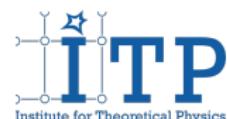


# Pre-clustering near the QCD critical point: nuclear correlations and light-nuclei production



Juan M. Torres-Rincon  
(Goethe University Frankfurt)



in collaboration with  
E. Shuryak (Stony Brook Uni.)

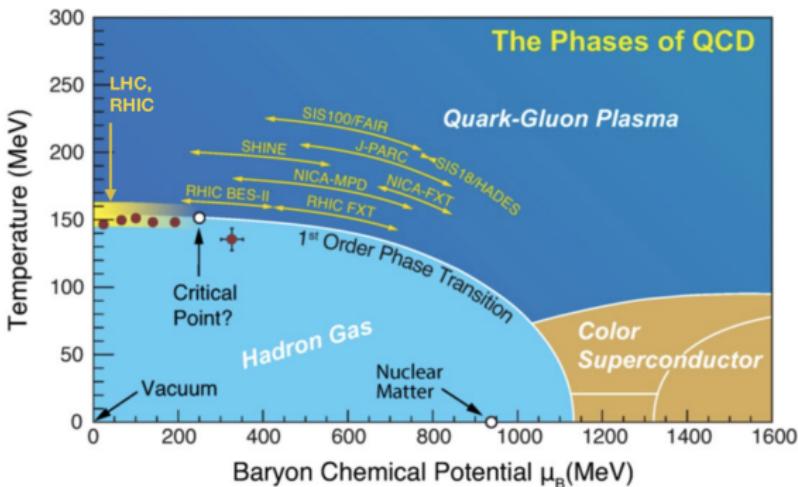


XXXII International (ONLINE) Workshop on High Energy Physics  
Protvino, Moscow region. Nov. 13, 2020

**HOT PROBLEMS OF STRONG INTERACTIONS**

# QCD phase diagram and critical point

from Bazavov et al., 1904.09951



- Hypothetical QCD critical point in baryon-rich region
- Lattice-QCD favors  $\mu_B/T > 2 - 3$  (e.g. Bazavov et al., PRD95, 054504 (2017).  
 $\mu_B/T = 4 - 5$  in DSE/FRG (C. Fischer and J. Pawłowski talks)
- Nucleon physics relevant in this region → Net-proton observables

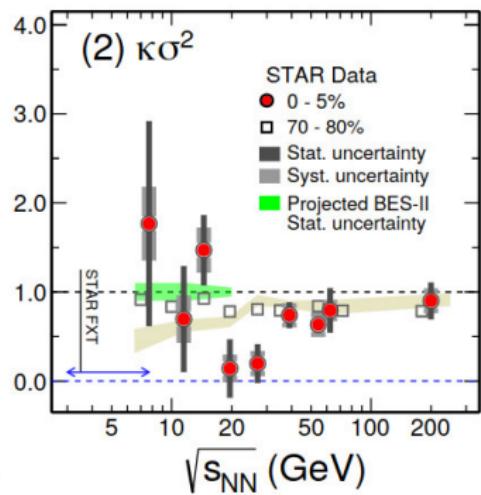
# Net-proton kurtosis

$$C_2 = \langle \delta N_{p-\bar{p}}^2 \rangle$$

$$C_4 = \langle \delta N_{p-\bar{p}}^4 \rangle - 3\langle \delta N_{p-\bar{p}}^2 \rangle^2$$

$$\kappa\sigma^2 = C_4/C_2$$

- Proton-proton interaction contained in cumulants of (net) proton distribution



STAR Collab.,  
2001.02852v2

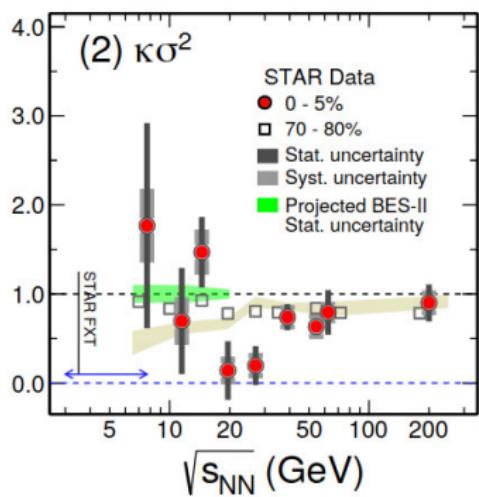
Au+Au Collisions  
 $0.4 < p_T < 2.0 \text{ (GeV/c), } |y| < 0.5$

