XXXII International (ONLINE) Workshop on High Energy Physics
"Hot problems of Strong Interactions"

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Universal (?) scaling of QCD from Wilson fermions

Maria Paola Lombardo

INFN Firenze

based on:

G. Aarts, C. Allton, J. Glesaaen, S. Hands, B. Jäger, S. Kim, MpL, A. A. Nikolaev, S. M. Ryan, J.-I. Skullerud, L.-K. Wu **arXiv:2007.041**

A. Yu. Kotov, MpL, A. M. Trunin **Eur.Phys.J.A 56 (2020) 8, 203** *and work in progress*

Which symmetry is restored in Hot QCD?

Shuryak(1993)

$$SU(N_f)_L \times SU(N_f)_R \times U(1)_A \times U(1)_B.$$

Breaking/restoration
at Tc
studied a lot
on the lattice

Always broken if topological charge fluctuates!

DOES IT?

BUT:
the 'amount' of breaking,
may
depend on temperature!

HOW ARE THESE RELATED??

IMPLICATIONS?

Which symmetry is restored in Hot QCD?

Shuryak(1993)

- if $U(1)_A$ breaking insensitive to the chiral restoration:
  $SU(2)_L \times SU(2)_R \to SU(2)_V$ or $O(4) \to O(3)$ **Pisarski-Wilczek(1983).**

  Second order transition with known exponents and equation of state.

- $U(1)_A$ restored at T_c,
  $U(2)_L \times U(2)_R \to U(2)_V$

  First or even a second order transition with different exponents

  Pelissetto et al(2013)

- Strategy:

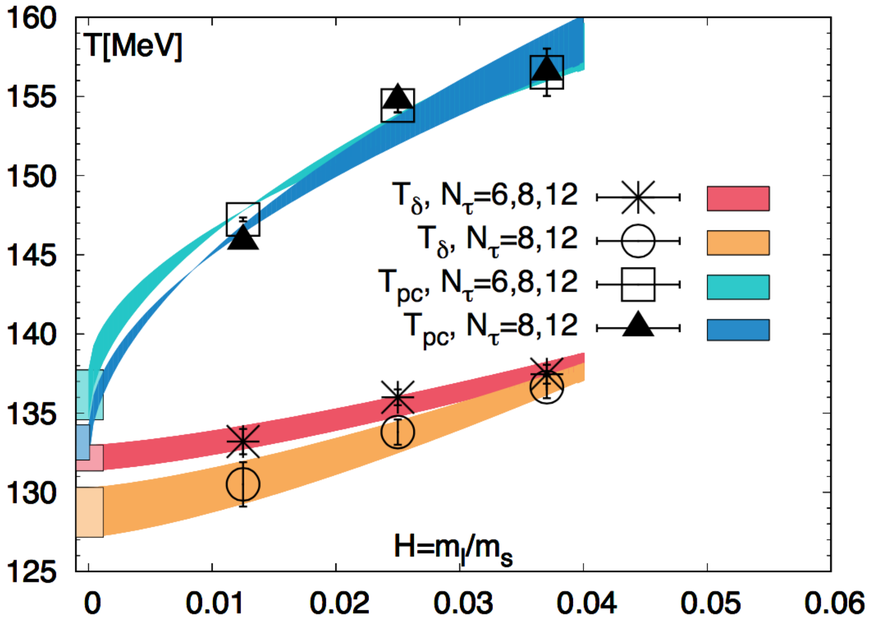
$$T_c(m_\pi) = T_c(0) + AT^{2/\beta\delta} \qquad \text{O(4)?}$$

+ scaling violating terms – from derivatives of $\bar{\psi}\psi$

$$T_c(m_\pi) = Constant$$

+ scaling violating terms – from RG invariant quantities

Status:



Staggered fermions, next Talk by A. Lahiri

Other recent works on Thermodynamics with Wilson fermions:

WB collaboration, JHEP08, 126 (2012), Phys. Rev.D92, 014505 (2015)

WHOT-QCD, Phys. Rev.D85, 094508 (2012),Phys. Rev. D96, 014509 (2017),

POSLATTICE2019, 088 (2019),arXiv:2005.00251 [hep-lat].

Twisted Mass at FT Phys. Rev.D98, 094501 (2018),

FASTSUM POSLATTICE2019 + early work on parity doubling, transport, …

Setup

FASTSUM

Nf = 2 + 1 $m_\pi = 384(4)\mathrm{MeV}; \quad 236(2)\ \mathrm{MeV}$

Anisotropy $\xi \equiv \frac{a_s}{a_t} = 3.453(6) \quad a_t = 0.033\ \mathrm{fm}$

Twisted mass - Maximal twist

Nf = 2 +1 + 1, $m_\pi^{phys} < m_\pi < 470\ \mathrm{MeV} \quad a = 0.06 - 0.09\ \mathrm{fm}$

Fixed scale approach - Temperature range $130\ \mathrm{MeV} < T < 500\ \mathrm{MeV}$

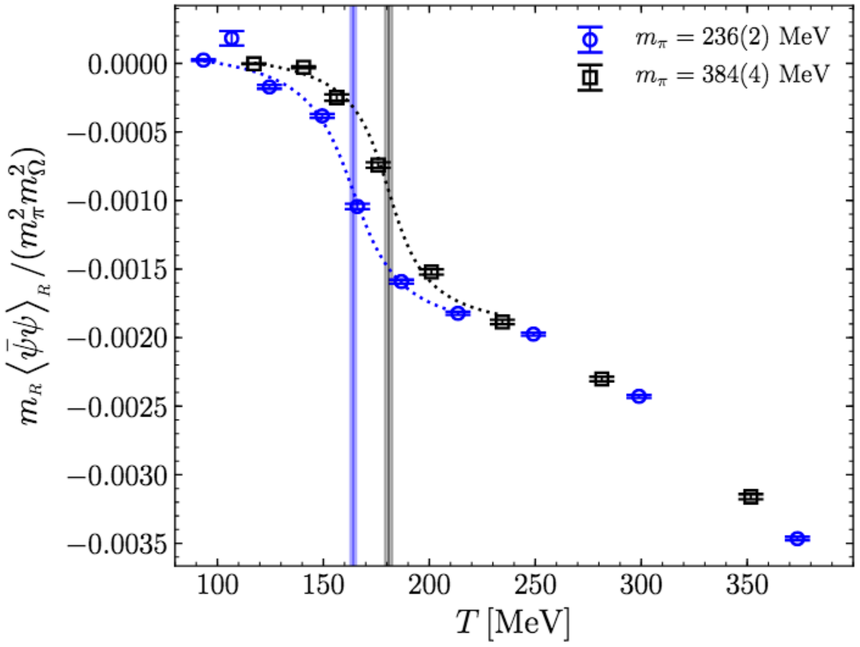
Observables: Chiral condensate and Susceptibility, (light mesons' screening masses)

Topological susceptibility, η' not discussed here.

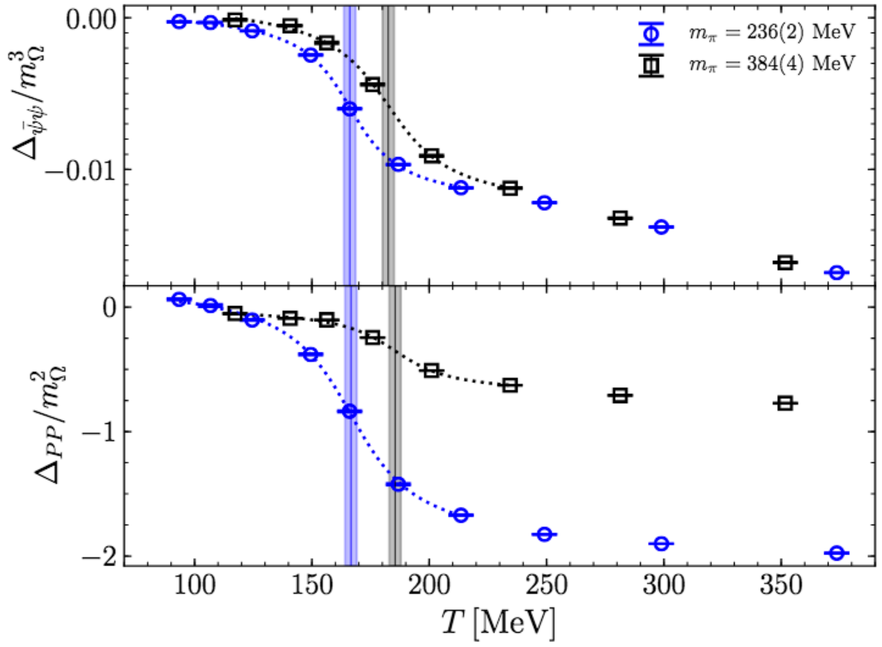
Renormalized chiral condensate

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = 2N_f m_{PCAC}^2 Z_A^2 \Delta_{PP}(T),$$
$$m_R\langle\bar{\psi}\psi\rangle_R(T) = m_{PCAC} Z_A \Delta_{\bar{\psi}\psi}(T) + \ldots$$

$$m_R\langle\bar{\psi}\psi\rangle_R(T) = \frac{\Delta^2_{\bar{\psi}\psi}(T)}{2N_f\Delta_{PP}(T)} + \quad \textbf{WB 2012}$$

