Phase diagram of strongly interacting matter and how should we study it in heavy ion collisions

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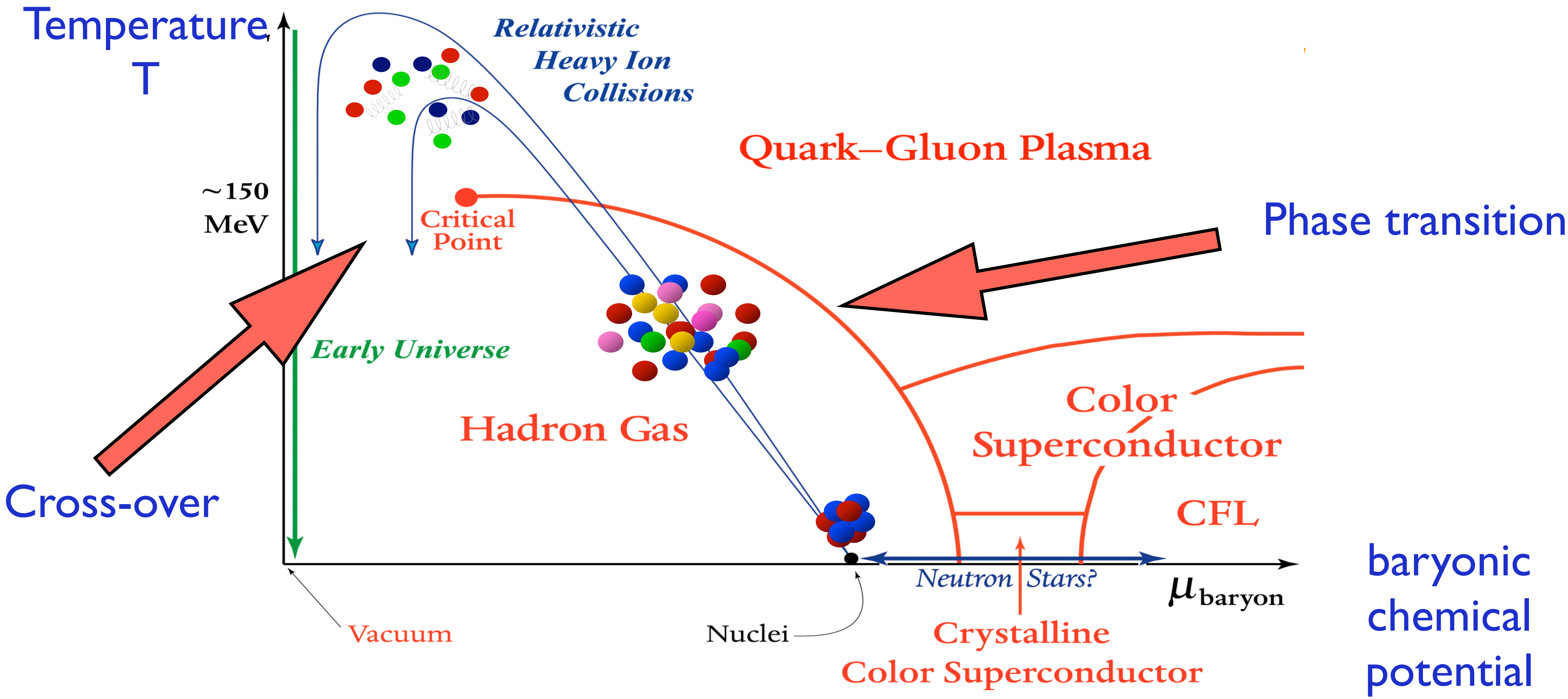
Hot Problems of Strong Interactions, November 13, 2020

Outline

1. Our results vs problems on a BIG scale
2. Examples of crude approximations
3. A few words about the fate of nuclear multifragmentation community
4. Exactly solvable statistical models: some results and history
5. IST and ISCT concepts and their relation to morphological thermodynamics
6. Conclusions

QCD EoS is unknown at & beyond CEP

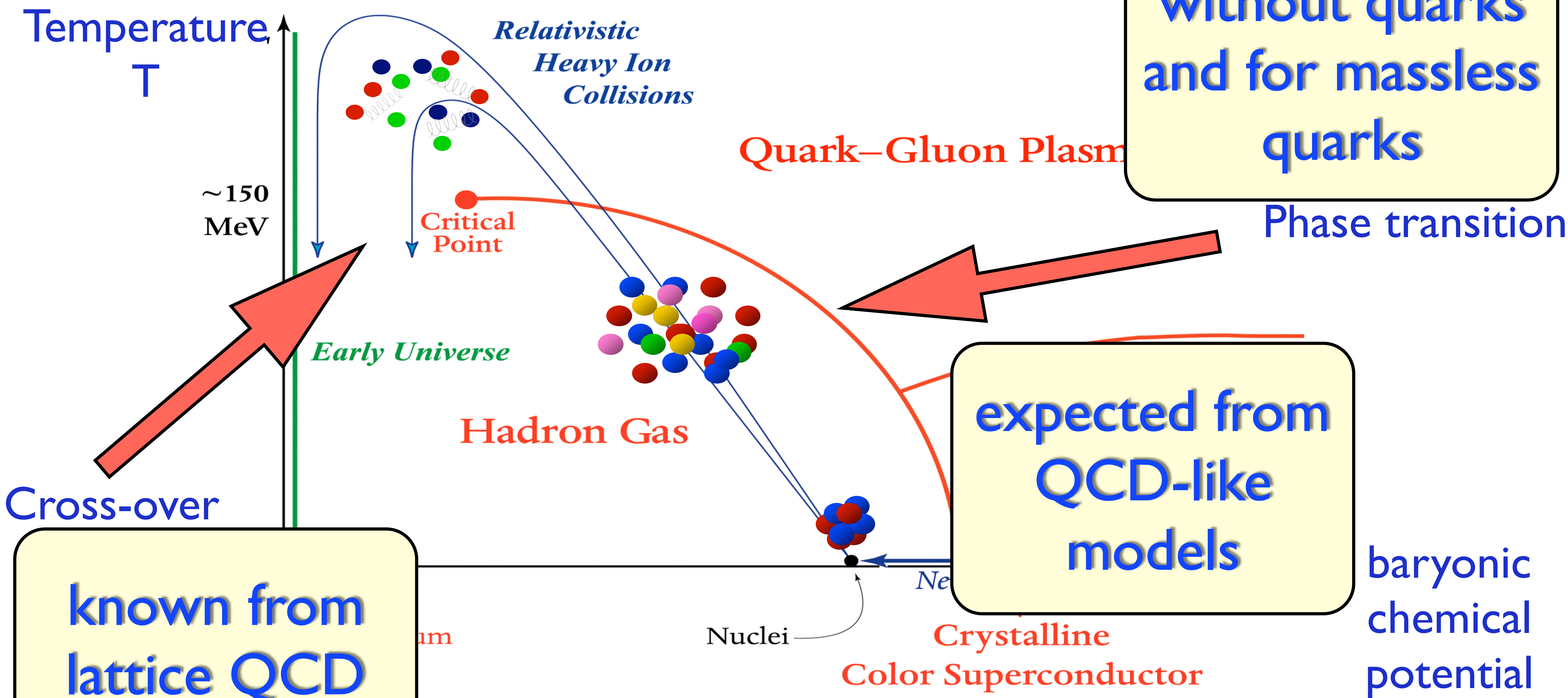


QGP is a dense phase, i.e. it is liquid-like!

