Pasta phase in hybrid hadron-quark stars

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RSF project 17-12-01427

Hot problems of Strong Interactions 2020 (Online)

Based on Phys.Rev.C 100 (2019)

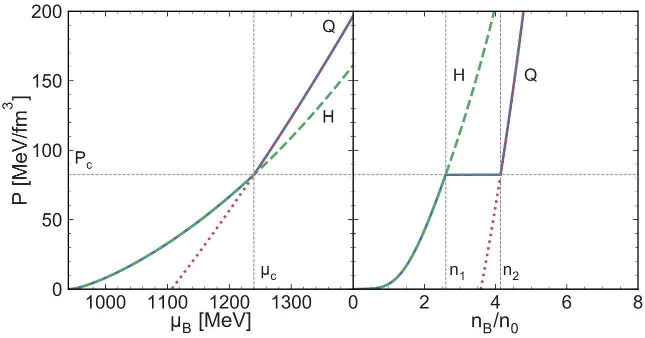
Introduction

- Neutron stars are natural laboratories for studying the matter at low temperatures, large baryon densities $n_B = (5-10)n_0$ and large isospin asymmetries $\beta = \frac{n_n - n_p}{n_B} \sim 1$, where n_0 – nuclear saturation density and n_n, n_p - neutron and proton number densities
- Many possible phases of dense matter are relevant for NS physics:
  - Liquid-gas phase transition (PT) at the inner crust-core boundary
  - Hadronic phase with more baryons species - hyperons, Δ
  - Meson $(K$, π, $\rho^-)$ condensates
  - Quark-hadron (QH) PT $\leftarrow$ *this work*
  - Phases of quark matter - color superconductivity, dual chiral density wave, etc.
- The most restrictive constraint comes from the maximum precisely measured NS mass of $2.14^{+0.10}_{-0.09}\,M_\odot$
- A lot of new data is expected from modern tools $\Rightarrow$ new constraints:
  - Simultaneous measurements of NS masses and radii
  - New gravitational wave detections

Enforced local neutrality – Maxwell construction

Local electric neutrality condition requires a specific $\mu_e = \mu_e(\mu_B)$, so only one chemical potential μ_B is left to fulfill the Gibbs conditions

$$\mu_B^{(H)} = \mu_B^{(Q)} \equiv \mu_c, \quad P^{(Q)}(\mu_c) = P^{(H)}(\mu_c) \equiv P_c$$

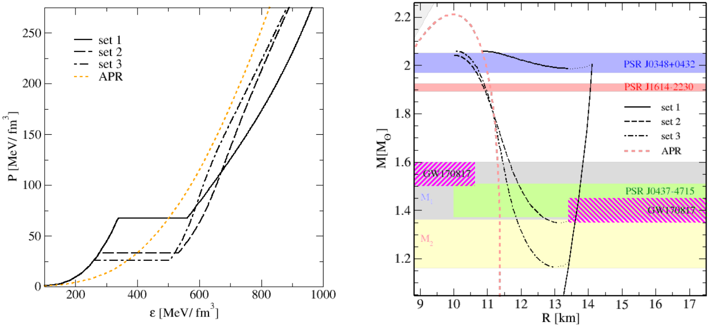


Baryon density and energy density ε is discontinuous across the phase transition with the energy jump $\Delta\varepsilon = \varepsilon(n_2) - \varepsilon(n_1)$.

Third family of compact stars (twin stars)

With a Maxwell construction a disconnected branch of compact stars may appear [U. Gerlach Phys.Rev. 172 1235 (1968)] if the quark matter is sufficiently stiff at large n_B, with the condition for the instability: [Z.F. Seidov Sov.Astron. 15 347 (1971)])

$$\frac{\Delta\varepsilon}{\varepsilon_c} \geq \frac{1}{2} + \frac{3}{2}\frac{P_c}{\varepsilon_c}, \quad \varepsilon_c \equiv \varepsilon(n_1), \quad \Delta\varepsilon = \varepsilon(n_2) - \varepsilon_c$$



[D. Alvarez-Castillo,D. Blaschke,A. Grunfeld,V. Pagura Phys.Rev.D 99 (2019)] - for QH PT

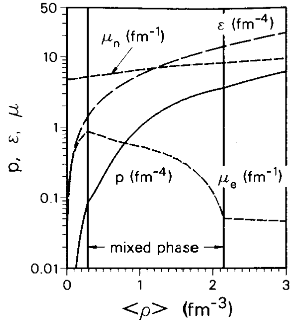
A measurement of NSs with same masses and different radii would prove the existence of a first-order PT, e.g. QH PT ⇒ QCD critical point, meson condensation, etc.

