Transport and spectral properties of heavy quarks from LQCD

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Hot problems of Strong Interactions
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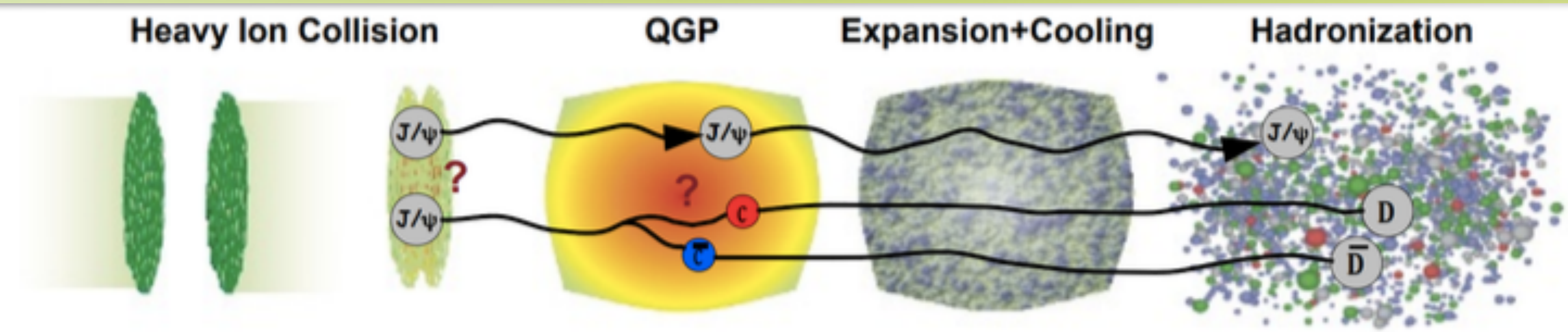
Outline

- Motivation
- Spectral properties of heavy quarks

  [H.-T. Ding, O. Kaczmarek, A.-L. Lorenz, R. Larsen, S. Mukherjee, H. Ohno, H. Sandmeyer, HTS, work in progress]
- Transport properties of heavy quarks

  [L. Altenkort, A. M. Eller, O. Kaczmarek, L. Mazur, G. D. Moore, HTS, 2009.13553]
- Conclusion & Outlook

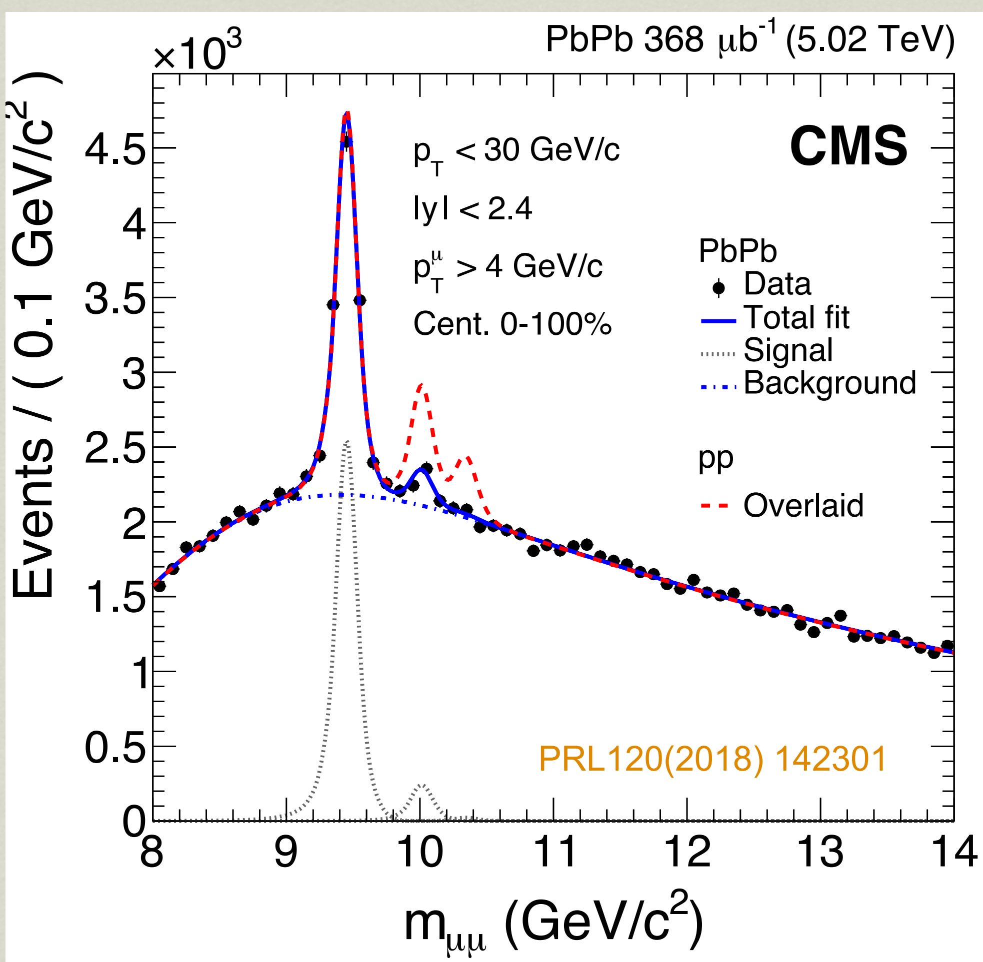
Motivation



Quarkonia produced in the early stage of collisions

- Remain as bound state in the whole evolution
- Release constituents and form open charm/bottom mesons

- Dissociation temperatures



- Heavy quark diffusion

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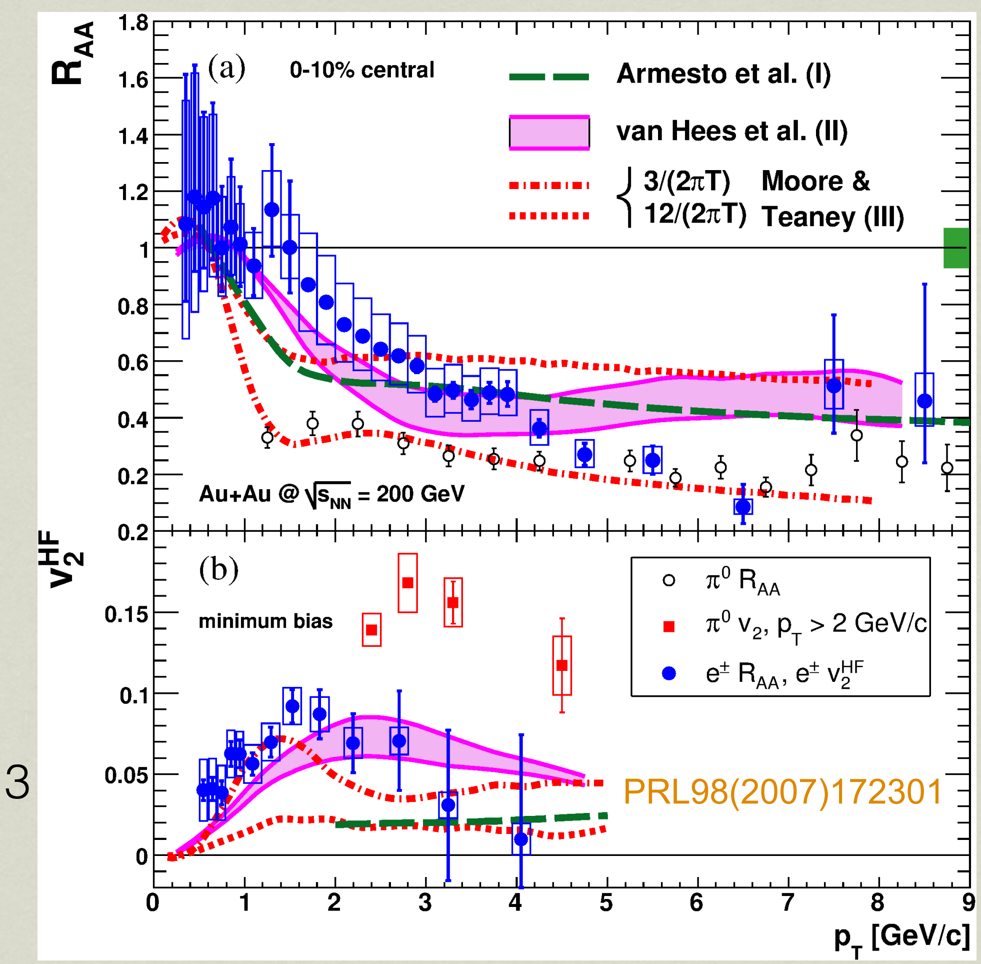
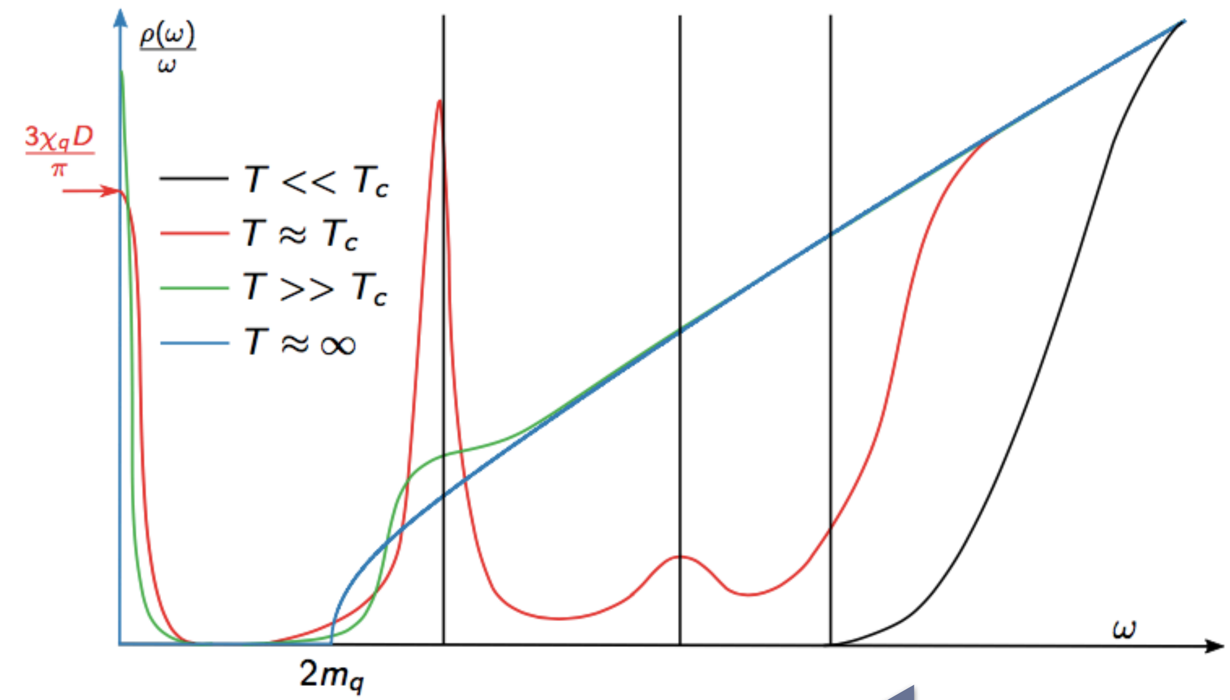


Hadron spectral functions

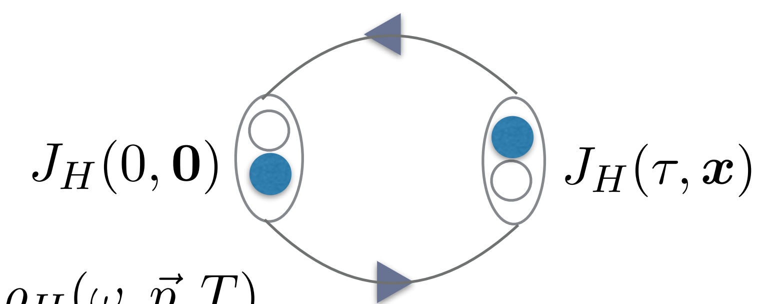
- Carry all information about the in-medium properties of quarkonia

  ✶ Deformation of SPF
  —> dissociation temperature

  ✶ Transport peak:
  —> heavy quark diffusion coefficient



- Analytic continuation and spectral reconstruction



$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^{\dagger}(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T - 1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

✶ New Bayesian Method — Y. Burnier and A. Rothkopf, PRL 111,18,182003
✶ Backus-Gilbert Method — B. B. Brandt, et al., PRD93, 054510(2016)
✶ Maximum Entropy Method — M. Asakawa, et al., PPNP. 46(2001) 445-508
✶ Stochastic Approaches — H.-T. Ding, et al., PRD97, 094503
✶ …
✶ Fit with theoretically inspired ansatz