But in contrast to our everyday experience (boiling water) QGP appears at higher temperatures!

QCD EoS is unknown at & be

known from lattice QCD without quarks and for massless quarks



expected from QCD-like models

known from lattice QCD for realistic quark m

ase, i.e. it is liquid-like!

r everyday experience (boiling water) QGP appears at higher temperatures!

Why Do We Study the QCD Phase Diagram?

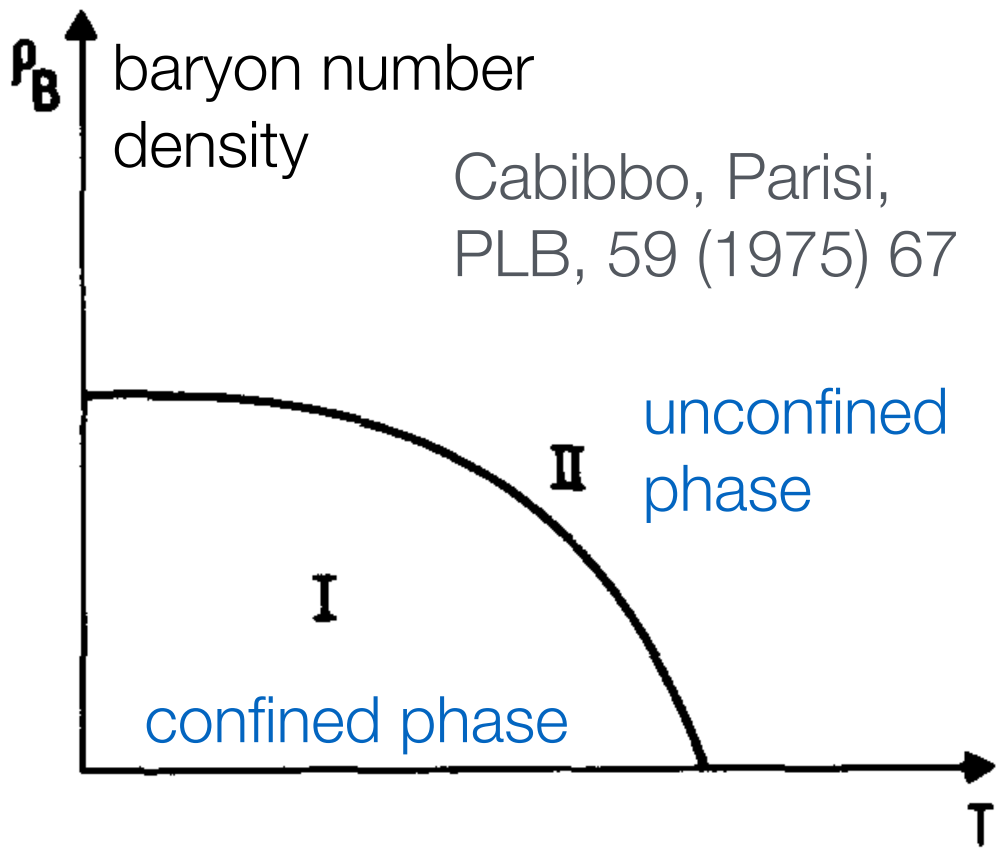
1. We want to find the QCD phase transition(s) experimentally

2. We want to locate (tri)CEP experimentally

3. We want to convince the colleagues from our community and physicists from other communities that goals 1. and 2. are achieved

Compare the Present QCD Phase Diagram

With the first one **(45 years old)**



assumed that deconfinement is a 2-nd order PT

Cabibbo, Parisi, PLB, 59 (1975) 67

- Exponential hadron spectrum not necessarily connected with a limiting temperature
- Rather: Different phase in which quarks are not confined

Have we really gone far From Cabibbo-Parisi Phase diagram?

Our Tools to Study QCD Phase Diagram

Experiments on HIC — about 40 years old

Lattice QCD
(computations, analytical) — about 40 years old

pre-QCD and QCD phenomenology — about 55 years old

Don't you think that after so long time

1. We did not find much in the experiments to reach our original goals 1 and 2?
2. We did not learn the hard lessons and, hence, we do not care why is it so?

Objective Reasons

- The first reason is that in the presence of realistic quarks the deconfinement PT **HAS NO well defined ORDER PARAMETER!** Thus, we have to study what is not well defined.

- The second reason is that we are dealing with finite and short living systems => **What is a PT in finite and short living system = ?**

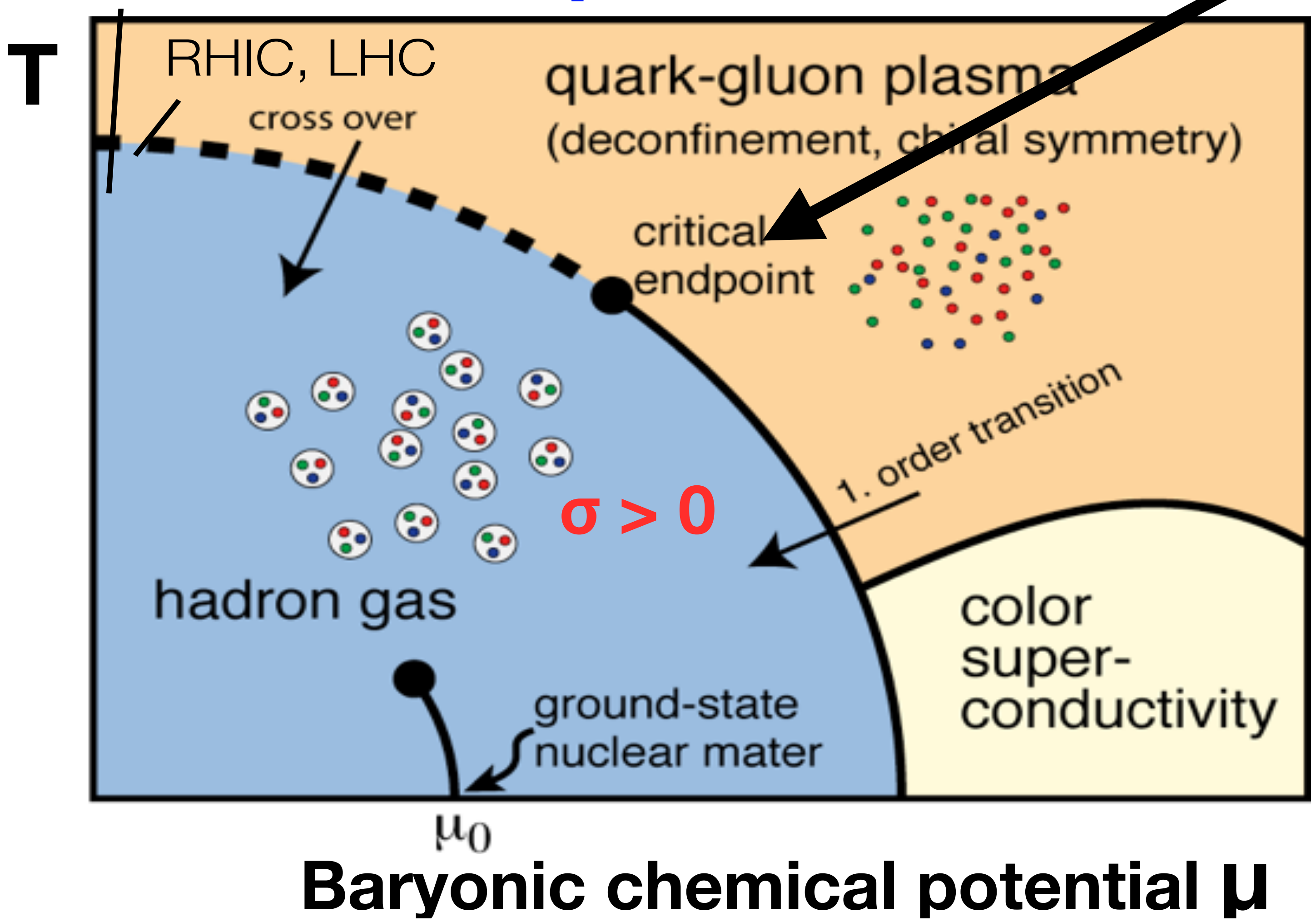
- The third reason is due to **TREMENDOUS COMPLEXITY** of the phenomena to be modeled and understood!

