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Neutral Meson properties in hot and magnetized quark matter

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“Hot problems of Strong Interactions”

Outline

- Motivation
- Thermo-magnetic effects on the coupling constants - IMC
- The importance of implementing a proper regularization procedure in order to treat thermo and magnetic contributions within non renormalizable theories
- Neutral meson pole mass in a magnetized and thermal medium within the SU(2) NJL model
- Neutral meson pole mass in a magnetized medium within the Linear Sigma model with quarks
- Conclusions and perspectives

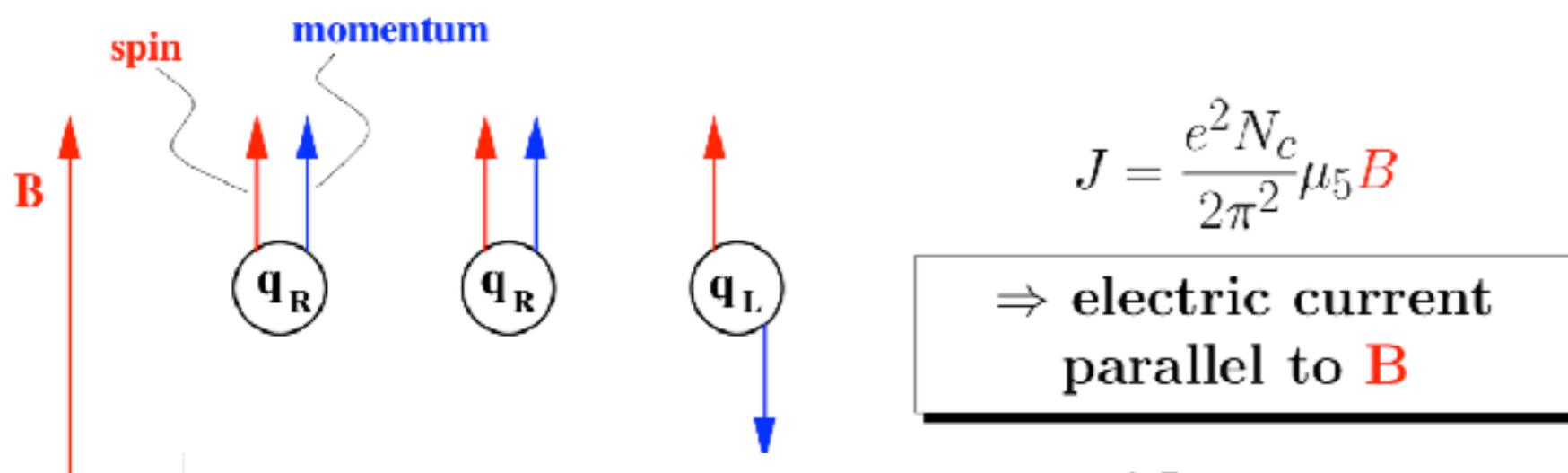
Strong magnetic fields may be produced in non central heavy ion collisions

K. Fukushima, D. E. Kharzeev, and H. J. Warringa, Phys. Rev. D 78, 074033 (2008). D. E. Kharzeev and H. J. Warringa, Phys. Rev. D 80, 0304028 (2009). D. E. Kharzeev, Nucl. Phys. A 830, 543c (2009).

○ Chiral magnetic effect

D.E. Kharzeev, L.D. McLerran, H.J. Warringa NPA 803, 227 (2008)

heavy-ion collisions:
temporarily $B \lesssim 10^{19}$ G
Skokov, Illarionov, Toneev,
Int. J. Mod. Phys. A 24, 5925 (2009)



NATURE PHYSICS | LETTER

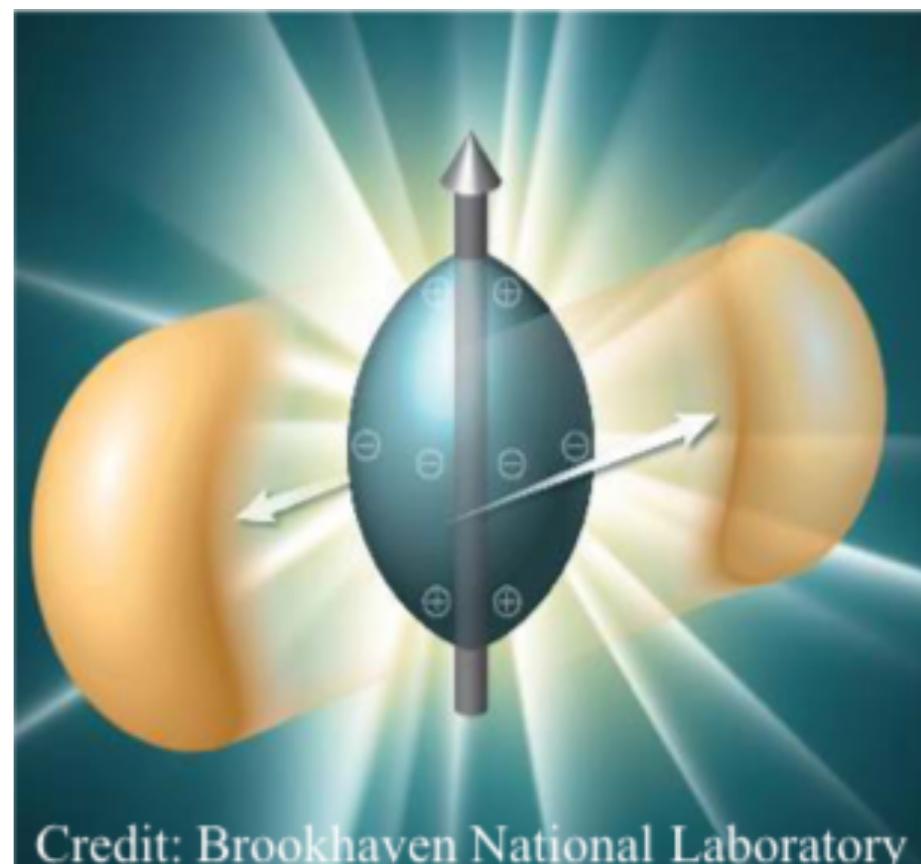
Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla

Affiliations | Contributions Corresponding authors

Nature Physics (2016) | doi:10.1038/nphys3848

Received 19 December 2014 | Accepted 04 January 2016 | Published online 08 February 2016



Credit: Brookhaven National Laboratory

Motivation

- Strong magnetic fields are also present in magnetars:

C. Kouveliotou et al., Nature 393, 235 (1998).

magnetars:

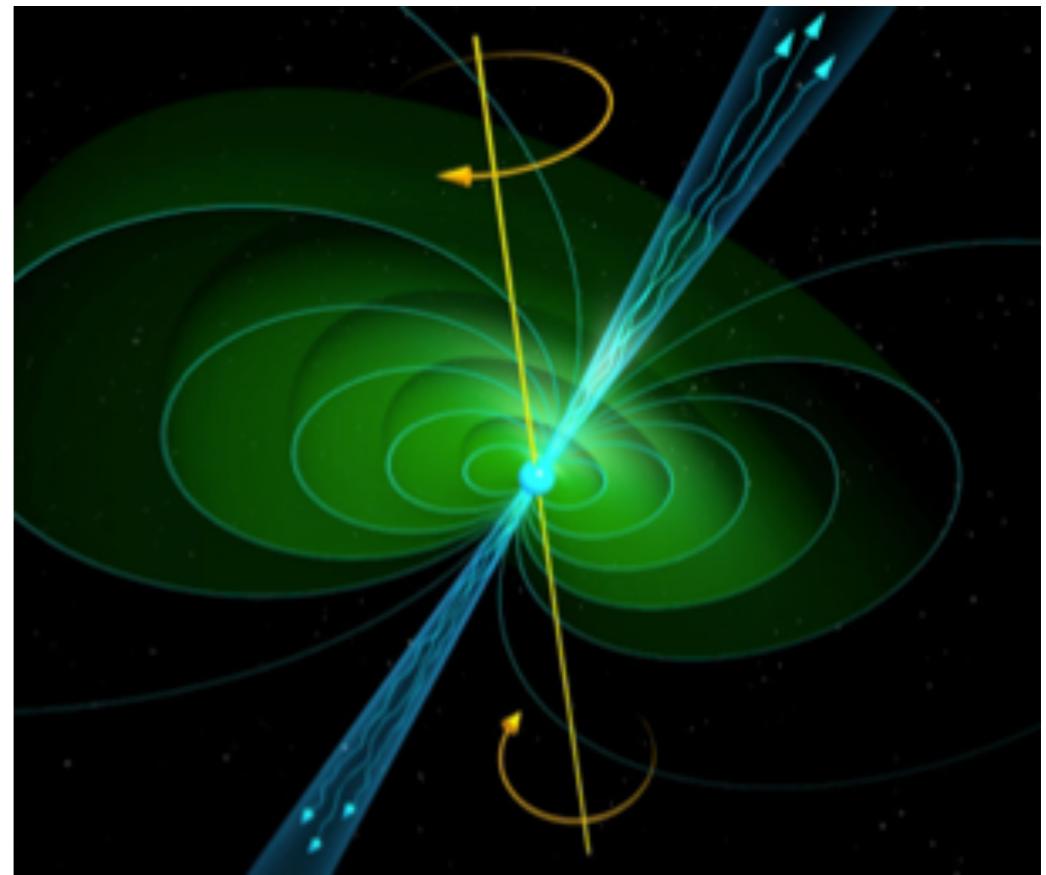
at surface $B \lesssim 10^{15}$ G

Duncan, Thompson, Astrophys.J. 392, L9 (1992)

larger in the interior,
 $B \sim 10^{18-20}$ G?

Lai, Shapiro, Astrophys.J. 383, 745 (1991)

E. J. Ferrer *et al.*, PRC 82, 065802 (2010)



A. K. Harding, D. Lai, Rept. Prog. Phys. 69, 2631 (2006)

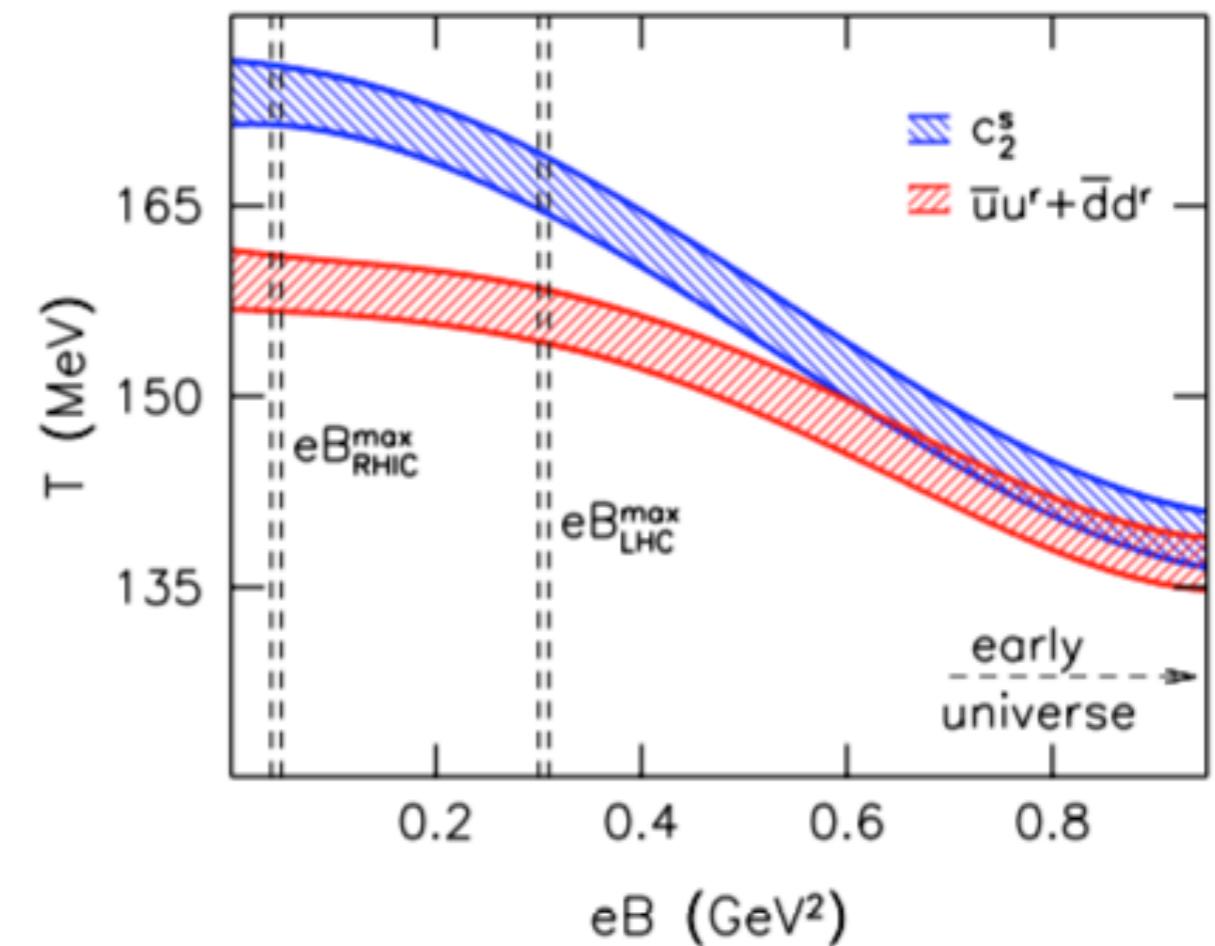
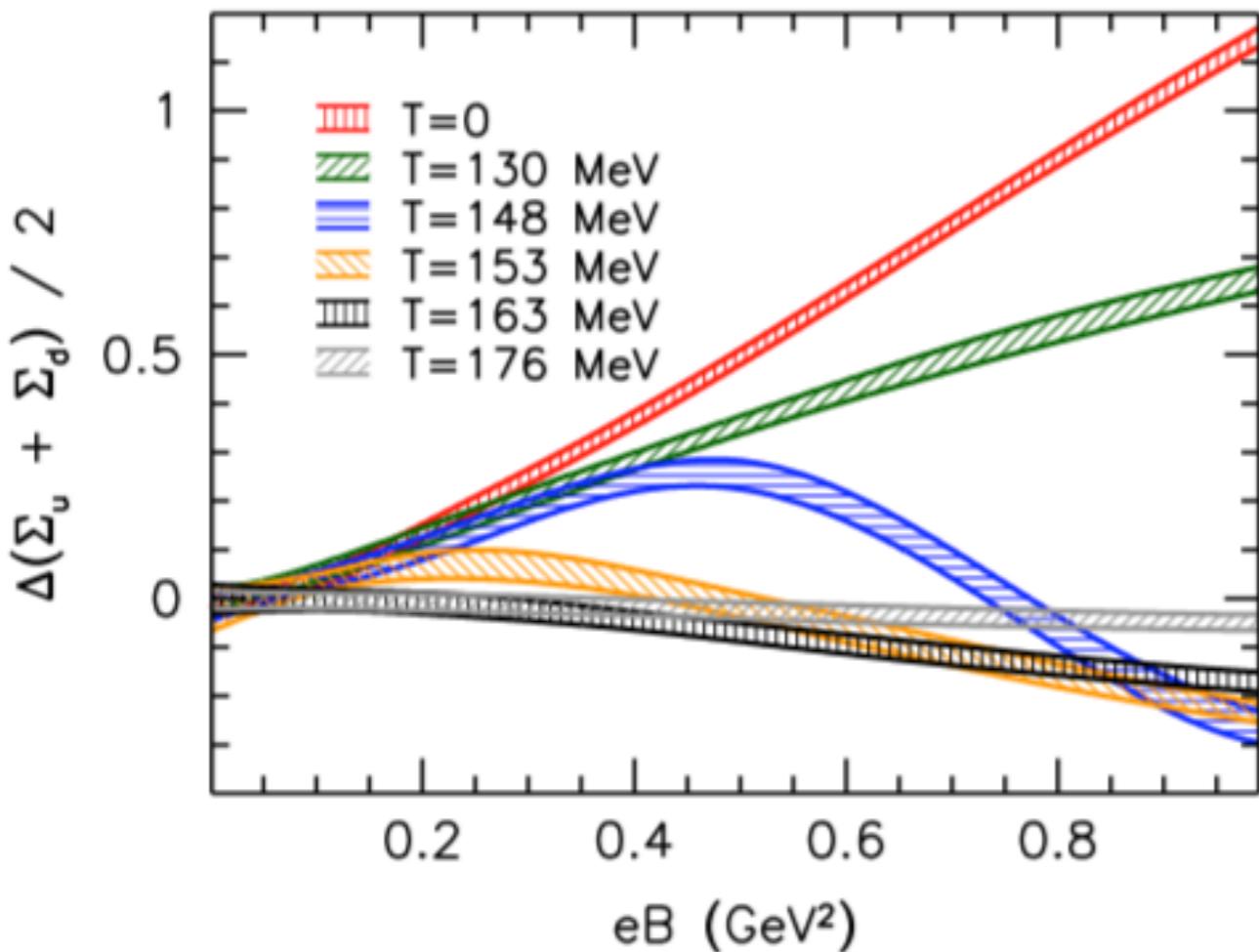
- and might have played an important role in the physics of the early universe. T. Vaschapati, Phys. Lett. B 265, 258 (1991).
D. Grasso and H.R. Rubinstein, Phys. Rep. 348, 163 (2001).

Effect on QCD phase transitions?

$$\Lambda_{\text{QCD}}^2 \sim (200 \text{ MeV})^2 \sim 2 \times 10^{18} \text{ G}$$

IMC mini-review
Aritra Bandyopadhyay, R.L.S. Farias,
e-Print: 2003.11054 [hep-ph]

MC and IMC



SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L}_{NJL} = \bar{\psi} (\not{D} - m) \psi + \boxed{G [(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2]} - \frac{1}{4} F^{\mu\nu} F_{\mu\nu}$$

$$D^\mu = (i\partial^\mu - Q A^\mu)$$

good **chiral** physics, pions,...