# Net-proton kurtosis

$$C_2 = \langle \delta N_{p-\bar{p}}^2 \rangle$$

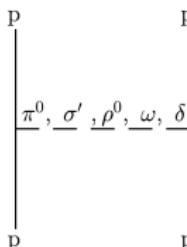
$$C_4 = \langle \delta N_{p-\bar{p}}^4 \rangle - 3\langle \delta N_{p-\bar{p}}^2 \rangle^2$$

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STAR Collab., Au+Au Collisions  
2001.02852v2  $0.4 < p_T < 2.0$  (GeV/c),  $|y| < 0.5$

- Proton-proton interaction contained in cumulants of (net) proton distribution
- Baryon interaction potential  $V_{NN}$  modeled via boson exchange



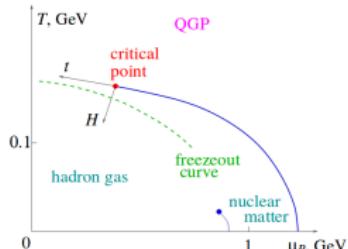
(from Machleidt and Slas, J.Phys.G:Nucl.Part.Phys.27 R69)

- $T \simeq m_0$ , in-medium modifications
- Main effect of critical point on  $V_{NN}$ ?

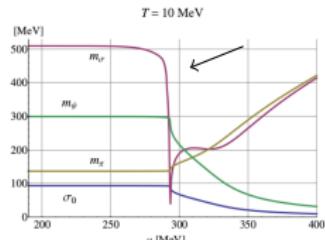
# QCD critical mode

$\sigma$  mass drops at critical point

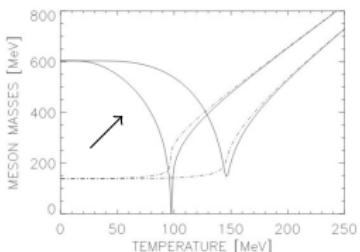
$$m_\sigma \sim \frac{1}{\xi} \sim \left( \frac{|T - T_c|}{T_c} \right)^\nu$$



Stephanov, *PRL* 102 (2009)  
032301, *PRL* 107 (2011) 052301



Tripolt et al., *PRD* 89 (2014), 034010



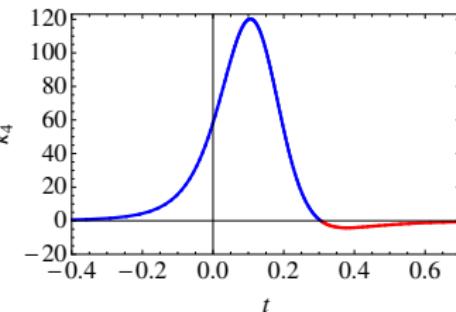
Scavenius et al., *PRC* 64 (2001) 045202

$$P[\sigma] \sim \exp(-\Omega/T)$$

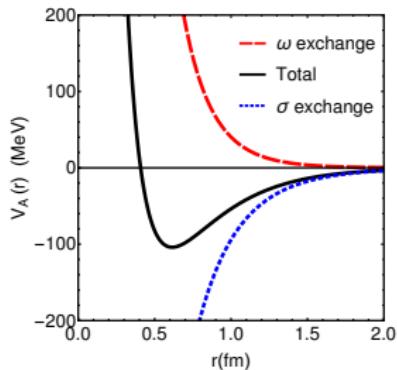
$$\Omega = \int d^3x \frac{(\nabla\sigma)^2}{2} + \frac{m_\sigma^2}{2}\sigma^2 + \frac{\lambda_3}{3}\sigma^3 + \frac{\lambda_4}{4}\sigma^4$$

$$\kappa_2 = \langle \sigma_0^2 \rangle \quad , \quad \kappa_4 = \langle \sigma_0^4 \rangle - 3\langle \sigma_0^2 \rangle^2$$

$$\text{Kurtosis} = \kappa_4 / \kappa_2^2$$



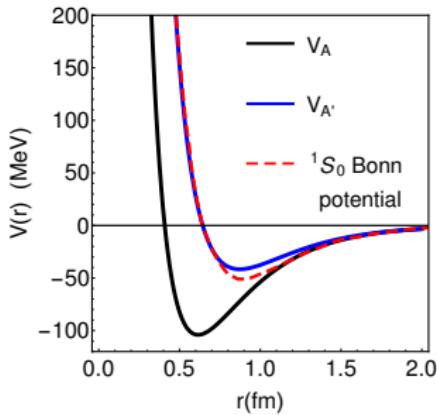
Serot-Walecka model for NN interaction (*Adv.Nucl.Phys.* 16 (1986) 1)



