Universal scaling close to chiral limit of QCD

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BMBF project

ALICE Germany

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CRC-TR 211
Strong-interaction matter under extreme conditions

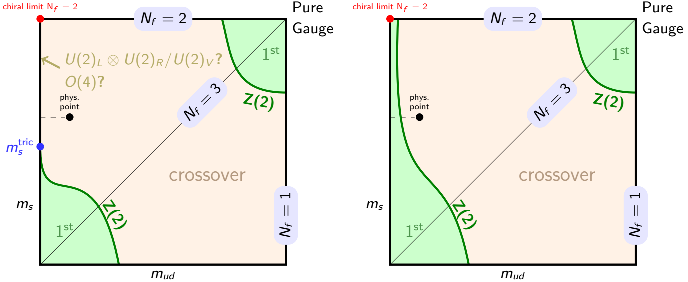
Questions to be answered

- Key question: What is the chiral transition temperature, T_c^0?
- Possibly another question: What is the nature of the chiral phase transition?

- $N_f = 2+1$: Two possible scenarios depending on the effective restoration of the $U_A(1)$ in the chiral limit at T_c.

  Pisarski and Wilczek. Phys. Rev. D29, 338, 1983.

- $N_f = 3$: No direct evidence of 1^{st} order transition down to $m_\pi = 80$ MeV. Scaling argument pushes it further to $m_\pi = 50$ MeV. A. Bazavov *et. al.* Phys. Rev. D95, 074505 (2017).



[O. Philipsen and C. Pinke. Phys. Rev. D93, 114507, 2016.]

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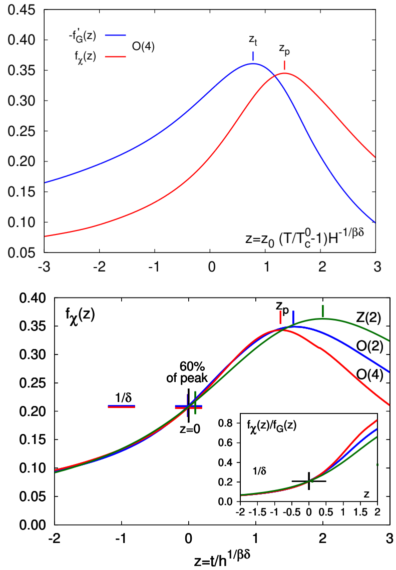
Scaling functions: Some intriguing facts



Mass scaling of the pseudo-critical estimators for any fixed z_X (in absence of sub-leading contributions):

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

Our approach: Use z_X at or close to 0. We choose to work with $X = \delta$ and 60:

$$\frac{H\chi_M(T_\delta, H)}{M(T_\delta, H)} = \frac{1}{\delta}$$

$$\chi_M(T_{60}, H) = 0.6\chi_M^{max}$$

Dependence on quark mass ($H = m_l/m_s$) reduced by two orders of magnitude

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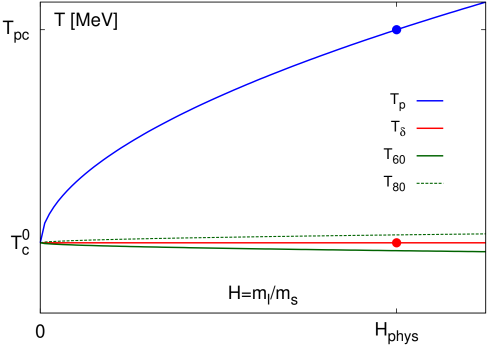
Improved estimators: basic philosophy

Mass scaling of the pseudo-critical estimators for any fixed z_X (in absence of sub-leading contributions):

$$T_X(H) = T_c^0 \left(1 + \frac{z_X}{z_0} H^{1/\beta\delta}\right)$$

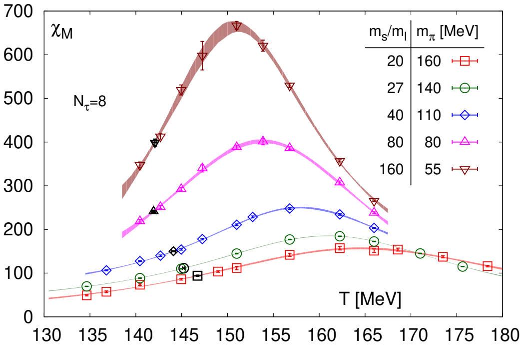
- Our approach: Use z_X at or close to 0.
- Because of the reduced variation w.r.t. H, up to the regular contributions, the pseudo-critical temperatures defined by the improved estimators at any finite value of H, *e.g.* H_{phys}, already gives a close estimate of T_c^0.