- The fourth reason is that we face **the fundamental problems which were not resolved by the other branches of physics!**

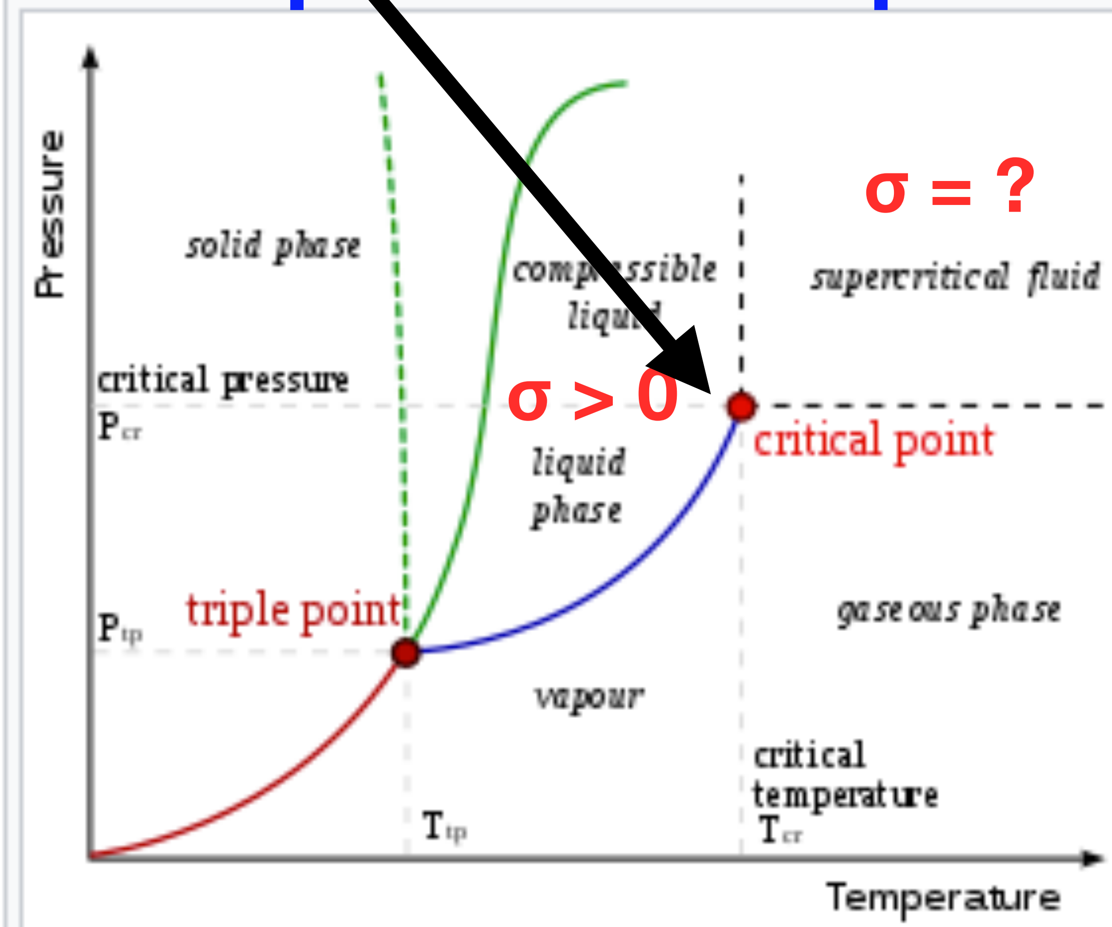
An Important Example of Objective Reason 4

What is the physical reason that the 1-st Order PT curve Is terminated? Experiments show that at (3)CEP the surface tension coefficient σ is 0, but what is it at T>Tcep?

Our expectations



Example from Wikipedia



A typical phase diagram for a single-component material, exhibiting solid, liquid and gaseous phases. The solid green line shows the usual shape of the liquid–solid phase line. The dotted green line shows the anomalous behavior of water when the pressure increases. The triple point and the critical point are shown as red dots.

So far, the only reason which may prevent the condensation of hadrons into a large bag is negative Surface tension coefficient for T>Tcep.

K.A.B. PRC(2007)

Subjective Reasons

We use very crude approximations which are not suited to model finite systems!

Typical example is the mean-field approximation

It can not be used to model phase transformations in finite systems, since it has the PT, even if the system volume contains just a single nucleon!

I have not seen the mean-field theoretical model which contains ALL existing hadrons and resonances!

Hence it is hard (if possible at all!) to exactly transform particles of mean-field models into the real particles measured by detectors!

Subjective Reasons 2

One cannot study the QCD (3)CEP with the mean-field approximation, since all mean-field models belong to the Universality class of the classical systems, like the Van Der Waals EoS.

=> Heat capacity, order parameter exponents behave differently

=> different predictions!

$$\textbf{VdWaals exponents:} \quad \alpha' = 0, \quad \beta = \tfrac{1}{2}, \quad \gamma' = 1, \quad \delta = 3$$

Relevant to QCD (3d Ising model)

	2d Ising model	Simple liquids	3d Ising model
α'	0	0.09-0.11	0.1096 ± 0.0005
β	$\frac{1}{8}$	0.32-0.35	0.3265 ± 0.0001
γ'	$\frac{7}{4}$	1.2-1.3	1.2373 ± 0.0002
δ	15	4.2-4.8	4.7893 ± 0.0008

Relevant to QCD

	O(3)	O(4)
α'	-0.115(9)	-0.19(6)
β	0.3645(25)	0.38(1)
γ'	1.386(4)	1.44(4)
δ	4.802(37)	4.82(5)

Moreover, up to now we do not know what is the finite volume analog of the CEP and how to define it!

What is to be done?

Theory of liquid-gas PT for Finite Systems Must Be Worked out

Otherwise our future will be similar to the nuclear liquid-gas PT related to a multifragmentation phenomenon

Multifragmentation was predicted in 1985
by J. Bondorf, I. Mishustin et al.
Experimentally it was discovered in 1995 by ALADIN

Till now they are discussing the ultimate signals of PT!

