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# Heavy-ion physics:
# Interplay of statistical and field-theoretic approches

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# Motivation

15 years of discovery of Quark-Gluon-Plasma (QGP), or substance with lowest viscosity ($\eta/s$ ratio) ever observed.

The discovery triggered reshuffle of theoretical disciplines, Emphasizing theory of relativistic quantum fluids in extremal conditions (rotation, accelaration...)

Traditionally, heavy-ion physics belongs to statistical theory
However, 'rotation, acceleration' belong to GR

Literature on the interplay of the two approaches is huge
Recollect, e.g., Luttinger's remark (1964),

$$\vec{a} \ \rightarrow \ -\frac{\vec{\nabla} T}{T}$$

# Outline of the talk

'kinematical acceleration' enters density operator $\hat{\rho}$
'gravitational acceleration' enters metric $g_{\mu\nu}$

Look for matching of one description with the other,
Or, trying to map heavy-ion physics into black-hole physics

Two parts:

I. From gauge chiral anomaly to chemical potential
II. From grav. chiral anomaly to horizon

First part is mostly review, second part is mostly due to collaboration with G.Yu. Prokhorov and O.V. Teryaev.
References are mostly omitted in the talk.

# From quantum anomaly to macroscopic flow

Chapter on ‘quantum-anomalous fluids’
started with (T.D. Son+P. Surowka (2009))
Hydro is a universal framework, based on conservation laws and gradient expansion. The chiral anomaly enters through:

$$\partial_\alpha J^{5,\alpha} \;=\; \alpha_{el}(const)F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Applying standard machinary of hydro

$$J^{5,\alpha} \;=\; n^5 u^\alpha \;+\; \frac{\mu^2}{2\pi^2}\epsilon^{\alpha\beta\gamma\delta}u_\beta\partial_\gamma u_\delta \;+\; O(\sqrt{\alpha_{el}})$$

where $n^5$ is density of microscopic carriers, $u^\alpha$ is 4-velocity of fluid, $\mu$ is the chemical potential, viscosity $\eta$ is neglected
Second term in r.h.s. is the Chiral Vortical Effect (CVE):

$$\vec{J}^5 = \mu^2(const)\vec{\Omega}$$

# Standard statistical approach

Started by A. Vilenkin (1980) , in case of rotation:

$$< \hat{J}^\mu(\vec{x}) > = \ Tr\Big(\hat{\rho}\hat{J}^\mu(\vec{x},t)\Big)$$

$\hat{\rho} = \frac{1}{Z} e^{-\beta\left(\hat{H} - \vec{\Omega}\hat{\vec{M}} - \mu\hat{N}\right)}$ with "effective" $\delta\hat{H} \ \sim \ \vec{\Omega}\vec{M} + \mu\hat{Q}$

$< J^5 > = \int_{-\infty}^{+\infty} \frac{\epsilon^2 d\epsilon}{4\pi^2}\Big(\big(\frac{1}{(1+e^{\beta(\epsilon-(\mu+\Omega/2))})} - \frac{1}{(1+e^{\beta(\epsilon-(\mu-\Omega/2))})}\Big)$

$< \vec{J}(0) > = \vec{\Omega}\big(\mu^2/(4\pi^2) + T^2/12\big)$ i.e. same ChVE

For energy density of massless particles similar expression:

$$< T_{00} > = .... \int_{-\infty}^{+\infty} \frac{\epsilon^3 d\epsilon}{4\pi^2} ....$$

# Trading Statistics for Effective FT

In thermodynamics one introduces

$\delta \hat{H} \ = \ \mu \hat{Q} \ + ...$

No actual interaction, rather a trick to maximize entropy

No problem to promote, at a price, $\delta \hat{H}$ to a "local" 4d FT:

$\delta L \ = \ \mu u^{\alpha} j_{\alpha}$ (Sadofyev et al. (2011)

4-velocity $u_{\alpha}$ plays a role of external field

$$eA_{\alpha} \ \rightarrow \ eA_{\alpha} + \mu u_{\alpha}$$

Starting from the chiral anomaly $\partial_{\alpha} J^{5,\alpha} = (const) F\tilde{F}$

reproduce the $J^5$ currenti, including CVE

# Specific features of statistical effective FT

In the original field theory

$$Q^5_{conserved} = Q^5_{naive} + \mathcal{H}_{magn\ hel}$$

where $\mathcal{H}_{magn\ hel.} \sim \alpha_{el} \int d^3x \vec{A} \cdot \vec{H}$