- We choose to work with $X = \delta$ and 60.



Chiral susceptibility

- No direct evidence of a 1^{st}-order phase transition down to $m_\pi = 80$ MeV.
- The increase of χ_M^{max} is apparently consistent with $H^{1/\delta - 1}$ with $\delta \approx 4.8$.
- Precise determination of δ is not possible with the present data.
- Preliminary analyses with H_c being a free parameter gives a quite uncertain estimate of H_c with 0 within the range.



- Saturating trend of T_{60} towards chiral limit even at $N_\tau = 8$ already puts this as an improved estimator.
- There is no strong evidence for H_c being non-zero.

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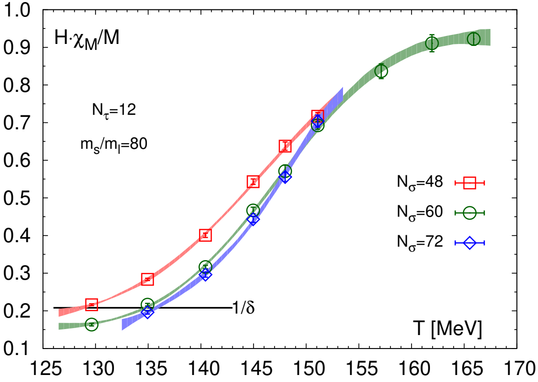
Ratio



- The intersection point of the ratio with the line at $1/\delta$ defines $T_\delta(H, L)$.
- $T_\delta(H, L)$ increases towards thermodynamic limit.
- Results for fixed H have been extrapolated to thermodynamic limit using $O(4)$ as well as $1/V$ ansatz.
- Then continuum and chiral extrapolation has been performed.

- We also tried, for a fixed N_τ, a joint chiral and thermodynamic limit extrapolation using $O(4)$ finite size scaling function and then took the continuum limits and this "improper limit" produces compatible results.

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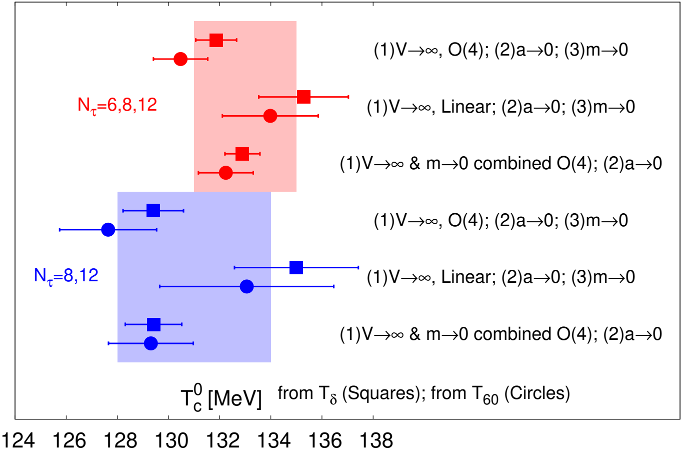
T_c^0: A single number



Final number we have quoted: $T_c^0 = 132^{+3}_{-6}$ MeV.

HotQCD; Phys. Rev. Lett. 123, 062002 (2019).

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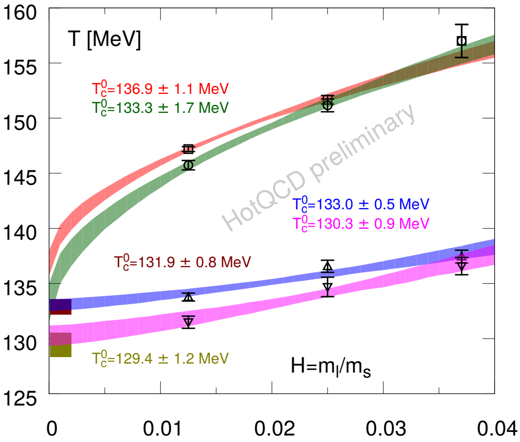
Preliminary comparison with conventional estimator

- Disclaimer: All T_p numbers and T_δ for $H = 1/27$ are not infinite volume extrapolated.
- Finite volume effect on T_p was estimated from joint thermodynamic and chiral extrapolation using $O(4)$ finite size scaling functions.
- T_c^0 from from T_p agree with those from T_δ within 95% CI.



- Stability of new estimators are vivid.