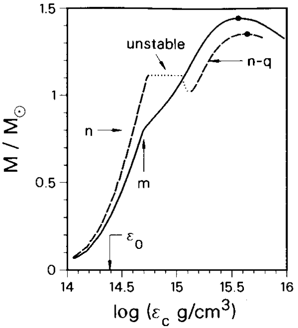
Phase transition with two conserved charges

In general, the conservation laws can be obeyed globally, not locally
[N.K. Glendenning Phys.Rev. D46 (1992) 1274-1287]
The Gibbs conditions

$$\mu_B^{(H)} = \mu_B^{(Q)}, \quad \mu_e^{(H)} = \mu_e^{(Q)}, \quad P^{(H)}(\mu_B^{(H)}, \mu_e^{(H)}) = P^{(Q)}(\mu_B^{(Q)}, \mu_e^{(Q)})$$

now have solutions over a range of μ_B





A puzzle - mixed phase or Maxwell construction?

Finite-size effects

Coulomb interaction

Tends to break up the like-charged regions into smaller ones

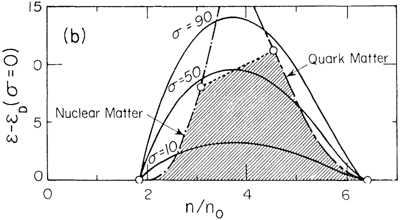
VS

Surface tension

Requires minimization of the surface

⇒ formation of structures with d = 3,2,1 (droplets, rods, slabs)

- Thin diffuseness layer $\sim$ 1 fm $\Rightarrow$ can describe the surface contribution using the surface tension parameter σ – not without a model of both phases; treated as a parameter
- For the surface tension σ larger than some critical σ_c formation of structures becomes energetically unfavorable



solid lines - energy density of the droplets for various σ [MeV/fm^2] relative to $\sigma = 0$

dashed line - Maxwell construction

adapted from [Heiselberg Pethick Staubo Phys.Rev.Lett. 70 (1993) 1355-1359]

Critical surface tension σ_c depends on the model

Solution to the puzzle - treatment of the electric field

Wigner-Seitz approximation with the cell radius R_W (consider d=3)
Self-consistent treatment of electrostatic potential: $\mu_e \to \mu_e - V(r)$
Equations of motion for the electric field potential in a phase $p = H, Q$

$$\Delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V^{(p)}(r)]$$

$\Rightarrow$ nonuniform electron density distribution and charge screening
Linearized version defines Debye screening lengths in a phase $p = H, Q$

$$\Delta \delta V^{(p)}(r) = 4\pi e^2 n_{\rm ch}^{(p)}[\mu_B, \mu_e - V_{\rm ref}] + (\lambda_D^{(p)})^{-2} \delta V(r),$$

$$\delta V^{(p)}(r) = V(r) - V_{\rm ref}^{(p)}, \quad (\lambda_D^{(p)})^{-2} = -4\pi e^2 \left(\frac{\partial n_{\rm ch}^{(p)}}{\partial \mu_e}\right)_{\mu_B}$$

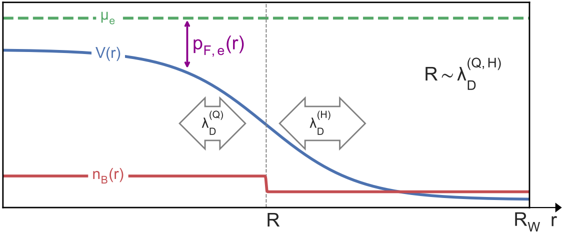
Linearized equation can be solved analytically with matching and boundary conditions (R - radius of a droplet)

$$V^{(Q)}(R) = V^{(H)}(R), \quad (\frac{d}{dr}V^{(Q)})(R) = (\frac{d}{dr}V^{(H)})(R), \quad (\frac{d}{dr}V^{(H)})(R_W) = 0$$