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

BUT no confinement

$$Q=\text{diag}(q_u=2e/3, q_d=-e/3)$$

✓ strong magnetic field background
that is constant and homogeneous!

$$G, \Lambda \text{ and } m_c \rightarrow m_\pi, f_\pi \text{ and } \langle \bar{\psi}\psi \rangle$$

natural units: $1\text{GeV}^2 \approx 5.34 \times 10^{19} \text{ G}$ and $e = \sqrt{\frac{4\pi}{137}}$

NJL at finite B

At B=0

$$\mathcal{F} = \frac{(M - m_c)^2}{4G} - N_c \sum_{f,s} \int \frac{d^4 p}{(2\pi)^4} \ln [p^2 + M^2]$$

By using the replacement $\vec{p}^2 \rightarrow p_3^2 + 2k|q_f|B$

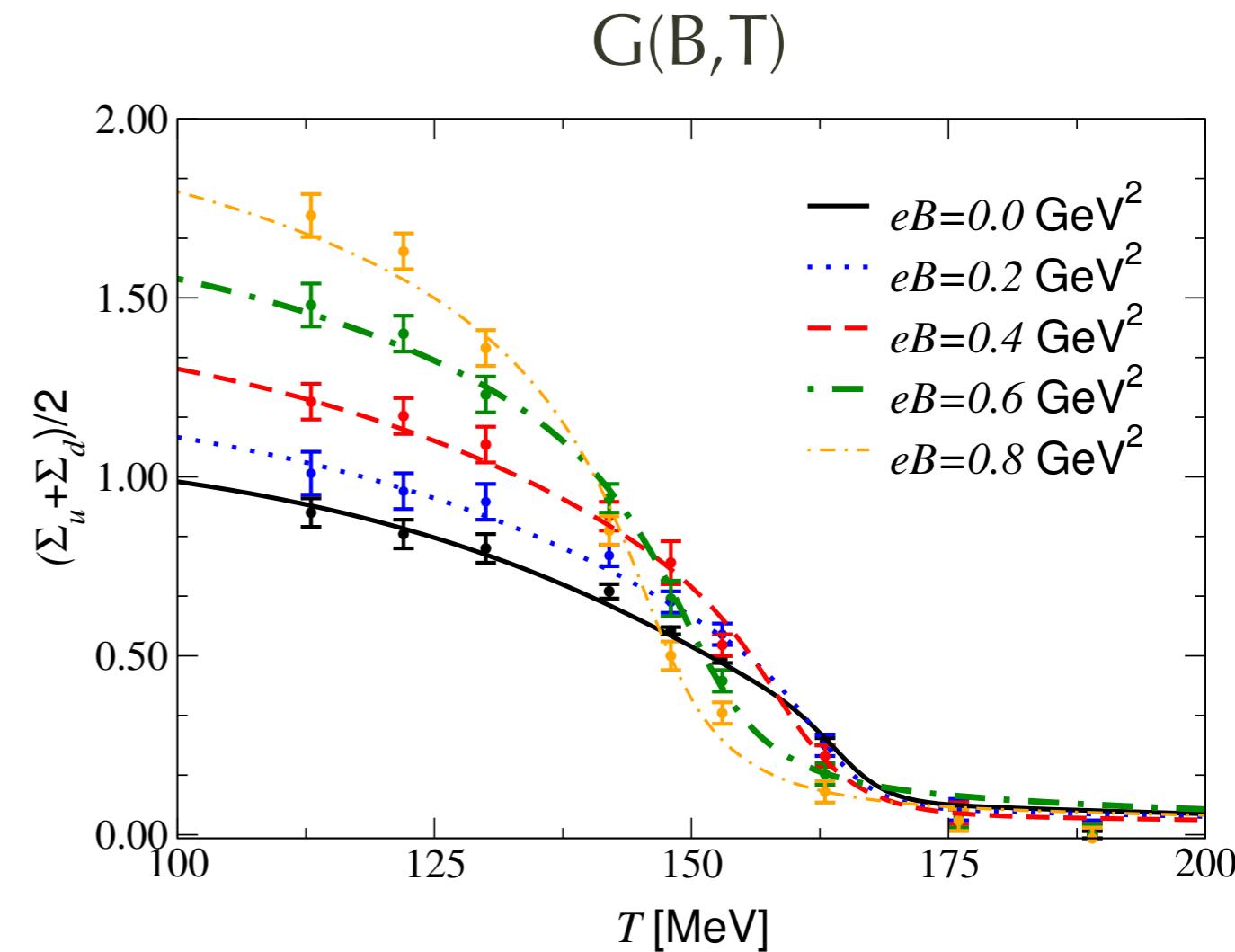
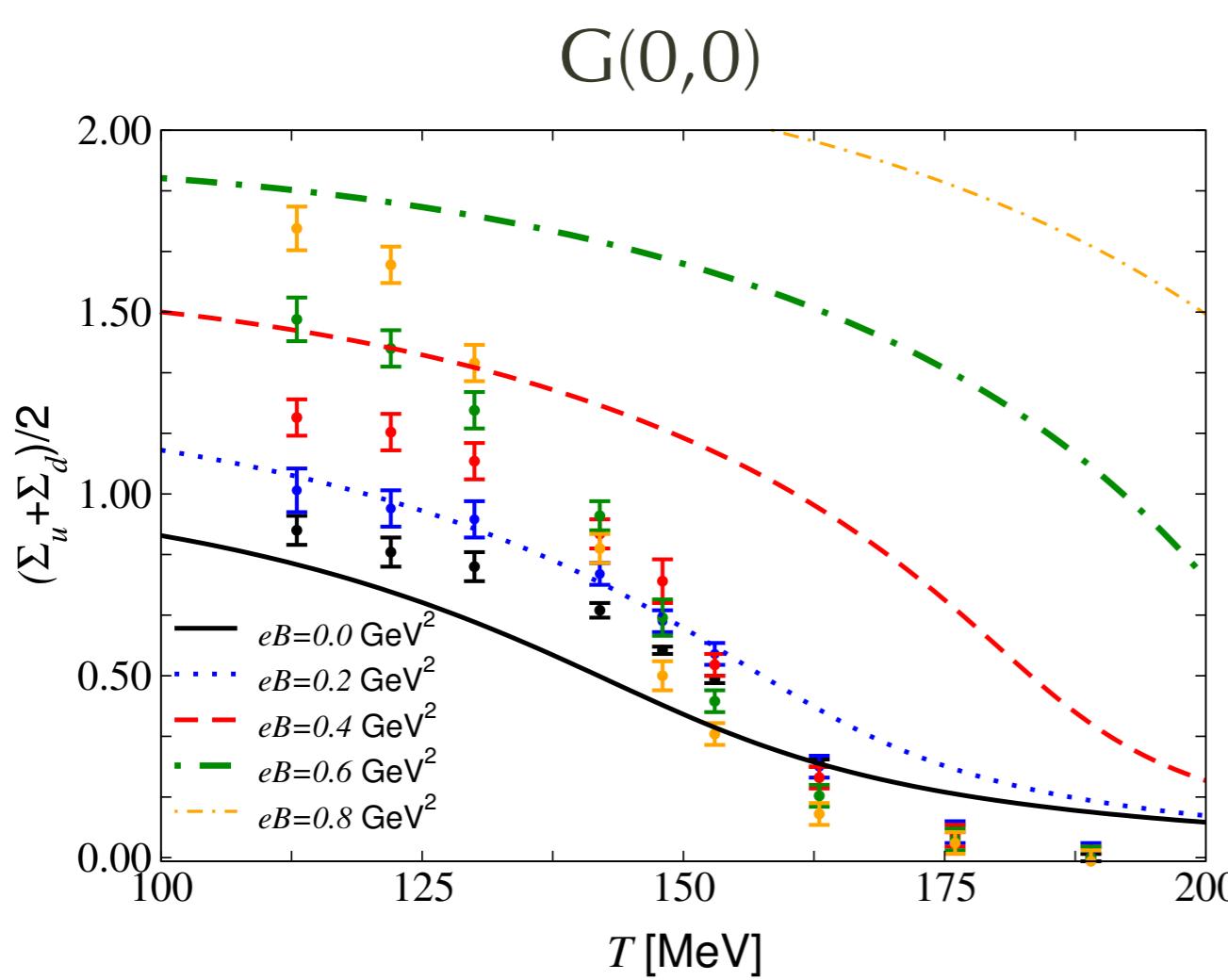
$$\int \frac{d^4 p}{(2\pi)^4} \rightarrow \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \sum_{k=0}^{\infty} \alpha_k$$

$$\alpha_k = 2 - \delta_{k0}$$

$$\begin{aligned} \mathcal{F} &= \frac{(M - m_c)^2}{4G} \\ &- N_c \sum_f \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \ln [p_4^2 + p_3^2 + 2k|q_f|B + M^2] \end{aligned}$$

And the gap equation: $\partial \mathcal{F} / \partial M = 0 \longrightarrow \infty$

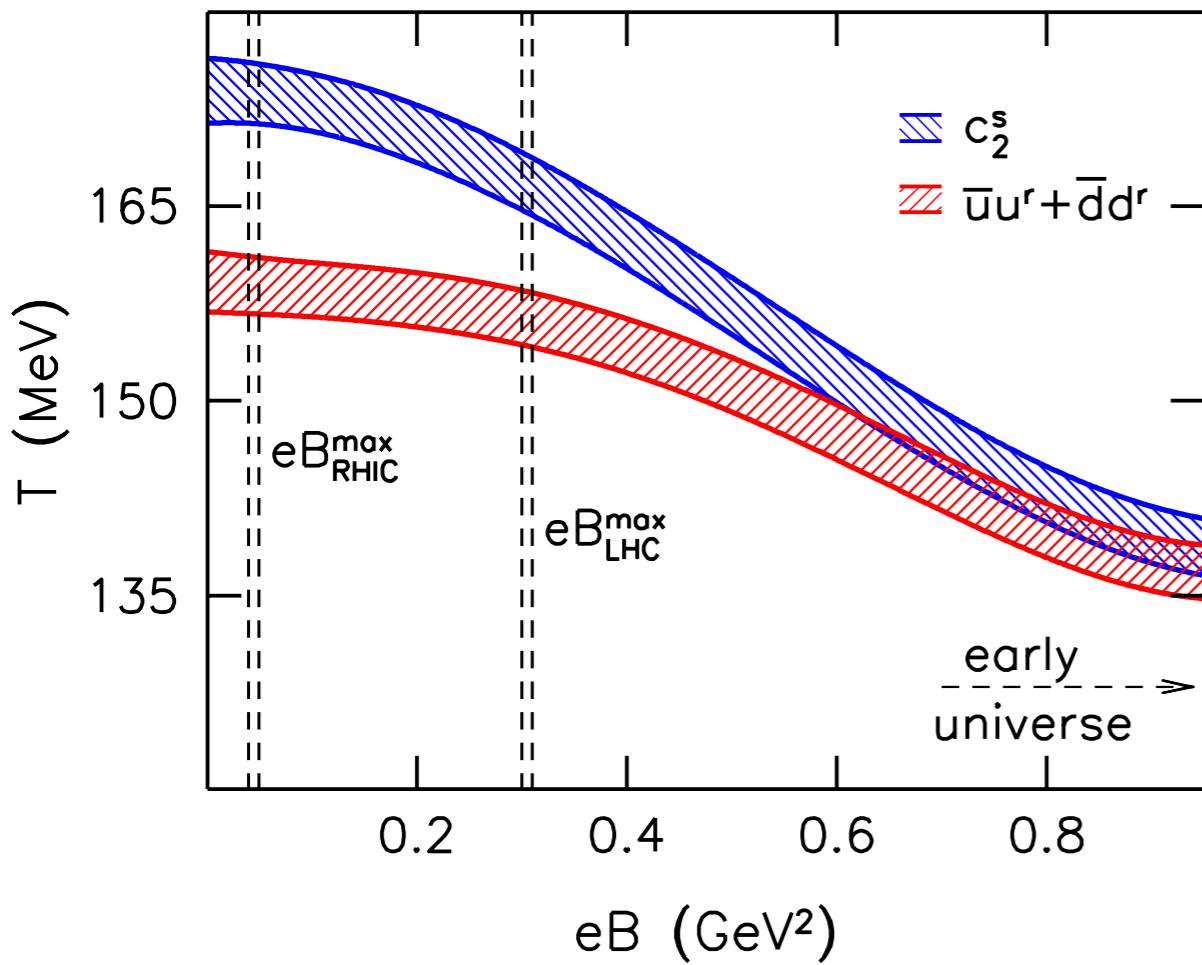
NJL + Thermo-Magnetic effects $G(B, T)$



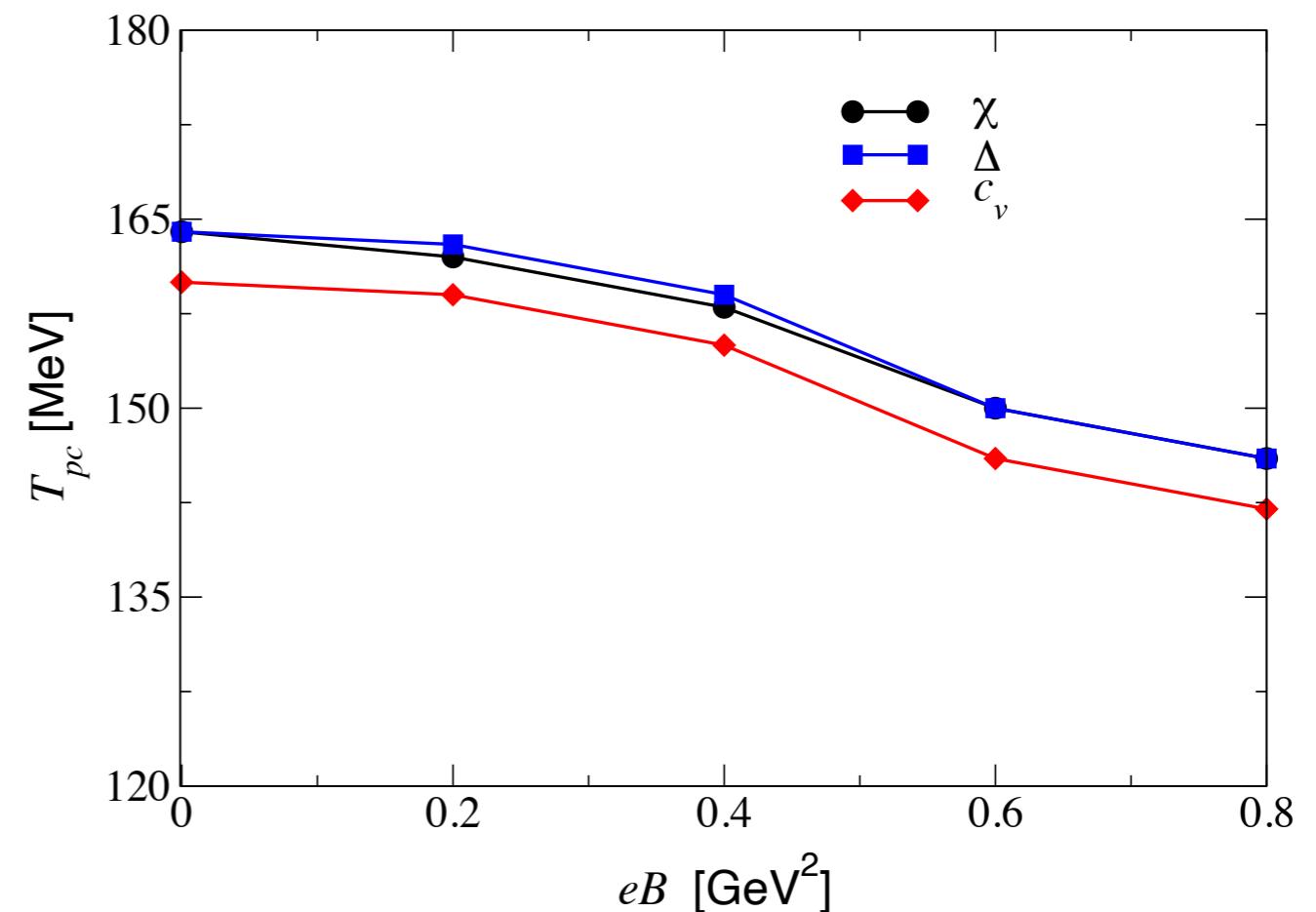
$$\Sigma_f(B, T) = \frac{2m_f}{m_\pi^2 f_\pi^2} \left[\langle \bar{\psi}_f \psi_f \rangle - \langle \bar{\psi}_f \psi_f \rangle_0 \right] + 1$$

T_{pc} lattice X NJL

Lattice



$G(B, T)$



JHEP 1207 (2012) 056

[Farias at all, Eur. Phys. J. A \(2017\) 53: 101](#)

G(B, T) Thermo-magnetic effects!

We need a regularization
procedure!

Which procedure/method
is more appropriate?

Is there any criteria?

MFIR - Magnetic Field Independent Regularization

- ✓ D. Ebert and K.G. Klimenko, **Nucl. Phys. A728, 203 (2003)**.
- ✓ D. P. Menezes, M. B. Pinto, S. S. Avancini, A. P. Martínez, and C. Providênci, **Phys. Rev. C 79, 035807 (2009)**.
- ✓ P. G. Allen, A. G. Grunfeld, and N. N. Scoccola, **Phys. Rev. D 92, 074041 (2015)**.
- ✓ D.C.Duarte,P.G.Allen,R.L.S.Farias,P.H.A.Manso,R.O.Ramos, and N. N. Scoccola, **Phys. Rev. D 93, 025017 (2016)**.
- ✓ S. S. Avancini, W. R. Tavares, and M. B. Pinto, Phys. Rev. D 93, **Phys. Rev. D 014010 (2016)**
- ✓ ...