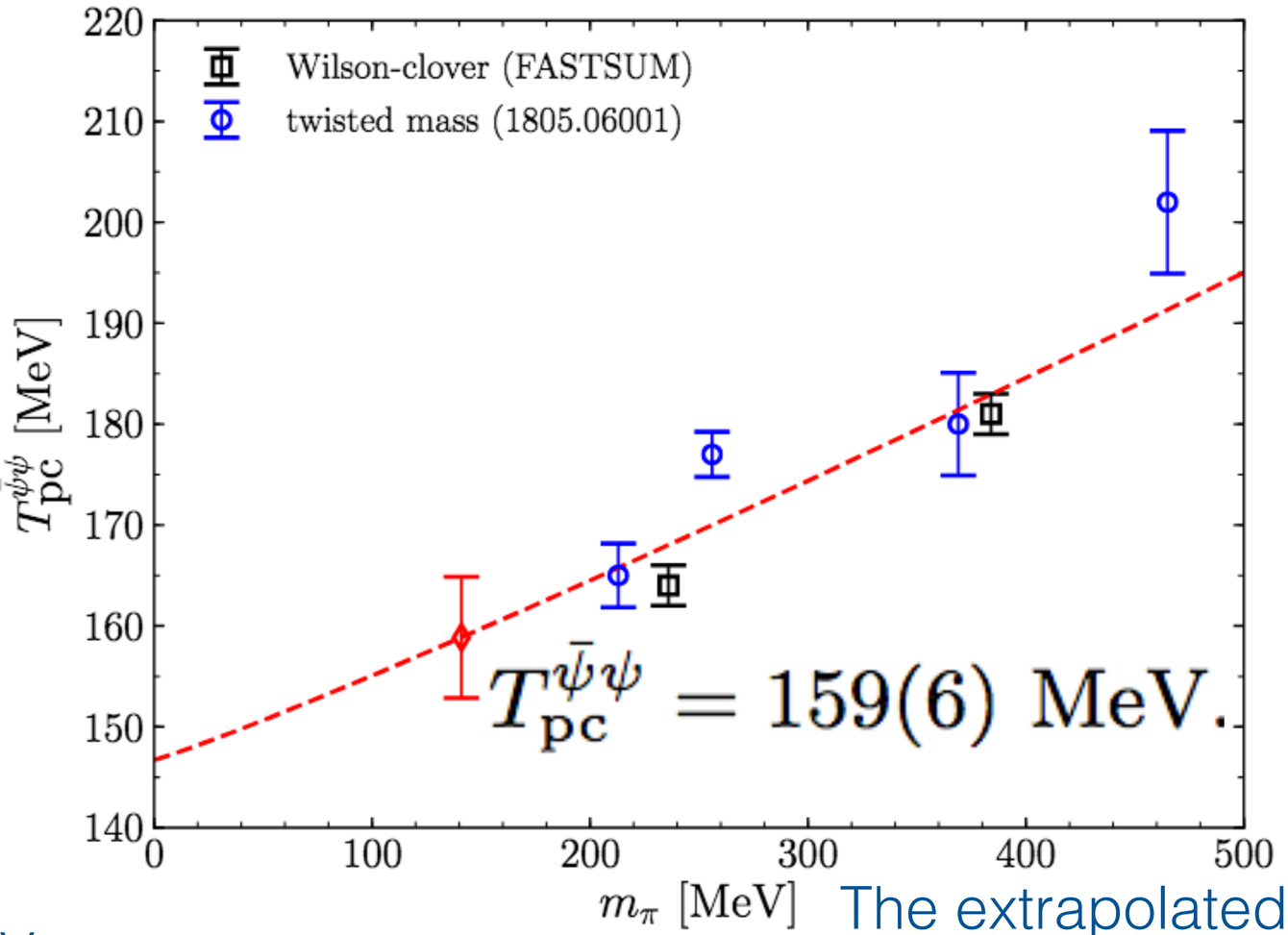


N.B. We work at fixed scale!
Tc is renormalization independent!!



$$\frac{m_R\langle\bar{\psi}\psi\rangle_R(T)}{m_\pi^2 m_\Omega^2} = c_0 + c_1 \arctan\left[c_2(T - T_{\rm pc})\right],$$

O4 fit to the chiral condensate works



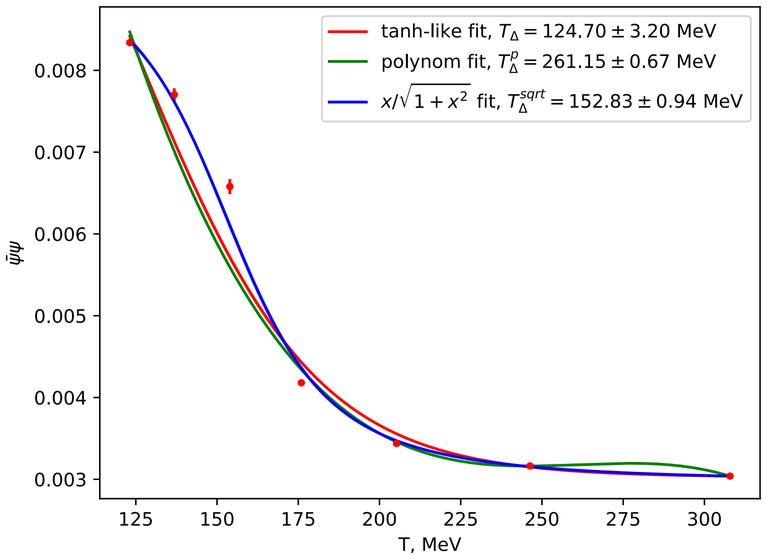
The extrapolated pseudo critical temperatu
for physical pion mass
compares well with HotQCD
result

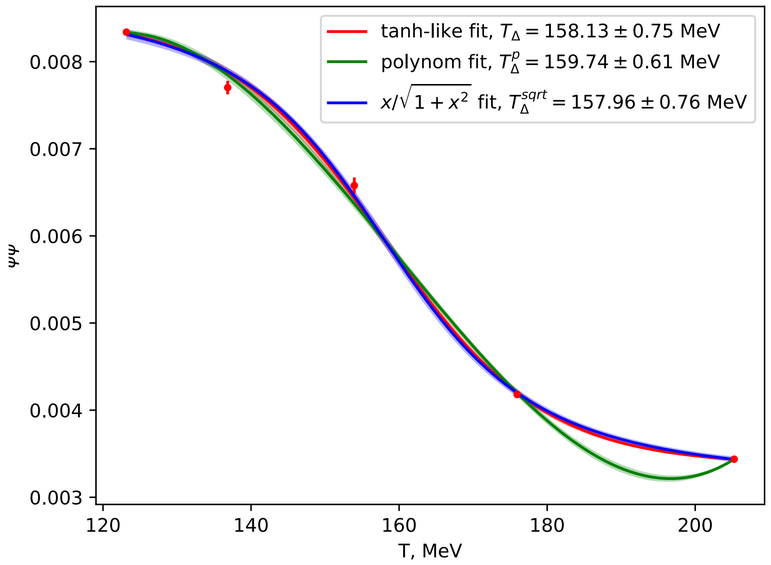
T_0 = 147(4) MeV

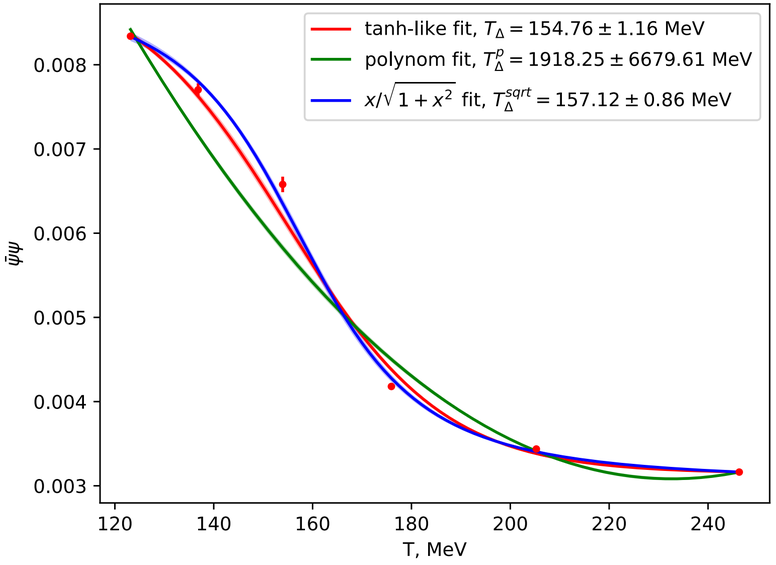
(no extrapolation to continuum, large errors)