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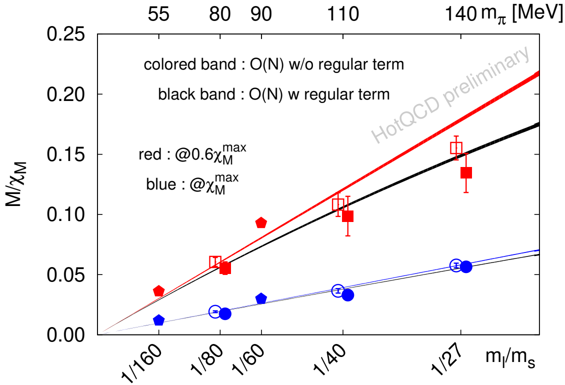
Order of the chiral transition

$$\frac{M}{\chi_M} = H\frac{f_G(z)}{f_\chi(z)}$$

- For small H the data seems to be linear.
- Lines are NOT fitted curves rather expectations for $O(2)$ and $O(4)$.
- Regular term $\propto H^{2-1/\delta}$.
- Coefficient of the regular term is NOT fitted, rather taken from MEoS fits.



- $Z(2)$ transition, at some finite H_c, will results into a sudden drop in the ratio at $H_c \Rightarrow$ 1^{st} order transition is unlikely for $m_\pi > 55$ MeV.
- Additional low H measurements: slope can be directly determined as a fit parameter.

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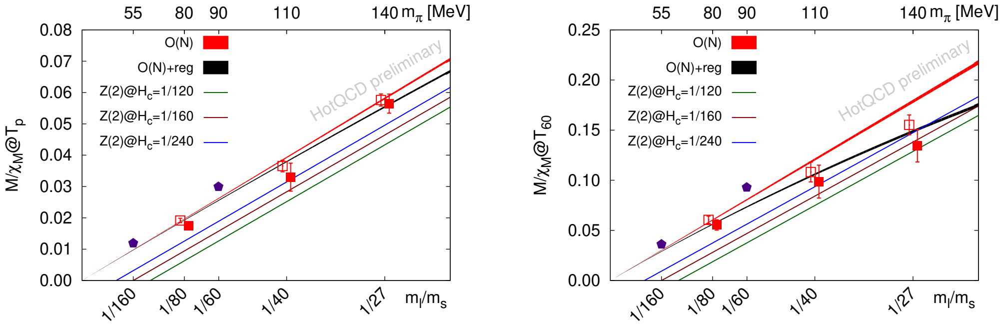
Order of the chiral transition



- $Z(2)$ lines are schematic: $\dfrac{M}{\chi_M} = (H - H_c)\,\dfrac{f_G(z)}{f_\chi(z)}$
- If M is not exactly order parameter then the $Z(2)$ lines will have a curvature.
- Mixing becomes weak as H_c becomes small.
- Our calculation seems to favor $O(N)$ compared to $Z(2)$. Kaczmarek *et. al.*, arXiv:2010.15593.

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Gluonic observables towards chiral limit

- Wilson's RG approach: thermodynamics in the vicinity of a critical point can be described by an effective Hamiltonian.
- Two types of operators: ones which respect the symmetry and others don't; termed as energy-like and magnetization-like.
- Being gluonic, Polyakov loop (PL) and heavy quark free energy (HQFE) are both expected to be energy-like operators w.r.t. chiral phase transition.

$$F_q(T,H)/T = AH^{(1-\alpha)/\beta\delta} f_f'(z) + f_{\mathsf{reg}}(T,H)$$

- HQFE doesn't diverge at chiral critical point, so importance of the regular terms could be higher. Let's calculate the mixed susceptibility

$$\frac{\partial F_q(T,H)/T}{\partial H} = -AH^{(\beta-1)/\beta\delta} f_G'(z) + \frac{\partial f_{\mathrm{reg}}(T,H)}{\partial H}$$

which has a divergent behavior.

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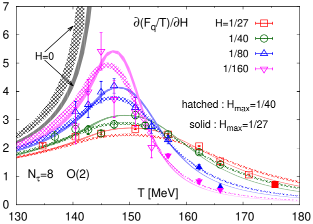
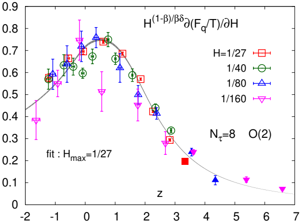
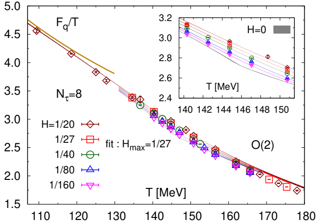
Gluonic observables towards chiral limit





- Fit with singular terms only.
- Determined singular part compares well with other determinations.