Due to absence of strong hydro flow the signals are much more clean!

=> The quality of their data predictions is beyond our dreams in HIC!

But due to a strong role of Coulomb interaction at late stage they have no thermodynamic limit and, hence, no proof of PT existence.

Actually, presence of a Coulomb interaction will be our headache soon!

Nuclear Multifragmentation

Nuclear caloric curve

The quality of their data predictions is beyond our dreams in HIC!

Predicted in 1985 within the SMM
Bondorf, Donangelo, Mishustin, Schulz
NPA 444 (1985) 460

Experimental discovery
Pochodzalla and ALADIN collaboration,
PRL 75 (1995) 1040

J. Bondorf et al. / Statistical multifragmentation 469

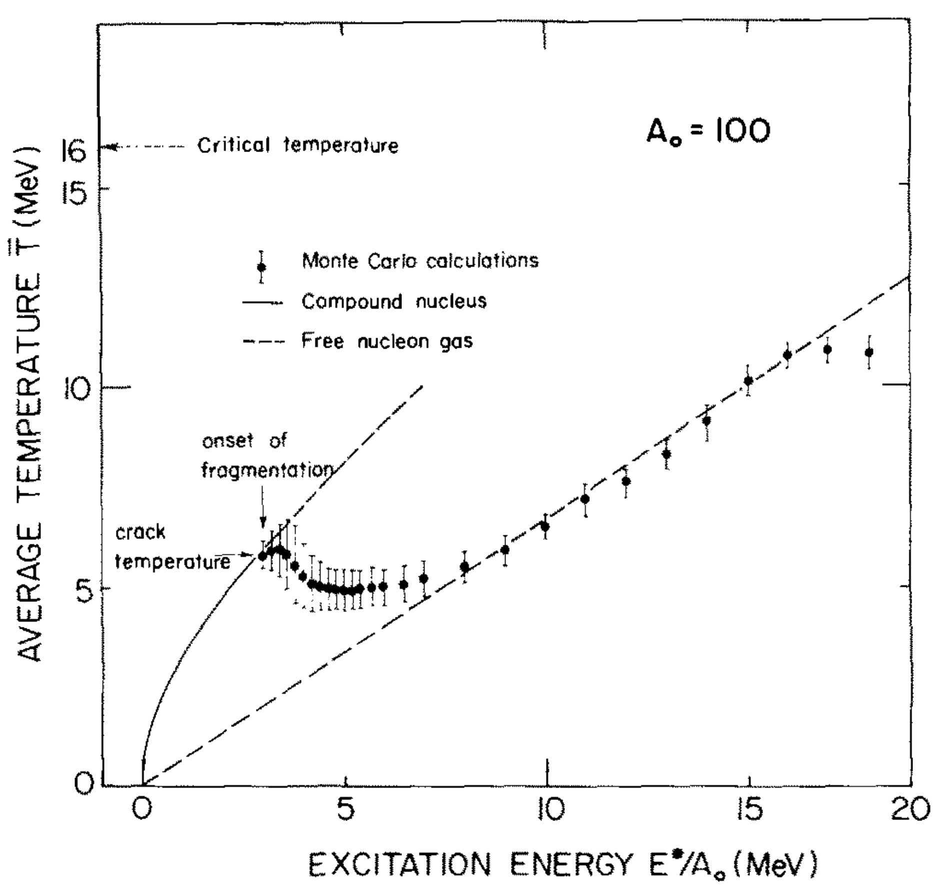


Fig. 4. The average temperature T as a function of the excitation energy E^*/A_0. The dashed line illustrates the temperature of a free nucleon gas.

