Universality driven analytic structure of QCD crossover

Vladi Skokov

Based on:

S. Mukherjee and V.S.: 1909.04639

A. Connelly, G. Johnson, F. Rennecke, and V.S.: Phys.Rev.Lett. 125 (2020) 19, 191602

RBRC RIKEN BNL Research Center

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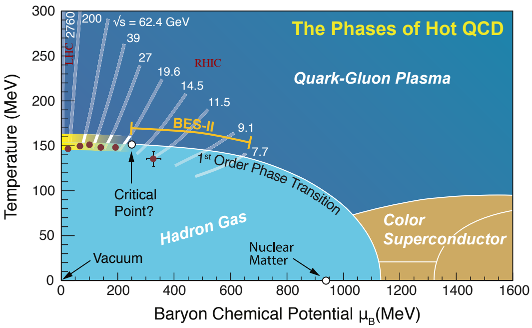
BEST COLLABORATION

NC STATE UNIVERSITY

Intro



- LQCD + universality argument $\rightsquigarrow$ new input on QCD phase diagram

Summary

Consider a system close to transition. There is a universal function of scaling variable which describe properties of this transitions. There is a non-universal map between parameters of the theory (T, μ, m) and the scaling variable. The analytic structure (location/type of singularities and cuts) of this universal function defines thermodynamics of the system. This function has poles: Yang-Lee Edge singularities. This singularities define the radius of convergence. The location of Yang-Lee Edge singularities z_c was not known until recently.

QCD crossover for small pion mass is described by the universal function. Thus there are singularities at complex μ. Their location can be found from lattice input on mapping $(T, \mu, m) \to$ the scaling variable and the value of z_c.

Universal function: computed for (simpler) theory in the same universality class as QCD.

Radius of convergence

Conventionally:

- LQCD has access to zero chemical potential (and imaginary)
- Taylor series: radius of convergence of power series is radius of largest disk in which the series converges
- Radius of convergence of power series is defined by closest singularity
- Example:

$$\frac{1}{1+x^2} = \sum_{n=0}^{\infty} (-1)^n x^{2n}$$

Radius of convergence

- Singularities of thermodynamic functions $\rightsquigarrow$ signal critical point, phase transitions

- Example 1: in chiral limit for O(4) line $\alpha < 0$

$$p \propto \#(T - T_c + \kappa\mu^2)^{2-\alpha} + p_{\text{reg}}$$

  Branch point singularity (when the argument $T - T_c + \kappa\mu^2 = 0$) on the real μ axis

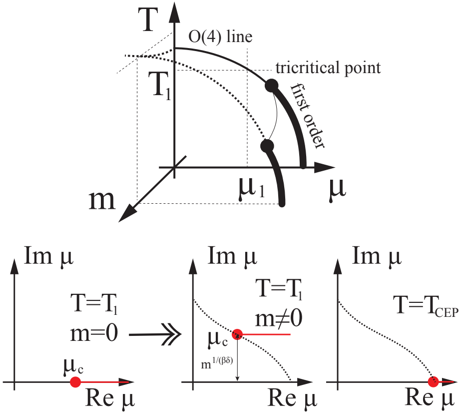
- Example 2: thermal singularity

$$f_{\text{FD}} = [e^{\frac{\omega}{T} - \frac{\mu}{T}} + 1]^{-1}$$

  Singularity on lines Im $\mu/T = \pm\pi$. Singularity at complex value of μ.

Radius of convergence

- Example 3: non-zero quark mass. At given temperature (above CP)

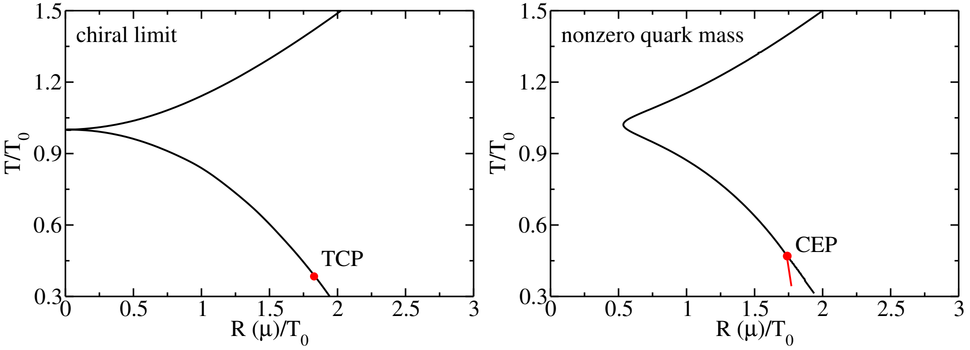


$$p \propto \#(\mu - \mu_{\rm YL})^{1+\sigma} + p_{\rm reg}$$

$\mu_{\rm YL}$ is complex number; $\sigma \approx 0.1$ is Yang-Lee edge singularity

