based on 1805.08779 and 2006.03591

Zilch Vortical Effect, Berry Phase, and Kinetic Theory

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ITEP and USC

in collaboration with X.-G. Huang, P. Mitkin, E. Speranza

Celebrating Valentin's birthday



AS, V.I. Shevchenko, V.I. Zakharov, PRD, 2011 V.P. Kirilin, AS, V.I. Zakharov, PRD, 2012 V.P. Kirilin, AS, V.I. Zakharov, 2013 A. Avdoshkin, V.P. Kirilin, AS, V.I. Zakharov, PLB, 2016 Z.V. Khaidukov, V.P. Kirilin, AS, V.I. Zakharov, NPB, 2018 A. Avdoshkin, AS, V.I. Zakharov, PRD, 2018

The anomaly and chiral effects, generalized axial charge and instabilities, defects in chiral media, IR effects in the anomalous transport, etc.

CVE of photons, chiral drag force, chiral instabilities, chiral superfluidity, vortices in chiral media, etc.

one of my takeaways – "if you calculate something in 10 different ways and you get the same unexpected answer probably you should check your reasoning and expectations"

International Theoretical Physics Olympiad (http://thworldcup.com)

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$$\begin{split} \bigvee_{\substack{k=0}}^{\mathsf{N}} (\mathsf{t}_{d/2,1}x_{1}) & \Rightarrow (\mathsf{t}_{d/2,1}x_{4}) \underbrace{(\mathsf{t}_{d/2,1}x_{4})}_{\substack{k=0}} \underbrace{(\mathsf{t}_{d/2}^{\mathsf{L}})^{-1}}_{\substack{k=0}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}_{\substack{k=0}}^{\mathsf{m}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}_{\substack{k=0}}^{\mathsf{m}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}_{\substack{k=0}}^{\mathsf{m}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}_{\substack{k=0}}^{\mathsf{m}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}_{\substack{k=0}}^{\mathsf{m}} \mathfrak{s} \xrightarrow{\mathsf{m}}} (\mathfrak{s}) \xrightarrow{\mathsf{m}} (\mathfrak{s}) \xrightarrow{\mathsf{m}}} (\mathfrak{s}) \xrightarrow{\mathsf{m}} \mathfrak{s} \xrightarrow$$

Outline

- Introduction: polarization and chiral effects
- Chiral vortical effect of photons
- Chiral kinetic theory for an arbitrary spin
- Wigner function for photons
- Zilch vortical effect

Spin polarization in HIC



Spin polarization by rotation and magnetic fields



Chiral Vortical Effect (CVE) $\partial_{\mu}J_{5}^{\mu} = cR \cdot \tilde{R}$



Chiral Vortical Effect (CVE) $\partial_{\mu}J_{5}^{\mu} = cR \cdot \tilde{R}$



Chiral Vortical Effect (CVE)

$$S = \int_{V} d^{4}x \, \bar{\psi} \left[\gamma_{\alpha} \left(i\partial^{\alpha} + A^{\alpha} \right) + \mu \gamma^{0} + \mu_{5} \gamma^{0} \gamma_{5} \right] \psi$$
take the fermionic action at finite densities

$$A^{\alpha} \to A^{\alpha} + \mu u^{\alpha} + \mu_5 \gamma_5 u^{\alpha}$$

and introduce an effective gauge field allow the velocity to change in space and time

$$S = \int_{V} d^{4}x \, \bar{\psi} \gamma_{\alpha} \left[i\partial^{\alpha} + A^{\alpha} + \mu u^{\alpha} + \mu_{5} \gamma_{5} u^{\alpha} \right] \psi$$

AS, V.I. Shevchenko, V.I. Zakharov, PRD, 2011



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Anomaly for photons

$$\nabla_{\alpha}J_{5}^{a} = \frac{1}{192\pi^{2}}\epsilon^{\mu\nu\alpha\beta}R^{\lambda}_{\ \rho\mu\nu}R^{\rho}_{\ \lambda\alpha\beta}$$

known from 1969, Kimura

$$\nabla_{\alpha}K^{a} = \frac{1}{96\pi^{2}} \epsilon^{\mu\nu\alpha\beta} R^{\lambda}_{\ \rho\mu\nu} R^{\rho}_{\ \lambda\alpha\beta}$$