Twisted mass results - chiral condensate for physical pion mass

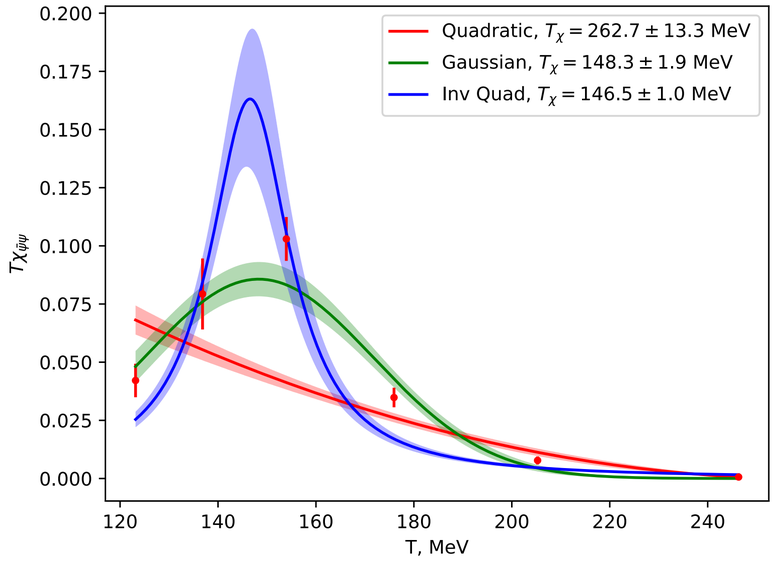
PRELIMINARY

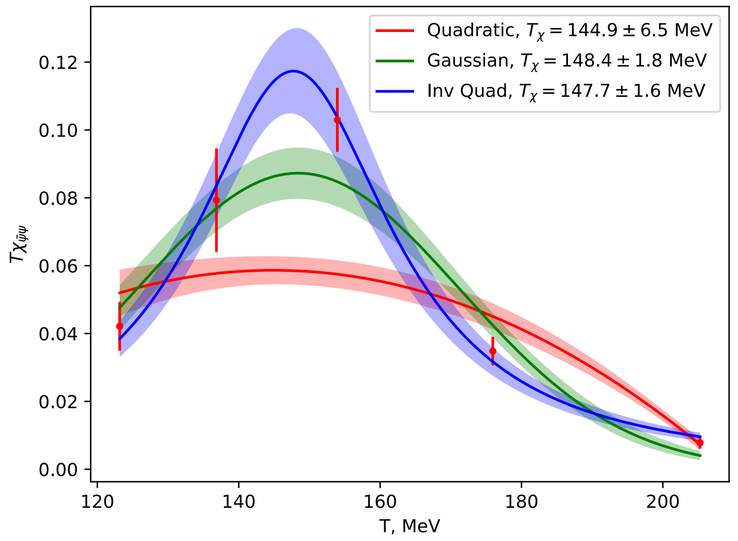


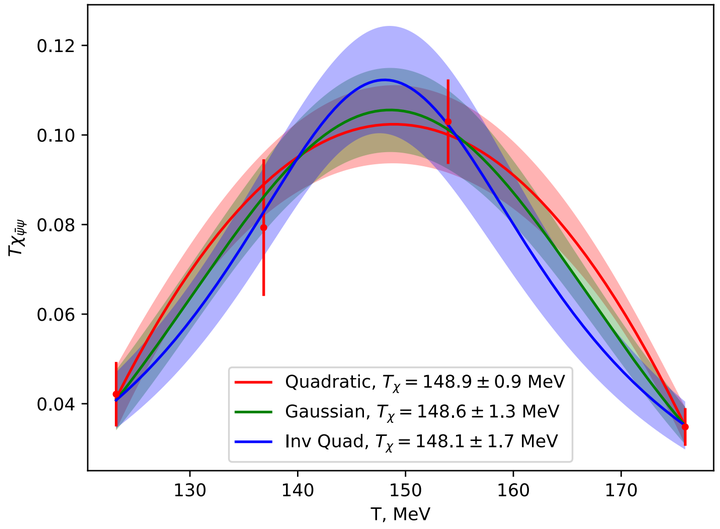




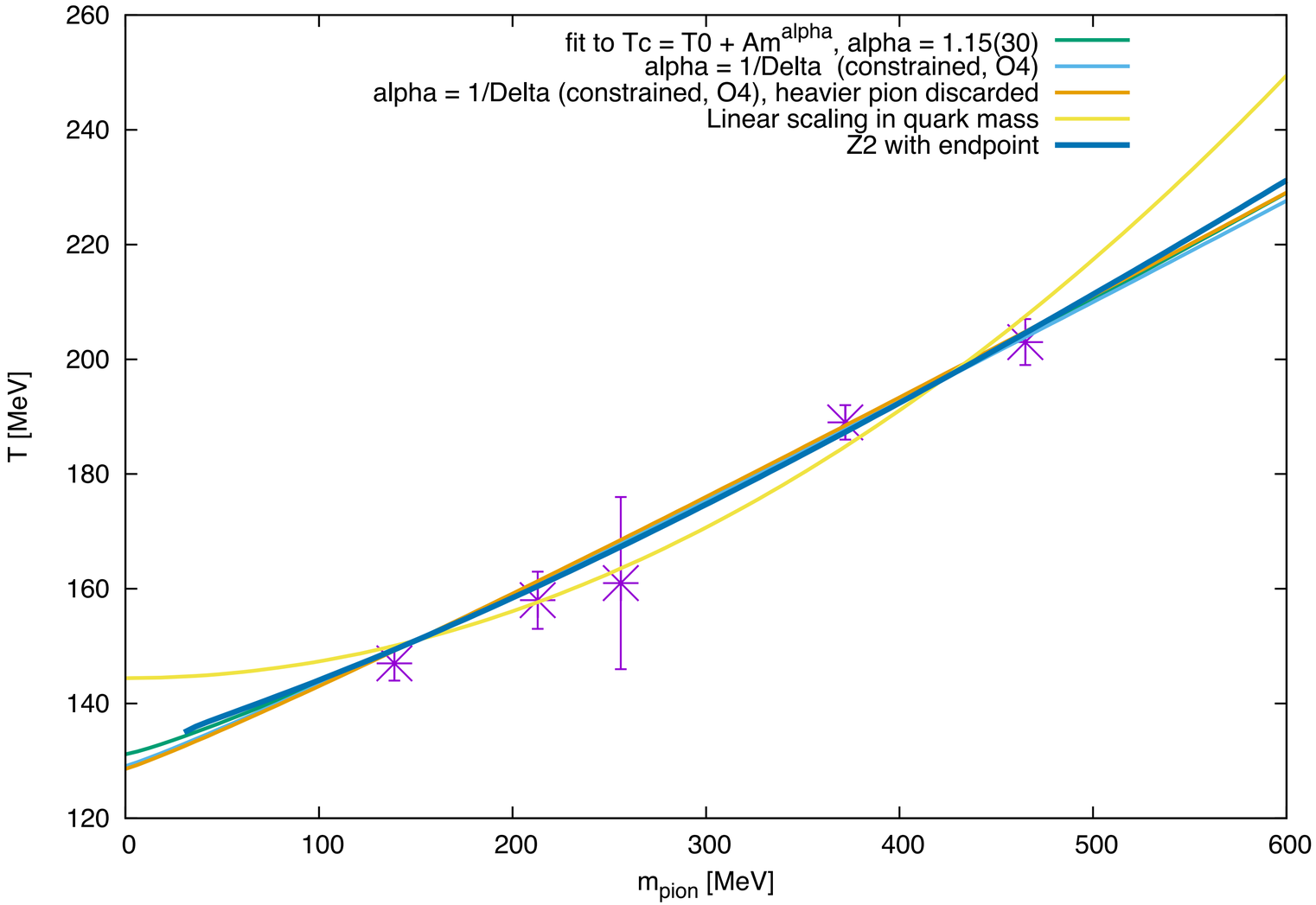
Twisted mass results - disconnected susceptibility for physical pion mass