MFIR - Magnetic Field Independent Regularization

$$\begin{aligned}\frac{\partial \mathcal{F}}{\partial M} &= \frac{M - m_0}{2G} - 2MN_c \sum_f \frac{|q_f|B}{2\pi} \\ &\times \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \frac{1}{p_4^2 + p_3^2 + 2k|q_f|B + M^2}\end{aligned}$$

We add and subtract the B=0 contribution

$$I_1 = 4 \int \frac{dp^4}{(2\pi)^4} \frac{1}{p^2 + M^2}$$

$$\frac{\partial \mathcal{F}}{\partial M} = \frac{M - m_0}{2G} - 2MN_c \left[I_1 + \sum_f I_f \right]$$

MFIR

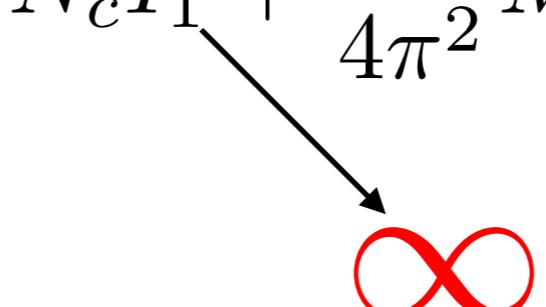
$$I_f = \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \left[\frac{|q_f|B}{2\pi} \sum_{k=0} \alpha_k \frac{1}{p_3^2 + p_4^2 + 2k|q_f|B + M^2} - 2 \int_{-\infty}^{\infty} \frac{dp_1}{2\pi} \int_{-\infty}^{\infty} \frac{dp_2}{2\pi} \frac{1}{p_1^2 + p_2^2 + p_3^2 + p_4^2 + M^2} \right]$$

$$I_f = \frac{M^2}{8\pi^2} \eta(x_f)$$

Where: $\eta(x) = \frac{\ln \Gamma(x)}{x} - \frac{\ln 2\pi}{2x} + 1 - \left(1 - \frac{1}{2x}\right) \ln x$

$$\frac{\partial \mathcal{F}}{\partial M} = \frac{M - m_0}{2G} - 2MN_c I_1 + \frac{N_c}{4\pi^2} M^3 \sum_f \eta(x_f)$$

$x_f = M^2 / (2|q_f|B)$



Noncovariant Regularizations

Form factors: $\sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \rightarrow \sum_{k=0}^{\infty} \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} U_{\Lambda}(p_3^2 + 2k|q_f|B)$

✓ Lorenztian: $U_{\Lambda}^{(LorN)}(x) = \left[1 + \left(\frac{x}{\Lambda^2} \right)^N \right]^{-1}$

✓ Wood-Saxon: $U_{\Lambda}^{(WS\alpha)}(x) = \left[1 + \exp \left(\frac{x/\Lambda^2 - 1}{\alpha} \right) \right]^{-1}$

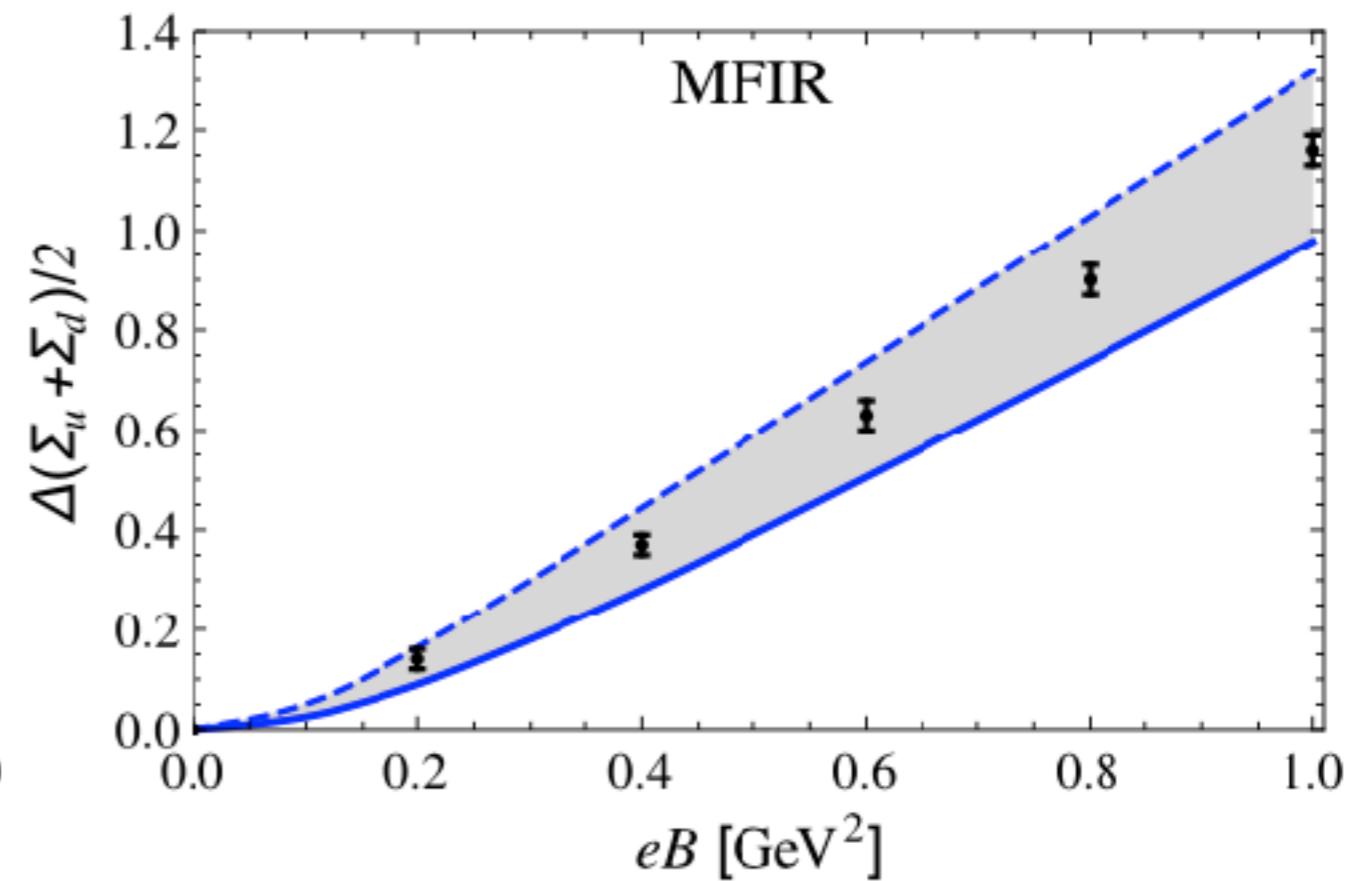
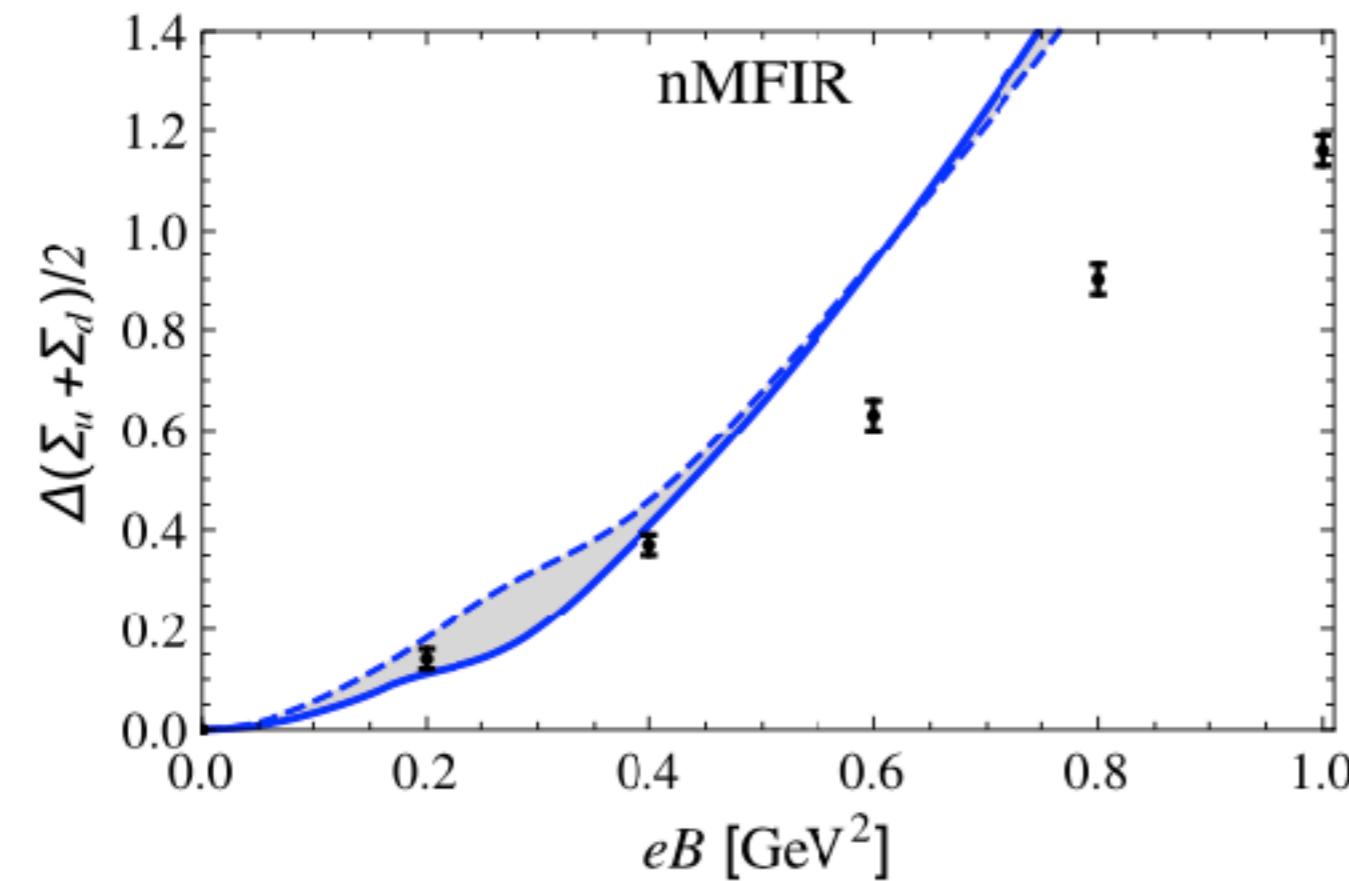
✓ Fermi-Dirac: $U_{\Lambda}^{\text{FD}}(x) = \frac{1}{2} \left[1 - \tanh \left(\frac{\frac{x}{\Lambda} - 1}{\alpha} \right) \right]$

✓ 3D sharp cutoff

Lorentzian Form Factor

$$U_{\Lambda}^{(LorN)}(x) = \left[1 + \left(\frac{x}{\Lambda^2} \right)^N \right]^{-1}$$

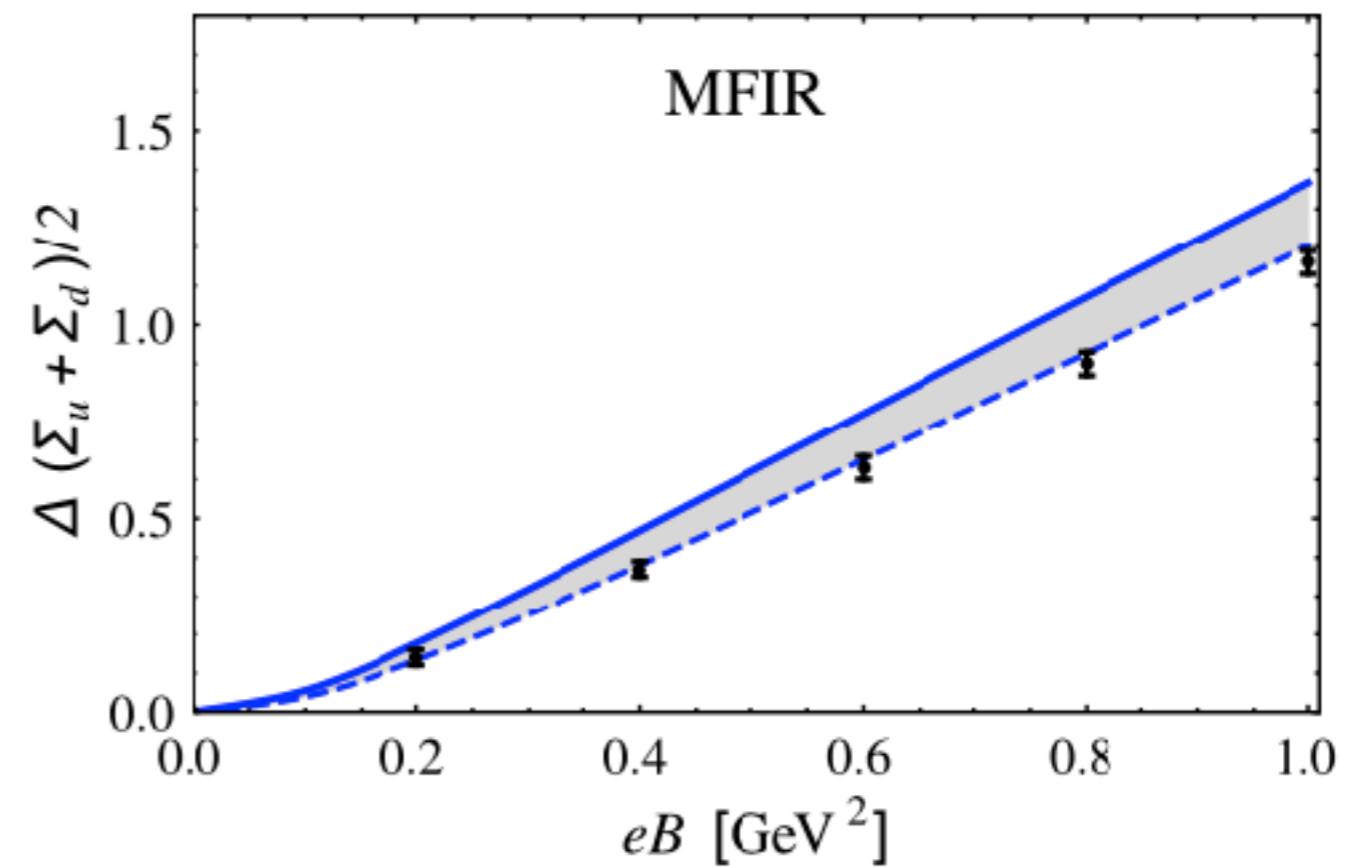
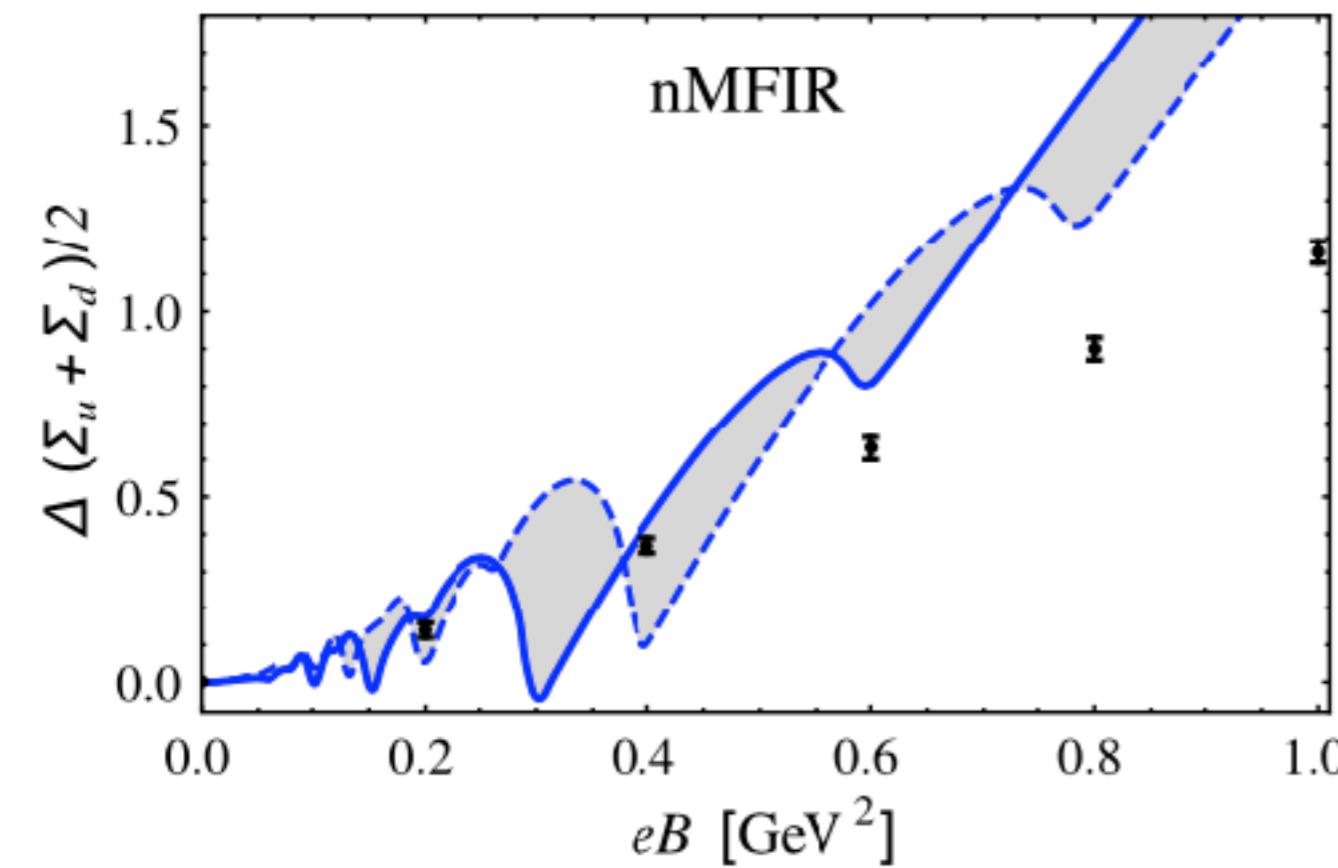
$$245 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$



Fermi-Dirac Form Factor

$$U_{\Lambda}^{\text{FD}}(x) = \frac{1}{2} \left[1 - \tanh \left(\frac{\frac{x}{\Lambda} - 1}{\alpha} \right) \right]$$

