Fermion helicity vs. chirality:

transport, thermodynamic, and spin-polarization effects

Maxim Chernodub

Institut Denis Poisson, CNRS, Tours, France
Pacific Quantum Center, Vladivostok, Russia

in collaboration with

Victor Ambruş

West University of Timișoara, Romania
Goethe University Frankfurt, Germany

ArXiv
1912.09977
1912.11034
2005.03575
2010.05831

Partially supported by grant No. 0657-2020-0015 of the Ministry of Science and Higher Education of Russia.

Helicity vs. Chirality for Dirac fermions

Motivation

(for simplicity, we consider massless fermions **unless explicitly noted**)

— Chirality is an important property of a massless fermion

(relevant example: massless QED)

- **a conserved quantity up to anomalies**
- **essential for a number of anomalous transport effects**

(example: chiral vortical effects)

— Helicity:

- **similar but <u>not always</u> equal to chirality**

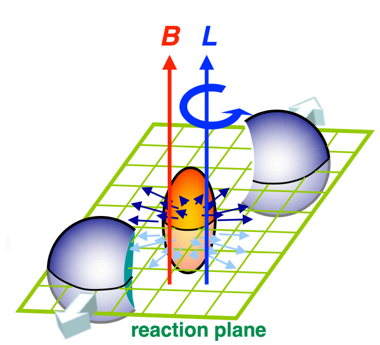
(in other words: helicity ≠ chirality)

- **a conserved number in elastic collisions**

(for massless fermions, any order)

— Questions:

Q1: is helicity as significant as chirality?
Q2: any new transport effects due to helicity?
Q3: important for quark-gluon plasma?



Helicity vs. Chirality for Dirac fermions

Lagrangian: $$\mathcal{L} = \frac{i}{2}(\overline{\psi}\gamma^\mu\partial_\mu\psi - \partial_\mu\overline{\psi}\gamma^\mu\psi)$$

---

spin: $s^i = \frac{1}{2}\varepsilon^{0ijk}\Sigma_{jk}$ with $\Sigma^{\mu\nu} = \frac{i}{4}[\gamma^\mu, \gamma^\nu])$

Hamiltonian

$$\widehat{H} = \boldsymbol{\alpha}\cdot\boldsymbol{p}$$

$\boldsymbol{\alpha} \equiv \gamma^0\boldsymbol{\gamma}$

Helicity: $$h = \frac{\boldsymbol{s}\cdot\boldsymbol{p}}{p} \equiv \frac{1}{2}\gamma^5 \mathrm{sign}\,(\hat{H})$$

momentum

Chirality: $$\gamma^5\psi_R = +\psi_R, \qquad \gamma^5\psi_L = -\psi_L$$

right-handed chirality   left-handed chirality

Chirality defines the axial charge

Helicity vs. Chirality for Dirac fermions

Helicity: $$h = \frac{\boldsymbol{s} \cdot \boldsymbol{p}}{p} \equiv \frac{1}{2} \gamma^5 \mathrm{sign}\,(\hat{H})$$

(massless fermions)

Two definitions: 1 2

1 **Helicity = projection of the spin of a fermion on its momentum (direction of motion)** **[intuitive]**

2 **Helicity = + axial charge (for a fermion) = - axial charge (for an anti-fermion)** **[practical]**

The helical charge of an ensemble of Dirac fermions equals to the total axial charge carried by particles minus the total axial charge carried by the anti-particles

(again: helicity ≠ chirality)

Vector, axial and helical charges

Currents: $J_V^\mu = \overline{\psi}\gamma^\mu\psi$ $\quad J_A^\mu = \overline{\psi}\gamma^\mu\gamma^5\psi$ $\quad J_H^\mu = 2\overline{\psi}\gamma^\mu h\psi$

$\underbrace{\text{vector} \qquad \text{axial = chiral}}_{\text{known effects on transport, thermodynamics, ...}}$ $\qquad \underbrace{\text{helical}}_{\text{new phenomena?}}$

Helicity ("helical charge") is similar, but not identical to chirality ("chiral charge")

Common eigensystem:

$\underbrace{\underset{\text{right-handed}}{\gamma^5\psi_R = +\psi_R,} \qquad \underset{\text{left-handed}}{\gamma^5\psi_L = -\psi_L}}_{\text{axial}}$ $\qquad \underbrace{\underset{\text{right-handed}}{2h\psi_\uparrow = +\psi_\uparrow,} \qquad \underset{\text{left-handed}}{2h\psi_\downarrow = -\psi_\downarrow}}_{\text{helical}}$

Conservation (free theory)

$$[\gamma^5, h] = 0, \qquad [\widehat{H}, \gamma^5] = 0, \qquad [\widehat{H}, h] = 0$$

Hamiltonian

$$\widehat{H} = \boldsymbol{\alpha}\cdot\boldsymbol{p}$$

helicity operator

$$h = \frac{\boldsymbol{s}\cdot\boldsymbol{p}}{p} \equiv \frac{1}{2}\gamma^5 \mathrm{sign}\,(\hat{H})$$

Vector vs. Axial vs. Helical: *C*, *P* and *T*

Charges:

$$\begin{pmatrix} Q_V \\ Q_A \\ Q_H \end{pmatrix} = \int d^3x\, \overline{\psi}\gamma^0 \begin{pmatrix} 1 \\ \gamma^5 \\ 2h \end{pmatrix} \psi$$

(similarly for currents)

Discrete symmetries:

vorticity

	charges			currents			
	Q_V	Q_A	Q_H	$\boldsymbol{J}_V$	$\boldsymbol{J}_A$	$\boldsymbol{J}_H$	$\boldsymbol{\omega}$
C	$-$	$+$	$-$	$-$	$+$	$-$	$+$
P	$+$	$-$	$-$	$-$	$+$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$	$-$

Different

Different

The signs $+/-$ denote the even/odd parities.

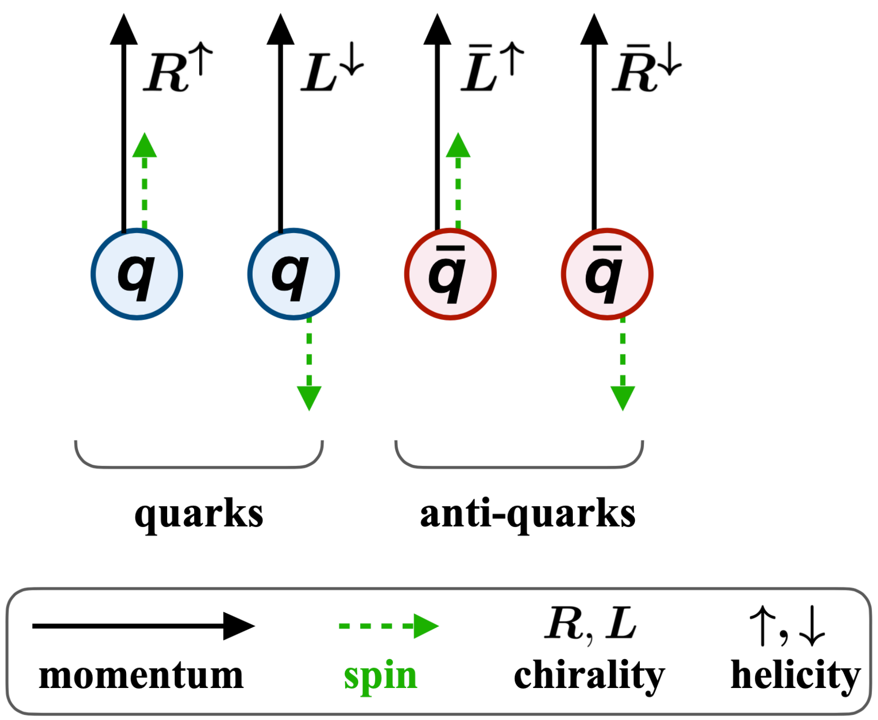
In this talk: vortical effects and thermodynamics (no magnetic field)

Effects in the magnetic-field background: in preparation [V. Ambrus and M.Ch. (2020)]

Interesting acceleration effects for the axial current: Prokhorov, Teryaev, Zakharov, JHEP 02, 146 (2019)