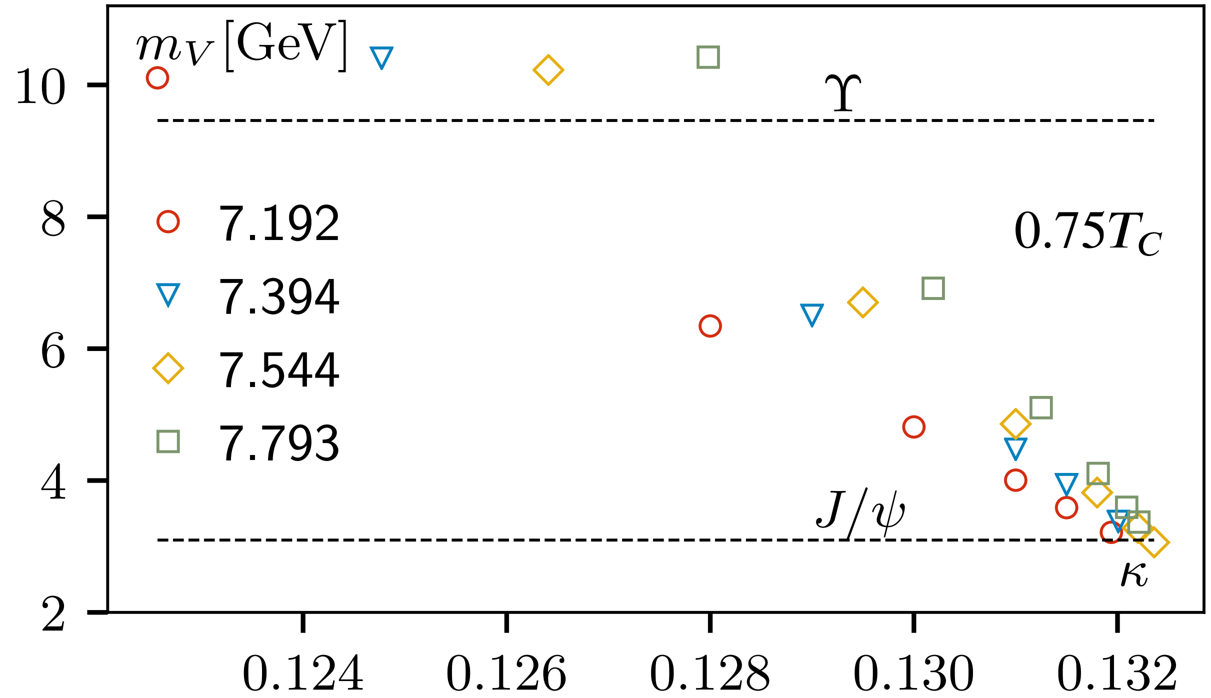
Lattice setup

β	r_0/a	a[fm](a^{-1}[GeV])	N_σ	N_τ	T/T_c	# confs
7.192	26.6	0.018(11.19)	96	48	0.75	237
				32	1.1	476
				28	1.3	336
				24	1.5	336
				16	2.25	237
7.394	33.8	0.014(14.24)	120	60	0.75	171
				40	1.1	141
				30	1.5	247
				20	2.25	226
7.544	40.4	0.012(17.01)	144	72	0.75	221
				48	1.1	462
				42	1.3	660
				36	1.5	288
				24	2.25	237
7.793	54.1	0.009(22.78)	192	96	0.75	224
				64	1.1	291
				56	1.3	291
				48	1.5	348
				32	2.25	235

β	κ	m_V[GeV]	β	κ	m_V[GeV]
7.192	0.13194	3.21(1)	7.394	0.132008	3.38(2)
	0.1315	3.59(1)		0.1315	3.94(2)
	0.131	4.01(1)		0.131	4.47(2)
	0.13	4.81(1)		0.129	6.50(2)
	0.128	6.34(1)		0.124772	10.04(1)
	0.12257	10.11(1)			
7.544	0.13236	3.06(2)	7.793	0.13221	3.37(1)
	0.1322	3.28(1)		0.13209	3.59(1)
	0.1318	3.82(2)		0.13181	4.11(1)
	0.131	4.86(2)		0.13125	5.11(1)
	0.1295	6.70(2)		0.13019	6.92(1)
	0.12641	10.23(2)		0.12798	10.42(1)

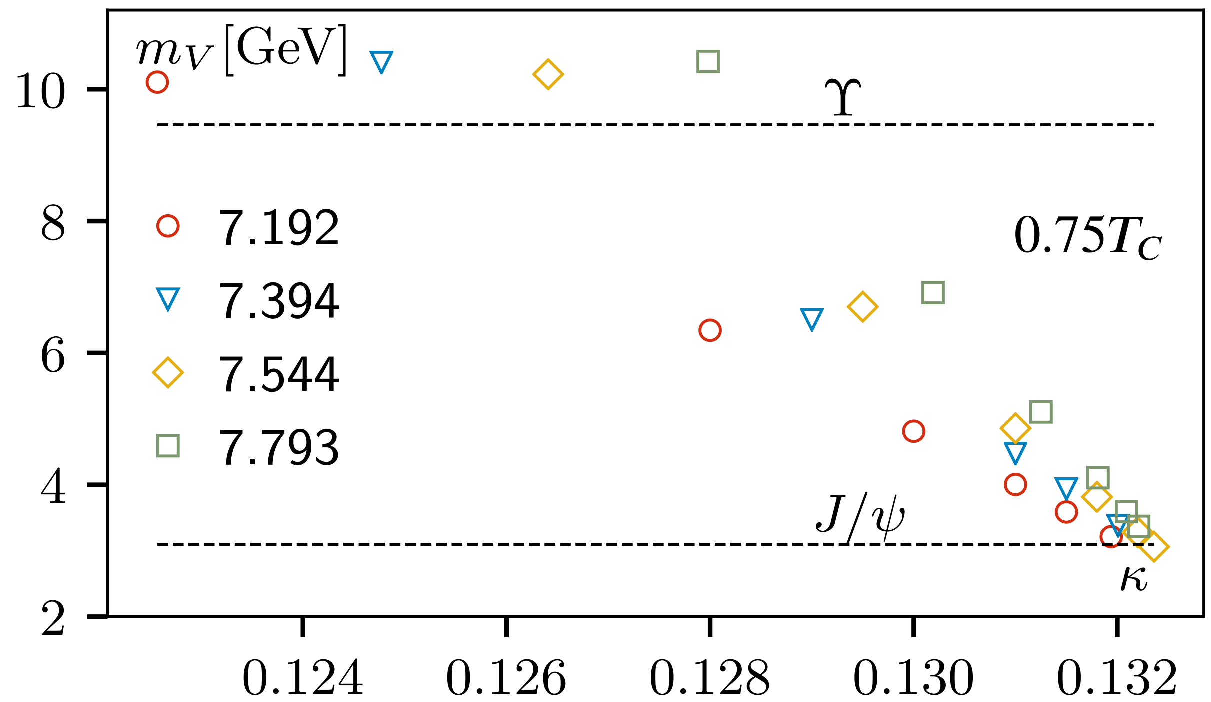
- Large, fine, isotropic lattices in the quenched approximation
- Five different temperatures
- Clover improved Wilson fermions
- Wide quark mass range

Meson correlators — mass interpolation

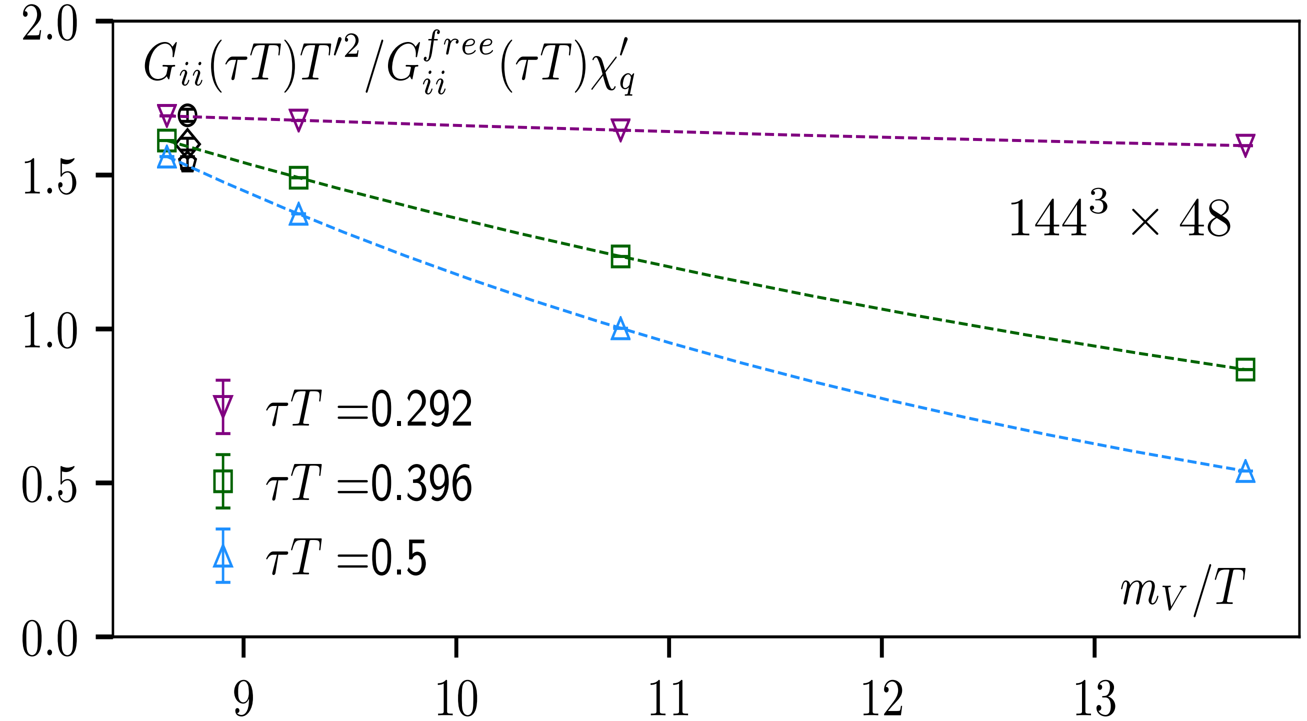


- Vector meson masses at different κ

Meson correlators — mass interpolation



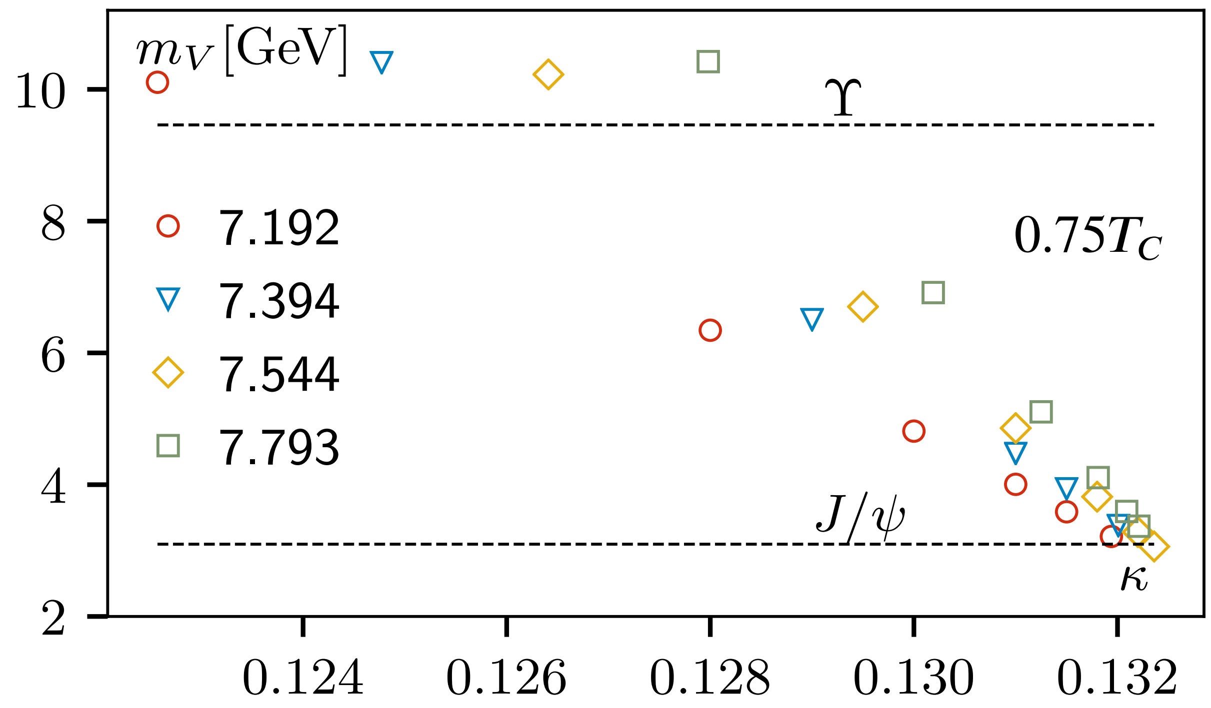
- Vector meson masses at different κ



- Mass interpolation:

$$G_{ii}(\tau T, \frac{m_V}{T}) \sim \exp\left(p(\frac{m_V}{T})^2 + q\frac{m_V}{T} + r\right)$$

