Transport properties of the hot and dense QGP

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In collaboration with David Fuseau, Joerg Aichelin

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"Hot problems of Strong interactions"
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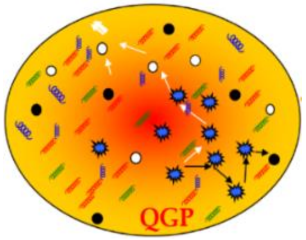

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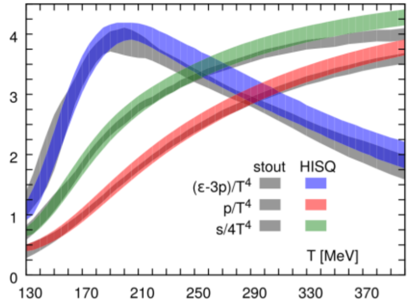
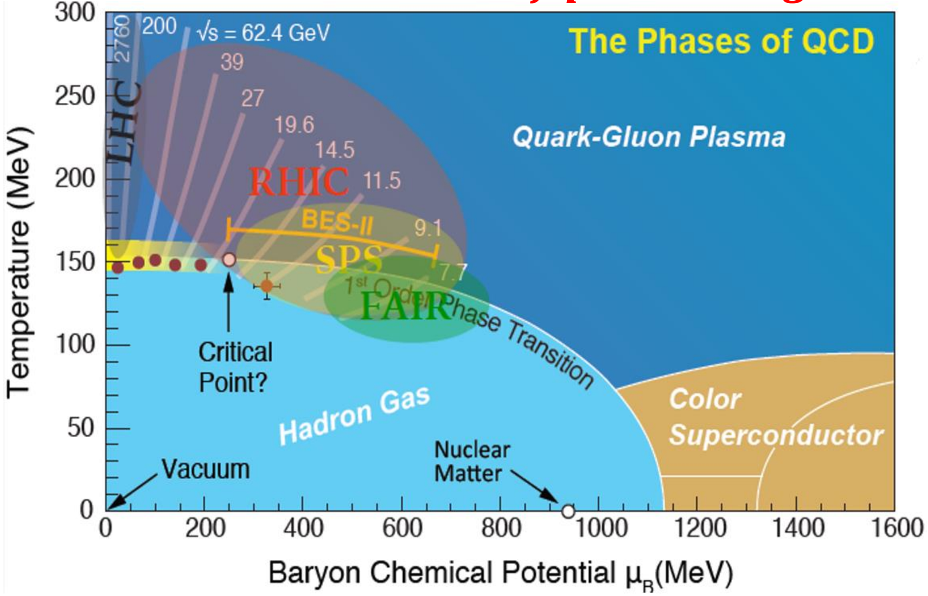
Motivation: QGP at finite baryon density

- **Explore the QCD phase diagram at finite temperature and chemical potential through heavy-ion collisions**
- **Available information:**
  - **Experimental data at SPS, BES at RHIC**
  - **Lattice QCD calculations**



EoS

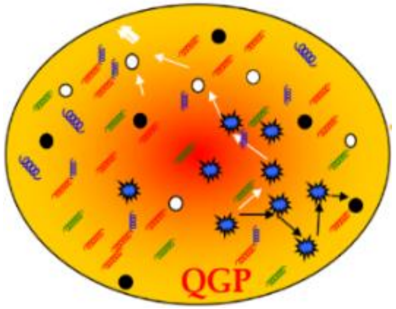
Theoretical scketch of phase diagram



❖ **How to learn about degrees-of-freedom of QGP ? ➔**

HIC simulations – transport models

! Transport models need an input for the partonic phase: cross-sections, masses,..



QGP in equilibrium: DQPM and PNJL

Transport coefficients at finite T and μ_B

1.) crossover (DQPM model)

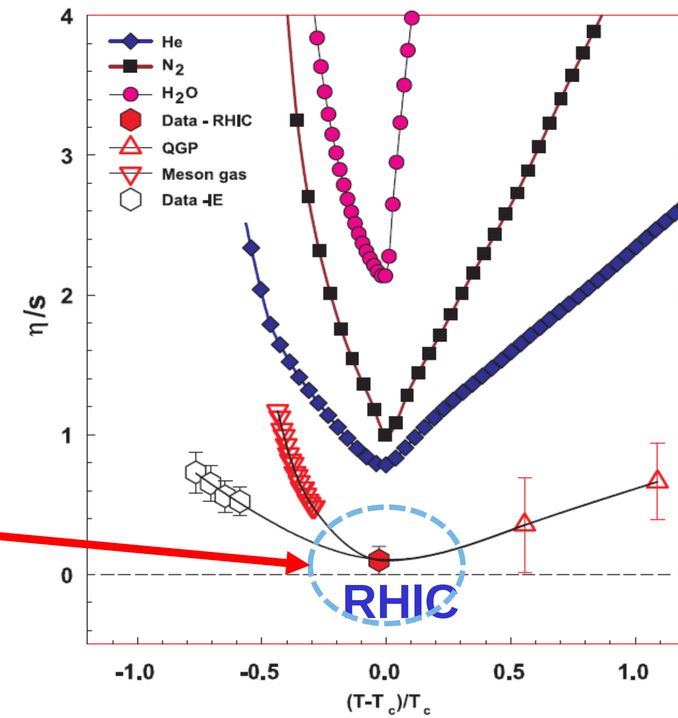
2.) CEP and 1st order phase transition (PNJL model)

Transport coefficients of QGP: model predictions

Hydrodynamical model

$$\Delta T^{\mu\nu} = \boxed{\eta}\left(D^{\mu}u^{\nu} + D^{\nu}u^{\mu} + \tfrac{2}{3}\Delta^{\mu\nu}\partial_{\rho}u^{\rho}\right) - \boxed{\zeta}\Delta^{\mu\nu}\partial_{\rho}u^{\rho}$$

input for hydro simulations

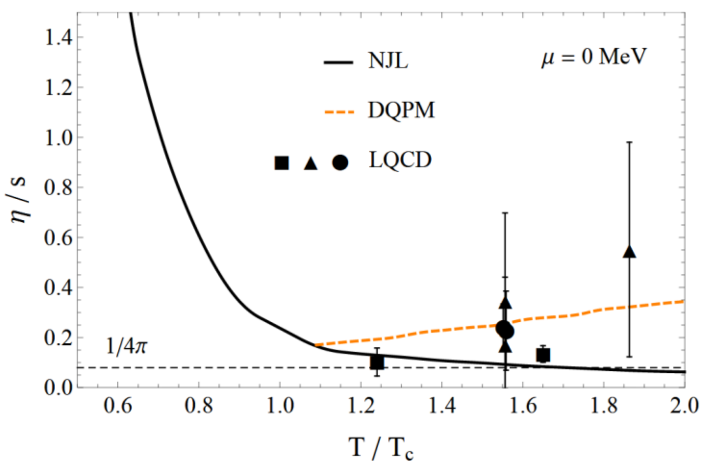


R.A. Lacey, A. Taranenko, PoS CFRNC 2006 (2006) 021

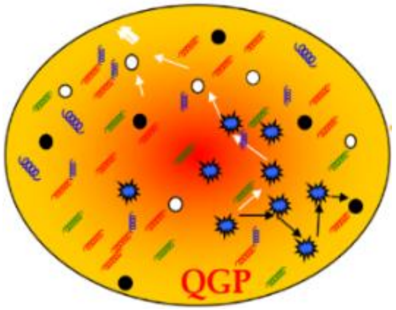
Shear viscosity to entropy density ratio is extremely small

QGP is the most ideal liquid!

Model predictions:



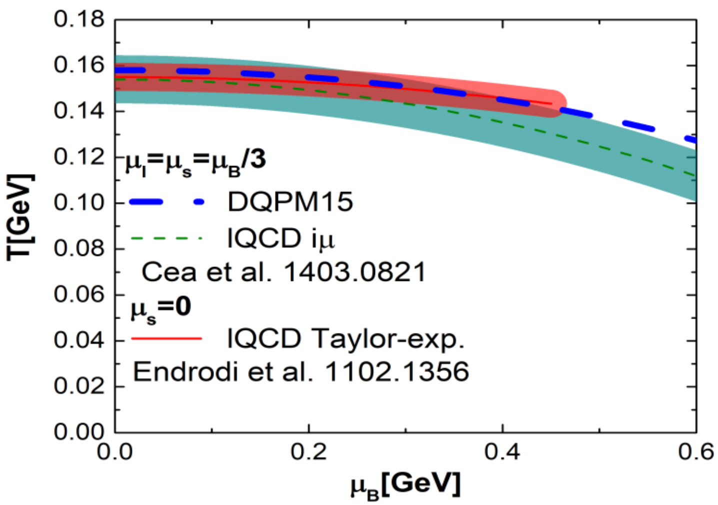
! Different models using the same EoS can have completely different transport coefficients!



QGP in equilibrium:

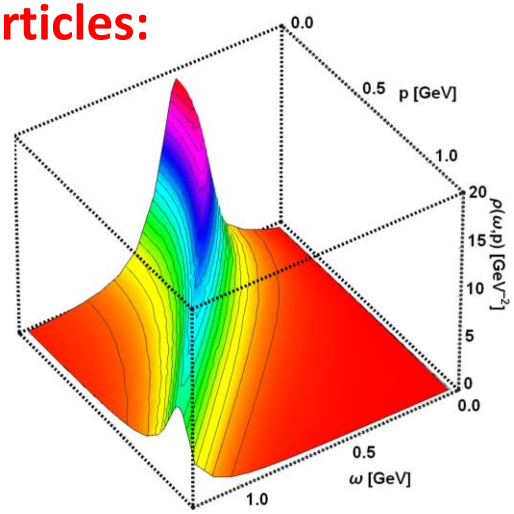
Dynamical QuasiParticle Model (DQPM)

DQPM: consider the effects of the nonperturbative nature of the strongly interacting quark-gluon plasma (sQGP) constituents (vs. pQCD models)



Dynamical QuasiParticle Model (DQPM)

- **The QGP phase is described in terms of interacting quasiparticles: quarks and gluons with Lorentzian spectral functions:**



$$\rho_j(\omega,\mathbf{p}) = \frac{\gamma_j}{\tilde{E}_j}\left(\frac{1}{(\omega-\tilde{E}_j)^2+\gamma_j^2} - \frac{1}{(\omega+\tilde{E}_j)^2+\gamma_j^2}\right)$$
$$\equiv \frac{4\omega\gamma_j}{\left(\omega^2-\mathbf{p}^2-M_j^2\right)^2+4\gamma_j^2\omega^2}$$

- **Resummed properties of the quasiparticles are specified by scalar complex self-energies:**

gluon propagator: $\Delta^{-1} = P^2 - \Pi$	&	**quark propagator** $S_q^{-1} = P^2 - \Sigma_q$
gluon self-energy: $\Pi = M_g^2 - i2\gamma_g\omega$	&	**quark self-energy:** $\Sigma_q = M_q^2 - i2\,\gamma_q\omega$

  - **Real part of the self-energy: thermal mass** (M_g, M_q)
  - **Imaginary part of the self-energy: interaction width of partons** (γ_g, γ_q)

Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

Parton properties

- **Modeling of the quark/gluon masses and widths (inspired by HTL calculations)**

