Rotational effect versus finite-size effect on chiral phase transition

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S. Ebihara, K. Fukushima, KM, PLB 764, 94 (2017)
HL. Chen, K. Fukushima, XG. Huang, KM PRD 96, 054032 (2017)

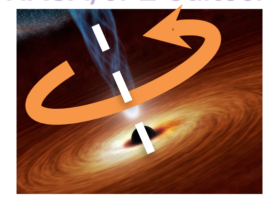
Rotating Relativistic Systems

- binary star merger

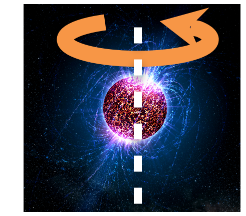
NASA/Tod Strohmayer

- black hole

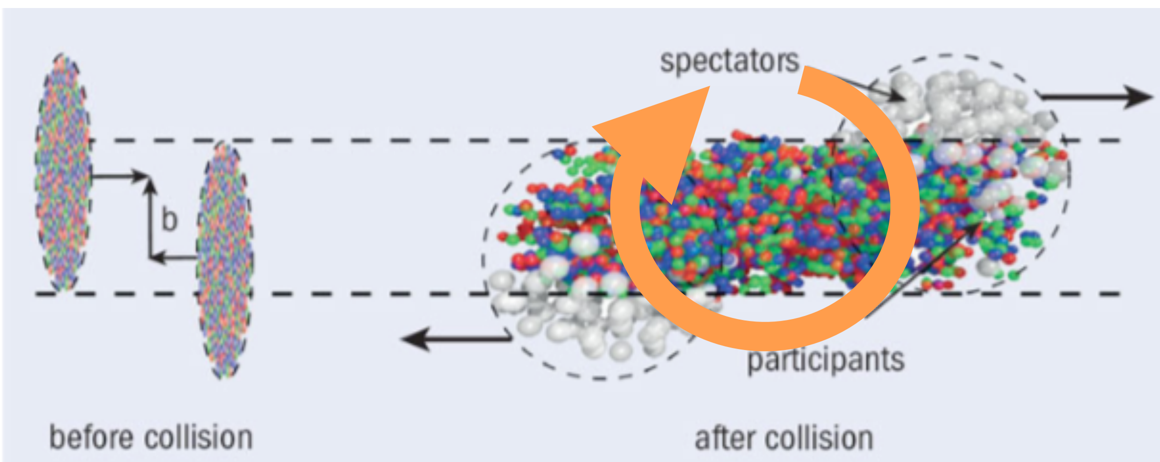
NASA/JPL-Caltech



- neutron star



- **heavy-ion collision**

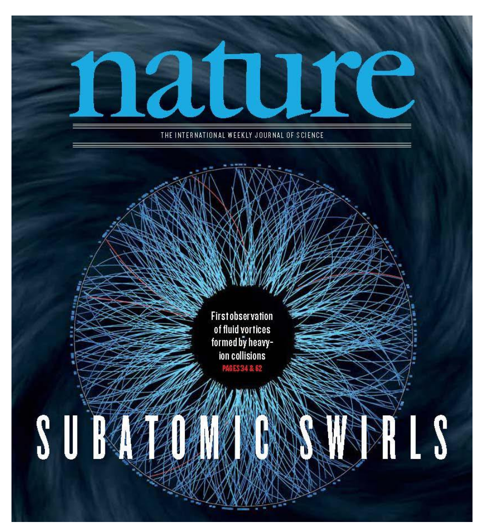


Angular momentum of heavy-ions

⟶ **Vorticity** (or **Rotation**)

First evidence:
global Λ-polarization (2017)

$$\Omega \gtrsim 10^{21}\ \mathrm{Hz}$$



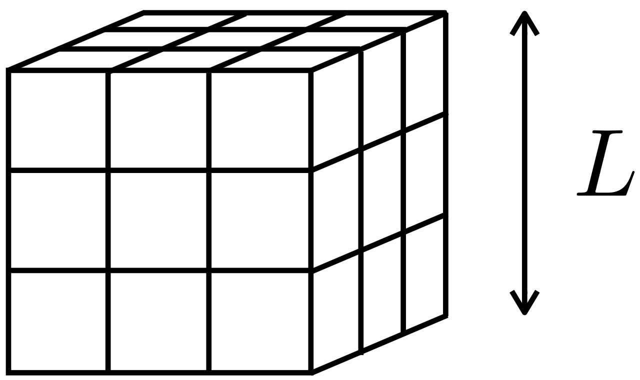
#38

The Fastest Fluid
by Sylvia Morrow
Superhot material spins at an incredible rate.

Necessary to know **Rotational effect on QCD**

Why finite-sized system?

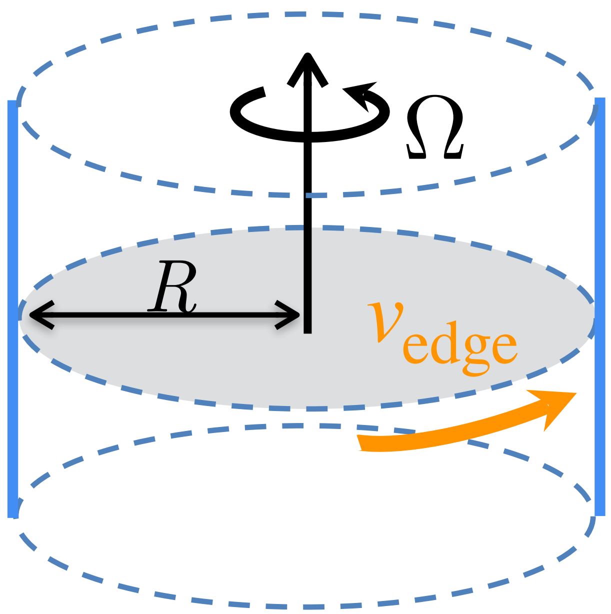
usual systems



thermodynamic limit (after evaluation)

$$L \to \infty$$

rotating system



causality $v_{\mathrm{edge}} = \Omega R \leq 1$

$$R \leq 1/\Omega < \infty$$

→ crucial for IR physics, e.g.,

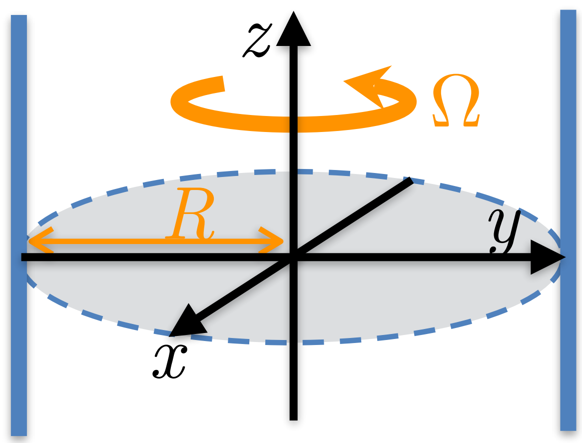
phase structure and **anomalous transport**

1. Rotating Fermions in Finite-sized Cylinder

S. Ebihara, K. Fukushima, KM, PLB 764, 94 (2017)

Rotating Fermions

Dirac eq. $$[i\gamma^\mu(\partial_\mu + \Gamma_\mu) - m]\psi = 0$$



$$g_{\mu\nu} = \begin{pmatrix} 1 - r^2\Omega^2 & y\Omega & -x\Omega & 0 \\ y\Omega & -1 & 0 & 0 \\ -x\Omega & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\left[i\gamma^0\left\{\partial_0 + \Omega\left(-x\partial_2 + y\partial_1 - \frac{i}{2}\sigma^{12}\right)\right\} + i\gamma^1\partial_1 + i\gamma^2\partial_2 + i\gamma^3\partial_3 - m\right]\psi = 0$$