Meson correlators — mass interpolation



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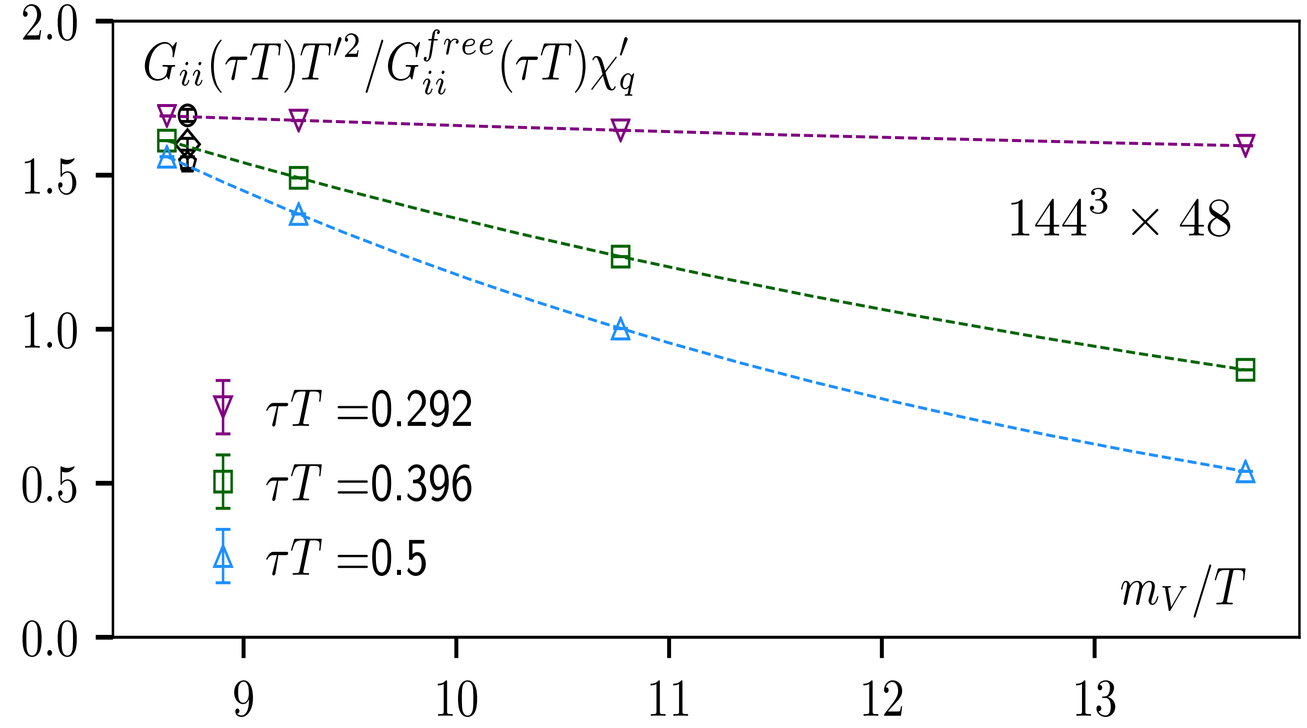

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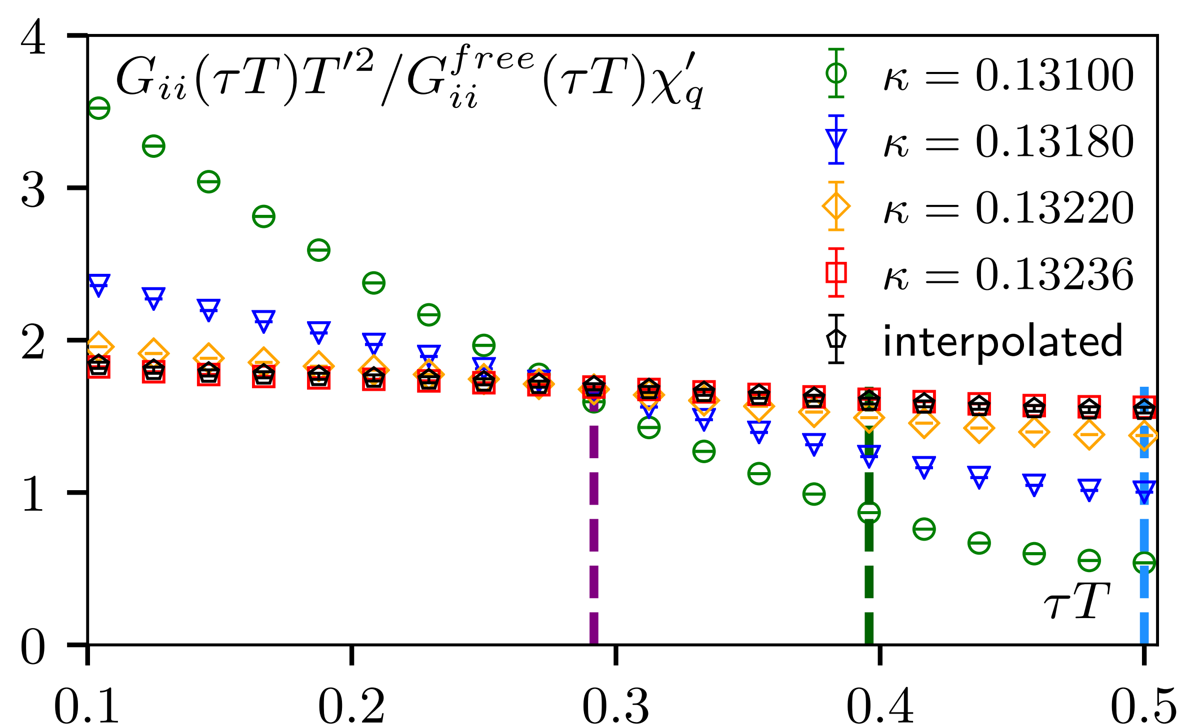
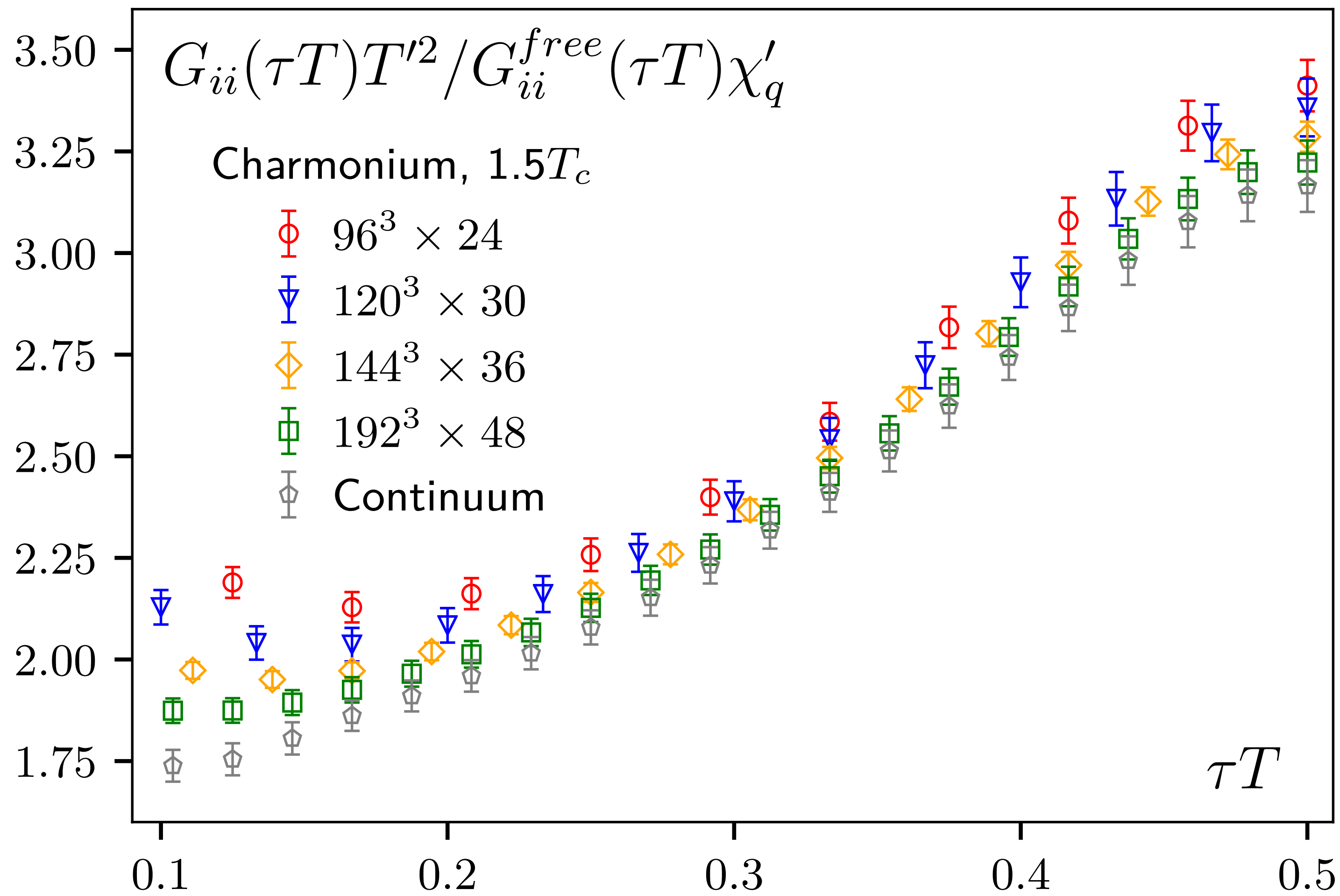
$$G_{ii}(\tau T, \frac{m_V}{T}) \sim \exp\left(p(\frac{m_V}{T})^2 + q\frac{m_V}{T} + r\right)$$



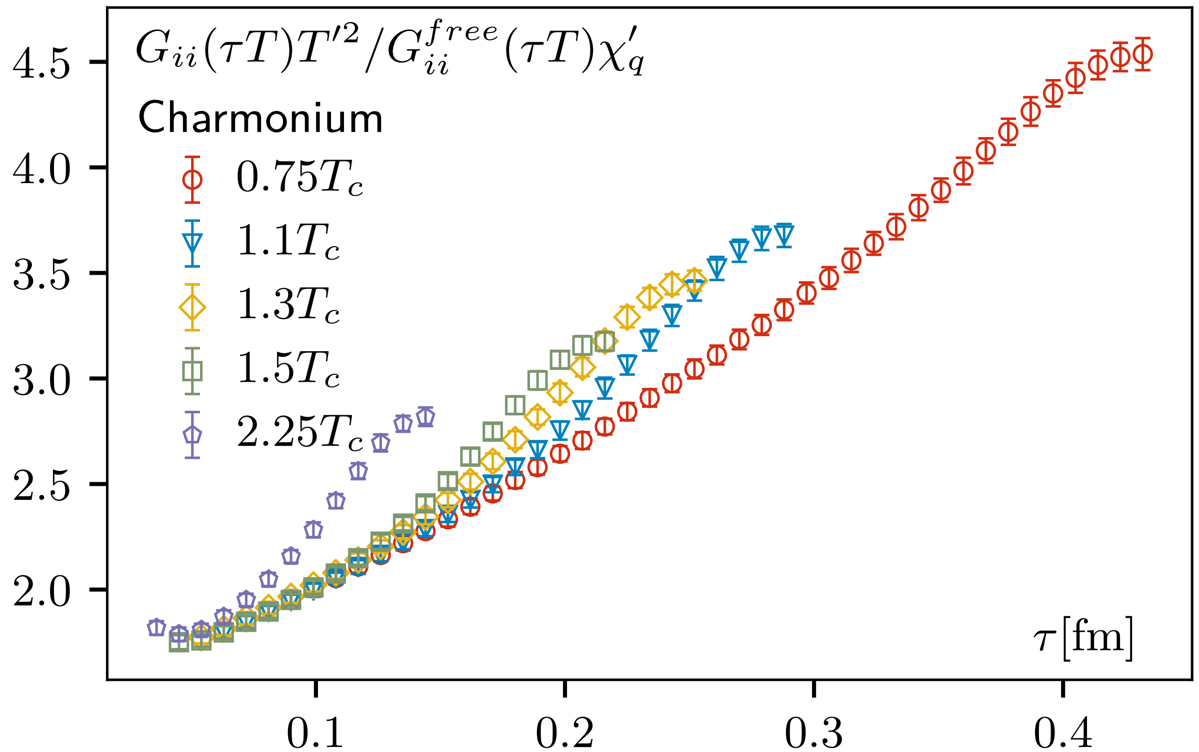
Meson correlators — continuum extrapolation

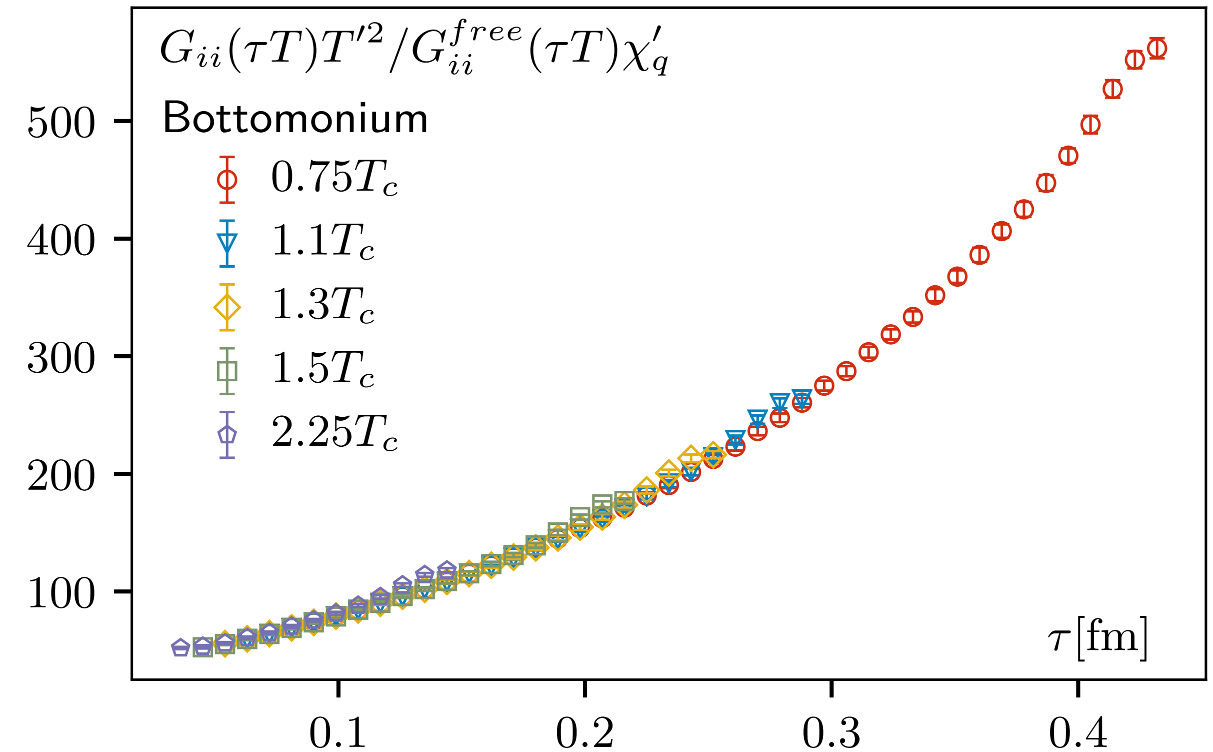


- Well controlled continuum extrapolation via ansatz:

$$G_{ii}(\tau T) = \frac{a}{N_\tau^2} + b$$

Meson correlators — temperature dependence





$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$

- ✶ UV part from “ultraviolet asymptotics + pNRQCD”:

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$

- ✶ IR part in the vector channel from effective Langevin theory:

$$\rho_{ii}^{trans}(\omega) = 3\chi_q\,\frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \quad \boldsymbol{\rightarrow} \quad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega}$$

- ✶ Almost constant transport contribution for narrow transport peak (high $\boldsymbol{T}$ limit)

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$

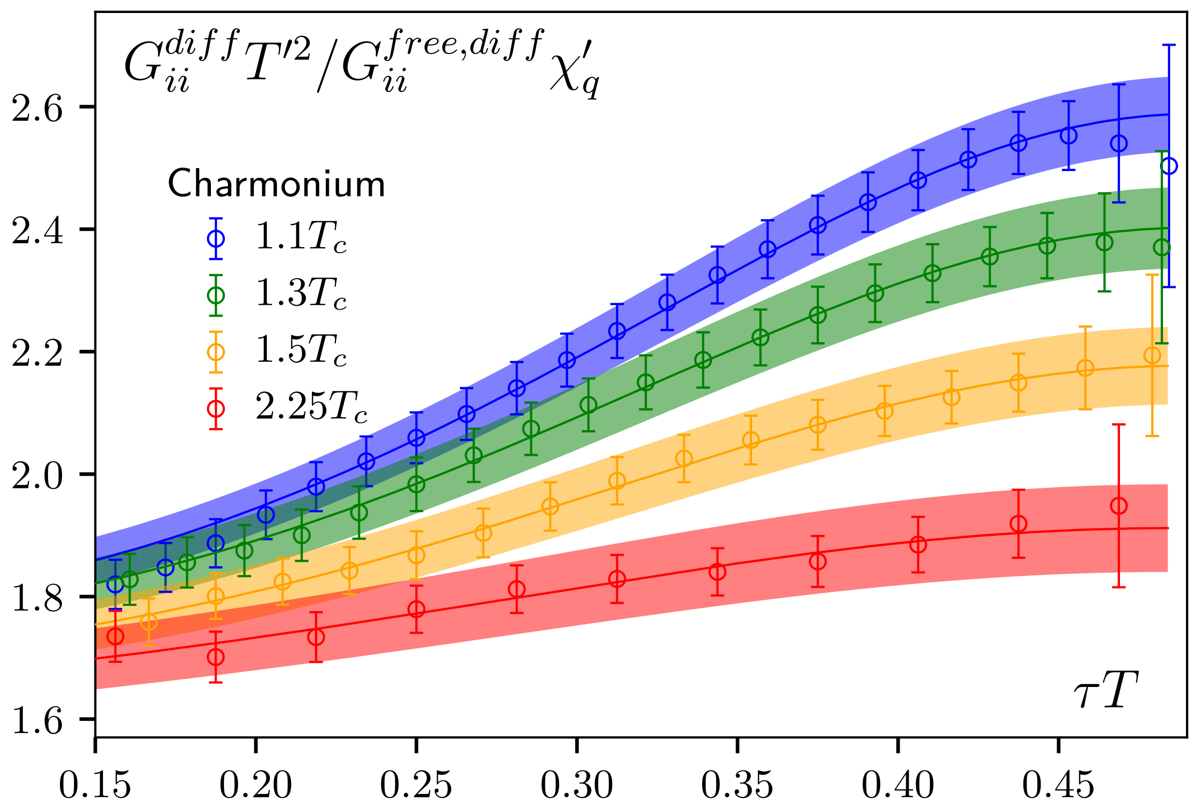
Meson correlators — IR subtraction

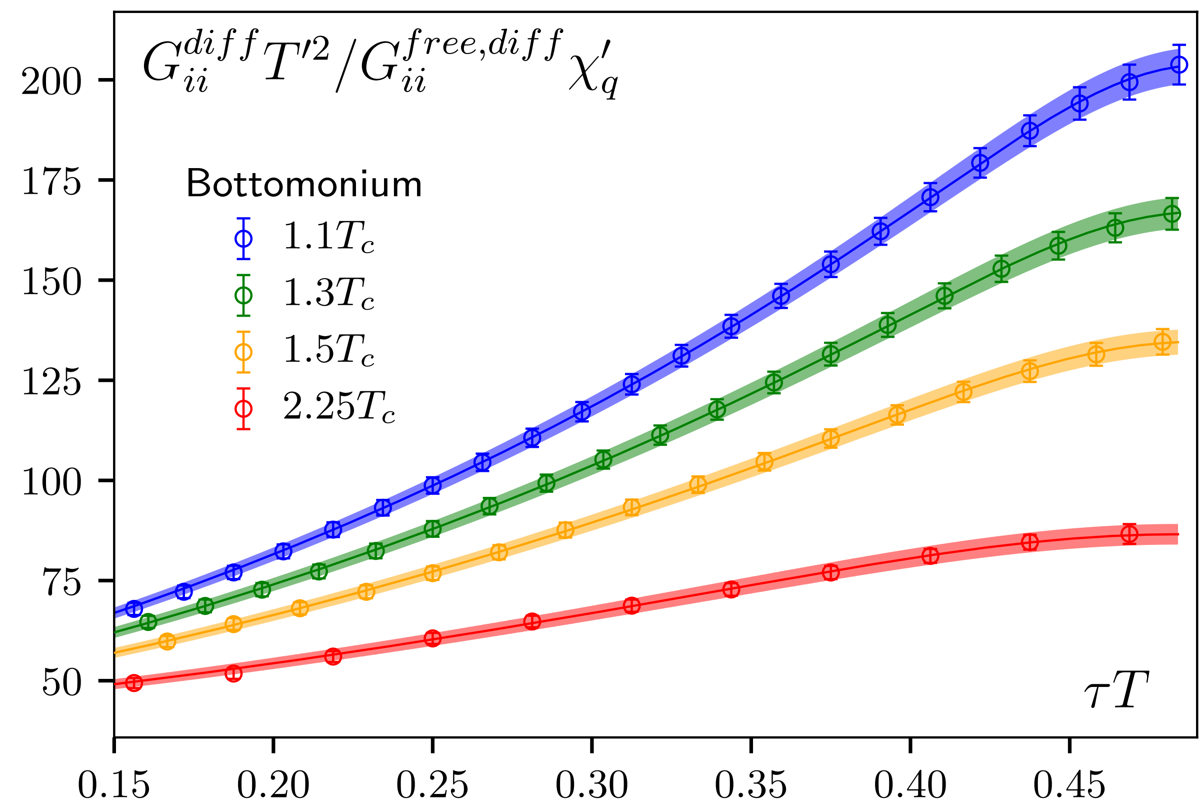
- Taking the difference of adjacent correlators effectively removes the IR contribution:

$$G_{ii}^{diff}(\tau/a) = G_{ii}(\tau/a+1) - G_{ii}(\tau/a)$$

- Small modifications needed for the perturbative model to describe lattice data:

$$\rho_{ii}^{mod}(\omega) = A\rho_V^{pert}(\omega - B)$$



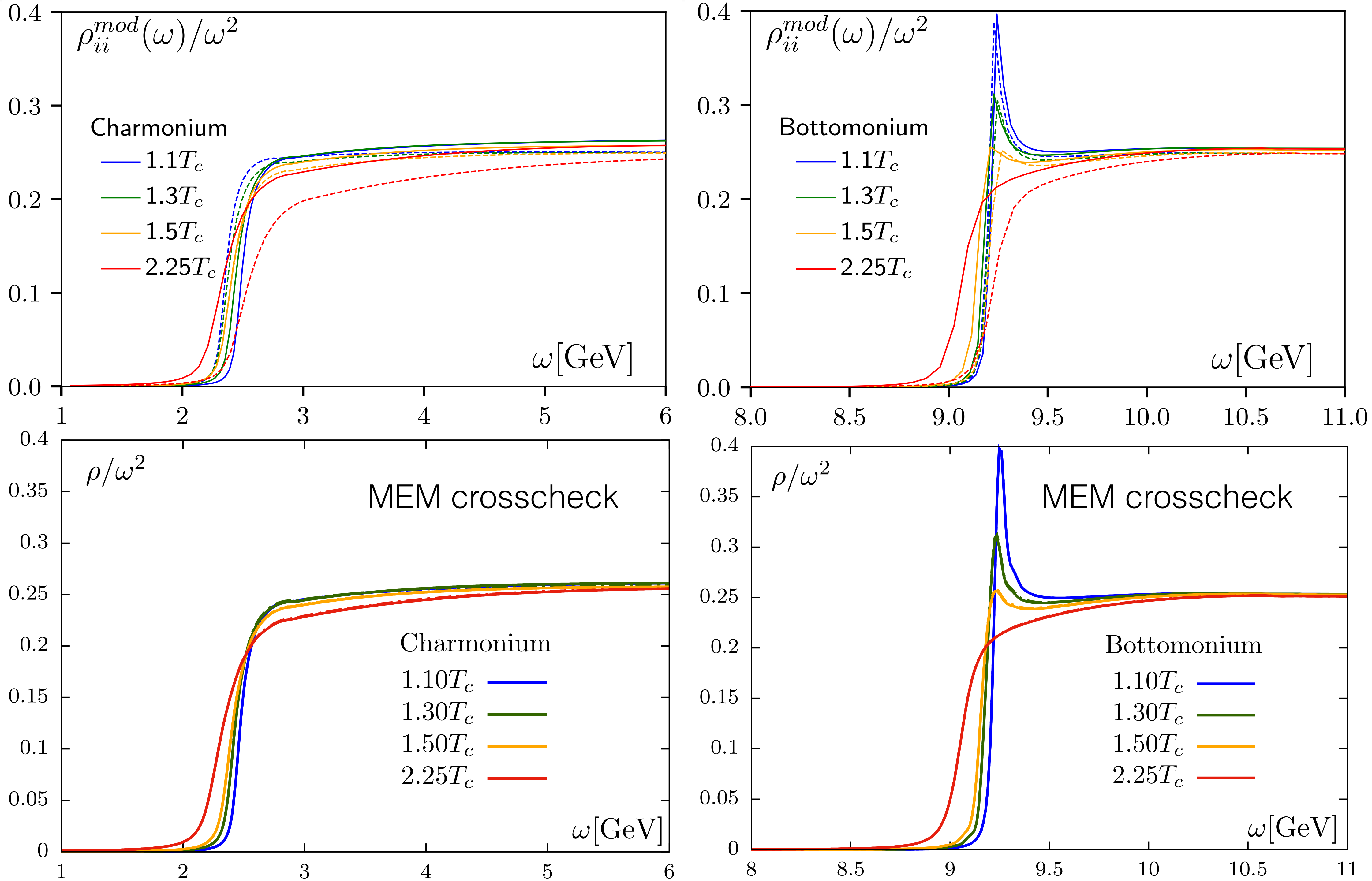


	Charmonium		Bottomonium	
T/T_c	A	B/T	A	B/T
1.1	1.09(2)	0.37(4)	1.03(2)	0.04(2)
1.3	1.07(2)	0.16(5)	1.01(1)	-0.05(2)
1.5	1.03(2)	0.01(6)	1.00(2)	-0.12(2)
2.25	0.99(3)	-0.27(9)	0.99(2)	-0.23(4)

The possible source for the difference between perturbative and lattice calculations:

- ✳ uncertainties in perturbative renormalization
- ✳ uncertainties in pole mass determination