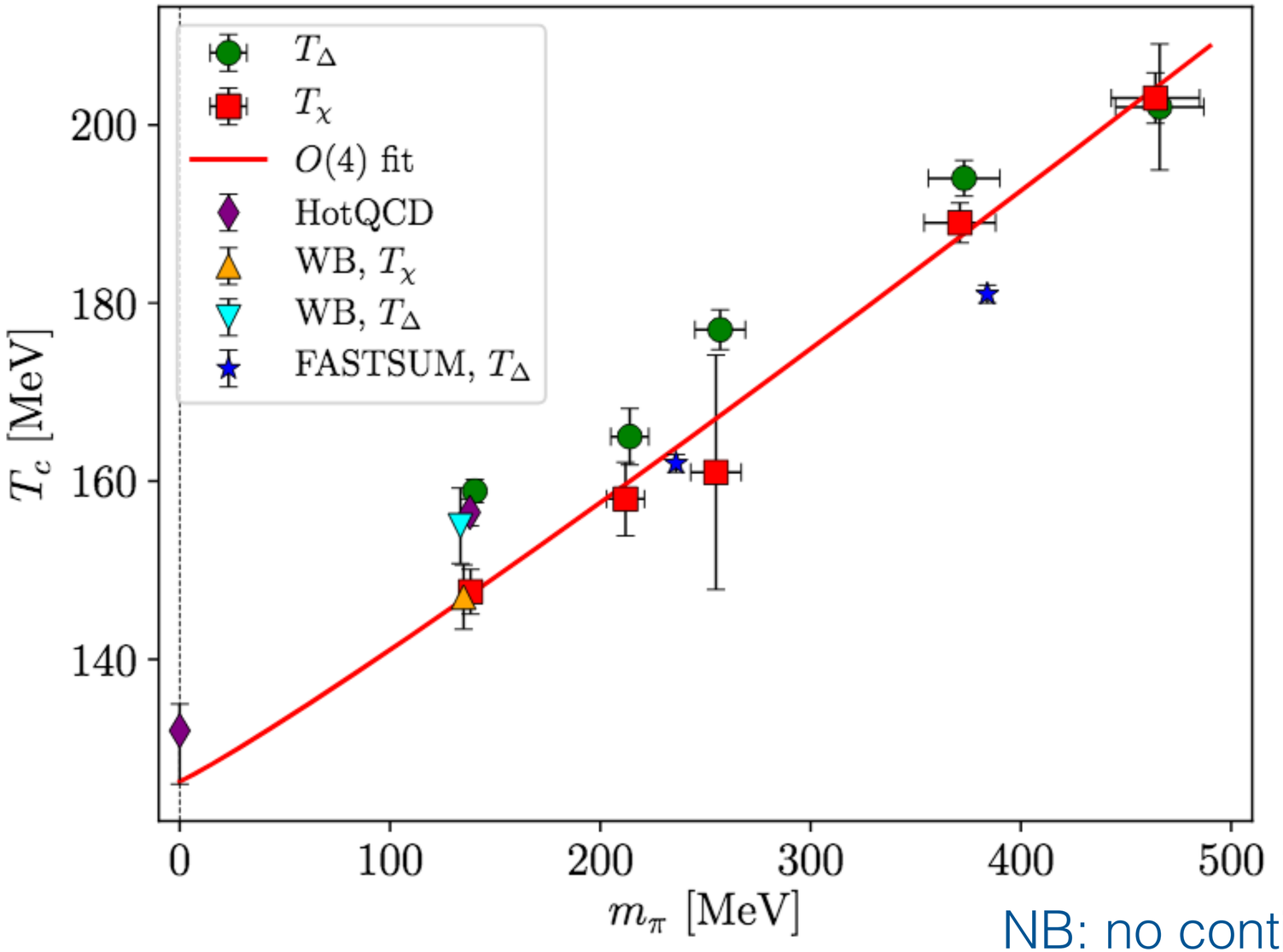






Direct measure of the pseudo critical temperature at the physical point consistent with staggered. Other models - besides O4 - work as well, and all give the same Tphys





NB: no continuum extrapolation at the physical point

An alternative combination of chiral observables: 'Beating' the regular terms/additive renormalization for more stringent universality checks

$$O_R \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m}) \equiv m(\chi_T - \chi_L)$$

$$h = M^\delta f(t/M^{1/\beta})$$

Magnetic equation of State

$M \equiv \bar{\psi}\psi$, $h \equiv m_q$, $t \equiv T - T_c$, m_q is the quark mass and T_c is the critical temperature

$$\chi_T = \frac{\bar{\psi}\psi}{m}$$

$$\chi_L = \frac{\partial\bar{\psi}\psi}{\partial m}.$$

Transverse and longitudinal susceptibilities

$$R_\pi \equiv \chi_T^{-1}/\chi_L^{-1}$$

$$\frac{1}{R_\pi(t,m)} = \delta - \frac{x}{\beta}\frac{f'(x)}{f(x)},$$

$$R_\pi(0,m) = \frac{1}{\delta}$$

Kocic, Kogut, MpL;
Karsch, Laermann

- linear terms in m drop in $O_R \equiv (\bar{\psi}\psi - m\chi_L) \equiv (\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$

- O_R order parameter, no leading order additive renormalization

Use: $M = h^{1/\delta} f_G(t/h^{1/\beta\delta})$ (parametrization in:

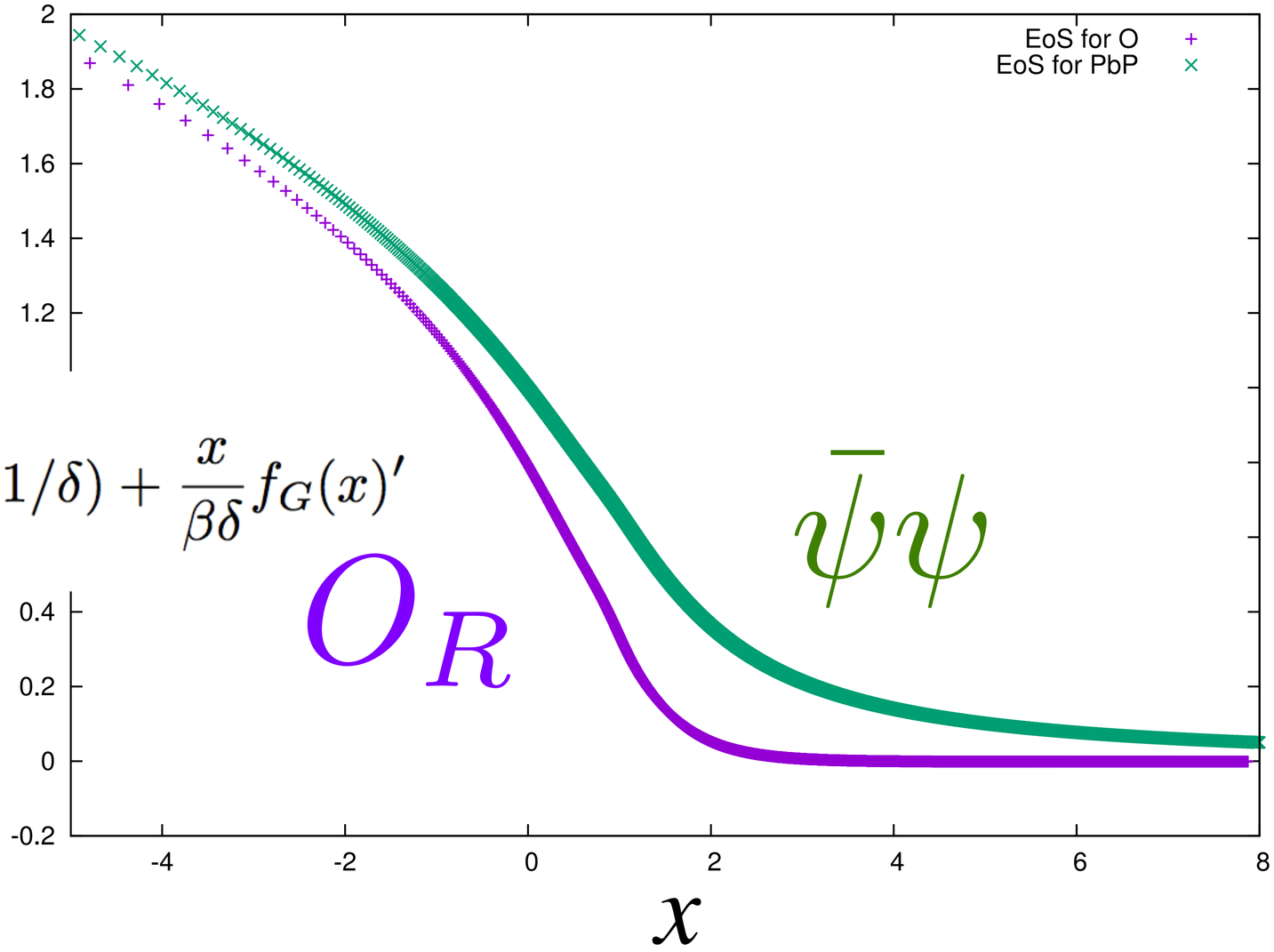
J.Engels and F.Karsch, Phys. Rev. D 85, (2012)