- Regular part is then determined from HQFE keeping the singular part fixed at the value determined from $\partial(F_q/T)/\partial H$ fit. Clarke *et. al.*, arXiv:2008.11678.

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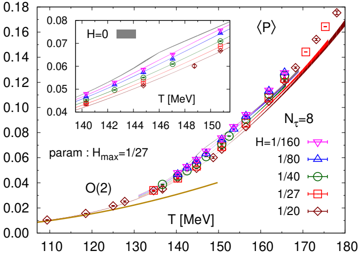
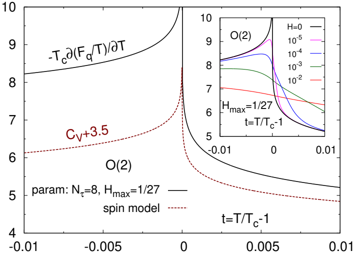
Gluonic observables towards chiral limit





- PL behaves as an energy-like observable towards chiral limit.
- No inflection point in PL can be identified in the chiral crossover region.
- In the chiral limit: Clarke *et. al.*, arXiv:2008.11678.

$$T_c \frac{\partial(F_q(T,0)/T)}{\partial T} = a^r_{1,0}\left(1 + R^{\pm}|t|^{-\alpha}\right)$$

- Peak develops only in a very tiny interval around T_c towards chiral limit.
- Peak height is non-universal.
- Identifying a peak in C_V is hard because of the rising regular background in QCD.

Gupta and Sharma, PoS CPOD2014 (2015) 011.

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Conserved charge fluctuations towards chiral limit

- μ_B does not break chiral symmetry explicitly.
- for finite μ_B definition of the $O(4)$ scaling fields to the leading order:

$$t = \frac{1}{t_0}\left(\frac{T - T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right) \quad \text{and} \quad h = \frac{1}{h_0}\frac{m_l}{m_s}$$

- Close to chiral limit: $\frac{\partial}{\partial T} \sim \frac{\partial}{\partial \mu_B^2} \Rightarrow \chi_2^B$ is expected to be energy-like observable.

$$\chi_2^B(T, H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(z) + \text{regular terms.}$$

- We check this at $T = T_c$. Sarkar *et. al.*, arXiv:2011.00240.

$$\chi_2^B(T_c, H) = -A\kappa_2^B H^{(1-\alpha)/\beta\delta} f_f'(0) + \text{constant regular term.}$$

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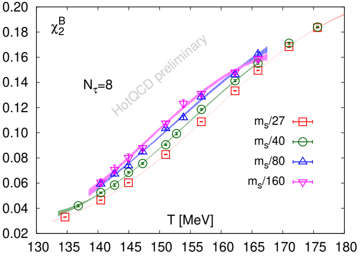

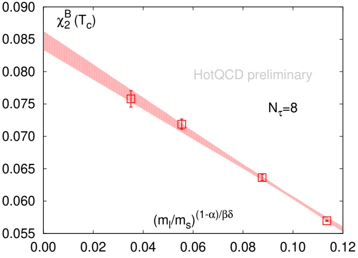

- Linear fit in $H^{(1-\alpha)/\beta\delta}$ works quite well.
- Singular part vanishes at the chiral limit.

Sarkar *et. al.*, arXiv:2011.00240.

- $\chi_2^B(T_c, 0) - \chi_2^B(T_c, H)$ gives the singular part for any finite mass.
- Ratio of singular parts of of conserved charge X and Y is same as κ_2^X/κ_2^Y.
- Preliminary calculations: $\kappa_2^B/\kappa_2^S = 1.0$ and $\kappa_2^Q/\kappa_2^B = 2.6$.
- Consistent with physical mass results for ratios of κ's. [HotQCD; PLB 795 15 (2019)].
- Curvature does not change much towards chiral limit.

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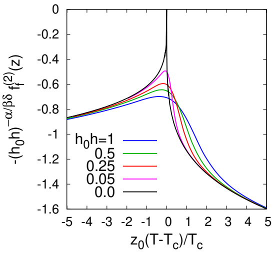
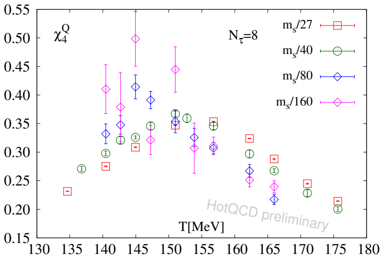
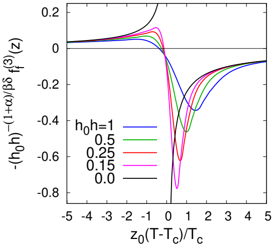
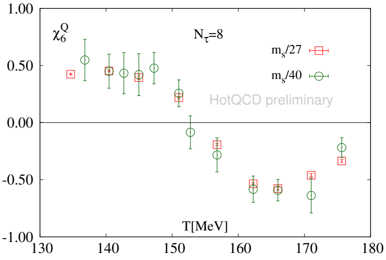
Conserved charge fluctuations towards chiral limit









