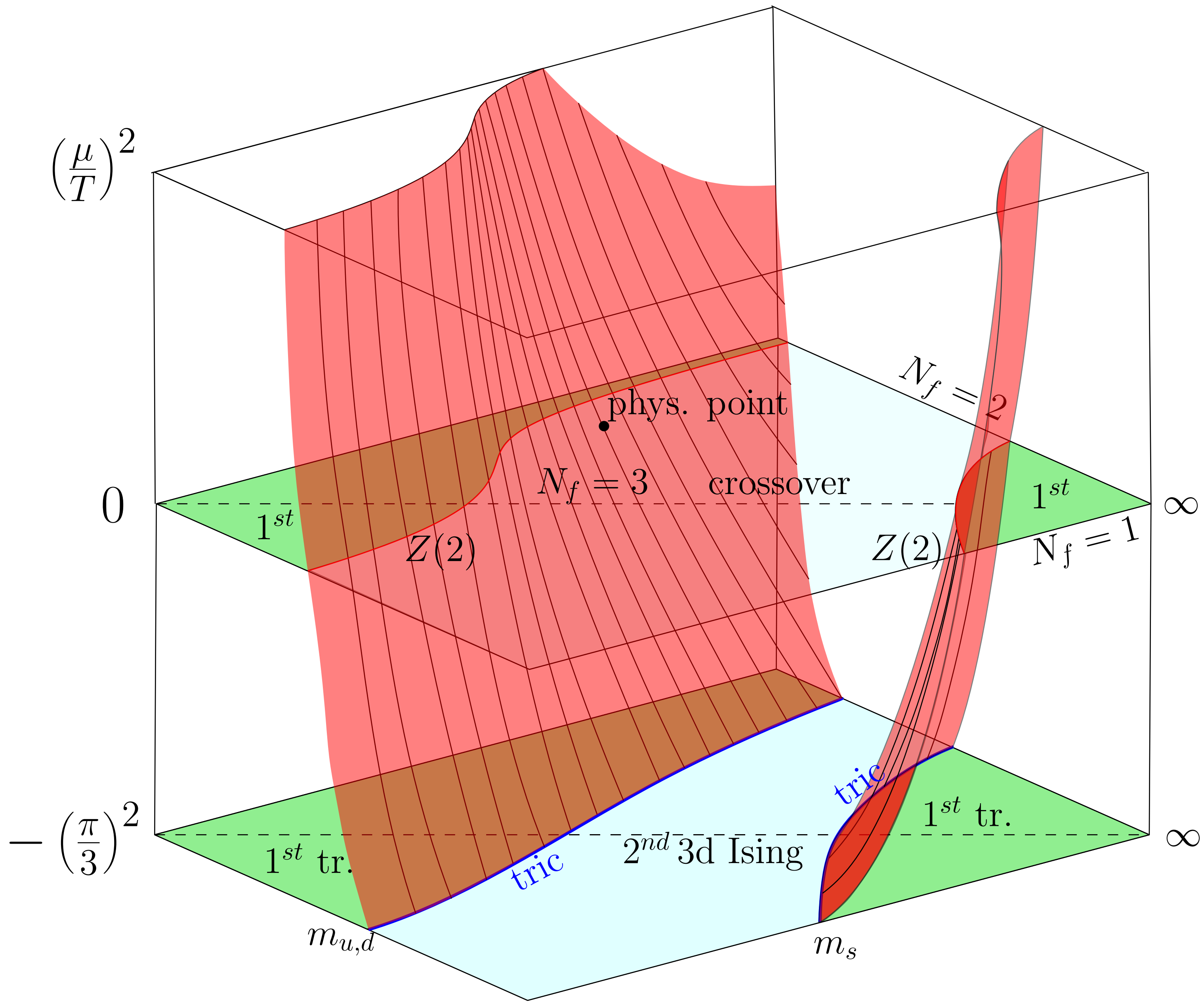
Pisarski, Wilczek, PRD 29 (1984)

Work from Pisarski and Wilczek suggests that the chiral transition ($N_f = 2 + 1, \mu = 0$) belongs to the $O(4)$ universality class if $U(1)_A$ is still broken at T_c . [MPL, AL, AGN....]

- A few Lattice actions show a first order chiral transition but…..

1. **These results are done on fixed N_τ. Not continuum extrapolated.**
2. **They use unimproved actions which have large cut-off effects, means the calculations are done away from the continuum.**
3. **The order of the transition can change if calculations are done close to continuum.**

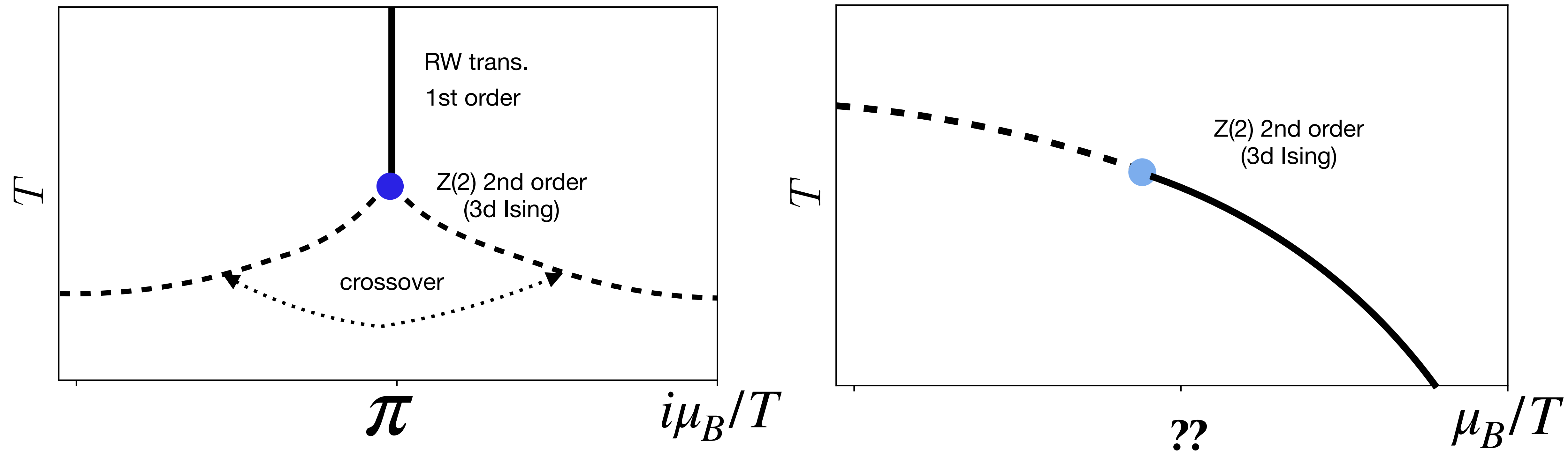
Order of the transition in the chiral limit($m_u = m_d = 0$)?



Possible Solutions:

1. **Verify with actions which allows us to do the calculations close to the continuum i.e. with improved actions.[See talk by A. Lahiri]**
2. **Simulation with imaginary chemical potential.**

Phase diagram in the $i\mu_B$ plane



Phase diagram, m_u, m_d at physical value, $m_\pi \sim 135$ MeV

Free energy and the universal functions for second order transition near the critical point can be written as,[$t \to 0, h \to 0, t = (T - T_c)/T_c$]

$$f = b^{-d} f_s(b^{y_t} t, b^{y_h} h, b^{-1} N_\sigma) + f_{ns} \,, \qquad z_f = z_0 t N_\sigma^{1/\nu}$$

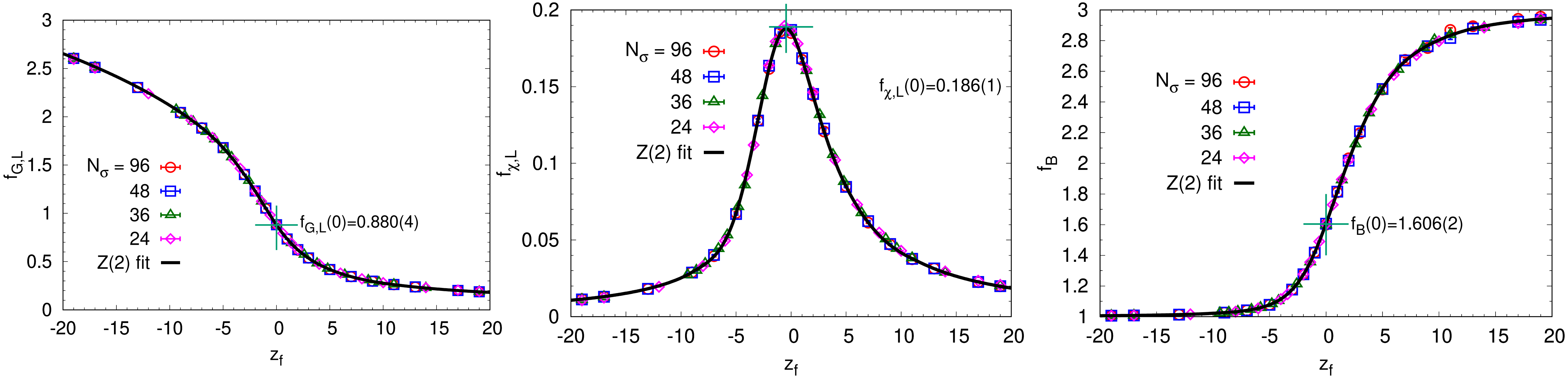
$$M = \langle |Im\ L| \rangle = \left.\frac{\partial f}{\partial h}\right|_{h\to 0} \sim N_\sigma^{-\beta/\nu} f_h(z_f)$$

$$B_4 \sim f_B(z_f)$$

$$\chi_h = \left.\frac{\partial^2 f}{\partial h^2}\right|_{h\to 0} \sim N_\sigma^{\gamma/\nu}\ f_\chi(z_f)$$

In our case β, ν, α and γ *are Z(2) critical exponents*

Scaling analysis at physical values of m_u, m_d



$$M = A_M N_\sigma^{-\beta/\nu} f_{G,L}(z_f)$$

$$\chi_M = A_M^2 N_\sigma^{\gamma/\nu} f_{\chi,L}(z_f) + \chi_{reg}(t)$$

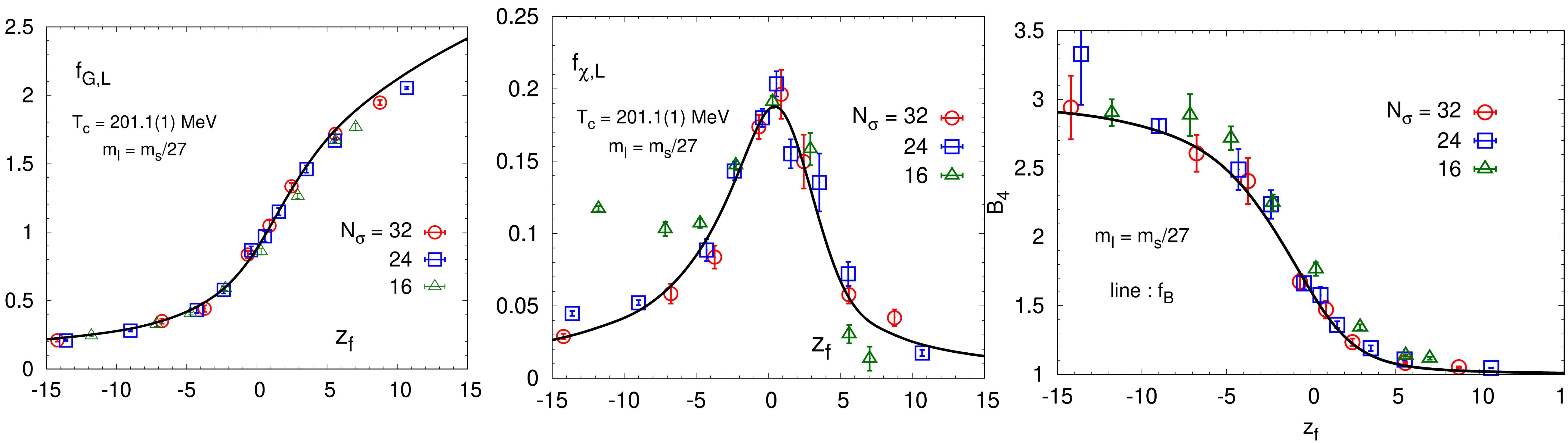
$$B_4 = f_{B,L}(z_f) + \frac{d}{N_\sigma^3}$$