To get EoS for O_R

$$O_R = m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) - 1/\delta m^{1/\delta-1} f_G(t/m^{1/\beta\delta}) + m^{1/\beta\delta+1} f'_G((t/m^{1/\beta\delta})$$

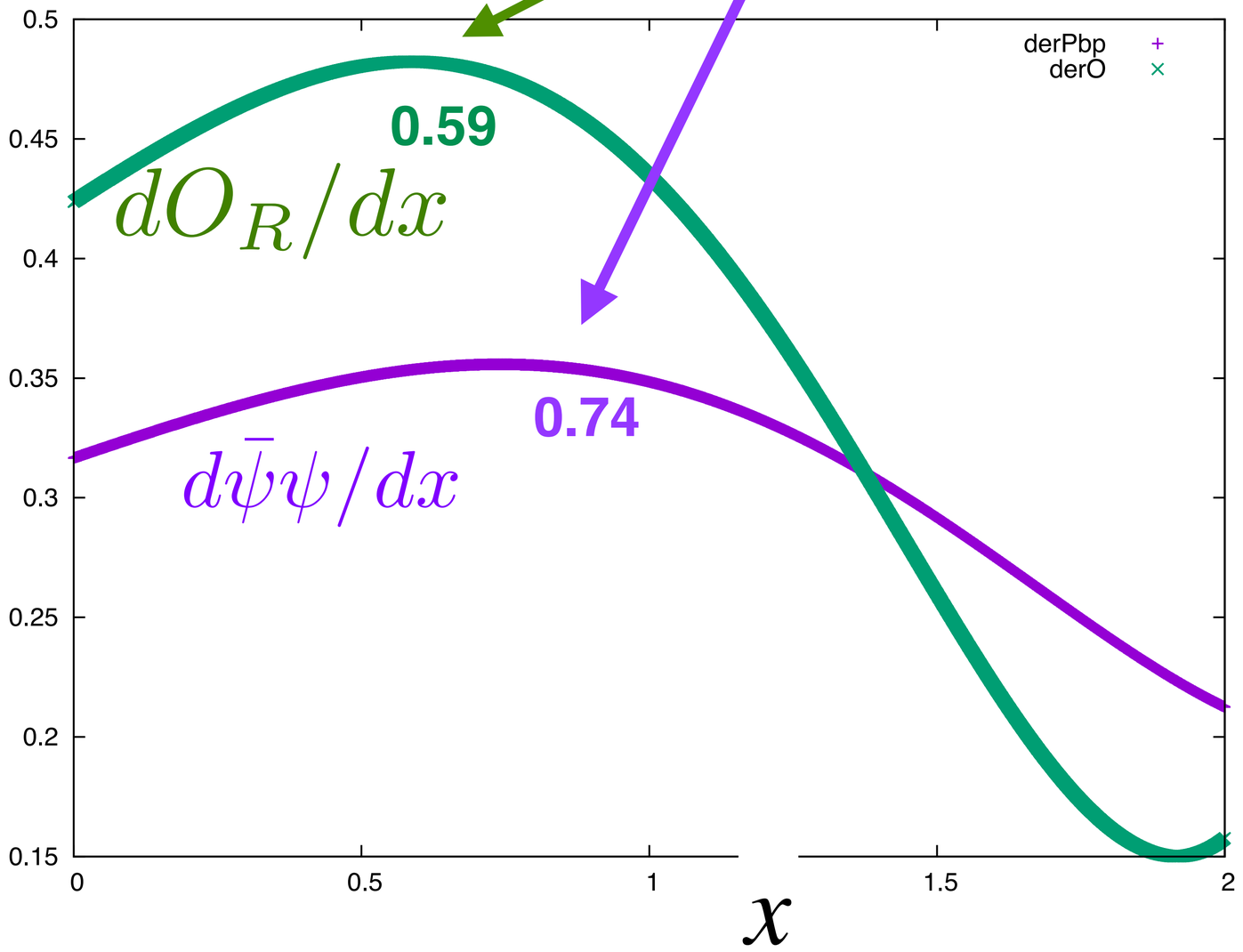
$$\frac{O_R}{m^{1/\delta}} = f_G(x)(1-1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$

$$\frac{O_R}{m^{1/\delta}} = f_G(x)(1-1/\delta) + \frac{x}{\beta\delta} f_G(x)'$$



Derivatives: give scaling of pseudo critical temperature Tc with mass

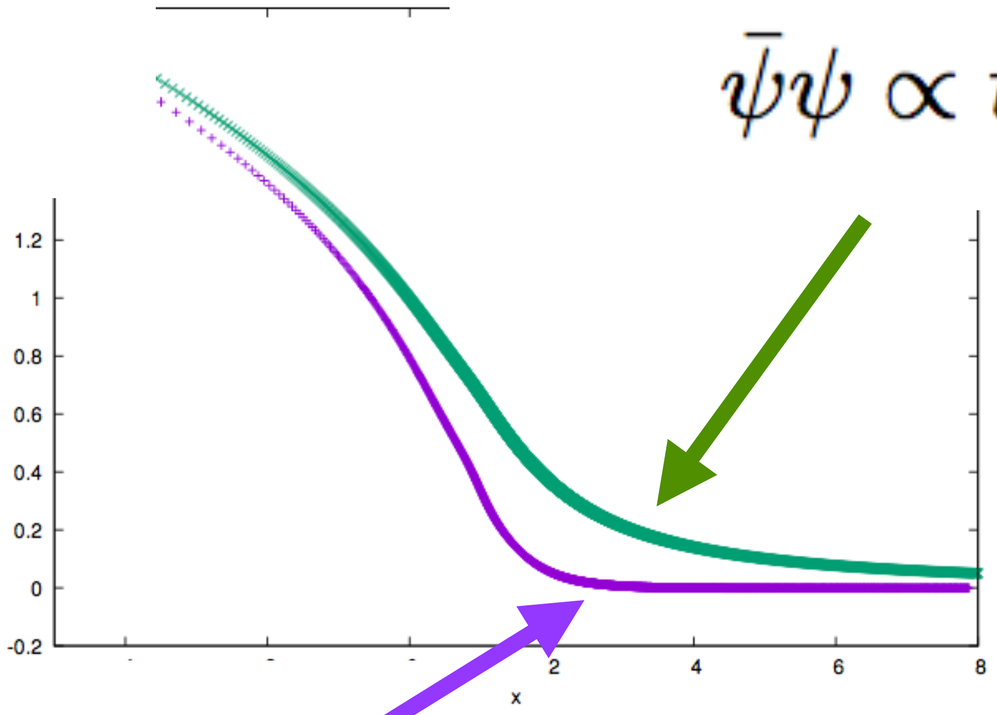
$$T_c = T_c(0) + k_s m_\pi^{2/\beta\delta}$$



Asymptotic behavior - high T expansion

$$f_G(x) = x^{-\gamma} \sum_{n=0}^{\infty} d_n x^{-2n\Delta}$$

again, linear term drops in O_R:

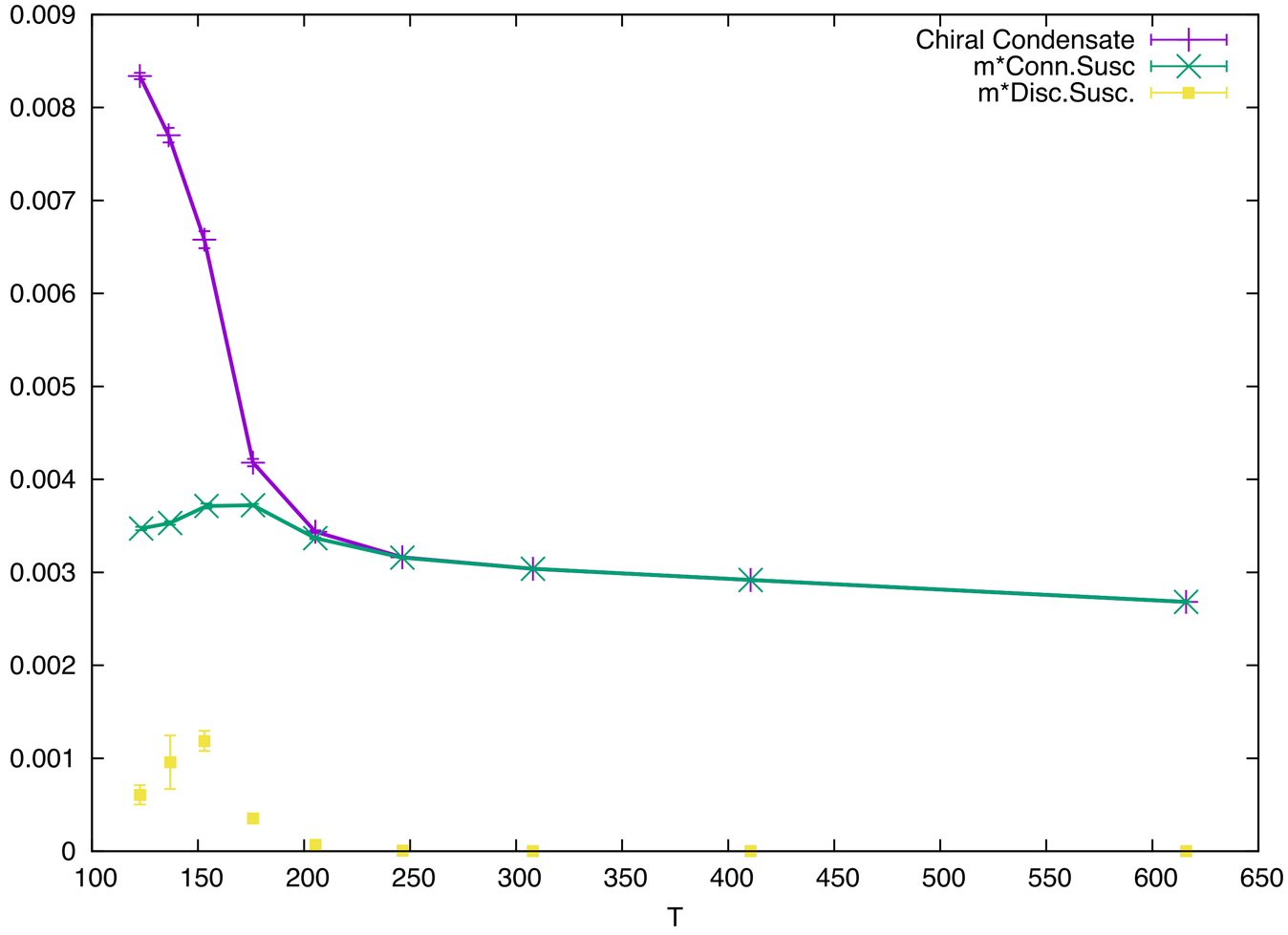


$$\bar{\psi}\psi \propto t^{-\gamma}$$

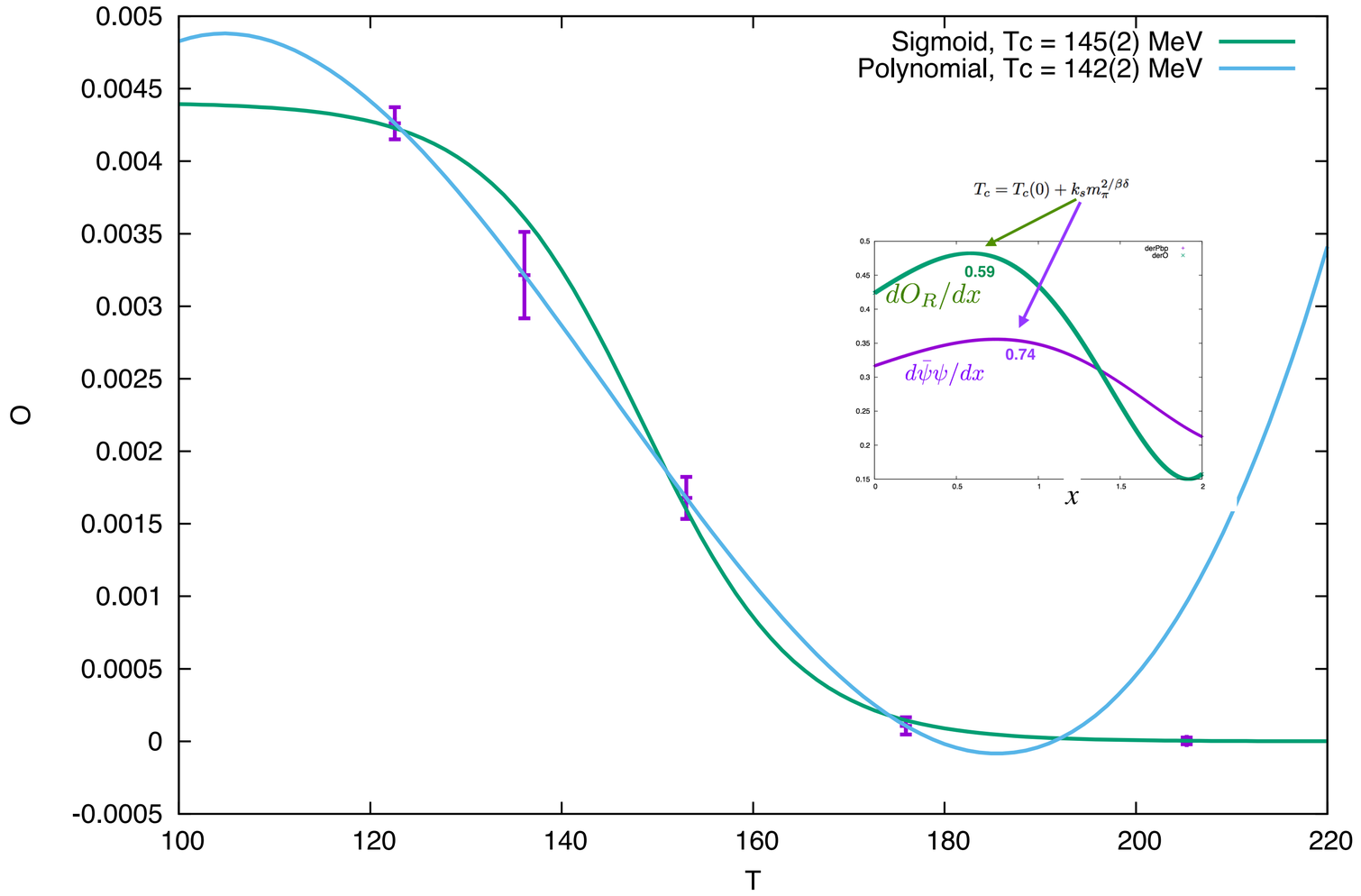
$$O_R \propto t^{-\gamma-2\beta\delta}$$