- 4^{th}-order cumulants do not diverge rather expected to show characterisitc cusp.
- For low masses there seems to be a cusp developing.
- Regular terms are different in various energy-like observables.
- 6^{th} and higher order cumulants diverge at chiral limit.
- Negative trough of 6^{th}-order is expected to have a factor 1.27 when going from $H = 1/27$ to $H = 1/40$.

Friman *et. al.*, Eur. Phys. J. C71 (2011) 1694.   Sarkar *et. al.*, arXiv:2011.00240.

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Summary and Outlook

- Summary:
  1. Scaling fits with $O(4)$ exponents worked reasonably.
  2. T_c^0 obtained taking chiral and continuum extrapolations in different order, agrees well.
  3. Current estimate of T_c^0, in continuum, is 132^{+3}_{-6} MeV.
  4. Comparison with conventional estimators has been done.
  5. Preliminary calculations seem to show no evidence of a Z_2 transition at non-vanishing H_c and thus favor $O(N)$ in the chiral limit.
  6. Polyakov Loop and second order cumulants of conserved charge fluctuations behave as energy-like observables w.r.t. chiral phase transition.

- Outlook:
  1. Additional low H measurements will help us to be confident about the scaling behavior.
  2. Ongoing calculations of disconnected part of chiral susceptibilities and $\chi_\pi - \chi_\delta$ at high temperatures can throw some light into $U_A(1)$ restoration in chiral limit.
  3. Exploration of specific-heat like observables are ongoing.

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Basic quantities

In terms of temperature T and symmetry breaking field $H = m_l/m_s$ the scaling variables are defined as:

$$t = \frac{1}{t_0}\frac{T - T_c^0}{T_c^0} \qquad \text{and} \qquad h = \frac{1}{h_0}\frac{m_l}{m_s} = \frac{1}{h_0}H$$

Scaling variable:

$$z = \frac{t}{h^{\frac{1}{\beta\delta}}} = z_0\left(\frac{T - T_c^0}{T_c^0}\right)\left(\frac{1}{H^{1/\beta\delta}}\right); \quad z_0 = \frac{h_0^{\frac{1}{\beta\delta}}}{t_0}$$

Chiral condensate : $\langle\bar{\psi}\psi\rangle_f = \dfrac{T}{V}\dfrac{\partial \ln Z}{\partial m_f}$

Chiral susceptibility : $\chi_m^{fg} = \dfrac{\partial}{\partial m_g}\langle\bar{\psi}\psi\rangle_f$

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Scaling relations

Renormalization group invariant (RGI) definition of order parameter:

$$M = \frac{m_s}{f_K^4}\left(\left(\langle\bar{\psi}\psi\rangle_u + \langle\bar{\psi}\psi\rangle_d\right) - \frac{m_u + m_d}{m_s}\langle\bar{\psi}\psi\rangle_s\right) \equiv \frac{\Sigma_{\mathsf{sub}}}{f_K^4}$$

RGI definition of order parameter susceptibility:

$$\chi_M = \frac{T}{V} m_s \left(\frac{\partial}{\partial m_u} + \frac{\partial}{\partial m_d}\right) M$$

Close to chiral limit, singular part behaves as:

$$\begin{aligned} M &= h^{1/\delta} f_G(z) \\ \chi_M &= \frac{1}{h_0} h^{1/\delta - 1} f_\chi(z) \end{aligned}$$

$f_G(z)$ and $f_\chi(z)$ are universal scaling functions which have been precisely determined from various spin models.

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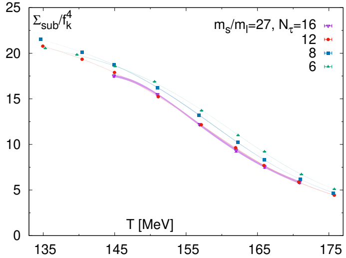
Scaling at physical Pion mass?