- $\sigma$  controls attraction
- Cancellation between attraction and repulsion leads to bound nuclear matter
- Small imbalance would substantially modify physics

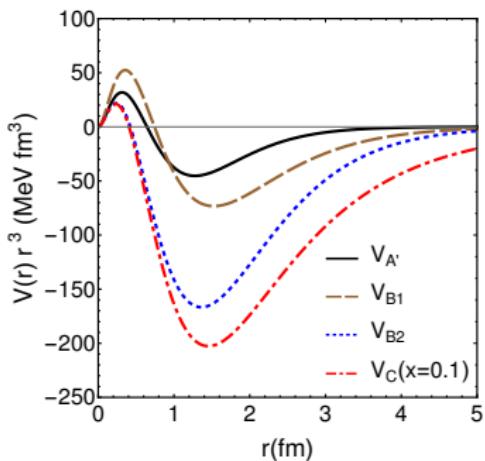
$$V_A(r) = -\frac{\alpha_\sigma}{r} e^{-m_\sigma r} + \frac{\alpha_\omega}{r} e^{-m_\omega r}$$

What happens to  $V_{NN}$  around  $T_c$ ?



Shuryak and JMT-R, PRC 100 (2019), 024903, PRC 101 (2020) 034914

- Decrease of  $m_\sigma$  not compensated by repulsion
- Net nuclear attraction generates proton-proton correlations



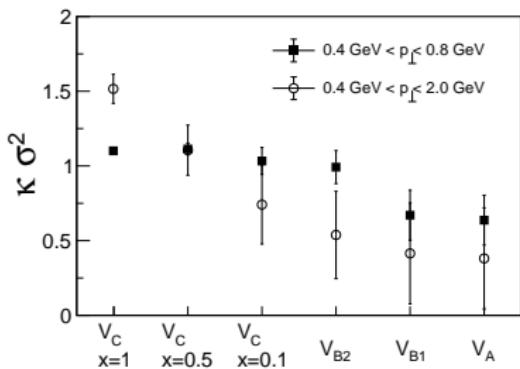
- $V_{A'}$ : Serot-Walecka with extra repulsion ( $\sim$  Bonn potential)
- $V_{B1}$ :  $\{m_\sigma^2, \alpha_\sigma\} = \{m_{\sigma,0}^2/2, \alpha_{\sigma,0}/2\}$
- $V_{B2}$ :  $m_\sigma^2 = m_{\sigma,0}^2/2$
- $V_C$ :  $m_\sigma^2 \in (m_{\sigma,0}^2/2, m_{\sigma,0}^2/6)$

## Implications in dynamics?

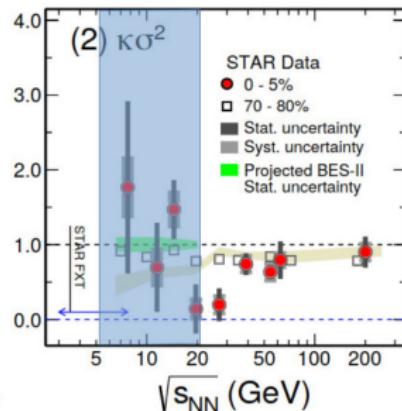
Caveat very close to  $T_c$ :  $\sigma$  self-interaction can lead to repulsion (DeMartini, Shuryak, 2010.02785)

# Higher-order moments

- Molecular Dynamics of a nucleon gas in thermal equilibrium (Shuryak and JMT-R, *PRC100* (2019), 024903)
- Mimicking STAR freeze-out conditions for  $\sqrt{s_{NN}} < 19.6$  GeV



Shuryak, JMT-R, *PRC100* (2019), 024903



STAR Collab., 2001.02852v2

Results of **simple** model same order as STAR data (also skewness).  
**STAR data might be showing us effects of modified potential**

# Pre-cluster vs nuclei

## Pre-clusters

- Formed by strong nuclear correlations (modified  $NN$  potential) until freeze-out.
- $T \sim 120$  MeV. Mostly classical states.
- Broad ( $\Delta E \sim T$ ) states with characterized by Wigner distribution

## Nuclei

- Final state, measured by experiment.
- $T \simeq 0$ . Quantum states. No critical modification.
- Characterized by wave function  $\Psi_{\text{nuclei}}$

Pre-clusters can decay into ground state, into stable products or survive as excited states until final decay.