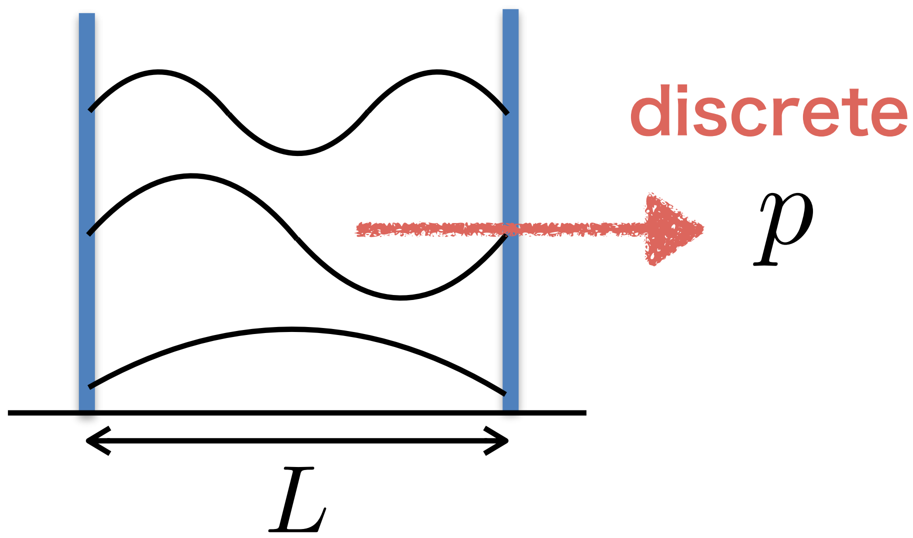
distribution $$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon - \Omega(l+1/2)\}} + 1}$$ l : eigenvalue of $\hat{L}_z$

rotational effect ≈ density effect

discretized (finite size effect) $$\varepsilon = \sqrt{p_\perp^2 + p_z^2}$$ continuous

Momentum Discretization

Bosons in a well



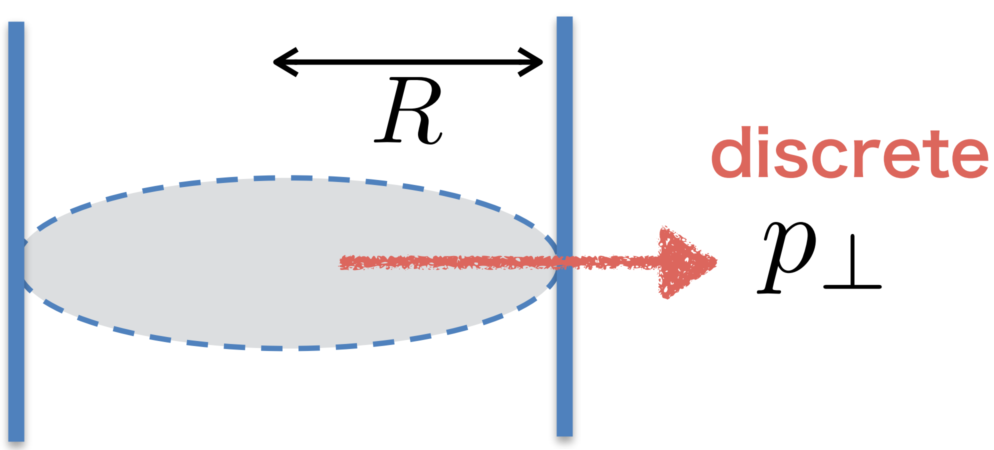
Dirichlet type

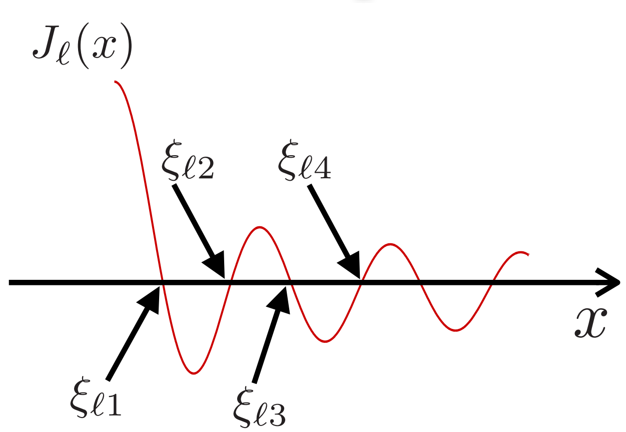
$$\sin(px)|_{x=L} = 0$$

$$\longrightarrow \quad p = \frac{n\pi}{L} \geq \frac{\pi}{L}$$

IR gapped mode

Fermions in a cylinder





NO incoming current

$$\hat{e}_r \bar{\psi} \gamma^r \psi \Big|_{r=R} = 0$$

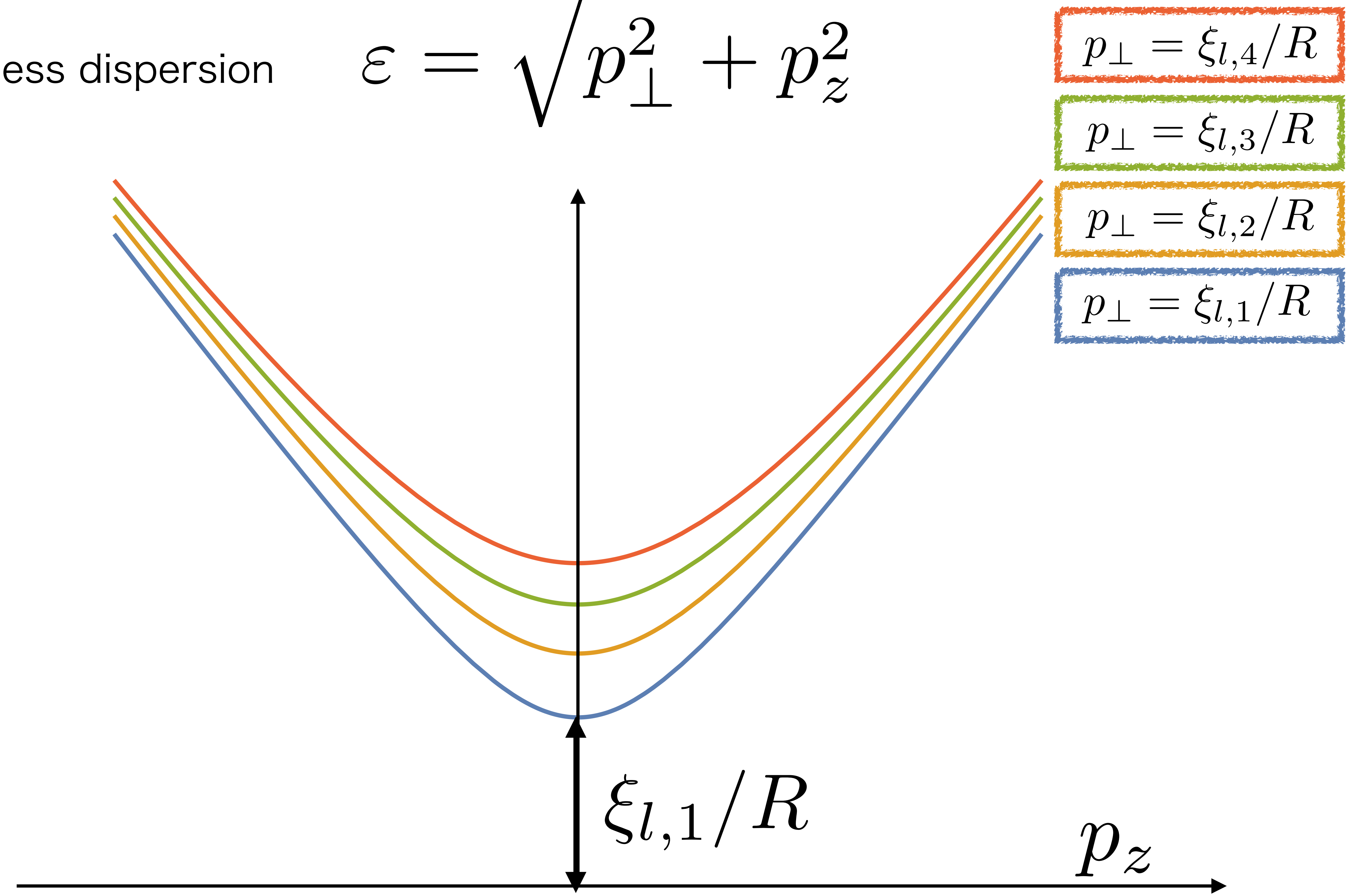
$$\longrightarrow \quad p_\perp = \frac{\xi_{l,k}}{R} \geq \frac{\xi_{l,1}}{R}$$