3-d Ising model Universal Scaling functions

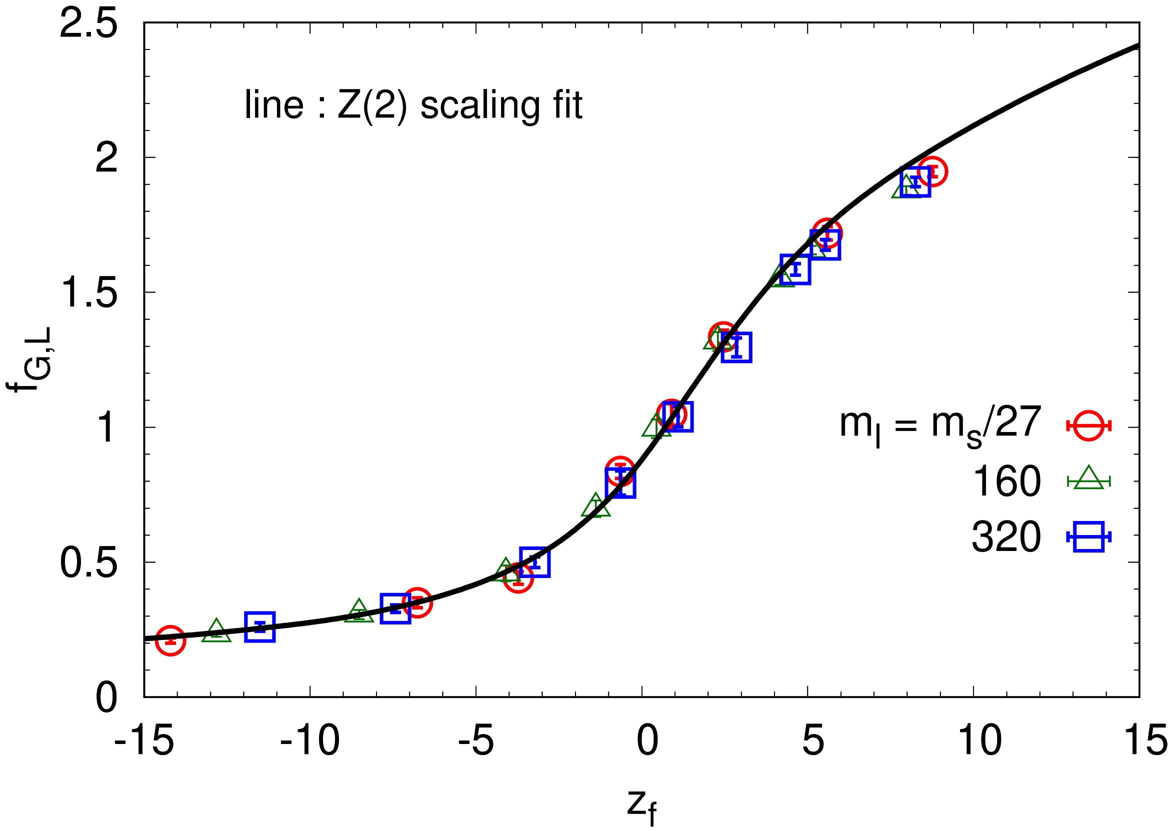
$$z_f \equiv z_0 t N_\sigma^{1/\nu}, t = (T - T_c)/T_c$$

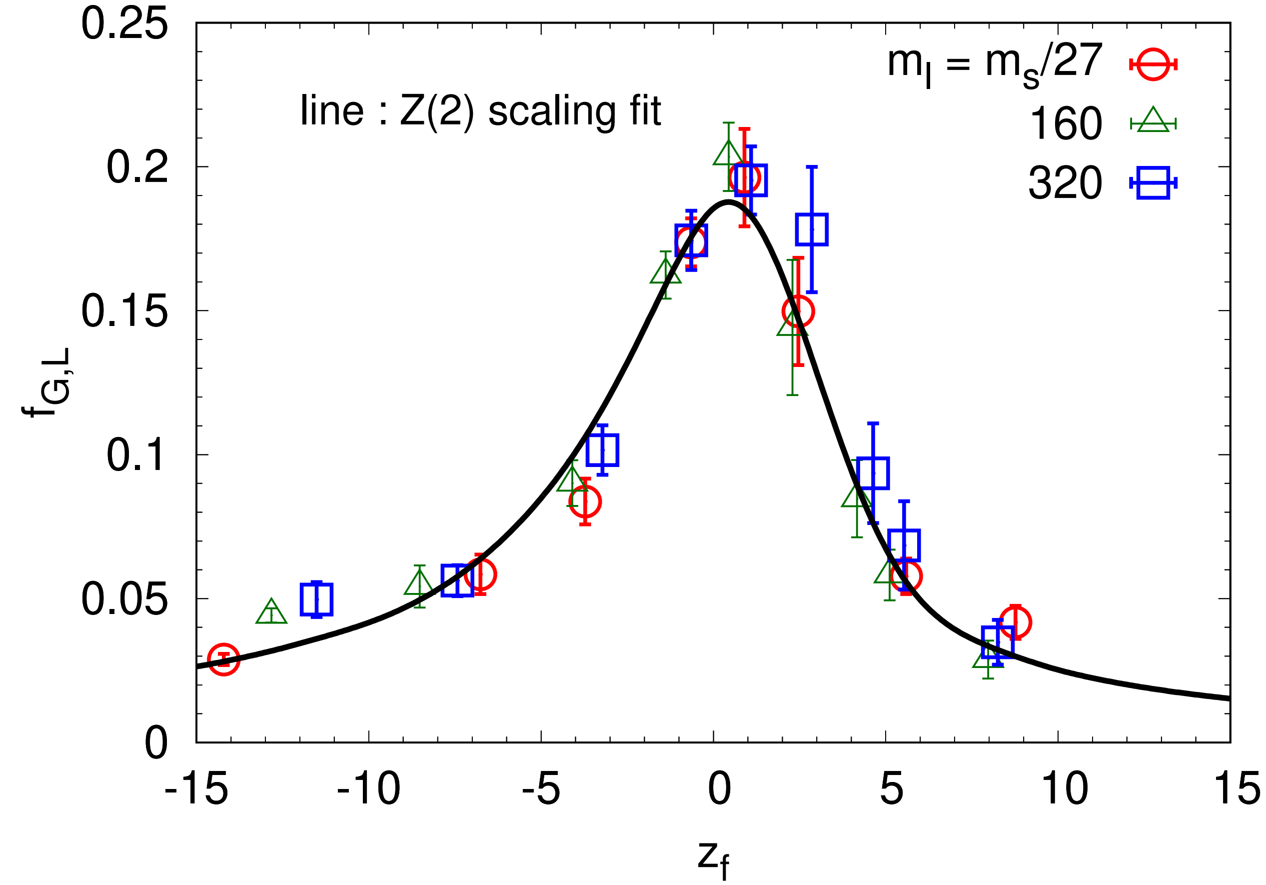
T_c, z_0, A_M **are non-universal parameters.**

Lattice Size = $N_\sigma^3 \times 4$



Fate of the RW transition in the chiral limit ($m_u, m_d \to 0$)





- **Order parameter and its susceptibility shows good agreement with the expected finite size $Z(2)$ scaling functions.**
- **RW transition remains $Z(2)$ for $m_\pi \geq 40$ MeV, No sign of first order!!**

Consistent with: Claudio Bonati,arXiv:1807.02106 [hep-lat]

Chiral phase transition in the chiral limit($m_u, m_d \to 0$)

- The definition of order parameter and its susceptibility of chiral phase transition, $\Delta_{ls} = \dfrac{2m_s}{f_k^4}(\langle\bar{\psi}\psi\rangle_l - \dfrac{m_l}{m_s}\langle\bar{\psi}\psi\rangle_s),$

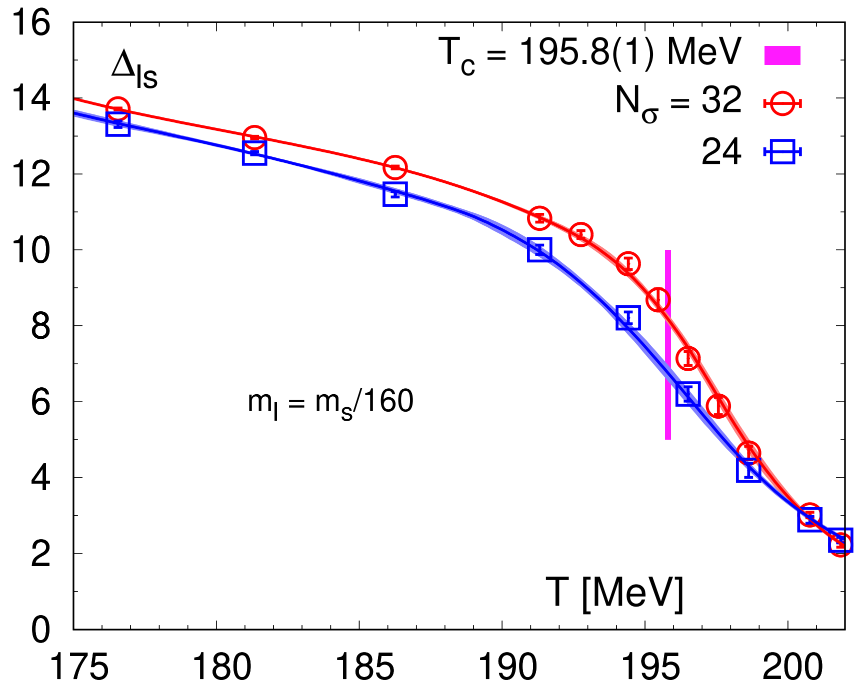
  $\langle\bar{\psi}\psi\rangle_l = (\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d)/2, \langle\bar{\psi}\psi\rangle_f = \dfrac{1}{4}\dfrac{1}{N_\sigma^3 N_\tau}\langle TrM_f^{-1}\rangle$

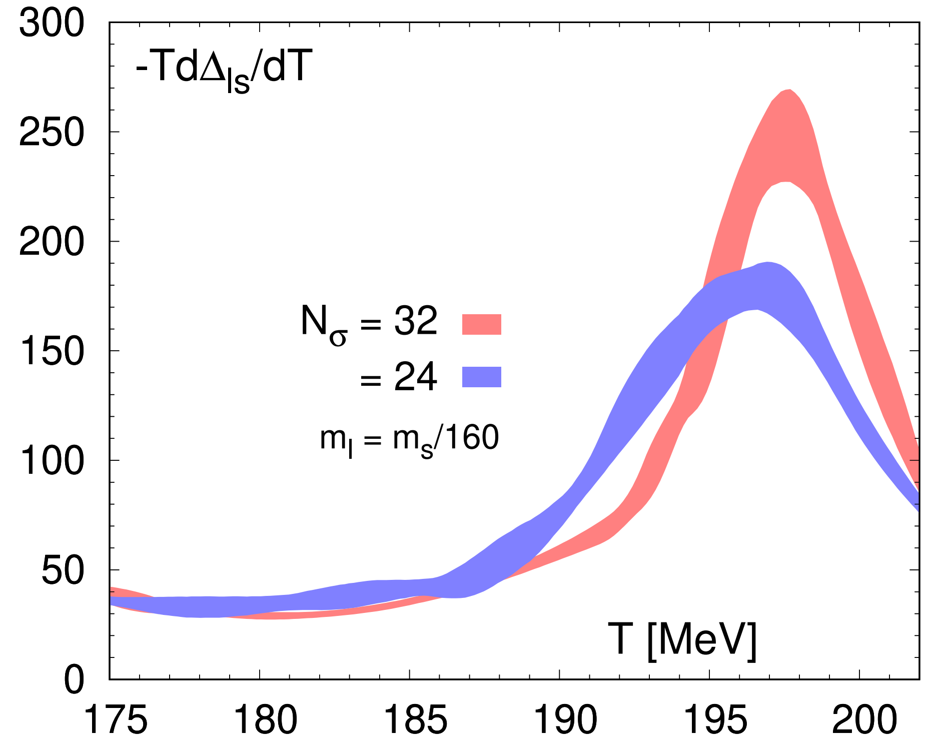
  $\chi_{dis} = \dfrac{1}{4}\dfrac{m_s^2}{N_\sigma^3 N_\tau}\left(\langle(\mathrm{Tr}M_l^{-1})^2\rangle - \langle\mathrm{Tr}M_l^{-1}\rangle^2\right)/f_K^4,$