known from 1987, V. Zakharov et al

A. Avkhadiev, AS, PRD, 2017

CVE of photons

 $K^{\mu}=\epsilon^{\mu
ulphaeta}A_{
u}\partial_{lpha}A_{eta}$ -- photon helicity current



$$K^{\mu} = \sigma_{K} \omega^{\mu}$$
, $\sigma_{K} = \lim_{q \to 0} \frac{-i}{q_{k}} \epsilon_{ijk} \left\langle K^{i} T^{0j} \right\rangle |_{\omega=0}$
linear response (Kubo)

$$K^{\mu}=c_{\gamma}T^{2}\omega^{\mu}$$

CVE of photons

A. Avkhadiev, AS, PRD, 2017 N. Yamamoto, PRD, 2017

- Are the fermionic and photonic vortical responses related to an anomaly? What anomaly is it?
- Is the photonic CVE related to the topological properties of the theory as it is for the other chiral effects (**Berry phase**)?
- Could we extend the notion of chiral effects to the general case of massless particles?
- Is there local, gauge invariant, Lorentz covariant measure of photon polarization current?

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D.T. Son, N. Yamamoto, PRL+PRD, 2012

$$S = \int dt [\dot{\mathbf{x}} \cdot \mathbf{p} + \mathbf{A} \cdot \dot{\mathbf{x}} - \mathbf{a}_{p} \cdot \dot{\mathbf{p}} - \varepsilon_{p} - \mathbf{A}^{0}]$$

$$\dot{\mathbf{x}} = \frac{\partial \varepsilon}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{b} \quad , \quad \dot{\mathbf{p}} = \mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}$$

$$\mathbf{b} = \nabla \times \mathbf{a}_{p} = \pm \frac{\hat{p}}{2|\mathbf{p}|^{2}}$$

$$\downarrow$$

$$\mathbf{J} = \int_{p} \sqrt{G} \dot{\mathbf{x}} f(\mathbf{p}, \mathbf{x})$$

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$$\int$$

$$J_{el} = \frac{\mu_{5}}{2\pi^{2}} \mathbf{B}$$

M. Stephanov, Y, Yin, PRL, 2012

$$\dot{\mathbf{x}} = rac{\partial arepsilon}{\partial \mathbf{p}} + \dot{\mathbf{p}} \times \mathbf{b}$$
, $\dot{\mathbf{p}} = \mathbf{E} + \dot{\mathbf{x}} \times \mathbf{B}$
 $\mathbf{B}
ightarrow 2|\mathbf{p}|\Omega$
 \downarrow
 $\mathbf{J}_5 = \left(rac{\mu^2 + \mu_5^2}{2\pi^2} + rac{T^2}{6}
ight)\Omega$



Berry phase of photons

V.S. Liberman, B.Ya. Zel'dovich, PRA, 1992 K.Yu. Bliokh, Yu.P. Bliokh, PLA, 2004



X.-G. Huang, AS, JHEP, 2019

Chiral kinetic theory $H = \frac{1}{s} \mathbf{S} \cdot \mathbf{p} - \Phi - \mathbf{\Omega} \cdot (\mathbf{x} \times \mathbf{p} + \hbar \mathbf{S})$

the Hamiltonian of a spin S particle in a rotating frame, where S is $(2s + 1) \times (2s + 1)$ matrix

$$egin{aligned} \dot{m{x}} &= rac{1}{s}m{S} - m{\Omega} imes m{x} \ \dot{m{p}} &= m{
abla}_x \Phi + m{p} imes m{\Omega} \ \dot{m{S}} &= rac{1}{s\hbar}m{p} imes m{S} - m{\Omega} imes m{S} \end{aligned}$$

X.-G. Huang, AS, JHEP, 2019

After the spin dof's are integrated out, the semi-classical EOMs read

 $\dot{\boldsymbol{x}} = \hat{\boldsymbol{p}} - \boldsymbol{\Omega} \times \boldsymbol{x} + \hbar \boldsymbol{\nabla}_{x} \Phi \times \boldsymbol{b} \quad , \quad \dot{\boldsymbol{p}} = \boldsymbol{\nabla}_{x} \Phi + \boldsymbol{p} \times \boldsymbol{\Omega}$ where $b = \pm s \frac{\hat{p}}{|p|^{2}}$ is an emergent "magnetic" field due to the Berry phase and the dispersion reads

$$arepsilon = |m{p}| - \Phi - m{\Omega} \cdot (m{x} imes m{p}) - s\hbar m{\Omega} \cdot \hat{m{p}}$$