IR gapped mode

$\xi_{l,k}$: the kth root of $J_l(x)$

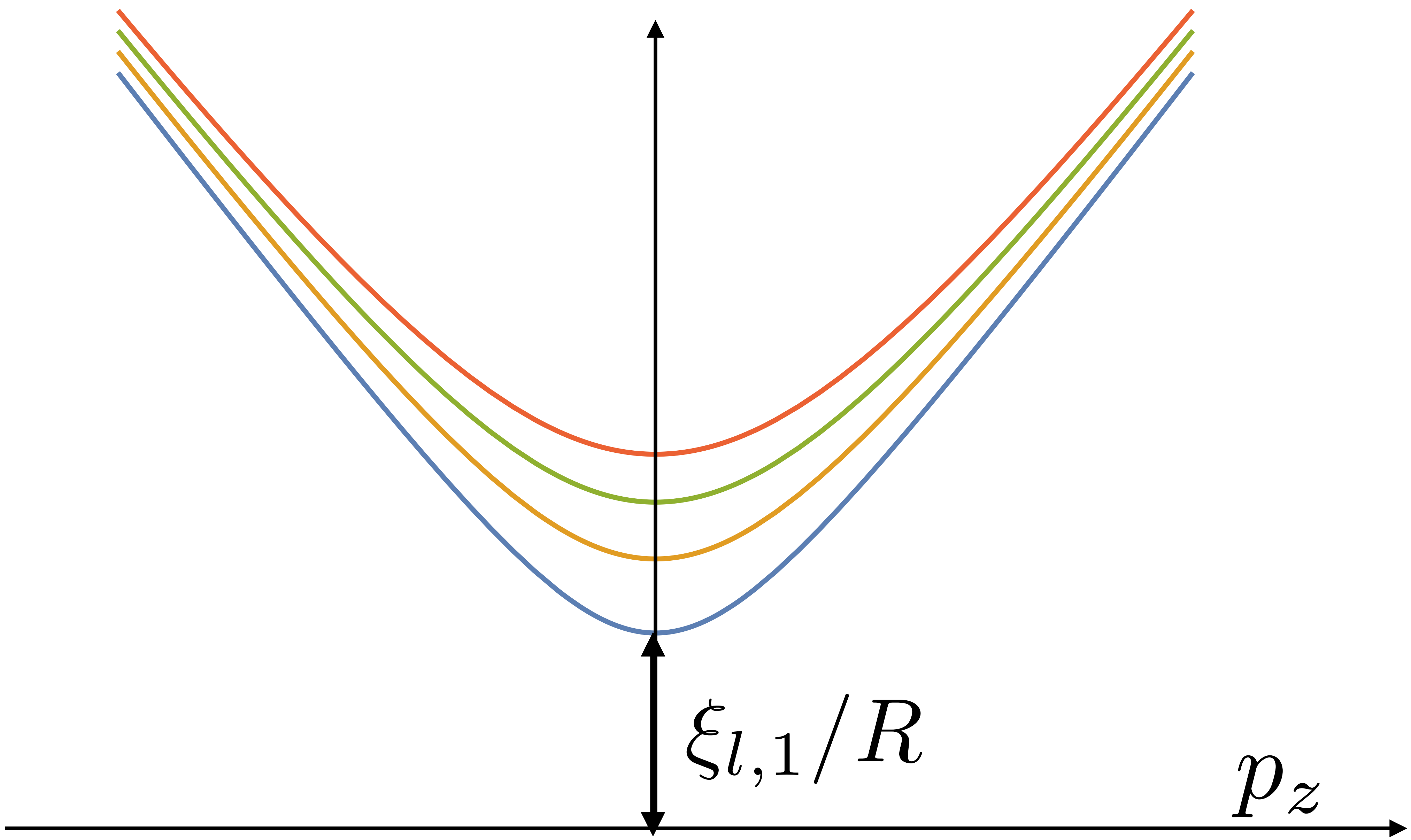
Rotational Effects at T=0

massless dispersion $$\varepsilon = \sqrt{p_\perp^2 + p_z^2}$$



Rotational Effects at T=0

$$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon - \Omega(l+1/2)\}} + 1} \longrightarrow f(\varepsilon) = \theta\big(\Omega(l+1/2) - \varepsilon\big)$$



