HEAVY QUARKONIUM PRODUCTION IN PNRQCD

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OUTLINE

- Inclusive heavy quarkonium production in NRQCD
- Potential NRQCD formalism for NRQCD matrix elements
- Production of *P*-wave quarkonium
- Summary and outlook

INCLUSIVE PRODUCTION OF QUARKONIUM

- Inclusive production cross sections of heavy quarkonium in colliders are important observables in QCD.
  Many measurements are available and will continue to be made at current and future colliders.

- Quarkonium production is expected to be a useful probe for various areas of QCD such as the QGP.
  For this to work, it is important to understand the quarkonium production mechanism based on QCD.

- Nonrelativistic EFTs provide a factorization formalism for quarkonium production. **Bodwin, Braaten, Lepage, PRD51, 1125 (1995)**

NRQCD FACTORIZATION

- NRQCD provides a factorization formalism for inclusive production cross sections.

NRQCD matrix elements

$$\sigma = \sum_n \sigma_{Q\bar{Q}(n)} \langle \Omega | \mathcal{O}_n | \Omega \rangle$$

Perturbatively calculable
***$Q\bar{Q}$* cross sections**

Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

- The matrix elements are usually determined from measured cross sections, because in general it is not known how to compute them from first principles. So far this approach has not lead to a comprehensive description of measurements.
- We aim to ***compute*** the matrix elements in ***potential NRQCD***, which is obtained by integrating out scales above mv^2.

POTENTIAL NRQCD

- We work in the strong coupling regime where $mv^2 \ll \Lambda_{\rm QCD}$, which is valid for non-Coulombic quarkonia.
  The degree of freedom is the singlet field $S(x_1,x_2)$, which describe Q and $\bar{Q}$ at positions x_1 and x_2 in a color-singlet state.

$$\mathcal{L}_{\rm pNRQCD} = {\rm Tr}\{S^\dagger (i\partial_0 - h) S\}$$

Pineda, Soto, NPB Proc. Suppl. 64, 428 (1998)
Brambilla, Pineda, Soto, Vairo, NPB566, 275 (2000)
Brambilla, Pineda, Soto, Vairo, Rev. Mod. Phys. 77, 1423 (2005)

- Matching to NRQCD is done nonperturbatively.
- The equation of motion is the Schrödinger equation.
  Decay matrix elements are expressed in terms of wavefunctions at the origin and universal gluonic correlators.
- ***We extend the formalism for production matrix elements.***

DECAY/PRODUCTION MATRIX ELEMENTS

- ***Production*** and *decay* matrix elements have different forms.

Projection onto states containing quarkonium

Production : $\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$ **Decay :** $\langle \mathcal{Q} | \psi^\dagger \mathcal{K} \chi \chi^\dagger \mathcal{K}' \psi | \mathcal{Q} \rangle$

color/spin matrices and covariant derivatives

- While *decay* matrix elements are *expectation values on a heavy quarkonium state*,
  production matrix elements involve projection onto states that contain a ***quarkonium + anything***.

$$\mathcal{P}_{\mathcal{Q}} = \sum_X |\mathcal{Q} + X\rangle\langle \mathcal{Q} + X| = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$$

- In order to compute production matrix elements, we need to ***identify the projection operator*** $\mathcal{P}_{\mathcal{Q}} = a^\dagger_{\mathcal{Q}} a_{\mathcal{Q}}$.

DECAY MATRIX ELEMENTS IN PNRQCD

- We briefly review the formalism for computing decay matrix elements in strongly coupled pNRQCD.

Brambilla, Eiras, Pineda, Soto, Vairo, PRL88, 012003 (2002)
Brambilla, Eiras, Pineda, Soto, Vairo, PRD67, 034018 (2003)
Brambilla, HSC, Müller, Vairo, JHEP04 (2020) 095

- Matrix elements are computed in expansion in $1/m$:

NRQCD Hamiltonian $$H_{\rm NRQCD} = H^{(0)}_{\rm NRQCD} + H^{(1)}_{\rm NRQCD}/m + \ldots$$

Eigenstates $$|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle = |\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(0)} + \frac{1}{m}|\underline{\rm n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle^{(1)} + \ldots$$

- $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ is the ground state, excited states have energy gaps of order $\Lambda_{\rm QCD}$, and x_1 and x_2 are positions of Q and $\bar{Q}$.

- Leading order in $1/m$ corresponds to the static limit.

DECAY MATRIX ELEMENTS IN PNRQCD

- A heavy quarkonium state is given by the ground state $|\underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle$, because excited states $|\underline{\mathrm{n}};\boldsymbol{x}_1,\boldsymbol{x}_2\rangle$ with $n{>}0$ are integrated out.