then the equilibrium number currents for \pm polarizations are given by

$$m{J}_{\pm} = \int_{m{
ho}} \left(\dot{m{x}} - |m{
ho}| m{b} imes m{
abla}_{ imes}
ight) f_{\pm}(m{
ho}, m{x}) = rac{s m{\Omega}}{2\pi^2} \int 2 f_{\pm}(arepsilon) arepsilon \, darepsilon$$

and the gradient term corresponds to the magnetization

X.-G. Huang, AS, JHEP, 2019

CVE for an arbitrary spin

CVE appears to be a general phenomenon for massless particles with spin



X.-G. Huang, AS, JHEP, 2019 A. Avkhadiev, AS, PRD, 2017

- We have constructed a chiral kinetic theory for a rotating system of massless particles with spin;
- We have seen that CVE is a general phenomenon taking place for particles of an arbitrary spin;
- If one argues that CVE is related to an anomaly for fermions, the same should be checked for s>1 (e.g. grav. anomaly for photons);
- Chiral kinetic theory for fermions was used to study polarization and transport in hot QCD matter, what's about vector particles?

$$W^{\mu\nu}(x,p) = \int \frac{d^4y}{(2\pi\hbar)^4} e^{-\frac{i}{\hbar}p \cdot y} \langle :A^{\mu}\left(x+\frac{y}{2}\right)A^{\nu}\left(x-\frac{y}{2}\right): \rangle$$
$$\Box A^{\mu} = 0 \qquad \longrightarrow \qquad \left(p^2 - \frac{\hbar^2}{4}\partial^2\right)W^{\mu\nu}(x,p) = 0,$$
$$\hbar p \cdot \partial W^{\mu\nu}(x,p) = 0.$$
$$\partial_{\mu}A^{\mu} = 0 \qquad \longrightarrow \qquad \left(p_{\alpha} + i\frac{\hbar}{2}\partial_{\alpha}\right)W^{\mu\alpha}(x,p) = 0$$

One has to impose an extra gauge freedom, c.f. Coulomb gauge:

$$n_{\alpha}W^{\alpha\mu}(x,p) = n_{\alpha}W^{\mu\alpha}(x,p) = 0$$

and then the Wigner function can be solved for order by order in the semi-classical expansion

$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \dots$$

At the zeroth order:

$$p^2 W^{(0)\mu\nu}(x,p) = p \cdot \partial W^{(0)\mu\nu}(x,p) = 0$$
$$p_\alpha W^{(0)\alpha\mu}(x,p) = n_\alpha W^{\alpha\mu}(x,p) = 0$$

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$$W^{\mu\nu} = W^{(0)\mu\nu} + \hbar W^{(1)\mu\nu} + \dots$$

At the zeroth order:

$$W^{(0)\mu\nu}(x,p) = P_n^{\mu\nu} F(x,p)\delta(p^2)$$
$$P_n^{\mu\nu} = -g^{\mu\nu} + \frac{p^{\mu}n^{\nu} + p^{\nu}n^{\mu}}{p \cdot n} - \frac{p^{\mu}p^{\nu}}{(p \cdot n)^2}$$

At the first order:

$$p^{2}W^{(1)\mu\nu}(x,p) = p \cdot \partial W^{(1)\mu\nu}(x,p) = 0$$

plus the gauge constraints

•

$$p_{\alpha}W^{(1)\alpha\mu}(x,p) - \frac{i}{2}\partial_{\alpha}W^{(0)\alpha\mu}(x,p) = 0,$$

$$p_{\alpha}W^{(1)\mu\alpha}(x,p) + \frac{i}{2}\partial_{\alpha}W^{(0)\mu\alpha}(x,p) = 0,$$

$$n_{\alpha}W^{(1)\alpha\mu}(x,p) = n_{\alpha}W^{(1)\mu\alpha}(x,p) = 0.$$

At the first order:

$$p^2 W^{(1)\mu\nu}(x,p) = p \cdot \partial W^{(1)\mu\nu}(x,p) = 0$$

plus the gauge constraints

$$p_{\alpha}W_{S}^{(1)\alpha\mu} = 0, \qquad p_{\alpha}W_{A}^{(1)\alpha\mu} = \frac{i}{2}P_{n}^{\mu\alpha}\partial_{\alpha}F(x,p)\delta(p^{2})$$