Helicity as a degree of freedom

Four degrees of freedom



Four combinations

Conservation
in a realistic theory

– Total particle number:

$$N_{\rm tot} = N_{\uparrow} + N_{\downarrow} + \bar{N}_{\uparrow} + \bar{N}_{\downarrow}$$

No

– Vector ("electric") charge:

$$N_V = N_{\uparrow} + N_{\downarrow} - \bar{N}_{\uparrow} - \bar{N}_{\downarrow}$$

Yes

– Axial ("chiral") charge:

$$N_A = N_{\uparrow} + \bar{N}_{\uparrow} - N_{\downarrow} - \bar{N}_{\downarrow}$$

No

Difference in axial charges carried by particles and by anti-particles

– Helical charge:

$$N_H = N_{\uparrow} + \bar{N}_{\downarrow} - N_{\downarrow} - \bar{N}_{\uparrow}$$

No

Anomalous transport

Known example

Axial anomaly

$$\partial_\mu j^\mu_A = \frac{e^2}{16\pi^2} F_{\mu\nu}\tilde{F}^{\mu\nu}$$

Magnetic field

Transport

$$j_A = \frac{\mu_V}{2\pi^2} eB, \qquad j_V = \frac{\mu_A}{2\pi^2} eB$$

Usual (vector) chemical potential

Chiral (axial) chemical potential

Known example

Mixed axial-gravitational anomaly

$$\partial_\mu j^\mu_A = -\frac{1}{384\pi^2} R_{\mu\nu\alpha\beta}\tilde{R}^{\mu\nu\alpha\beta}$$

Vorticity (angular velocity)

Transport

$$j_V = \frac{\mu_V \mu_A}{\pi^2}\Omega, \qquad j_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}\right)\Omega$$

Axial anomaly

Mixed axial-gravitational anomaly

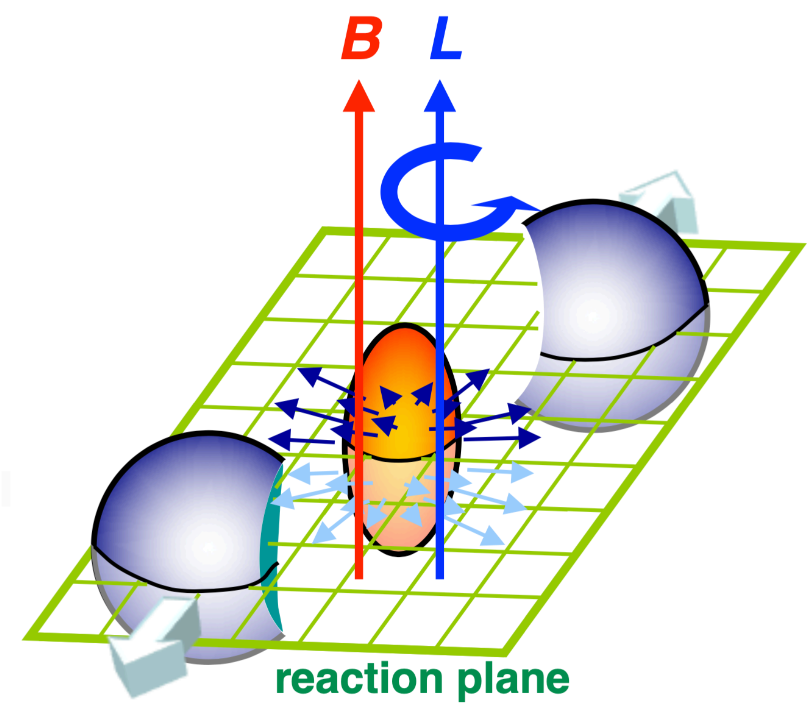
Axial anomaly

And if one considers the fermion’s helicity?

Transport? Anomalous (???) transport?

ArXiv:1912.09977
ArXiv:1912.11034

CPT-check for vortical effects



non-central heavy-ion collisions

We know the chiral vortical effects:

$$\boldsymbol{J}_V = \frac{\mu_V \mu_A}{\pi^2}\Omega, \qquad \boldsymbol{J}_A = \left(\frac{T^2}{6} + \frac{\mu_V^2 + \mu_A^2}{2\pi^2}\right)\Omega$$

where

$$\boldsymbol{J} = \sigma\,\boldsymbol{\Omega}$$

conductivity

current

background

	charges (determine conductivity)			currents			background
	Q_V	Q_A	Q_H	$\boldsymbol{J}_V$	$\boldsymbol{J}_A$	$\boldsymbol{J}_H$	$\boldsymbol{\Omega}$
C	$-$	$+$	$-$	$-$	$+$	$-$	$+$
P	$+$	$-$	$-$	$-$	$+$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$	$-$

chemical potentials → charges

The signs $+/-$ denote the even/odd parities.

Vector, axial and helical charges

determined by appropriate chemical potentials

Introduce chemical potentials:

$$
\begin{aligned}
\delta\mathcal{L}_Q &= \mu_V Q_V + \mu_A Q_A + \mu_H Q_H \\
&= \mu^R_\uparrow N^R_\uparrow + \mu^L_\downarrow N^L_\downarrow + \bar{\mu}^R_\downarrow \bar{N}^R_\downarrow + \bar{\mu}^L_\uparrow \bar{N}^L_\uparrow
\end{aligned}
$$

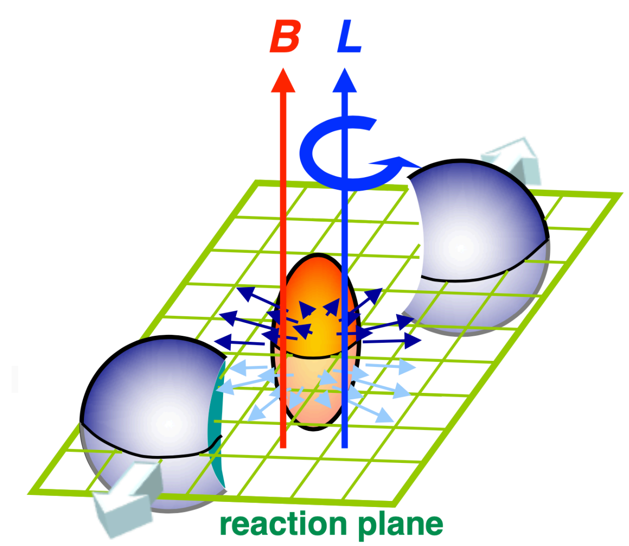
In components (formally):

$$\mu_V = \left[\left(\mu^R_\uparrow + \mu^L_\downarrow\right) - \left(\bar{\mu}^R_\downarrow + \bar{\mu}^L_\uparrow\right)\right]/4 \qquad \text{vector}$$

$$\mu_A = \left[\left(\mu^R_\uparrow + \bar{\mu}^L_\uparrow\right) - \left(\mu^L_\downarrow + \bar{\mu}^R_\downarrow\right)\right]/4 \qquad \text{axial}$$

$$\mu_H = \left[\left(\mu^R_\uparrow + \bar{\mu}^R_\downarrow\right) - \left(\mu^L_\downarrow + \bar{\mu}^L_\uparrow\right)\right]/4 \qquad \text{helical}$$

Vector, axial and helical transport



Our strategy:

0. think about a rotating quark-gluon plasma

1. consider a fluid of Dirac fermions

2. which possesses nonzero charges of all three types: vector, axial, and helical

2' in other words: we have an electrically charged fluid with both chiral imbalance and helical imbalance

They may be created by fluctuations at the initial stages of the collisions

3. then rotate the fluid and calculate the currents (along the vorticity axis) of all three types: vector, axial, and helical

3' Bonus: calculate the energy flux along Ω

Vector, axial and helical transport

(in a leading order of a high-temperature expansion)

$$\mathcal{J}_\ell = \sigma^z_\ell \, \boldsymbol{\Omega}$$

ArXiv:1912.09977, 1912.11034

Currents: **Conductivities:**

vector $$\sigma^z_V = \frac{2\mu_H T}{\pi^2}\ln 2 + \frac{\mu_V \mu_A}{\pi^2},$$

axial $$\sigma^z_A = \frac{T^2}{6} + \frac{\boldsymbol{\mu}^2}{2\pi^2},$$

helical $$\sigma^z_H = \frac{2\mu_V T}{\pi^2}\ln 2 + \frac{\mu_H \mu_A}{\pi^2},$$

energy $$\sigma^z_\varepsilon = \frac{\mu_A T^2}{3} + \frac{4\mu_V\mu_H T}{\pi^2}\ln 2 + \frac{\mu_A(3\boldsymbol{\mu}^2 - 2\mu_A^2)}{3\pi^2}$$

Known results

axial anomaly

mixed axial-gravitational anomaly

NEW: helical vortical effects

where

$$\boldsymbol{\mu}^2 = \mu_V^2 + \mu_A^2 + \mu_H^2$$

Helical transport, unusual feature:

Conductivity is linear in temperature (forbidden for chirality due to CPT)

vector $$\sigma^z_V = \boxed{\frac{2\mu_H T}{\pi^2}\ln 2} + \frac{\mu_V\mu_A}{\pi^2},$$

axial $$\sigma^z_A = \frac{T^2}{6} + \frac{\mu^2}{2\pi^2},$$

helical $$\boxed{\sigma^z_H = \frac{2\mu_V T}{\pi^2}\ln 2} + \frac{\mu_H\mu_A}{\pi^2},$$

impossible for vector/axial degrees of freedom only!