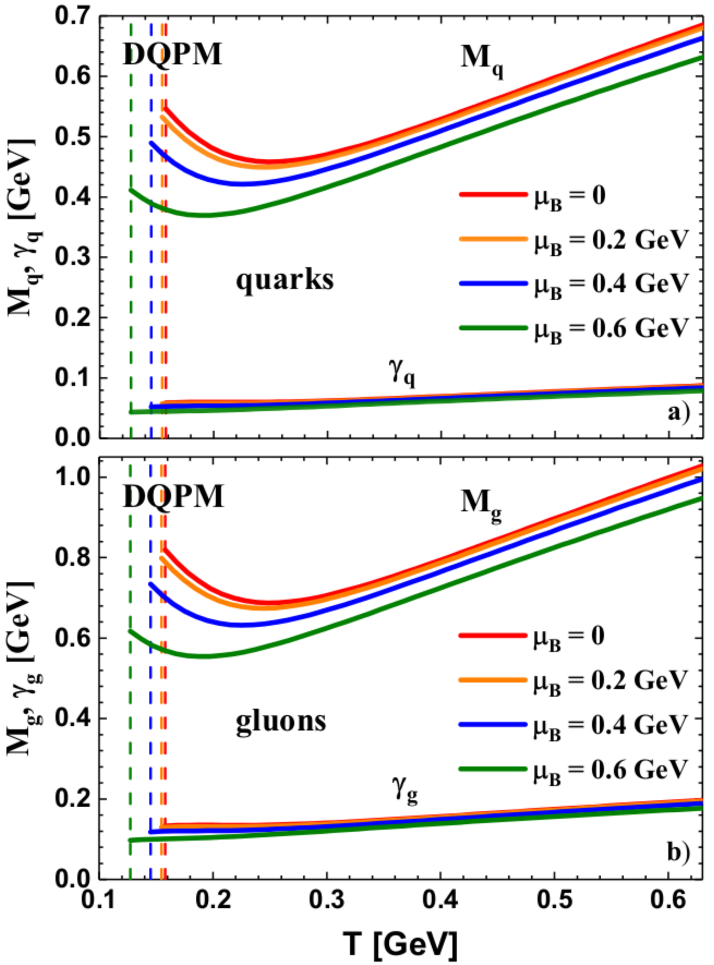
$$M^2_{q(\bar{q})}(T,\mu_B) = \frac{N_c^2-1}{8N_c} g^2(T,\mu_B) \left(T^2 + \frac{\mu_q^2}{\pi^2}\right)$$

$$\gamma_{q,g}(T,\mu_B) = \frac{c_{A,F}}{3} \frac{g^2(T,\mu_B) T}{8\pi} \ln\left(\frac{2c}{g^2(T,\mu_B)} + 1\right)$$

- **Only one parameter (c = 14.4) + (T, μ_B)-dependent coupling constant to determine from lattice results**

- **Entropy and baryon density in the quasiparticle limit (G. Baym 1998, Blaizot et al. 2001)**

$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma), n_B^{DQPM}(\Pi, \Delta, S_q, \Sigma)$$



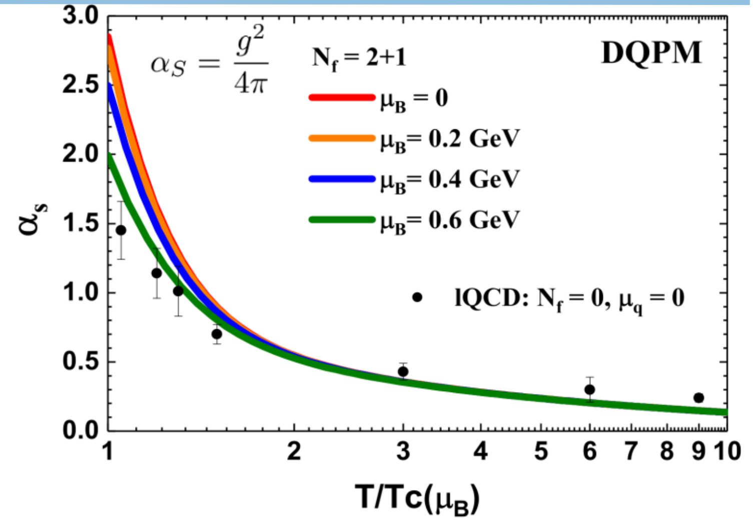
Peshier, Cassing, PRL 94 (2005) 172301; Cassing, NPA 791 (2007) 365: NPA 793 (2007)

DQPM coupling constant

- **Input: entropy density as a f(T , $\mu_B = 0$)**

$$g^2(s/s_{SB}) = d\left((s/s_{SB})^e - 1\right)^f$$
$$s_{SB}^{QCD} = 19/9\pi^2 T^3$$
$$s^{DQPM}(\Pi, \Delta, S_q, \Sigma) = s^{lattice}$$

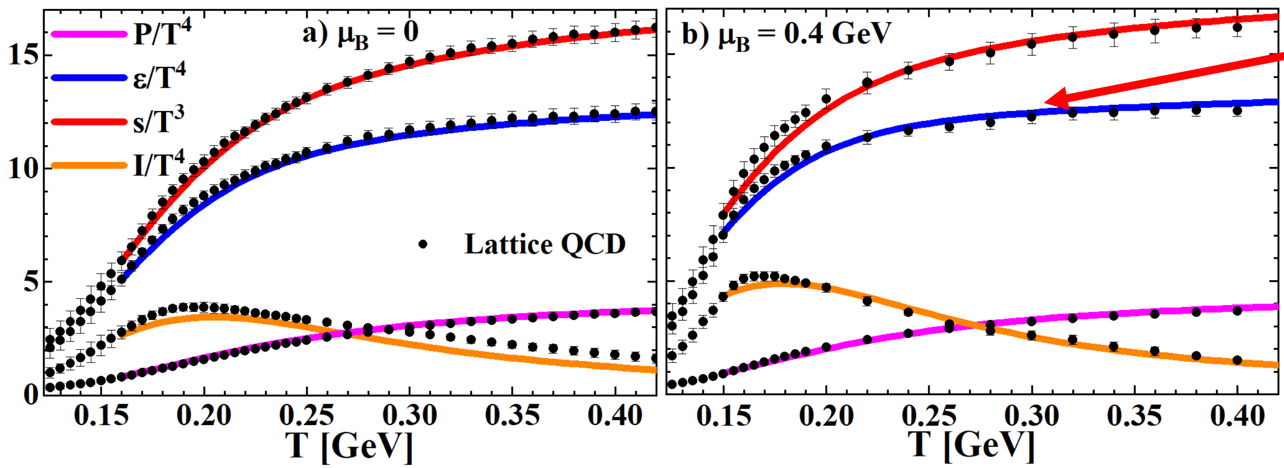
fit S from QP to S from lQCD ➔ fix the model parameters



- **Scaling hypothesis at finite $\mu_B \approx 3\mu_q$**

$$g^2(T/T_c, \mu_B) = g^2\left(\frac{T^*}{T_c(\mu_B)}, \mu_B = 0\right)$$ **with the effective temperature** $$T^* = \sqrt{T^2 + \mu_q^2/\pi^2}$$

Input: lattice EoS $\mu_B = 0$ (dots)



Output: (lines) DQPM EoS $\mu_B > 0$

Relaxation Time Approximation

- **Boltzmann equation** $f_a = f_a^{\rm eq}(1+\phi_a)$

$$\frac{df_a^{\rm eq}}{dt} = \mathcal{C}_a = -\frac{f_a^{\rm eq}\phi_a}{\tau_a}$$

RTA: system equilibrates within the relax time τ, Express collisional Integral via τ and f_a

- **Relaxation times:**

$$\frac{1+d_a f_a^{\rm eq}}{\tau_a(E_a^*)} = \sum_{bcd} \frac{1}{1+\delta_{ab}} \int d\Gamma_b^* \, d\Gamma_c^* \, d\Gamma_d^* \, W(a,b|c,d) f_b^{\rm eq} \left(1+d_c f_c^{\rm eq}\right)\left(1+d_d f_d^{\rm eq}\right) + (cd), (bc)$$

$$T^{\mu\nu} = -Pg^{\mu\nu} + wu^\mu u^\nu + \Delta T^{\mu\nu} \qquad J_B^\mu = n_B u^\mu + \Delta J_B^\mu$$

$$\Delta T^{\mu\nu} = \eta\left(D^\mu u^\nu + D^\nu u^\mu + \tfrac{2}{3}\Delta^{\mu\nu}\partial_\rho u^\rho\right) - \zeta \Delta^{\mu\nu}\partial_\rho u^\rho$$

$$\Delta J_B^\mu = \lambda \left(\frac{n_B T}{w}\right)^2 D^\mu \left(\frac{\mu_B}{T}\right)$$

hydrodynamics

Energy-momentum tensor and baryon diffusion current can be expressed using f_a :
$T^{\mu\nu}(f_a, m_{q,g}), J_B^\mu(f_a, m_{q,g})$

Obtain the transport coefficients using conservation laws, and f_a:

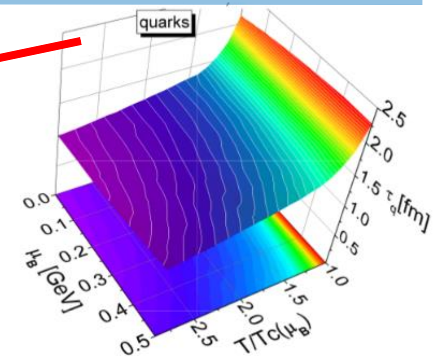
$$\begin{cases} \partial_\mu J_B^\mu = 0 \\ \partial_\mu T^{\mu\nu} = 0 \end{cases} \Rightarrow \eta^{\rm RTA}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B) \, d_i(1\pm f_i)f_i$$

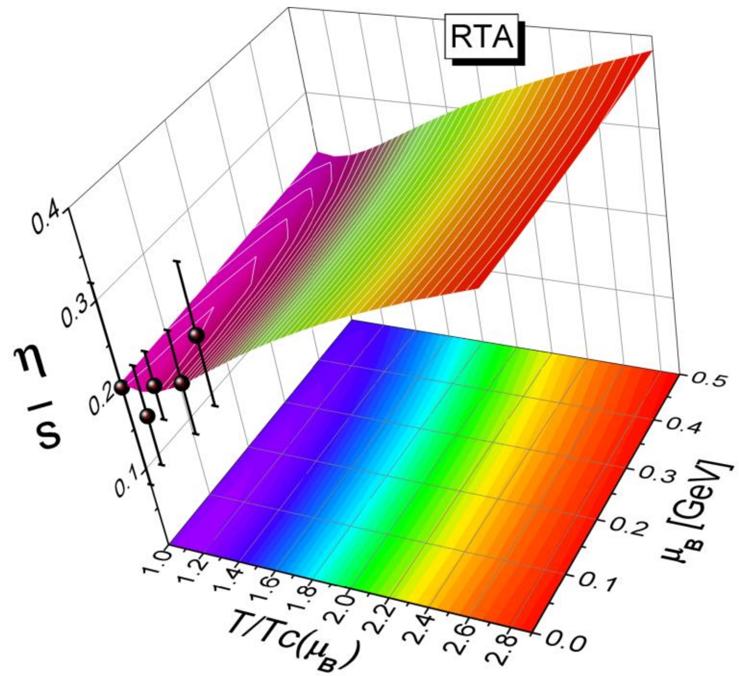
P. Chakraborty and J. I. Kapusta, PRC 83,014906 (2011).