UV parts of spectral functions of J/ψ and Υ

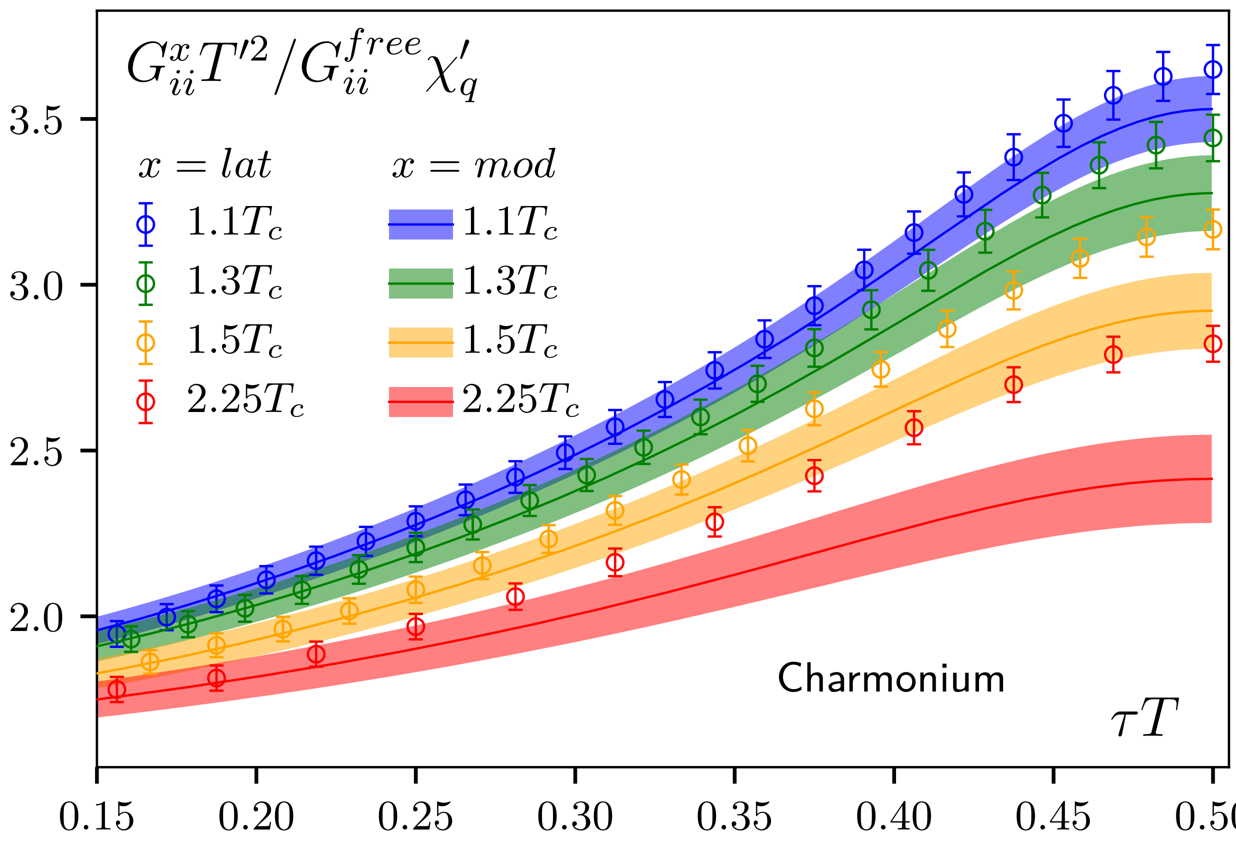


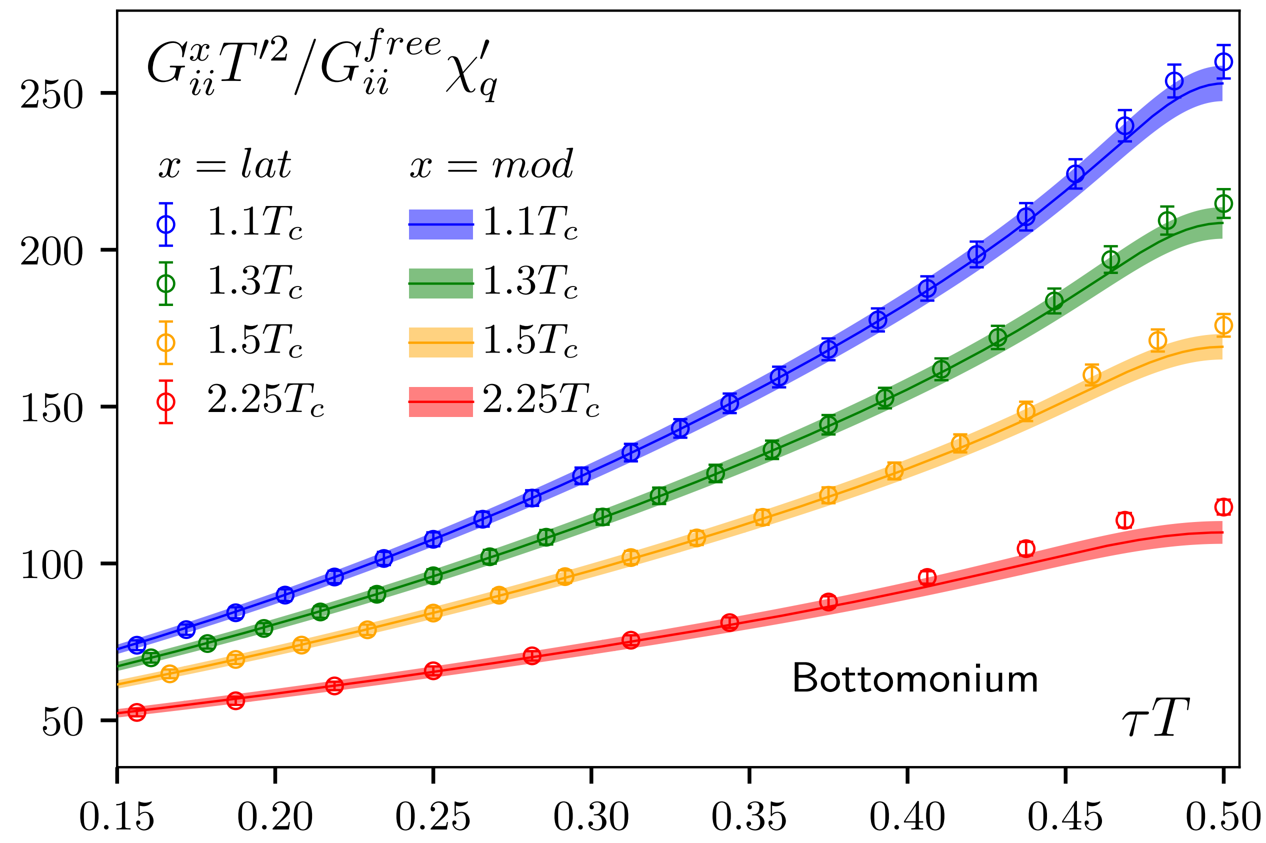
- For J/ψ no resonance peak is needed to describe the lattice data even at $1.1T_C$
- For Υ the the resonance peak persists to $1.5T_C$

Meson correlators — transport contribution

- Compare the model correlators to the full lattice correlators

$$\rho_{ii}(\omega) = \rho_{ii}^{trans}(\omega) + \rho_{ii}^{mod}(\omega) \qquad G_{ii}(\tau T) = G_{ii}^{trans}(\tau T) + G_{ii}^{mod}(\tau T)$$





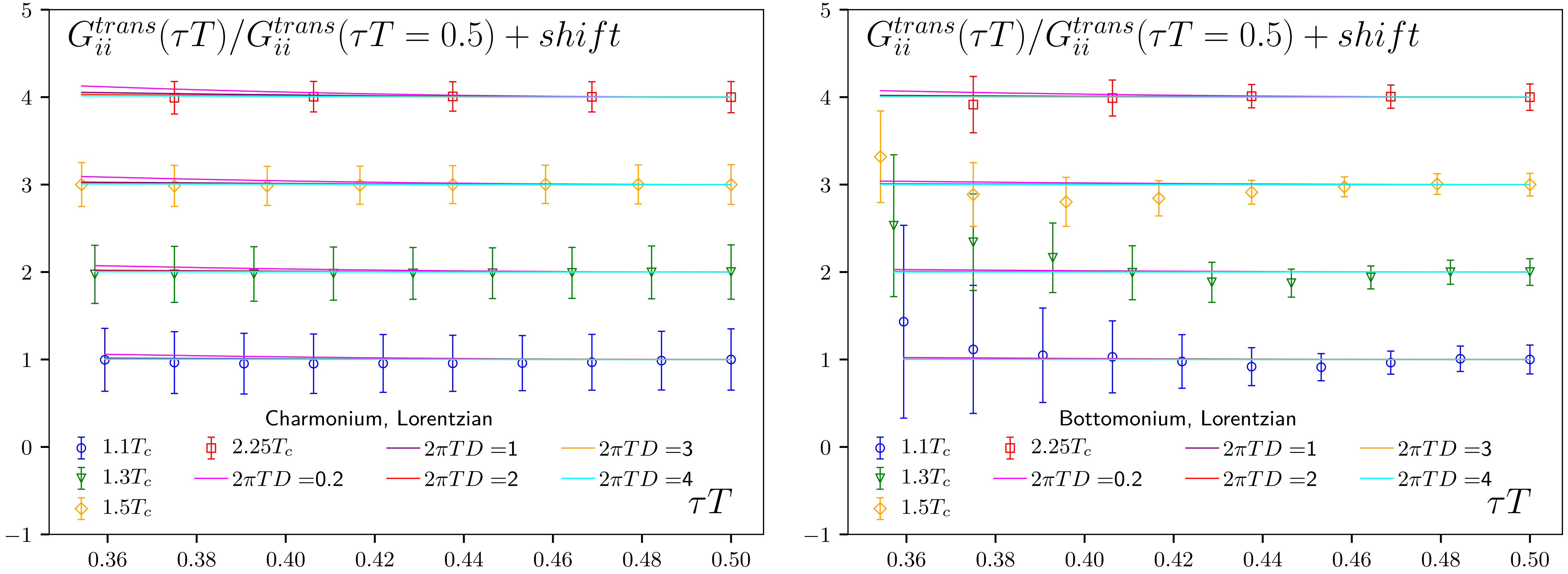
Deviation between lattice correlators and model correlators
—> small transport contribution exists and needs careful examination

Heavy quark diffusion from transport peak

- Lorentzian transport peak enters the description of full lattice correlators

$$\rho_{ii}^{trans}(\omega) = 3\chi_q \frac{T}{M}\frac{\omega\eta}{\omega^2+\eta^2} \qquad D = \frac{1}{3\chi_q}\lim_{\omega\to 0}\frac{\rho_{ii}^{trans}(\omega)}{\omega} \qquad \eta = \frac{T}{MD}$$

$$G_H(\tau,\vec{p}) = \sum_{x,y,z} exp(-i\,\vec{p}\cdot\vec{x})\langle J_H(0,\vec{0})J_H^\dagger(\tau,\vec{x})\rangle = \int \frac{d\omega}{2\pi}\frac{\cosh(\omega(\tau T-1/2)/T)}{\sinh(\omega/2T)}\rho_H(\omega,\vec{p},T)$$



- ✳ Tiny curvature in transport contribution
- ✳ Difficult to resolve D from mesonic correlators for now
- ✳ Easier in the heavy quark mass limit

From mesonic corr. to color-electric corr.

- Construct a kinetic mass dependent momentum diffusion coefficient

$$\kappa^{(M)} \equiv \frac{M^2\omega^2}{3T\chi_q} \sum_i \frac{2T\rho_V^{ii}(\omega)}{\omega}\Big|^{\eta \ll |\omega| \lesssim \omega_{\rm UV}} \qquad D = \frac{2T^2}{\kappa^{(M)}} \qquad \eta = \frac{\kappa^{(M)}}{2MT}$$

- Large quark mass limit in Langevin theory [S. Caron-Huot et al., JHEP 0904 (2009) 053]

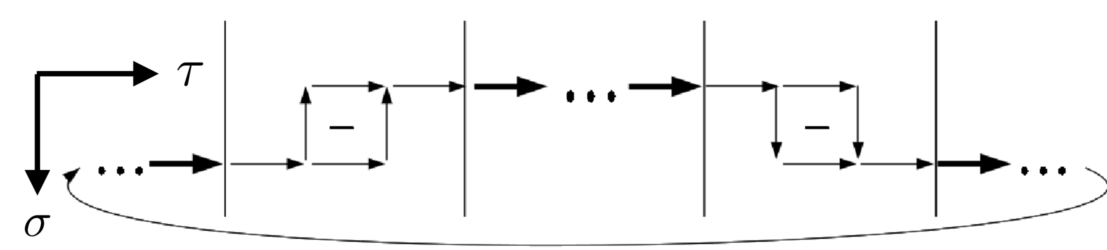
$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3} \lim_{\omega\to 0}\left[\lim_{M\to\infty} \frac{M^2}{\chi_q}\int_{-\infty}^{\infty} \mathrm{d}t\ e^{i\omega t}\int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\{\mathcal{J}^i(t,\vec{x}),\mathcal{J}^i(0,\vec{0})\}\right\rangle\right]$$

- Carry out large quark mass limit for the operators

$$\kappa \equiv \frac{\beta}{3}\sum_{i=1}^{3} \lim_{\omega\to 0}\left[\lim_{M\to\infty} \frac{M^2}{\chi_q}\int_{-\infty}^{\infty} \mathrm{d}t \int \mathrm{d}^3\vec{x}\left\langle \frac{1}{2}\left\{\left[\hat\phi^\dagger gE^i\hat\phi - \hat\theta^\dagger gE^i\hat\theta\right](t,\vec{x}),\left[\hat\phi^\dagger gE^i\hat\phi - \hat\theta^\dagger gE^i\hat\theta\right](0,\vec{0})\right\}\right\rangle\right]$$

- Perform analytic continuation and discretize the operator on the lattice

$$G_{EE}(\tau) = -\frac{1}{3}\sum_{ii=1}^{3} \frac{\langle \mathrm{Re\ Tr}[U(\beta,\tau)gE_i(\tau,\vec{0})U(\tau,0)gE_i(0,\vec{0})]\rangle}{\langle \mathrm{Re\ Tr}[U(\beta,0)]\rangle}$$