- This leads to the following formula

Quarkonium wavefunction

$$\langle\mathcal{Q}|\mathcal{O}|\mathcal{Q}\rangle = \frac{1}{\langle\boldsymbol{P}{=}\boldsymbol{0}|\boldsymbol{P}{=}\boldsymbol{0}\rangle}\int d^3r\int d^3r'\int d^3R\int d^3R'\,\langle\boldsymbol{P}{=}\boldsymbol{0}|\boldsymbol{R}\rangle\phi^*_{\mathcal{Q}}(\boldsymbol{r})$$
$$\times\left[\langle\underline{0};\boldsymbol{x}_1,\boldsymbol{x}_2|\int d^3\xi\,\mathcal{O}(\boldsymbol{\xi})|\underline{0};\boldsymbol{x}'_1,\boldsymbol{x}'_2\rangle\right]\langle\boldsymbol{R}'|\boldsymbol{P}{=}\boldsymbol{0}\rangle\phi_{\mathcal{Q}}(\boldsymbol{r}')$$

Contact term, computed order by order in $1/m$

$$\boldsymbol{r}=\boldsymbol{x}_1-\boldsymbol{x}_2,\quad \boldsymbol{R}=(\boldsymbol{x}_1+\boldsymbol{x}_2)/2$$

- The contact term is proportional to $\delta^{(3)}(\boldsymbol{r})$, and in general involves derivatives and universal gluonic correlators.

QUARKONIUM PROJECTION OPERATOR

- The projection operator $\mathcal{P}_{\mathcal{Q}} = a_{\mathcal{Q}}^{\dagger} a_{\mathcal{Q}}$ is essentially a number operator. If we neglect decay rates, $\mathcal{P}_{\mathcal{Q}}$ and the NRQCD Hamiltonian are ***simultaneously diagonalizable***.
- The matrix elements of $\mathcal{P}_{\mathcal{Q}}$ should involve $|\underline{0}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$, because this describes a ***quarkonium in vacuum***.
- Matrix elements of $\mathcal{P}_{\mathcal{Q}}$ can also involve $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ with $n>0$, if this describes a ***quarkonium + light particles***.

QUARKONIUM PROJECTION OPERATOR

- A ***simultaneous eigenstate*** of $\mathcal{P}_{\mathcal{Q}}$ and the Hamiltonian is

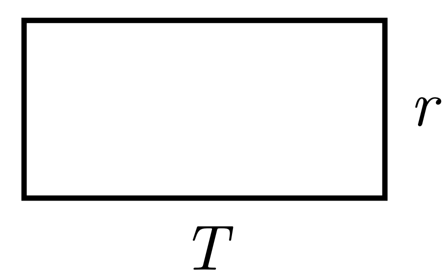
$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2\, \phi_{\mathcal{Q}(n)}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

- For $n{=}0$, this is just the quarkonium in vacuum and ϕ is the usual quarkonium wavefunction.
- For $n{>}0$, the "wavefunctions" ϕ are in general unknown.
- The projection operator is then $\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$.

The sum is restricted to states that reduce to color-singlet $Q\bar{Q}$ at leading order in $1/m$ and at $x_1{=}x_2$.

QUARKONIUM WAVEFUNCTIONS

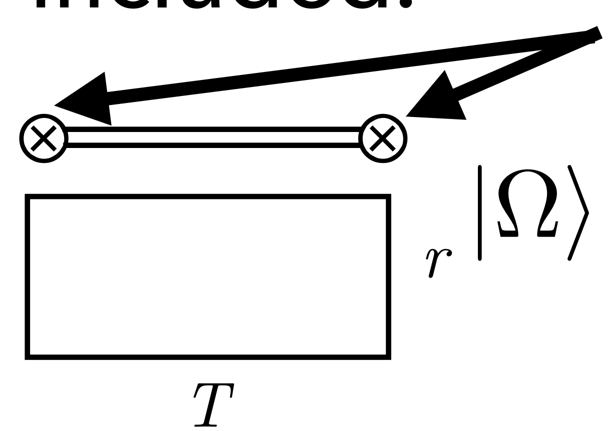
- Quarkonium wavefunctions are determined from a Schrödinger equation, where the potential is a vacuum expectation value of a Wilson loop: at leading order in $1/m$ (static potential for $n{=}0$),

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$$


$$|\Omega\rangle$$

- For the potential for the $n{>}0$ states, the light excitations in the $|\underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$ states should be included.

gluonic operators representing the light excitations

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log\langle\Omega|$$


$$|\Omega\rangle$$

QUARKONIUM WAVEFUNCTIONS

- In general, VEVs of products of color-singlet operators factorize into products of VEVs of individual operators.

$$\langle \Omega | AB | \Omega \rangle = \langle \Omega | A | \Omega \rangle \langle \Omega | B | \Omega \rangle [1 + O(1/N_c^2)]$$

Makeenko, Migdal, PLB88, 135 (1979)
Witten, NATO Sci. Ser. B 59, 403 (1980)

- So the $n>0$ potentials reduce to

$$V(r) = \lim_{T\to\infty} \frac{i}{T} \log \left(\langle \Omega | \underset{T}{\boxed{}}\, r | \Omega \rangle \times \langle \Omega | \otimes\!\!=\!\!=\!\!=\!\!\otimes | \Omega \rangle \right) + O(\tfrac{1}{N_c^2})$$

$$= \lim_{T\to\infty} \frac{i}{T} \log \langle \Omega | \underset{T}{\boxed{}}\, r \, | \Omega \rangle + \text{constant} + O(\tfrac{1}{N_c^2})$$

which differ from the $n=0$ potential by only a constant.