Transport coefficients: shear viscosity

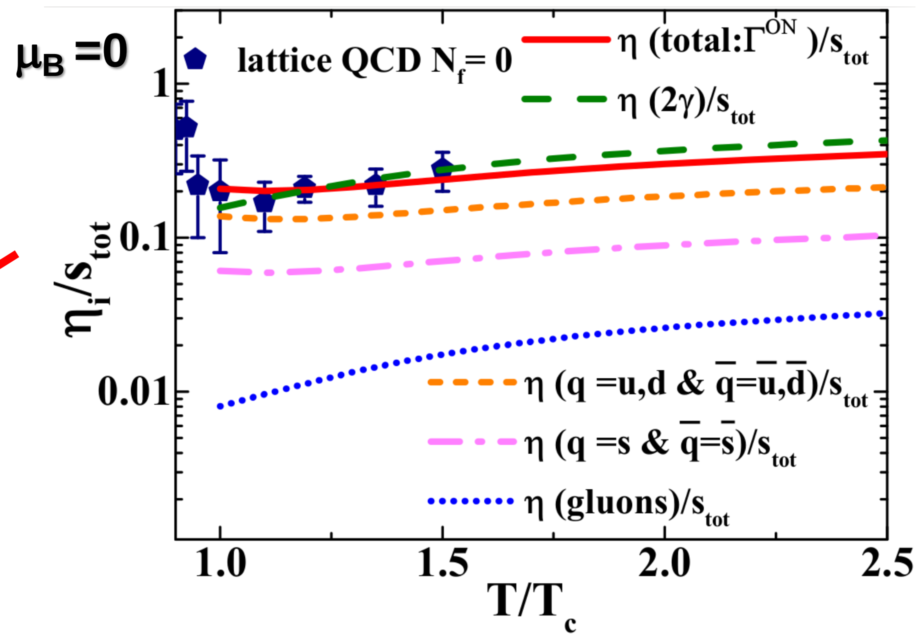
Relaxation times

$$\eta^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\tau_i(\mathbf{p},T,\mu_B)\, d_i(1\pm f_i)f_i$$





μ_B



Lattice: N. Astrakhantsev, V. Braguta, A. Kotov JHEP 1704 (2017) 101

- Main contribution comes from light quarks and anti-quarks

Transport coefficients: bulk viscosity

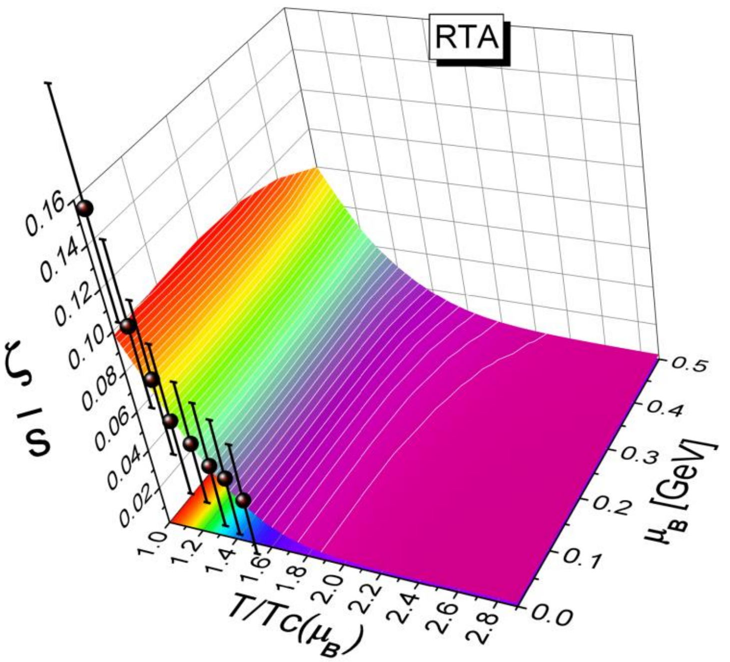
$$\zeta^{\rm RTA}(T,\mu_B) = \frac{1}{9T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\tau_i(\mathbf{p},T,\mu_B)\,\frac{d_i(1\pm f_i)f_i}{E_i^2}\left(\mathbf{p}^2 - 3c_s^2\left(E_i^2 - T^2\frac{dm_i^2}{dT^2}\right)\right)^2$$

Relaxation times

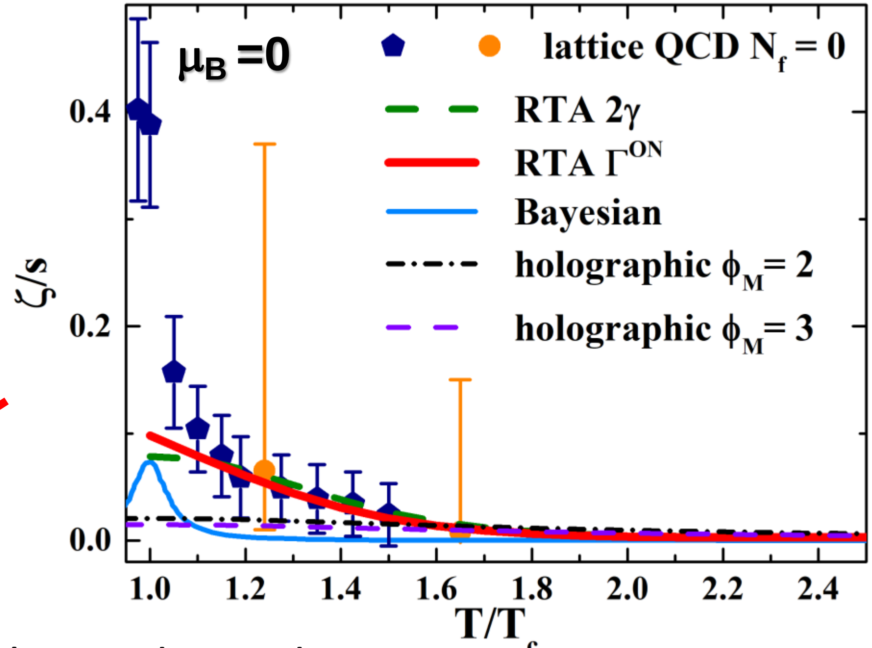
$$1)\,\tau_i(\mathbf{p},T,\mu_B) = \frac{1}{\Gamma_i(\mathbf{p},T,\mu_B)}$$

$$2)\,\tau_i(T,\mu_B) = \frac{1}{2\gamma_i(T,\mu_B)}$$

From DQPM parametrization



μ_B



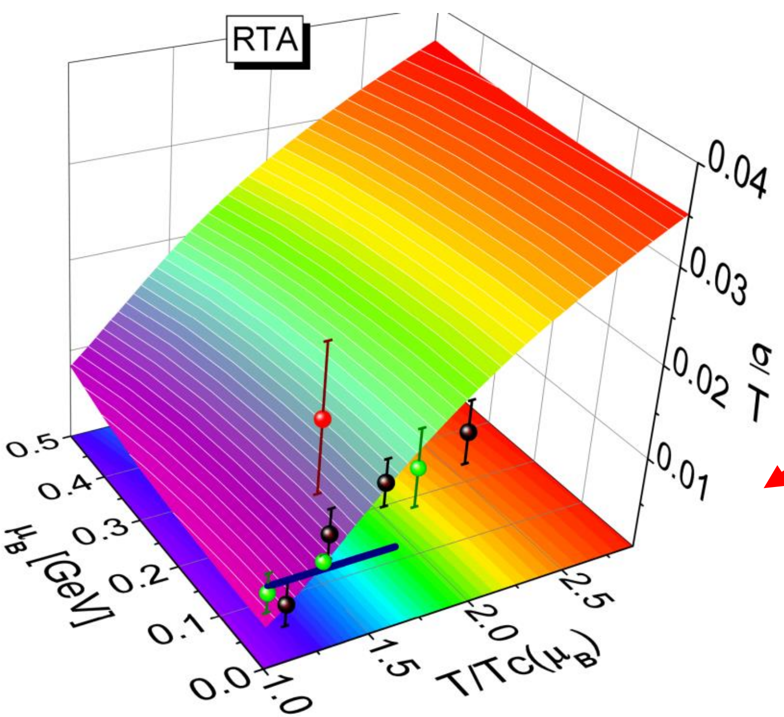
- weak μ_B dependence

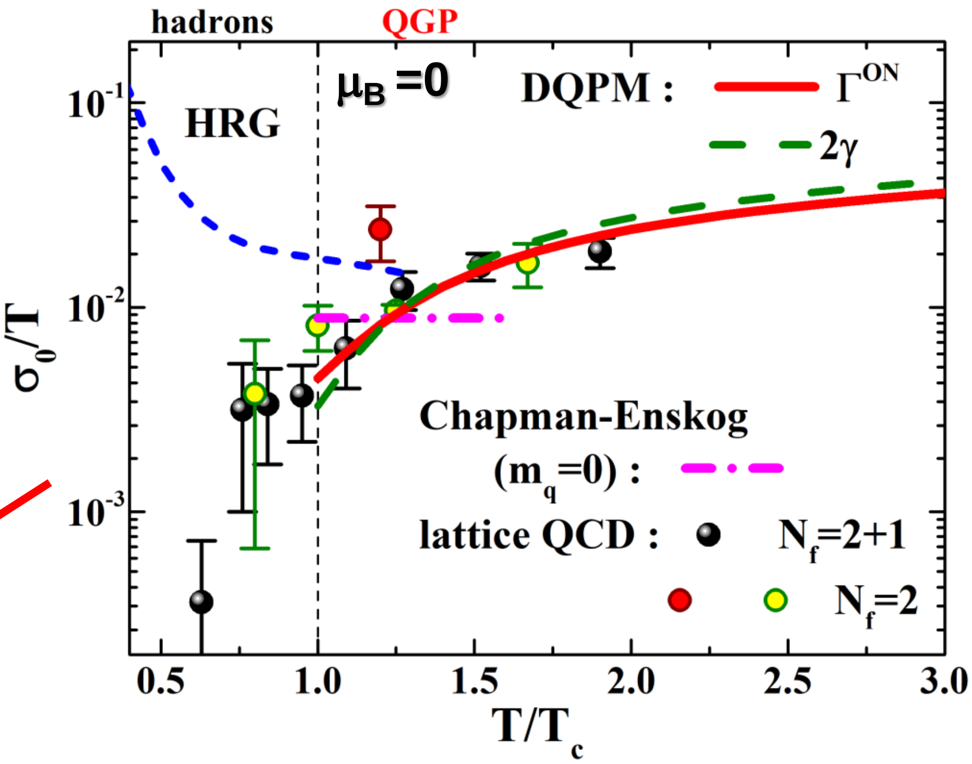
Lattice: N. Astrakhantsev et al, PRD 98 (2018) no.5, 054515
Holographic model: M. Attems et al,JHEP 10 (2016), 155

Transport coefficients: electric conductivity

$$\sigma_0^{\mathrm{RTA}}(T,\mu_B) = \frac{e^2}{3T}\sum_{i=q,\bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\,\tau_i(\mathbf{p},T,\mu_B)\,d_i(1-f_i)f_i$$

Relaxation times





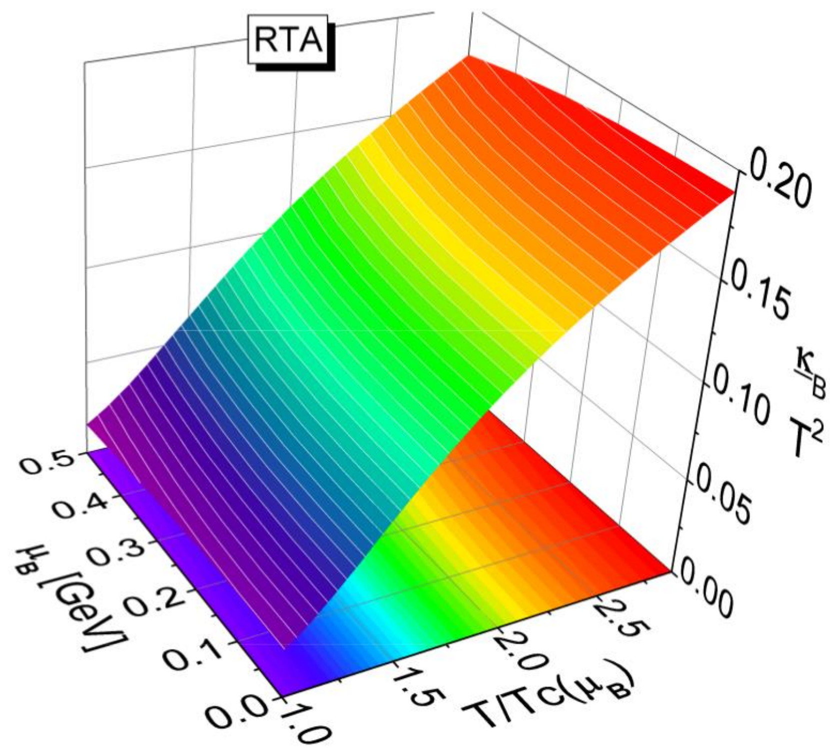
HRG: J. A. Fotakis et al, PRD 101 (2020) 7, 076007