$E$ (MeV)	$J^P$	$\Gamma$ (MeV)	decay modes, in %
20.21	$0^+$	0.50	$p = 100$
21.01	$0^-$	0.84	$n = 24, p = 76$
21.84	$2^-$	2.01	$n = 37, p = 63$
23.33	$2^-$	5.01	$n = 47, p = 53$
23.64	$1^-$	6.20	$n = 45, p = 55$
24.25	$1^-$	6.10	$n = 47, p = 50, d = 3$
25.28	$0^-$	7.97	$n = 48, p = 52$
25.95	$1^-$	12.66	$n = 48, p = 52$
27.42	$2^+$	8.69	$n = 3, p = 3, d = 94$
28.31	$1^+$	9.89	$n = 47, p = 48, d = 5$
28.37	$1^-$	3.92	$n = 2, p = 2, d = 96$
28.39	$2^-$	8.75	$n = 0.2, p = 0.2, d = 99.6$
28.64	$0^-$	4.89	$d = 100$
28.67	$2^+$	3.78	$d = 100$
29.89	$2^+$	9.72	$n = 0.4, p = 0.4, d = 99.2$

Excited states of  ${}^4\text{He}$   
([www.nndc.bnl.gov/nudat2/](http://www.nndc.bnl.gov/nudat2/))

# Excited states of Helium-4

$E$ (MeV)	$J^P$	$\Gamma$ (MeV)	decay modes, in %
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- **Statistical thermal model:** all states equally populated at finite  $T$
- They necessarily account for feed-down in  $t, d, p$  yields
- Practical implementation by Vovchenko et al. *PLB 809 (2020) 135746* using Thermal-FIST: 5-10 % effect at RHIC/SPS and  $\sim 60$  % effect GSI/FAIR for final  $t, {}^3He, {}^4He$

Feed down particularly important if  $V_{NN}$  is modified

How to observe the effect when overall multiplicity is low?

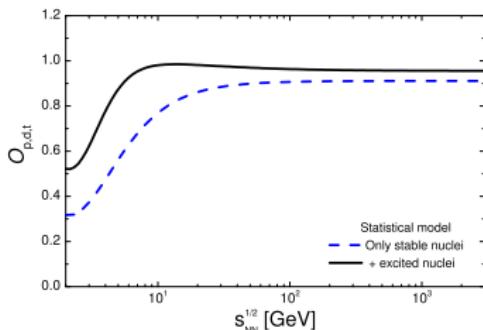
# Pre-clusters to light nuclei: Triton-proton/deuteron ratio

$$\frac{N_t N_p}{N_d^2} = g \quad (g = 0.29)$$

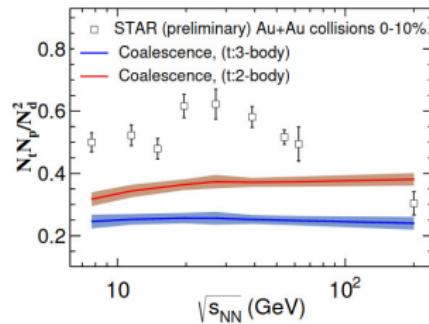
- Assumption: **Statistical thermal model** provides a fair description

$$N = \mathcal{V} \frac{(2S+1)}{2\pi^2} m^2 T K_2(m/T) \exp\left(\frac{B\mu_B + q\mu_q}{T}\right)$$

- Feed down corrections are also important
- Ratio introduced by Sun et al. *PLB 774, 103 (2017)* in coalescence model