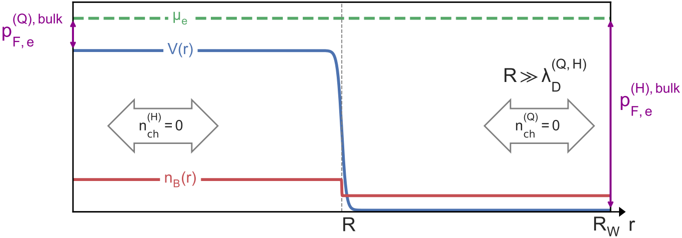
[D.N. Voskresensky, M. Yasuhira, T.Tatsumi PLB 541 (2002) 93-100, NPA 723 (2003) 291-339]

Schematic effect of the screening

1. Case $R \sim \lambda_D$: smooth non-uniform electron density distribution
$\lambda_D \sim 1/e^2 \gg$ diffuseness layer thickness $l \sim 1$ fm $\Rightarrow$ neglected in $n_B(r)$ profile



2. Case of large droplets $R \gg \lambda_D^{(Q,H)}$: electric field contributes only in the thin border layer $\Rightarrow$ contribution to the effective surface tension



Critical surface tension

In the large droplet limit the energy per cell is (in terms of $\xi = R/\lambda_D^{(Q)}$)

$$\epsilon \simeq \frac{3}{\beta\lambda_D^{(Q)}}\frac{\sigma - \sigma_c}{\xi}$$

$$\sigma_c = \frac{(U_0^{\rm II} - U_0^{\rm I})^2}{8\pi e^2(\lambda_D^{(H)} + \lambda_D^{(Q)})}, \quad U_0^{\rm II} \simeq -\mu_{e,\rm bulk}^{(H)}, \quad U_0^{\rm I} = -4\pi e^2 (\lambda_D^{(Q)})^2 n_{\rm ch}^{(Q)}.$$

The critical value σ_c comes entirely from the electrostatic contribution
For $\sigma > \sigma_c$ energy minimization leads to
$\xi \to \infty \Rightarrow$ Maxwell construction
Model dependence resides in $\mu_e^{(H)}(\mu_B)$, $\lambda_D^{(Q,H)}(\mu_B, \mu_e)$

Recent work

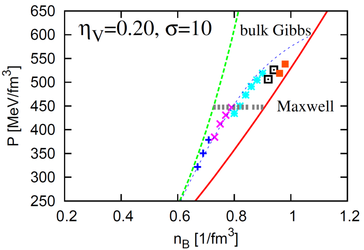
Similar structure formation has been studied for many phase transitions:

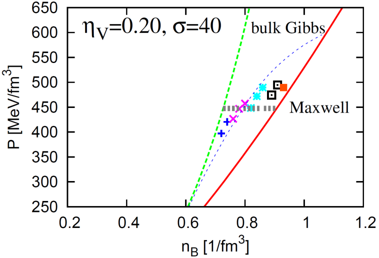
- Nuclear liquid-gas phase transition in NS crust
  Both phases within the same model $\Rightarrow$ no need to introduce the surface tension parameter [T. Maruyama et al. PRC72 (2005) 015802]
  Can be also studied using molecular dynamics
  [A.S. Scheider et al. PRC88 (2013) no.6, 065807]
  + many, many more works...
- Kaon condensation
  [T. Maruyama et al. PRC73 (2006) 035802]
  Pasta within a nucleon-meson model
  [A. Schmitt Phys.Rev.D 101 (2020)]
- Quark-hadron phase transition
  [N. Yasutake et al. PRC89 (2014) 065803,
  X. Wu, H. Shen PRC96 (2017) no.2, 025802]

Current work: quark-hadron phase transition
How does the pasta affect the third family?
How strong is the model dependence?