Transport coefficients: baryon diffusion coefficient

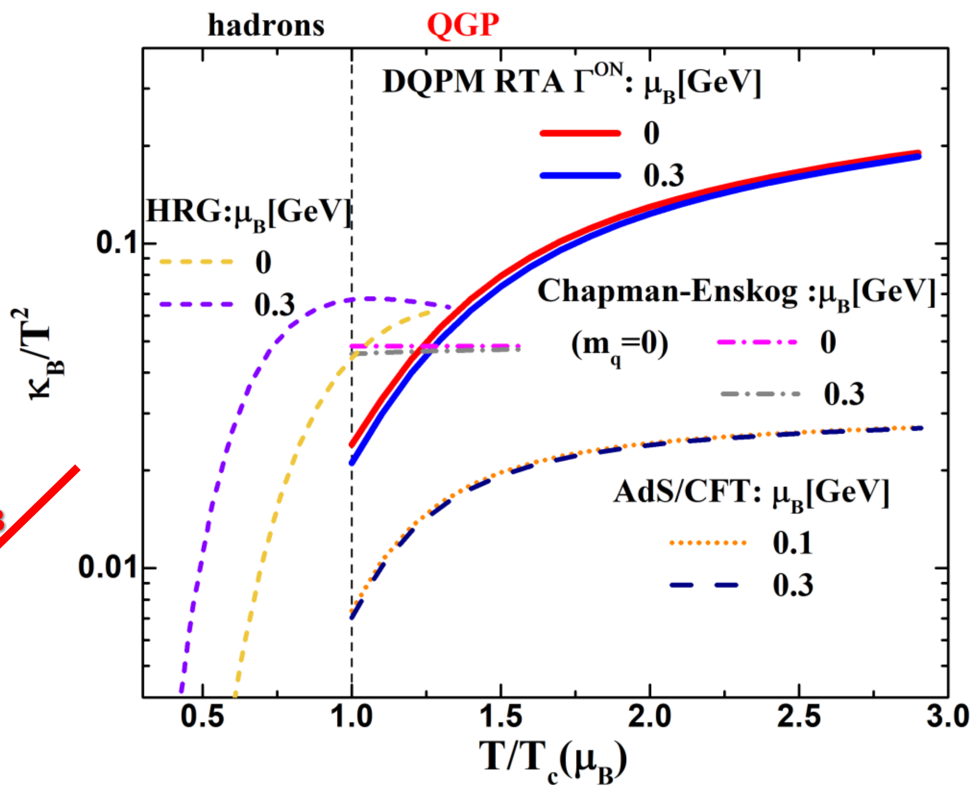
$$\kappa_B^{\mathrm{RTA}}(T,\mu_B) = \frac{1}{3}\sum_{i=q,\bar{q}}\int \frac{d^3p}{(2\pi)^3}\mathbf{p}^4 \tau_i(\mathbf{p},T,\mu_B) \frac{d_i(1\pm f_i)f_i}{E_i^2}\left(b_a - \frac{n_B E_i}{\epsilon + p}\right)^2$$

Relaxation times

DQPM EoS



μ_B



HRG: J. A. Fotakis et al, PRD 101 (2020) 7, 076007
AdS/CFT: T. Son and A. O. Starinets, JHEP 0603, 052 (2006)

Polyakov Nambu Jona-Lasinio (PNJL)

- **Effective lagrangian with the same symmetries for the quark dof as QCD**

$$\mathscr{L}_{PNJL} = \sum_i \bar{\psi}_i (iD - m_{0i} + \mu_i \gamma_0)\psi_i$$
$$+ G \sum_a \sum_{ijkl} \left[(\bar{\psi}_i \; i\gamma_5 \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \; i\gamma_5 \tau^a_{kl} \psi_l) + (\bar{\psi}_i \tau^a_{ij} \psi_j) \; (\bar{\psi}_k \tau^a_{kl} \psi_l) \right]$$
$$- K \det_{ij} \left[\bar{\psi}_i \; (-\gamma_5) \psi_j \right] - K \det_{ij} \left[\bar{\psi}_i \; (+\gamma_5) \psi_j \right]$$
$$- \mathcal{U}(T; \Phi, \bar{\Phi})$$

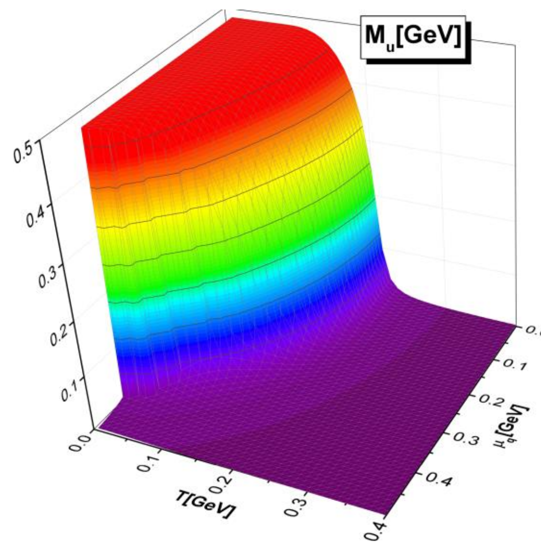
5 parameters fixed by vacuum values K,π masses, η-η'mass splitting , π decay constant,
Chiral condensate

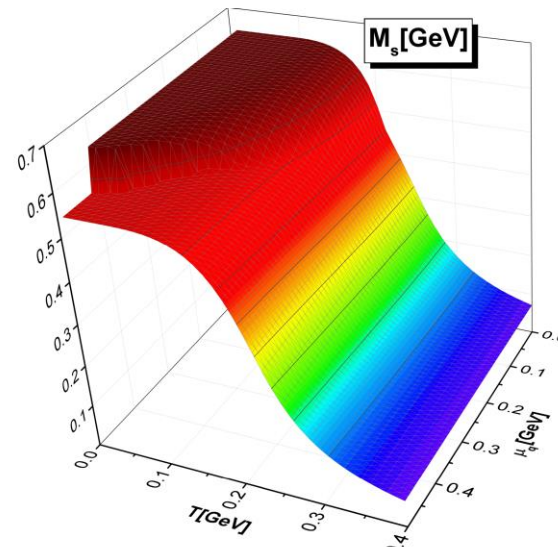
Polyakov potential fitted to the YM

D.Fuseau, T.Steinernert, J.Aichelin PRC 101 (2020) 6 065203

- **Gap equation + minimization of the grand potential → Chiral masses (M_l, M_s)**

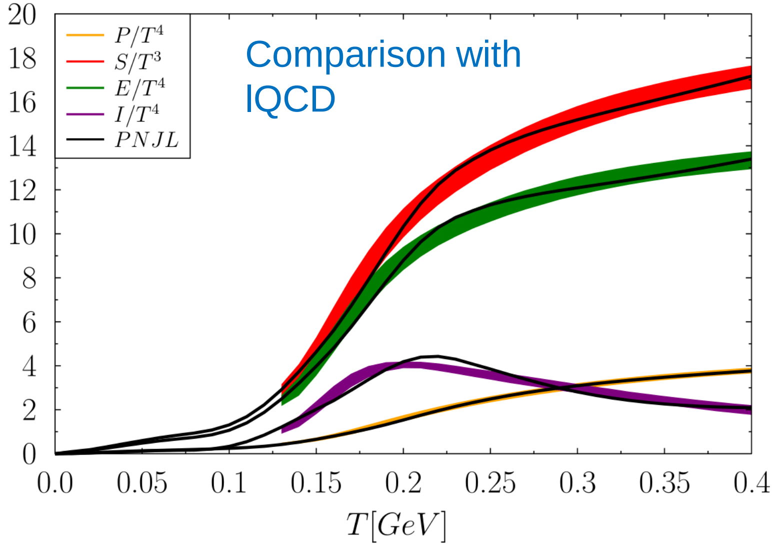
- **1[st] order PT at high μ_B (sudden change of q and meson masses)**





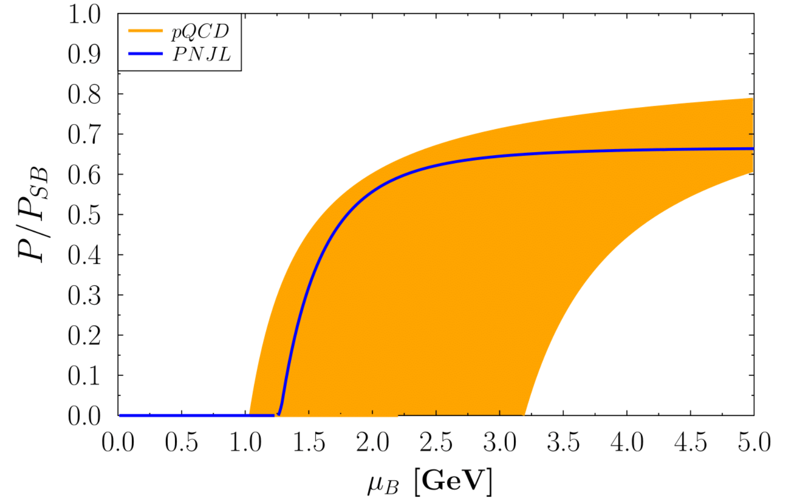
Polyakov Nambu Jona-Lasinio (PNJL):EOS

- **PNJL allow for predictions for finite T and μ_B:** D. Fuseau, T. Steinernert, J.Aichelin PRC 101 (2020) 6 065203
- **Parameters fixed, EoS at $\mu_B = 0$:**



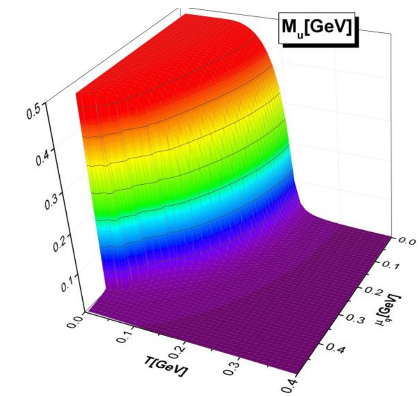
HotQCD Phys.Rev. D90 (2014) 094503

- **EoS at high μ_B :**



pQCD: A.Kurkela, A.Vuorinen, PRL 117 (2016)4 042501

- **CEP: (T , μ_B) = (110,960) MeV , μ_B/T = 8.73**
- **1st order PT at high μ_B (sudden change of q and**
- **meson masses)**

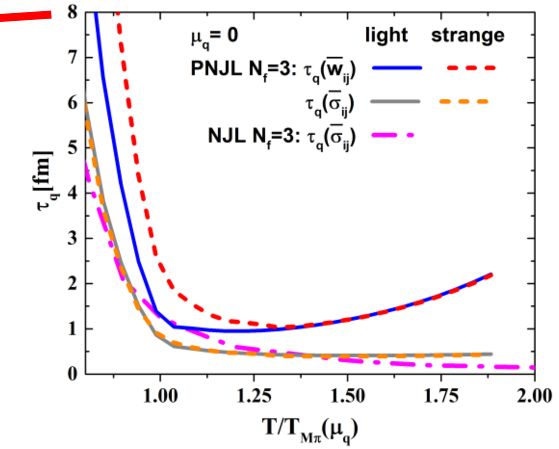


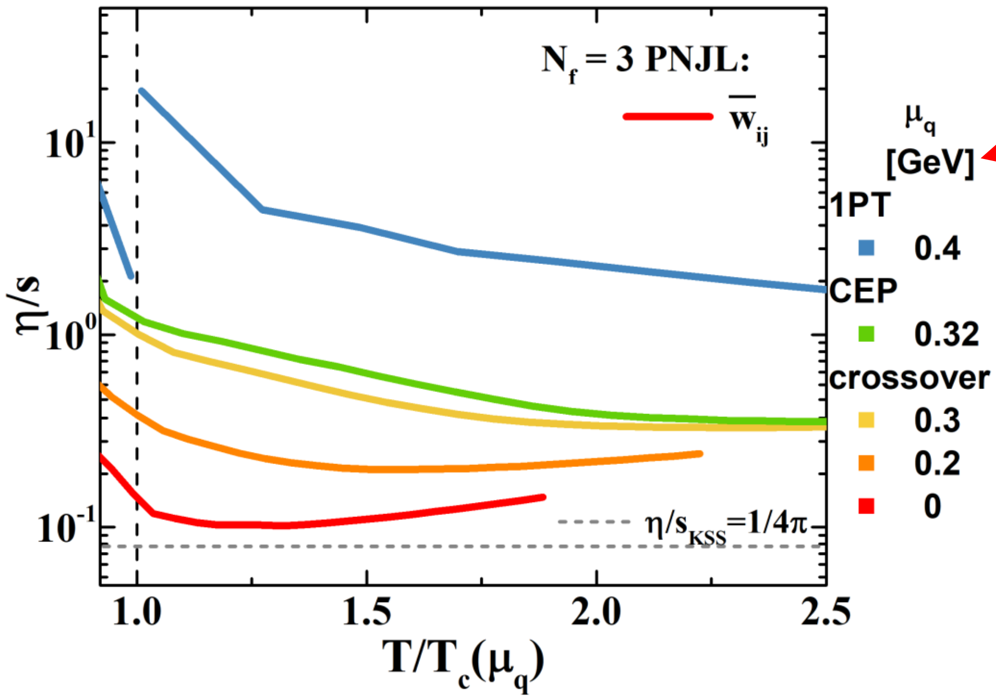
Specific shear viscosity at high μ_B

$$\eta^{\rm RTA}(T,\mu_B) = \frac{1}{15T}\sum_{i=q,\bar{q},g}\int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^4}{E_i^2}\,\tau_i(\mathbf{p},T,\mu_B)\,d_q f_i^{\phi}$$