QUARKONIUM PROJECTION OPERATOR

- Hence, the $n>0$ *potentials are just the* $n=0$ *potential*, plus constants that have no effect to the wavefunctions.
- Therefore, the ***wavefunctions*** ϕ ***are independent of*** n, and the projection operator is just given by

$$\mathcal{P}_{\mathcal{Q}} = \sum_n |\mathcal{Q}(n)\rangle\langle\mathcal{Q}(n)|$$

$$|\mathcal{Q}(n)\rangle = \int d^3x_1 d^3x_2 \, \phi_{\mathcal{Q}}(\boldsymbol{x}_1, \boldsymbol{x}_2)|\underline{n}; \boldsymbol{x}_1, \boldsymbol{x}_2\rangle$$

  Now ϕ is the usual quarkonium wavefunction.

- This is valid up to corrections of relative order $1/N_c^2$.

PRODUCTION MATRIX ELEMENTS IN PNRQCD

- Now we can ***compute the production matrix elements***

$$\langle \Omega | \chi^\dagger \mathcal{K} \psi \mathcal{P}_{\mathcal{Q}} \psi^\dagger \mathcal{K}' \chi | \Omega \rangle = \int d^3x_1 d^3x_2 \int d^3x_1' d^3x_2' \phi(\boldsymbol{x}_1, \boldsymbol{x}_2) \phi(\boldsymbol{x}_1', \boldsymbol{x}_2') \sum_n \langle \Omega | \chi^\dagger \mathcal{K} \psi | \underline{\mathrm{n}}; \boldsymbol{x}_1, \boldsymbol{x}_2 \rangle \langle \underline{\mathrm{n}}; \boldsymbol{x}_1', \boldsymbol{x}_2' | \psi^\dagger \mathcal{K}' \chi | \Omega \rangle$$

Contact term,
computed order by order in $1/m$

- This allows ***calculation of production matrix elements*** in ***strongly coupled pNRQCD***, by computing the contact terms in the same way as the decay matrix elements.

PRODUCTION OF P-WAVE QUARKONIA

- We apply this formalism for ***production of*** χ_{QJ} $(Q{=}c,b,\ J{=}1{,}2)$
- At leading order in v, the cross section is given by

$$\sigma_{\chi_{QJ}+X} = (2J+1)\sigma_{Q\bar{Q}(^3P_J^{[1]})}\langle \mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]})\rangle + (2J+1)\sigma_{Q\bar{Q}(^3S_1^{[8]})}\langle \mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]})\rangle$$

Bodwin, Braaten, Yuan, Lepage, PRD46, R3703 (1992)
Bodwin, Braaten, Lepage, PRD51, 1125 (1995)

color singlet : $$\mathcal{O}^{\chi_{Q0}}(^3P_0^{[1]}) = \frac{1}{3}\chi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\psi\mathcal{P}_{\chi_{Q0}}\psi^\dagger\left(-\frac{i}{2}\overleftrightarrow{\boldsymbol{D}}\cdot\boldsymbol{\sigma}\right)\chi$$

color octet : $$\mathcal{O}^{\chi_{Q0}}(^3S_1^{[8]}) = \chi^\dagger\sigma^i T^a\psi\Phi_\ell^{\dagger ab}\mathcal{P}_{\chi_{Q0}}\Phi_\ell^{bc}\psi^\dagger\sigma^i T^c\chi$$

Nayak, Qiu, Sterman, PLB613, 45 (2005)

- ***We compute both color singlet and color octet matrix elements in strongly coupled pNRQCD***.

P-WAVE PRODUCTION MATRIX ELEMENTS

- ***Color-singlet*** matrix element: we *reproduce the known result in the vacuum-saturation approximation*.

$$\langle \mathcal{O}^{\chi_{Q0}}({}^3P_0^{[1]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2$$

- ***Color-octet*** matrix element: ***result is given in terms of the wavefunction and a universal gluonic correlator***.

$$\langle \mathcal{O}^{\chi_{Q0}}({}^3S_1^{[8]})\rangle = \frac{3N_c}{2\pi}|R^{(0)\prime}_{\chi_{Q0}}(0)|^2 \, \frac{\mathcal{E}}{9N_c m^2}$$

- $\mathcal{E}$ is a universal quantity that ***does not depend on quark flavor or radial excitation***. Determination of $\mathcal{E}$ directly leads to ***determination of all χ_{cJ} and $\chi_{bJ}(nP)$ cross sections, as well as h_c and h_b production rates***.