Effect on the EoS

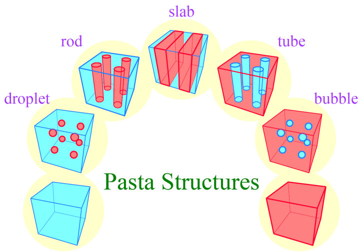
Typical result for the pressure with pasta:





[N. Yasutake et al. PRC89 (2014) 065803]

Different symbols - different structures (figure from [N. Yasutake et al. Phys.Rev. D80 (2009) 123009])



Pressure goes between bulk Gibbs ($\sigma = 0$) and Maxwell ($\sigma > \sigma_c$) constructions

Possible effect results in blurring of the phase transition

Its effect on the third family can be studied phenomenologically

Phenomenological description

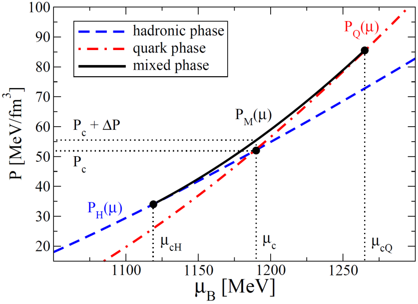
[A.Ayriyan et al. Phys.Rev. C97 (2018) no.4, 045802, EPJ Web Conf. 173 (2018) 03003]

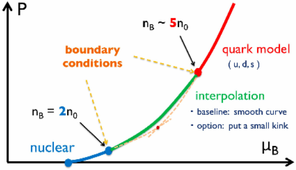
Simple parabolic interpolating construction in terms of $P(\mu)$ (here and below $\mu \equiv \mu_B$):

$$P(\mu) = \begin{cases} P^{(H)}(\mu), & \mu < \mu_{cH}, \\ a(\mu - \mu_c)^2 + b(\mu - \mu_c) + P_c + \Delta P, & \mu_{cH} < \mu < \mu_{cQ}, \\ P^{(Q)}(\mu), & \mu_{cQ} < \mu, \end{cases}$$

ΔP - the only parameter of the construction

$\mu_{cH}, \mu_{cQ}, a, b \Leftrightarrow$ continuity of the pressure and its first derivative over μ



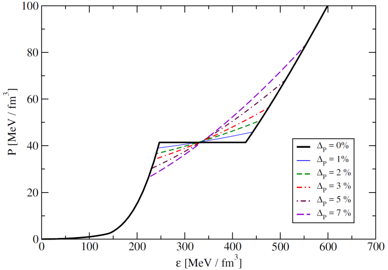


[T. Kojo 1912.05326]

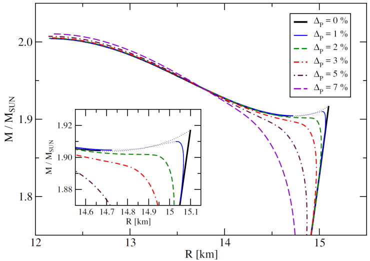
Implementation of the quark-hadron continuity

Effect on the third family

The construction in terms of $P(n)$ for a given pair of hadronic and quark models for various $\Delta_P \equiv \frac{\Delta P}{P_c}$



Disconnected third branch can disappear if the PT is blurred for a large enough Δ_P



Does the construction describe the pasta adequately?
What is the relation between Δ_P and the surface tension σ?

Need for a complicated model: contradicting constraints

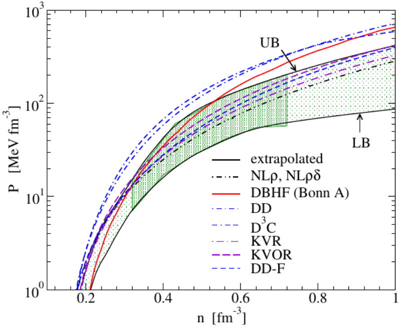
Constraint for the pressure, obtained from analyses of transverse and elliptic flows in heavy-ion collisions

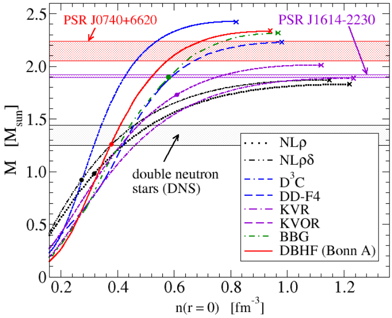
Passed by rather soft EoSs

[P. Danielewicz, R. Lacey, W.G. Lynch, Science 298 (2002)]

The maximum NS mass constraint favors stiff EoS

NS cooling data ⇒ direct URCA (DU) is not operative for most stars ⇒ constraint for the proton fraction





figures adated from [T. Klahn et al. PRC74 (2006)]

Hadronic models: framework

E. E. Kolomeitsev and D. N. Voskresensky NPA 759 (2005) 373

- KVOR – Walecka-type model with in-medium change of masses and coupling constants of all hadrons in terms of the scalar field σ:

$$m_i^* = m_i \Phi_i(\sigma),\ g_{mB}^* = g_{mB}\chi_m(\sigma),$$
$$m = \{\text{mesons}\},\ B = \{\text{baryons}\},\ i = B \cup m$$