In hydro extra terms:

$$\Delta Q^5_{conserved} = \mathcal{H}_{fluid\ hel.} + \sqrt{\alpha_{el}} \left( \mathcal{H}_{fluid-magn} + \mathcal{H}_{magn-fluid} \right)$$

where $\mathcal{H}_{fluid-hel} \sim \int d^3x \vec{v} \cdot [\vec{\nabla} \times \vec{v}]$

For consistency of theory extra conservation laws.
Neglecting $\alpha_{el}$:

$$\frac{d\mathcal{H}_{fluid\ hel}}{dt} = 0$$

'External field' $u_\alpha$ is not arbitrary, but constrained

# From Theory to Phenomenology

- Ideal-fluid solution to the constraint:

  $$\Sigma\big(\mathcal{H}_{fluid} + \mathcal{H}_{fluid-magn} + \mathcal{H}_{magn-fluid} + \mathcal{H}_{magn}\big) = const$$
  $$Q^5_{naive} = const'$$

  astro-applicatuions
- $ChVE \ = \ \Sigma(Spin\ of\ the\ cores\ of\ vortices)$

  true in case of superfluid. Heavy d.o.f = core of vortices
- Historically first attemt to evaluate polarization of heavy particles:

  O. Rogachevsky, A. Sorin, O. Teryaev “Chiral vortaic effect and neutron asymmetries in heavy-ion collisions” PHYSICAL REVIEW C 82, 054910 (2010).

# Terms of first order in acceleration

In flat space ($\alpha_{el} = 0$) extra conservation law:
$\partial_\alpha J^\alpha_{vortical} = 0$
In curved space becomes
$\nabla_\alpha \mathcal{J}^\alpha_{vortical} = 0$
which again can be rewritten in flat space as
$\partial_\alpha J^\alpha_{vortical} \neq 0$
In particular, in our case:
$\partial_\alpha J^{\alpha,5} = (const)\mu^2(\vec{a}\cdot\vec{\Omega})$
In this way we rediscover 'gravimagnetic anomaly':
$\partial_\alpha J^{\alpha,5} = (const)(\vec{B}_g \cdot \vec{E}_g)$ (D.Kharzeev et al. (2013))

In our language this is no anomaly at all, but rather analogy to $a_{kin} = a_{grav}$

# Part II. Strong gravity. Density operator

As a response to phenomenology, novel density operator (beyond Landau & Lifshitz) is highlighted (F. Becattini et al. (2017))

$$\hat{\rho} \; = \; \frac{1}{Z}\left(-b_\alpha \hat{P}^\alpha + \frac{1}{2}\varpi_{\alpha\beta}\hat{J}^{\alpha\beta} + \xi\hat{Q}\right)$$

$$\varpi_{\alpha\beta}\hat{J}^{\alpha\beta} \; = \; -2\alpha_\rho \hat{K}^\rho - 2w^\rho \hat{J}_\rho$$

$T \cdot \alpha_\rho$ is 4-vector of acceleration, $T \cdot \alpha_\rho \equiv u^\sigma \partial_\sigma u_\rho$

$T \cdot w^\rho$ is vorticity, $T \cdot w^\rho \equiv \frac{1}{2}\epsilon^{\rho\sigma\alpha\beta}u_\sigma \partial_\alpha u_\beta$

$\hat{K}^\rho$ is boost operator

$\hat{J}^\rho$ is the angular momentum operator

Boost operator is conserved but does not commute with $\hat{H}$

Much more time consuming calculations

# Acceleration as imaginary chemical potenial

For massless spin-1/2 particles rotation is reduced to an extention of chemical potential:

$$\mu \;\to\; \mu \;\pm\; \frac{\Omega}{2}$$

Adding acceleration $\vec{a} \neq 0$ involves boost operators $\hat{\vec{K}}$
However, if $\vec{a}\,||\vec{\Omega}$ non-commutativity can be avoided by introducing imaginary acceleration
Similar trick is in textbooks on FT:
$\hat{\vec{J}} \sim \hat{\sigma}/2, \hat{\vec{K}} \sim i\hat{\sigma}/2$