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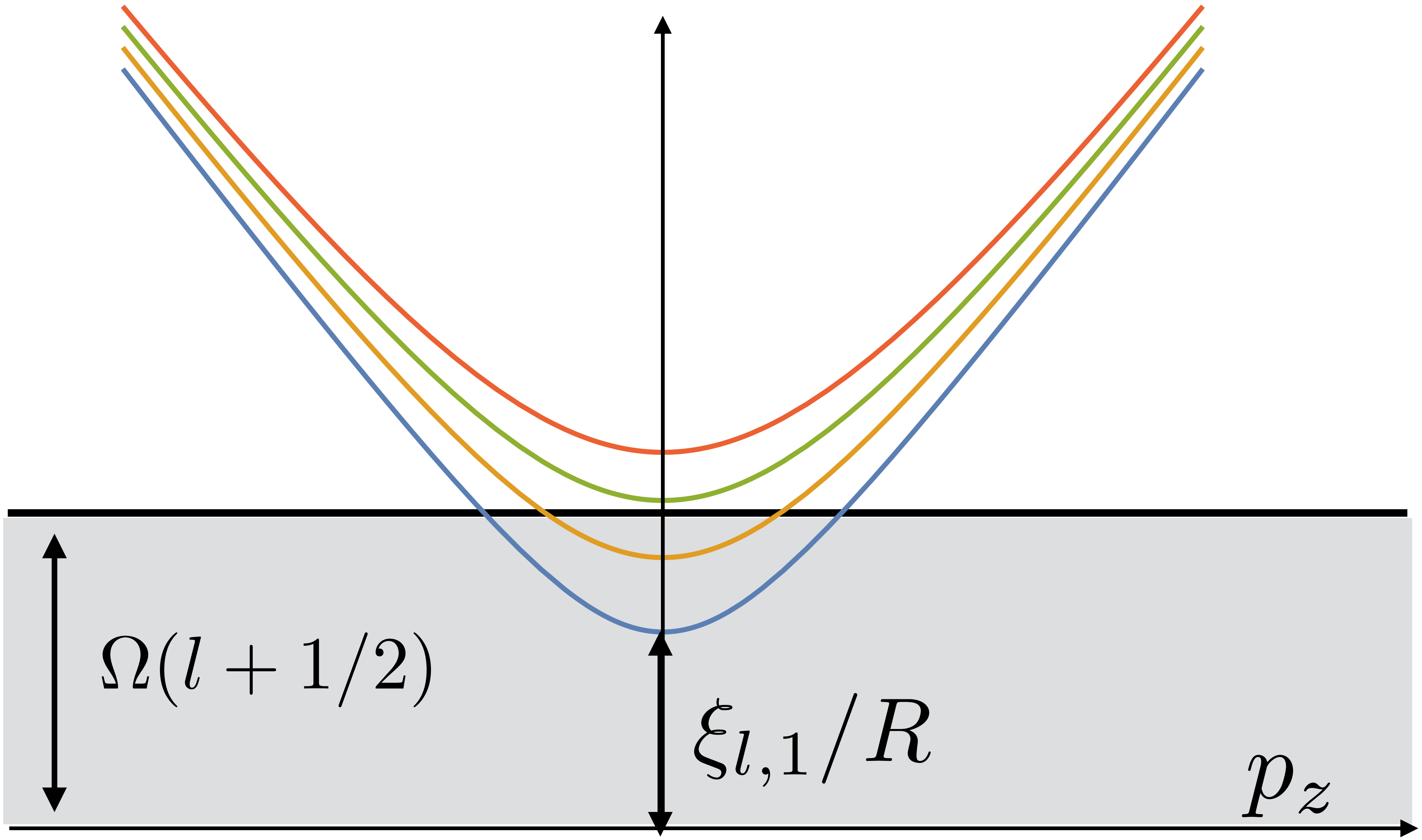
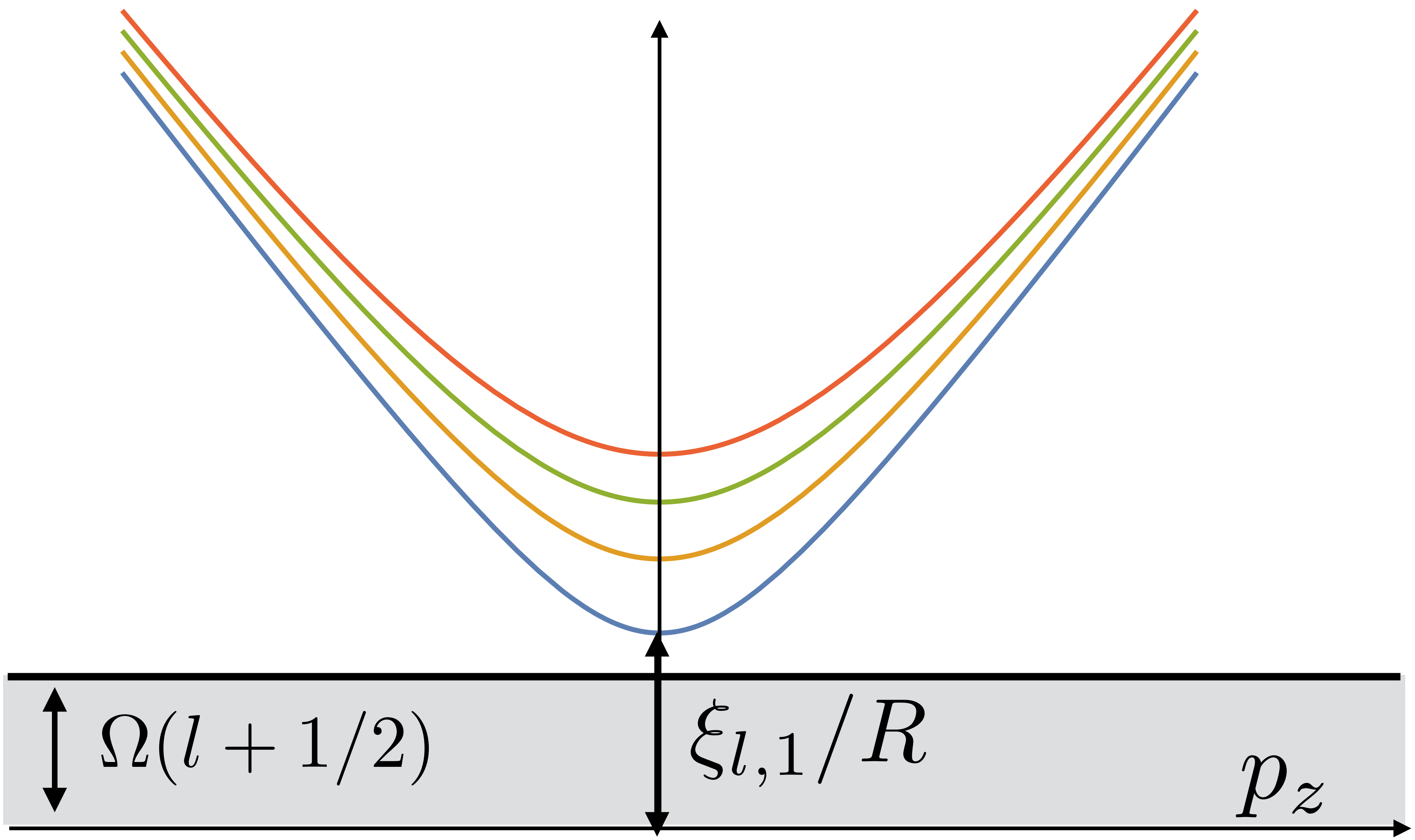


visible

Rotational Effects at T=0

$$f(\varepsilon) = \frac{1}{e^{\beta\{\varepsilon-\Omega(l+1/2)\}}+1} \longrightarrow f(\varepsilon) = \theta\big(\Omega(l+1/2)-\varepsilon\big)$$



invisible

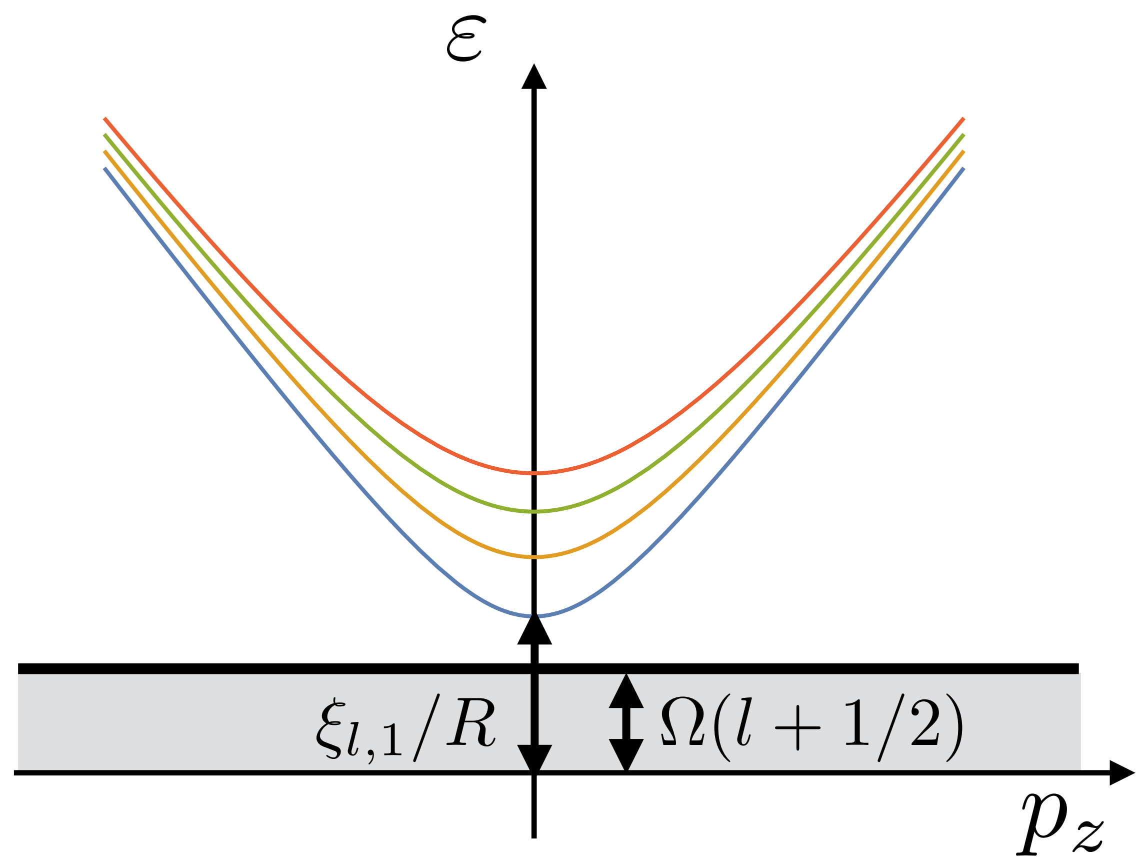
cf. Silver Blaze

Rotational Effects at T=0

One can prove $\xi_{l,1} > l + 1/2$ for arbitrary l Giordano, Laforgia (1983)