- Extensions: [K.A.M, E.E.Kolomeitsev, D.N.Voskresensky PRC 92 (2015)]

KVORcut03 – H1 (softer)	**KVORcut02 – H2 (stiffer)**
Based on KVOR Sharp decrease in $m_\omega^*(\sigma)$ (increase in $g_\omega^*(\sigma)$) Stiff in NS matter and symmetric matter	
Flow constraint +	Flow constraint −
Twins -	Twins +

- They the maximum NS mass constraint with both **hyperons** (with help of ϕ-meson) and Δ-**isobars** included

[E.E.Kolomeitsev, K.A.M., D.N.Voskresensky Nucl.Phys. A961 (2017) 106-141]

- Many other constraints are also satisfied

Quark models

[M.Kaltenborn et al. Phys.Rev. D96 (2017) no.5, 056024]

$$\mathcal{L} = \mathcal{L}_{\text{free}} - U(\bar{q}q, \bar{q}\gamma_0 q)$$

expand around the mean-field $\langle \bar{q}q \rangle = n_S, \langle \bar{q}\gamma_0 q \rangle = n_V$, neglect fluctuations:

$$P = \sum_{f=u,d} P_{\text{quasi}}(\{\mu_f^*\}, \{m_f^*\}) - (U - n_S\Sigma_S - n_V\Sigma_V),$$

$$\Sigma_S(n_S, n_V) = \frac{\partial U}{\partial n_S}, \quad \Sigma_V(n_S, n_V) = \frac{\partial U}{\partial n_V}, \quad m_f^* = m_f + \Sigma_S, \quad \mu_f^* = \mu_f - \Sigma_V$$

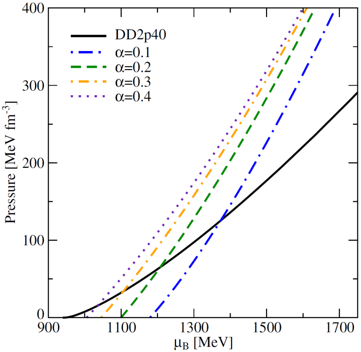
Values of the n_S, n_V for given μ_f – from the self-consistency conditions

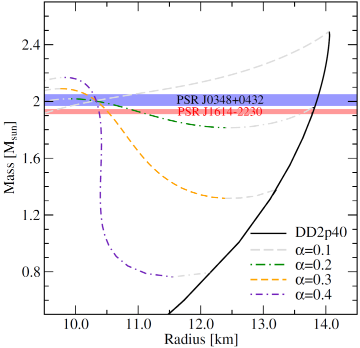
$$U(n_S, n_V) = D(n_V)n_S^{2/3} + an_V^2 + \frac{bn_V^4}{1 + cn_V^2}.$$

- $\Sigma_S = D(n_V)n_S^{-1/3}$ – quark effective mass diverges at $n_B \to 0$: simulation of the confinement; $D(n_V) = D_0 \exp(-\alpha(n_V \cdot \text{fm}^{-3})$ – reduction of the effective string tension in the medium.
- an_V^2 – ordinary vector repulsion
- Nonlinear repulsion – needed to control the existence of the third family.

Parameterization of the effective potential

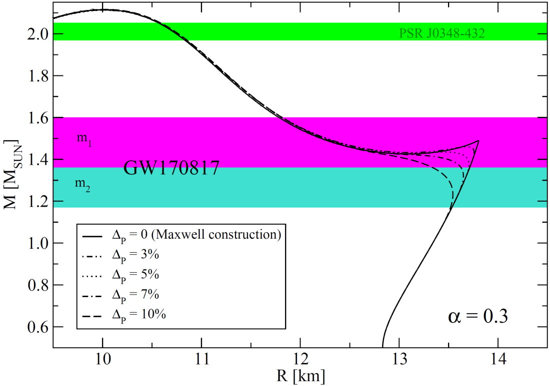
Two models we use differ by the value of α:
Q1: $\alpha = 0.2$ and Q2: $\alpha = 0.3$