P-WAVE PRODUCTION MATRIX ELEMENTS

- The dimensionless correlator $\mathcal{E}$ is defined in terms of chromoelectric fields gE with Wilson lines Φ extending to infinity in the ℓ direction.

$$\mathcal{E} = \frac{3}{N_c} \int_0^\infty t\, dt \int_0^\infty t'\, dt' \langle \Omega | \Phi_\ell^{\dagger ab} \Phi_0^{\dagger da}(0,t) gE^{d,i}(t) gE^{e,i}(t') \Phi_0^{ec}(t',0) \Phi_\ell^{bc} | \Omega \rangle.$$

- $\mathcal{E}$ has a ***one-loop scale dependence*** that is ***consistent with the evolution equation for NRQCD matrix elements***

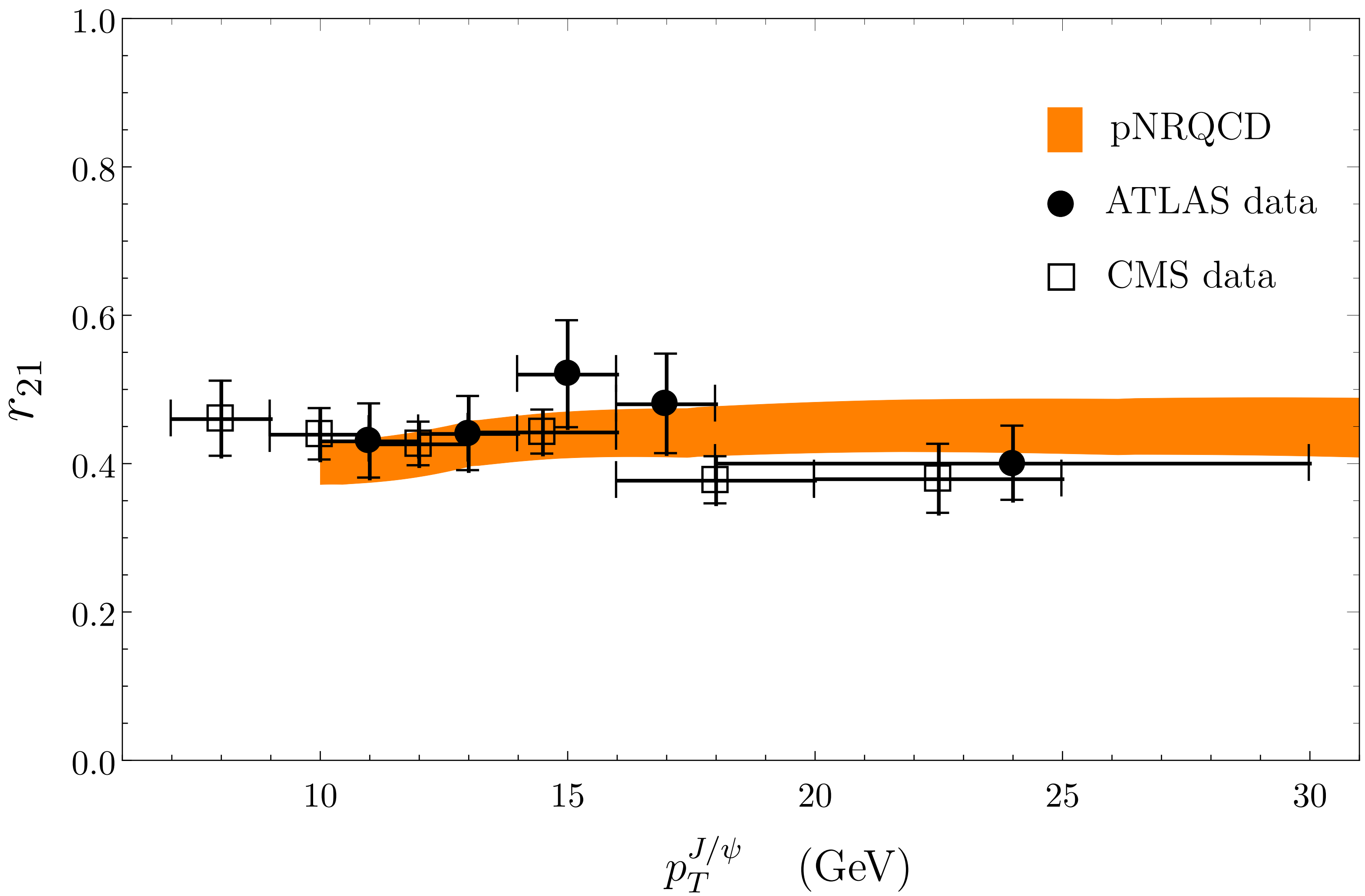
$$\frac{d}{d \log \Lambda} \mathcal{E}(\Lambda) = 12 C_F \frac{\alpha_s}{\pi} \qquad \frac{d}{d \log \Lambda} \langle \mathcal{O}^{\chi_{QJ}}({}^3S_1^{[8]}) \rangle = \frac{4 C_F \alpha_s}{3 N_c \pi m^2} \langle \mathcal{O}^{\chi_{QJ}}({}^3P_J^{[1]}) \rangle$$