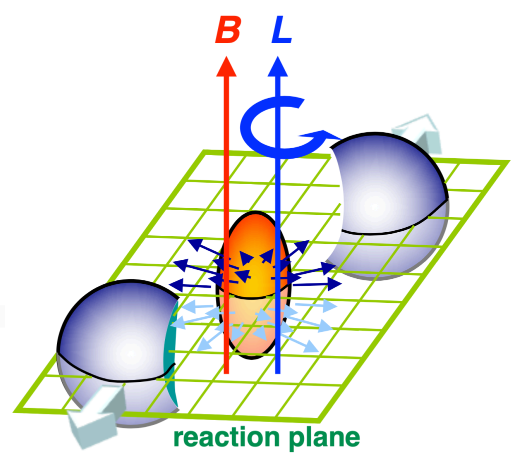
consistent with all symmetries

	Q_V	Q_A	Q_H	J_V	J_A	J_H	Ω
C	$-$	$+$	$-$	$-$	$+$	$-$	$+$
P	$+$	$-$	$-$	$-$	$+$	$+$	$+$
T	$+$	$+$	$+$	$-$	$-$	$-$	$-$

ArXiv:2010.05831

Is helicity important? (1)

Helical vortical effect vs. chiral vortical effect



$$\boldsymbol{J}_V = \frac{1}{\pi^2}\mu_V\mu_A\boldsymbol{\Omega} + \frac{2\ln 2}{\pi^2}\mu_H T\boldsymbol{\Omega}$$

chiral vortical   helical vortical

effects

Compare the magnitudes of the current

— spin flips induce same-order fluctuations in axial and chemical densities

take for estimation

$$\mu_A = \mu_H$$

— at the chemical freeze-out:

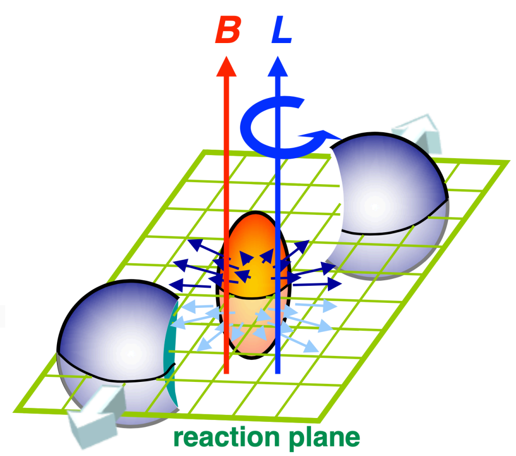
Seems important!

$$\frac{(J_V)_{\mathrm{HVE}}}{(J_V)_{\mathrm{CVE}}} = 6\ln 2\frac{T}{\mu_B} \simeq 30, \qquad (\sqrt{s_{NN}} = 200\,\mathrm{GeV})$$

ArXiv:2010.05831

Is helicity important? (2)

Helical vortical effect vs. chiral magnetic effect



$$\boldsymbol{J}_V = \frac{\mu_A}{2\pi^2} e\boldsymbol{B}$$

chiral magnetic effect

$$\boldsymbol{J}_V = \frac{2\ln 2}{\pi^2} \mu_H T \boldsymbol{\Omega}$$

helical vortical effect

Compare the magnitudes of the current

—at the chemical freeze-out:

$$T = 166\,\mathrm{MeV} \qquad \Omega = 6.6\,\mathrm{MeV} \qquad B = 0.05\, m_\pi^2$$

STAR collaboration, Nature 548, 62 (2017); L. McLerran and V. Skokov, Nucl. Phys. A 929, 184 (2014).
The magnetic–field estimations are not final, depend on conductivity effects, vortical swirls etc.
e.g., X. Guo, J. Liao and E. Wang, Sci. Rep. 10, 2196 (2020).

Of the same order as CME!

$$\frac{(J_V)_{\mathrm{HVE}}}{(J_V)_{\mathrm{CME}}} = 4\ln 2 \cdot \frac{T\Omega}{eB} \simeq 3, \qquad (\sqrt{s_{NN}} = 200\,\mathrm{GeV})$$

ArXiv:2005.03575

Thermodynamics at a finite helical density

- the presence of a finite density of the chiral charge (a finite chiral chemical potential) affects the phase diagram

- The relevant question: what are the effects of the presence of a finite helicity on the phase diagram of QCD?

- Is the mass gap generation/chiral symmetry breaking affected?

Surprise:

- the helical chemical potential is a thermodynamically consistent quantity in theories with the mass gap generation …
  … contrary to the axial chemical potential (!)

- The helicity is frame-dependent but conserved (!) quantity for massive fermions (we take the ``temperature” frame)

$$\mathcal{L} = \overline{\psi}\left(i\partial\!\!\!/ + m\right)\psi$$

$$H = -i\gamma^0\boldsymbol{\gamma}\cdot\boldsymbol{\nabla} + m\gamma^0$$

$$[\mathsf{h}, H] = 0$$

QCD thermodynamics and helicity imbalance

Take a linear sigma model coupled to quarks

$$\mathcal{L} = \mathcal{L}_q(\bar{\psi}, \psi, \sigma, \vec{\pi}, L) + \mathcal{L}_\sigma(\sigma, \vec{\pi})$$

$$\mathcal{L}_q = \overline{\psi}\left[i\partial\!\!\!/ - g(\sigma + i\gamma^5\vec{\tau}\cdot\vec{\pi})\right]\psi$$

One of the simplest, standard mean-field approaches to probe chiral symmetry breaking

$$\mathcal{L}_\sigma(\sigma, \vec{\pi}) = \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\pi^0\partial^\mu\pi^0\right) + \partial_\mu\pi^+\partial_\mu\pi^- - V(\sigma, \vec{\pi})$$

$$V(\sigma, \vec{\pi}) = \frac{\lambda}{4}\left(\sigma^2 + \vec{\pi}^2 - v^2\right)^2 - h\sigma$$