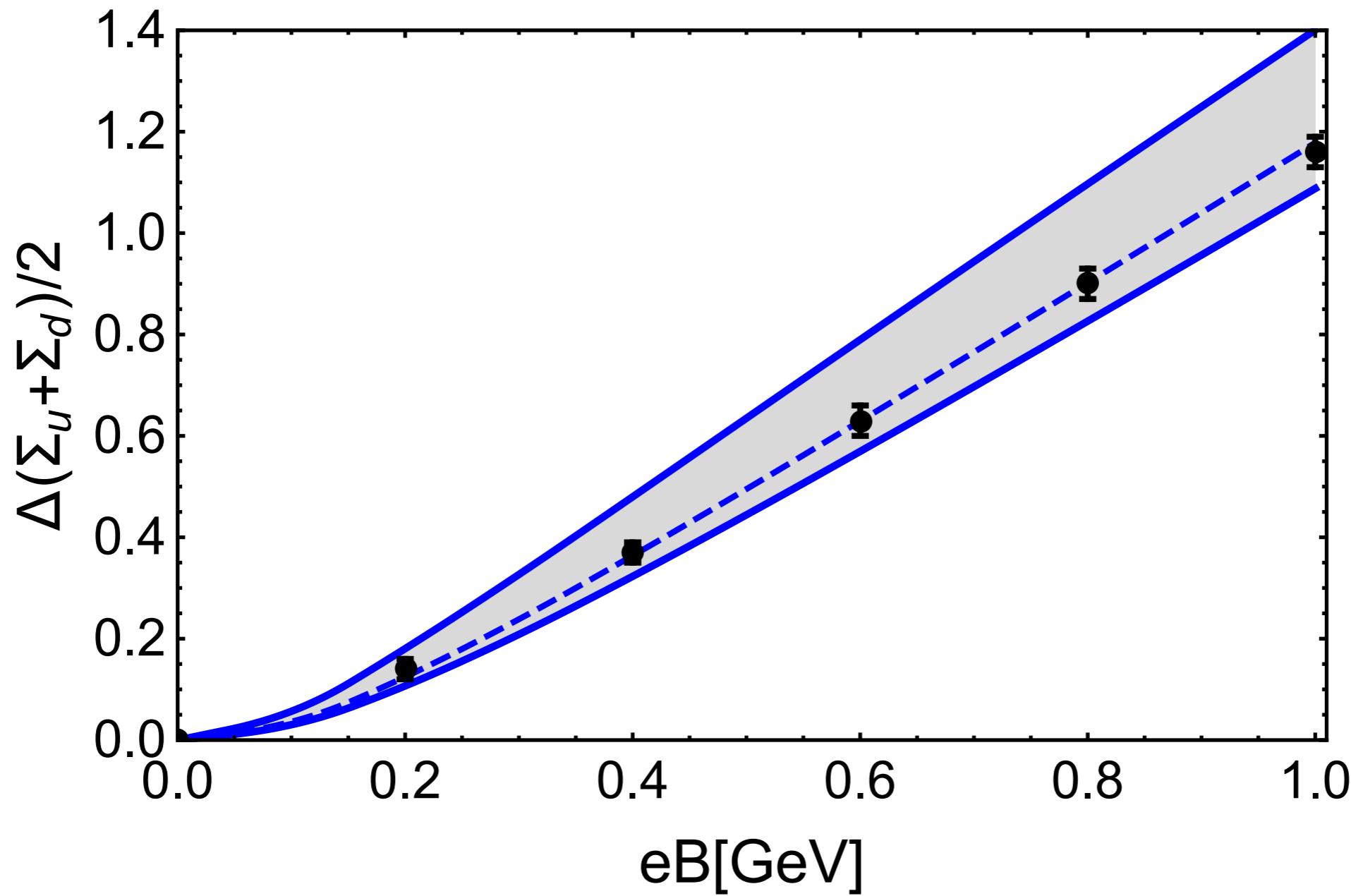
$$245 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$



Cutoff 3D + MFIR

$$241 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$

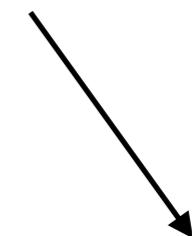
$$I_1^{3D} = \frac{\Lambda^2}{2\pi^2} \left[\sqrt{1 + M_\Lambda^2} + M_\Lambda^2 \ln \frac{M_\Lambda}{1 + \sqrt{1 + M_\Lambda^2}} \right]$$



Covariant Regularizations:

- ✓ 4D sharp cutoff
- ✓ Proper time
- ✓ Pauli-Villars

$$\begin{aligned}\mathcal{F} = & \frac{(M - m_c)^2}{4G} \\ - & N_c \sum_f \frac{|q_f|B}{2\pi} \sum_{k=0}^{\infty} \alpha_k \int_{-\infty}^{\infty} \frac{dp_3}{2\pi} \int_{-\infty}^{\infty} \frac{dp_4}{2\pi} \ln [p_4^2 + p_3^2 + 2k|q_f|B + M^2]\end{aligned}$$

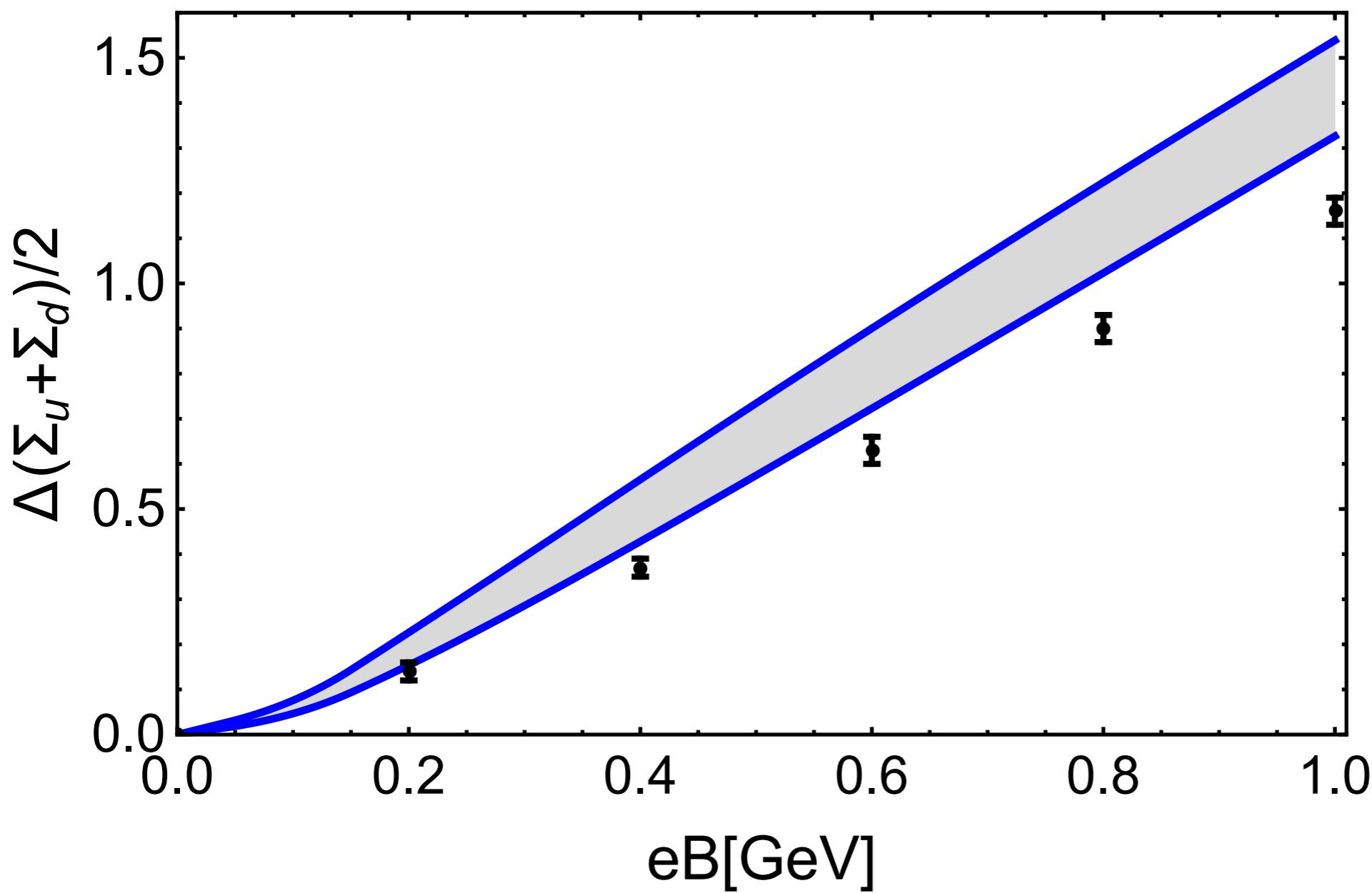


And the gap equation: $\partial\mathcal{F}/\partial M = 0 \longrightarrow \infty$

Cutoff 4D + MFIR

$245 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$

$$I_1^{4D} = \frac{\Lambda^2}{4\pi^2} \left[1 + M_\Lambda^2 \ln \frac{M_\Lambda^2}{1 + M_\Lambda^2} \right]$$



$$M_\Lambda = \frac{M}{\Lambda}$$

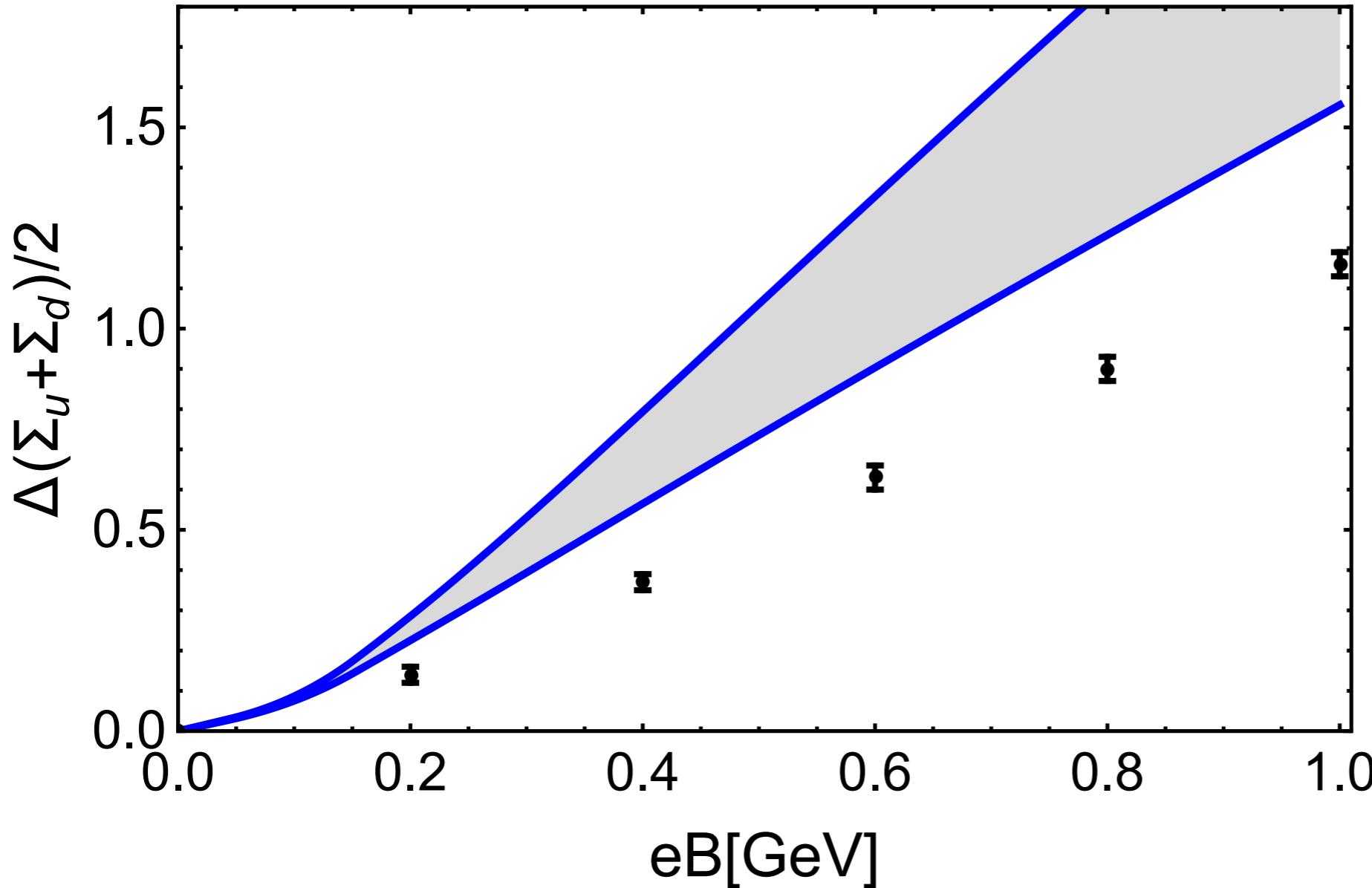
Proper Time + MFIR

$$\frac{1}{p^2 + M^2} = \int_0^\infty e^{-\tau(p^2 + M^2)} d\tau$$

$$220 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$

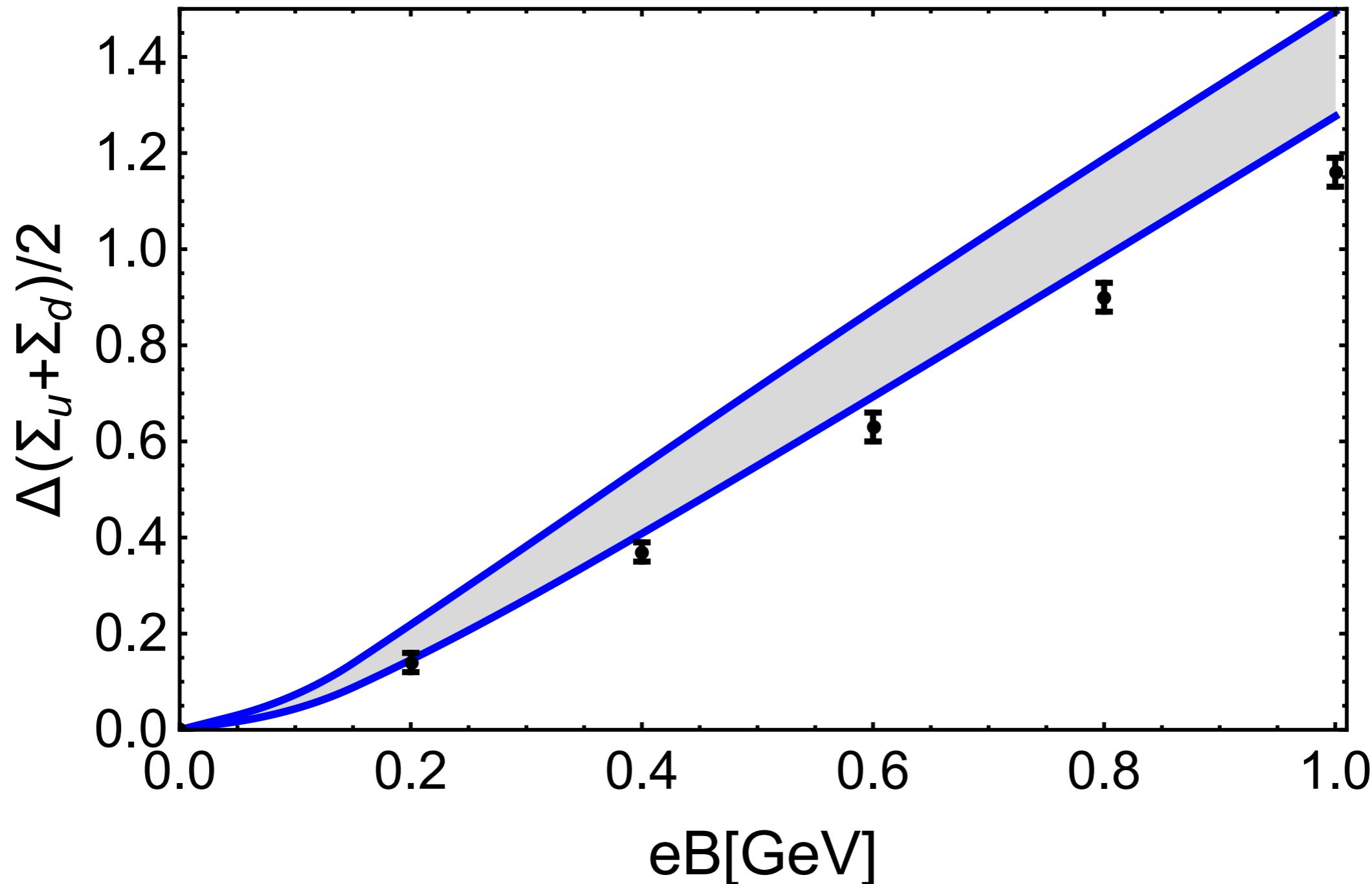
$$I_1^{PT} = \frac{\Lambda^2}{4\pi^2} E_2(M_\Lambda^2)$$

$$E_n(x) = \int_1^\infty d\tau \tau^{-n} e^{-\tau x}$$



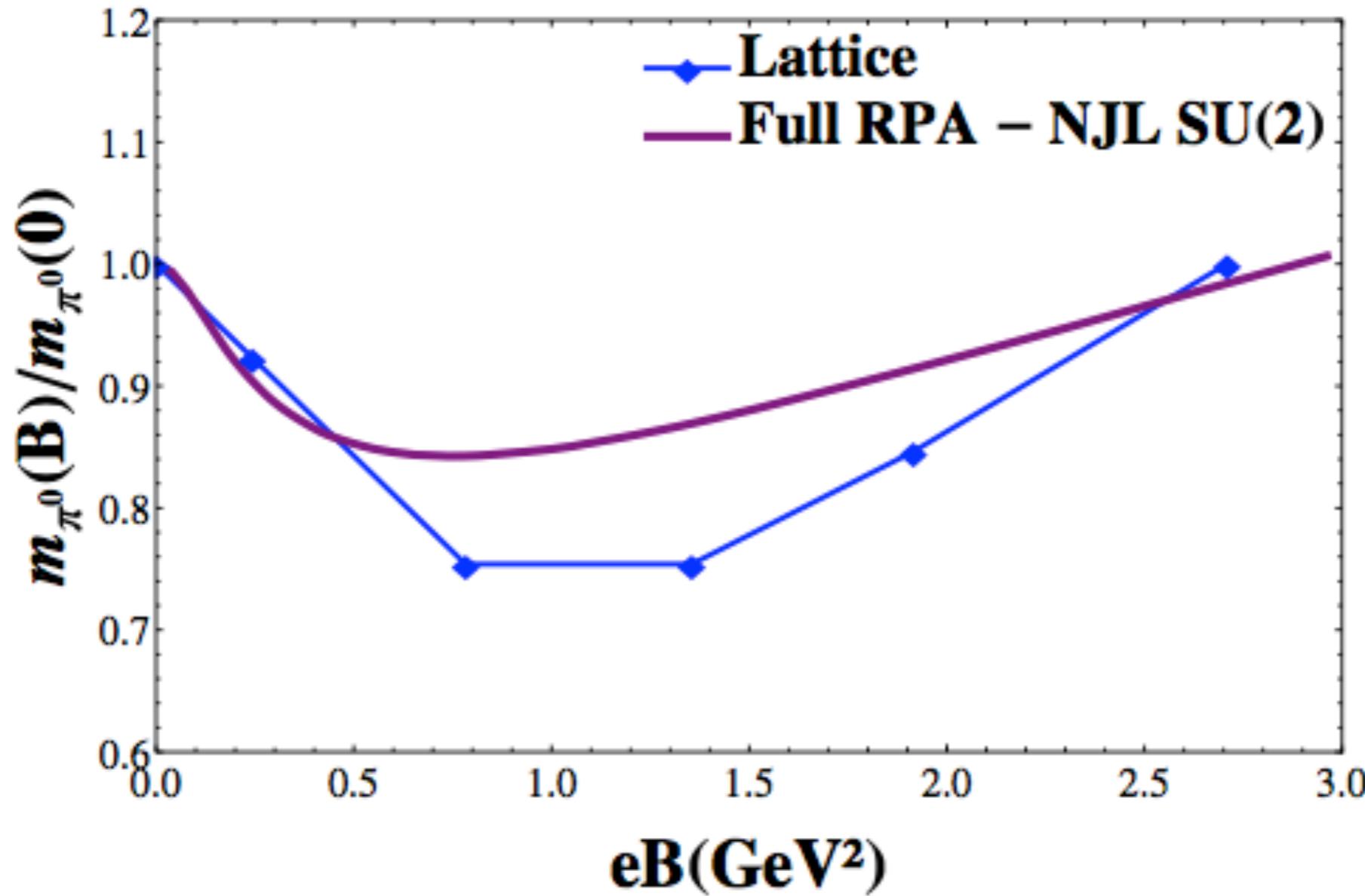
Pauli-Villars + MFIR

$$220 \text{ MeV} < -\bar{\Phi}_0^{1/3} < 260 \text{ MeV}$$



Meson masses

Lattice Results (2013)