- In principle, $\mathcal{E}$ can be determined from lattice QCD. Since a lattice calculation is unavailable, we determine $\mathcal{E}$ from measured χ_{cJ} cross section ratios to obtain $\mathcal{E}(\Lambda = 1.5 \text{ GeV}) = 1.97 \pm 0.06$

P-WAVE CHARMONIUM PRODUCTION

- Cross section ratio $r_{21} = \sigma(\chi_{c2})/\sigma(\chi_{c1})$ at the LHC compared to ATLAS and CMS data.

CMS, EPJC72, 2251 (2012)
ATLAS, JHEP07, 154 (2014)



Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

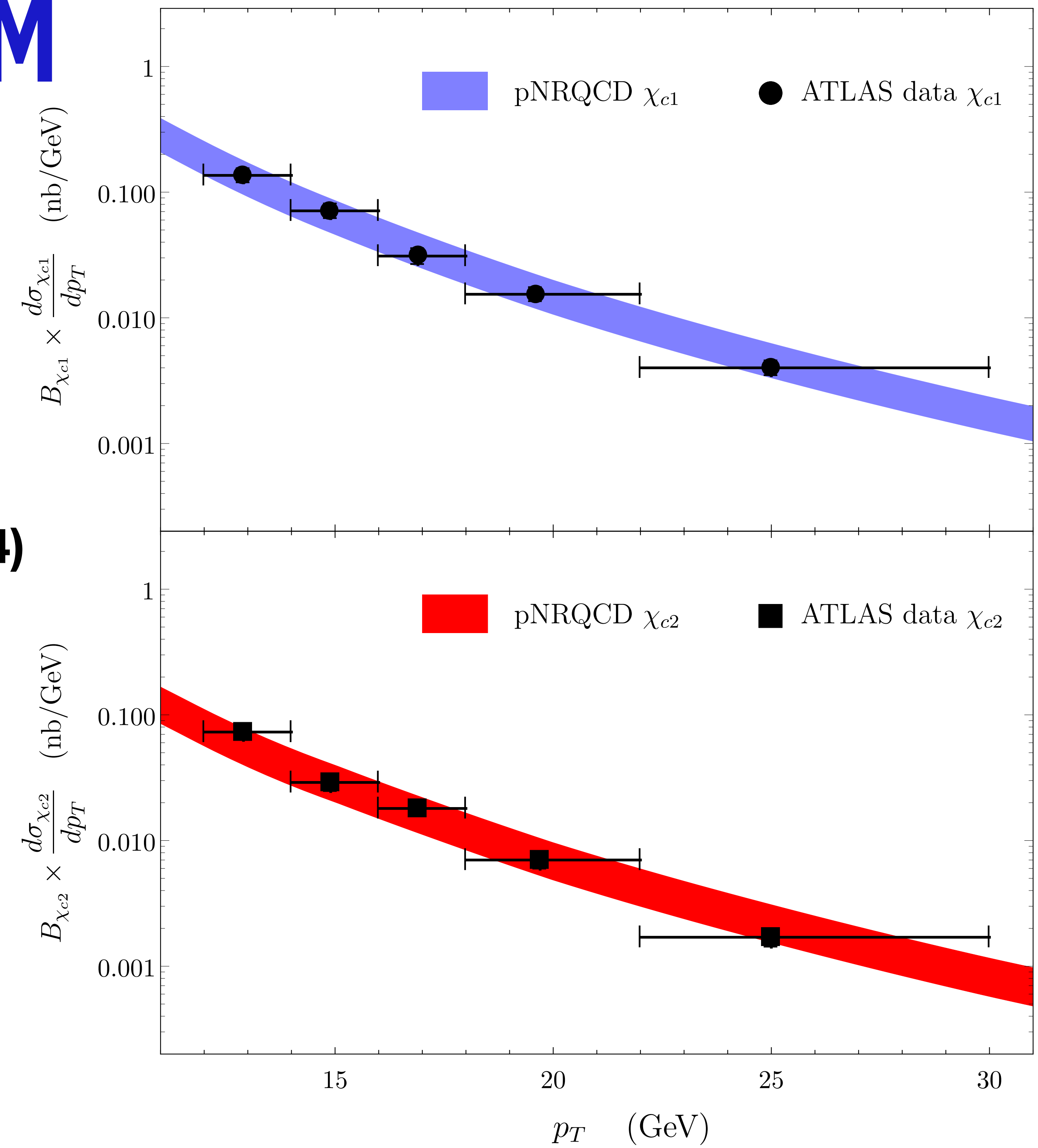
P-WAVE CHARMONIUM PRODUCTION

- χ_{c2} and χ_{c1} cross sections at the LHC, compared to ATLAS data.