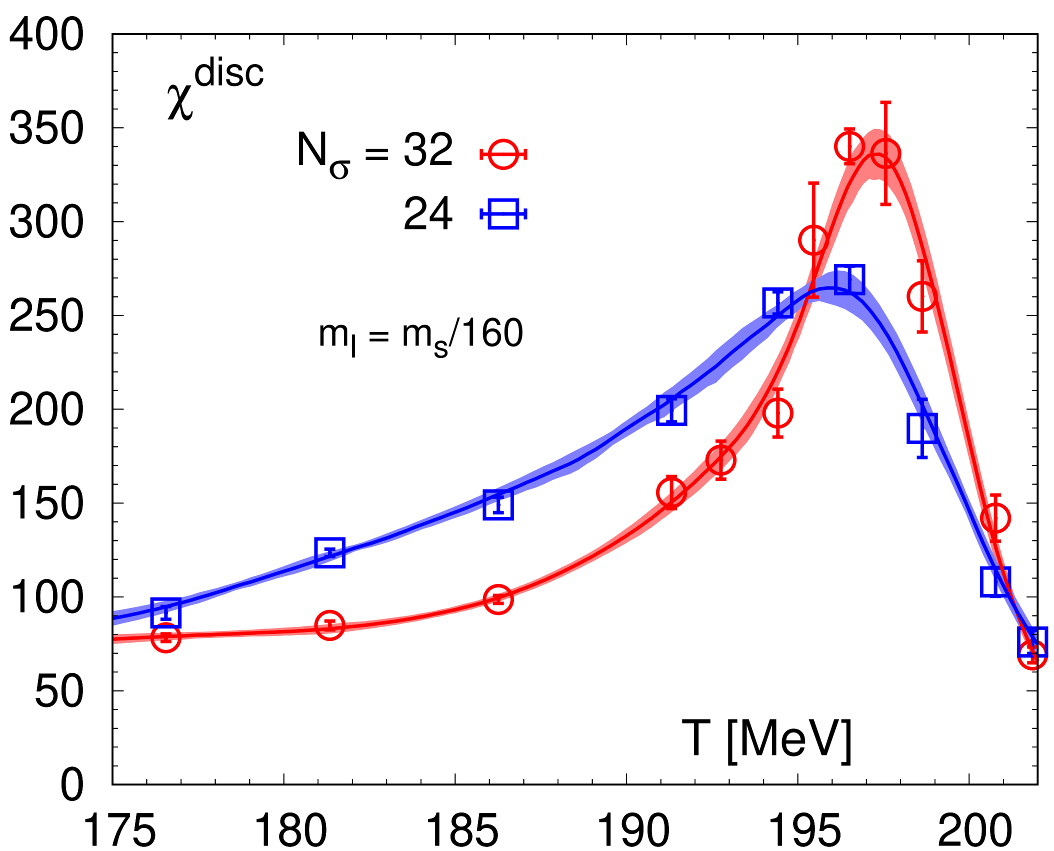
  M_f is the HISQ Dirac operator for quark flavors, $f = u, d, s$.

- For , $i\mu_B/T \neq \pi, m_u, m_d \to 0$, Δ_{ls} and χ_{disc} will follow the finite size behaviour of 3d, $O(N)$ model.

- For, $i\mu_B/T = \pi$, influence of additional $Z(2)$ for any value of m_u, m_d on the Δ_{ls} and χ_{disc} ??







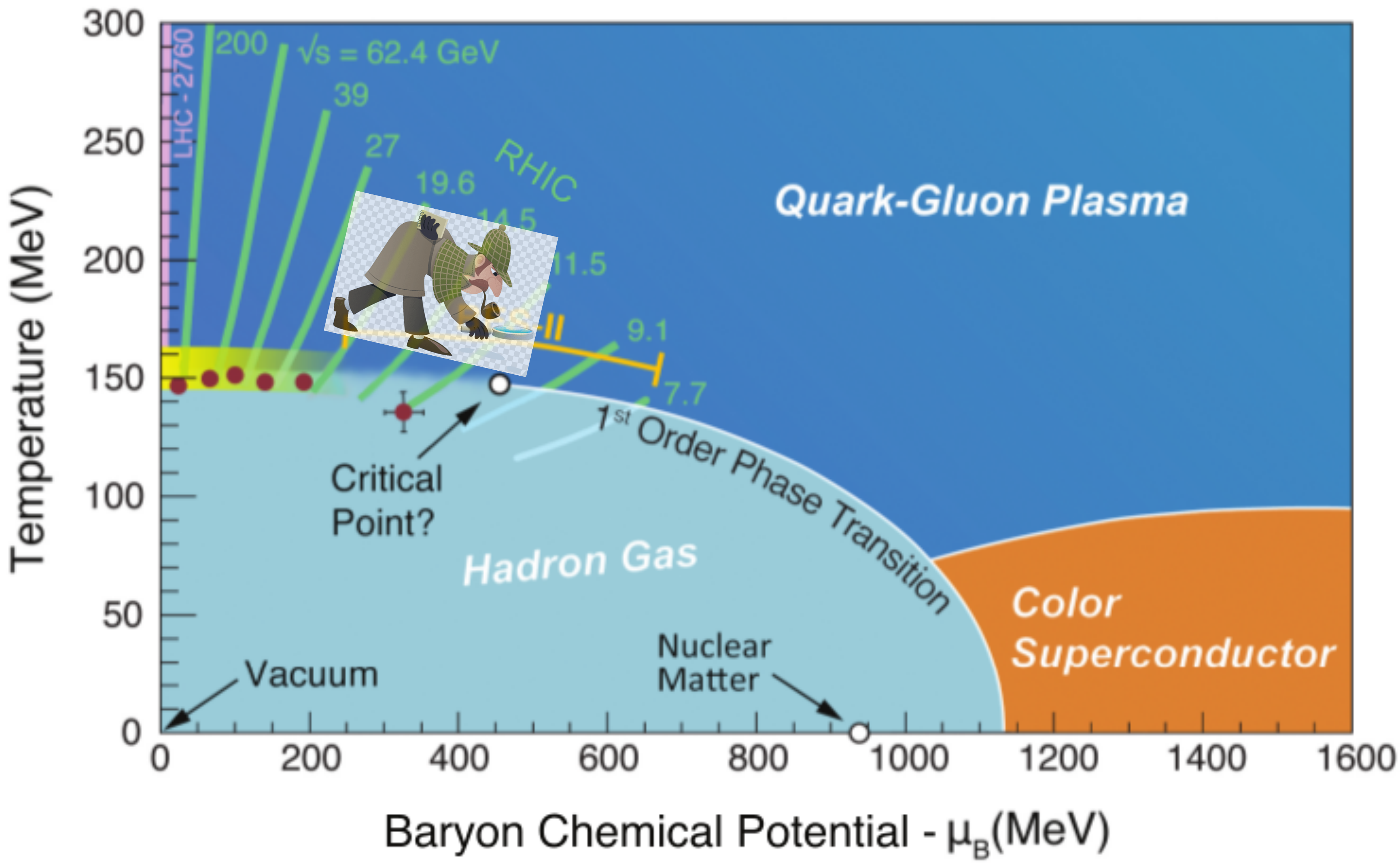
$$\chi_t = \frac{\partial^2 f}{\partial t^2}\bigg|_{h\to 0} \sim N_\sigma^{\alpha/\nu}\ f_c(z_f)$$

- Behaviour of Δ_{ls} is consistent with the order parameter in $O(N)$ models. The temperature derivative of Δ_{ls} suggests that it will have an infinite slope in the infinite volume.

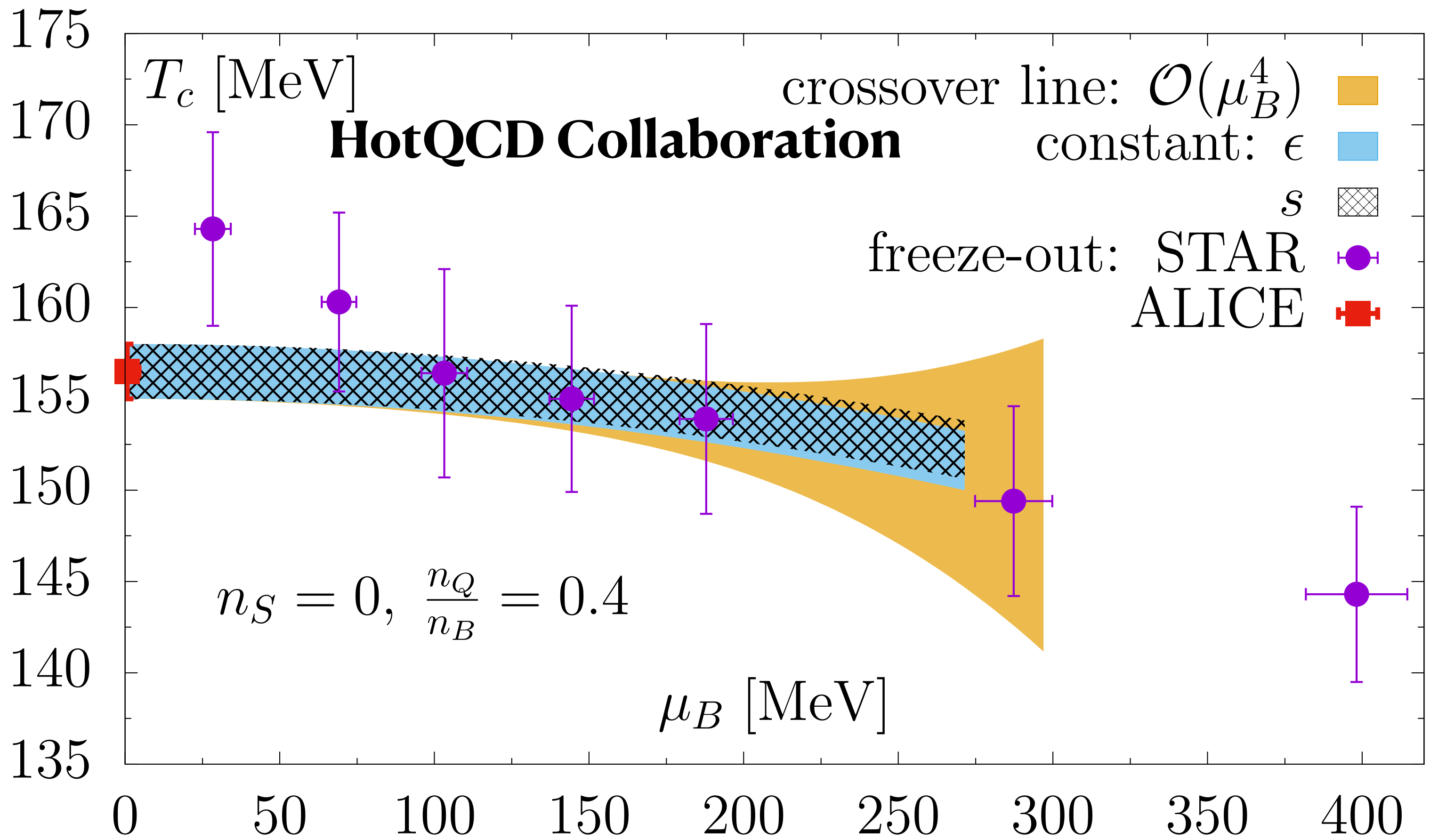
- However, volume dependence of χ_{disc} is consistent with the specific heat of $Z(2)$ transition. In that sense Δ_{ls} could also be an energy like observable for the RW transition.

Outline

Bigger Picture : Understand the thermodynamics at the QCD crossover, Study the QCD phase diagram, Indication on the location of the critical point.....



Curvature of the pseudo-critical line from LQCD



$$T_c(\mu_B) = T_{pc,0}\left[1 - \kappa_2\left(\frac{\mu_B}{T_{pc,0}}\right)^2 + \kappa_4\left(\frac{\mu_B}{T_{pc,0}}\right)^4\right]$$

$T_{pc,0} = 156.5(1.5)$ MeV $\kappa_2 = 0.12(4), \kappa_4 \sim 0.0$

$\epsilon = 0.42(6)$ GeV/fm^3 $s = 3.7(5)\mathrm{fm}^{-3}$

A. Bazavov et al, Phys.Lett.B 795 (2019) 15

- ALICE Freeze-out Temperature, $T_f = T_{pc,0}$.
- Thermodynamics of hot hadron gas at $T \leq T_{pc,0}$.
- Curvature of the pseudo-critical line is consistent with freeze-out line of STAR except for the $\sqrt{s} = 200$ GeV point.
- Location of the critical point $[\mu_B/T]_{cep} \geq 2$??