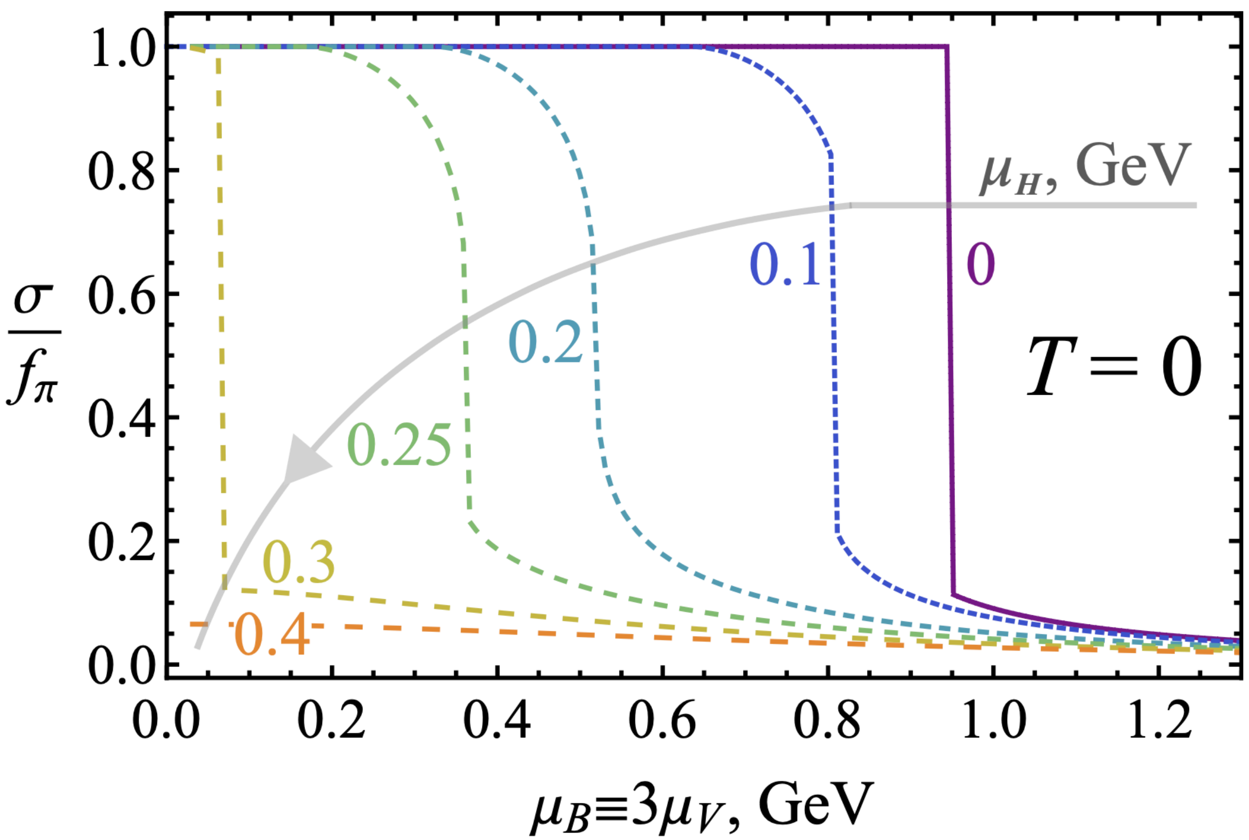
Three ingredients:

- finite temperature
- finite baryon density (baryon chemical potential)
- helicity imbalance (via helical chemical potential)

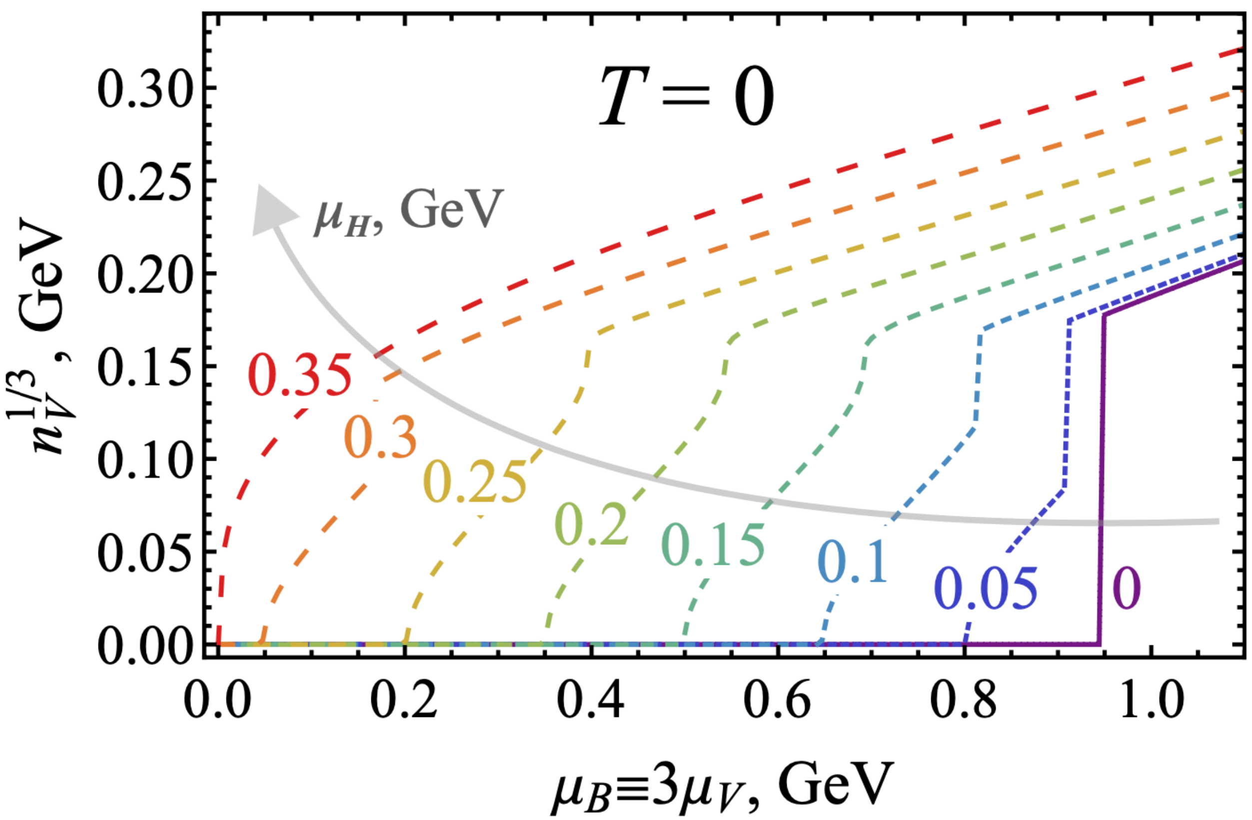
QCD thermodynamics with helicity imbalance