ATLAS, JHEP07, 154 (2014)

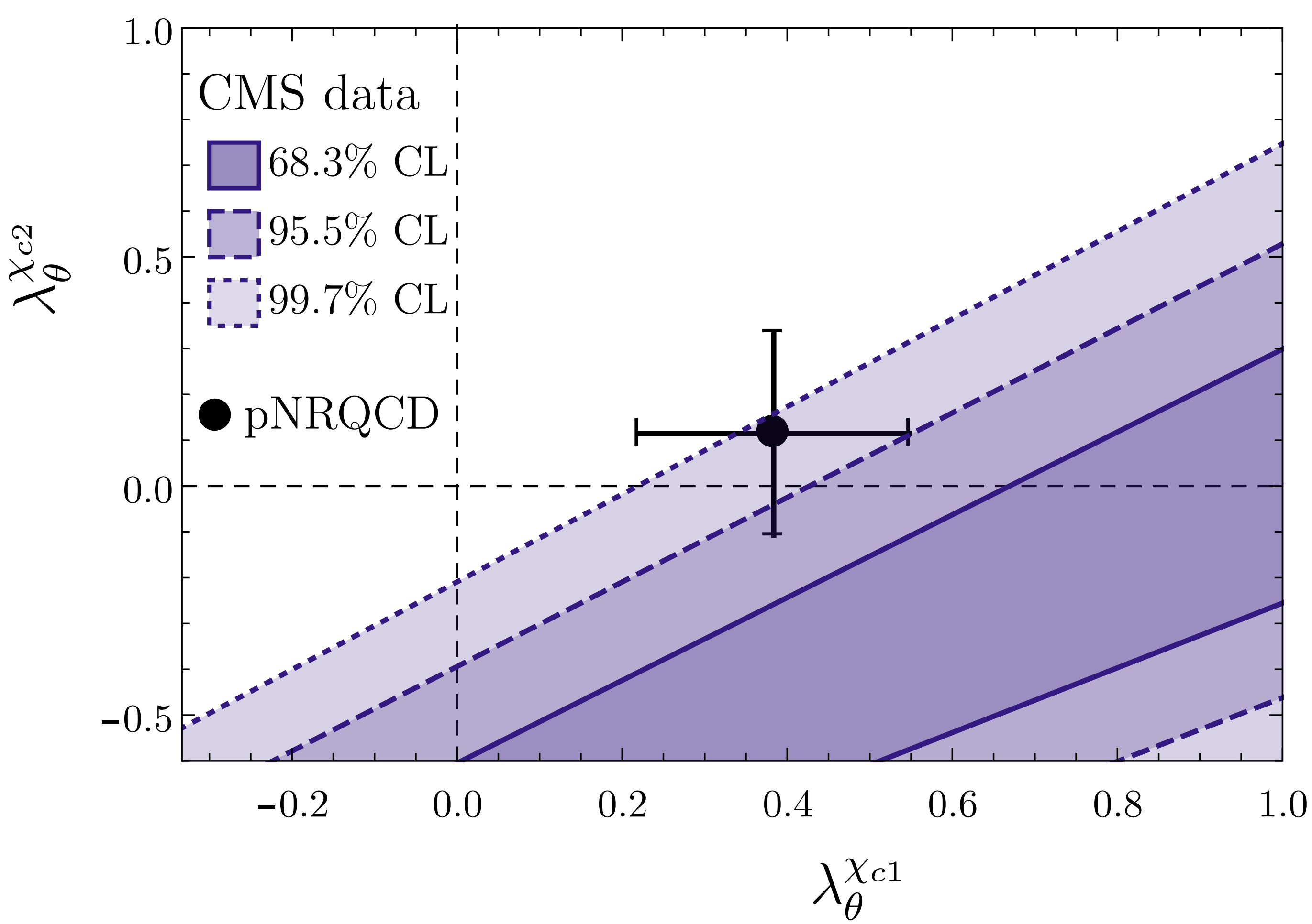
- Wavefunctions at the origin obtained from two-photon decay rates of χ_{c2} and χ_{c0}.