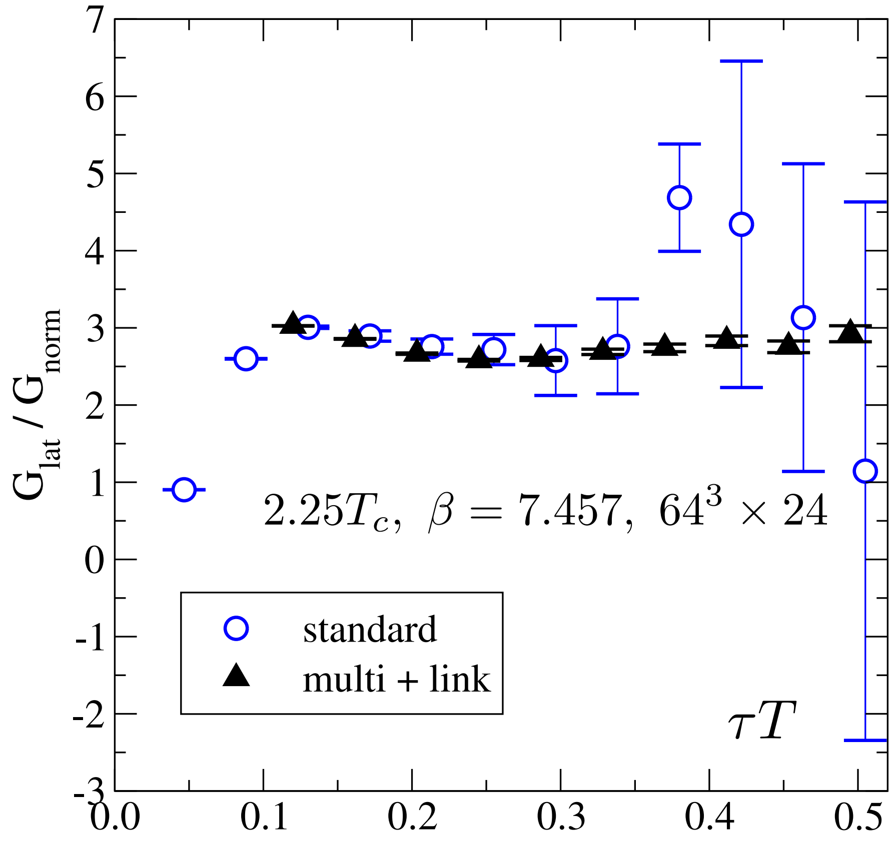


$$\Rightarrow \quad \kappa = \lim_{\omega\to 0} 2T\frac{\rho(\omega)}{\omega}$$

- correlators cheap on the lattice
- less structure in spectral functions

Gradient flow

- Color-electric correlators from multi-level and link-integration: [PRD92(2015)116003]



✤Multi-level [Luscher & Weisz, JHEP09 (2001)010]
✤Link-integration [Forcrand &Roiesnel, PLB151(1985)77]

However, only works in quenched approximation
Need Gradient Flow in full QCD

[Luscher & Weisz, JHEP1102(2011)051]
[Narayanan & Neuberger, JHEP0603(2006)064]

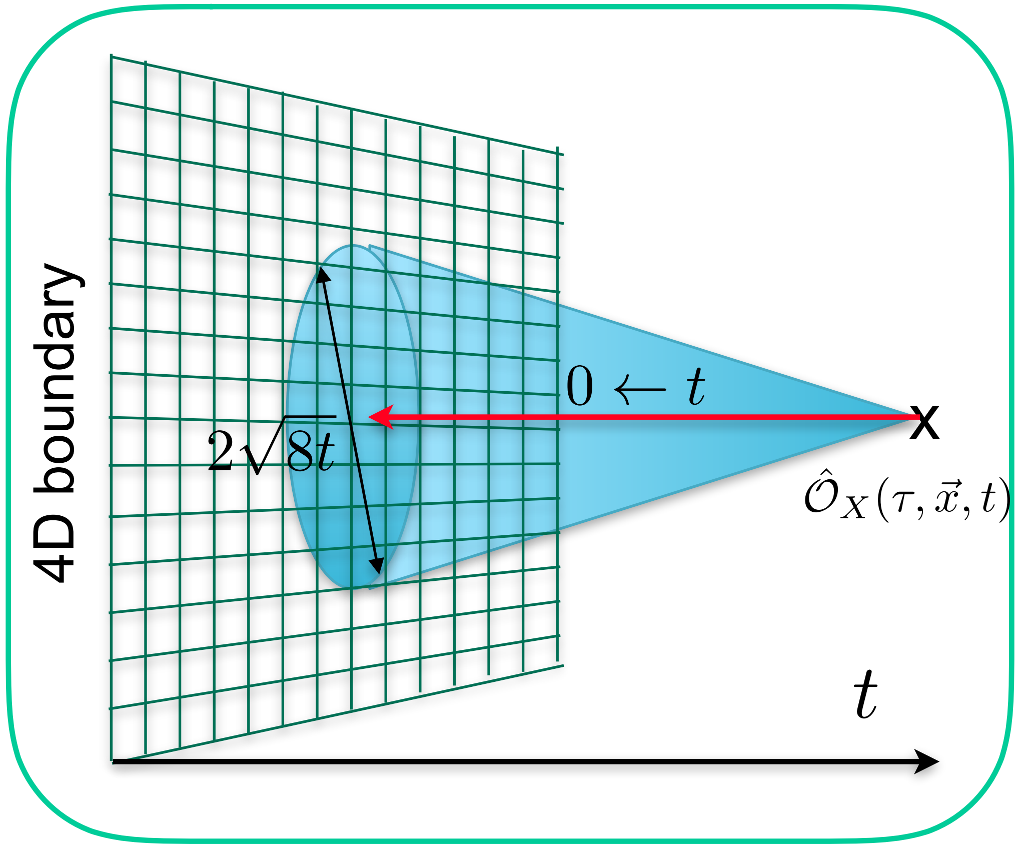
- Gradient flow as a "diffusion" equation along *t* :

$$\partial_t B(x,t) = D_\nu G_{\nu\mu}(x,t) \qquad B_\nu(x,t)|_{t=0} = A_\nu(x)$$

- Small *t* expansion: $\mathcal{O}(x,t) \xrightarrow{t \to 0} \sum_k c_k(t) \mathcal{O}_k^R(x)$

- Applications:

running coupling / topo. charge / scale setting
defining operators / noise reduction / ...



Lattice set-up

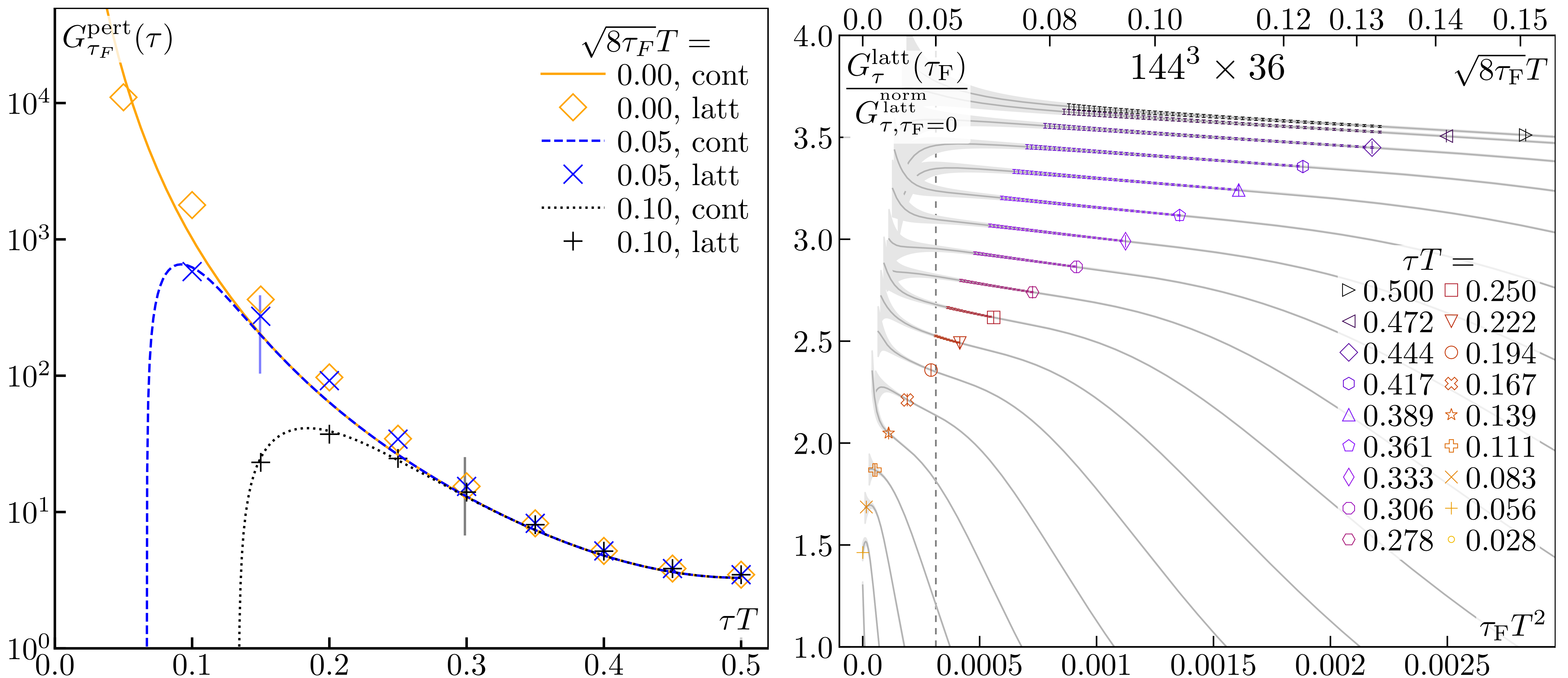
β	$a[\mathrm{fm}](a^{-1}[\mathrm{GeV}])$	N_σ	N_τ	T/T_c	#confs.
6.8736	0.026 (7.496)	64	16	1.50	10000
7.0350	0.022 (9.119)	80	20	1.50	10000
7.1920	0.018 (11.19)	96	24	1.50	10000
7.3940	0.014 (14.21)	120	30	1.50	10000
7.5440	0.012 (17.01)	144	36	1.50	10000

- Large quenched isotropic lattice
- Five different lattices
- Enough statistics
- Intensive discrete flow times

Data good enough for reliable $a \to 0$ extrapolation

and $t \to 0$ extrapolation !