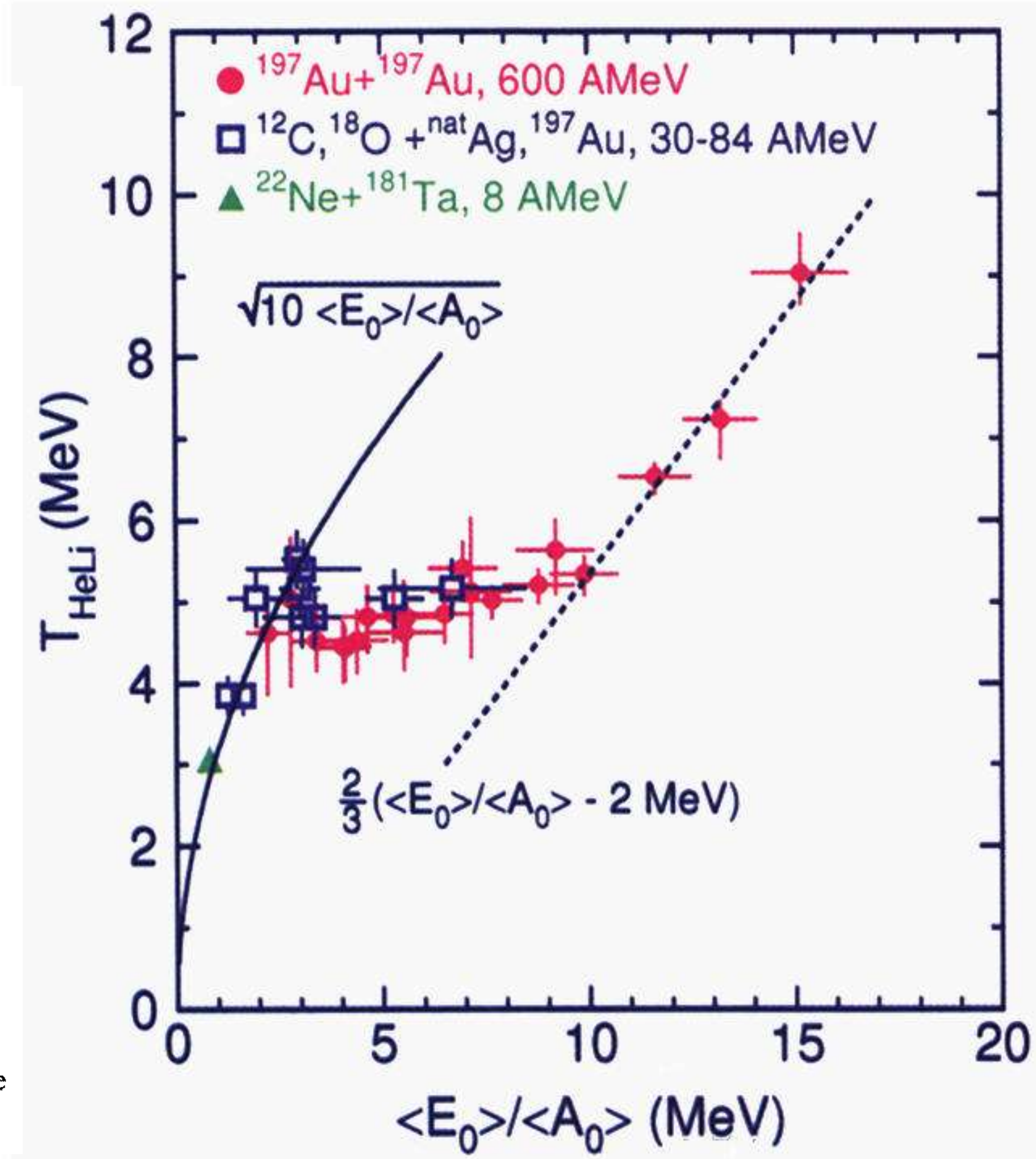
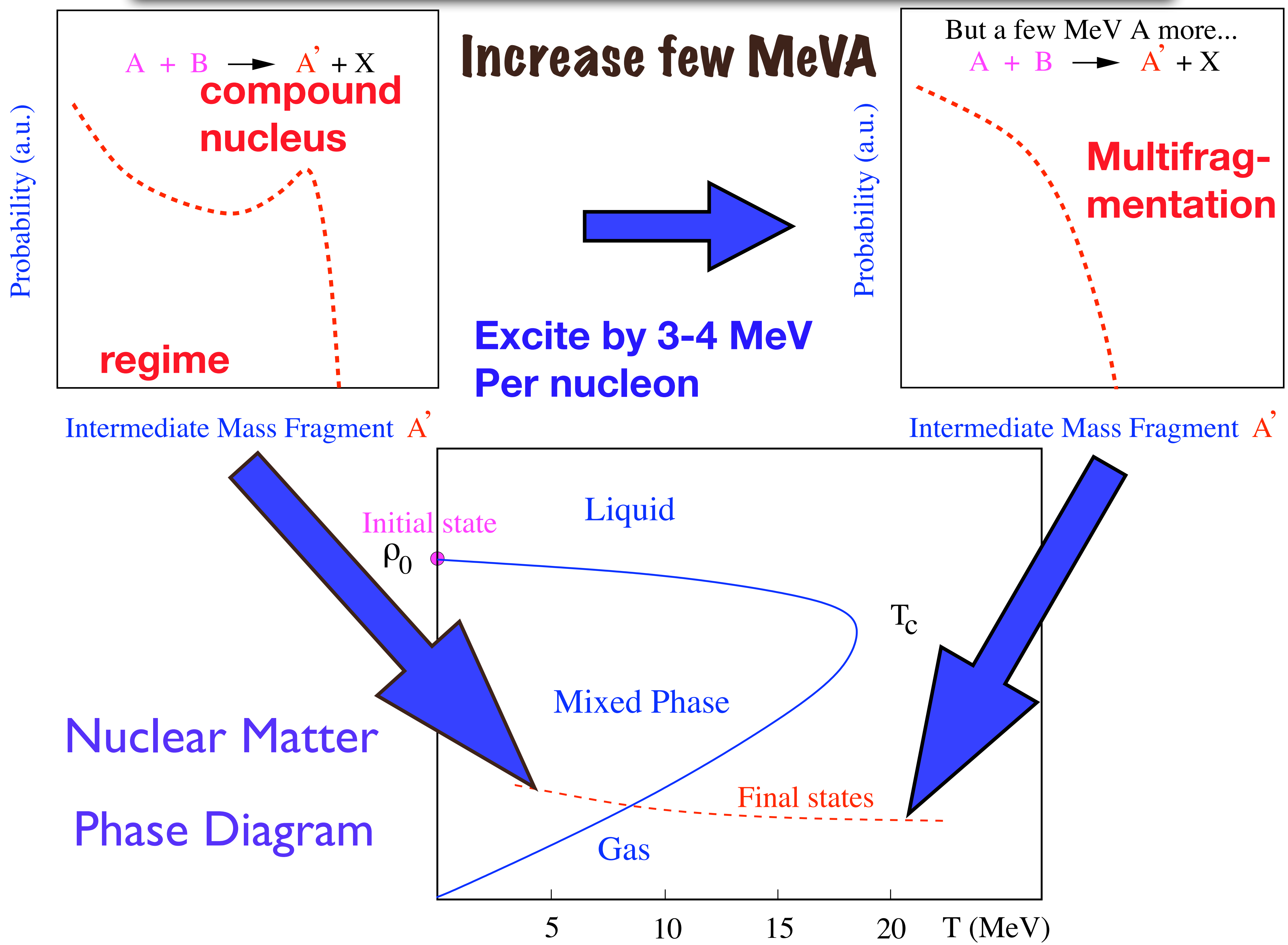


FIG. 2. Caloric curve of nuclei determined by the dependence of the isotope temperature T_{HeLi} on the excitation energy per nucleon. The lines are explained in the text.

Multifragmentation



Explanation: final state moves from mixed phase to the gas!

But there is NO such phase diagram in thermodynamic limit…

One Possibility is to Use Statistical Models (Short History)