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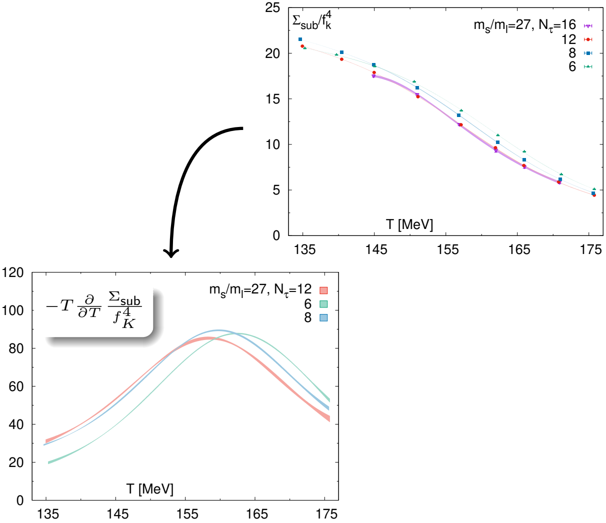

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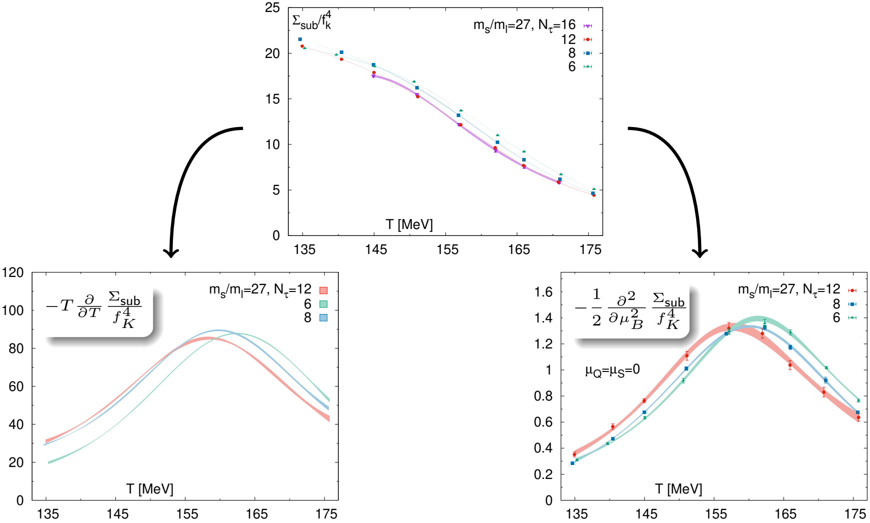

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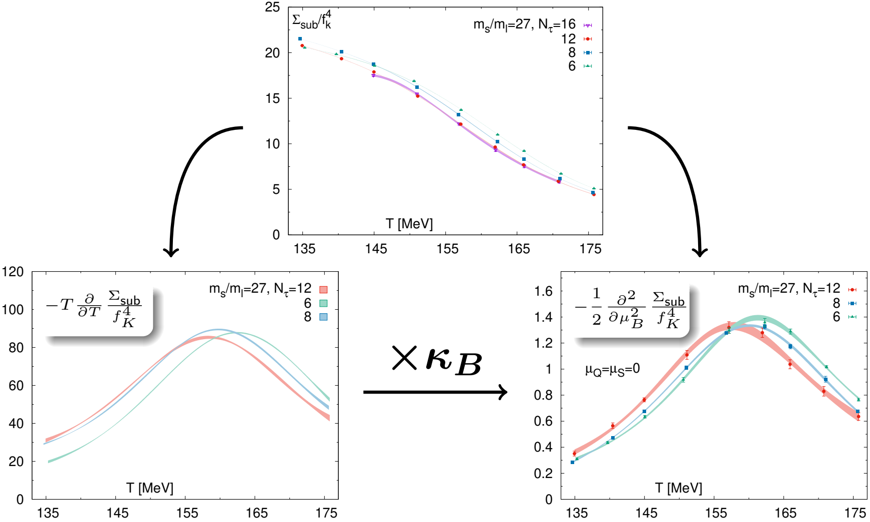
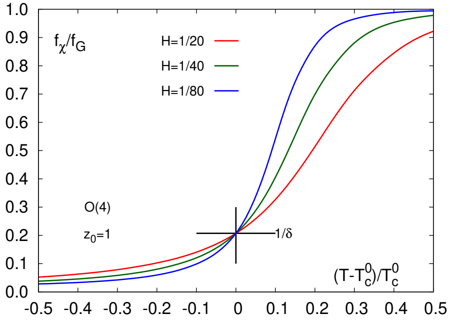


- Scaling expectation already holds for physical Pion.