Vovchenko et al., *PLB 809 (2020)  
135746*



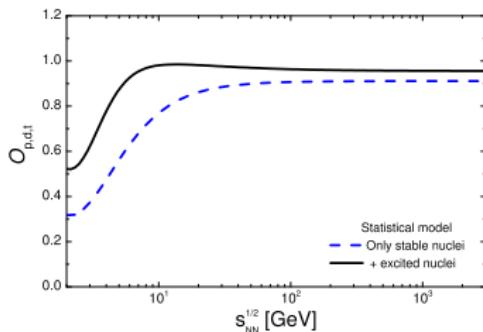
Zhao et al., *PRC 102, 044912  
(2020)*

# Triton-proton/deuteron ratio

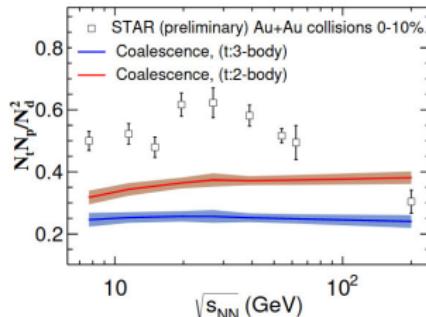
Non-ideal effects

$$\frac{N_t N_p}{N_d^2} \simeq g \frac{\langle \exp \left( -\frac{3V_{NN}(r)}{T} \right) \rangle}{\langle \exp \left( -\frac{V_{NN}(r)}{T} \right) \rangle^2} \sim g \left\langle e^{-\frac{V_{NN}(r)}{T}} \right\rangle$$

If  $|V_{NN}| \simeq T$  close to the critical point, then a nonmonotonous behavior for the ratio is to be expected (enhancement factor  $e$ )



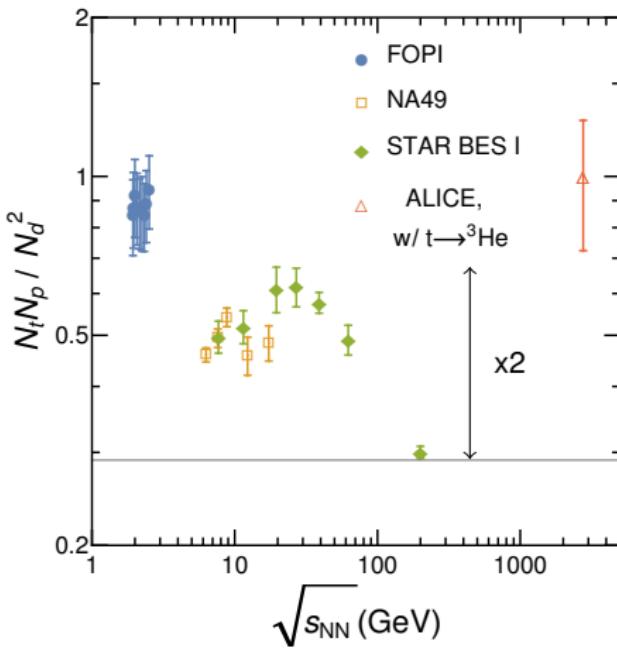
Vovchenko et al. *PLB 809 (2020)*  
135746



Zhao et al., *PRC 102, 044912 (2020)*

# Triton-proton/deuteron ratio

Some experimental data:



- FOPI, total yields,  $4\pi$  reconstruction, Au+Au collisions, most central [NPA848, (2010) 366-427]
- STAR BES-I, preliminary 0%-10% Au+Au, ratios based on extrapolated yields at midrapidity (arXiv:1909.07028)
- Sun et al. PLB 774, 103 (2017) based on NA49 data.  $dN/dy$  at midrapidity, Pb+Pb central (typically 0-7%)
- ALICE, Pb+Pb @  $\sqrt{s_{NN}} = 2.76$  TeV (data from several sources).  $dN/dy$  at midrapidity ( ${}^3\text{He}$  used instead triton)

## New light-nuclei yield ratios

Ratios involving  ${}^4\text{He}$  ( $=\alpha$ ) with higher global powers of  $e^{|V(r)/T|}$

$$\mathcal{O}_2 \equiv \frac{N_\alpha N_p}{N_{^3\text{He}} N_d} \simeq 0.18 \frac{\langle e^{-6V(r)/T} \rangle}{\langle e^{-3V(r)/T} \rangle \langle e^{-V(r)/T} \rangle}$$

$$\mathcal{O}_3 \equiv \frac{N_\alpha N_t N_p^2}{N_{^3\text{He}} N_d^3} \simeq 0.05 \frac{\langle e^{-6V(r)/T} \rangle}{\langle e^{-V(r)/T} \rangle^3}$$