At the first order:

$$p^{2}W^{(1)\mu\nu}(x,p) = p \cdot \partial W^{(1)\mu\nu}(x,p) = 0$$

and the solution reads

$$W_A^{(1)\mu\nu} = -\frac{i}{2} \frac{\tilde{p}^{[\mu} P_n^{\nu]\alpha}}{(p \cdot n)^2} \partial_\alpha F(x, p) \delta(p^2) + i\epsilon^{\mu\nu\rho\sigma} \frac{p_\rho n_\sigma}{p \cdot n} U(x, p) \delta(p^2)$$

a free function

$$W_A^{(1)\mu\nu} = -\frac{i}{2} \frac{\tilde{p}^{[\mu} P_n^{\nu]\alpha}}{(p\cdot n)^2} \partial_\alpha F(x,p)\delta(p^2) + i\epsilon^{\mu\nu\rho\sigma} \frac{p_\rho n_\sigma}{p\cdot n} U(x,p)\delta(p^2)$$

$$U = \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} \frac{p_{\rho} n_{\sigma}}{p \cdot n} \Omega_{\mu\nu} F'(\beta \cdot p) + U_0$$

- We have constructed a Wigner function for a rotating system of photons;
- It is still not fully constrained -- U_0 ;
- We can compare some expectation values to see if it can be further constrained;
- For instance we can try to figure out what operator corresponds to the helical current of photons we considered in the naïve CKT;

Search for a word	Q
zilch	
/ziICH/	
INFORMAL	
pronoun	
nothing. "I did absolutely zilch"	
determiner	
not any; no.	

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VOLUME 5, NUMBER 5

MAY 1964

Existence of a New Conservation Law in Electromagnetic Theory

DANIEL M. LIPKIN

American Electronic Laboratories, Inc., Colmar, Pennsylvania (Received 17 January 1964)

Ten new extensive quantities that appear to be independent of stress-energy, but that analogously characterize the physical state of an electromagnetic field, are exhibited and are shown to be conserved in vacuum because of Maxwell's equations. These new quantities are shown to be capable of retrograde flow in a circularly polarized plane-wave field.

div
$$\left[\mathbf{E} \times \frac{\partial \mathbf{E}}{\partial T} + \mathbf{H} \times \frac{\partial \mathbf{H}}{\partial T} \right]$$

 $+ \frac{\partial}{\partial T} \left[\mathbf{E} \cdot (\operatorname{curl} \mathbf{E}) + \mathbf{H} \cdot (\operatorname{curl} \mathbf{H}) \right] = 0.$ (1)

extra conservation law for free EM fields

$$\int d^3x \, K^0 = \int d^3x \, A \cdot B = \sum_k \left[n_R(k) - n_L(k) \right]$$

integral of CS density is gauge-inv. and measures polarization

weighted measure of the polarization transport with **gauge-inv. density**



$$K^{\mu} = \epsilon^{\mu
ulphaeta}A_{
u}\partial_{lpha}A_{eta}$$

vs.
 $\mathbf{Z} = rac{1}{2}\left(oldsymbol{E} imes \dot{oldsymbol{E}} + oldsymbol{B} imes \dot{oldsymbol{B}}
ight) \ , \ \ Z^{0} = rac{1}{2}\left(oldsymbol{B} \cdot \dot{oldsymbol{E}} - oldsymbol{E} \cdot \dot{oldsymbol{B}}
ight)$

$$oldsymbol{K}=rac{T^2}{6}oldsymbol{\Omega}$$
 -- CVE for photons (A. Avkhadiev, AS, PRD, 2017)

$$Z_{i00}=rac{8\pi^2T^4}{45}\Omega_i~$$
-- ZVE for photons (M. Chernodub, et al, PRD, 2018)

$$\bar{Z}_{\alpha_1..\alpha_s}^{(s)} = \tilde{F}_{\lambda\{\alpha_1} \overset{\leftrightarrow}{\partial}_{\alpha_2} ... \overset{\leftrightarrow}{\partial}_{\alpha_{s-1}} F_{\alpha_s\}}^{\lambda}$$

symmetric modification of the general zilch current (see e.g. C. Copetti, J. Fernandez-Pendas, 2018)

$$\bar{Z}_{30..0}^{(s)} = \frac{1}{\hbar^s} \sum_{\lambda} \lambda (-1)^{\frac{s+1}{2}} \int \frac{d^4p}{(2\pi)^3} \,\delta(p^2) \, p_{\{0}p_0...j_{3\}}$$

from the Lorentz structure we can deduce the zilch in CKT and in equilibrium it reads