Zero temperature

mass gap



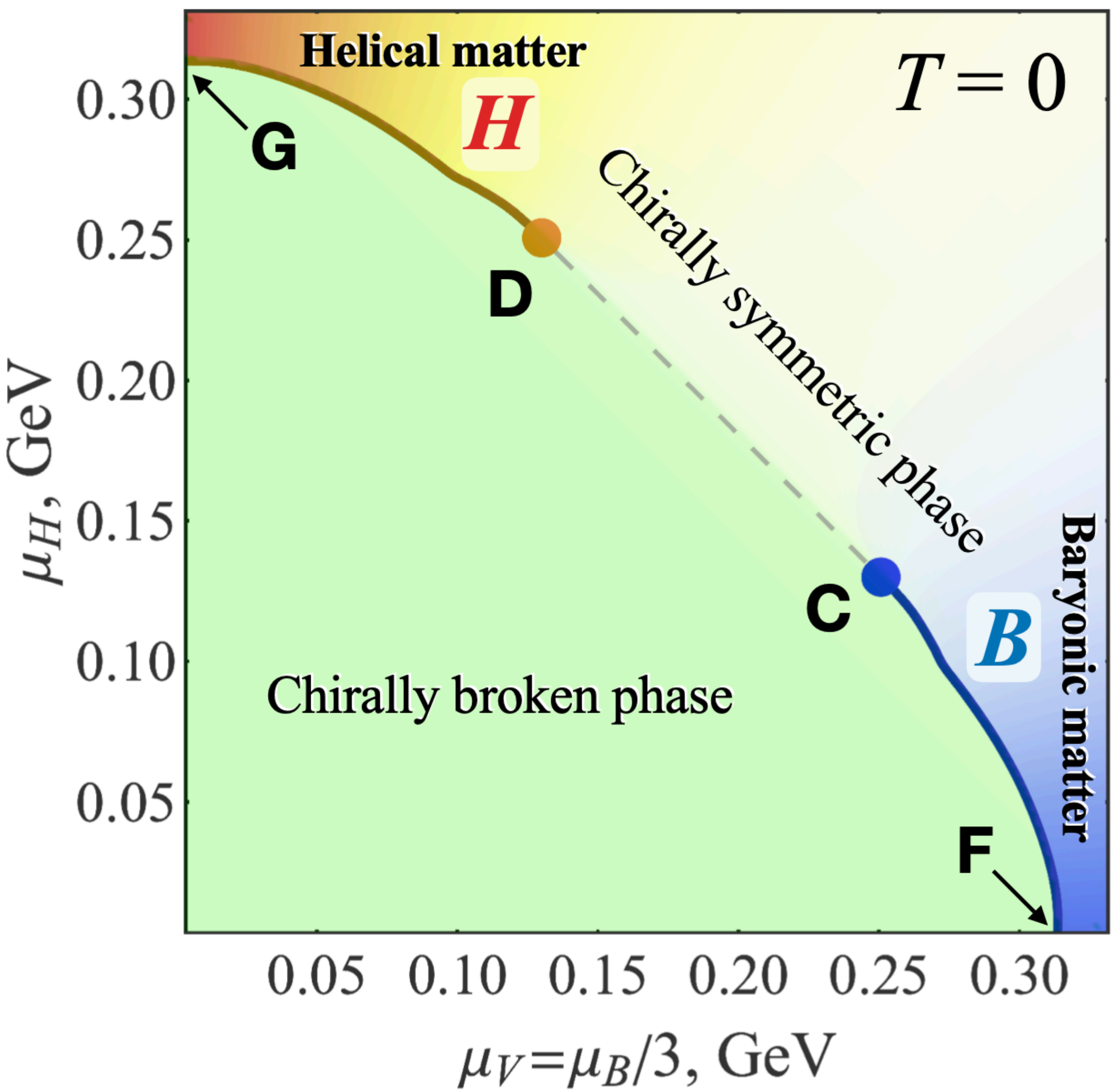
vector (baryon/3) density



evolution of the baryon-density-induced symmetry-restoration transition in the presence of the helical imbalance

QCD thermodynamics with helicity imbalance

Phase diagram at zero temperature and duality



helical imbalance

baryon density

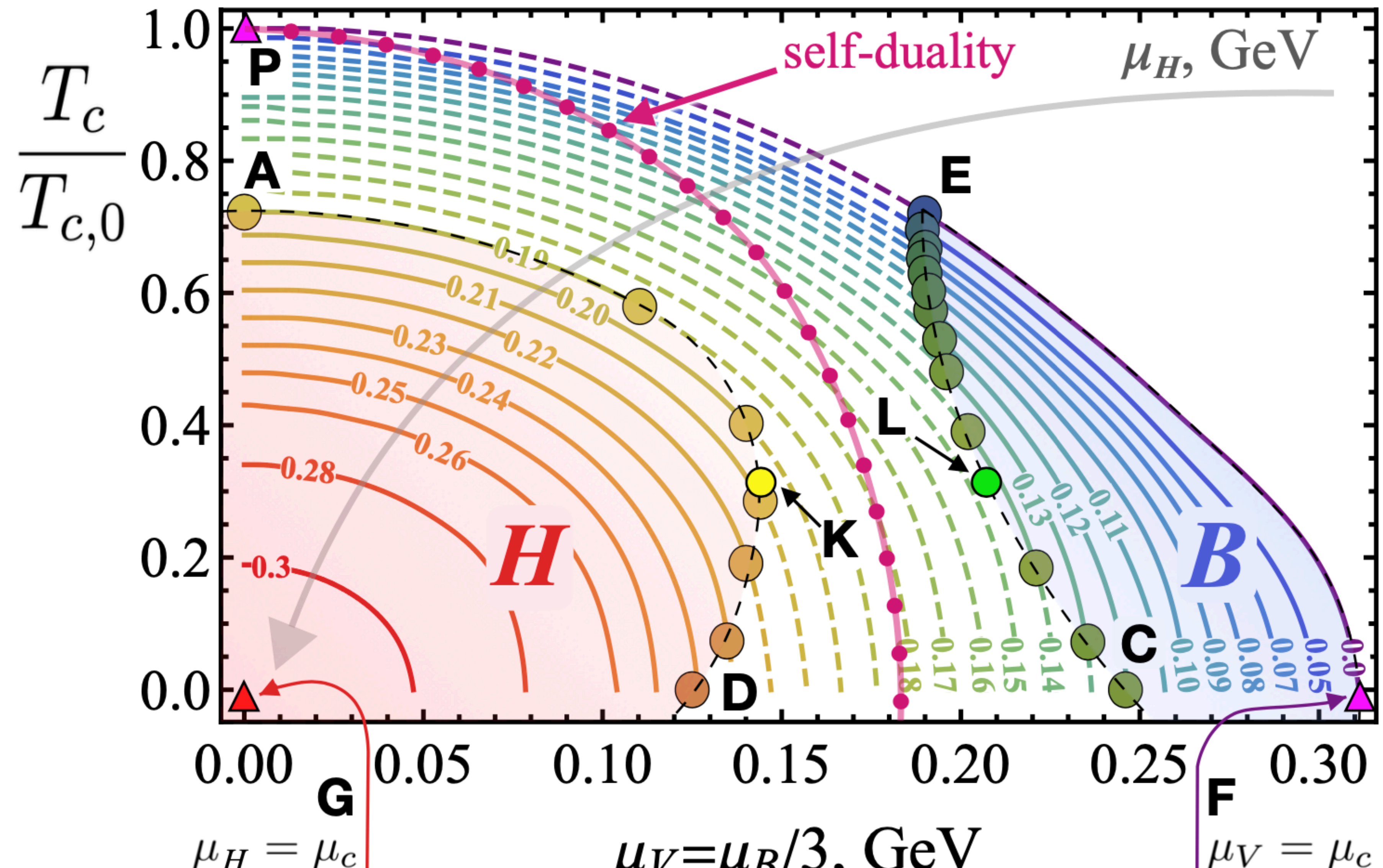
unexpected duality

$$\begin{pmatrix} \mu_V \\ \mu_H \end{pmatrix} \rightleftarrows \begin{pmatrix} \mu_H \\ \mu_V \end{pmatrix}$$

for thermodynamic potential

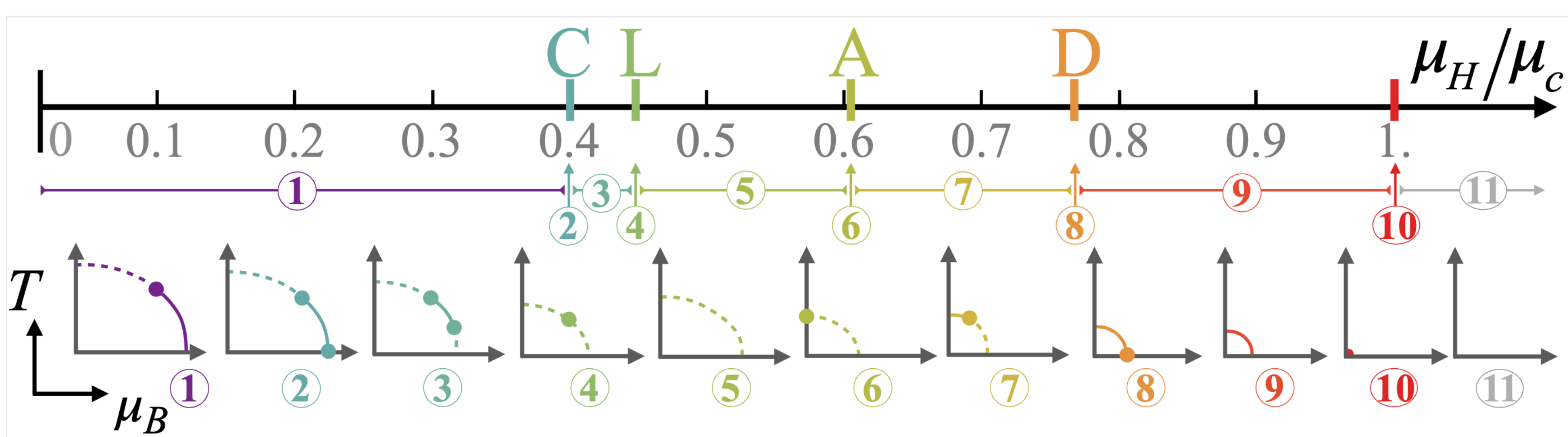
$$\Omega_T(\mu_V, \mu_H) = \Omega_T(\mu_H, \mu_V)$$

Phase diagram at nonzero temperature



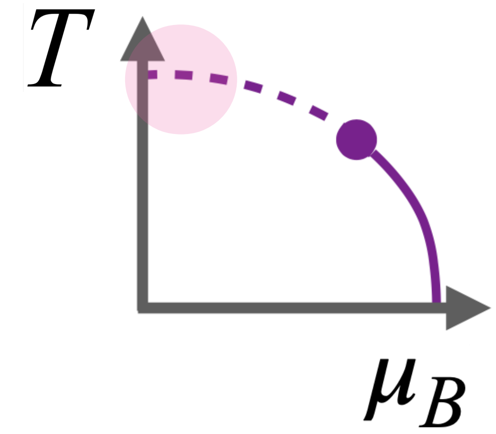
The helicity imbalance affects drastically the chiral symmetry restoration.