Stat.Bootstrap Model, S.Frautschi, 1971

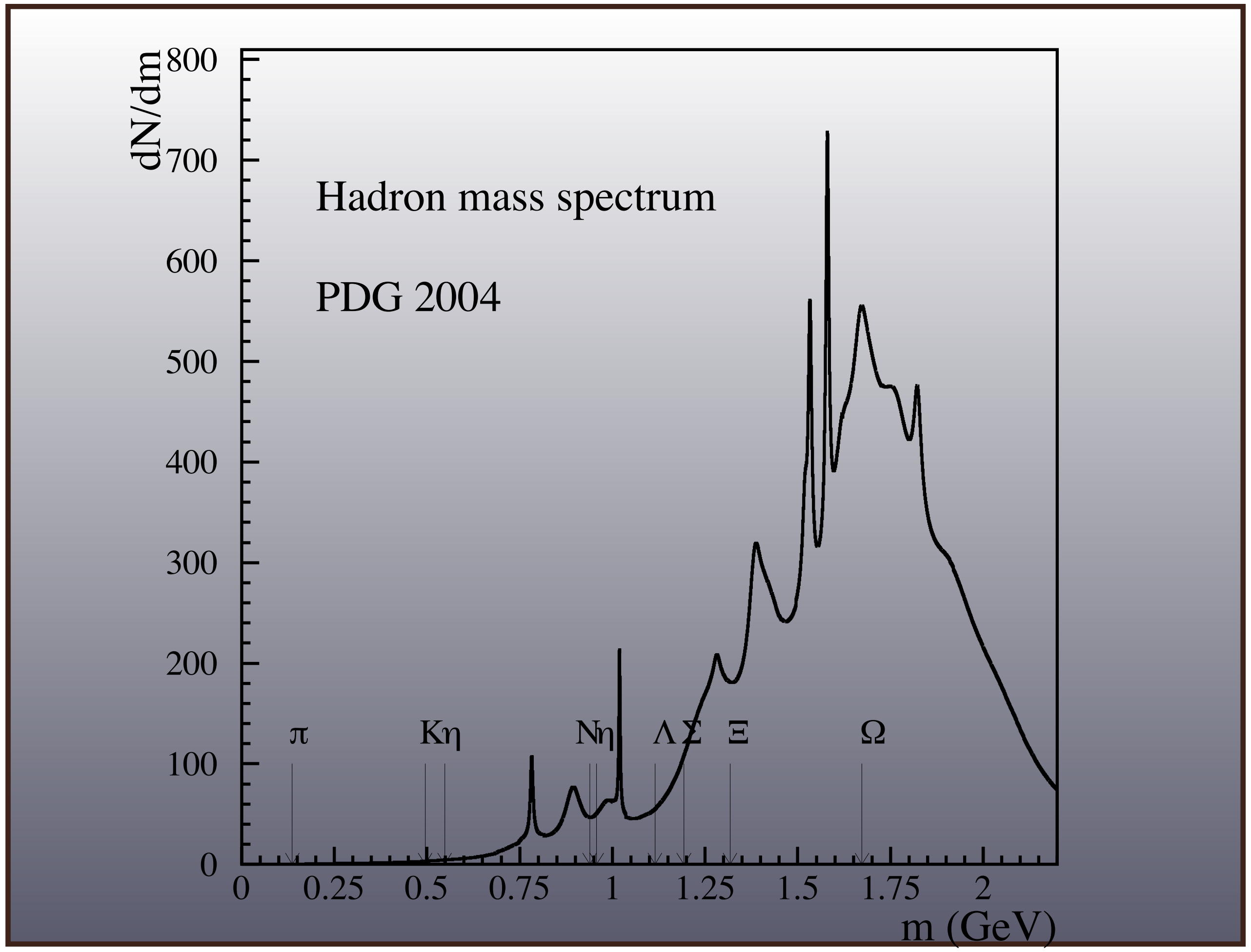
Hadrons are built from hadrons

Idea belongs to Rolf Hagedorn, but equation was written by Steven Frautschi

This model is really great and it leads to the Hagedorn (exponential) mass spectrum of heavy hadrons!

The Hagedorn mass spectrum follows from several models including string model and 3+1 QCD in Large Nc limit:

Large Nc limit of 3+1 QCD
T. Cohen, 2009



Unfortunately Hagedorn is not observed experimentally...?

The only reasonable explanation is that hadronic resonances or bags of mass M have a large width $\Gamma \sim \sqrt{M}$ and hence cannot be observed!

<= **K.A.B., V.K. Petrov, G.M. Zinovjev, EPL(2009)**

Short History 2 (realistic models)

M.I.T. Bag Model, J.Kapusta, 1981

Hadrons are quark-gluon bags

J. I. Kapusta, Phys. Rev. D 10, 2444 (1981)

=> This model cannot have the usual CEP or triCEP

Highly unrealistic model of the Gas of Bags, since the key Element of statistical approach is missing

Experiments and exactly solvable models of liquid states show that

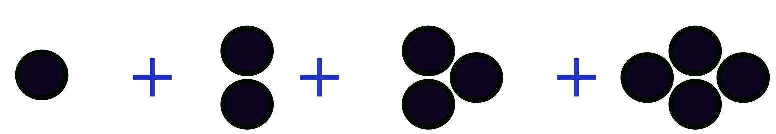
The real gases consist of droplets (clusters) of all possible sizes!

Fisher Droplet Model (FDM)-

Condensation of gases

M. Fisher, Physica 3 (1967);

J.B. Elliott et al, nucl-ex/0608022 (2006)

Real gas =  **...**

Statistical Multifragmentation Model (SMM) [without Coulomb interaction]- Liquid-Gas PT in nuclear matter

J. P. Bondorf et al, Phys. Rep. 257(1995);

K.A.B., Phys. Part. Nucl. 38 (2007);

Only this fact explains the reason of how the liquid appears from gas!

Short History 3 (Role of Surface Tension)

The key element of elaborate statistical models is the T-dependent surface tension $\sigma(T)$ of large droplets (or bags or nuclear clusters)

In Kapusta's Gas of Bags model PRD (1981) this element is missing!

$\sigma(T) > 0$ for $T < Tcep$
known from real gas-liquid PTs
$\sigma(T) = 0$ for $T = Tcep$

$\sigma(T) = ?$ for $T > Tcep$
NOT known from Experiments

Nice prediction: at $\sigma(Tcep)=0$ => large the mass distribution of droplets (or bags or nuclear clusters) has a power-law!

By 2005 I had an exactly solvable statistical model of surface partition of large clusters in which it was possible to show that degeneracy factor g(V) of large cluster of mean volume V is
(Vo is the volume of minimal cluster)

$$g(V) \approx 0.3814 \exp[\, 1.0609\, (V/Vo)^{2/3}\,]$$

=> $\sigma(T) = \sigma(0) - 1.0609\, T$

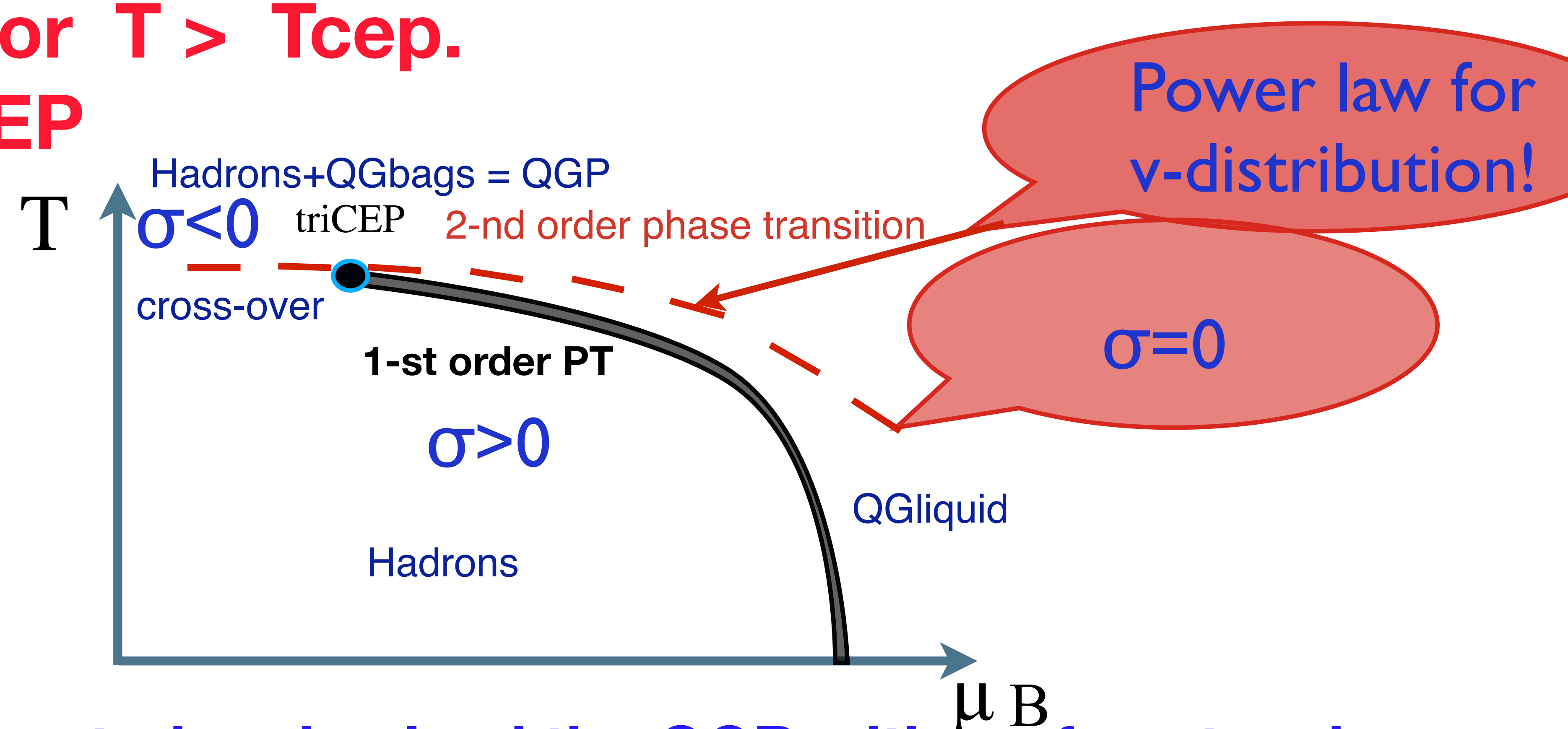
K.A.B., J.B. Elliott, L. Phair PRE (2005)

=> At high T the surface tension coefficient MUST be negative!