Numerical results for physical pion mass

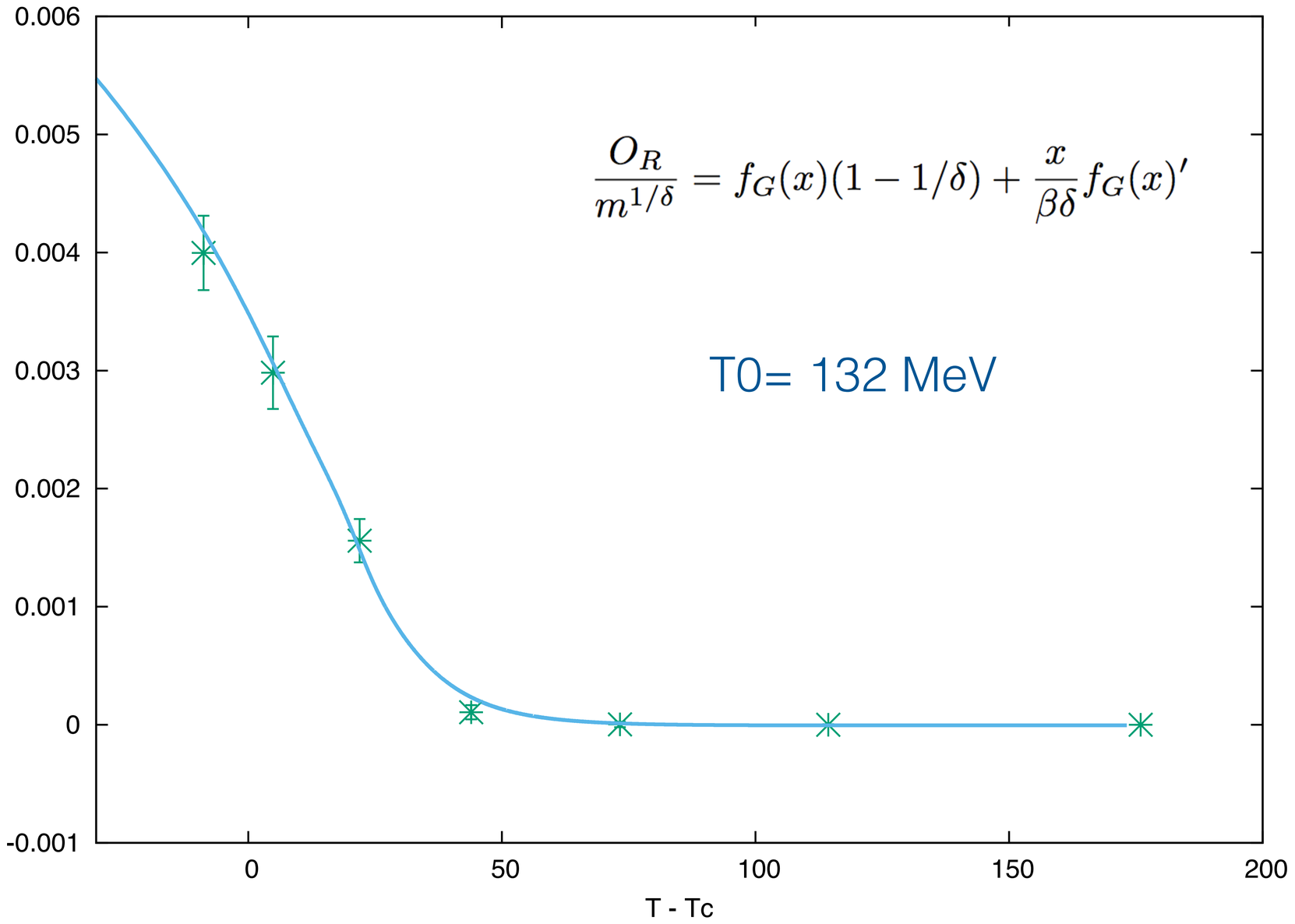
$$(\bar{\psi}\psi - m\frac{\partial\bar{\psi}\psi}{\partial m})$$