Rich phase structure



Curvature of the chiral transition and helicity imbalance

Curvature at small baryon density



$$\frac{T_c(\mu_B,\mu_H)}{T_{c,0}} = \frac{T_c(\mu_H)}{T_{c,0}} - \kappa(\mu_H)\left(\frac{\mu_B}{T_{c,0}}\right)^2 + \ldots$$

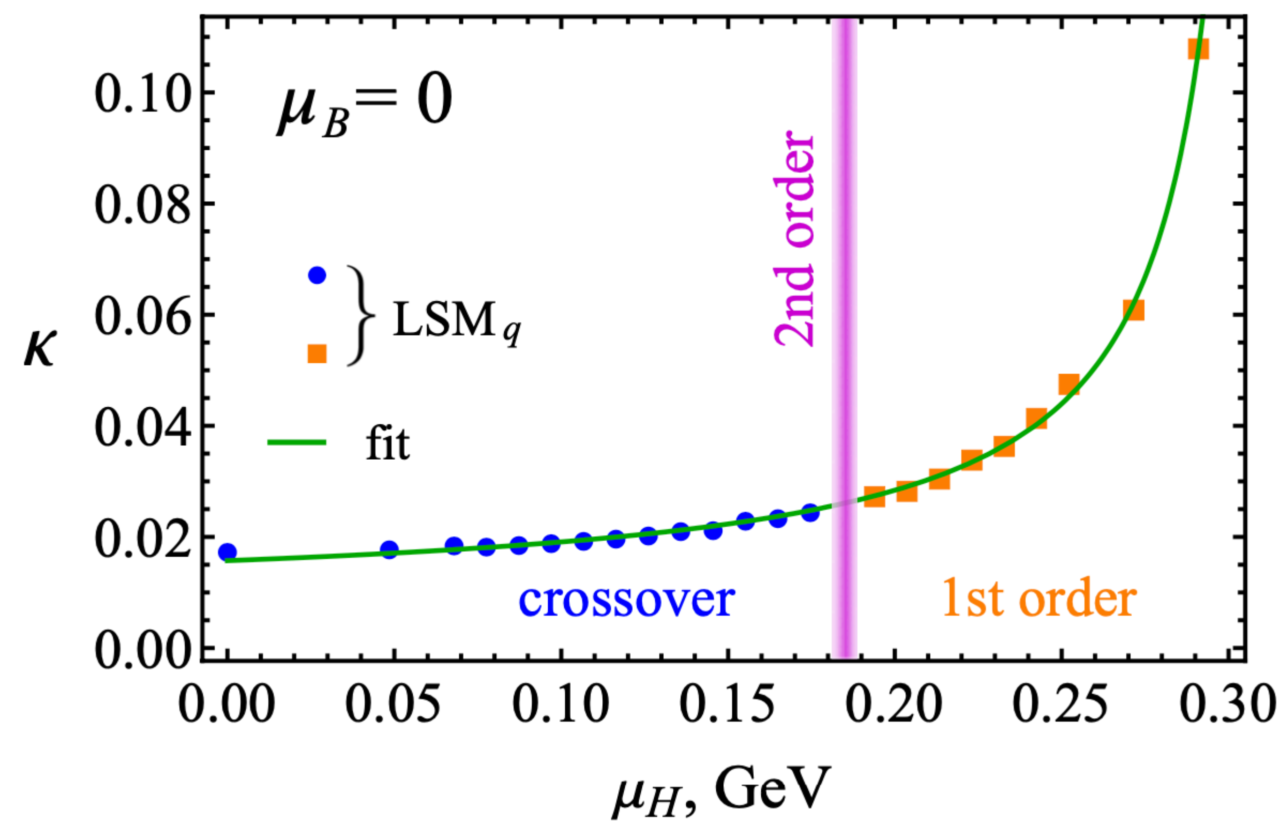
Exploding curvature:

$$\kappa^{\rm fit}(\mu_H) = \kappa_0\left(1+\alpha\frac{\mu_H}{\mu_{H,c}-\mu_H}\right)$$

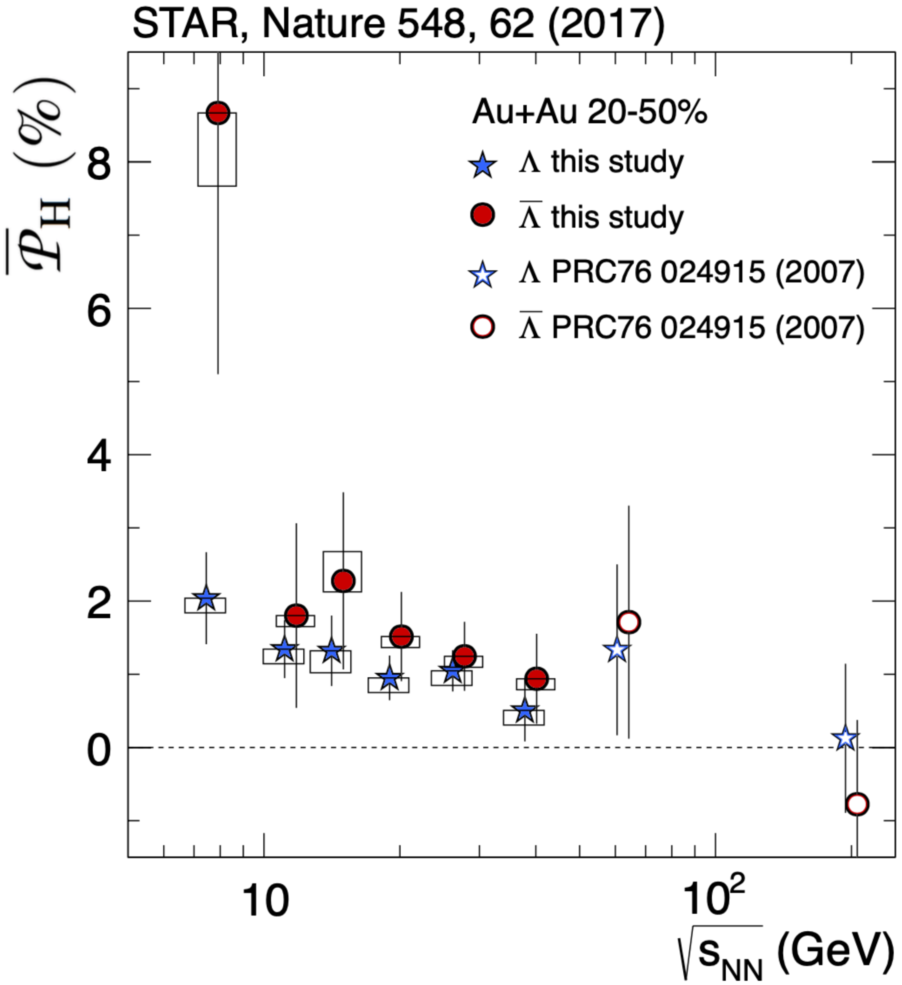
$\kappa_0 = 0.0158(3), \qquad \mu_{H,c} = 0.314(1)\,\mathrm{GeV}$