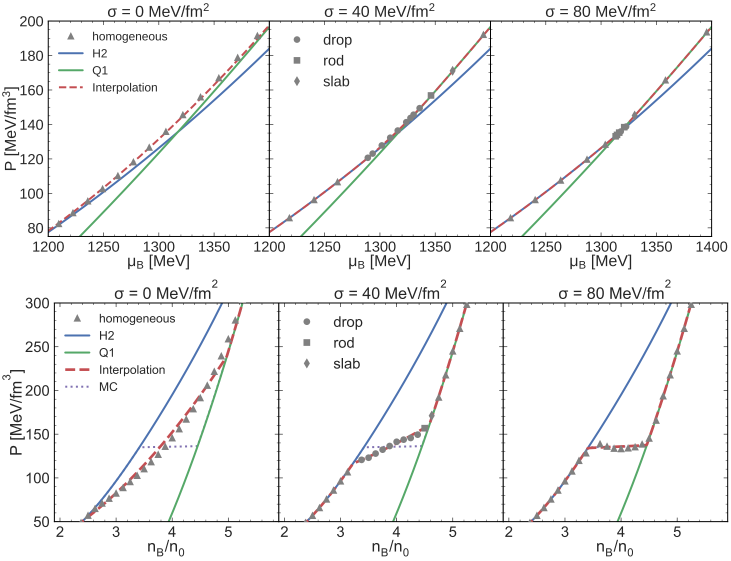
[M.Kaltenborn et al. Phys.Rev. D96 (2017) no.5, 056024]

Phenomenological results for models of this class



We found that the critical value of Δ_P for the twin disappearance is $6-7\%$ within these types of models [A.Ayriyan et al. Phys.Rev. C97 (2018) no.4, 045802]

Fitting the pasta results: example (Q1–H2)

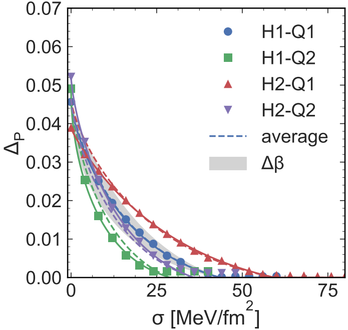


Fit works reasonably well, so the polynomial construction indeed can be used to describe the effect of the structures

Determination of the critical surface tension

Thus obtained $\Delta_P(\sigma)$ decreases with increase of σ – shown by symbols

Systematic error due to numeric limitations $\Rightarrow \Delta_P \neq 0$ for any σ



Determination of "numeric" critical surface tension σ_c: filter out the error using the smoothing fit function

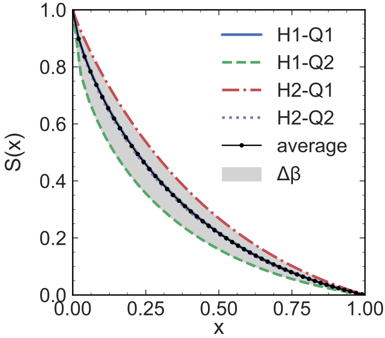
$$\Delta_P = \Delta_P^{(0)} S(\sigma/\sigma_c;\beta)$$

$$S(x;\beta) = e^{-x}(1 - x^{\beta})\theta(1 - x)$$

Result shown by lines

Dashed lines – using the same $\bar{\beta} = 0.68$ – almost coinciding

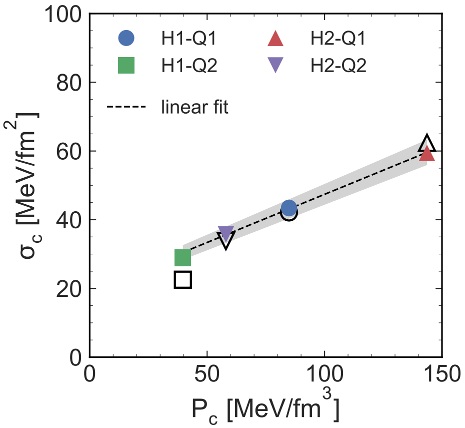
Shaded area – dispersion of $\bar{\beta}$



Universality?