causality $1/R \geq \Omega$

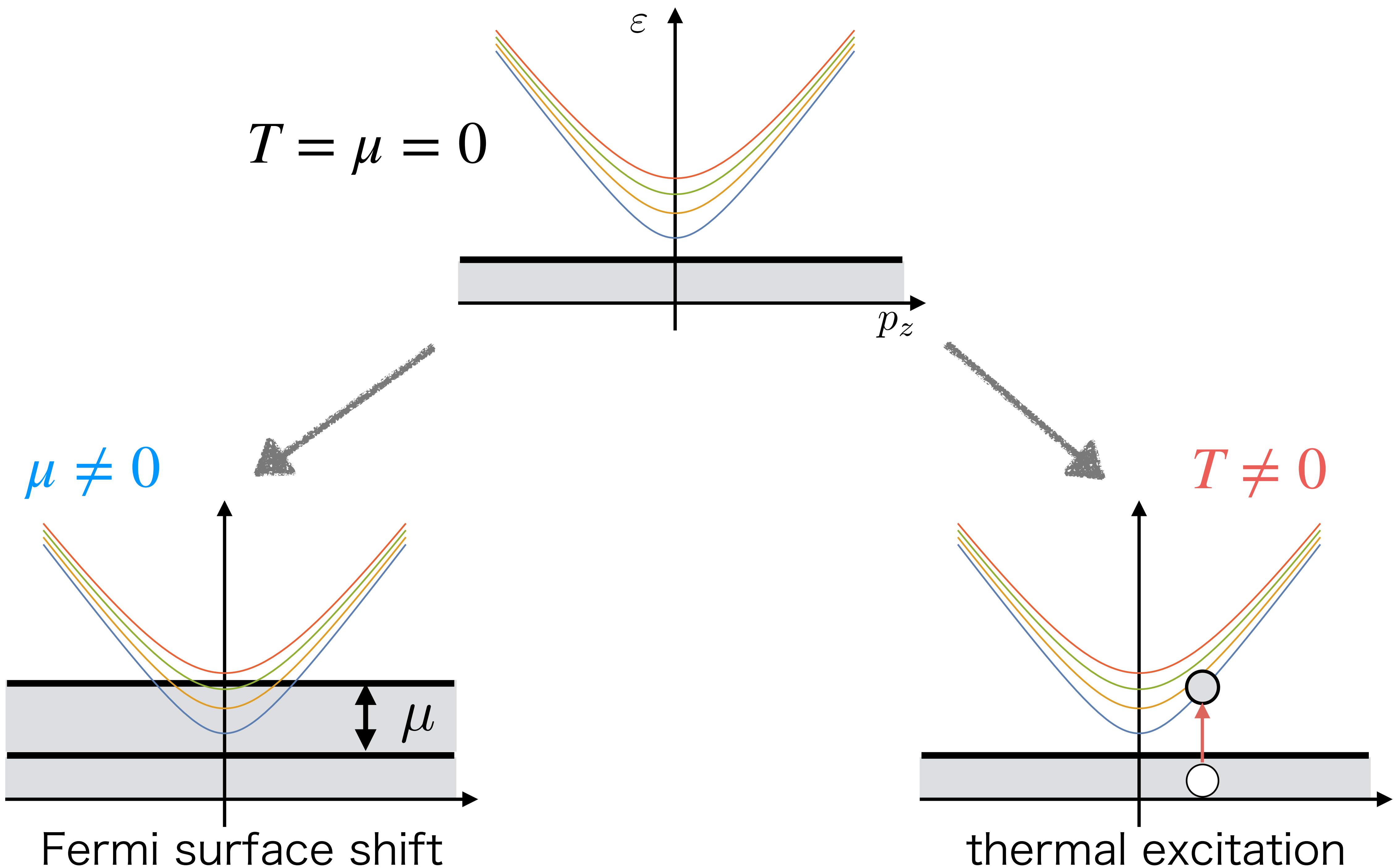
$$\xi_{l,1}/R > \Omega(l + 1/2)$$



NO rotational effect at $T = 0$ due to causality

Ebihara, Fukushima, KM (2017)

Not in Vacuum



ex.) chiral vortical effect
(for Weyl fermion)

$$J = \left(\frac{\mu^2}{4\pi^2} + \frac{T^2}{12} \right) \Omega$$

Vilenkin (1979)

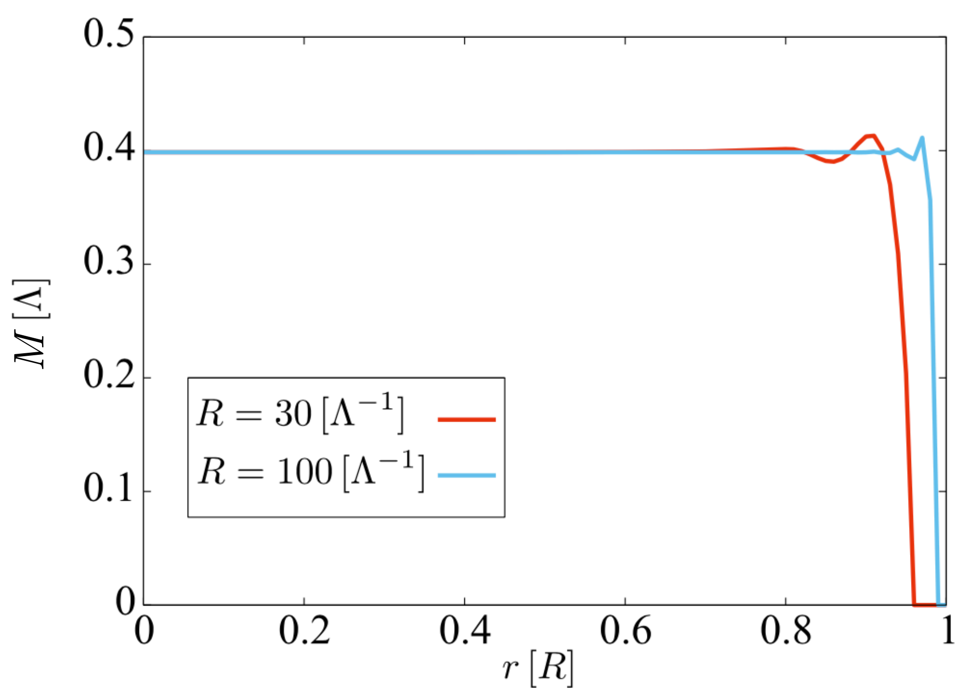
Dynamical Mass

Ebihara, Fukushima, KM (2016)

$$\mathcal{L} = \int d^4x\sqrt{-g}\left[\bar{\psi}\gamma^\mu\left(\partial_\mu + \Gamma_\mu\right)\psi + \frac{G}{2}\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2\right)\right]$$ w/ $|\partial_r M/M| \ll 1$

NJL model + rotation — local density approx.