(in practice,  $N_t \simeq N_{^3\text{He}}$ )

(Shuryak, JMT-R, *PRC 101 (2020) 034914*)

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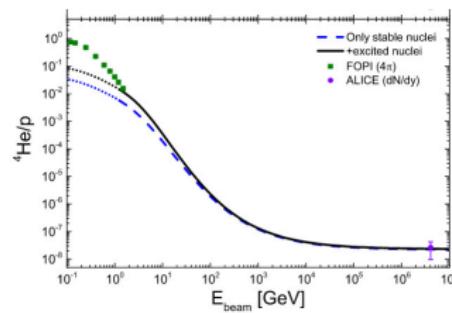
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(in practice,  $N_t \simeq N_{^3\text{He}}$ )

(Shuryak, JMT-R, *PRC 101 (2020) 034914*)

${}^4\text{He}$  production not negligible at low energies  
(Vovchenko et al. *PLB 809 (2020) 135746*)

Data exist at very low (FOPI) and high energies (ALICE)

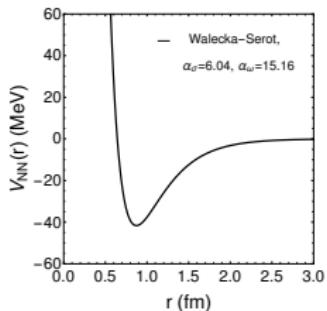


# Flucton solution for 2 particles

## Euclidean action

$$S_E = \int d\tau \left[ \frac{m_N}{2} (\dot{x}_1^2 + \dot{x}_2^2) + V_{NN}(|\mathbf{x}_1 - \mathbf{x}_2|) \right]$$

$$= \int d\tau \left( \frac{m_N}{4} \dot{r}^2 + V_{NN}(r) \right) + C.M.$$



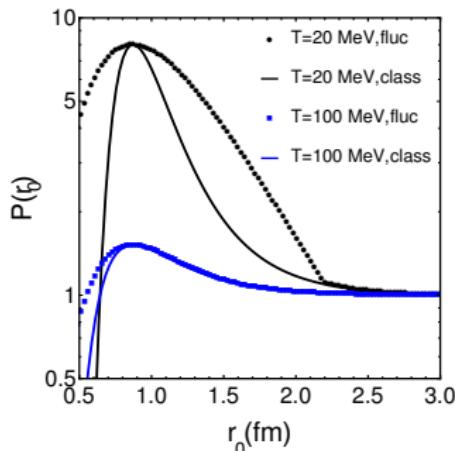
- 1 Experimental  $N_t N_p / N_d^2$  ratio fixes  $V_{NN}/T(r, m_\sigma, T)$  via flucton method
- 2 Spatial averages using

$$\langle A \rangle \equiv \frac{4\pi \int dr r^2 A(r)[P(r) - 1]}{4\pi \int dr r^2 [P(r) - 1]}$$

- 3 New ratios can be computed with calibrated  $V_{NN}/T(r, m_\sigma, T)$

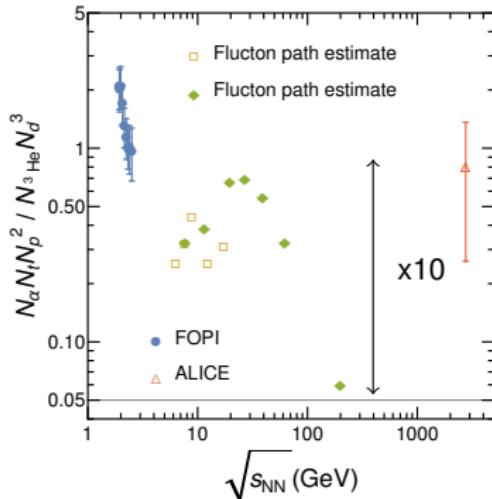
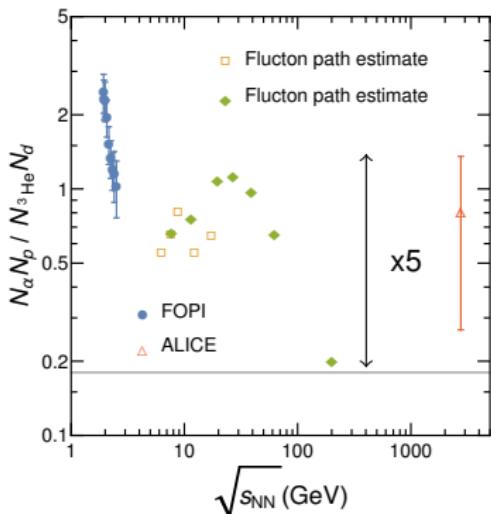
Shuryak, *NPB 302 (1988) 621*  
 Shuryak and JMT-R, *EPJA 56 (2020) 9, 24*