J.-Y. Chen et al, PRL, 2015

Side jumps

$$J^{\mu\nu} = x^{\mu}p^{\nu} - x^{\nu}p^{\mu} + S^{\mu\nu}$$

full angular momentum

$$p_\mu S^{\mu
u} = n_\mu S^{\mu
u} = 0$$
 fixing the spin part

$$j^{\mu} = p^{\mu}f + S_{n}^{\mu\nu}\partial_{\nu}f \quad \text{with} \quad S_{n}^{\mu\nu} = \lambda \hbar \frac{\epsilon^{\mu\nu\rho\sigma}p_{\rho}n_{\sigma}}{p \cdot n}$$

covariant current

$$j^{\mu} = p^{\mu} f(\beta_{\nu} p^{\nu}) - \frac{1}{2} \lambda \,\hbar \,\epsilon^{\mu\nu\rho\sigma} p_{\nu} \Omega_{\rho\sigma} f'(\beta \cdot p) + \mathcal{O}\left(\hbar^{2}\right)$$

 $J^{\mu
u}=x^{\mu}p^{
u}-x^{
u}p^{\mu}+S^{\mu
u}$ full angular momentum

$$p_\mu S^{\mu
u} = n_\mu S^{\mu
u} = 0$$
 fixing the spin part

$$j^{\mu} = p^{\mu}f + S_{n}^{\mu\nu}\partial_{\nu}f \quad \text{with} \quad S_{n}^{\mu\nu} = \lambda \,\hbar \, \frac{\epsilon^{\mu\nu\rho\sigma}p_{\rho}n_{\sigma}}{p \cdot n}$$

$$\bar{Z}_{30..0}^{(s)}\Big|_{r\to 0} = (-1)^{\frac{s-1}{2}} \frac{(s+2)(s+1)}{3s\hbar^{s-1}} \frac{\Omega}{\pi^2} \int d\omega \,\omega^s f_B(\beta\omega) + \mathcal{O}(\Omega^2)$$

$$W_{A}^{(1)\mu\nu} = -\frac{i}{2} \frac{\tilde{p}^{[\mu} P_{n}^{\nu]\alpha}}{(p \cdot n)^{2}} \partial_{\alpha} F(x, p) \delta(p^{2}) + i\epsilon^{\mu\nu\rho\sigma} \frac{p_{\rho} n_{\sigma}}{p \cdot n} U(x, p) \delta(p^{2})$$

$$\downarrow$$

$$\bar{Z}_{\alpha_{1}..\alpha_{s}}^{(s)} = \tilde{F}_{\lambda\{\alpha_{1}} \overleftrightarrow{\partial}_{\alpha_{2}}...\overleftrightarrow{\partial}_{\alpha_{s-1}} F_{\alpha_{s}}^{\lambda}$$

Now we can compare the two answers

The results for the ZVE obtained with the naïve CKT and the Wigner function agree if

$$U_0 = 0$$

and read

$$\bar{Z}_{30..0}^{(s)}\Big|_{r\to 0} = (-1)^{\frac{s-1}{2}} \frac{(s+2)(s+1)}{3s\hbar^{s-1}} \frac{\Omega}{\pi^2} \int d\omega \,\omega^s f_B(\beta\omega) + \mathcal{O}(\Omega^2)$$

Outlook -- kinetic theory

- We have constructed a naïve CKT for particles of an arbitrary spin;
- We have seen that the Wigner function formalism results in the same CKT for photons (at least at the level of some observables);
- The frame vector for photons is directly related to the residual gauge freedom;

Outlook -- chiral effects

- CVE takes place for massless particles of an arbitrary spin, in the general case it originates in the Berry phase;
- ZVE in the general zilch current originates in the Berry phase as well giving a further insight into the origin of CVE;
- CVE and ZVE can be used to study spin polarization of gauge bosons in rotating plasma (hydrodynamics with spin, evolution of spin dof's in HIC);

Some questions

- There is a large family of CVEs are they anomalous?
- Can the Wigner function for gauge fields be constrained in a more clear way? What's about massive vectors?
- Hydrodynamics from particles of different spin, can one further constrain its structure comparing these cases?
- Are there other gauge invariant measures of the gauge field polarization in interacting systems?