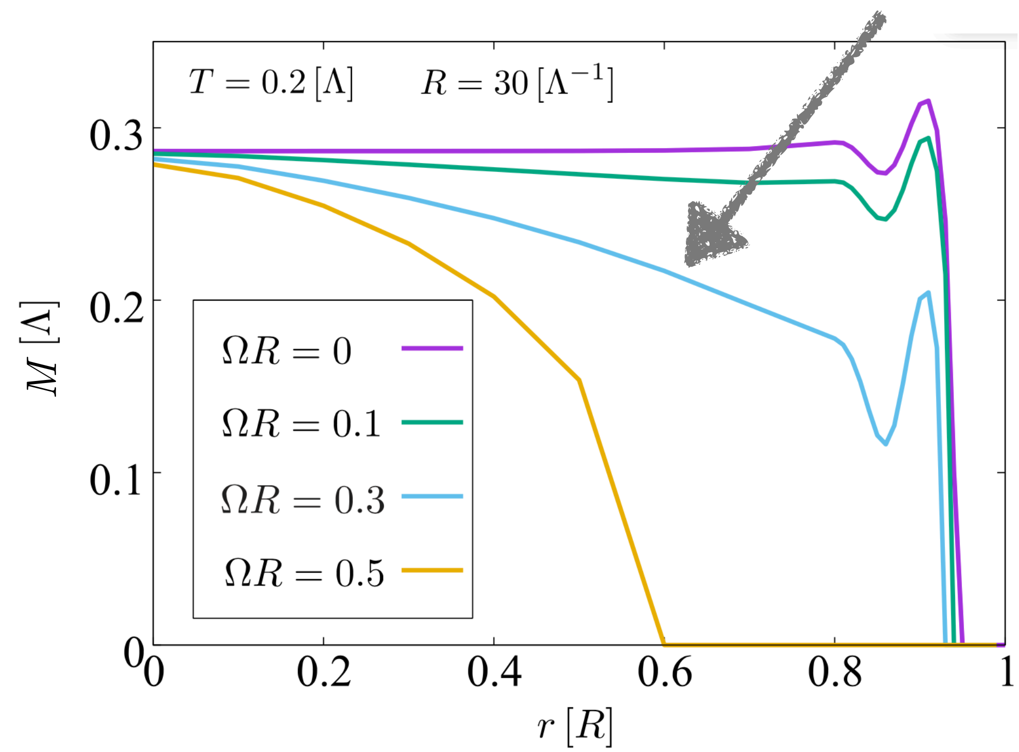
$T = 0$



$R = 30\Lambda^{-1} \simeq 6\,\mathrm{fm}$ for $\Lambda \simeq 1\,\mathrm{GeV}$

$T > 0$

$|\partial_r M|/M^2 \simeq 0.05$



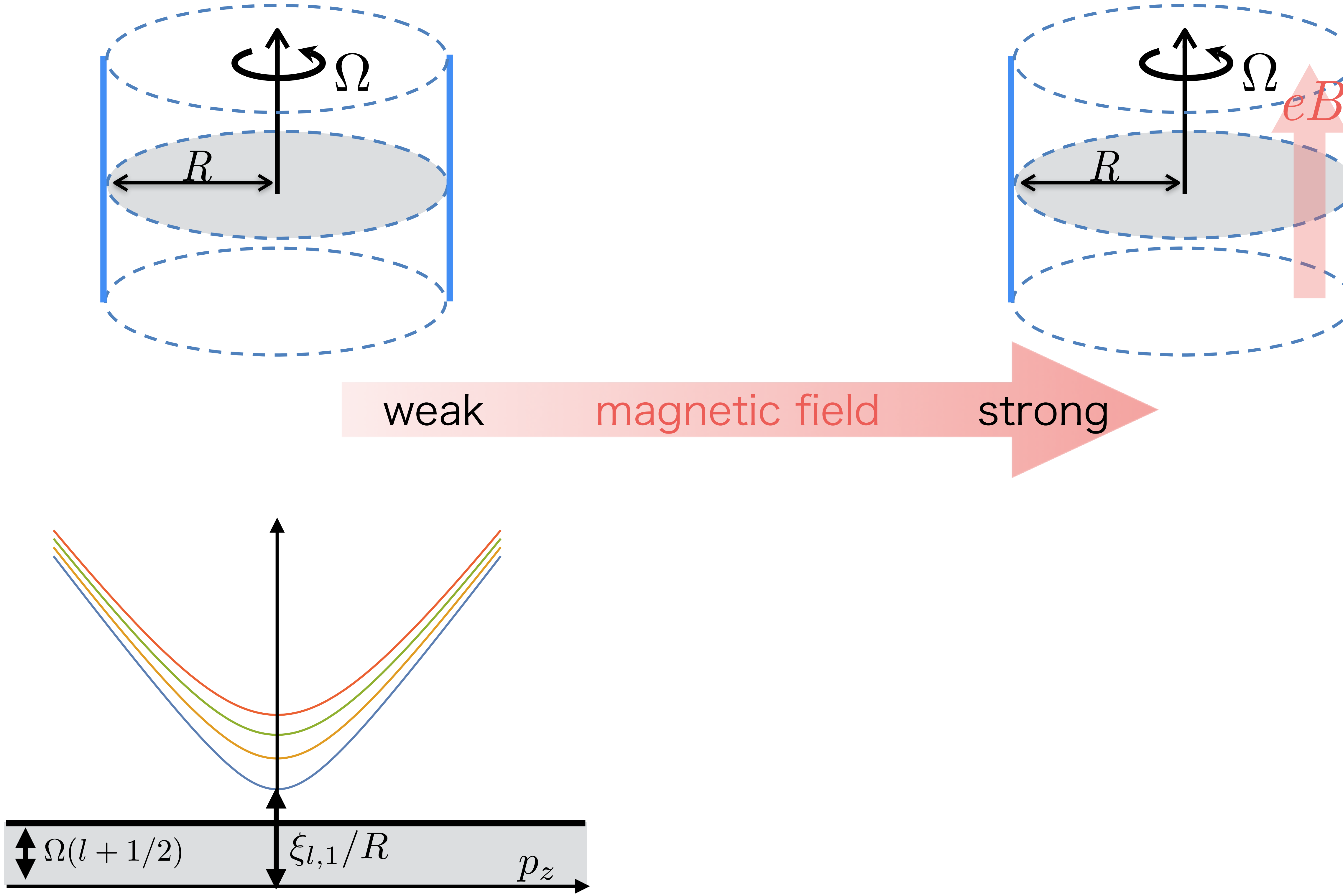
rotational effect ≈ density effect : melt condensate

similar results : Jiang, Liao (2016) Chernodub, Gongyo (2016)