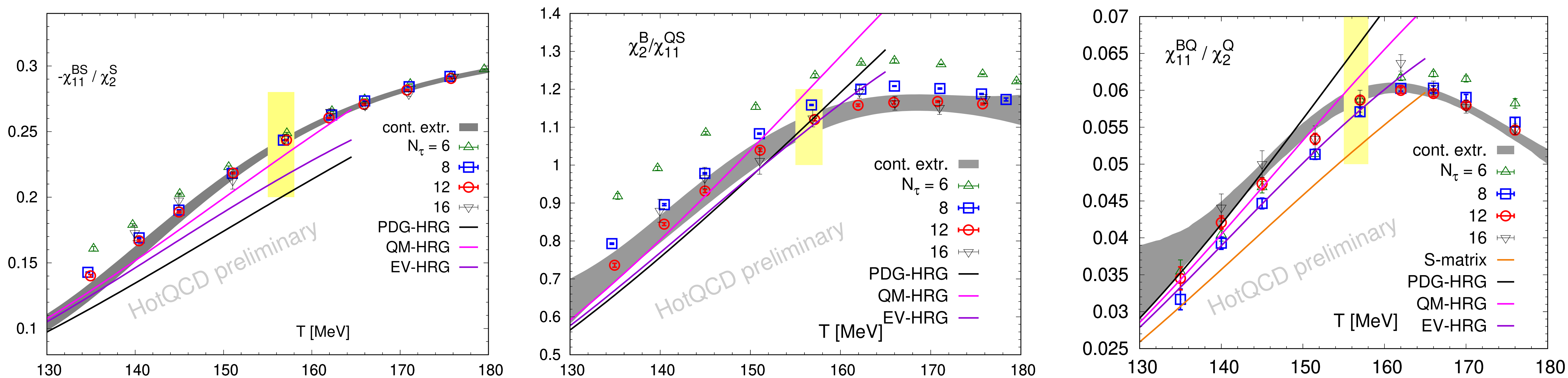
See Other talk of LQCD,DS... method for detailed comparison

LQCD and Hadron Resonance Gas

- In lattice QCD calculations, $m_u = m_d$ generates two constraints between second order cumulants,

$$\chi_2^S = 2\chi_{11}^{QS} - \chi_{11}^{BS} \;\; , \;\; \chi_2^B = 2\chi_{11}^{BQ} - \chi_{11}^{BS}$$

- HRG respects these constraints upto 1 % ,despite of the fact that $m_u \neq m_d$ reflect in the hadron masses.



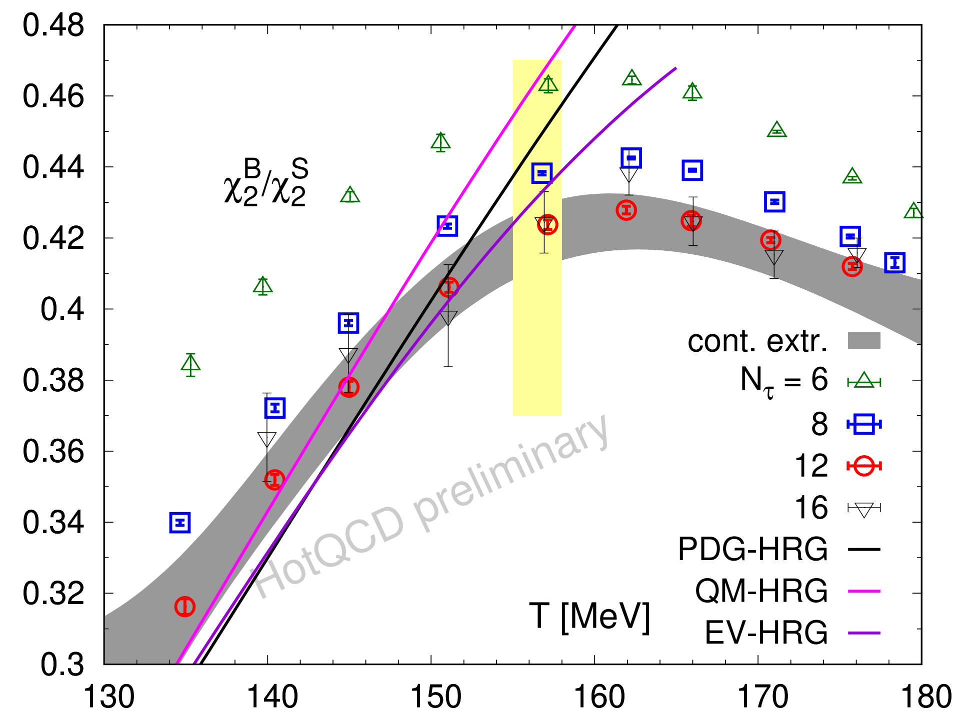
The three independent second order ratios

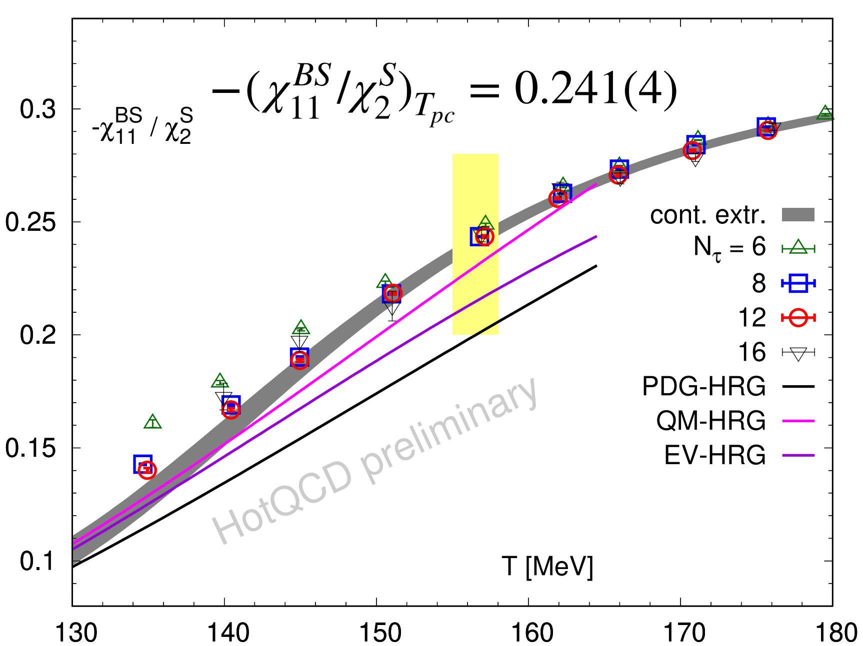
arXiv:2011.02812 [hep-lat]

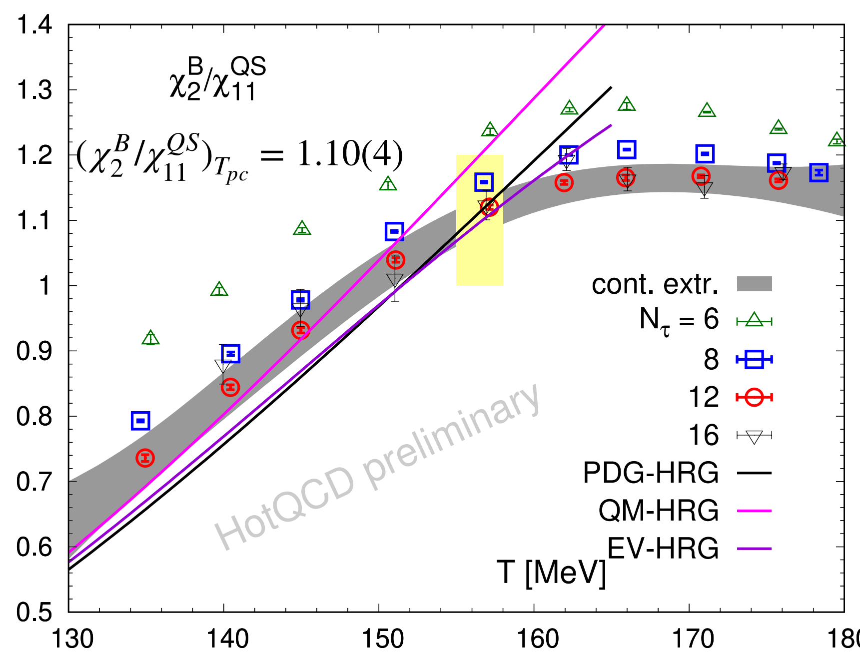
LQCD and Hadron Resonance Gas

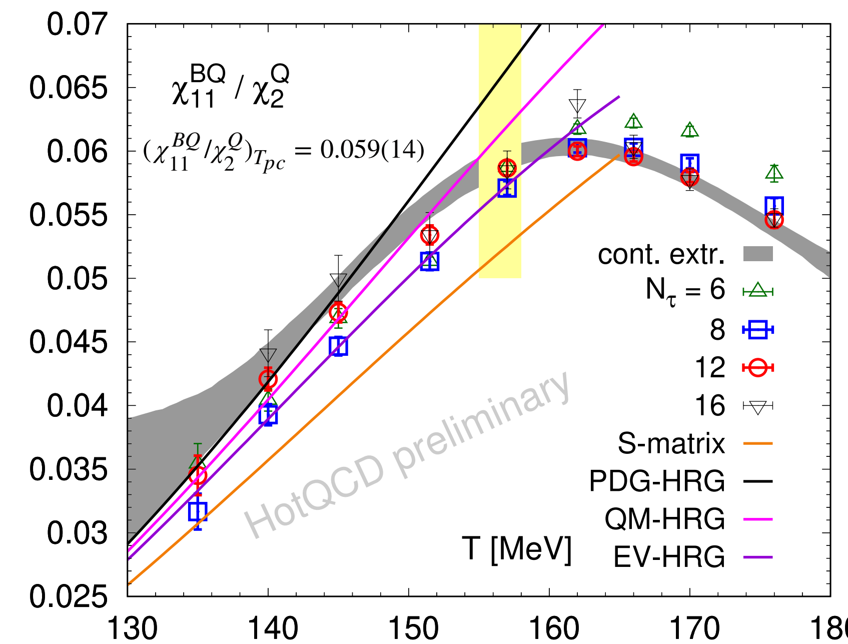
$$\frac{\chi_2^B}{\chi_2^S} = \frac{1}{2}\frac{\chi_2^B}{\chi_{11}^{QS}}\left(1 + \frac{\chi_{11}^{BS}}{\chi_2^S}\right)$$

$$(\chi_2^B/\chi_2^S)_{T_{pc}} = 0.417(18)$$









The three independent second order ratios

Any second order cumulant ratio can be obtained using these three ratios

μ_S/μ_B from Strange Baryons

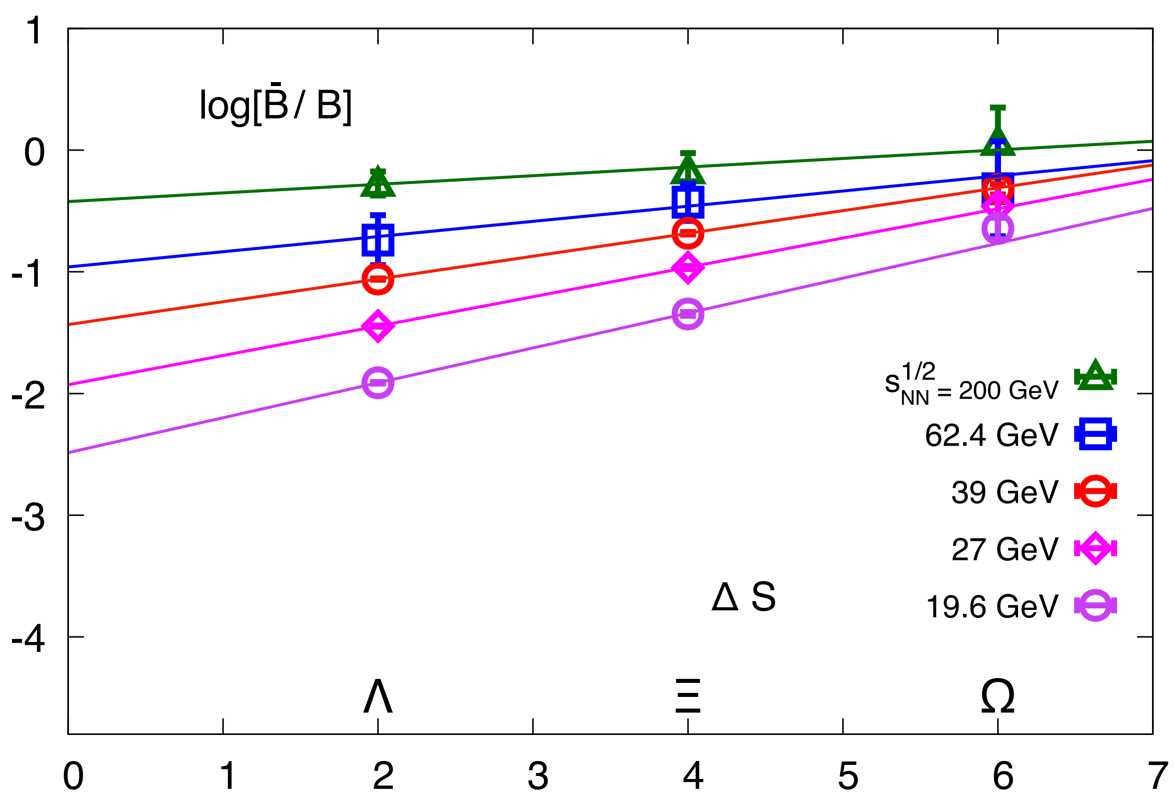
Particle yield ratios from Thermal models,