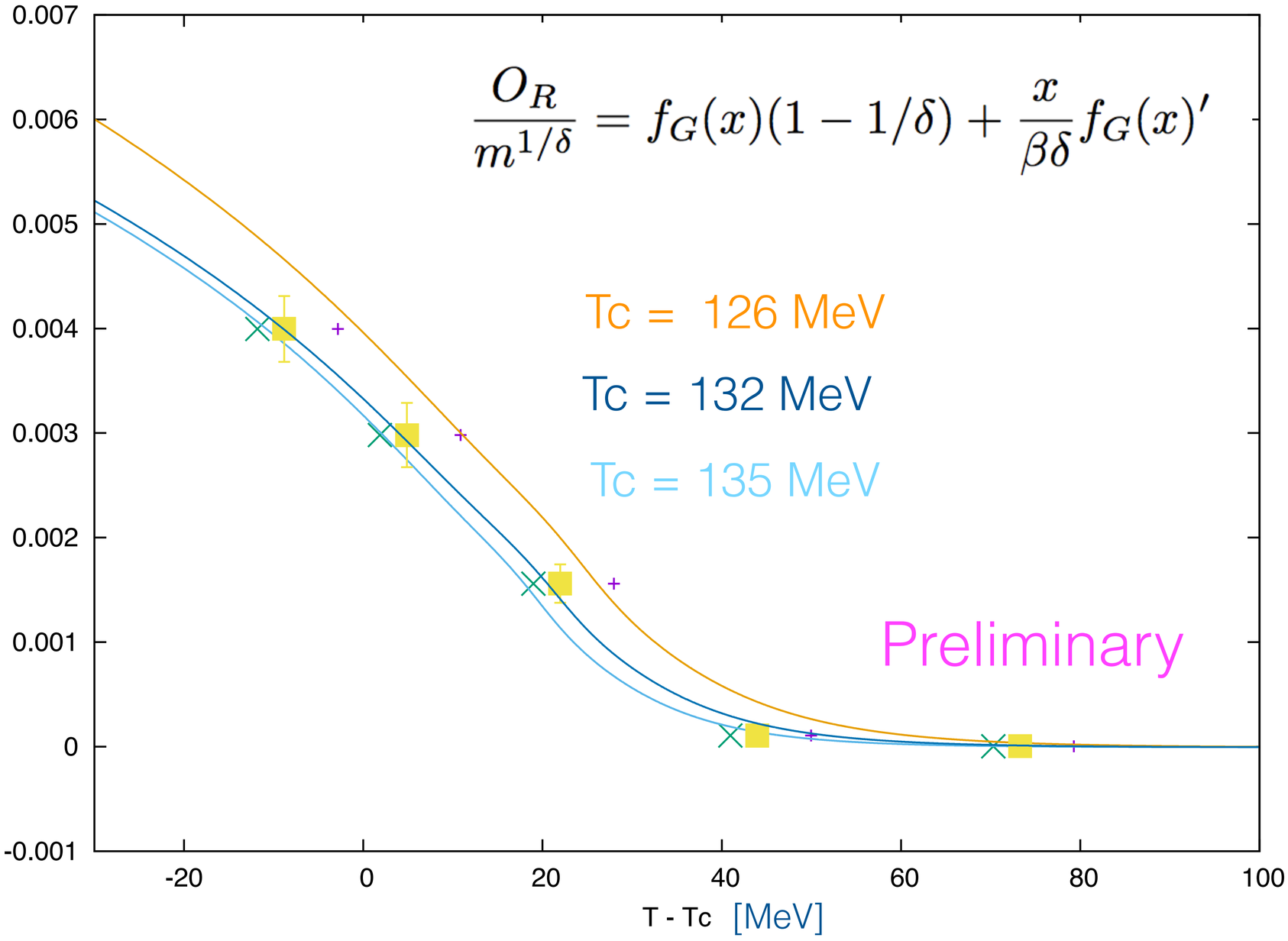


Pseudocritical point Tc < Tc from chiral condensate OK

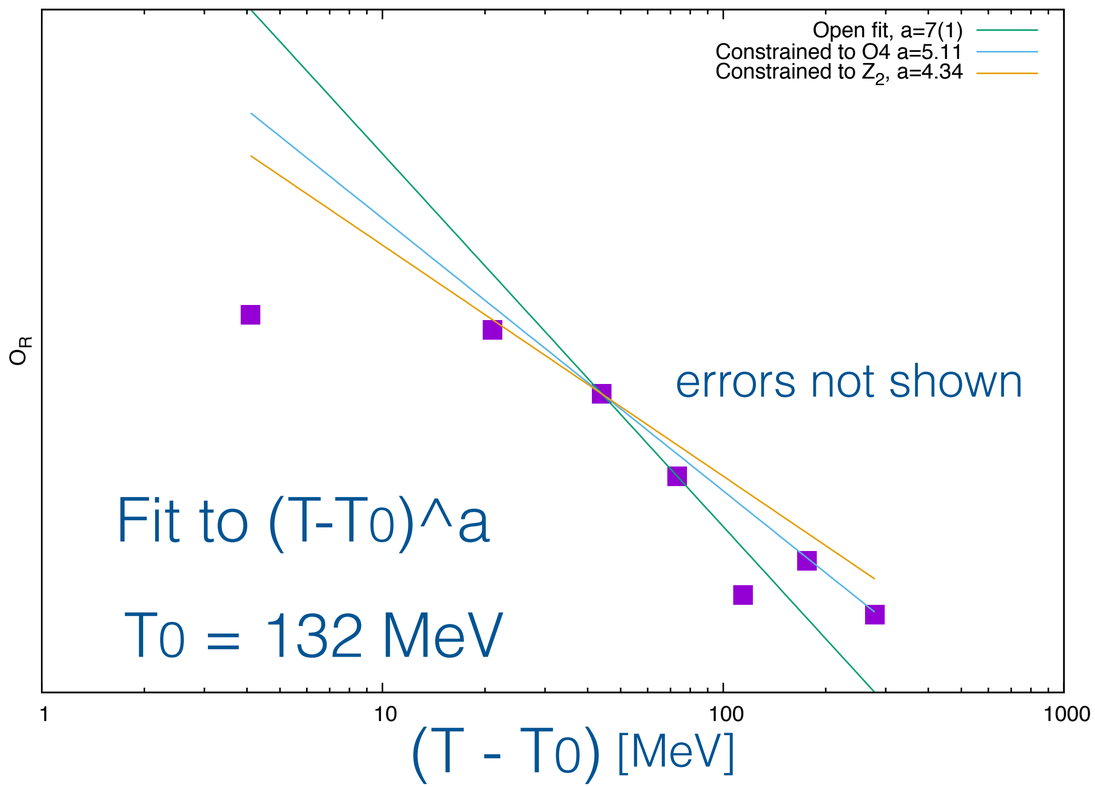


Match to the EoS with selected T0




$\frac{O_R}{m^{1/\delta}} = f_G(x)(1-1/\delta) + \frac{x}{\beta\delta} f_G(x)'$
Tc = 126 MeV
Tc = 132 MeV
Tc = 135 MeV
Preliminary
0.007
0.006
0.005
0.004
0.003
0.002
0.001
0
-0.001
-20
0
20
40
60
80
100
T - Tc [MeV]

High temperature behavior $O_R \propto t^{-\gamma-2\beta\delta}$



$\gamma - 2\beta\delta = 5.11(O4); 4.34(Z2)$

Some discriminating power..?

Preliminary

The physics behind: a quick look at the axial symmetry

Which symmetry is restored in Hot QCD? II

Correlation functions of local operators **Shuryak(1993),Buchoff(2013)**

$$\begin{aligned} \sigma &= \bar{\psi}_l \psi_l \\ \delta^i &= \bar{\psi}_l \tau^i \psi_l \\ \eta &= i\bar{\psi}_l \gamma^5 \psi_l \\ \pi^i &= i\bar{\psi}_l \tau^i \gamma^5 \psi_l . \end{aligned}$$

(σ, π^i) and (η, δ^i) are related by $SU(2)_L \times SU(2)_R$ transformations
(σ, η), $(\delta^i, \pi^i)_{1 \leq i \leq 3}$ by $U(1)_A$.
Several lattice studies in mesonic channels.

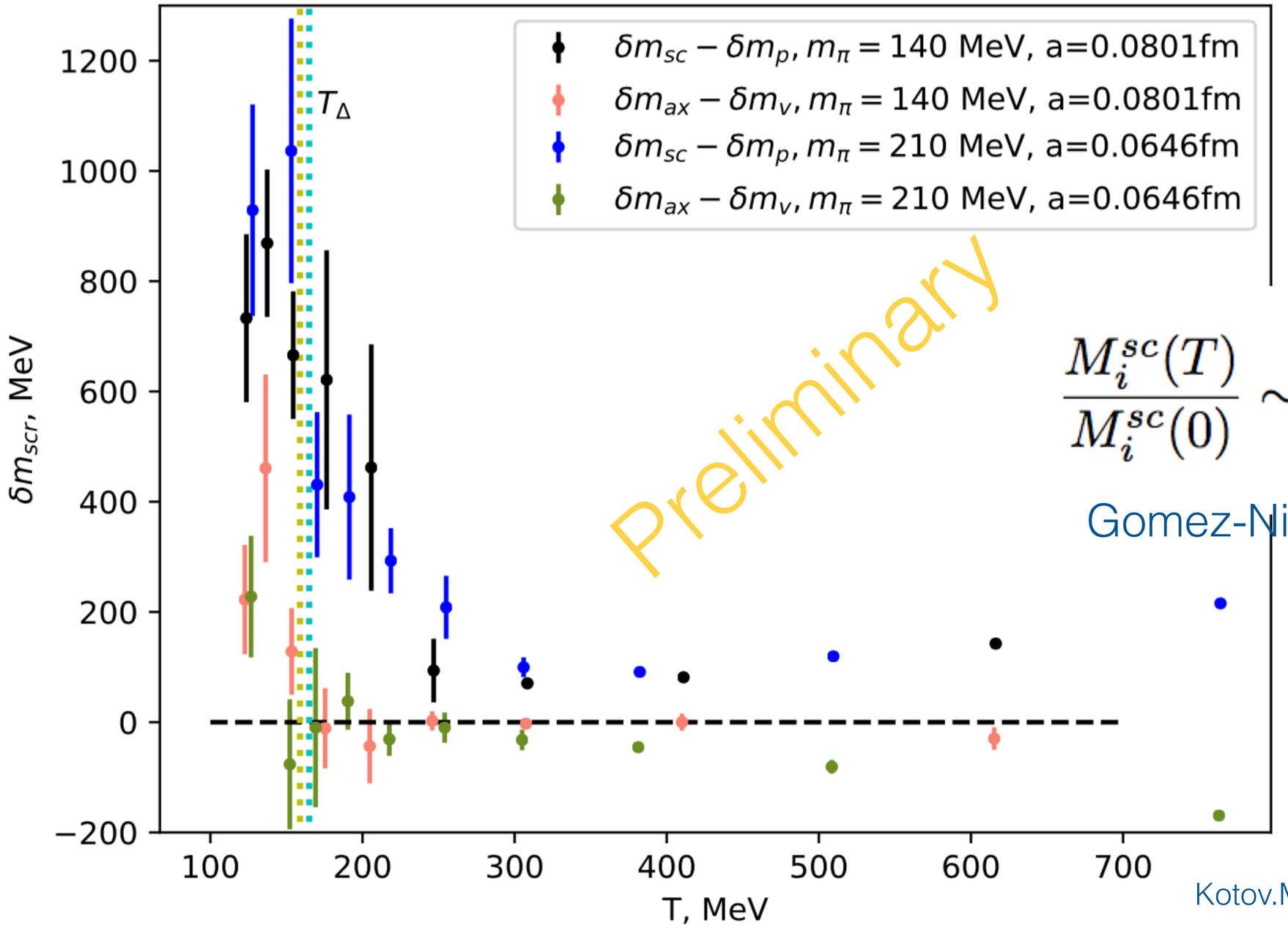
- Consensus: axial symmetry is effectively restored $T \simeq 1.2\, T_c$

  Kaczmarek(2020),Mazur(2018),Buchoff(2013),Suzuki(2020),Kanazawa(2015),Aoki(2012),

  Tomiya(2016),Brandt(2019),Cossu(2013),Chiu(2013),Tomiya(2016)

- Controversy: how close to T_c the effective restoration may happen.

  same refs as above



Preliminary

$$\frac{M_i^{sc}(T)}{M_i^{sc}(0)} \sim \left[\frac{\chi_i(0)}{\chi_i(T)}\right]^{-1/2}$$

Gomez-Nicola 2020

Kotov.MpL,Trunin, in progress

Summary and questions

Results from Wilson fermions have reached the physical point and are now confronting universality predictions.

Quantitative agreement with staggered results at the physical point.

Consistence with O4 Universality class, but hard to exclude other possibilities.

Main unknown: role of regular contribution/scaling violations

Proposed simple combination of chiral observables to ameliorate this problem - high T behavior may be a sensitive probe of universality

Axial partners, eta prime mesons, topology studies are needed for a coherent scenario

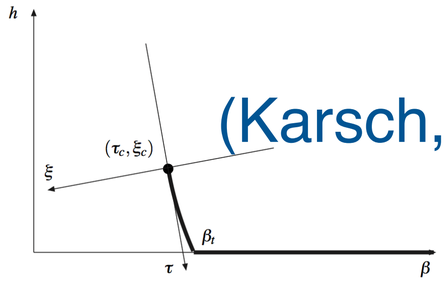
Questions:

General question: the ‘question mark’ in the title ..
interplay anomaly/chiral symmetries —> universal behavior

Specific questions:

Size of the scaling window - (Kogut, Kocic; Friman, Redlich, …)

High T asymptotic, merging with highT PT : ignoring change of variables for 1st order scenario?



(Karsch, Stephanov, Papa..)

Ultraviolet divergencies in susceptibilities? Wavefunction renormalization?

Motivated by Angel Gomez-Nicola proposal

$$\frac{M_i^{sc}(T)}{M_i^{sc}(0)} \sim \left[\frac{\chi_i(0)}{\chi_i(T)}\right]^{-1/2}$$