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)



P-WAVE CHARMONIUM POLARIZATION

- χ_{c2} and χ_{c1} polarization at the LHC compared to experimental constraints from CMS. **CMS, PRL124, 162002 (2020)**

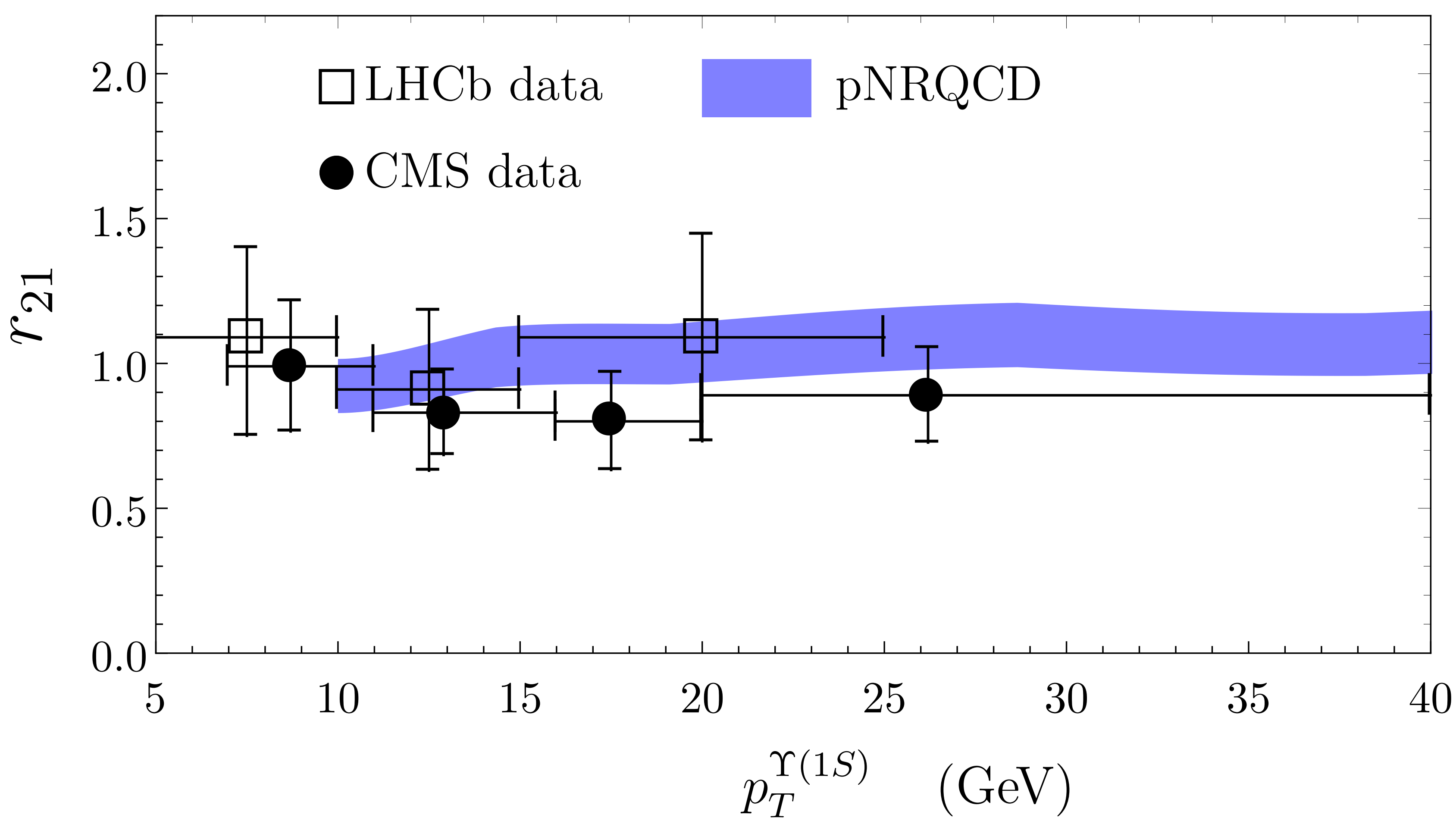


Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s + resummed logarithms from Bodwin, Chao, HSC, Kim, Lee, Ma, PRD93, 034041 (2016)

P-WAVE BOTTOMONIUM PRODUCTION

- Cross section ratio $r_{21} = \sigma(\chi_{b2})/\sigma(\chi_{b1})$ of $1P$ states at the LHC compared to LHCb and CMS measurements.

LHCb, JHEP10, 088 (2014)
CMS, PLB743, 383 (2015)