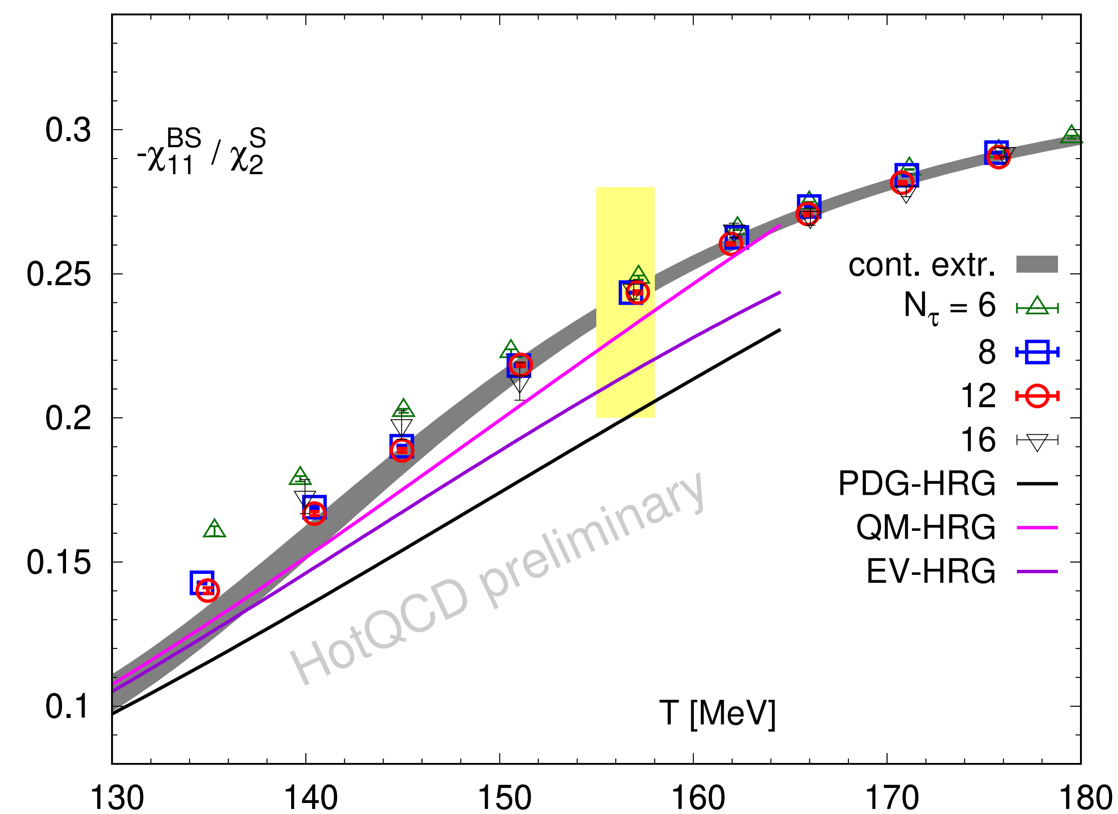
$$\log\frac{\bar{B}}{B} = -\frac{2\mu_B}{T} + \frac{\mu_S}{T}\Delta S + \frac{\mu_Q}{T}\Delta Q$$

$$\mu_Q/\mu_B \ll 1$$

$$\log\frac{\bar{B}}{B} = -\frac{2\mu_B}{T} + \frac{\mu_S}{T}\Delta S \equiv -\mu_B/T(2-\mu_s/\mu_B\Delta S)$$

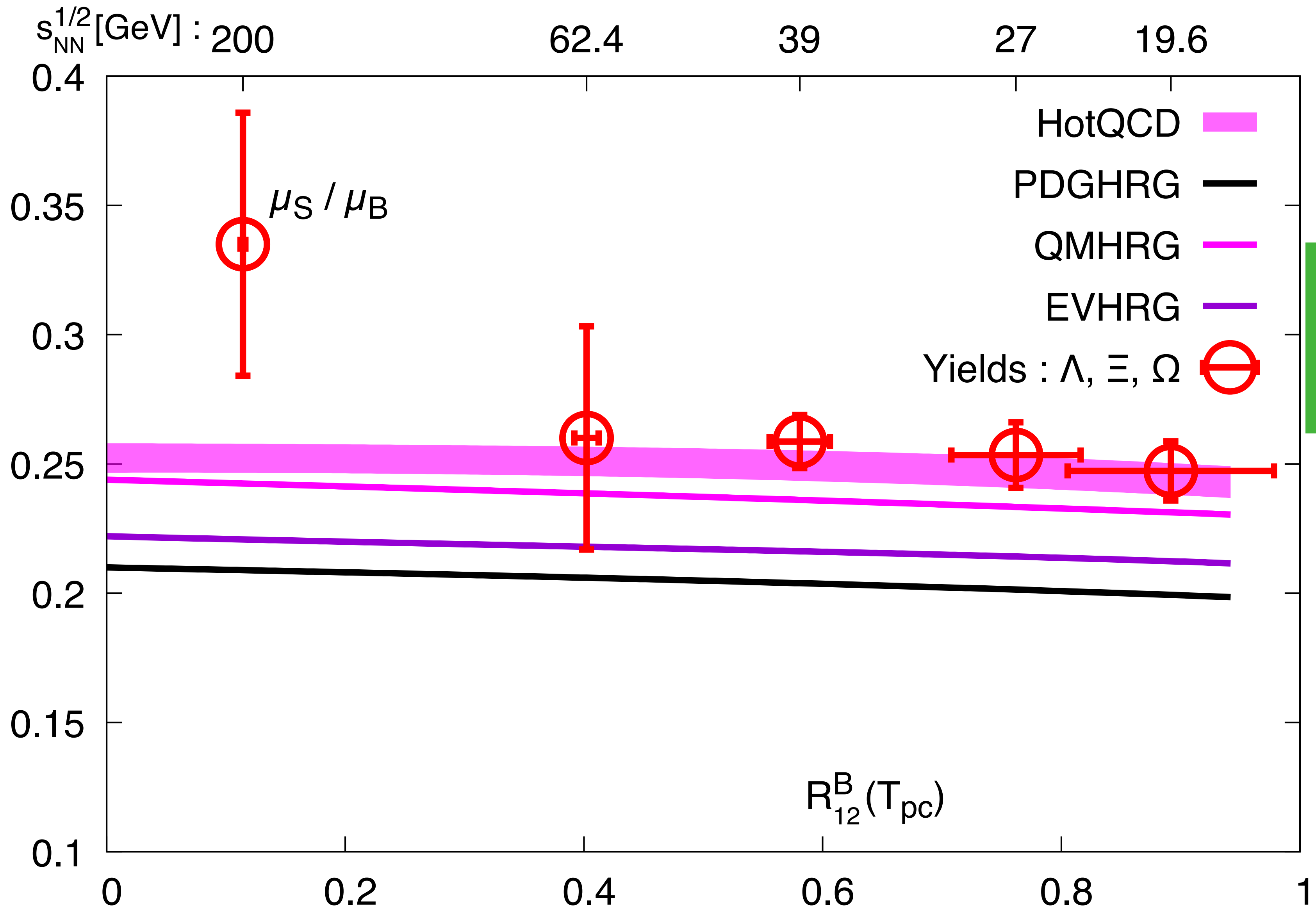
$$\mu_S/\mu_B|_{LO} \equiv -\chi^{BS}_{11}/\chi^S_2$$





Strange hadron production in Au+Au collisions at $\sqrt{S_{NN}}$= 7.7, 11.5, 19.6, 27, and 39 GeV, STAR Collaboration: J. Adam et. al, arXiv:1906.03732

Mapping LQCD to Experiment in the pseudo-critical line



D. Bollweg et al, arXiv:2010.15501

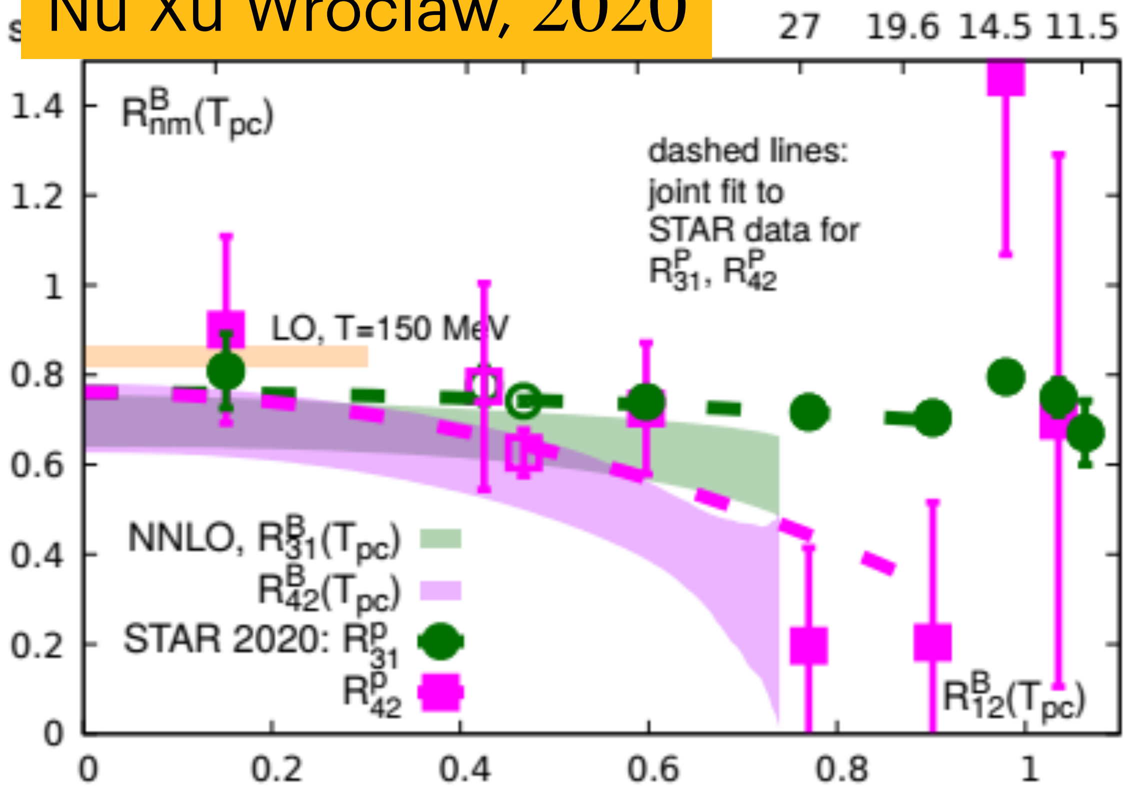
μ_S/μ_B, has to come close to 0.25 at $\sqrt{s} = 200\text{GeV}$

$R_{12}^p = \chi_1^p/\chi_2^p$ has been used as a proxy for R_{12}^B for plotting the STAR data.