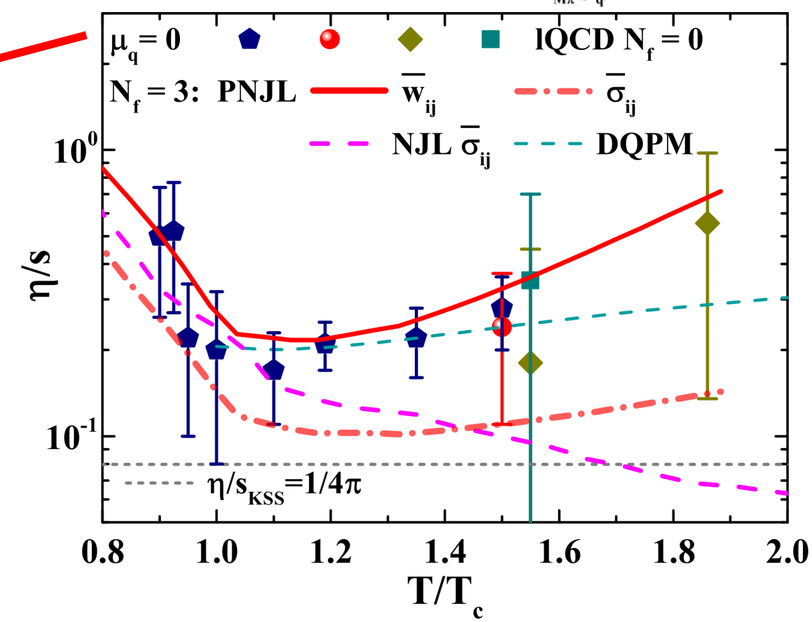
$$f_i^{\phi} = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1+3\phi e^{-(E_i\mp\mu)/T} + 3\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

with Polyakov loops





μ_B

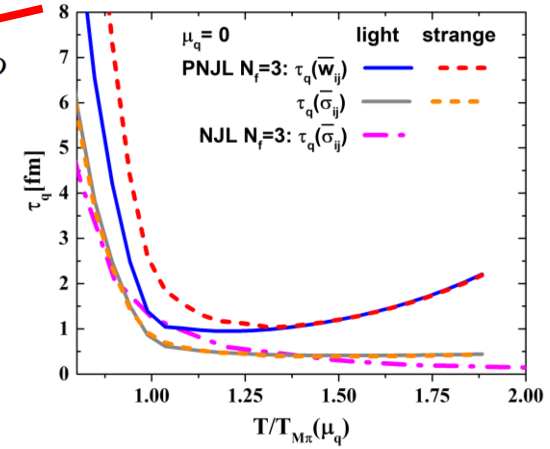


In agreement w Nf=2 NJL results C.Sasaki et al, NPA 832 (2010)

PNJL results:arXiv:2011.03505

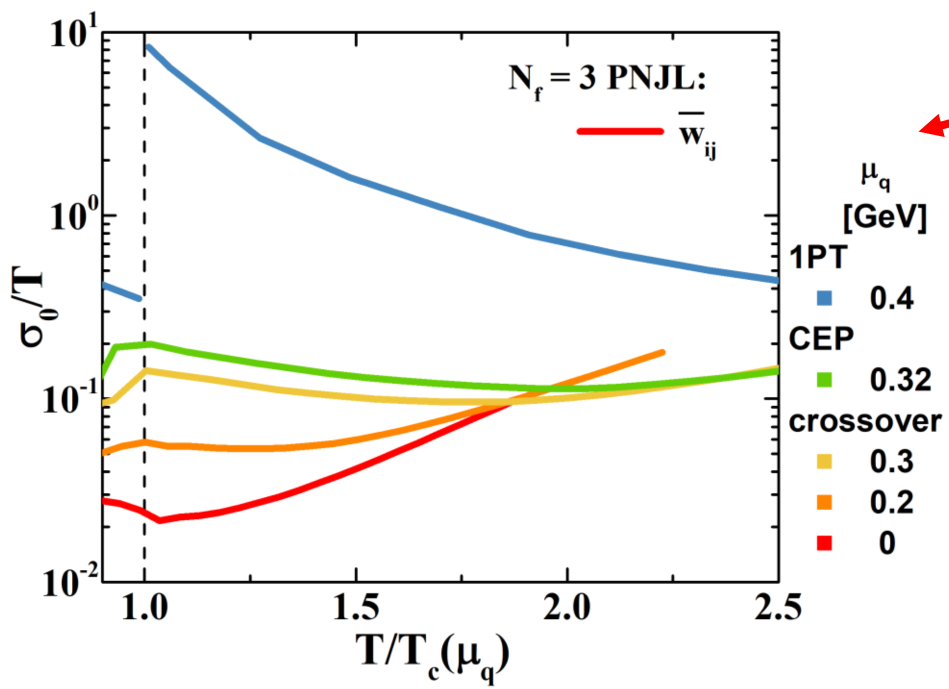
Electric conductivity at high μ_B

$$\sigma_0^{\mathrm{RTA}}(T,\mu_B) = \frac{e^2}{3T}\sum_{i=q,\bar{q}} q_i^2 \int \frac{d^3p}{(2\pi)^3}\frac{\mathbf{p}^2}{E_i^2}\,\tau_i(\mathbf{p},T,\mu_B)\, d_q f_i^{\phi}$$

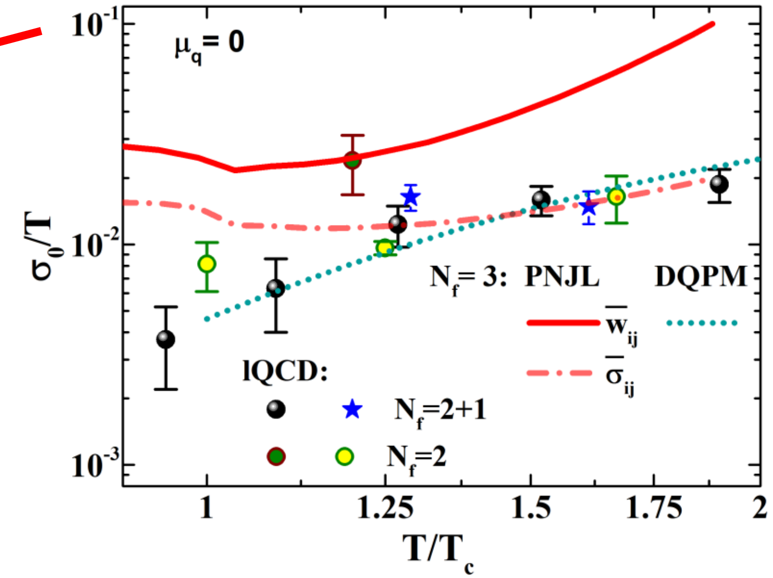


$$f_i^{\phi} = \frac{\phi e^{-(E_i\mp\mu)/T} + 2\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}{1 + 3\phi e^{-(E_i\mp\mu)/T} + 3\bar{\phi}e^{-2(E_i\mp\mu)/T} + e^{-3(E_i\mp\mu)/T}}$$

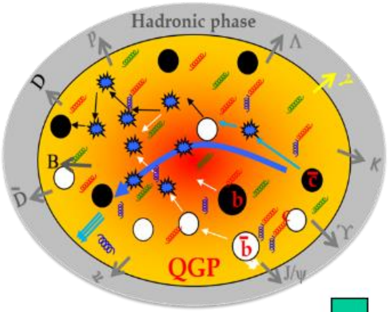
with Polyakov loops



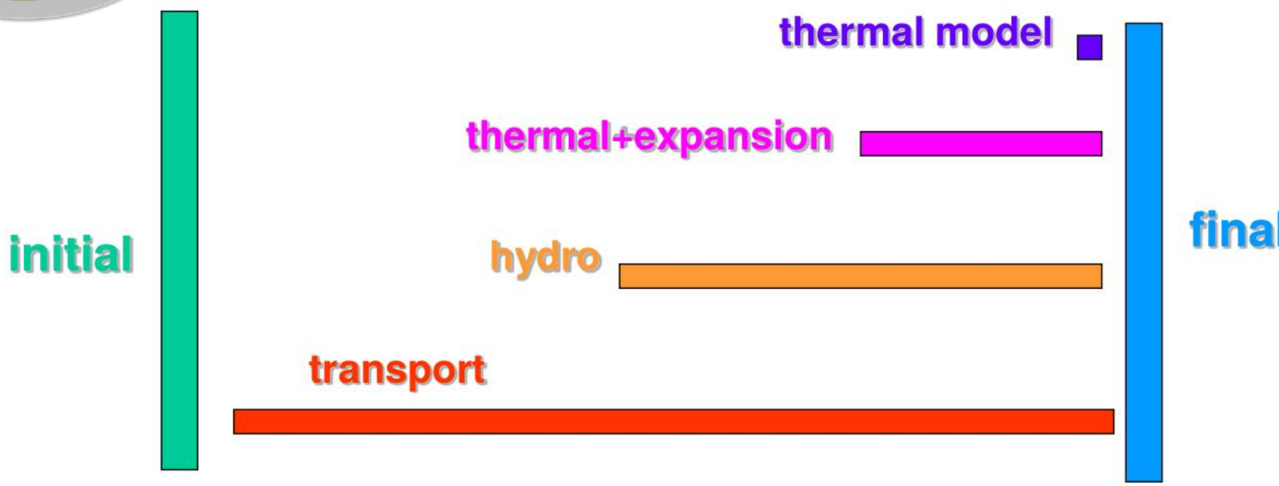
μ_B

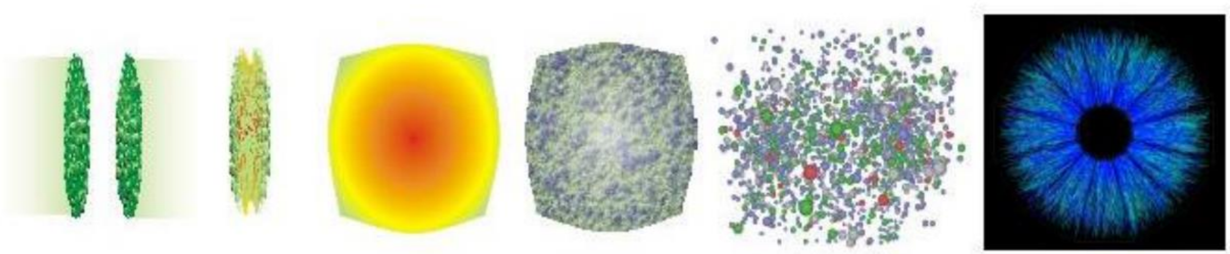


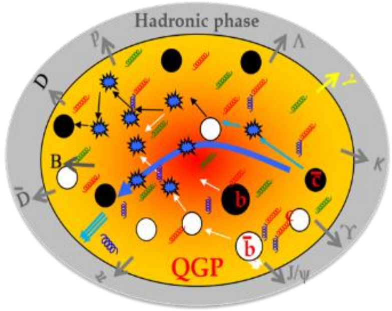
PNJL results:arXiv:2011.03505



QGP out-of equilibrium ←→ HIC



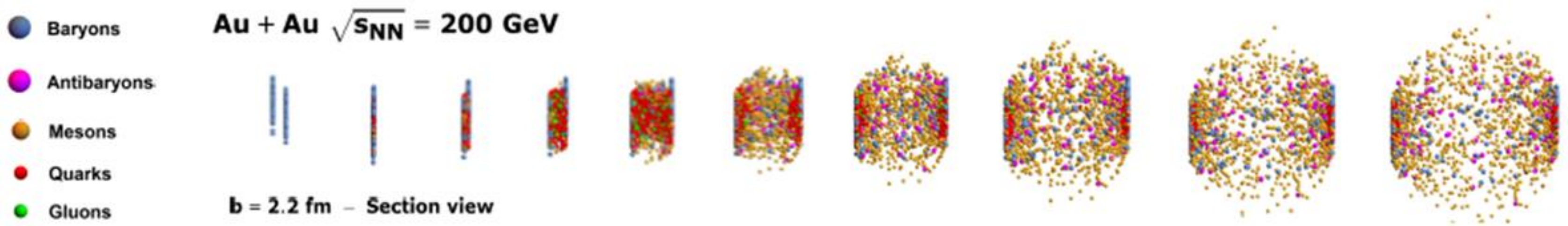




QGP out-of-equilibrium ←→ HIC

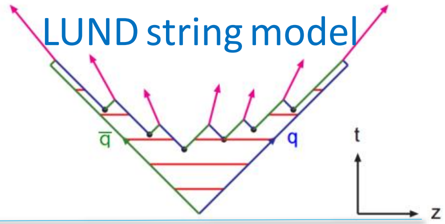
Parton-Hadron-String-Dynamics (PHSD)