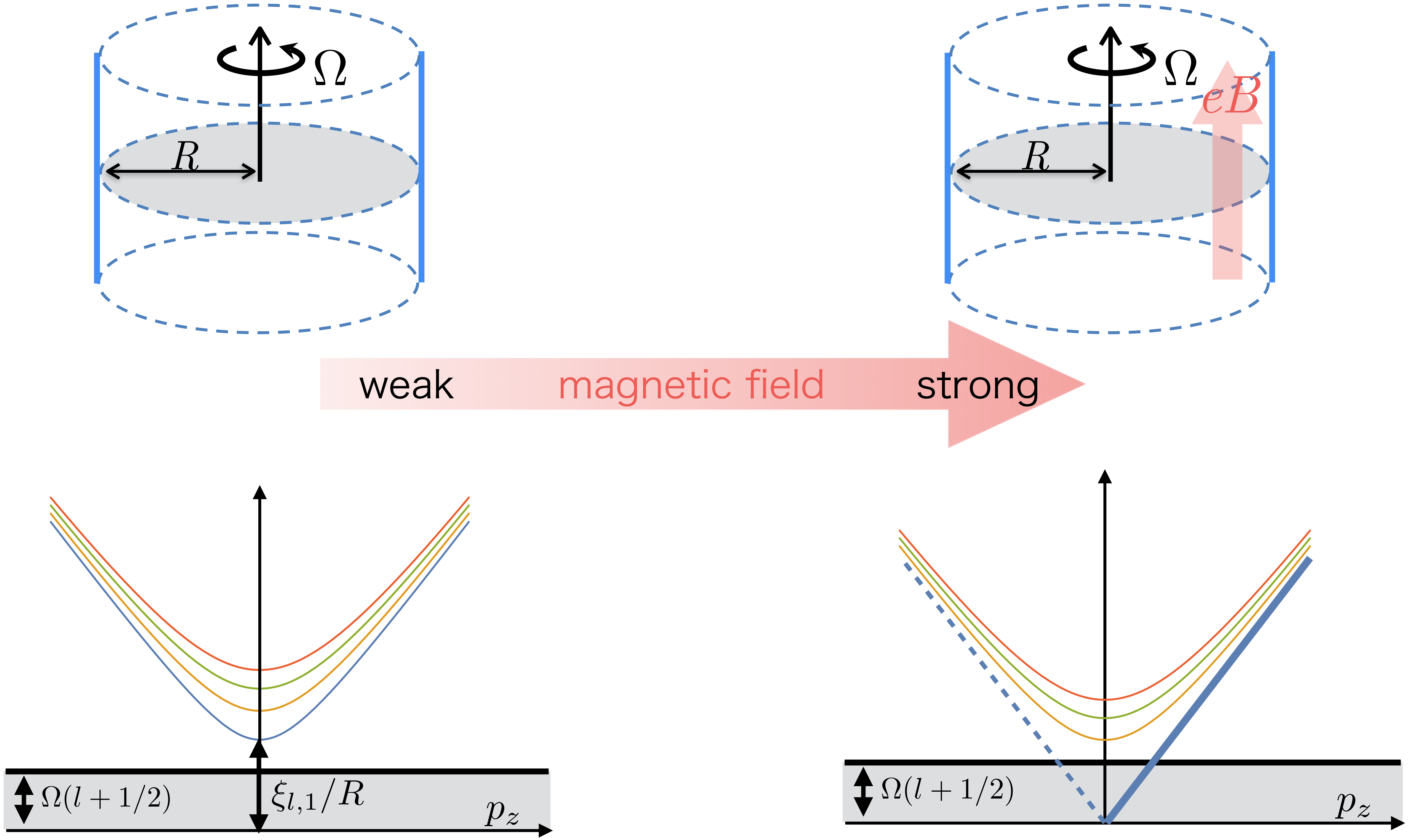
2. Magnetized Fermions in Rotating Cylinder

HL. Chen, K. Fukushima, XG. Huang, KM PRD 96, 054032 (2017)