$\alpha = 0.46(2) \approx 1/2$

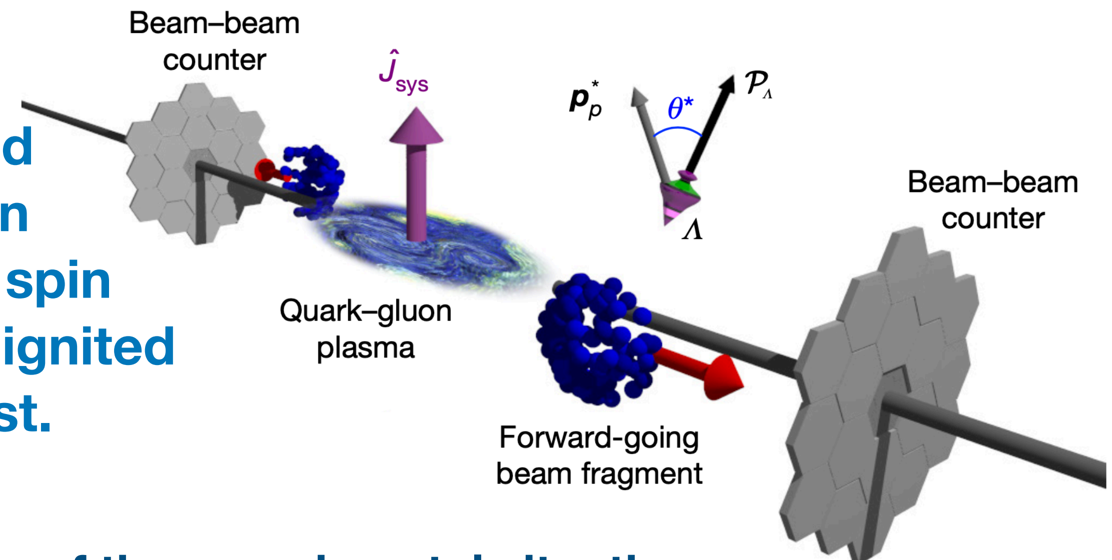
Helicity imbalance affects curvature of the finite-temperature transition of the chiral restoration



(Anti-)Hyperon spin polarizations via chiral and helical vortical effects



The observed asymmetry in the hyperon spin polarization ignited much interest.



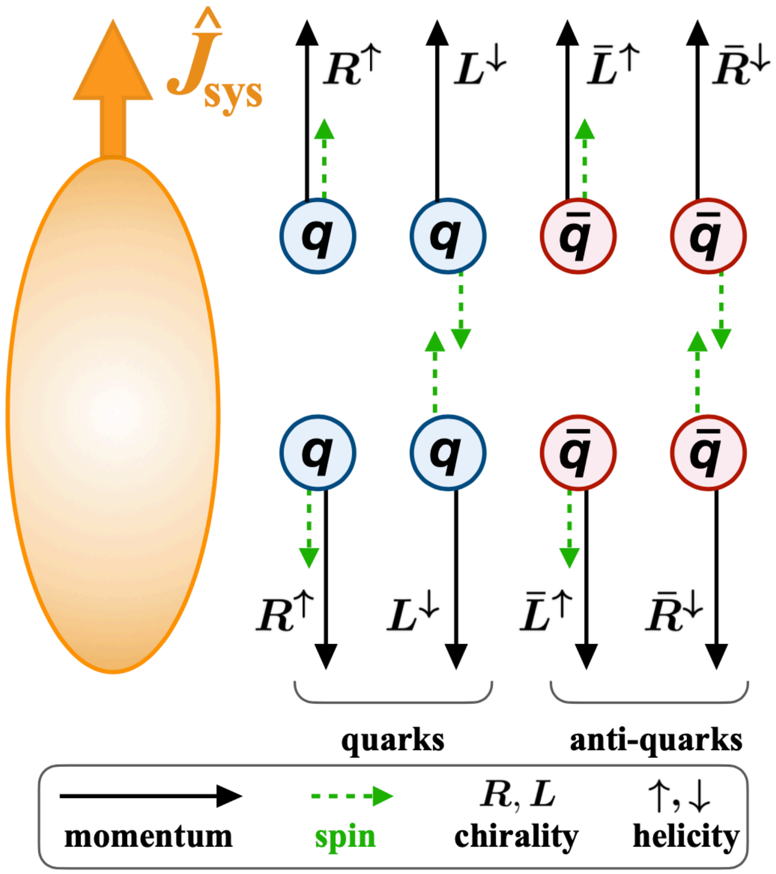
latest overview of the experimental situation:
T. Niida, talk at the workshop "Spin and hydrodynamics in relativistic nuclear collisions" ECT*, Trento, Italy, Oct. 05-16, 2020.

latest overview of the theoretical situation: "Vorticity and Spin Polarization in Heavy Ion Collisions: Transport Models", X.-G. Huang, J. Liao, Q. Wang, X.-L. Xia, arXiv:2010.08937

Baznat, Gudima, Sorin, Teryaev, Phys. Rev. C 88, 061901(R) (2013)
Sorin and Teryaev, Phys. Rev. C 95, no.1, 011902(R) (2017).
Becattini, Karpenko, Lisa, Upsal, Voloshin, Phys.Rev.C 95 (2017) 5, 054902
Teryaev, Zakharov, Phys.Rev.D 96 (2017) 9, 096023.
Baznat, Gudima, Sorin and Teryaev, Phys. Rev. C 97, no.4, 041902(R) (2018).
Csernai, Kapusta, and Welle, Phys. Rev. C 99, no.2, 021901(R) (2019).
D-Xian Wei, Wei-Tian Deng, and Xu-Guang Huang, Phys. Rev. C 99, 014905 (2019)
Vitiuk, Bravina and Zabrodin, Phys. Lett. B 803, 135298 (2020)
B. Fu, K. Xu, X.-G. Huang, H.Song, ArXiv:2011.03740 (today).

(Anti-)Hyperon spin polarizations via chiral and helical vortical effects

ArXiv:2010.05831



Global polarization: $$\overline{\mathcal{P}}_H = \langle \boldsymbol{P}_H \cdot \hat{\boldsymbol{J}}_{\text{sys}} \rangle$$

Anomalous currents along the global vorticity:

$$J_\ell = \boldsymbol{J}_\ell \cdot \boldsymbol{n}, \qquad \boldsymbol{n} = \frac{\hat{\boldsymbol{J}}_{\text{sys}}}{\hat{J}_{\text{sys}}} \equiv \frac{\boldsymbol{\omega}}{\omega}, \qquad \ell = V, A, H$$

Axial current: $$J_A = J_\uparrow + \bar{J}_\uparrow - J_\downarrow - \bar{J}_\downarrow$$

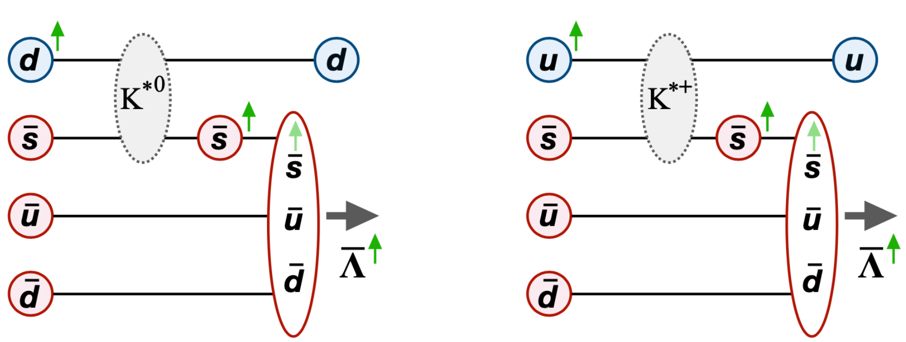
Helical current: $$J_H = J_\uparrow + \bar{J}_\downarrow - J_\downarrow - \bar{J}_\uparrow$$

Spins

Polarization of quarks: $$J_A + J_H = 2(J_\uparrow - J_\downarrow)$$

Polarization of anti-quarks: $$J_A - J_H = 2(\bar{J}_\uparrow - \bar{J}_\downarrow)$$

Mechanism



1) rotation effects polarize spins of light quarks
2) light quark transfer spin polarization to strange quarks
3) polarization of strange quarks is seen via hyperons

(Anti-)Hyperon spin polarizations via chiral and helical vortical effects

Polarization of quarks: $J_A + J_H = 2(J_\uparrow - J_\downarrow)$

Polarization of anti-quarks: $J_A - J_H = 2(\bar{J}_\uparrow - \bar{J}_\downarrow)$