Correlators under gradient flow

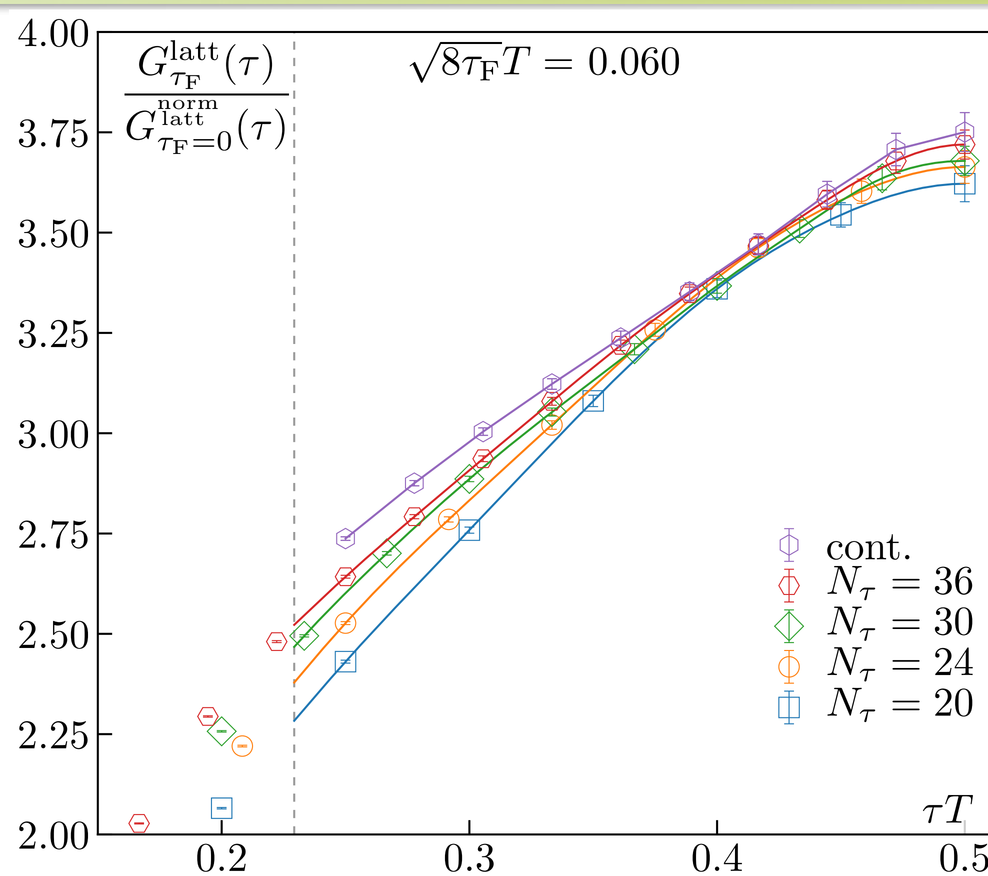


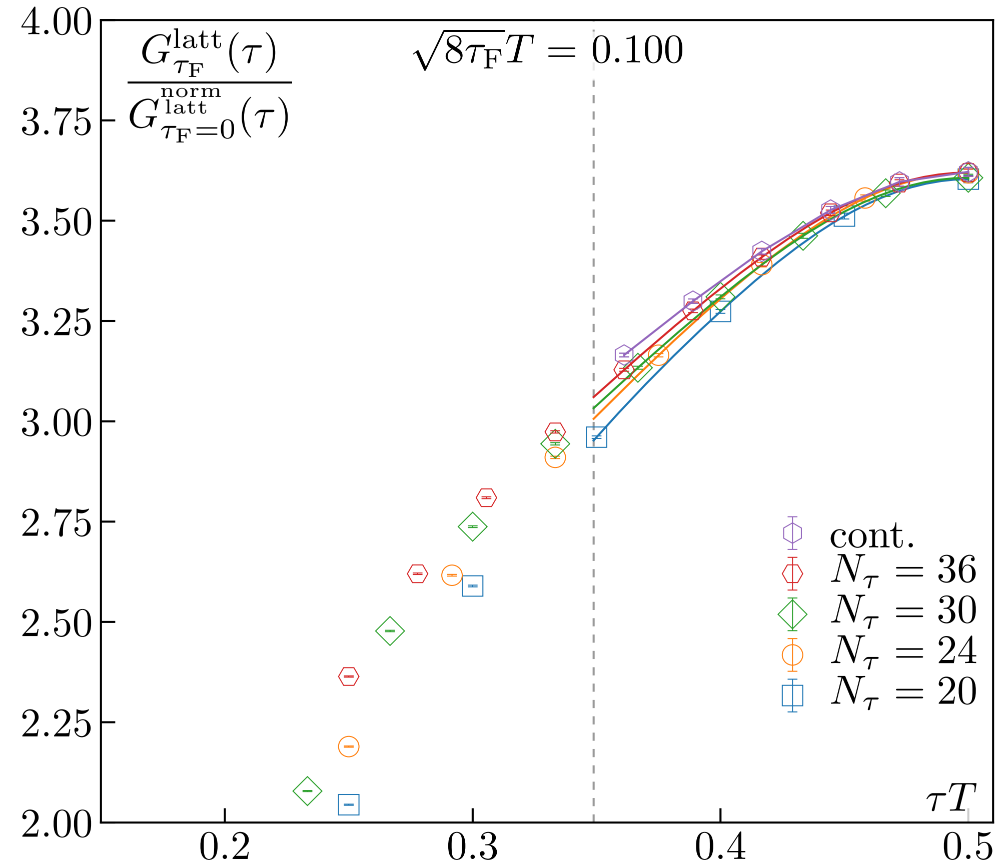
$$G_{\text{norm}}(\tau T) = \pi^2 T^4 \left[\frac{\cos^2(\pi\tau T)}{\sin^4(\pi\tau T)} + \frac{1}{3\sin^2(\pi\tau T)} \right]$$

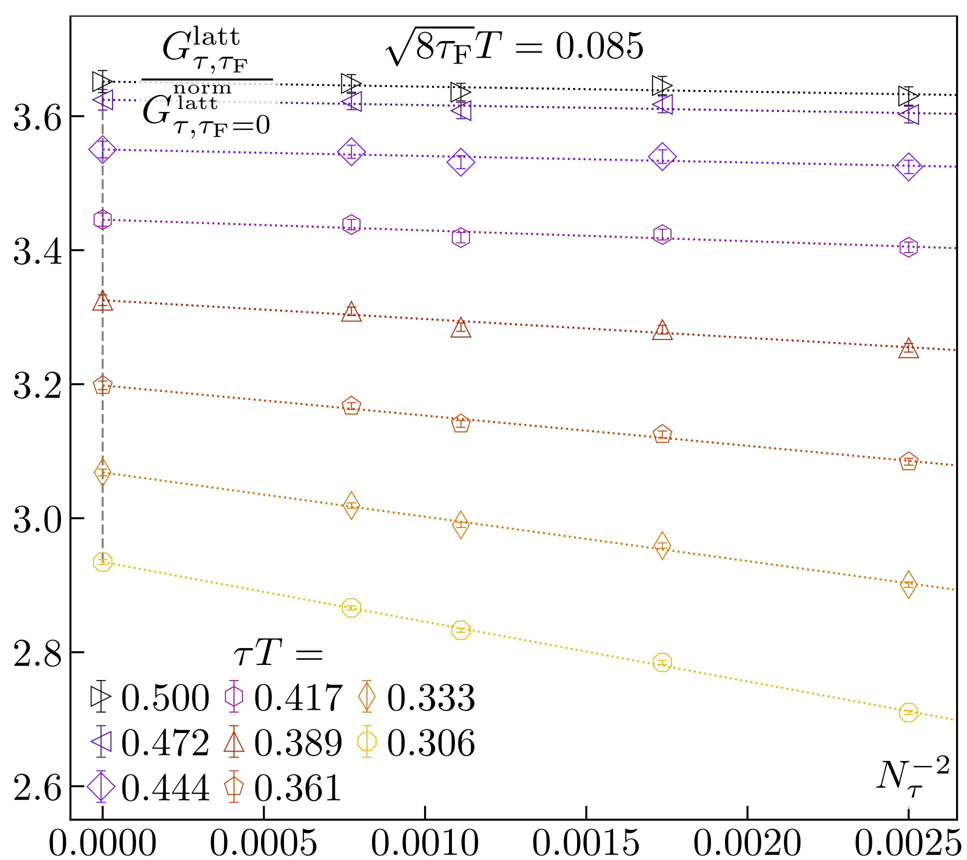
[S. Caron-Huot & M. Laine & G.D. Moore]

- Flow destroys the signal at small distances
- Large distance parts are not affected
- Flow reduces the error of data
- Perturbative flow time range is applicable on the lattice

Continuum extrapolation



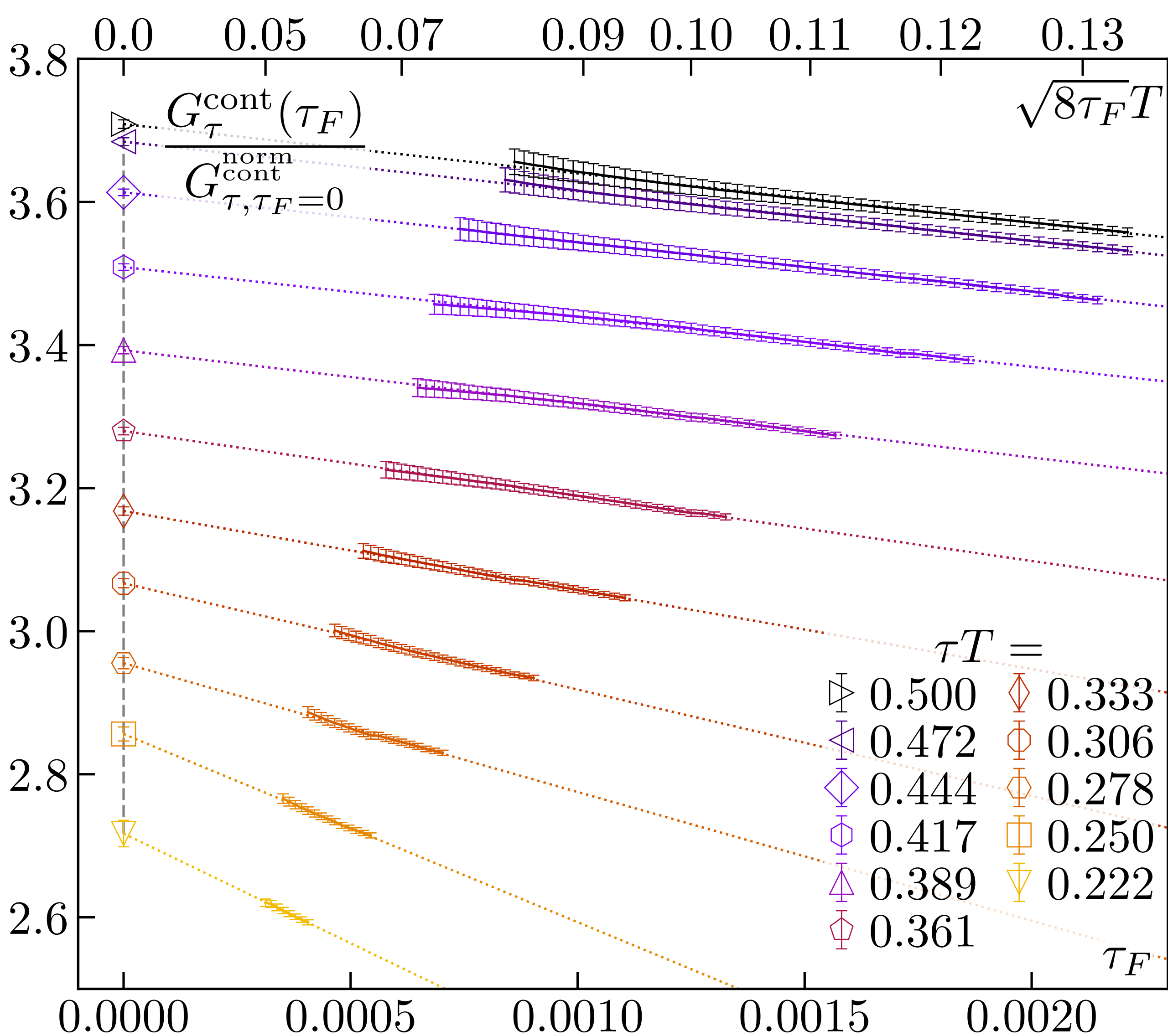


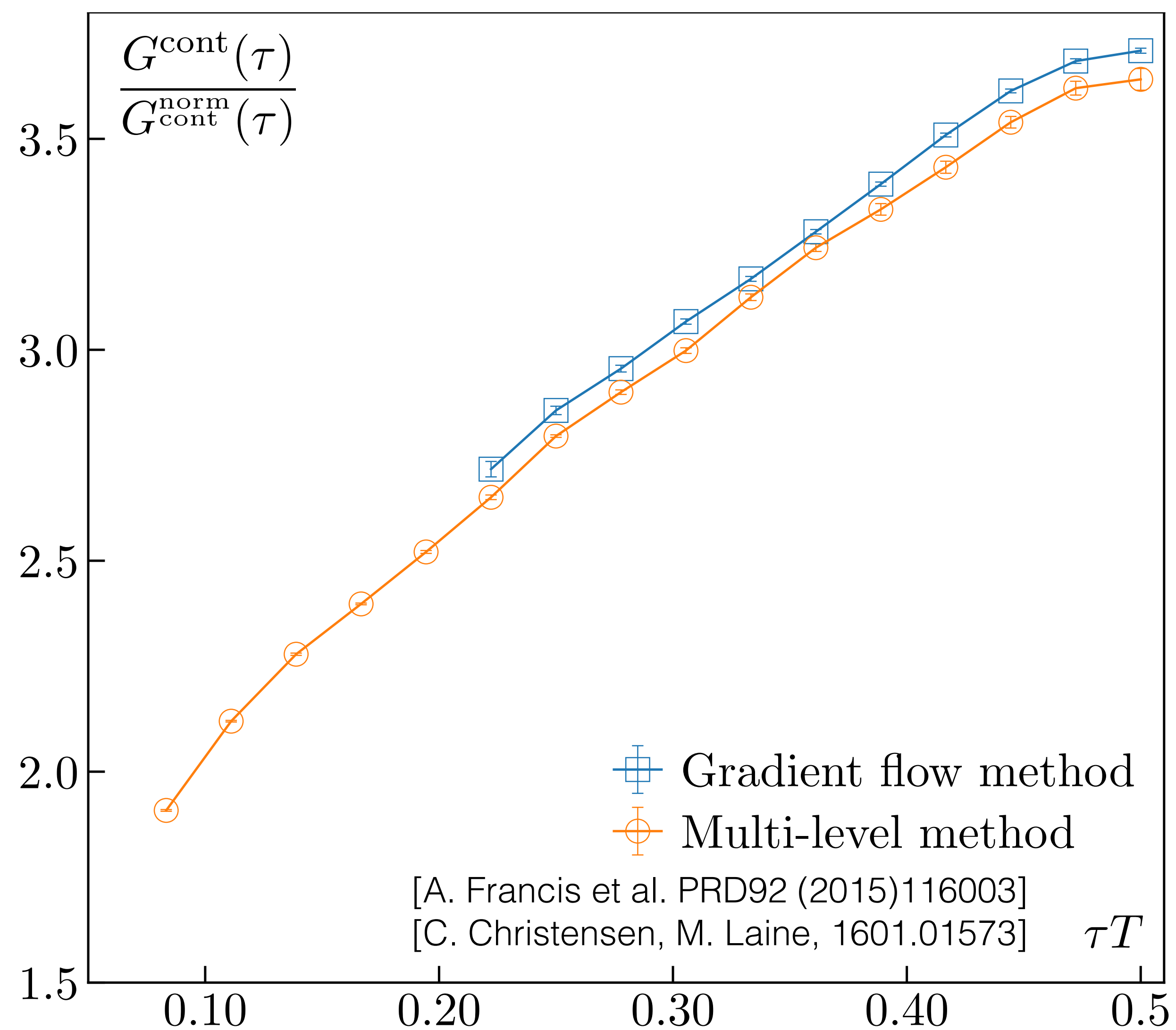


- Less τT available for larger t
- Flow removes the lattice effects
- Reliable and precise continuum can be achieved with ansatz:

$$G_{\tau,\tau_F}(N_\tau) = \frac{m}{N_\tau^2} + G^{\mathrm{cont}}_{\tau,\tau_F}$$