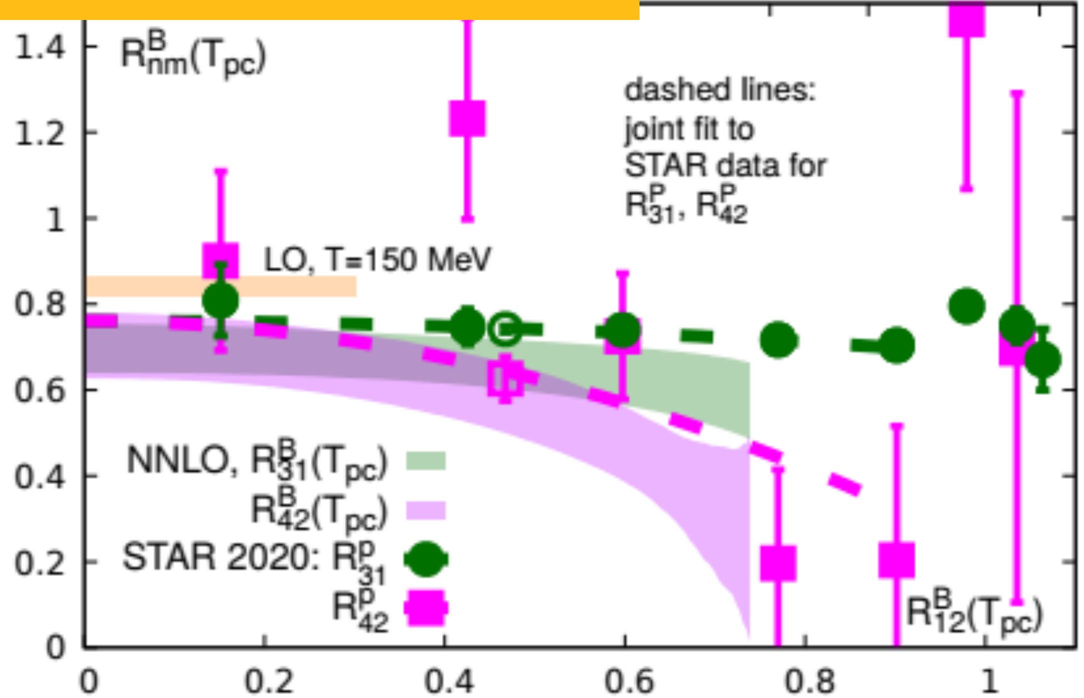
μ_S/μ_B is consistent with strange baryon freeze out at or close to crossover line!!

Mapping LQCD to Experiment in the pseudo-critical line

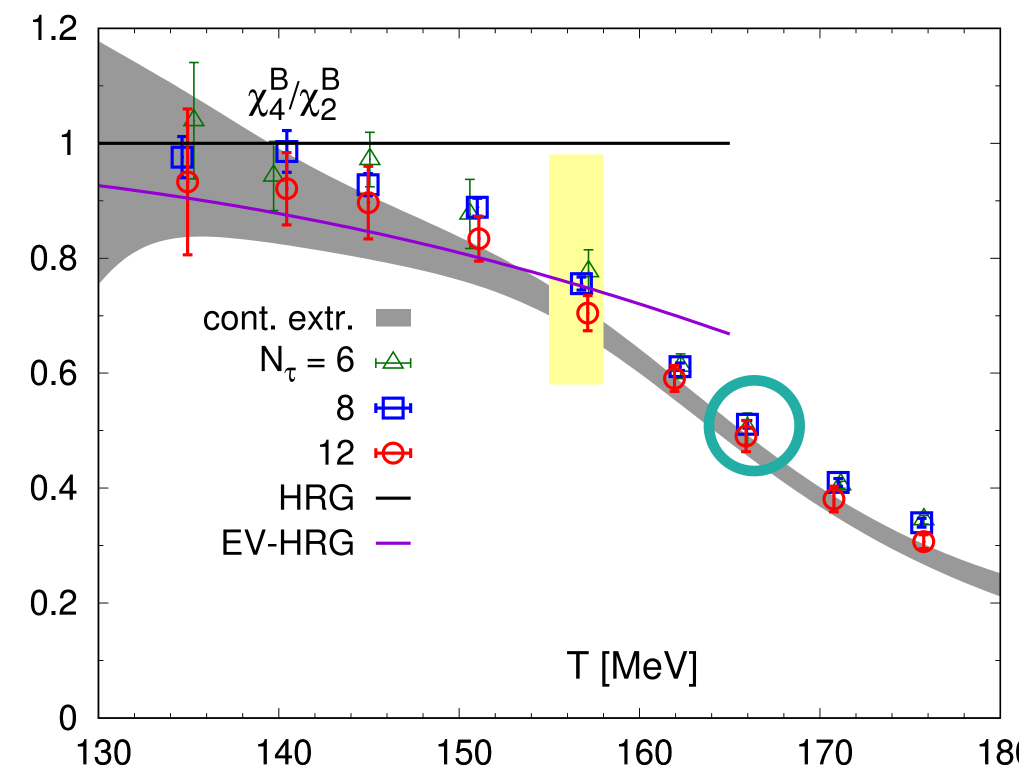
Nu Xu Wroclaw, 2020



STAR OLD



$$\left.\frac{\chi_4^B}{\chi_2^B}\right|_{T_{pc}} = 0.69(5) \qquad \left.\frac{\chi_4^B}{\chi_2^B}\right|_{T=165\text{MeV}} = 0.50(2)$$



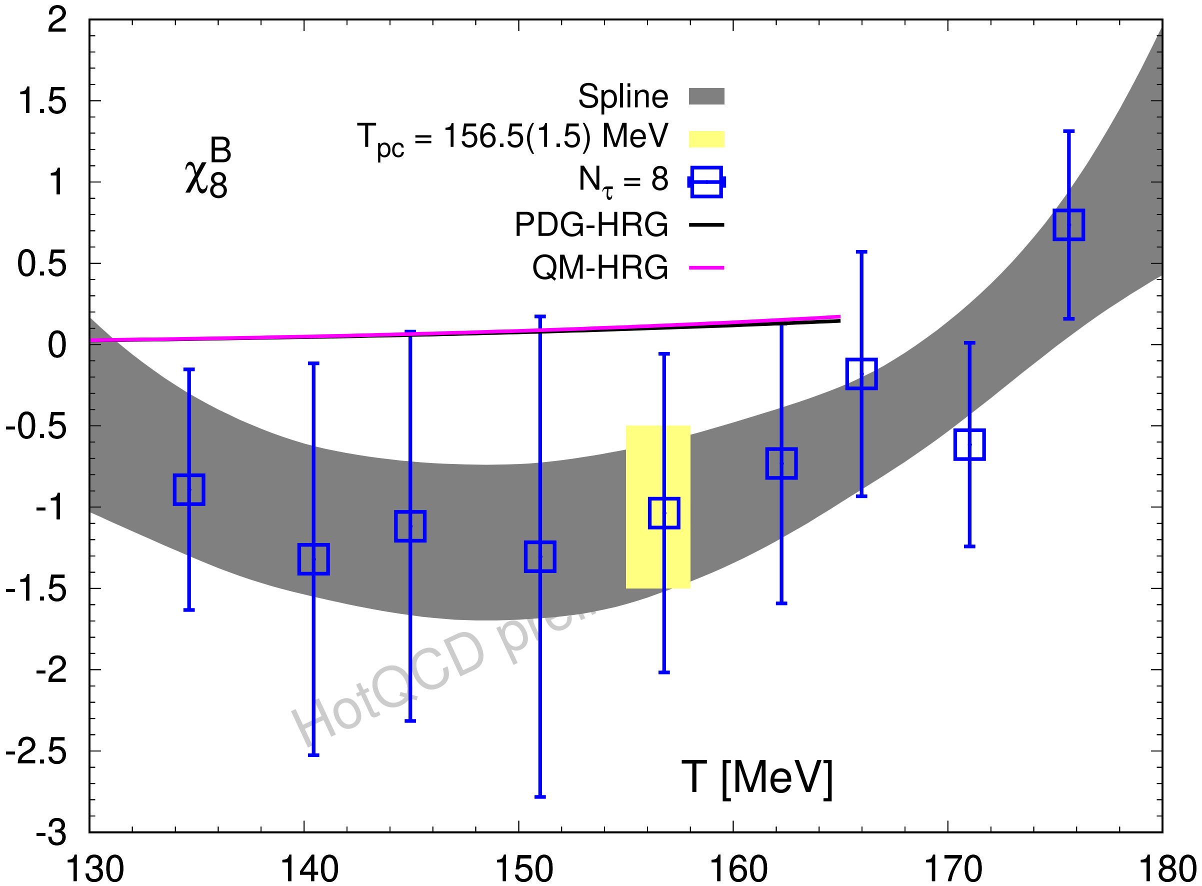
$$\frac{\chi_4^P}{\chi_2^P} \simeq \left.\frac{\chi_4^B}{\chi_2^B}\right|_{STAR} = 0.9(2)$$

Inconsistent with a freeze-out temperature,

$T_f = 165$ MeV.

HotQCD collaboration, arXiv:2001.08530 [hep-lat]

Radius of Convergence from higher order cumulants



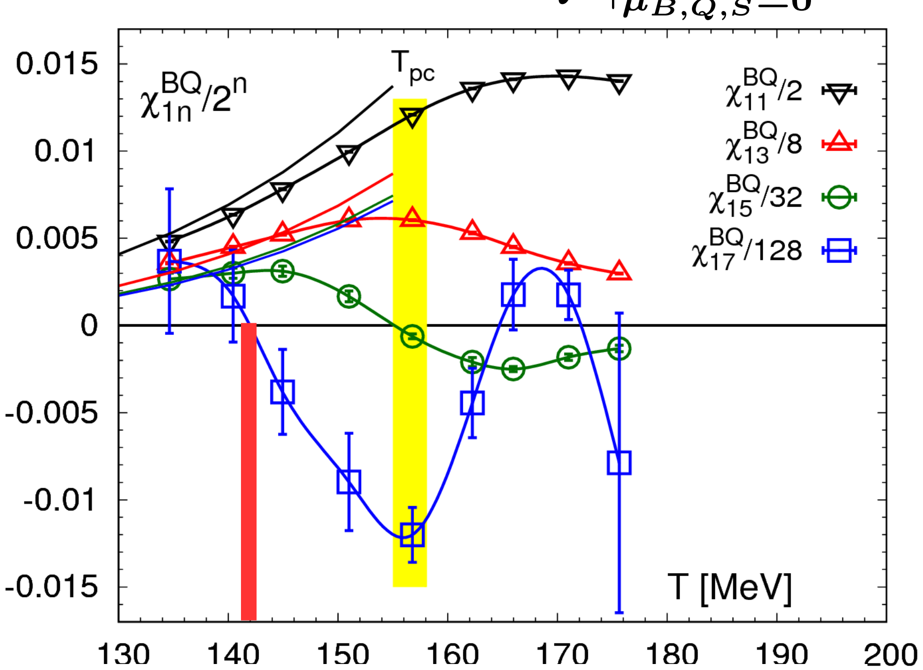
Many 8-th order cumulants turn negative for $T \sim (130 - 145)$ MeV. Which indicates that the limiting singularity of the Taylor series lies in the complex plane.