$$P(r_0) = e^{-S_E[r_{fluc}(\tau, r_0)]}; P_{class}(r_0) = e^{-V(r_0)/T}$$



# New light-nuclei yield ratios

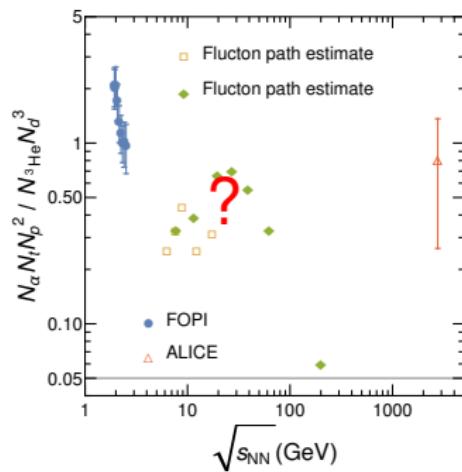
Shuryak and JMT-R, EPJA 56 (2020) 9, 24



- ${}^4\text{He}$  enhances the expected effect a factor 5 wrt  $N_t N_p / N_d^2$
- Good observable for testing nuclear modification
- **We need  ${}^4\text{He}$  measurement at intermediate energies!**

# Summary

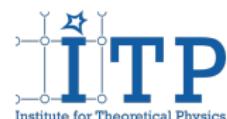
- Significant attractive and long-ranged  $NN$  potential near  $T_c$
- Increased correlations among nucleons  
Affect proton distribution probability and its cumulants
- Formation of pre-clusters (statistical correlations of nucleons)  
Generation and later decay of  ${}^4\text{He}$  (and other) excited states
- Possible enhanced production of light nuclei at “critical  $\sqrt{s_{NN}}$ ”  
Light-nuclei yield ratios using  ${}^4\text{He}$  to test the proposed effect



# Pre-clustering near the QCD critical point: nuclear correlations and light-nuclei production



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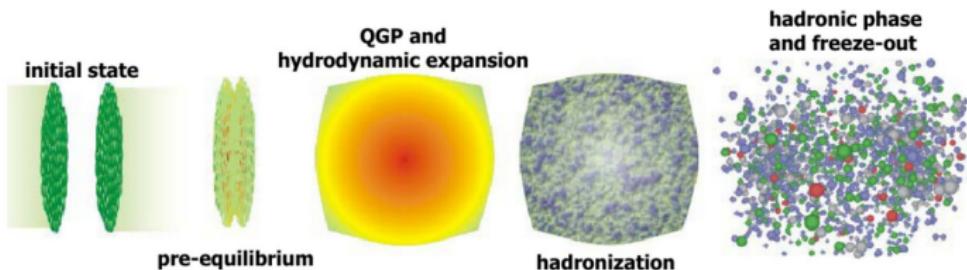
**HOT PROBLEMS OF STRONG INTERACTIONS**

# Approaching the physical case

## Effects preventing clustering

- Expansion, radial collective flow
- Freeze-out temperatures  $T \sim 150$  MeV
- Finite time effects (duration of hadronic phase)

We need to address these for RHIC collisions at the Beam Energy Scan



Focus on BES I at  $\sqrt{s_{NN}} < 19.6$  GeV, as measured by STAR @ RHIC (STAR Collab. 2016 & 2017)

We try to mimic as much as possible experimental situation in BES I, as measured by STAR @ RHIC (STAR Collab. 2016 & 2017)