- **Transport theory: off-shell transport equations in phase-space representation based on Kadanoff-Baym equations for the partonic and hadronic phase**



Stages of a collision in the PHSD

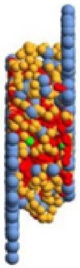
Initial A+A collision



- **String formation in primary NN collisions**

→ **decays to pre-hadrons (baryons and mesons)**

Partonic phase



- **Formation of a QGP state if $\varepsilon > \varepsilon_{critical}$:**
  Dissolution of pre-hadrons → DQPM

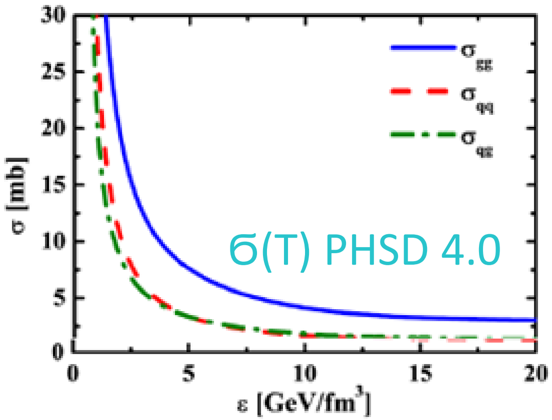
  → massive quarks/gluons and mean-field energy

(quasi-)elastic collisions :

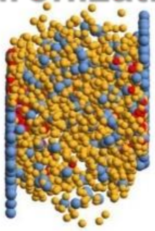
$$q+q \to q+q \quad g+q \to g+q$$
$$q+\bar{q} \to q+\bar{q} \quad g+\bar{q} \to g+\bar{q}$$
$$\bar{q}+\bar{q} \to \bar{q}+\bar{q} \quad g+g \to g+g$$

inelastic collisions:

$$q+\bar{q} \to g$$
$$g \to q+\bar{q}$$



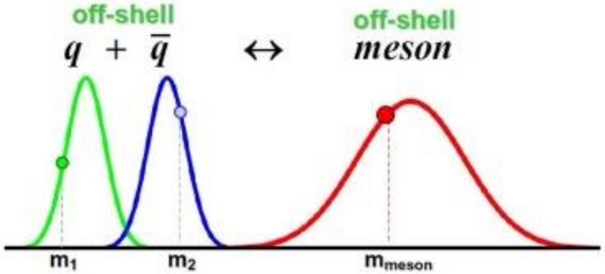
Hadronization



- **Hadronization to colorless off-shell mesons and baryons**

$$g \to q+\bar{q}, \quad q+\bar{q} \leftrightarrow meson \ ('\ string\ ')$$
$$q+q+q \leftrightarrow baryon \ ('\ string\ ')$$

Strict 4-momentum and quantum number conservation



- **Hadron-string interactions – off-shell HSD**

Extraction of (T, μ_B) in PHSD

For **each space-time cell** of the PHSD: $T^{\mu\nu} = \sum_i \frac{p_i^\mu p_i^\nu}{E_i}$ → Diagonalize in LRF → ε^{PHSD}

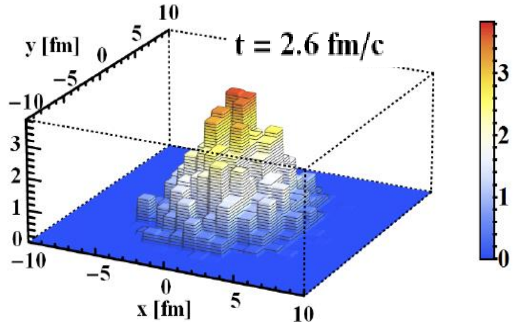
- Calculate the local energy density ε^{PHSD} and baryon density n_B^{PHSD}
- use lQCD relations (up to 6th order):

$$\begin{cases} \frac{n_B}{T^3} \approx \chi_2^B(T)\left(\frac{\mu_B}{T}\right) + \ldots \\ \Delta\epsilon/T^4 \approx \frac{1}{2}\left(T\frac{\partial\chi_2^B(T)}{\partial T} + 3\chi_2^B(T)\right)\left(\frac{\mu_B}{T}\right)^2 + \ldots \end{cases}$$

Use baryon number susceptibilities χ_n from lQCD

➔ obtain (T, μ_B) by solving the system of coupled equations using ε^{PHSD} and n_B^{PHSD}

$e(x, y, z = 0)$

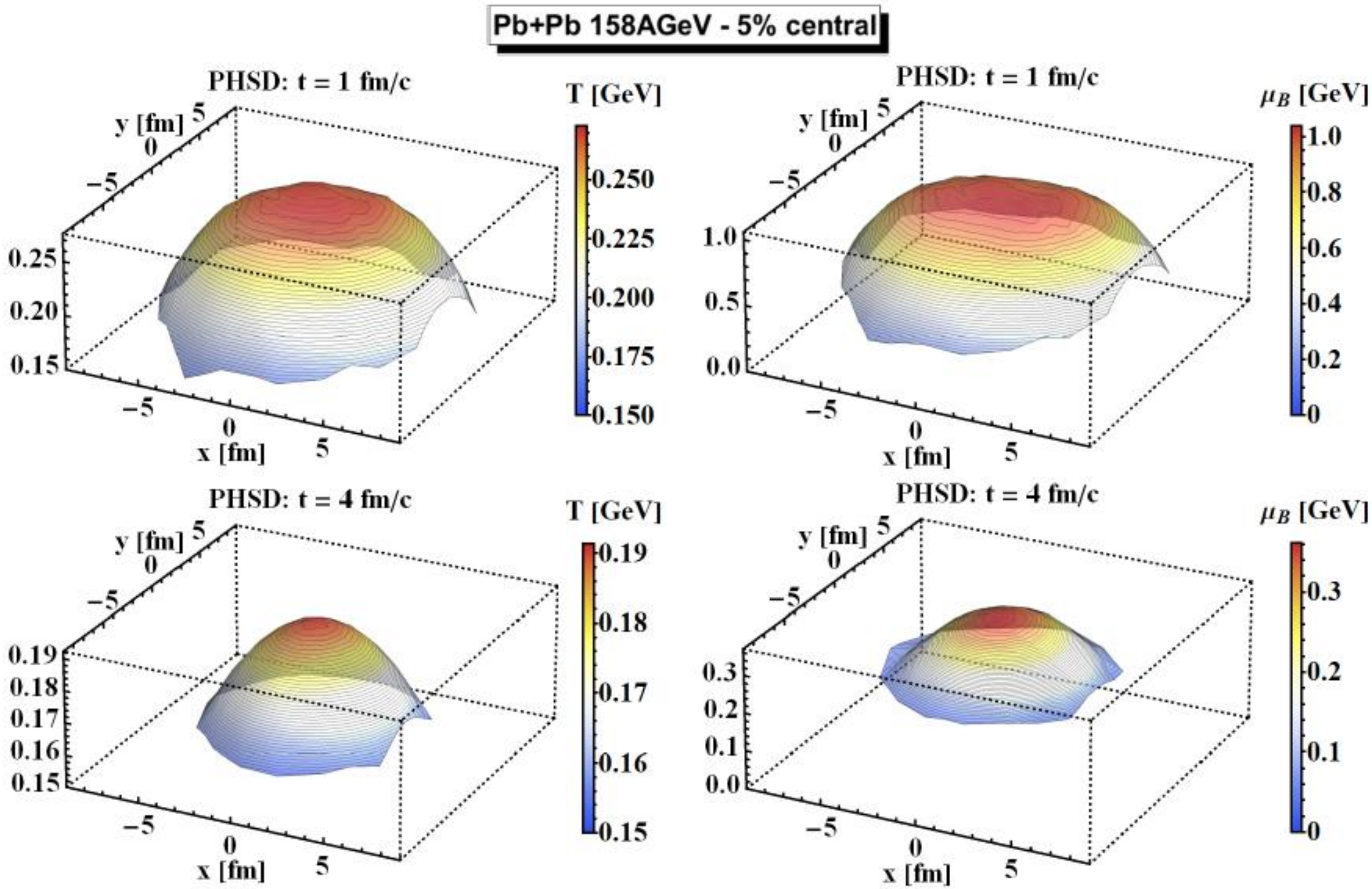


Input: ε^{PHSD} and n_B^{PHSD} → **Output:** T, μ_B

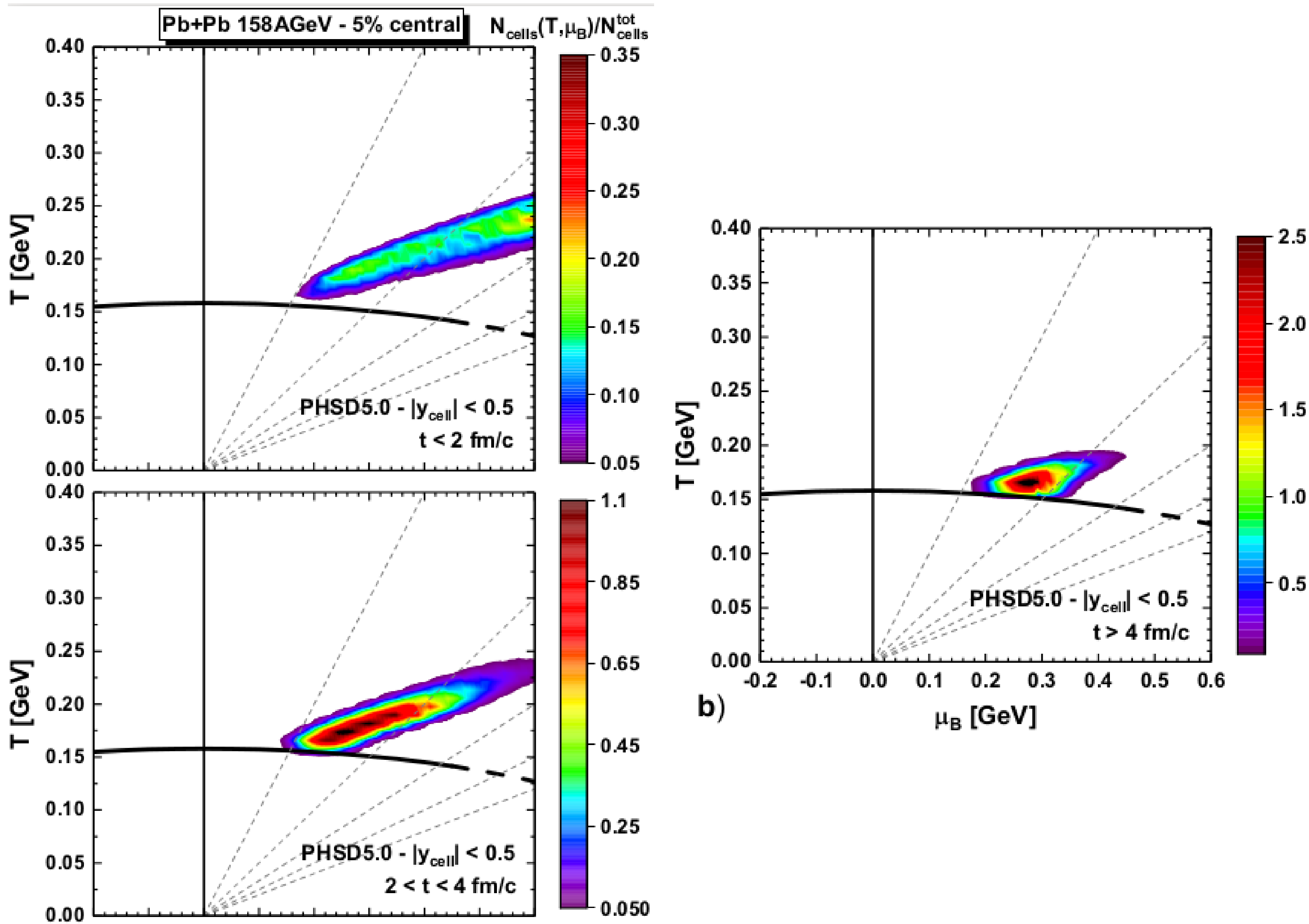
for details see P. Moreau, O. Soloveva, L. Oliva, T. Song, W. Cassing, E. Bratkovskaya arXiv:1903.10157, PRC 100 (2019) no. 1, 014911