No, critical point , $T > 140$ MeV

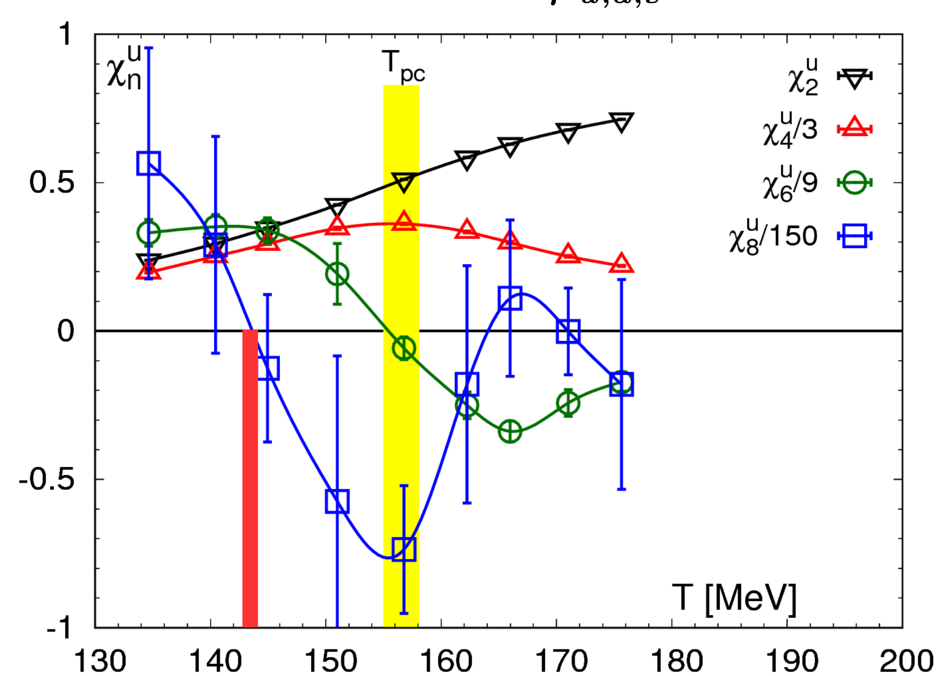
Consistent with the claim that , $T_{cep} < T_c \sim 132$ MeV

What about , μ_B/T??

$$\chi_{1n}^{BQ} = \left.\frac{\partial^{n+1} P/T^4}{\partial \hat{\mu}_B \partial \hat{\mu}_Q^n}\right|_{\mu_{B,Q,S}=0}$$

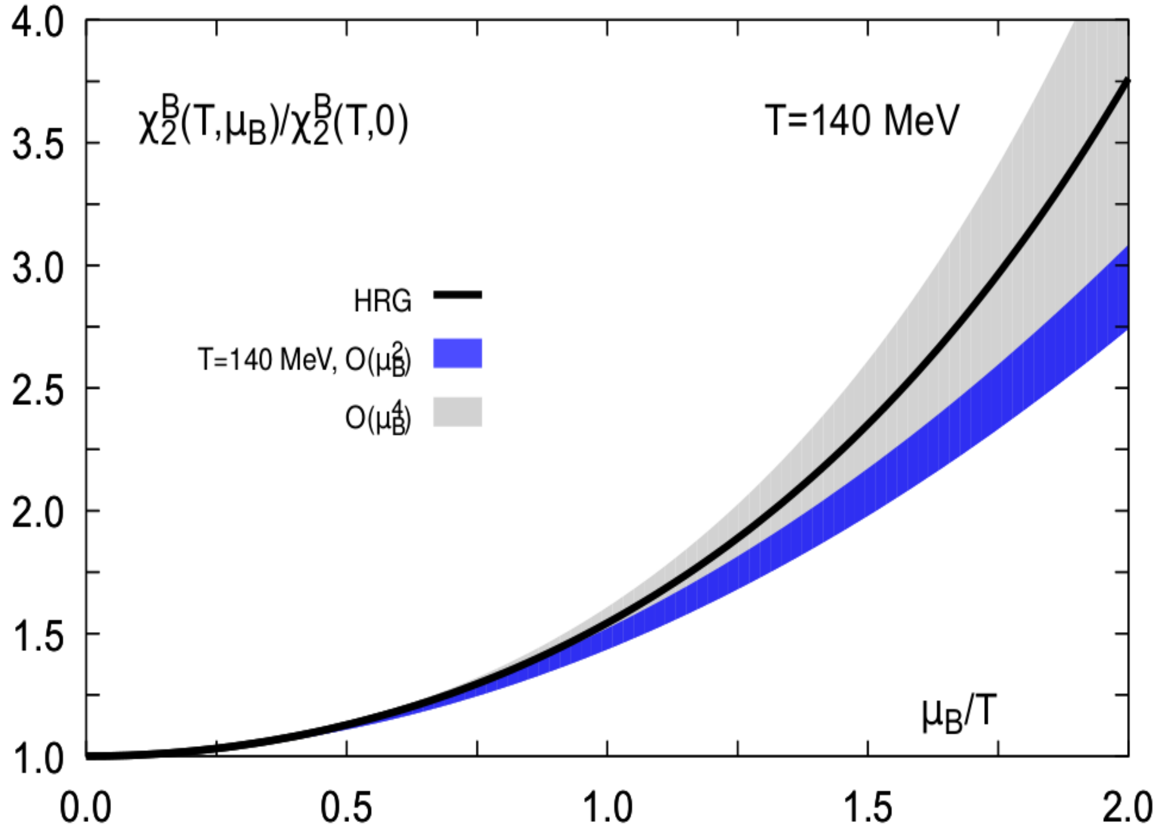


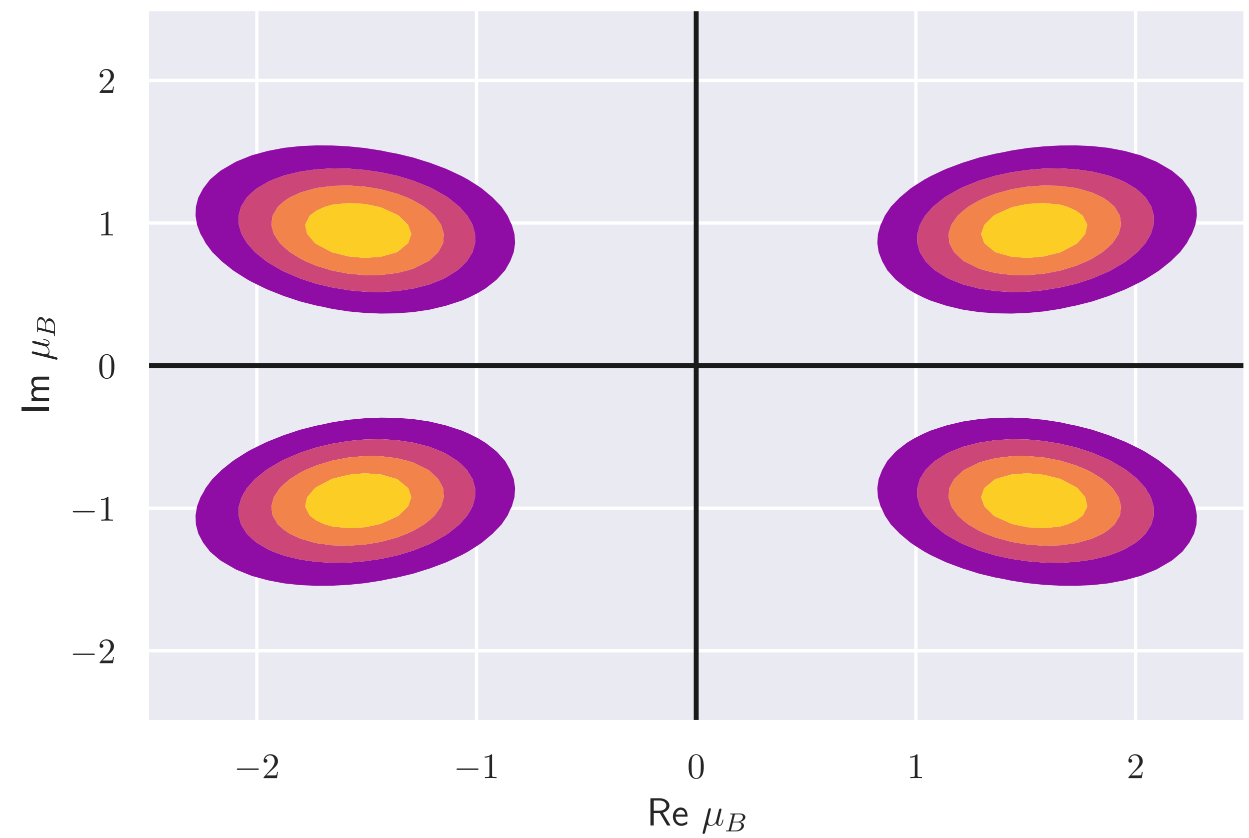
$$\chi_n^u = \left.\frac{\partial^n P/T^4}{\partial \hat{\mu}_u^n}\right|_{\mu_{u,d,s}=0}$$



Resummation of Taylor series

$$\chi_2^B(T,\mu_B) = \chi_2^B + \frac{1}{2!}\chi_4^B\mu_B^2 + \frac{1}{4!}\chi_6^B\mu_B^4 + \frac{1}{6!}\chi_8^B\mu_B^6 \dots\dots$$





No, critical point , $T > 140$ MeV, $(\mu_B/T) < 2$

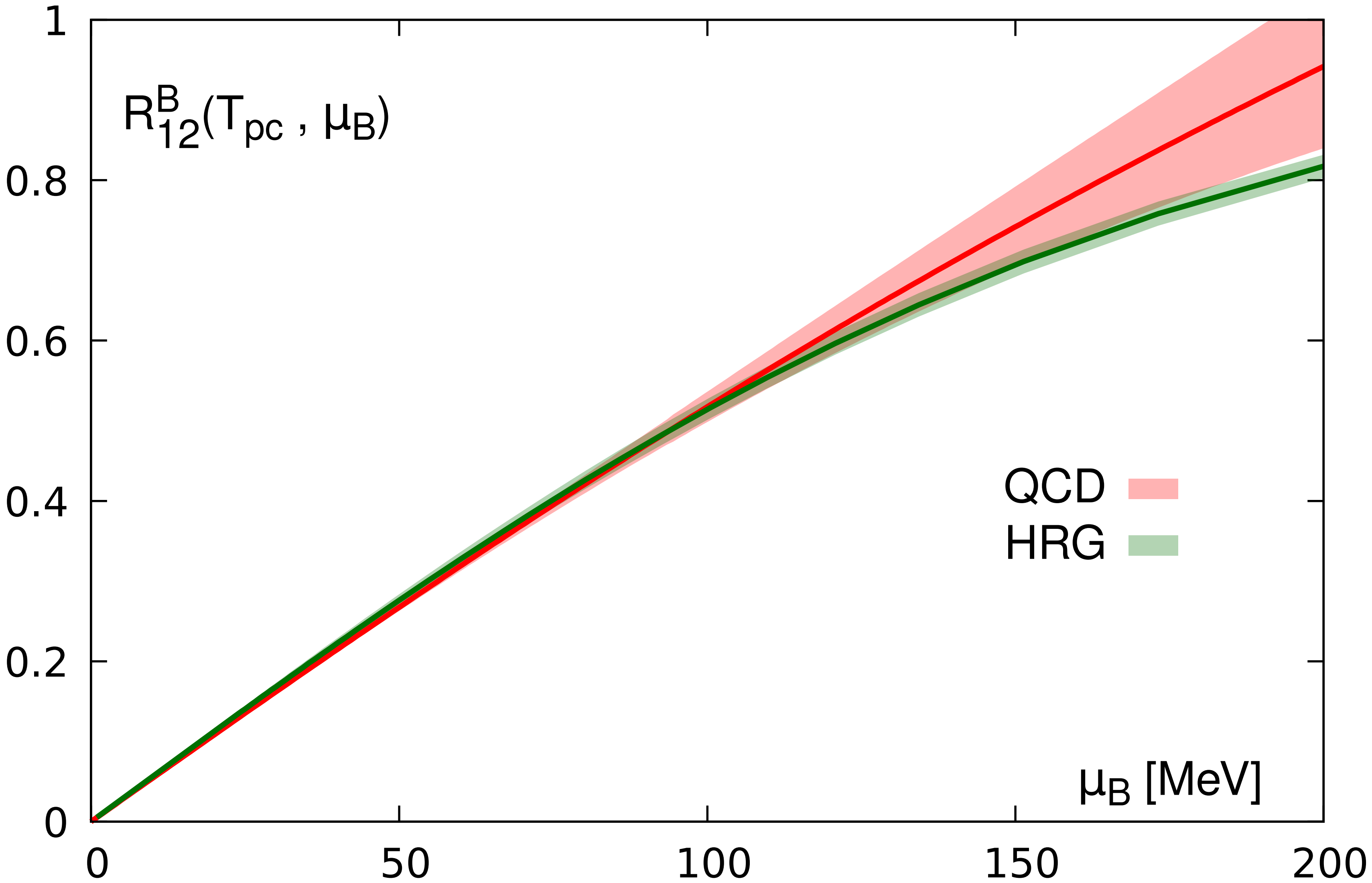
$$\chi_2^B = \frac{\sum_{l=0,1} X_{2l}\mu_B^{2l}}{1 + \sum_{m=1,2} Y_{2m}\mu_B^{2m}}$$

How do these poles move to the real axis when , $T \sim T_{cep}$??