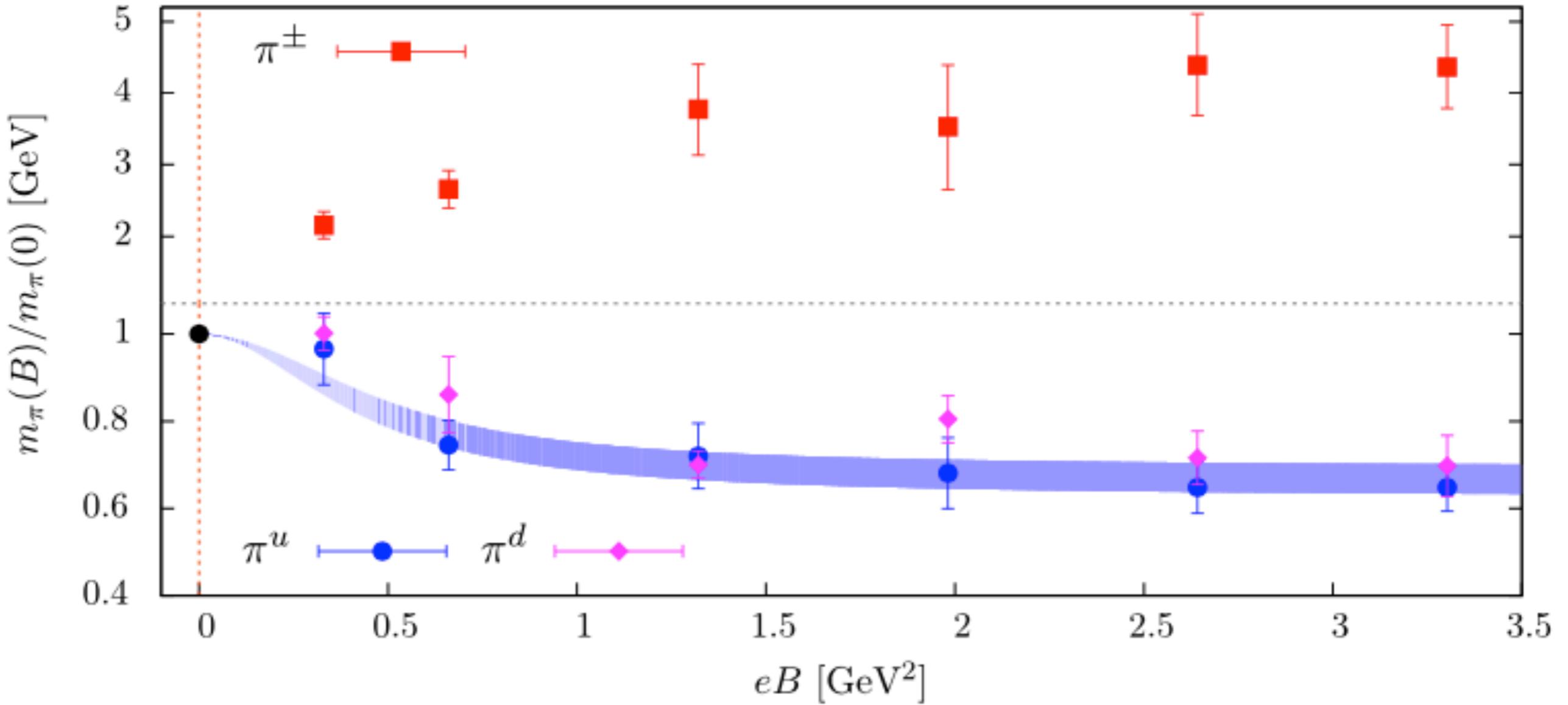


NJL + MFIR

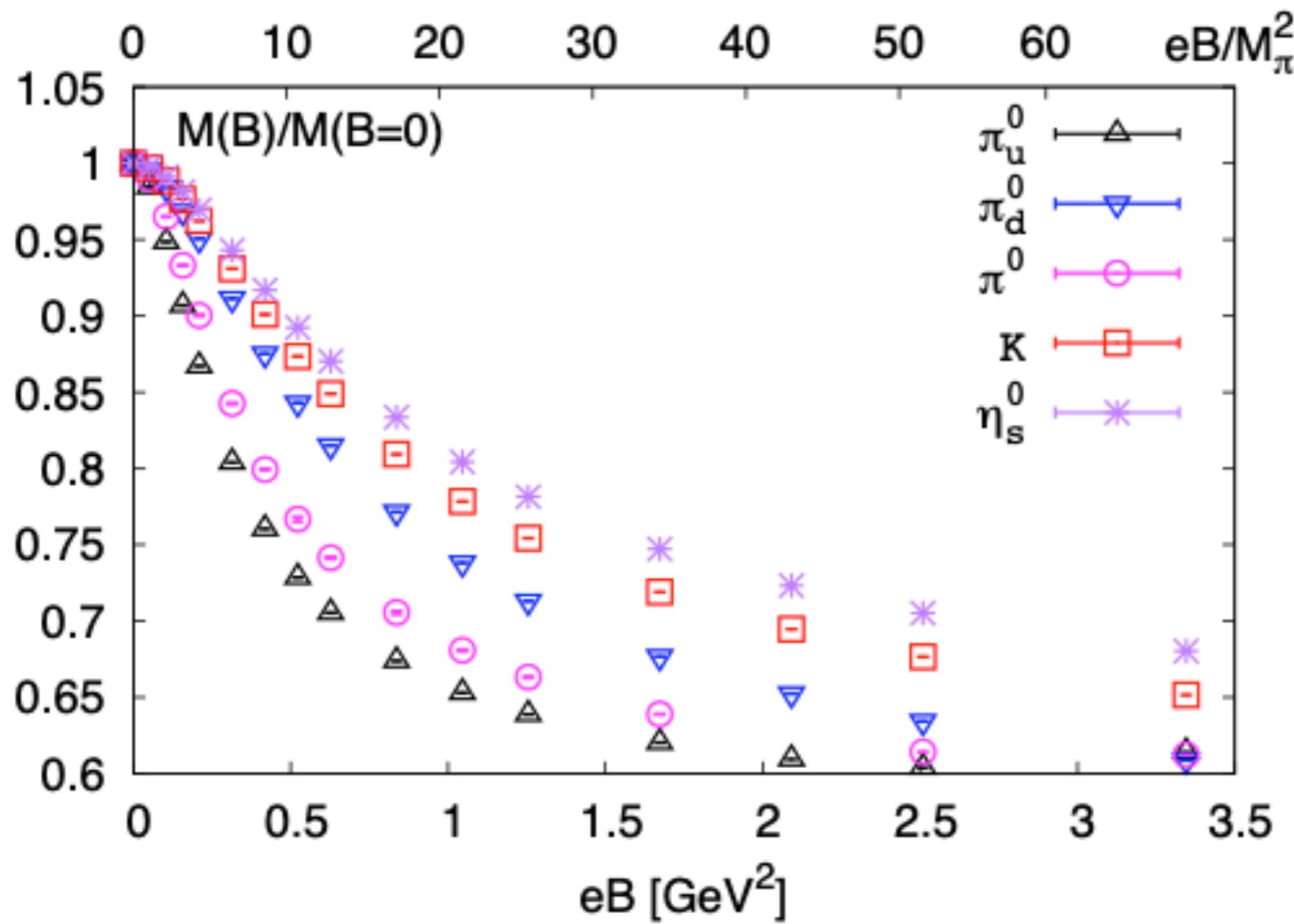
Y. Hidaka and A. Yamamoto,
Phys. Rev. D 87, 094502 (2013)

S.S. Avancini, W. R. Tavares, M.B. Pinto,
Phys. Rev. D 93, 014010 (2016)

Lattice Results (2018)



Lattice Results (2020)



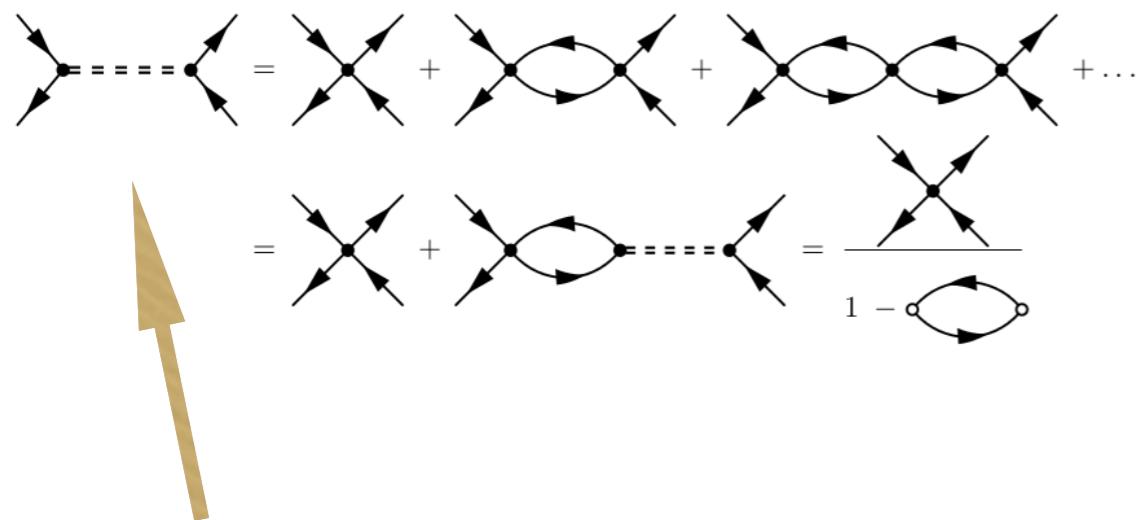
NJL - Meson properties at finite B

T matrix for the scattering of pairs of quarks, $q_1 q_2 \rightarrow q_1' q_2'$

can be calculated -> solving the Bethe-Salpeter equation in the ladder or random phase approximation (RPA)

$$m_{\pi^0}(B, T = 0)$$

only pionic degrees of freedom:



$$\mathcal{L}_{\pi qq} = ig_{\pi qq} \bar{\psi} \gamma_5 \vec{\tau} \cdot \vec{\pi} \psi$$

$$(ig_{\pi^0 qq})^2 i D_{\pi^0}(k^2) = \frac{2iG}{1 - 2G\Pi_{ps}(k^2)}$$

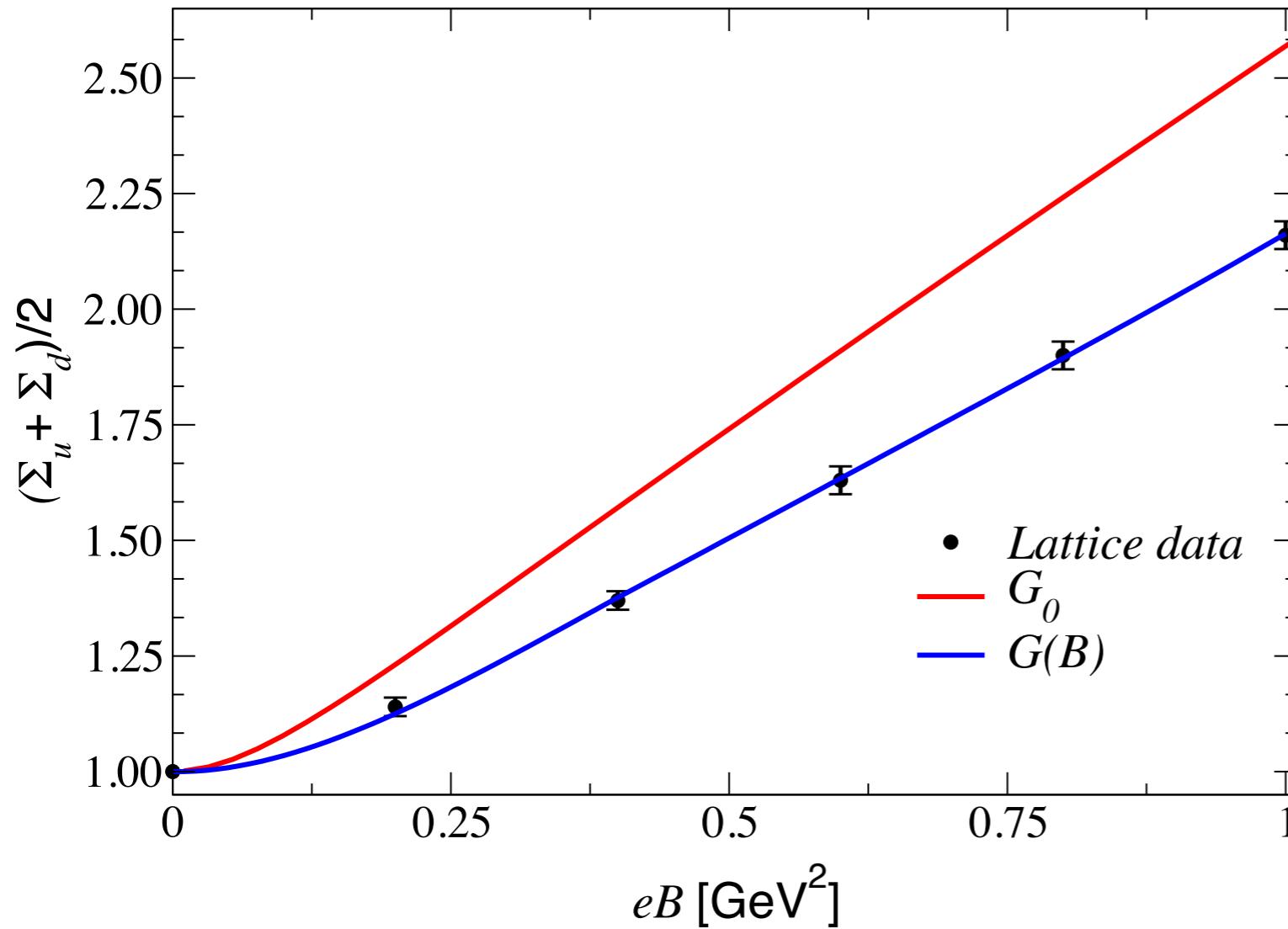
$$D_{\pi^0}(k^2) = \frac{1}{k^2 - m_{\pi^0}^2}$$

$$1 - 2G\Pi_{ps}(k^2)|_{k^2=m_{\pi^0}^2} = 0$$

✓ Magnetic Field Independent Regularization (MFIR)

Field dependent coupling $G(B, T=0)$

$$\Sigma_f(B) = \frac{2m}{m_\pi^2 f_\pi^2} [\langle \bar{\psi}_f \psi_f \rangle_B - \langle \bar{\psi}_f \psi_f \rangle_{00}] + 1$$