Illustration for HIC ($\sqrt{s_{NN}} = 17$ GeV)

The temperature profile in (x; y) Baryon chemical potential profile in (x;y) at midrapidity ($|y_{cell}| < 1$) at 1 and 4 fm/c

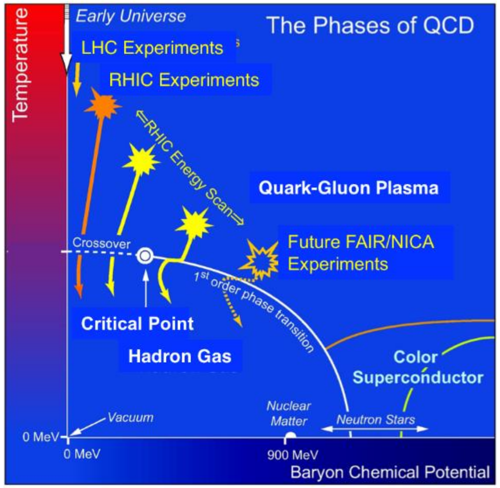


QGP evolution for HIC ($\sqrt{s_{NN}} = 17$ GeV)



Traces of the QGP at finite μ_B in observables in high energy heavy-ion collisions

PHSD



Results for HIC

- **Comparison between three different results:**

  - **PHSD 4.0 : only isotropic $\sigma(T)$ and ρ(T)**

    partonic cross sections

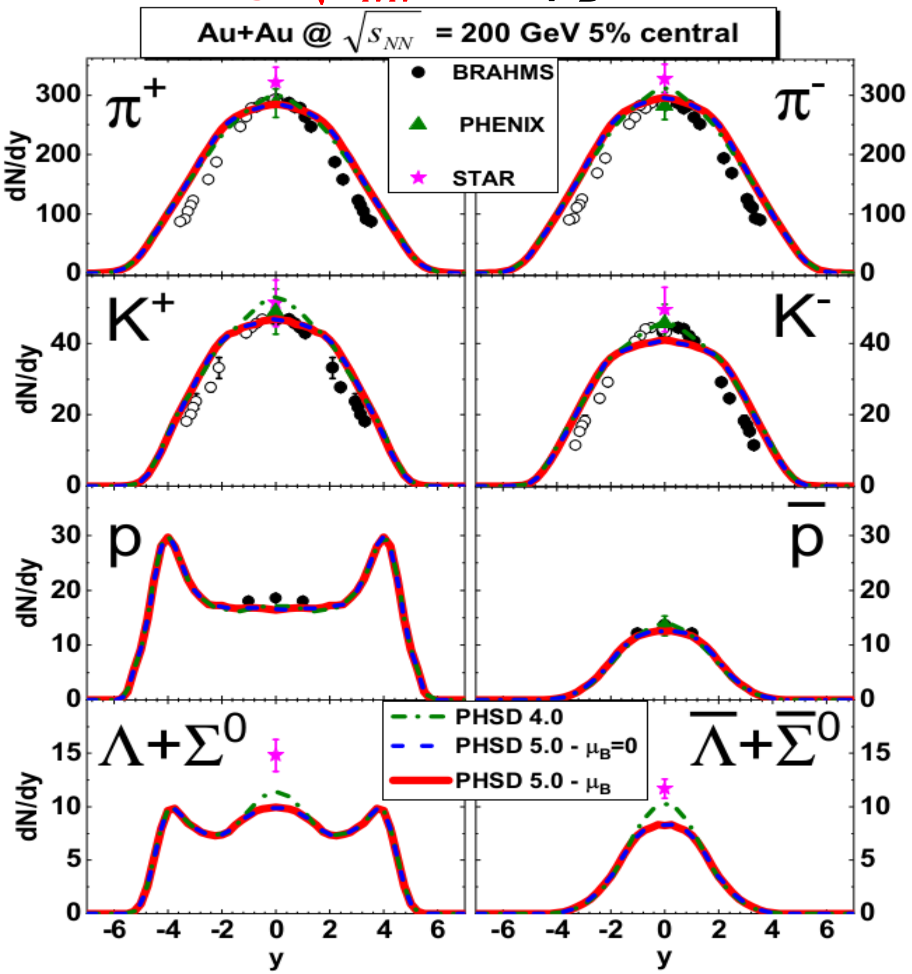
    parton spectral function (masses and widths)

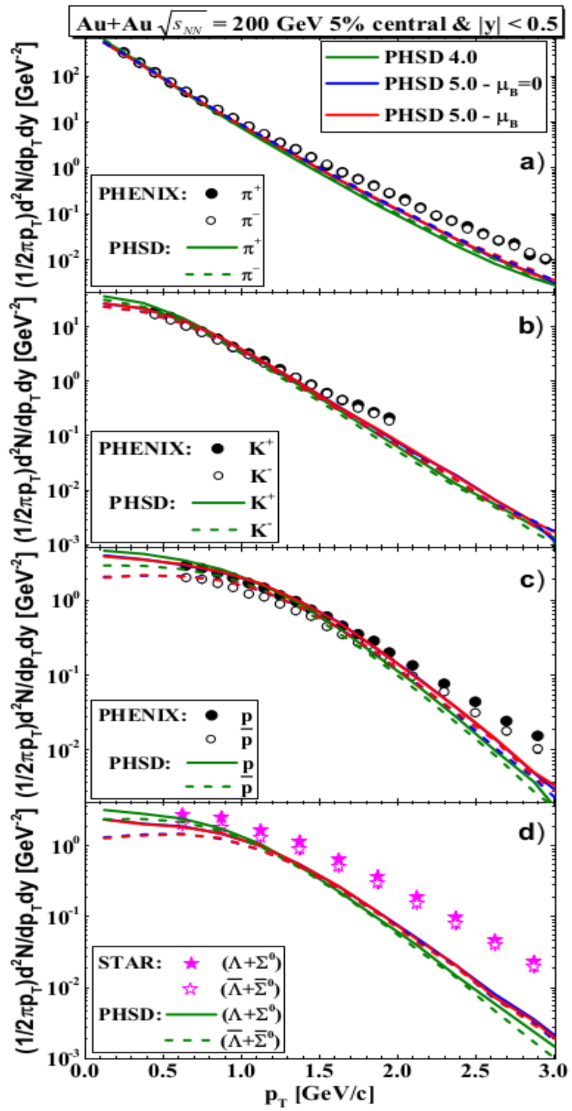
    new PHSD 5 : angular dependence of dσ/d cosθ

  - **PHSD 5.0 : with $\sigma\left(\sqrt{s}, m_1, m_2, T, \mu_B = 0\right)$ and ρ$(T, \mu_B = 0)$**
  - **PHSD 5.0 : with $\sigma\left(\sqrt{s}, m_1, m_2, T, \mu_B\right)$ and ρ(T, μ_B)**

Results for HIC ($\sqrt{s_{NN}} = 200$ GeV)

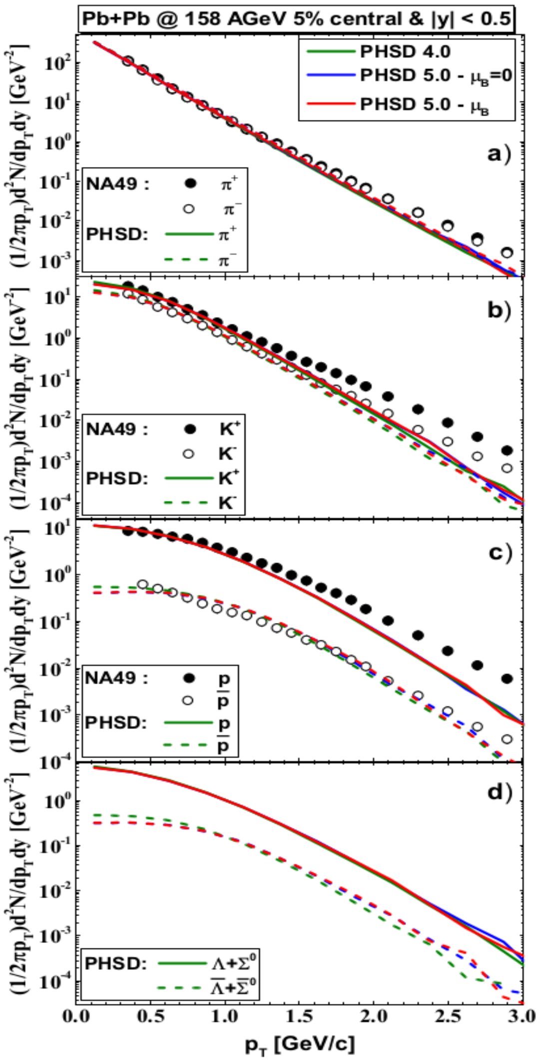
at high $\sqrt{s_{NN}}$ - low μ_B





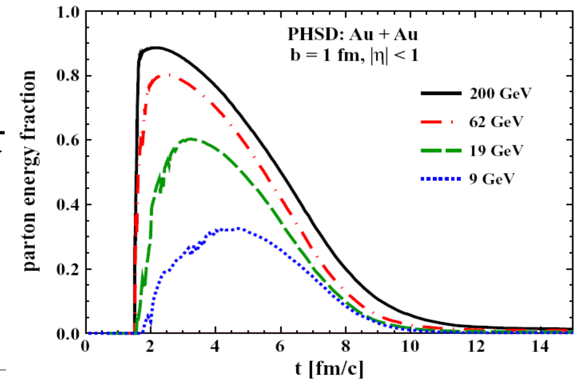
arXiv:1903.10157

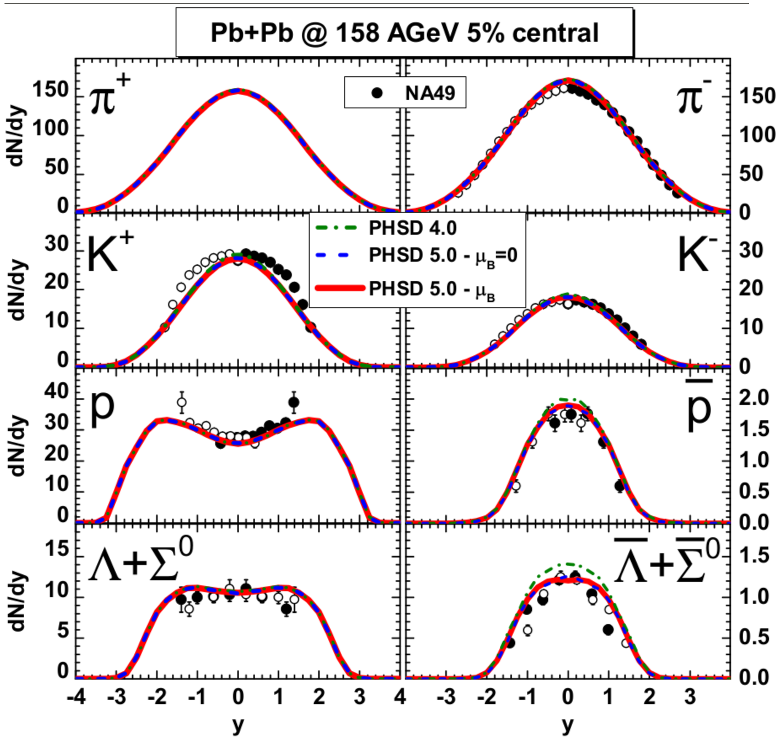
Results for HIC ($\sqrt{s_{NN}} = 17$ GeV)