Scaling functions: Some intriguing facts

$$\frac{f_\chi(z)}{f_G(z)} = \left\{ \begin{array}{ccc} 0 & , & z \to -\infty \\ 1/\delta & , & z = 0 \\ 1 & , & z \to +\infty \end{array} \right\}$$



- Behavior of $H\chi_M/M$ is like Binder cumulant at critical point. [F. Karsch and E. Laermann. Phys. Rev. D50, 6954, 1994.]
- Ratio is expected to have a constant value at the crossing point, $z = 0$, *i.e.* in chiral limit at T_c^0.
- Determine temperature $T_\delta(H)$ which satisfies:

$$\frac{H\chi_M\left(T_\delta, H\right)}{M\left(T_\delta, H\right)} = \frac{1}{\delta} \Rightarrow T_c^0 = \lim_{H\to 0} T_\delta(H)$$

.

- Uniqueness of the crossing point gets spoiled in presence of regular terms.

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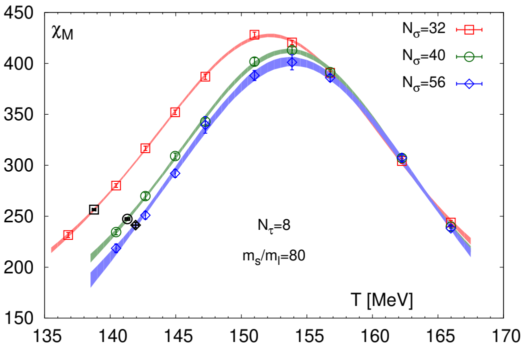
No evidence for 1^{st} order transition

- Volume dependence of χ_M is studied for $H = 1/80$ which corresponds to $m_\pi = 80$ MeV.
- $\chi_M^{\max}$ is NOT proportional to volume.
- $\chi_M^{\max}$ seems to saturate towards thermodynamic limit.
- T_{pc} and T_{60} increase towards thermodynamic limit.



- Possibility for 1^{st} order phase transition can be ruled out at $m_\pi = 80$ MeV for $N_\tau = 8$.
- Similar results are also obtained for $N_\tau = 6$ and 12.

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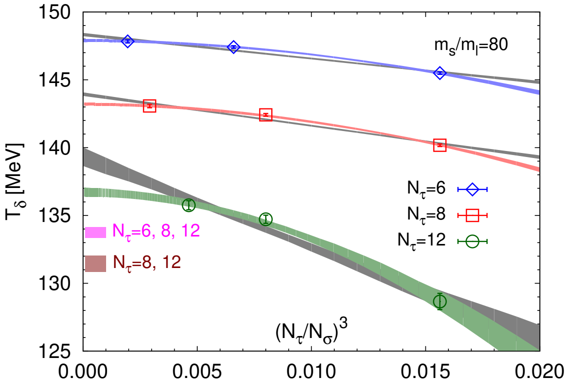
T_c^0 in continuum: 'Proper' limits



- Results for fixed H have been extrapolated to thermodynamic limit.
- Systematic uncertainty comes in form of difference between $O(4)$ and $1/V$ extrapolations.
- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

Chiral extrapolation: $T_X(H) = T_c^0 \left(1 + \dfrac{z_X}{z_0} H^{1/\beta\delta}\right) + c_X H^{1-1/\delta+1/\beta\delta}$

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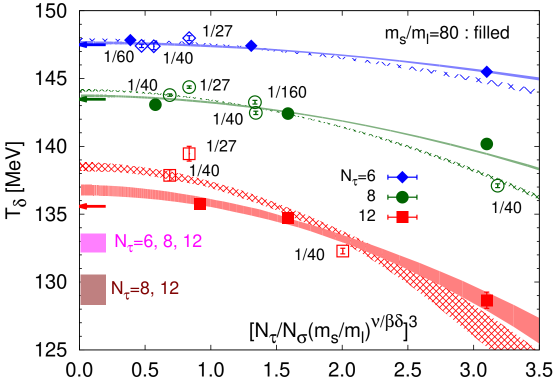
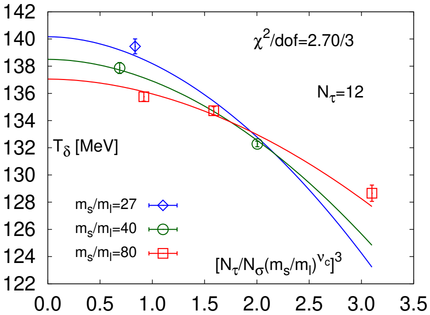
T_c^0 in continuum: 'Improper' limits



- Continuum extrapolation are performed with(out) $N_\tau = 6$ results which is another source of systematic uncertainty.

- Results for fixed N_τ have been extrapolated to thermodynamic limit and chiral limit simultaneously using $O(4)$ scaling functions.