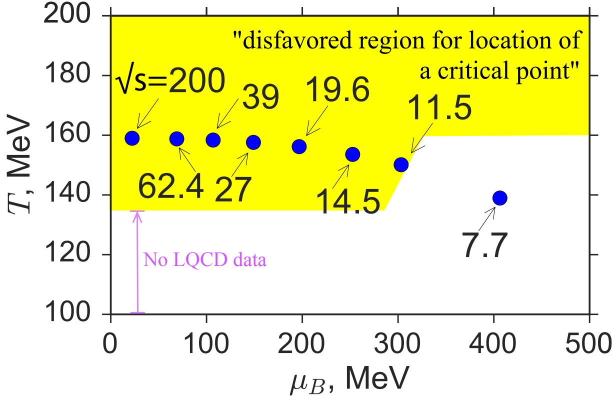
Radius of convergence

Mean-field quark-meson/NJL model:



Radius of convergence

- Analysis based on first few coefficients of Taylor series at $\mu = 0$:

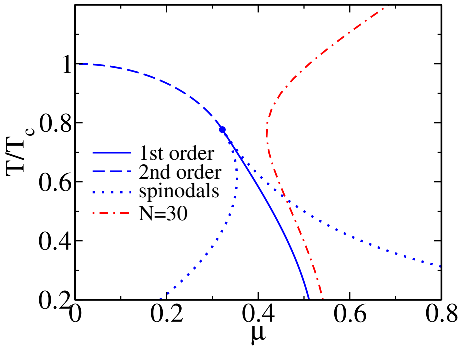


Replotted using data from BNL-Bi-CCNU Collaboration, arXiv:1701.04325

- Reversing the argument: Taylor expansion is reliable inside shaded region

Radius of convergence

- Precise mathematical statement about radius of convergence
  $\rightsquigarrow$ asymptotically high order coefficients



Even 30 Taylor coefficients are not sufficient to describe actual convergence radius

Radius of convergence

- Is there an alternative way?!
  - Based on first-principal QCD
  - Not relying on Taylor series expansion

- This talk: assuming that
  - QCD belongs to O(4) universality class in chiral limit *A. Lahiri's talk*
  - QCD is in scaling regime at physical pion mass

  it is possible to predict the radius of convergence based on universality argument and non-universal input from LQCD

Universality

- Scale invariance for free energy:

$$f = b^{-d} f(tb^{\lambda_t}, hb^{\lambda_h})$$

where $t = T - T_c$

- Inhomogeneity of this function leads to

$$f = h^{\frac{2-\alpha}{\beta\delta}} f_f(z = th^{-\frac{1}{\beta\delta}})$$

also often used “magnetic equation of state”

$$g = M/h^{1/\delta}$$

Universality

- In vicinity of QCD transition:

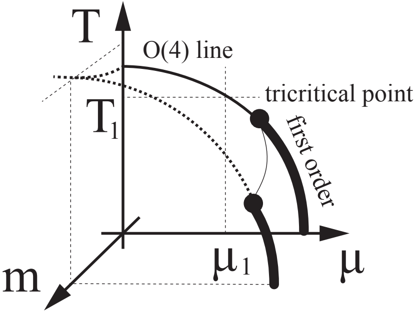
$$p(T,\mu_B)/T^4 = -\#h^{(2-\alpha)/\beta\delta} f_f(z) - f_{\text{regular}}(T,\mu_B)$$

$$z = z_0 t h^{-1/\beta\delta} = z_0 \left[\frac{T-T_c}{T_c} + \kappa_B \left(\frac{\mu_B}{T}\right)^2\right] \left(\frac{m_{u,d}}{m_s}\right)^{-1/\beta\delta}$$

$$t = \frac{T - T_c^0}{T_c^0} + \kappa_B \left(\frac{\mu_B}{T}\right)^2 \; ; \quad h = \frac{m_{u,d}}{m_s} \; .$$

O(4) critical exponents: $\alpha = -0.21$, $\beta = 0.38$, $\delta = 4.82$

- LQCD
  $T_c = 132^{+3}_{-6}$ MeV; $\kappa_B = 0.012(2)$; $z_0 = 1 - 2$



H. T. Ding et al. (2019), 1903.04801

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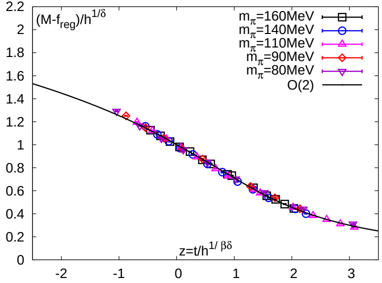