Then: $\mu \;\to\; \mu \;\pm\; \frac{\Omega}{2} \;\pm\; \frac{ia}{2}$

Expanded in $ia$, odd powers cancel

Prokhorov et al. (2018)

# Duality between statistics and field theory

Field theory: $\delta \hat{H} = \frac{1}{2} h_{\alpha\beta} \hat{T}^{\alpha\beta}$ ( $\Omega, a$ encoded in $h_{\alpha\beta}$ )

Statistics: $\Omega, a$ encoded in $\hat{\rho}$.

Duality: coincidence of final results as function of $a, \Omega$

Note: statistical approach is in flat space,
gravitational on non-trivial space (with horizon)

# An example of duality

Vacuum energy $T_{00}$ as function of independent $a, T$,

exploiting 'novel $\hat{\rho}$':

$$\epsilon_{vac} \;=\; \frac{7\pi^2 T^4}{60} \;+\; \frac{T^2 a^2}{24} \;-\; \frac{17 a^4}{960\pi^2}$$

Explicitly: $\epsilon_{vac}(T_U) \;=\; 0$

First ever evaluation of $\epsilon_{vac}$ without subtractions.
Standard way: impose $\epsilon_{vac}(T_U) \;=\; 0$

First, one-loop exact, evaluation
of the Unruh temperature statistically.

# On the other side of duality

The energy density in geometrical terms

Rindler space with conical singularity (see below) :

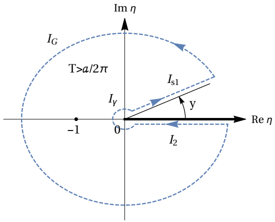
$$\epsilon_{vac} \;=\; \frac{1}{2}\left(\frac{7\pi^2 T^4}{60} \;+\; \frac{T^2}{24r^2} \;-\; \frac{17}{960\pi^2 r^4}\right)$$

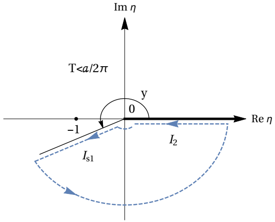
where $r$ is a geometric quantity (distance along the cone).
Exact duality with statistical approach in this case

However, the two approaches give typically not identical but rather complementary results

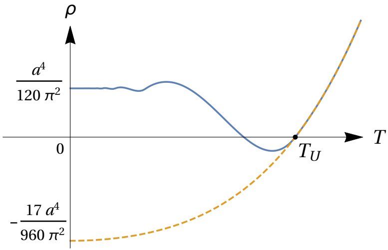
# Instability at $T < T_U$

In terms of complex plane integrals

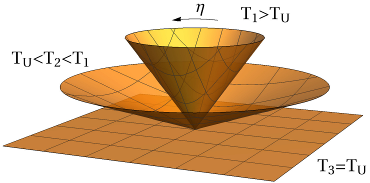



Energy density as a function of temperature




From the geometrical point of view



# Comments to the preceding slide

First ever evidence for statistical instability at $T < T_U$:
Behaviour of the energy density immediately below $T = T_U$ has been studied using the analyticity in the plane of generalized chemical potential, $Im\mu = a/2$.

With stretch of imagination,
Instability could be related to thermolization

Results for $T < T_U$ are in striking similarity with results for Black Holes obtained within field theory by Polyakov et al (1803.09168)

# Gravitational anomaly via duality

Axial current:

$$J^5 \ \sim \ c_T T^2 \Omega + c_a a^2 \Omega$$

$c_T$ fixed by statistical approach,
$c_a$ is fixed by the FT (anomaly)

Check of the consistency: vanishing of the current at $T_U$

By changing $T$ from $T \to \infty$ down to $T = T_U$
we tune to the horizon

# Conclusions

- Hydrodynamic analog of the gauge chiral anomaly has found its way to phenomenology of heavy-ion collisions
- Interplay of statistical and gravitational approches might allow to relate phenomenology of heavy-ion collisions to BH physics