Gapped to Gapless

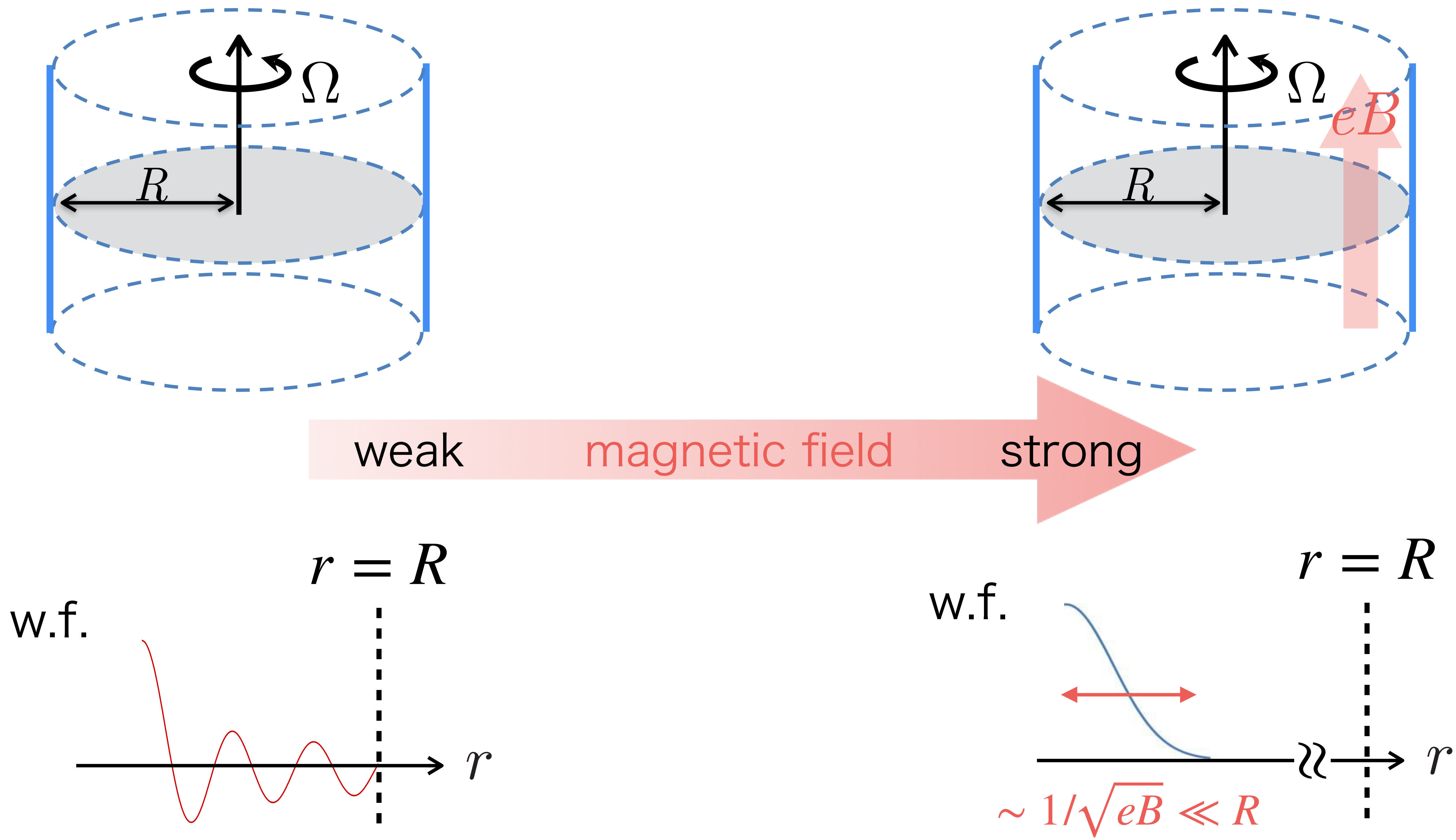


Gapped to Gapless



visible rotational effect

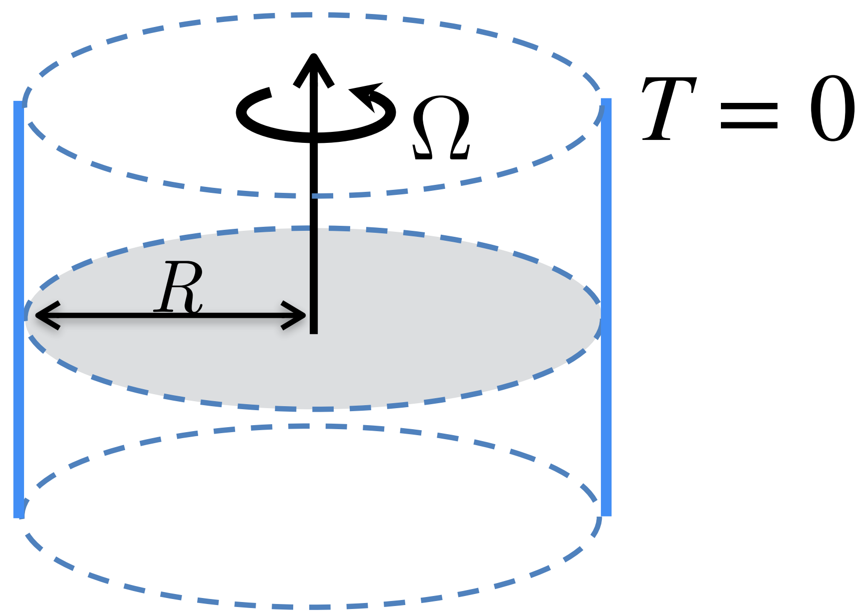
Gapped to Gapless



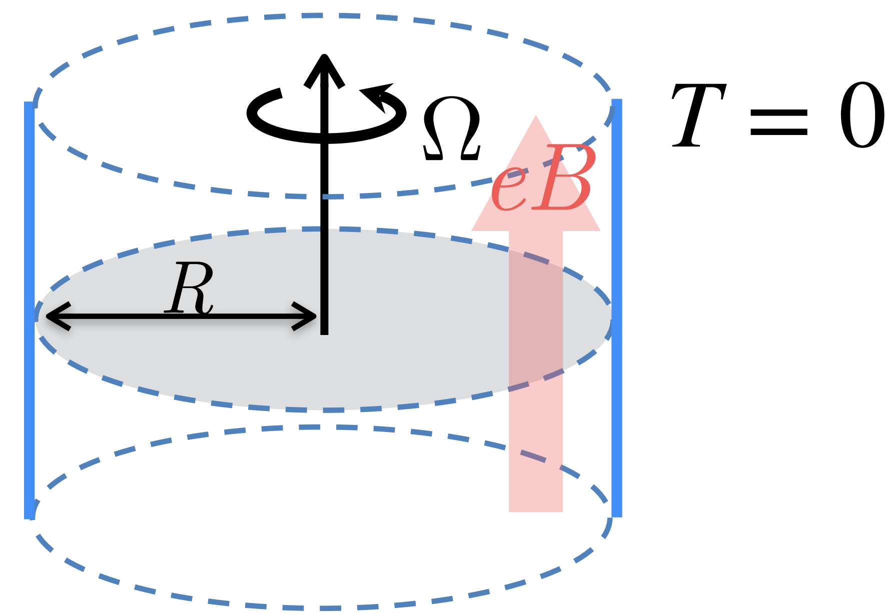
localization ⇒ boundary ignored (causality) ⇒ visible rotational effect

Ex.1) Density Induced by Rotation

rotational effect ≈ density effect



no density due to b.c



finite j^0 ?

$$\mathcal{L} = \int d^4x \sqrt{-g}\, \bar{\psi}\gamma^\mu(\partial_\mu + ieA_\mu + \Gamma_\mu)\psi$$

$$\longrightarrow \quad j^0 = \frac{eB\Omega}{4\pi^2}$$

Hattori, Yin (2016)

Ebihara, Fukushima, KM (2017)

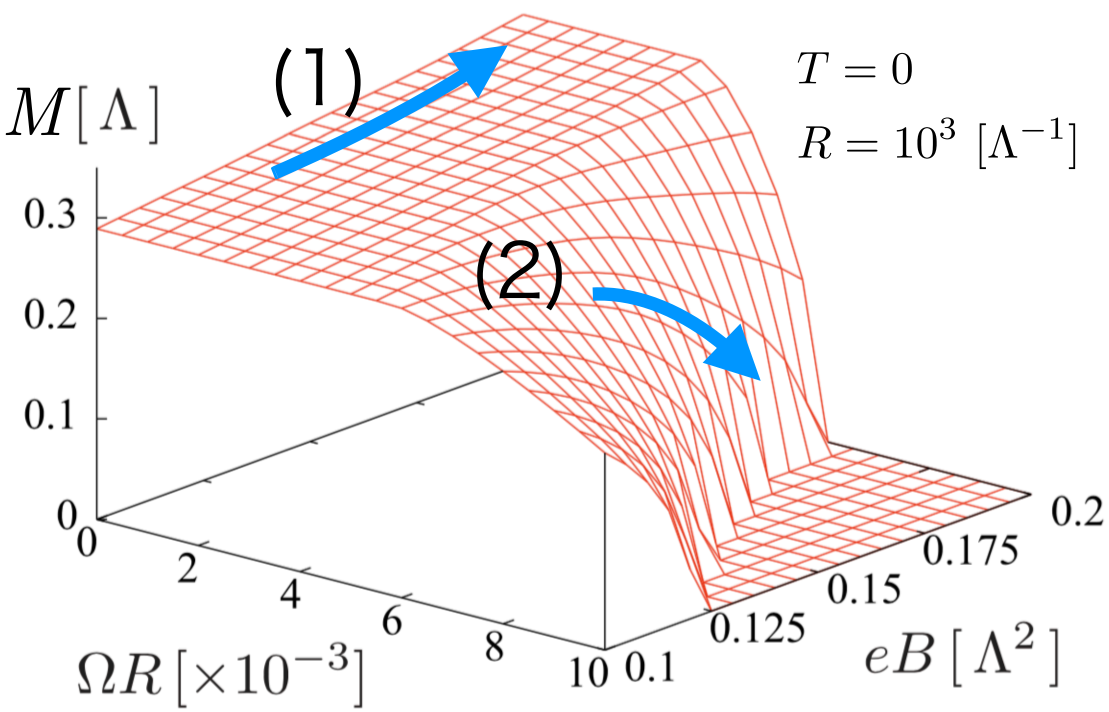
genuine density induced

Ex.2) Chiral Symmetry Breaking

Chen, Huang, Fukushima, KM (2016)

$$\mathcal{L} = \int d^4x \sqrt{-g} \left[\bar{\psi}\gamma^\mu \left(\partial_\mu + ieA_\mu + \Gamma_\mu \right) \psi + \frac{G}{2} \left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\psi)^2 \right) \right]$$

NJL model (mean field approx.) + homogeneity



(1) **magnetic catalysis**

(2) **inverse effect (like at finite μ)**

Ebert, Klimenko (1999)

Preis, Rebhan, Schmitt (2012)

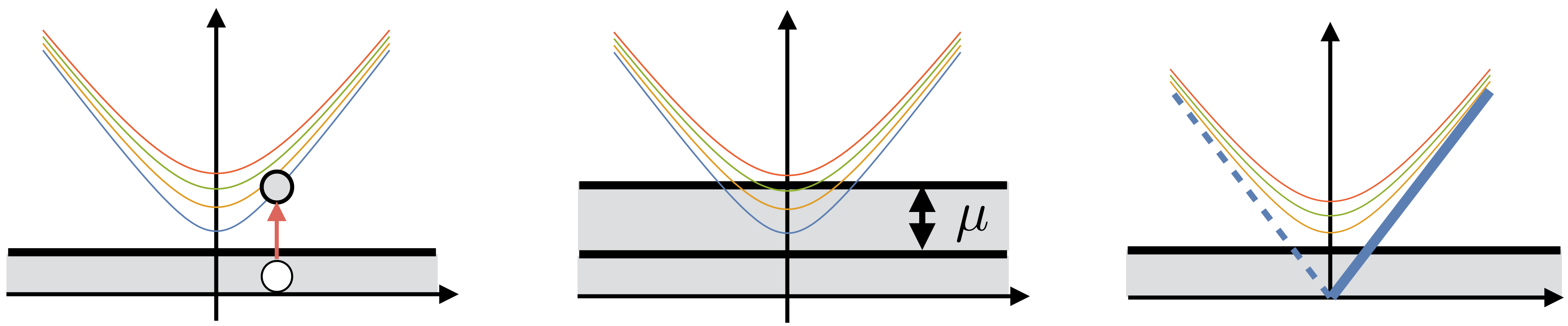
rotational effect ≈ density effect

"rotational magnetic inhibition"

Summary

[1] Rotation vs. Boundary effect

Visible rotational effect requires



- **temperature**
- **density**
- **magnetic field**

[2] Rotation yields abundant phase structures

rotational magnetic inhibition
(first demonstration of rotational effect)

chiral restoration
Jiang, Liao (2016)
Chernodub, Gongyo (2016)

neutral pion superfluid
Huang, Nishimura, Yamamoto (2017)

charged pion condensate?
Liu, Zahed (2017)
Cao, He (2019)/ Chen, Huang, KM (2019)

→ heavy-ion/neutron star?