Axial vortical effect $\boldsymbol{J}_A = \sigma_A \boldsymbol{\omega}, \qquad \sigma_A = \frac{T^2}{6} + \frac{\mu_B^2}{18\pi^2}$

Helical vortical effect $\boldsymbol{J}_H = \sigma_H \boldsymbol{\omega}, \qquad \sigma_H = \frac{2\ln 2}{3\pi^2}\mu_B T$

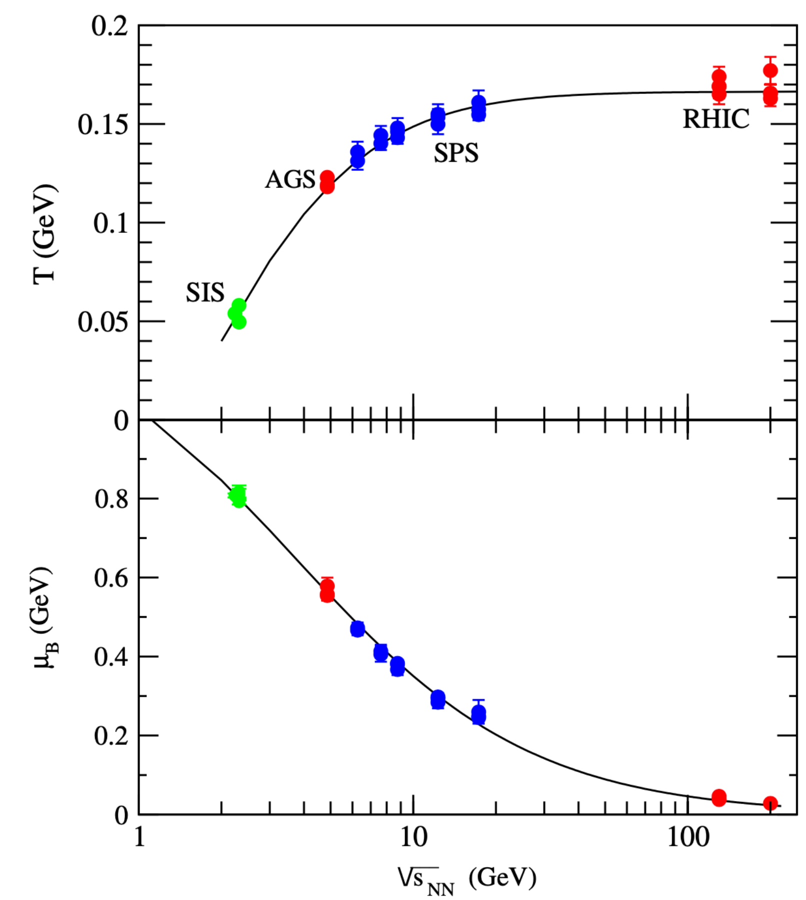
In our mechanism, the unknown (form)factors disappear in the ratio of the spin polarizations:

$$\mathcal{R}_{\bar{\Lambda}/\Lambda} = \frac{\overline{\mathcal{P}}_{\bar{\Lambda}'}}{\overline{\mathcal{P}}_{\Lambda'}}$$

Chiral/helical effects predict

$$\mathcal{R}_{\bar{\Lambda}/\Lambda} = \frac{\sigma_A + \sigma_H}{\sigma_A - \sigma_H}$$

(expressed via vortical conductivities only!)



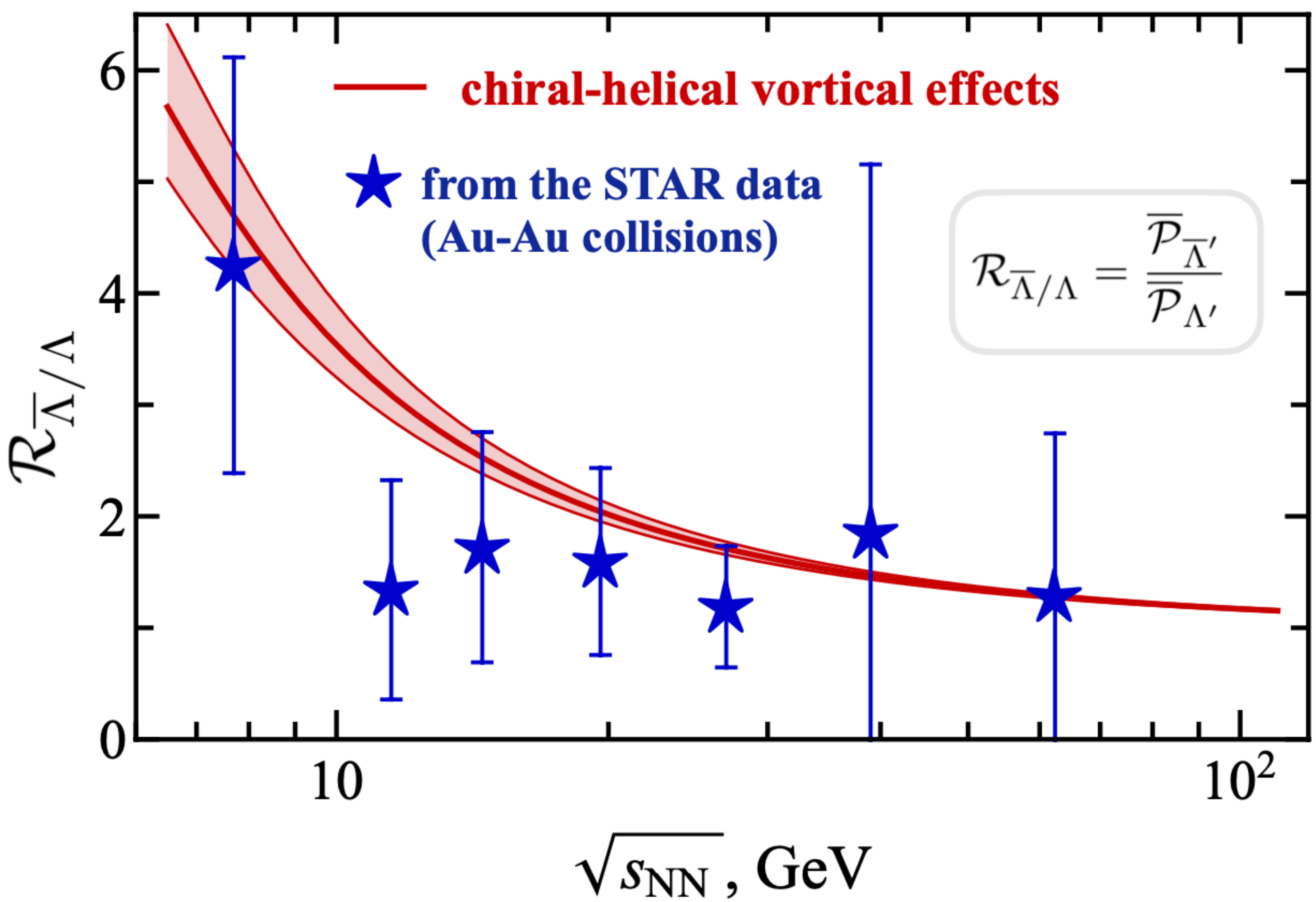
Cleymans, Oeschler, Redlich, Wheaton, PRC 73, 034905 (2006)

(no helical imbalance needed!)

Ratio of (anti)-hyperon spin polarizations: the prediction of chiral/helical vortical effects

$$\mathcal{R}_{\bar\Lambda/\Lambda} = \frac{\overline{\mathcal{P}}_{\bar\Lambda'}}{\overline{\mathcal{P}}_{\Lambda'}}$$

No fitting, no model parameters!



A combination of chiral and helical vortical conductivities

$$\mathcal{R}_{\bar\Lambda/\Lambda} = \frac{\sigma_A + \sigma_H}{\sigma_A - \sigma_H}$$

Summary

— Helicity is as important as (but not equal to) chirality.

— Helical imbalance modifies the phase diagram of QCD.

— Helical density is dual to the vector (baryon) density of quarks.

— The presence of helicity leads to new transport effects in vortical backgrounds.

— An interplay between chiral and helical vortical effects can explain the ratio of the hyperon spin polarizations. The prediction coincides with the data from the STAR experiment (with no model parameters required).