Critical surface tension: model dependence

Parameter of the phase transition, characterizing a pair of models: pressure on the Maxwell line P_c - different for all the models.
Filled symbols – numeric result,
Empty symbols – analytical result using the expression above



Observations:

- Critical surface tension grows almost linearly with P_c
  $\sigma_c \simeq 30 \mathrm{MeV/fm^2} + 0.28(P_c - 40\frac{\mathrm{MeV}}{\mathrm{fm}^3})$
- Numeric result agrees well with the analytical estimate
- If the energy jump is sufficiently large, low $P_c \Rightarrow$ low-mass twins. So these models should be less affected by the structures

Recipe of mimicking the pasta phase in the PT

For an arbitrary pair of EoSs:

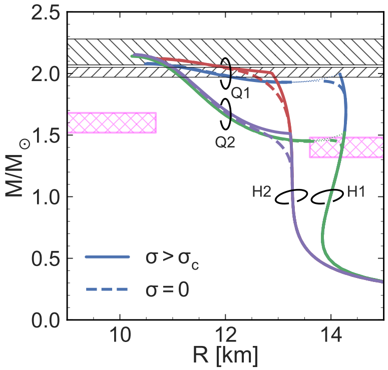
1. Prepare a Maxwell construction and the Glendenning construction $\Leftrightarrow P_c,\ \Delta_P^{(0)}$
2. Find $\sigma_c \simeq 30\mathrm{MeV/fm}^2 + 0.28(P_c - 40\frac{\mathrm{MeV}}{\mathrm{fm}^3})$
3. Use $\Delta_P = \Delta_P^{(0)} S(\sigma/\sigma_c;\bar{\beta})$ as the input for the interpolating construction $P = P(\mu, \Delta_P)$ to estimate the effect of the pasta for a given σ.

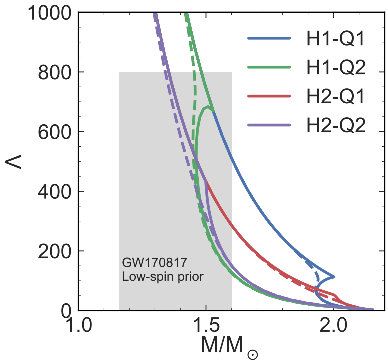
Still to be checked within different model classes

Δ_P^0 should be certainly dependent on the EoS – symmetry energy mostly

Compare the Maxwell case (solid) with the maximum Δ_P case (dashed)

- H1 model: The radius difference shrinks, but both high-mass and low-mass twins survive the inclusion of the pasta phases
  Consistent with [A.Ayriyan et al. Phys.Rev. C97 (2018)]

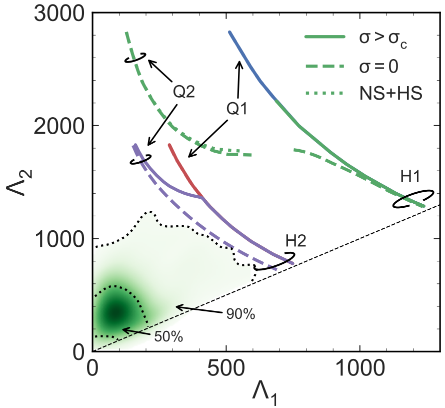




Tidal deformabilities

Can be constrained using gravitational-wave signals

Chirp mass $\mathcal{M} \equiv \frac{(M_1 M_2)^{3/5}}{(M_1 + M_2)^{1/5}} = 1.188\, M_\odot$ for GW170817



- Models not passing the constraint still can be reconciled with it by the mixed phase formation in the PT
- Inclusion of mixed phase $\Rightarrow$ the GW170817 could be a HS-HS merger

solid – Maxwell, dashed – bulk Gibbs
dotted line: NS-HS with Maxwell H1-Q2

Summary

Results and conclusions

- The effect of the structures can be approximately described by a simple phenomenological construction
- We found the critical surface tension for a class of models with inclusion of charge screening; consistent with the analytic result
- Critical surface tension is proportional to the Maxwell construction pressure
- Signs of a universal behavior – possible recipe to reproduce the effect of the pasta for an arbitrary EoS pair

Properties of hybrid stars

- Maximum possible effect of the structures does not destroy the third family
- GW170817 could be produced by two hybrid stars with mixed phase inside
- Mixed phase helps to describe $\Lambda_1 - \Lambda_2$ constraint

Outlook

- Inclusion of strangeness
- Dependence on the symmetry energy
- Verification for other classes of models
- Calculation of the surface tension – still needed to make a decisive conclusion