Short History 4 (3CEP vs CEP)

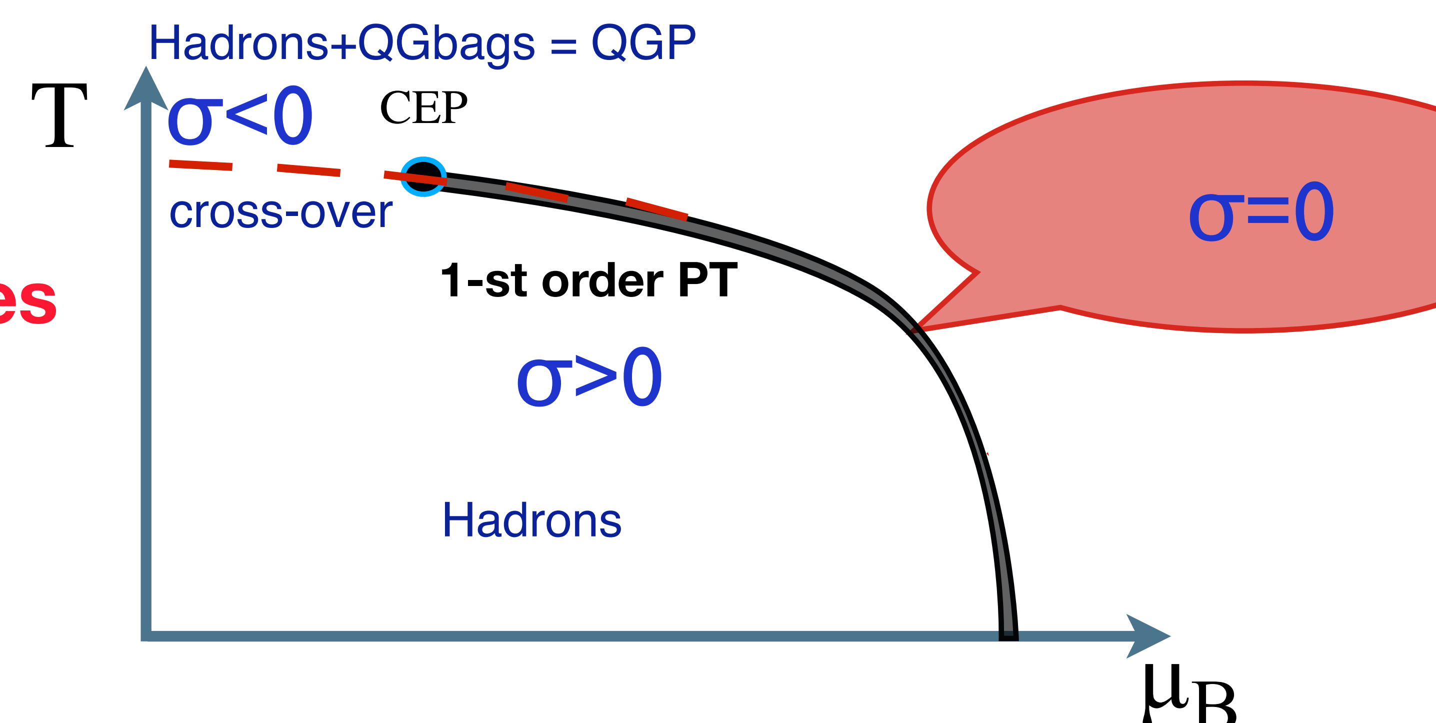
Hence in 2007 I suggested and solved the QGB with surface tension model with σ(T) < 0 for T > Tcep.

This model has 3CEP



K.A.B. PRC(2007)76

In 2012 our group suggested and solved the QGB with surface tension model with σ(T) < 0 for T > Tcep which has a CEP!



It was necessary to assume that Surface tension coefficient vanishes EXACTLY on 1-st order PT curve

K.A.B., V.K.Petrov, G.M.Zinovjev
Phys. Part. Nucl. Lett. (2012) 9

Short History 5 (new HRGM)

In a mean time I exactly solved a couple of statistical models for finite volumes, but no one was interested in exact solutions and in 2012 I decided to switch to an analysis of hadronic yields in HIC.

The main purpose was to improve the hadronic part of solvable statistical models. The problem to solve was to go beyond the Van Der Waals Approximation (2-nd virial coefficients for hard-core repulsion) for mixture of particles of different hard-core radii.

During 2013-2017 our group developed new Hadron Resonance Gas Mode which is a very accurate tool to analyze data with only 2 or 3 extra parameters compared to best GSI version of HRGM

D. Oliinychenko, KAB, A. Sorin, Ukr. J. Phys. 58 (2013)

KAB, D. Oliinychenko, A. Sorin, G.Zinovjev, EPJ A 49 (2013)

KAB et al., Europhys. Lett. 104 (2013)

KAB et al., Nucl. Phys. A 970 (2018)

Most successful version of the Hadron Resonance Gas Model (HRGM) although the colleagues from GSI ignore it!

Together with the young group members, Dima Oliinychenko, Oleksii Ivanytskyi and Violetta Sagun, we solved several hard problems Of hadrons data description which no one was able to resolve and

Short History 6 (works on PT Signals in QCD)

and in addition we found New Signals of two QCD phase transitions!

The high quality description of data allowed us to elucidate new irregularities at CFO from data and to formulate new signals of two QCD phase transitions

D. Oliinychenko et al., Ukr. J Phys. 59 (2014)

KAB et al., Phys. Part. Nucl. Lett. 12 (2015)

KAB et al., EPJ A 52 (2016) No 6

KAB et al., EPJ A 52 (2016) No 8

KAB et al., Phys. Part. Nucl. Lett. 15 (2018)

First work on evidence of two QCD phase transitions

Consistent with Giessen group results W. Cassing et al., Phys. Rev. C 93, 014902 (2016)

But the main outcome was that we derived (heuristically and rigorously) the HRGM which is based on the concept of induced surface tension = surface tension induced by interaction of each particle with a medium.

pressure

$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$$

induced surface tension

$$\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \overbrace{\exp\left(\frac{(1-\alpha)S_i\Sigma}{T}\right)}^{\text{new term}}$$

R_k, V_k and S_k are hard-core radius, eigenvolume and eigensurface of hadron of sort k

Short History 7 (IST EoS)

pressure

$$\frac{p}{T} = \sum_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right)$$

induced surface tension

$$\frac{\Sigma}{T} = \sum_i R_i \phi_i \exp\left(\frac{\mu_i - pV_i - \Sigma S_i}{T}\right) \cdot \overbrace{\exp\left(\frac{(1-\alpha) S_i \Sigma}{T}\right)}^{\text{new term}}$$

R_k, V_k and S_k are hard-core radius, eigenvolume and eigensurface of hadron of sort k

Great advantages!