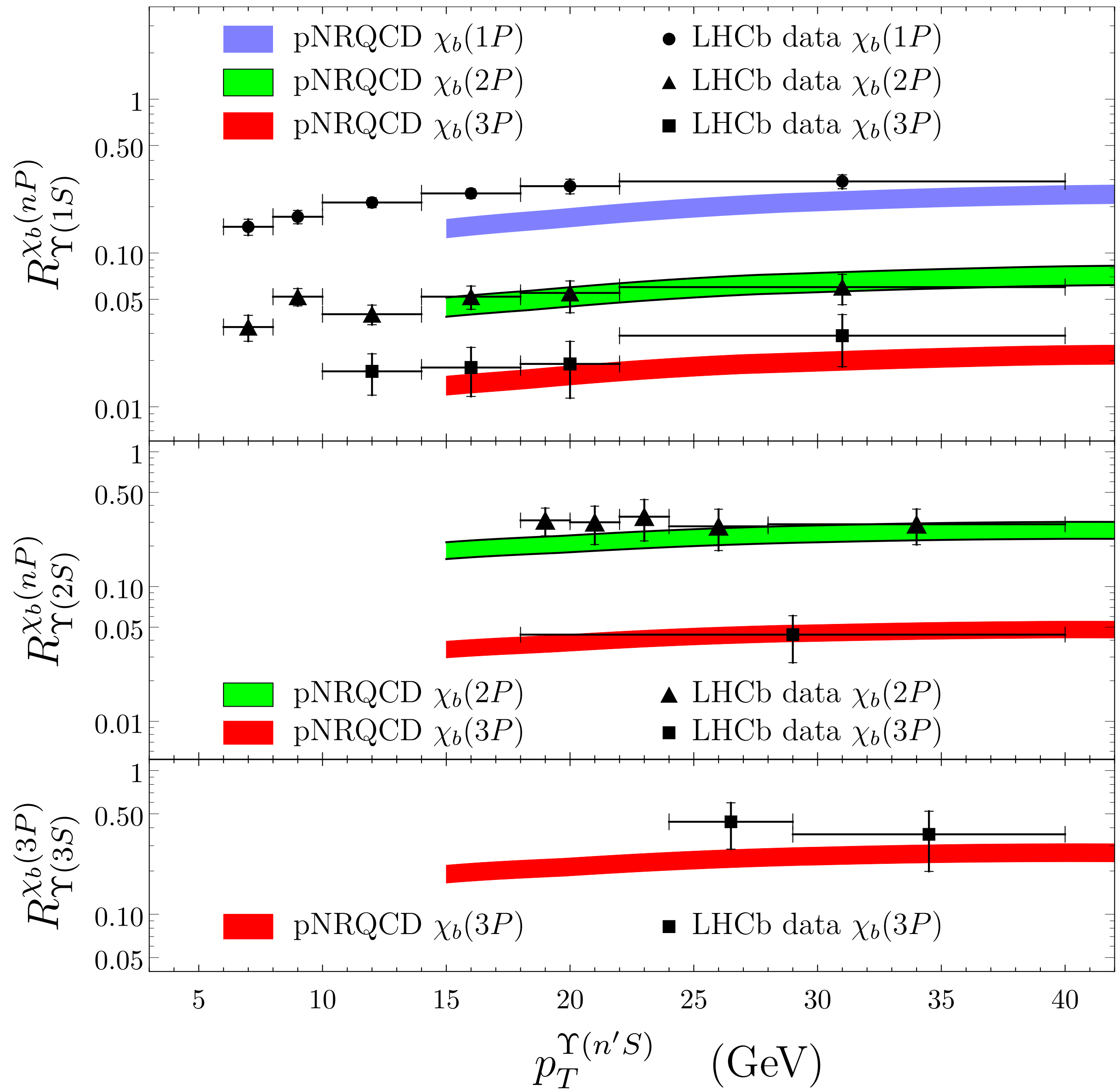


Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

P-WAVE BOTTOMONIUM PRODUCTION

- $\chi_{bJ}(nP)$ production rates relative to $\Upsilon(n'S)$ cross sections at the LHC compared to LHCb measurement of feeddown fractions.

LHCb, EPJC74, 3092 (2014)



$$R^{\chi_b(nP)}_{\Upsilon(n'S)} = \sum_{J=1,2} \frac{\sigma_{\chi_{bJ}(nP)} \times \mathrm{Br}_{\chi_{bJ}\to\Upsilon(nS)+\gamma}}{\sigma_{\Upsilon(n'S)}}$$

Perturbative $Q\bar{Q}$ cross sections computed at NLO in α_s using FDCHQHP Package from Wan and Wang, Comput. Phys. Commun. 185, 2939 (2014)

χ_{bJ} **wavefunctions from averages of potential-model calculations**

$\Upsilon(nS)$ **matrix elements taken from fits to data in Han, Ma, Meng, Shao, Zhang, Chao, PRD94, 014028 (2016)**

SUMMARY AND OUTLOOK

- We developed a formalism for ***inclusive production of heavy quarkonium*** in *strongly coupled* ***potential NRQCD***.
- For the ***first time***, ***first-principles determination*** of ***color-octet production matrix elements*** may become possible.
- A ***single gluonic correlator*** leads to determination of ***all P-wave charmonium and bottomonium*** cross sections, which are in ***good agreements with LHC measurements***.
- This formalism can also be applied to *S*-wave quarkonia $(J/\psi,\ \eta_c,\ \Upsilon,\ \eta_b)$, and may be extended for exotic states.
- Lattice determinations of gluonic correlators are desirable.