- Temperature  $T \simeq 150$  MeV
- Densities: 1-2  $n_0$
- Finite time evolution:  $t = 5$  fm
- Non-relativistic nucleon dynamics
- Fireball expansion: mapping of  $y$  and  $p_T$  distributions to experimental measured distributions
- Simulations: 32 nucleons,  $10^5$  events (similar to experiment for 5% most central events)
- Antinucleons: For  $\sqrt{s_{NN}} < 19.6$  GeV they are suppressed, at least, a factor of 10 w.r.t. protons

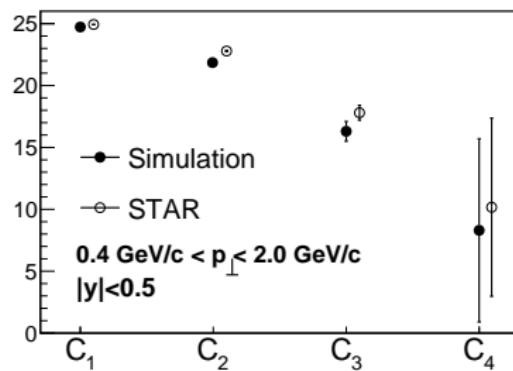
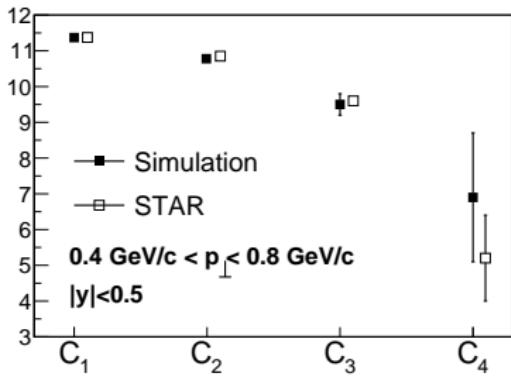
**Note:** It is a crude model and several effects not covered.  
Understand as a first approximation to the physical situation.

# Calibration at $\sqrt{s_{NN}} = 19.6$ GeV

Poisson distribution at  $\sqrt{s_{NN}} = 19.6$  GeV  $\leftrightarrow$  Noncritical potential  $V_{A'}$

- $|y| < 0.5, \quad 0.4 \text{ GeV}/c < p_\perp < 0.8 \text{ GeV}/c$
- $|y| < 0.5, \quad 0.4 \text{ GeV}/c < p_\perp < 2 \text{ GeV}/c$

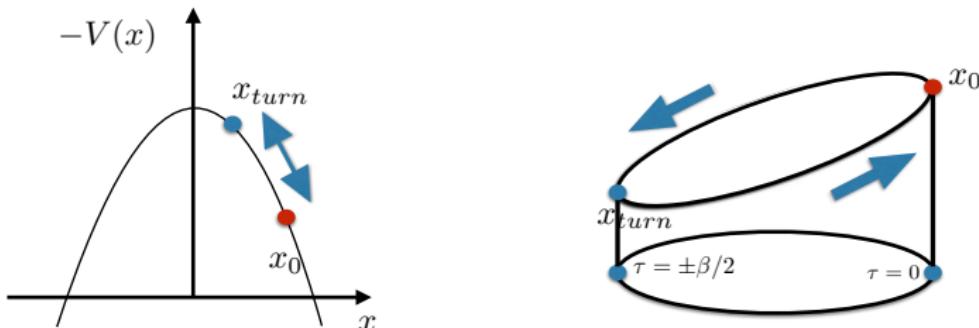
protons



$$C_1 = \langle N_p \rangle, \quad C_2 = \langle \delta N_p^2 \rangle, \quad C_3 = \langle \delta N_p^3 \rangle, \quad C_4 = \langle \delta N_p^4 \rangle - 3\langle \delta N_p^2 \rangle^2$$

# Quantum effects: Flucton solution

The flucton is a semiclassical solution of the EoMs in Euclidean time with period  $\beta = 1/T$  (Shuryak, 1988). Conceptually similar to the instanton.



Unlike the instanton it is periodic  $x(\beta) = x(0) = x_0$ , and it does not require a double well. We applied to 2,3,4-body systems at finite temperature

$$P(x_0) = \langle x_0 | e^{-\hat{H}\beta} | x_0 \rangle = \int_{x(0)=x_0}^{x(\beta)=x_0} \mathcal{D}x(\tau) e^{-S_E[x(\tau)]}$$