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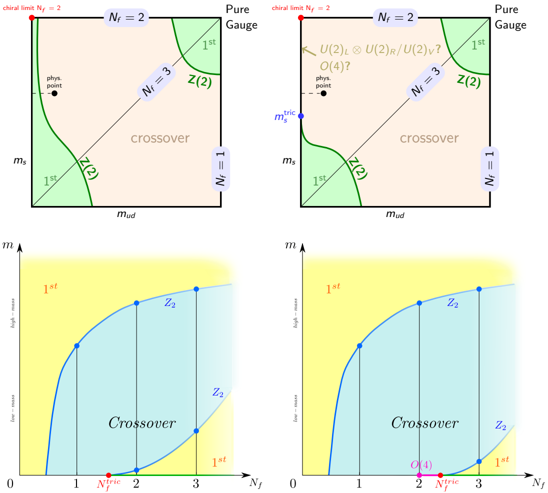
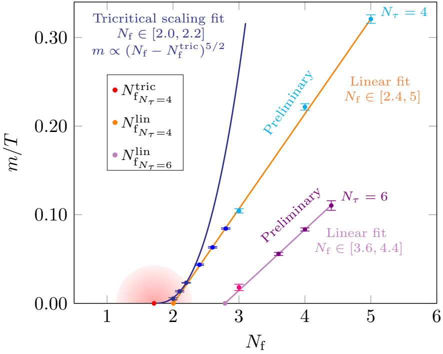
Order of the chiral transition



Cuteri *et. al.*,arXiv:1811.03840 [hep-lat].

Treat N_f as a continuous real parameter and look for tricritical scaling of m_{Z_2} towards $N_f = 2$.



First order chiral transition seems to be unlikely towards continuum.

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Symmetry transformations

$$
\begin{tikzcd}
\chi_{5,con} & \pi : \bar{q}\gamma_5 \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"] \arrow[d, leftrightarrow, "U(1)_A"'] & \sigma : \bar{q} q \arrow[d, leftrightarrow, "U(1)_A"] & \chi_{con} + \chi_{disc} \\
\chi_{con} & \delta : \bar{q} \frac{\tau}{2} q \arrow[r, leftrightarrow, "SU(2)_L \times SU(2)_R"'] & \eta : \bar{q}\gamma_5 q & \chi_{5,con} - \chi_{5,disc}
\end{tikzcd}
$$

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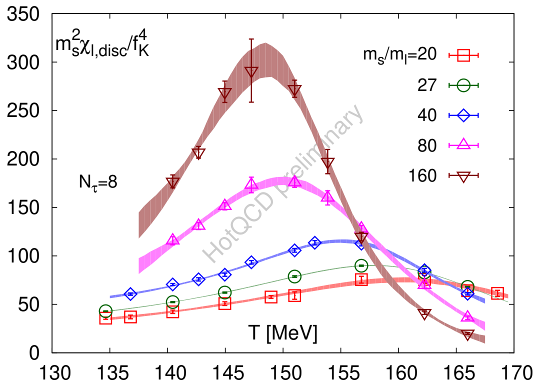
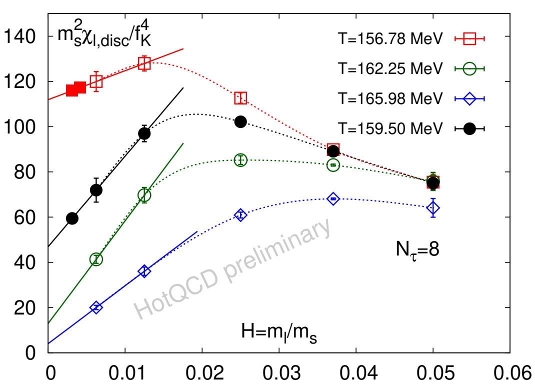
Disconnected chiral susceptibility





Kaczmarek *et. al.*, e-Print:2003.07920 [hep-lat].

Ward Identity

$$G_{\alpha\beta}(x) = \langle \phi_\alpha(x)\phi_\beta(0)\rangle - \langle\phi_\alpha\rangle\langle\phi_\beta\rangle$$

Define, $m_\sigma^{-2} = \int d^3x G_{00}$ and $m_\pi^{-2}\delta_{ij} = \int d^3x G_{ij}$ with $\phi_0 = \sigma$ and $\phi_i = \pi_i$.

From EoS

$$m_\sigma^2 = \frac{\partial H}{\partial M} \quad \text{and} \quad m_\pi^2 = \frac{H}{M}$$

Rajagopal and Wilczek, Nucl. Phys. B399 (1993) 395.