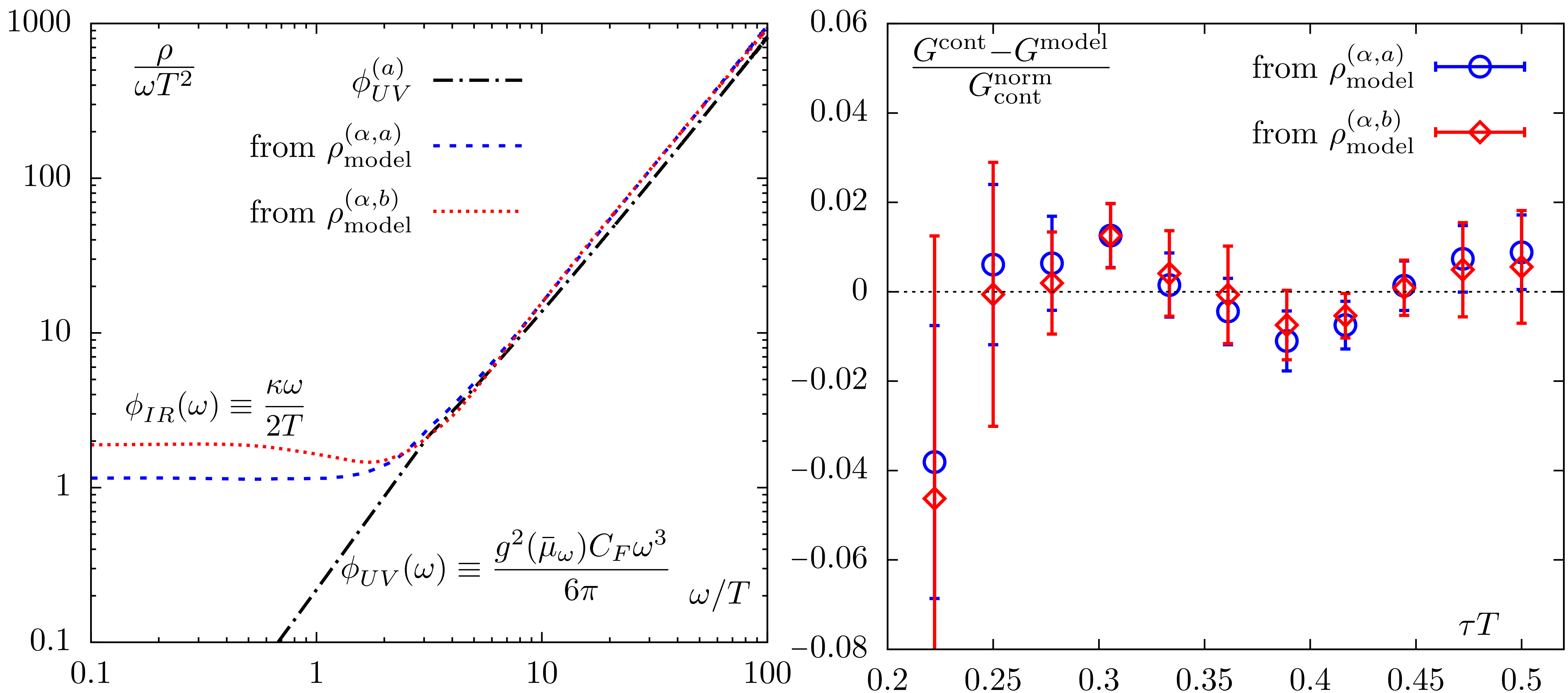
Flow time extrapolation





- Linear $t \to 0$ extrapolation after $a \to 0$ extrapolation
- An overall shift between correlators from GF and ML
- Non-pert. renormalization (GF) *v.s.* pert. renormalization (ML)

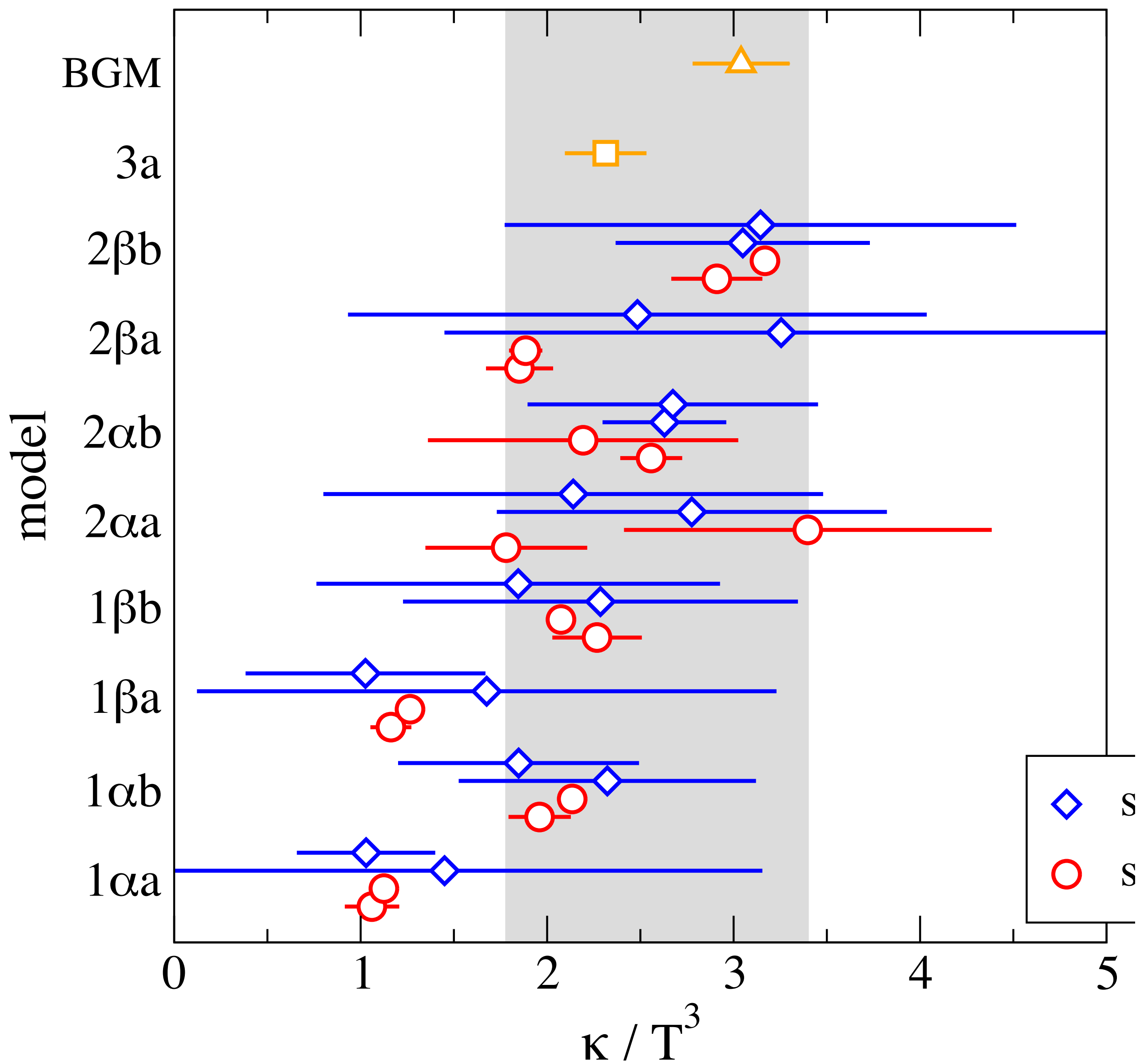
Spectral function reconstruction

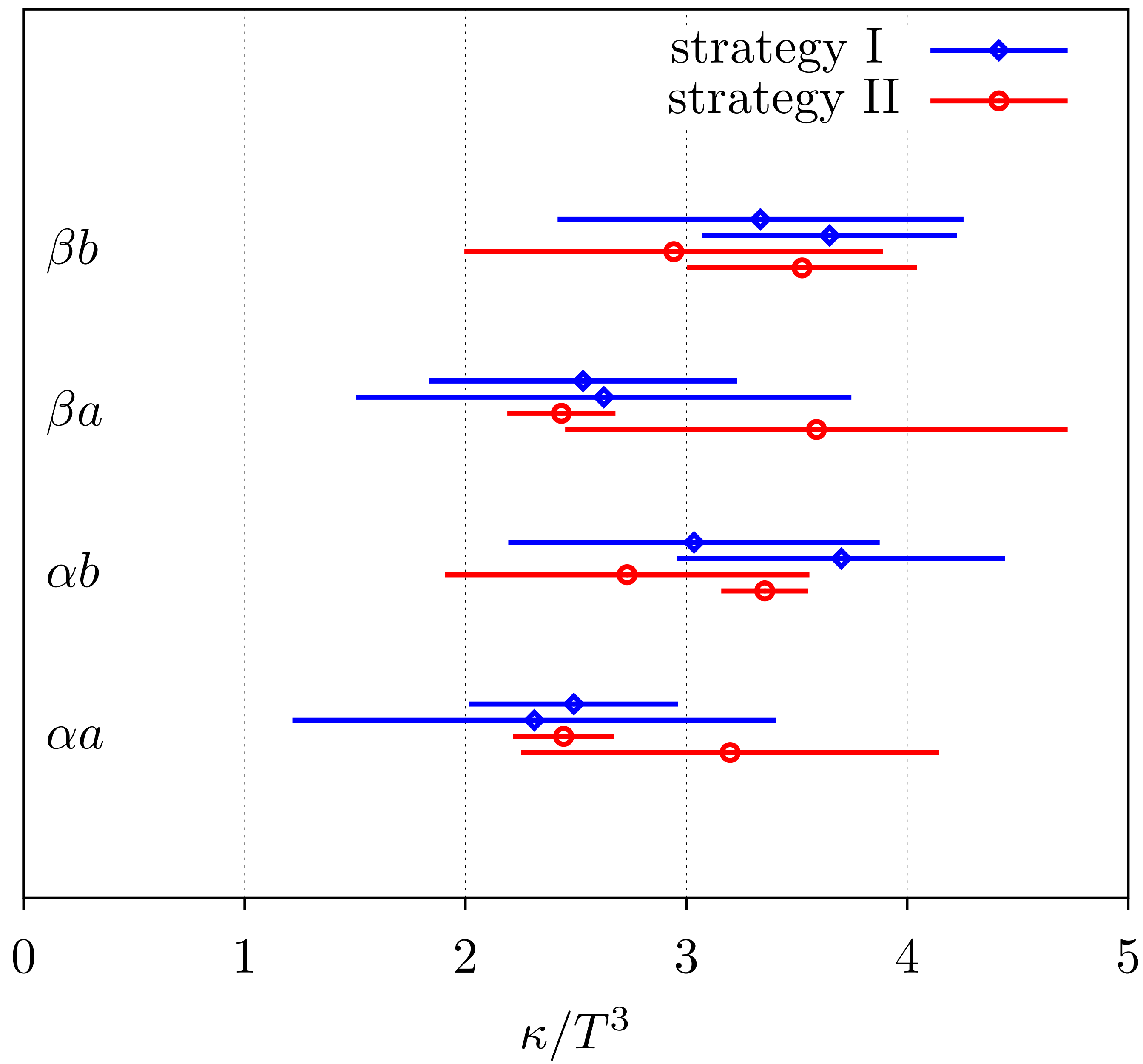


- Chi-square fitting with theoretically motivated models
- Interpolation between different regimes is needed
- Good agreement with data for different models

HQ momentum diffusion coefficient

[A. Francis et al. PRD92 (2015)116003]





$$\Rightarrow \quad \kappa/T^3 = 1.8 - 3.4$$
$$DT = 0.59 - 1.1$$

$$\Rightarrow \kappa/T^3 = 2.31 - 3.70$$
$$DT = 0.54 - 0.87$$

(this work)

(in agreement with TUMQCD Collaboration, 2007.10078 : $\kappa/T^3 = 1.31 - 3.64$)

Conclusion & Outlook

- pQCD-inspired models could well describe our lattice data at physical J/ψ and Υ mass
  - → For charmonium no resonance peak is needed even at $1.1T_C$
  - → For bottomonium the resonance peak persists to $1.5T_C$
- κ has been extracted from the non-perturbatively renormalized color-electric correlators and is consistent with other lattice studies

All results in this talk are in the continuum limit but quenched approximation!

✳ Extend to full QCD using large and fine 2+1-flavor HISQ lattices

✳ Include the $O(T/M)$ corrections to κ from color-magnetic correlators [A. Bouttefeux, M. Laine, 2010.07316]

$\Lambda_{\overline{\mathrm{MS}}}\big\vert_{N_f=0} \approx 255\mathrm{MeV}$	$\Lambda_{\overline{\mathrm{MS}}}\big\vert_{N_f=3} \approx 340\mathrm{MeV}$
$T_c\big\vert_{N_f=0} \approx 1.24\Lambda_{\overline{\mathrm{MS}}}\big\vert_{N_f=0}$	$T_c\big\vert_{N_f=3} \approx 0.45\Lambda_{\overline{\mathrm{MS}}}\big\vert_{N_f=3}$
$\alpha_s^{\mathrm{EQCD}}\big\vert_{T\simeq T_c} \simeq 0.2$	$\alpha_s^{\mathrm{EQCD}}\big\vert_{T\simeq T_c} > 0.3$
1^{st} order deconfinement transition	chiral crossover

In full QCD physics is more complicated:

→ pQCD-inspired models may be not applicable

→ flow of quark fields?

…but closer to the reality!

Thanks!