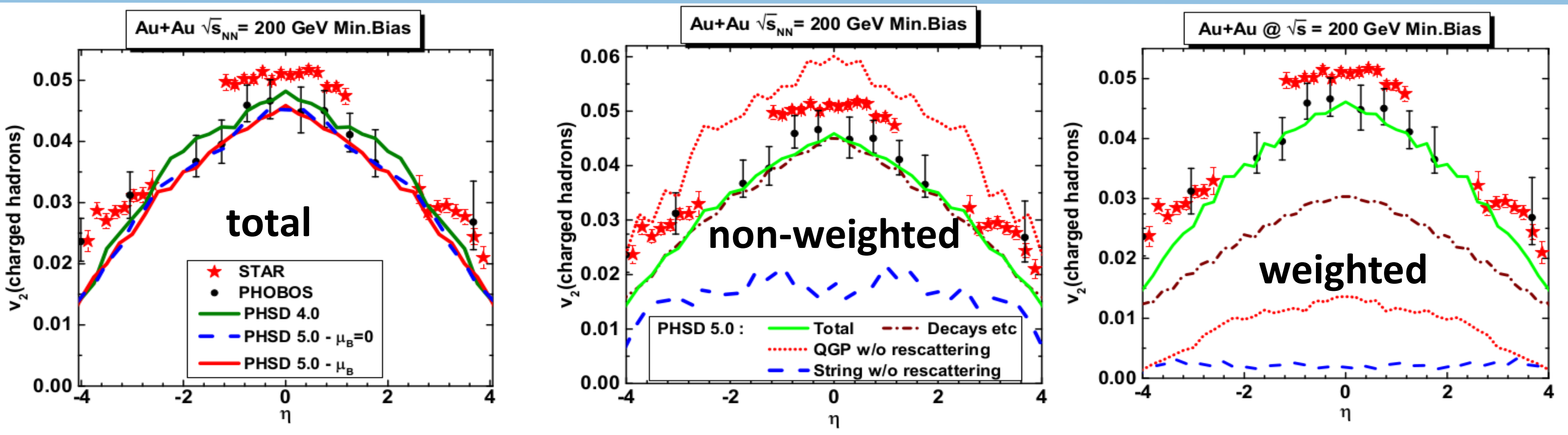
High-μ_B regions are probed at low $\sqrt{s_{NN}}$ or high rapidity regions

But, QGP fraction is small at low $\sqrt{s_{NN}}$

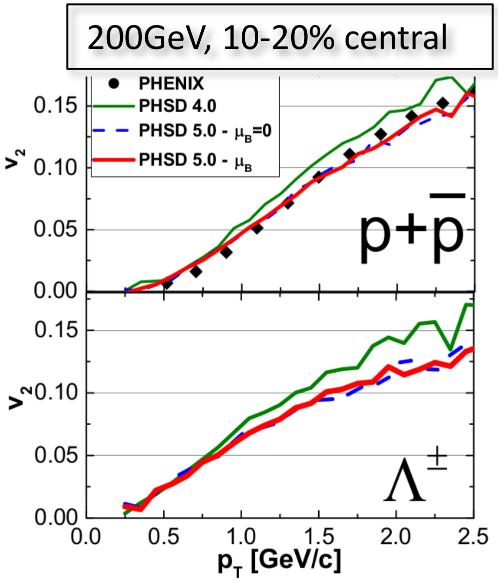


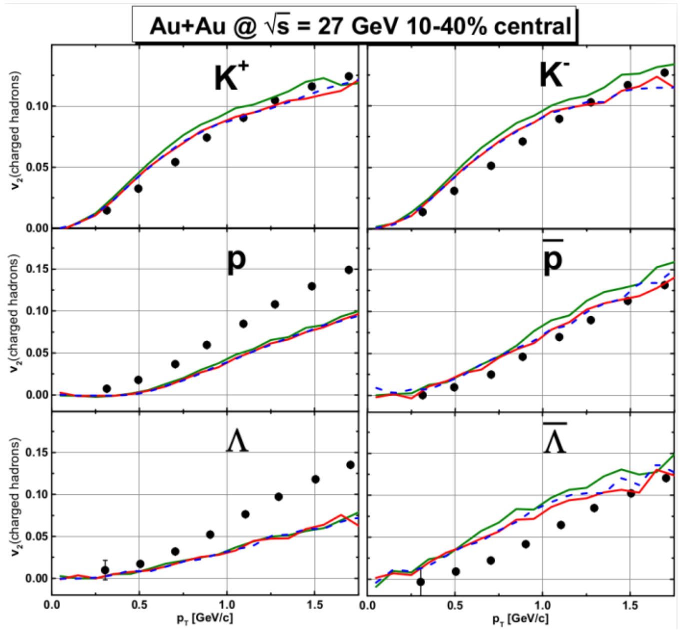


Elliptic flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27 GeV$)



a) b) c)

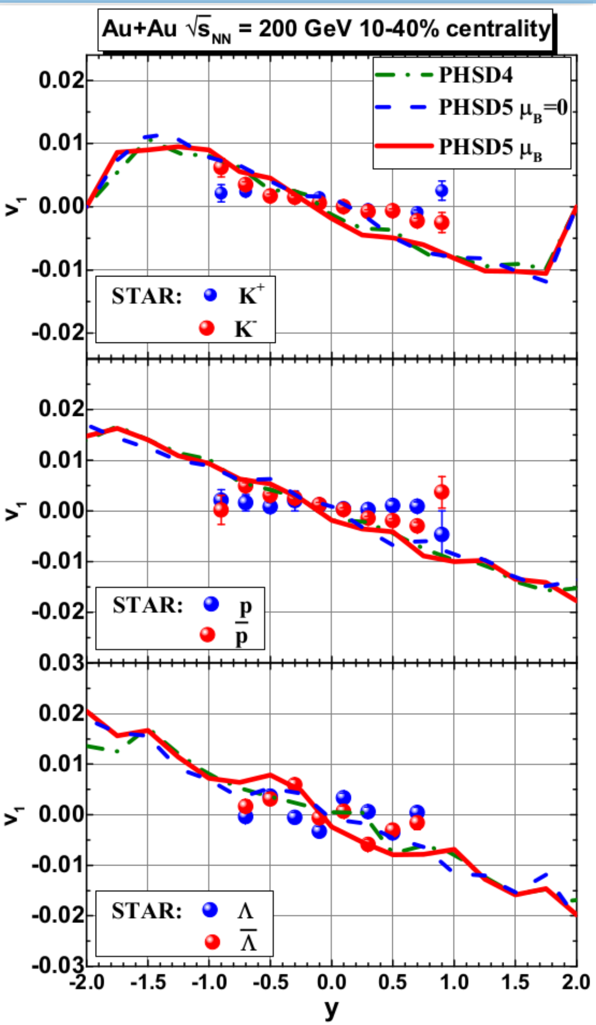


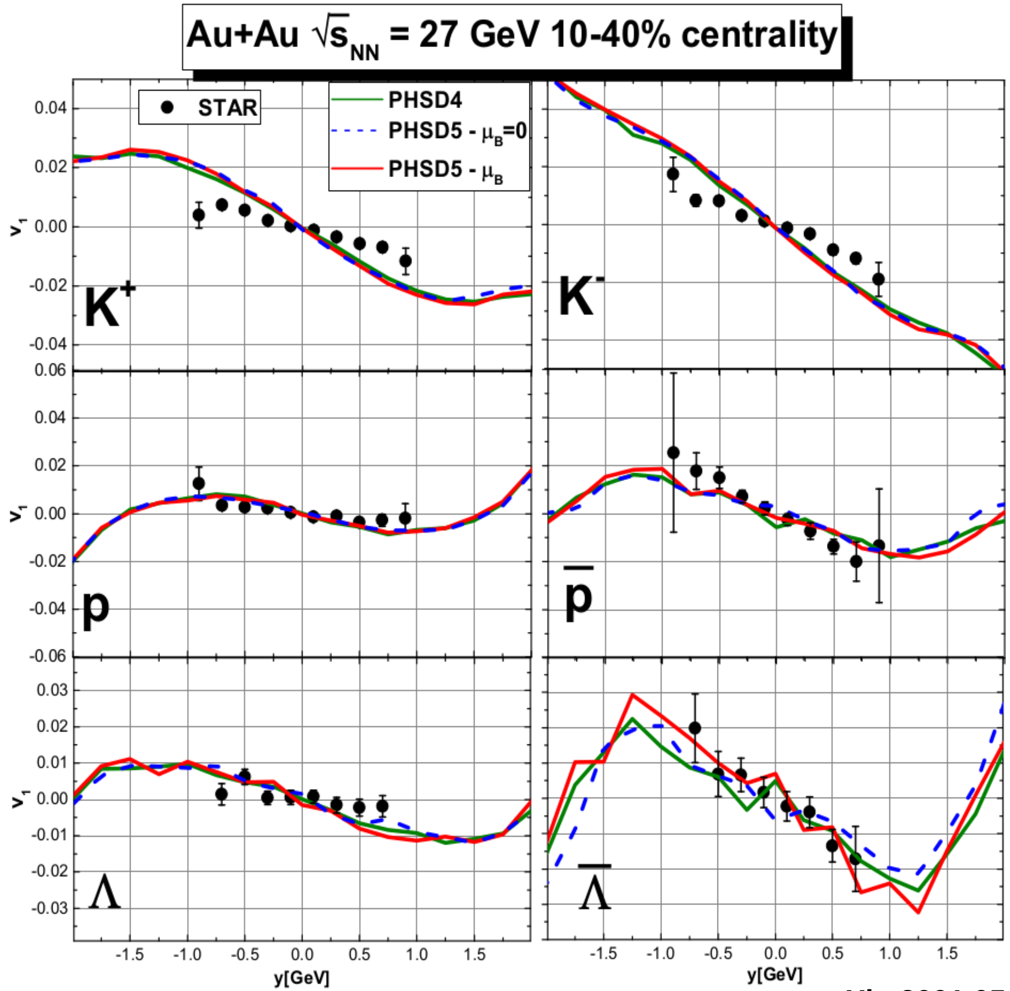


- No visible effects of μ_B dependence
- Small effect of the angular dependence of **dσ/dcosθ**

arXiv:2001.05395

Directed flow ($\sqrt{s_{NN}} = 200\ GeV\ vs\ 27\ GeV$)





arXiv:2001.05395

No visible effects of μ_B dependence or angular dependence

Summary / Outlook

- Transport coefficient at finite T and μ_B have been found using the (T, μ_B)-dependent cross sections(for cross-sections see DQPM[2] and PNJL[4])
- At μ_B= 0 good agreement with the Bayesian analisis estimations and gluodynamic lattice calculations of transport coefficients

- Bulk observables have been studied within the PHSD transport approach[2,3]
- High-μ_B regions are probed at low $\sqrt{s_{NN}}$ or high rapidity regions
- But, QGP fraction is small at low $\sqrt{s_{NN}}$: no effects seen in bulk observables[2,3]
- Directed and elliptic flows are also don't show μ_B dependence, while v_2 is sensitive to the explicit $\sqrt{s}$ dependence and angular dependence of dσ/dcosθ

- **Outlook:**
  - More precise EoS large μ_B
  - Possible 1st order phase transition at large μ_B, comparison w PNJL model

Summary / Outlook

Thank you for your attention!

[1] OS, P. Moreau, E. Bratkovskaya, arXiv:1911.08547 [nucl-th].
[2] P. Moreau, OS , L. Oliva, T. Song, W. Cassing, E. Bratkovskaya, arXiv:1903.10157
[3] OS, P. Moreau, L. Oliva, V. Voronyuk, V. Kireyeu, T. Song, E. Bratkovskaya, arXiv:2001.05395
[4] OS, D.Fuseau, J.Aichelin, E.Bratkovskaya, arXiv:2011.03505