Conclusions

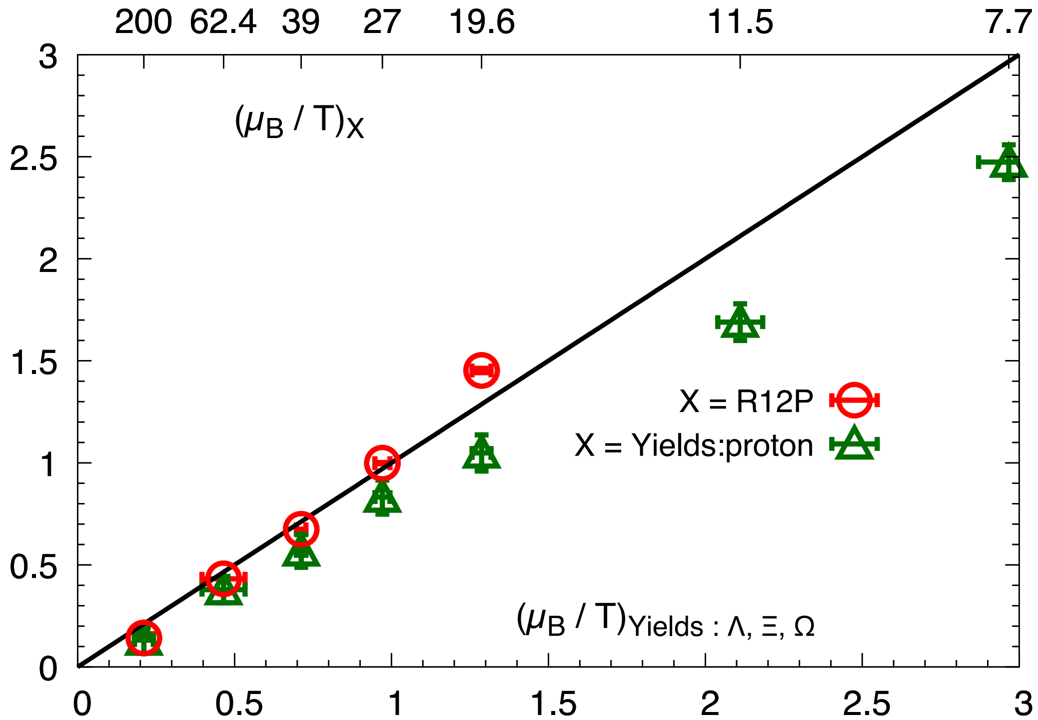
- RW end point remains as Z(2) second order for $m_\pi \geq 40$ MeV. Nature of the chiral transition in the RW plane favours 2nd order(O(N)).
- RW transition and chiral phase transition may coincide in the chiral limit ??
- The pseudo-critical line calculated in LQCD is consistent with the STAR freeze-out line and ALICE freeze-out temperature at $\mu_B = 0$.
- Qualitative features of 2^{nd} order cumulants of conserved charge fluctuations and correlations, calculated in lattice QCD, are reasonably well described by non-interacting HRG models up to the pseudo-critical temperature for the QCD transition.
- Higher order cumulants cannot be understood from HRG models and they will play a crucial role in the determination of $(\mu_B)_{cep}$.

Back up Slides


$R^B_{12}(T_{pc}, \mu_B)$
QCD
HRG
μ_B [MeV]
0
0.2
0.4
0.6
0.8
1
0
50
100
150
200

In a Non-interacting HRG,

$$\bar{S}\mu_S/T - \mu_B/T \equiv \frac{1}{2}\log\left[\frac{\bar{X}}{X}\right]_{primal} = -\,\mathrm{atanh}[\mathrm{R}_{12}^{\mathrm{X}}],$$ Where, X is any Baryon



A simple exercise,

if X (anti)baryon with Strange number S decay into (anti)proton at large μ_B,

$$\frac{p}{X} = \exp[\,|S|\mu_S]G(T),\ \frac{\bar{p}}{\bar{X}} = \exp[\,-|S|\mu_S]G'(T)$$

Calculation Details

- $$\frac{P(T,\vec{\mu})}{T^4} = \frac{1}{VT^3} ln \mathcal{Z}^{QCD} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \mu_B^i \mu_Q^j \mu_S^k$$
- $$\chi_{ijk}^{X} = \frac{\partial^{i+j+k} P/T^4}{\partial(\mu_X/T)^{i,j,k}}\Bigg|_{\mu_X=0}, \; X = B, Q, S$$

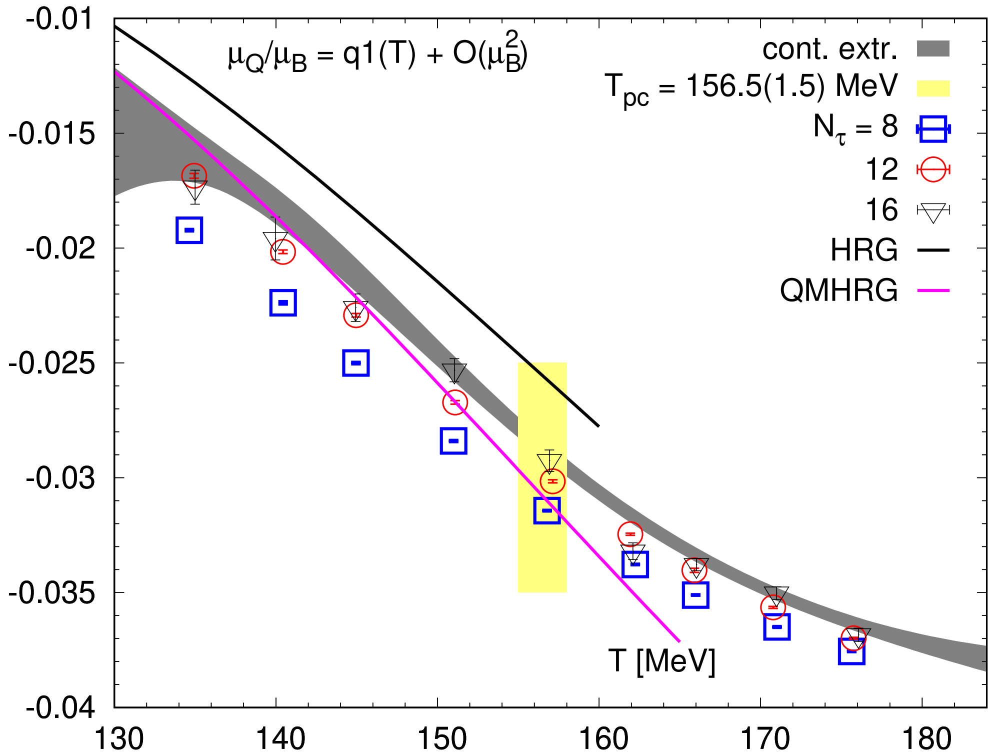
Cumulants at zero chemical potential

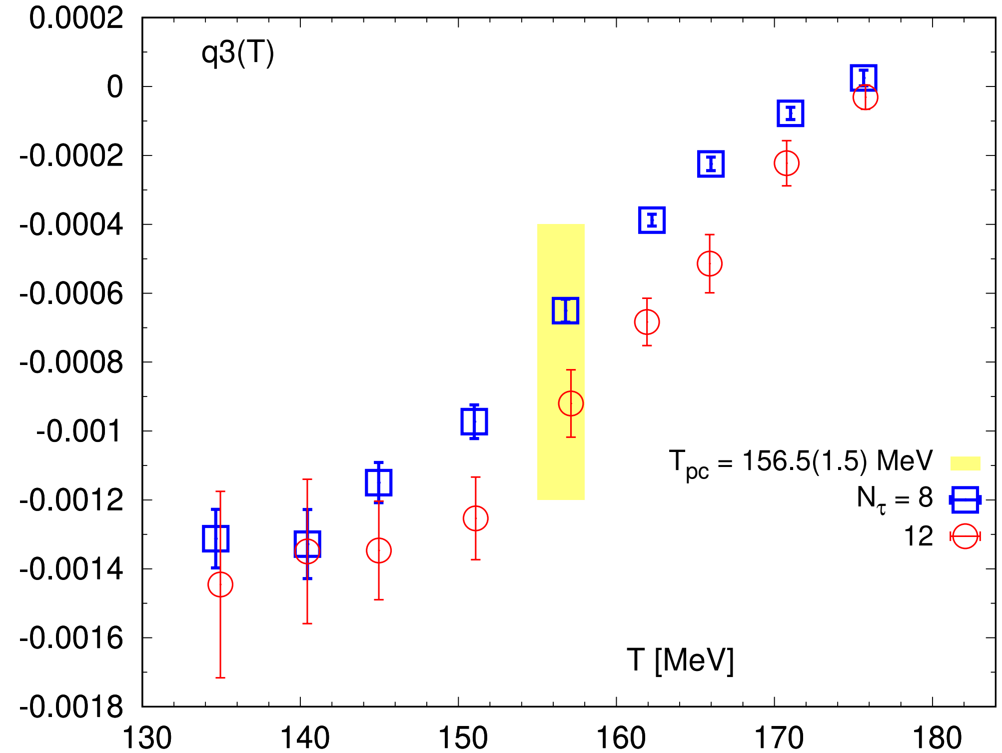
- $$\chi_{\mu_X}^{ijk} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \mu_B^i \mu_Q^j \mu_S^k,$$

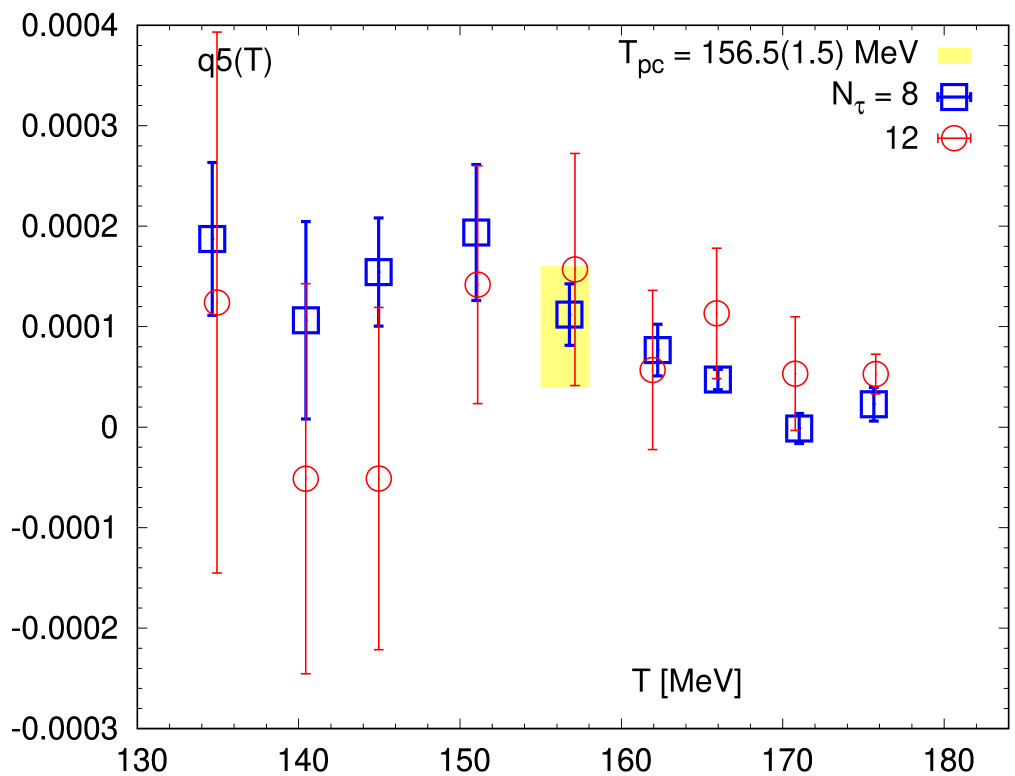
Strangeness neutrality condition

$n_Q/n_B = r,\ n_S = 0, \Rightarrow,\ \ \mu_Q = f(\mu_B),\ \mu_S = g(\mu_B), r = 0.4$ for Au+Au

- $$\chi_{\mu_X}^{ijk} = \sum_{i,j,k=0}^{\infty} \frac{1}{i!j!k!} \chi_{ijk}^{BQS} \mu_B^i f(\mu_B)^j g(\mu_B)^k$$
- where, $f(\mu_B) = \sum_{n=0}^{\infty} q_n \mu_B^{2n+1},\ \ g(\mu_B) = \sum_{n=0}^{\infty} s_n \mu_B^{2n+1}$







$$s1(T) \equiv -\frac{\chi_{11}^{BS}}{\chi_2^S} - \frac{\chi_{11}^{QS}}{\chi_2^S}\frac{\mu_Q}{\mu_B}$$

$$\mu_Q/\mu_B = q1(T) + q3(T)(\mu_B/T)^2 + q5(T)(\mu_B/T)^4 + \ldots$$