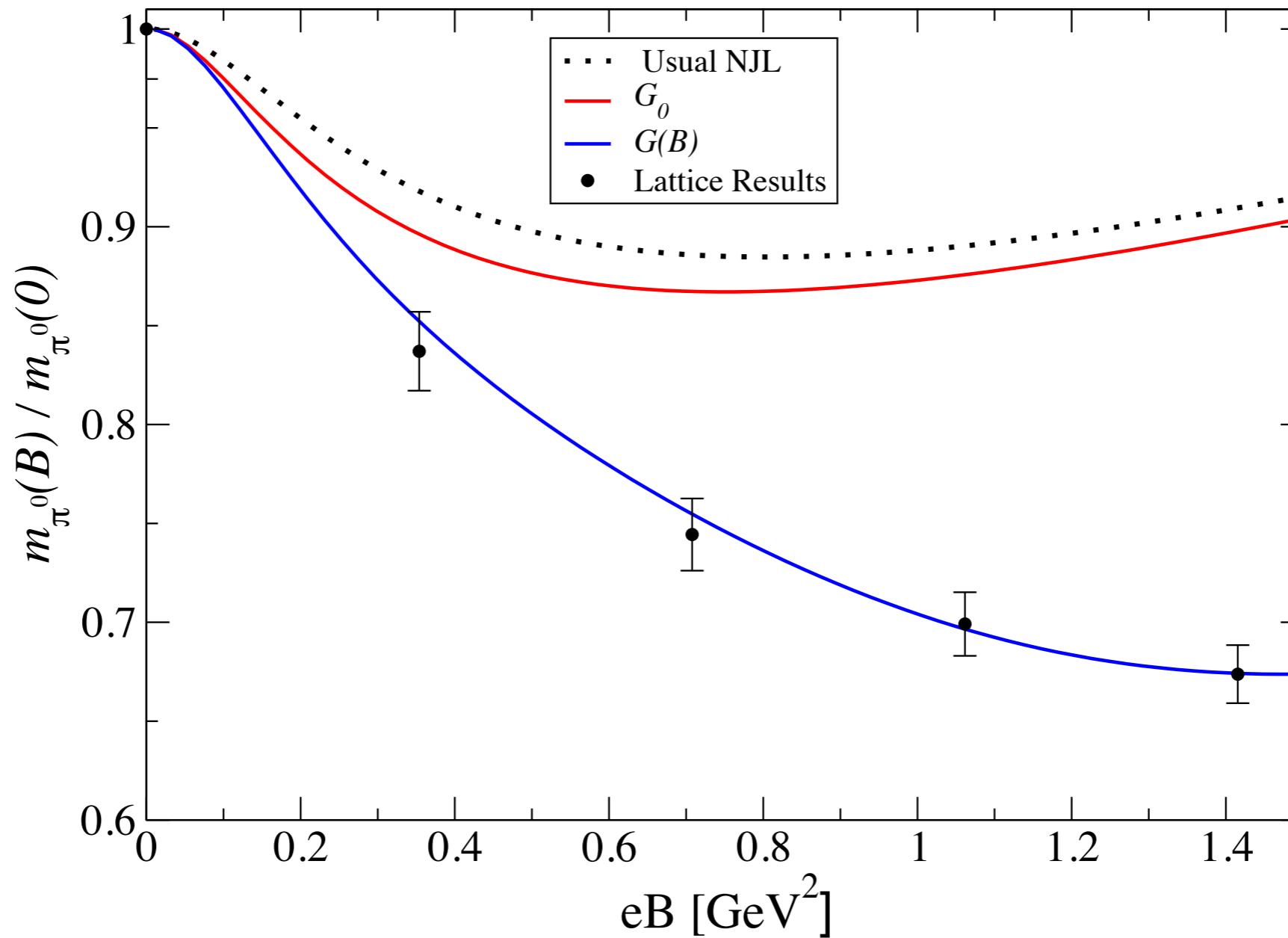


$$G(eB) = \alpha + \beta e^{-\gamma (eB)^2}$$

$$\begin{aligned}\alpha &= 1.44373 \text{ GeV}^{-2} \\ \beta &= 3.06 \text{ GeV}^{-2} \\ \gamma &= 1.31 \text{ GeV}^{-4}\end{aligned}$$

lattice results: Phys. Rev. D 86, 071502(R) (2012)

Meson masses at finite B



Farias at all, Phys. Lett. B 767 (2017) 247–252

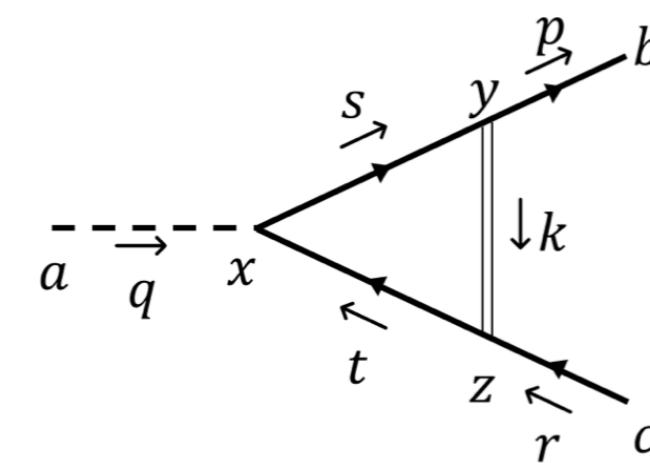
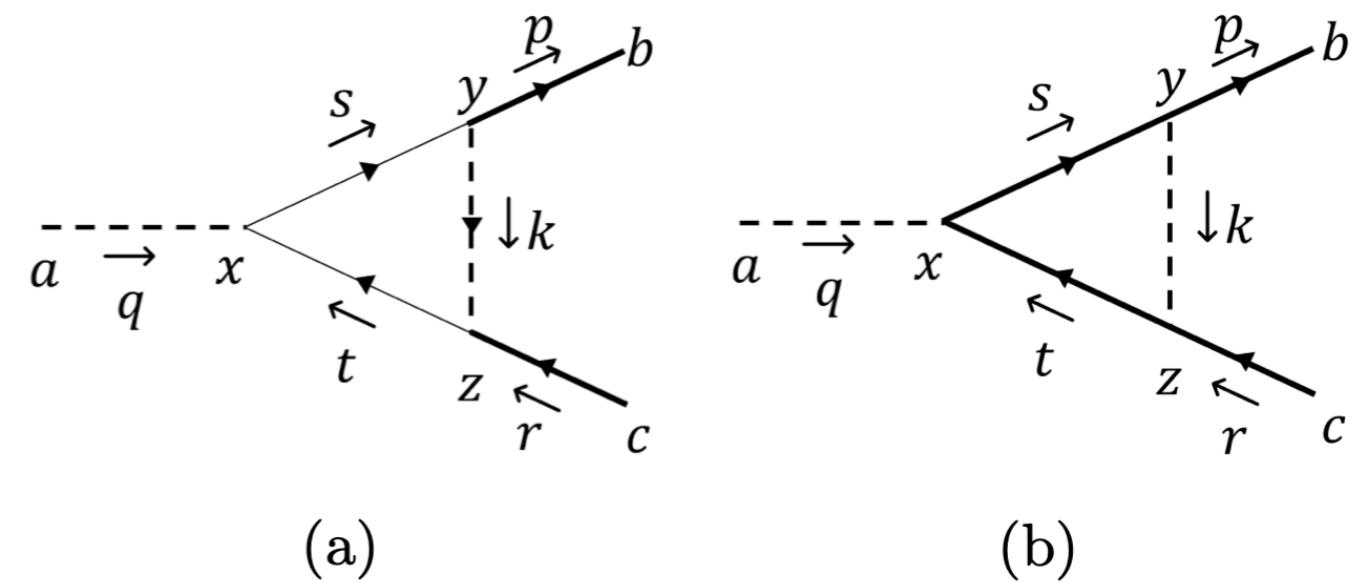
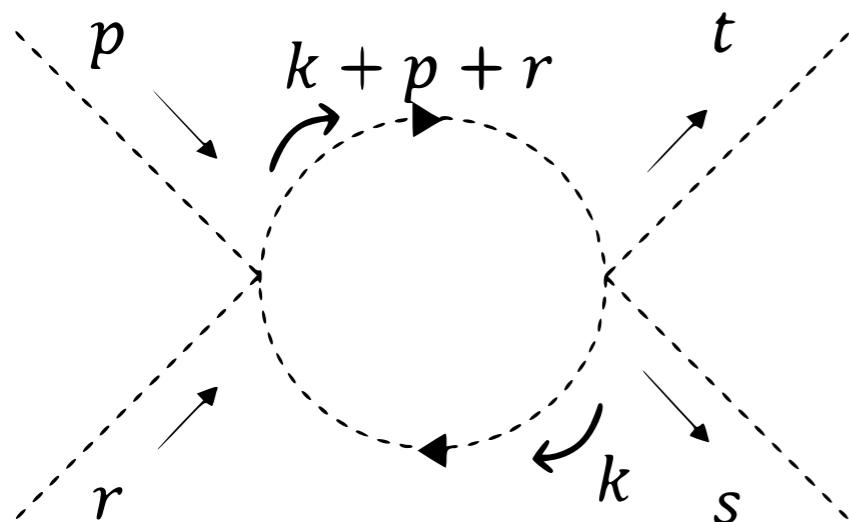
Lattice data: G. Bali, B.B. Brandt, G. Endrodi, B. Glaessle, arXiv:1510.03899 [hep-lat]

Linear Sigma model with quarks - LSMq

$$\begin{aligned}\mathcal{L} = & \frac{1}{2}(\partial_\mu \sigma)^2 + \frac{1}{2}(\partial_\mu \vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \\ & + i\bar{\psi}\gamma^\mu\partial_\mu\psi - ig\gamma^5\bar{\psi}\vec{\tau}\cdot\vec{\pi}\psi - g\bar{\psi}\psi\sigma\end{aligned}$$

- Pions are described by an isospin triplet, $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$;
- Two species of quarks are represented by an $SU(2)$ isospin doublet, ψ ;
- The σ scalar is included by means of an isospin singlet;
- λ is the boson self-coupling;
- g is the fermion-boson coupling;
- $a^2 > 0$ is the mass parameter.

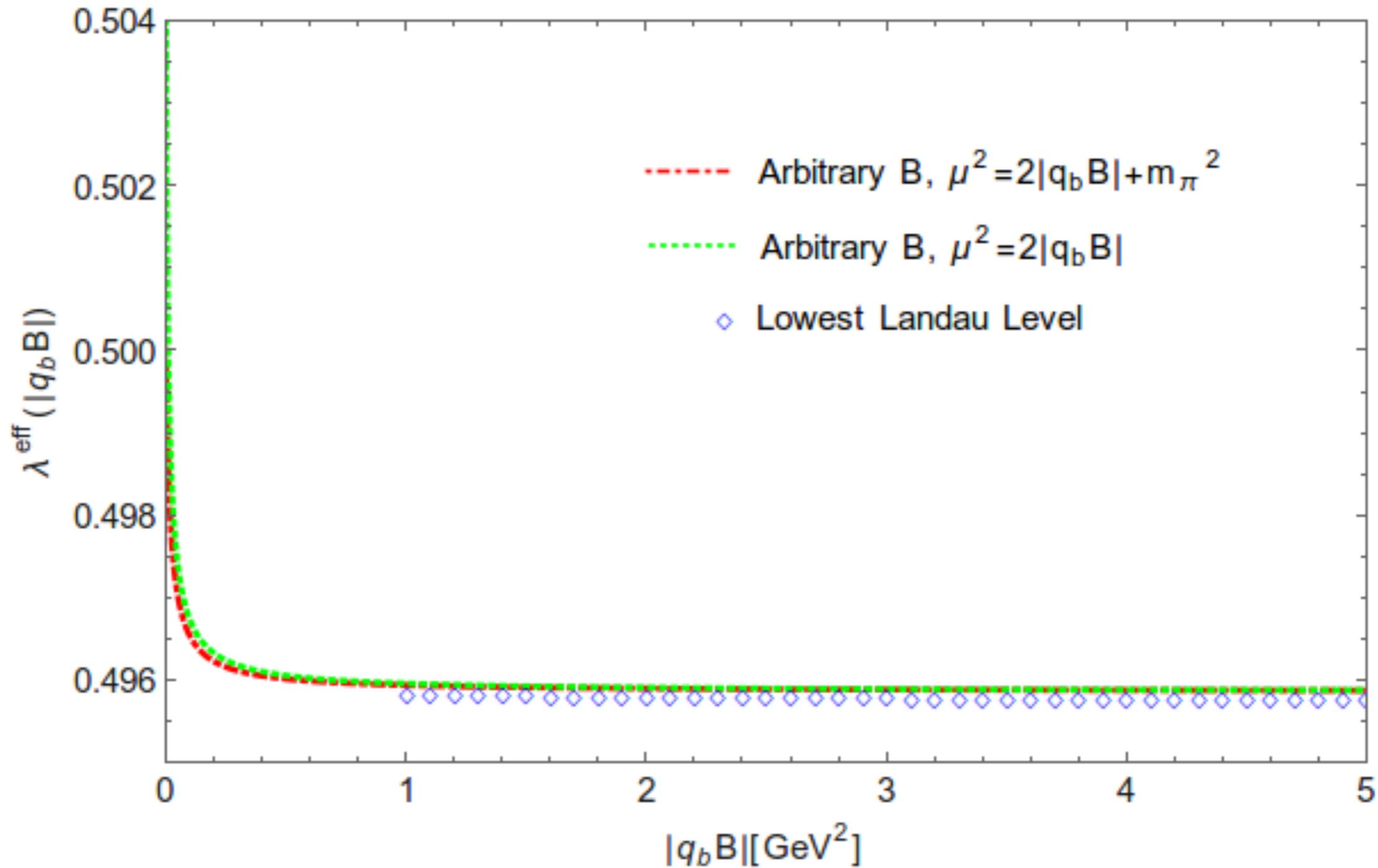
Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



arXiv:2009.13740 [hep-ph], A.Ayala,
J.L.Hernández, L. A. Hernández,
R.L. S. Farias, R. Zamora

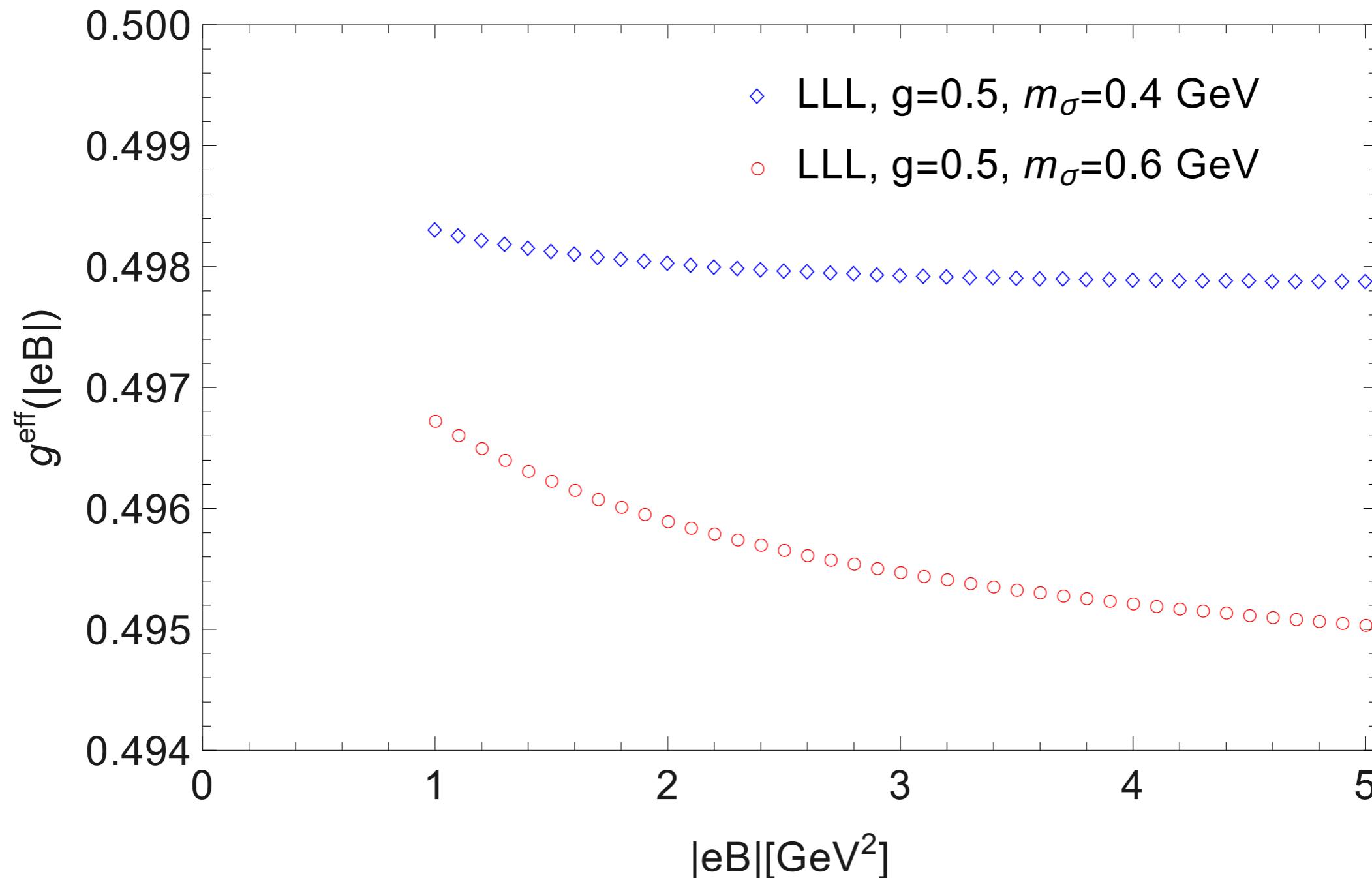
(c)

Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



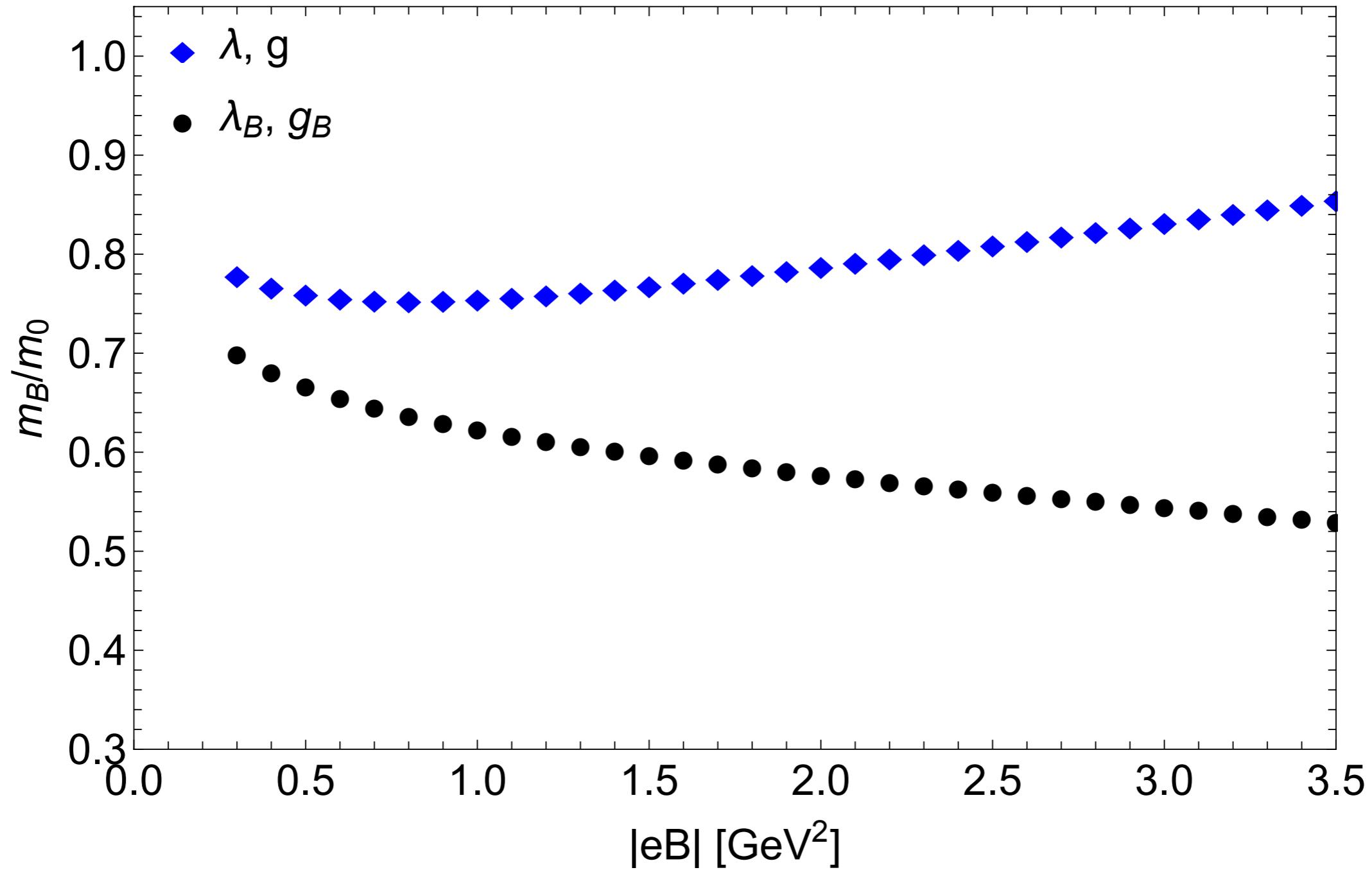
arXiv:2009.13740 [hep-ph], A.Ayala,
J.L.Hernández, L. A. Hernández,
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Magnetic corrections to the boson self-coupling and boson-fermion coupling in the LSMq



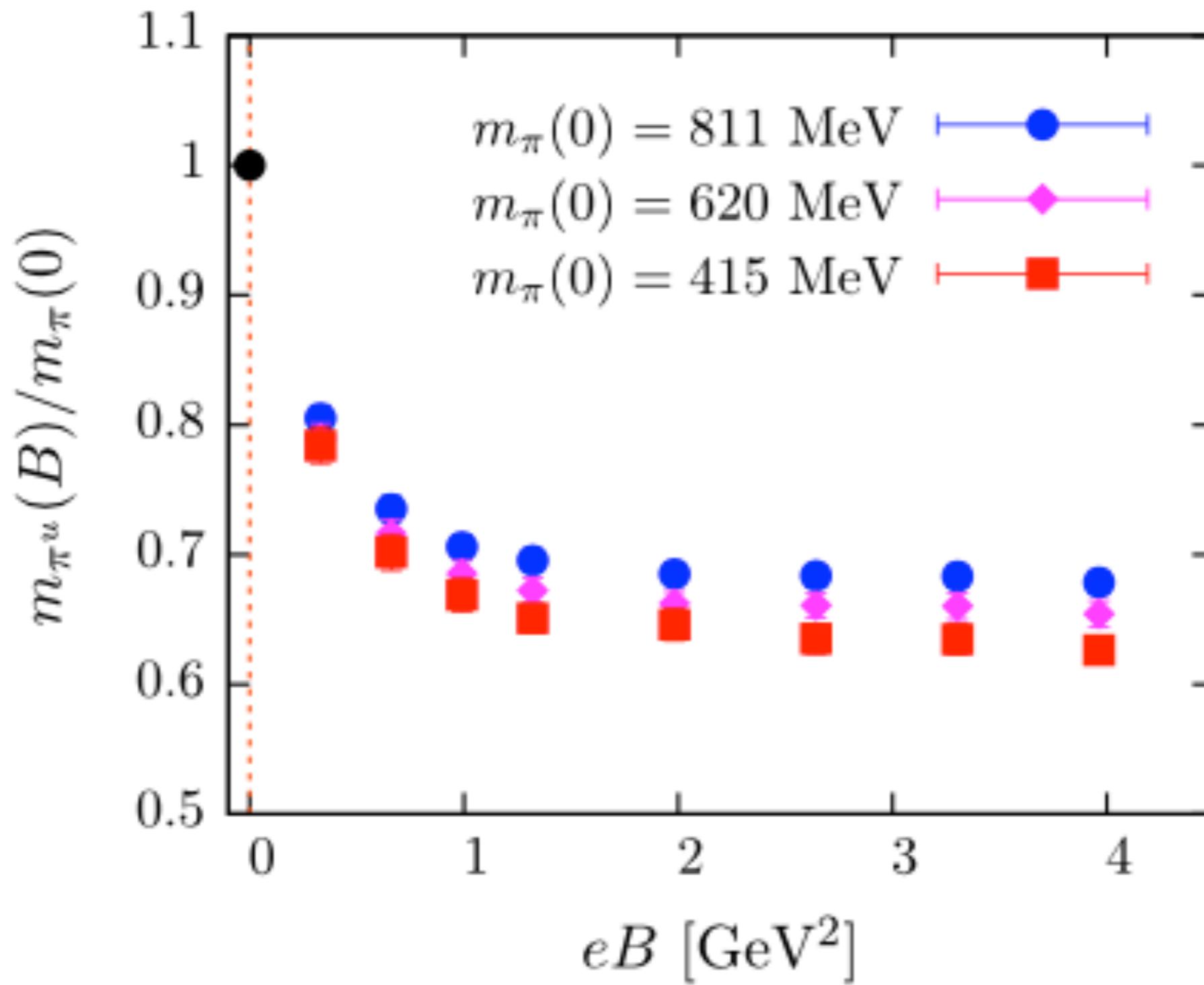
arXiv:2009.13740 [hep-ph], A.Ayala,
J.L.Hernández, L. A. Hernández,
R.L. S. Farias, R. Zamora

Neutral pion mass \times eB

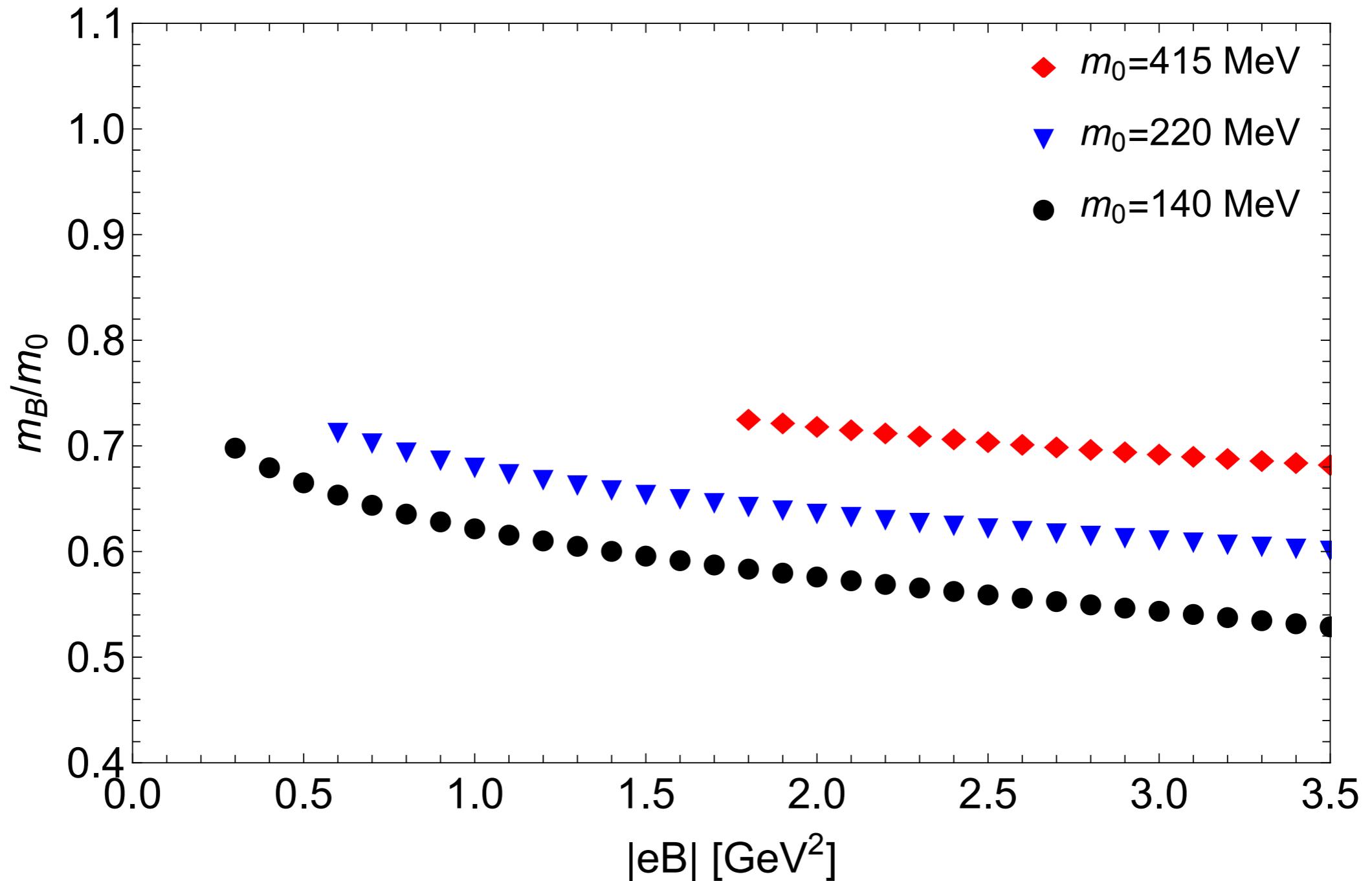


Tomorrow on arxivs, A. Ayala, J.L. Hernández,
L. A. Hernández, R.L. S. Farias, R. Zamora

Lattice

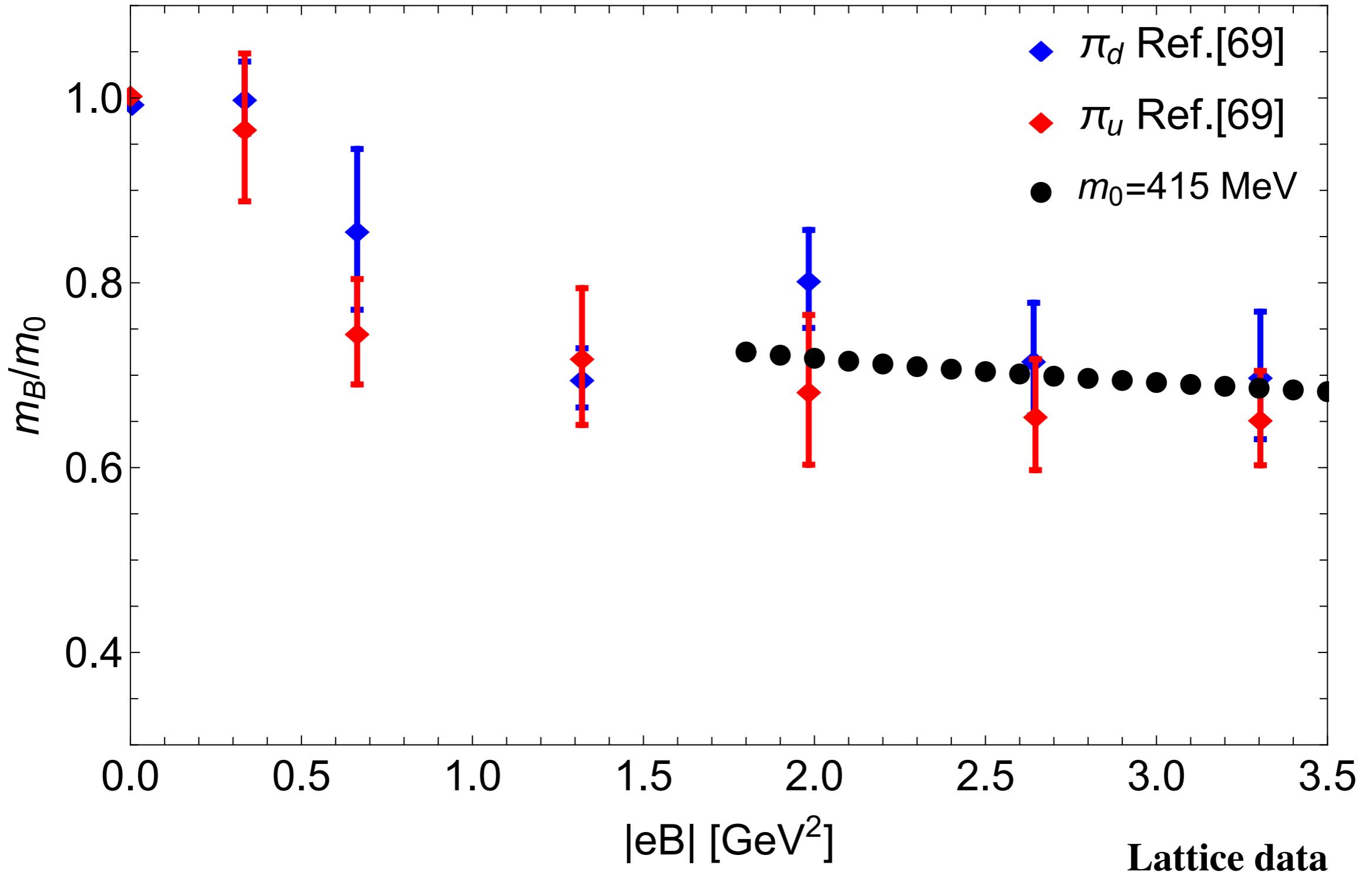


LSMq strong field



Tomorrow on arxivs, A. Ayala, J.L. Hernández,
L. A. Hernández, R.L. S. Farias, R. Zamora

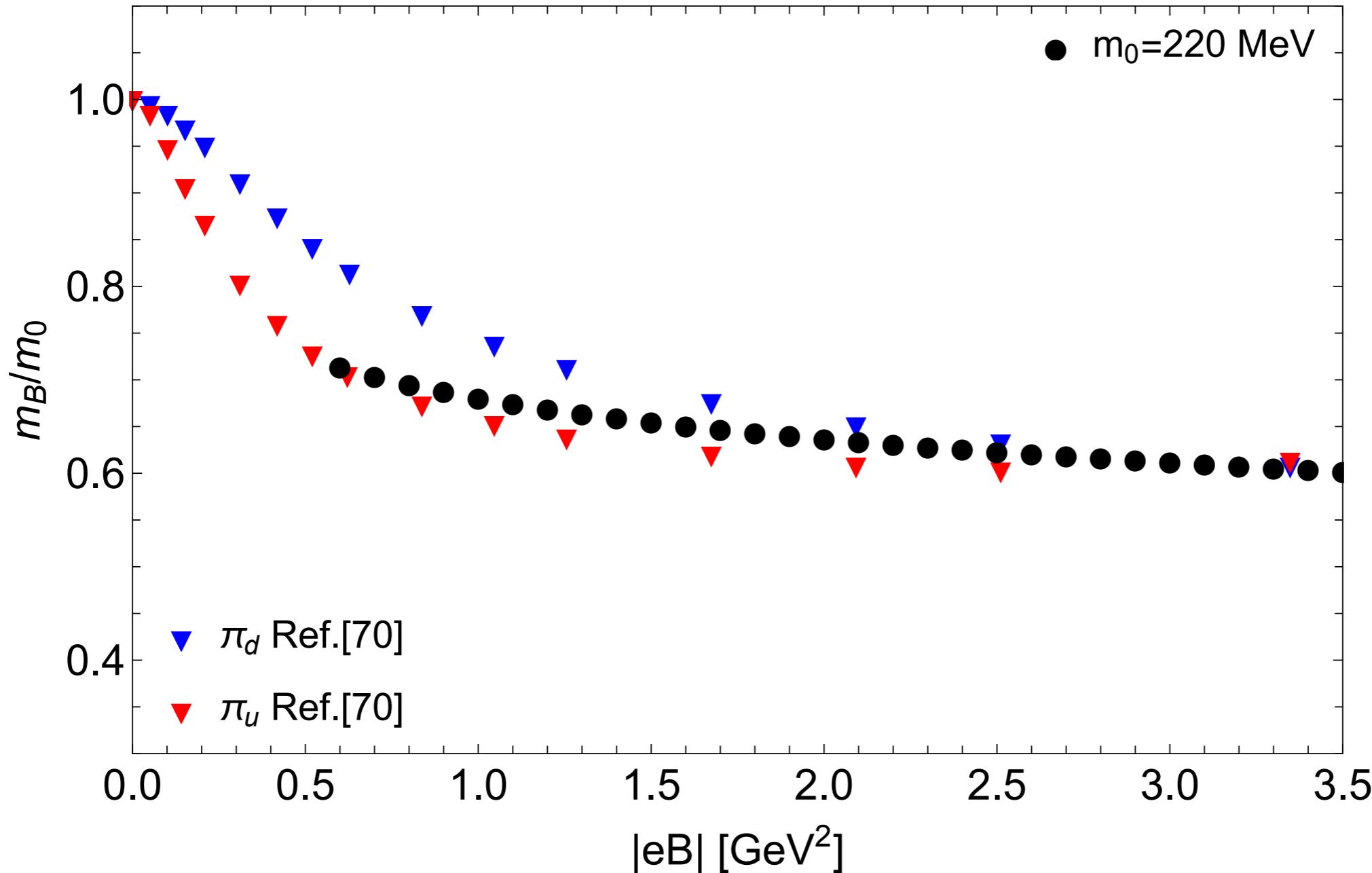
LSMq strong field



Tomorrow on arxivs, A. Ayala, J.L. Hernández,
L. A. Hernández, R.L. S. Farias, R. Zamora