1. It allows one to go beyond the Van der Waals approximation, since it reproduces 2-nd, 3-rd and 4-th virial coefficients of the gas of hard spheres for $\alpha = 1.245$.

2. Number of equations is 2 and it does not depend on the number of different hard-core radii!

V.V. Sagun, K.A.Bugaev, A.I. Ivanytskyi, D.R. Oliinychenko, EPJ Web Conf 137 (2017)

K.A.Bugaev, V.V. Sagun, A.I. Ivanytskyi, E. G. Nikonov, G.M. Zinovjev et. al., Nucl. Phys. A 970 (2018) 133-155

V.V. Sagun, K.A.Bugaev, A.I. Ivanytskyi, et al., Eur. Phys. J. A 54, 100 (2018).

Short History 7 (quantum ISCT EoS)

In 2019 I invented the method to rigorously deal with mixtures of particles having the multi-compomnent hard-core repulsion and derived the EoS directly from quantum GCE partition K.A.B., EPJ. A (2019) 55

Then we successfully generalized it to the mixtures of classical hard spheres and hard discs N. Yakovenko, K.A.B., L. Bravina, E. Zabrodin arXiv: 1910.04889 [nucl-th] (to appear in EPJ ST)

=> Generalization of the concept of induced surface tension
To induced surface and curvature tension

Presently our group is working on its extension to mixtures of hadrons
With light nuclei and bags

A few months ago I leaned a bare truth that…

Morphological Thermodynamics

… R. Roth and Co from Stuttgart came to a similar conclusion about the full free energy of the fluid existing in a convex container of volume V(shape), surface S(shape), mean curvature C(shape) and Gaussian curvature X(shape)

Ω(shape) = - Vp + Sσ + Cκ + Xψ (fluid in a convex container)

Grand potential Ω = (Landau) free energy

Taking below Tcep and above Triple for a convex shape

Here p is system pressure, σ mean surface tension coeff.,
κ mean curvature tension coeff. (bending rigidity) and
ψ mean Gaussian curvature tension(Gaussian bending rigidity).

P.-M. König, R. Roth, and K. R. Mecke, Phys. Rev. Lett. 93 (2004) 160601

P. Bryk, R. Roth, K.R. Mecke S. Dietrich, Phys. Rev. E 68 (2003) 031602

R. Roth, J. Phys.: Condens. Matter 17 (2005) S3463

Apply it to a large nucleus, bag or droplet in a vacuum (carefully)
=> Eigensurface tension, eigencurvature and eigen Gaussian curvature tensions of your system (although hard to calculate!).

Morphological Thermodynamics 2

Approach is based on H. Hadwiger theorem

H. Hadwiger, Vorlesungen uber Inhalt, Oberflache und Isoperimetrie (Springer, Berlin, Germany, 1957)

and it is valid for interaction of finite range.

For a convex rigid body r inside the fluid (<=> concave volume for fluid) of system with pressure p, (induced) mean surface tension coeff. Σ, (induced) mean curvature tension coeff. K (bending rigidity) and (induced) mean Gaussian curvature tension Ψ (Gaussian bending rigidity) one has:

$$\Delta\,\Omega_r = -\,V_r p - S_r \Sigma - C_r K - X_r \Psi \qquad \text{(rigid body inside fluid)}$$

Here V_r is eigenvolume, S_r eigensurface, C_r eigenperimeter, …

Coincides with our findings, but we always find Ψ =0

=> Explains why surface and curvature tensions must vanish at (tri)CEP!

=> Our group is on the right track!

In addition our derivations are valid for the mixtures of rigid bodies, and for quantum statistics. Generalization to an attraction is in progress

EoS for Finite Systems with PT

Hard-core repulsion, volume of largest cluster (bag, nucleus) is about the system volume V + surface tension (parameterized) of all clusters

$$p\,(T, \mu) = F(T, (p_{liq} - p)V)$$

K.A.B., Acta. Phys. Polon. B (2005) 36

K.A.B. and P.T. Reuter, Ukr. J. Phys. (2007) 52

For infinite system

∃ a single real solution $p\,(T, \mu)$: gas, liquid or gas-liquid PT with $(p_{liq} - p) = 0$ and $\mathrm{Im}\,(\,p\,(T, \mu)\,) = 0$

For finite systems

∃ a single real solution $p\,(T, \mu)$ for gaseous phase with

$(p_{liq} - p) > 0$ and $\mathrm{Im}\,(\,p\,(T, \mu)\,) = 0$

+ pairs of complex conjugate solutions $p_k(T, \mu)$ with

$\mathrm{Re}(p_{liq} - p_k) > 0$ and $\mathrm{Im}_k(\,p\,(T, \mu)\,) \neq 0$

Most important: in finite systems the gas phase can exists for T and μ Values which in the limit $V \longrightarrow \infty$ belong to mixed and liquid phases!

In Finite Systems the Location of CEP Depends on its Definition!

For found solutions in finite systems the gas phase can exists for T and μ Values which in the limit V—> ∞ belong to mixed and liquid phases…

K.A.B., Acta. Phys. Polon. B (2005) 36

K.A.B. and P.T. Reuter, Ukr. J. Phys. (2007) 52

=> the surface tension coefficient vanishes in gas phase and

=> the power law in size distribution of clusters must exists in gas phase

On the other hand, the location of the isothermal compressibility peak depends on the interplay of surface tension coefficient σ, value of Fisher exponent τ, value of $Re(p_{liq} - p_k)$ and the size of considered system!

=> No general conclusion can be made, unfortunately

For IST and ISCT EoS in finite volumes was not investigated yet, but qualitative it should be similar.

Conclusions

1. Development of statistical models of QG bags with surface tension is on the right track!

2. In a few years we will make them very accurate for infinite systems.

3. IST and ISCT concepts should be extended to finite volumes and the theory of PTs in finite system must be developed.
   If the Dyson-Schwinger eq. approach or FRG method will solve this problem, it will be great!

4. To employ the realistic EoS for finite systems the hydro/kinetic approaches must be extended further to use complex free energies!

THANK YOU VERY MUCH FOR YOUR ATTENTION!

Back up Slides

Main Results obtained by multicomponent HRGM

During 2013-2020 our group developed a very accurate tool to analyze data which allowed us

1. **To resolve the (anti) Λ-hyperon puzzle in HIC**

2. **To accurately describe the K+/π+ and Λ/π- ratios, i.e. to resolve horns puzzles (for the first time)**

3. **To independently explain that the strangeness suppression factor γ_S is equivalent to individual CFO hyper surface for strange hadrons (for the first time)**

4. **To demonstrate that existing HIC data favor 2 phase transitions in QCD (for the first time)**

5. **To demonstrate that there are NO (anti)proton «puzzle» at ALICE and highest RHIC energies**

6. **To accurately describe the problematic Hyper-triton ratios (for the first time)**