G.S. Bali, B.B. Brandt, G. Endrodi and
B.Glaessle, Phys. Rev D 97, 034505 (2018)

LSMq strong field



Lattice data

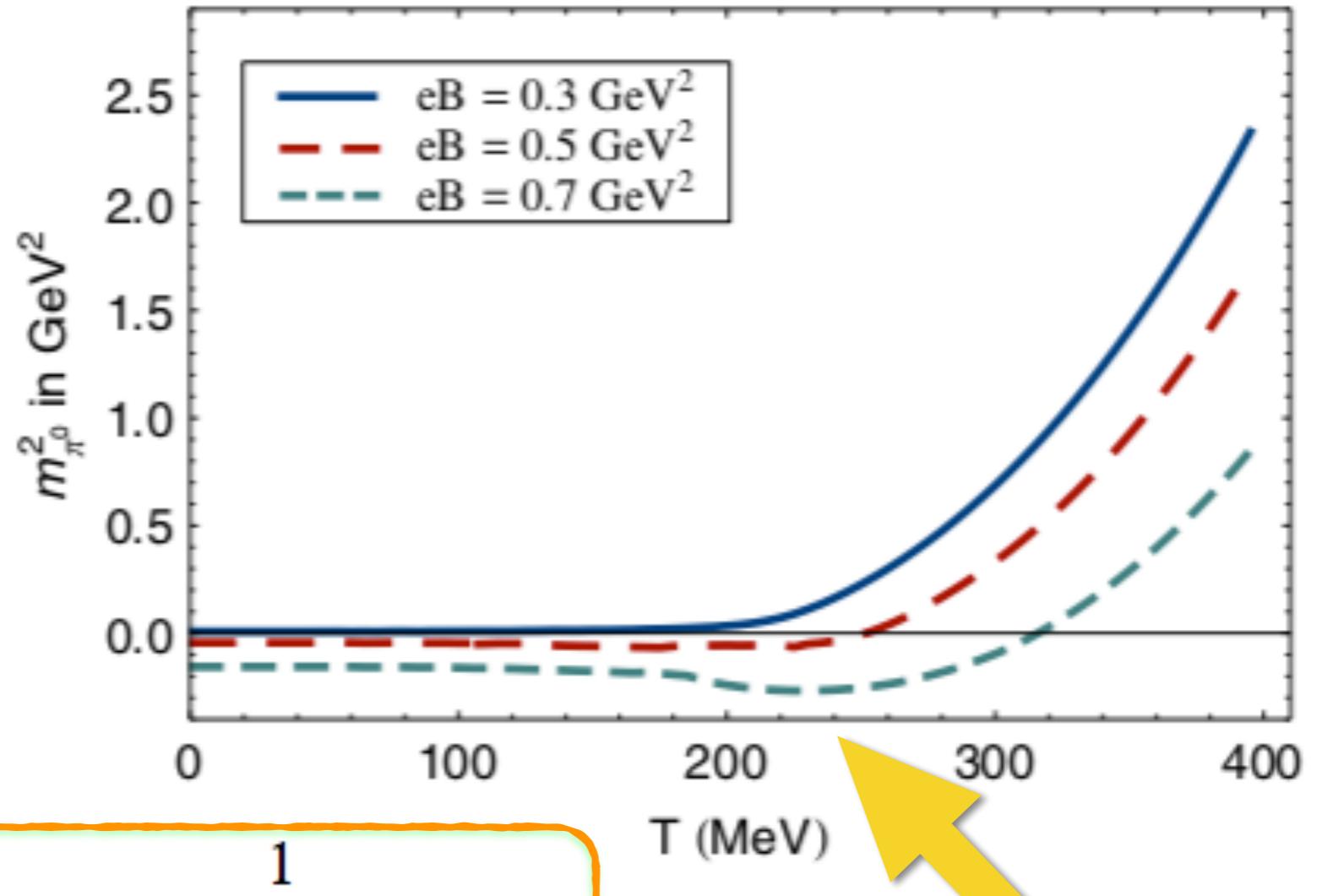
Tomorrow on arxivs, A. Ayala, J.L. Hernández,
L. A. Hernández, R.L. S. Farias, R. Zamora

H.-T. Ding, S.-T. Li, A. Tomiya, X.-D. Wang
and Y. Zhang, e-Print:2008.00493 [hep-lat].

Meson masses + T + B
using NJL

Neutral pion mass + B + T

SU(2) NJL
+ B + T

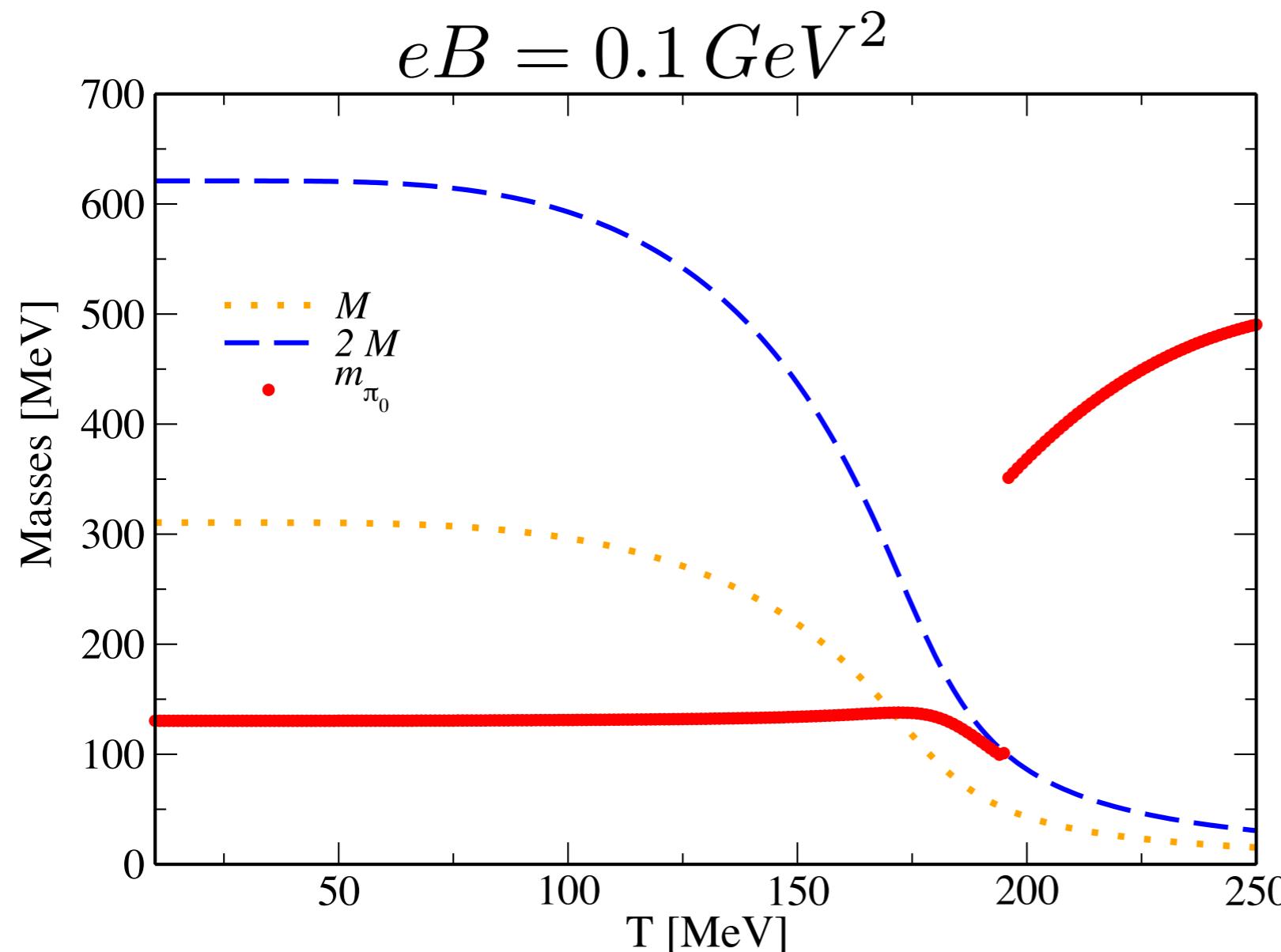


$$f_\Lambda = \frac{1}{1 + \exp\left(\frac{|\mathbf{p}| - \Lambda}{A}\right)},$$

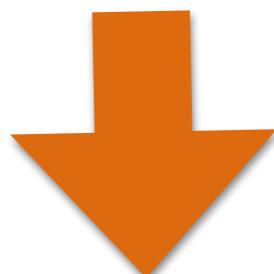
$$f_{\Lambda,B}^p = \frac{1}{1 + \exp\left(\frac{\sqrt{p_3^2 + 2|qeB|p} - \Lambda}{A}\right)}$$

Tachyonic Instabilities

Neutral pion mass X eB + MFIR



More energetic
resonances that we obtain
as we increase the
magnetic field



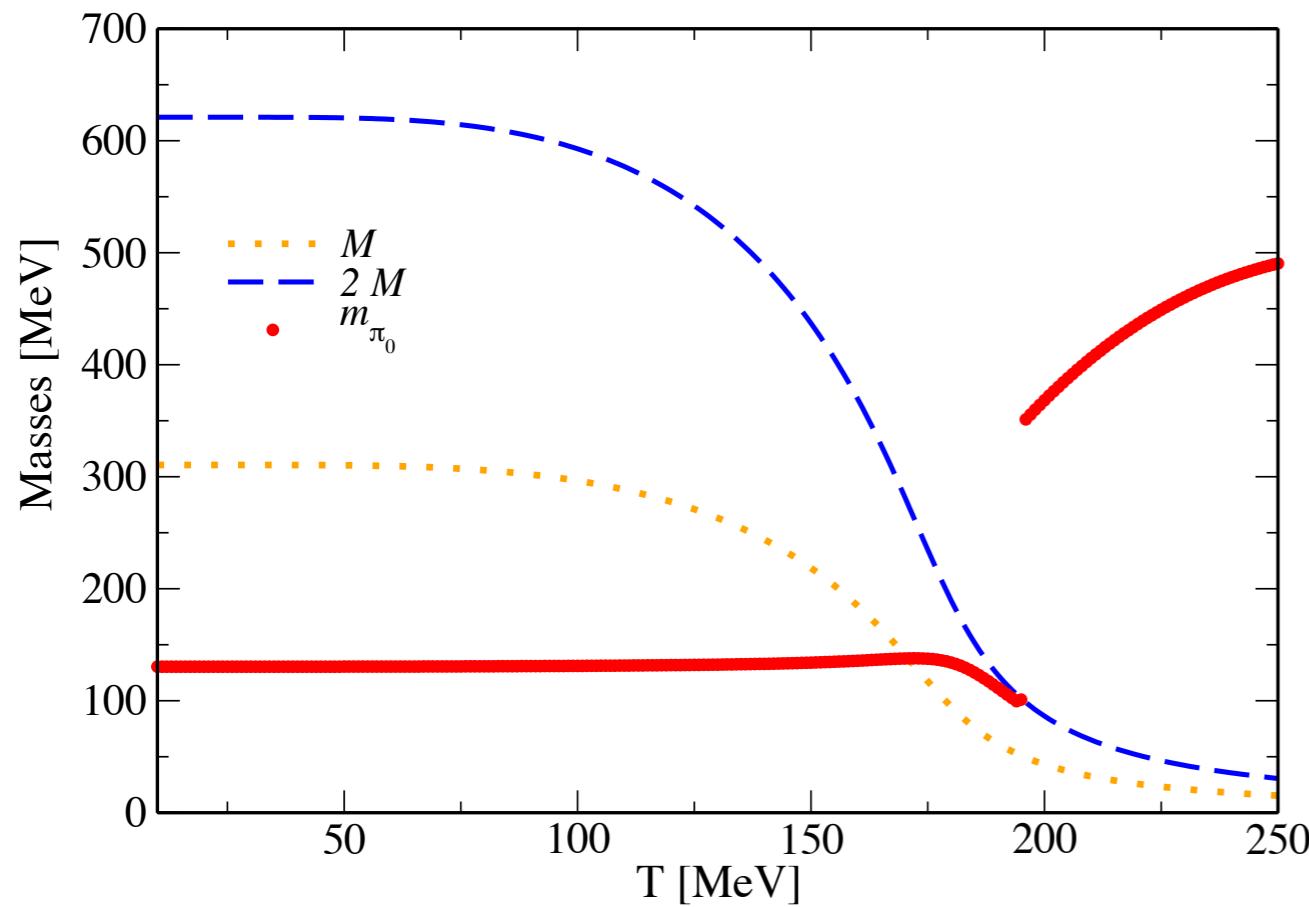
dimensional
reduction

No Tachyonic Instabilities

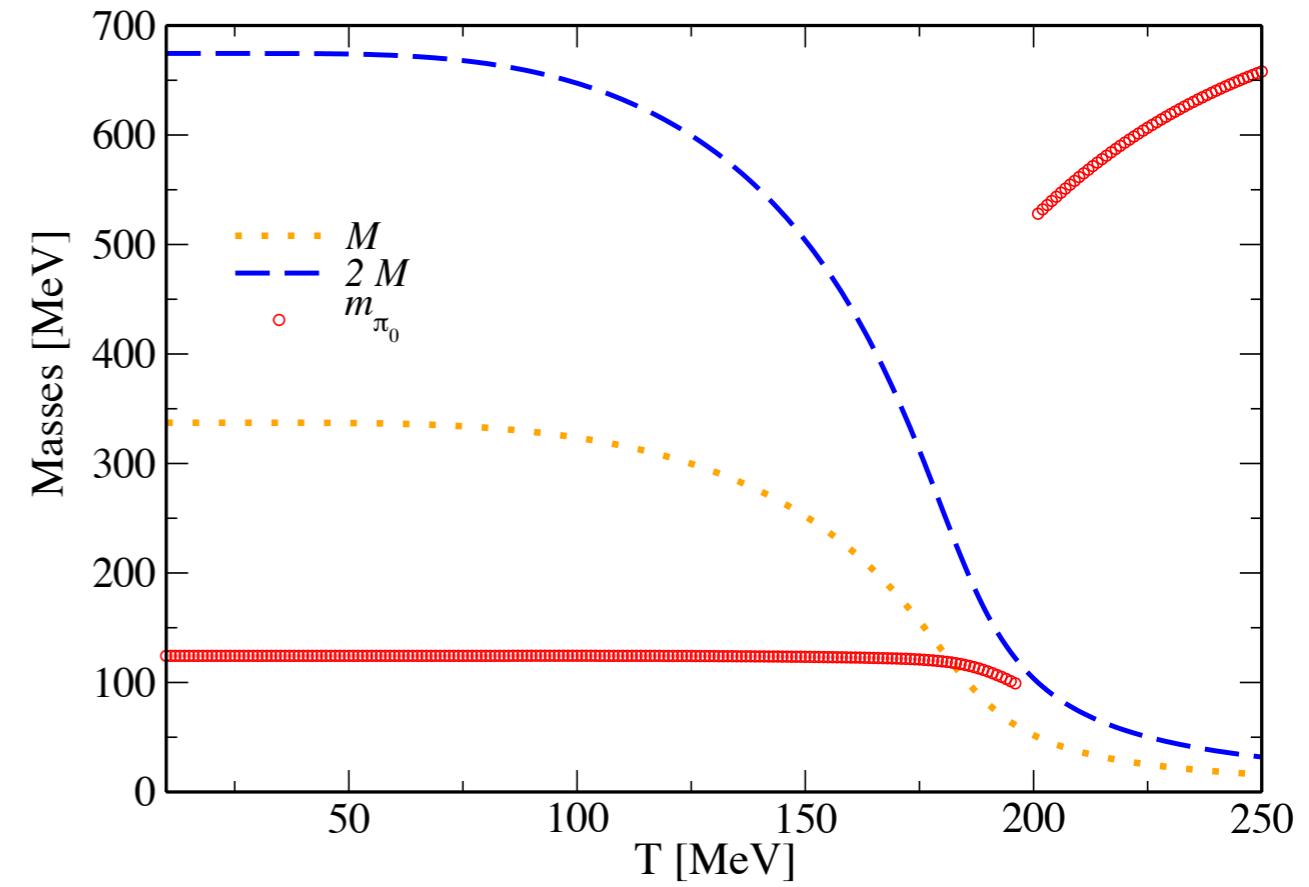
Neutral pion mass \times eB

No Tachyonic Instabilities!!!

$$eB = 0.1 \text{ GeV}^2$$



$$eB = 0.2 \text{ GeV}^2$$



T Mott increase with B using MFIR approach!

Conclusions

- ✓ We use results from lattice simulations of QCD in the presence of intense magnetic fields as a benchmark platform for comparing different regularization procedures used in the literature for the NJL type models MFIR X nMFIR
- ✓ MFIR scheme avoid some unphysical results, and this choice of regularization provide to us some different results from most of the regularizations prescriptions of the current literature.
- ✓ At T=0 meson masses evaluate using NJL and LSMq are in agreement with lattice when their coupling constants depend on B;

Conclusions

- ✓ Mott dissociation temperature is catalyzed with the increase of B
- ✓ The dramatic result is the more energetic resonances that we obtain as we increase the magnetic field. The π meson at the Mott dissociation temperature jumps to a resonance in a degenerate state with the σ meson.
- ✓ This is a direct result from the dimensional reduction of the system at strong magnetic fields that enforces the system to go to another state, since we have less states to the creation of the thermal $q - q^-$ excitation.

Perspectives

- ✓ Inclusion of thermo-magnetic effects in LSMq
- ✓ Charged mesons: NJL and LSMq

Thank you for your attention!