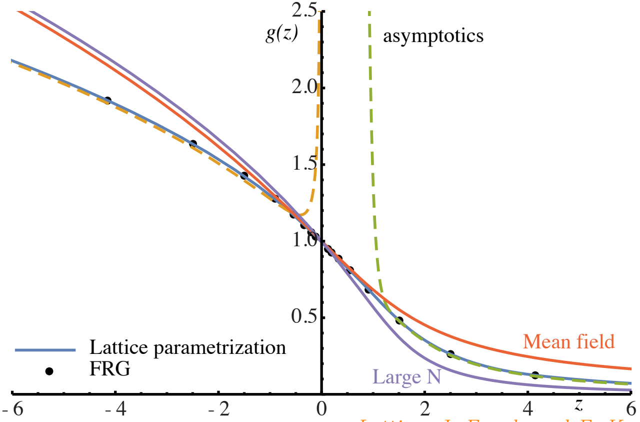
Sheng-Tai Li, Heng-Tong Ding, 1702.01294

- Recent progress from LQCD established:
  $T_c = 132^{+3}_{-6}$ MeV; $\kappa_B = 0.012(2)$; $z_0 = 1-2$

H. T. Ding et al. (2019), 1903.04801

O(4) scaling function: magnetic equation of state

$$M/h^{1/\delta} = g(z) \equiv \frac{z}{\beta\delta} f_f'(z) - \left(1 + \frac{1}{\delta}\right) f_f(z), \quad z = z_0 t / h^{1/\beta\delta}$$



Lattice: J. Engels and F. Karsch, arXiv:1105.0584
FRG: A. Connelly, G. Johnson, & V.S.

- The entire domain of this function is universal

Analytically computable models: mean-field and large N approximations

Analytical results for magnetic equation of state

$$g(z) \equiv \frac{z}{\beta\delta} f_f'(z) - \left(1 + \frac{1}{\delta}\right) f_f(z)$$

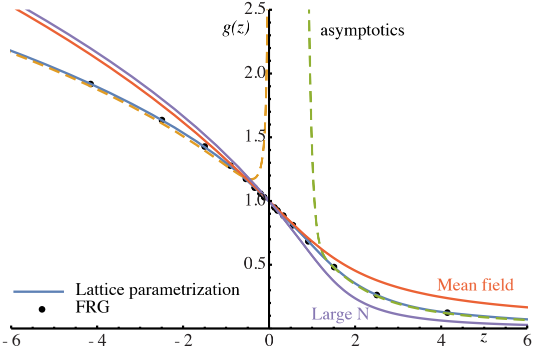
- Mean-field approximation starting from Landau functional $F(\phi) = t\phi^2/2 + \phi^4/4 - h\phi$ and equation of motion $F'(\phi) = t\phi + \phi^3 - h$ =0. Introducing $\phi = h^{1/3} g$ and $z = t/h^{2/3}$

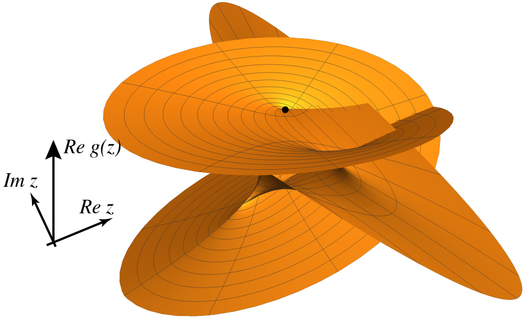
$$g(z)\left[z + g^2(z)\right] = 1, \quad z_c = \frac{3}{2^{2/3}}\, e^{\pm i \frac{\pi}{3}} \qquad |z_c| \approx 1.89$$

- Large N limit

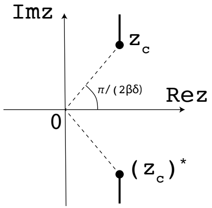
$$g(z)\left[z + g^2(z)\right]^2 = 1, \quad z_c = \frac{5}{2^{8/5}}\, e^{\pm i \frac{\pi}{5}} \qquad |z_c| \approx 1.649$$

Mean-field approximation: complex plane





Yang-Lee edge singularity I



Location of the singularity in complex z plane is universal
Know as Yang-Lee edge singularity (M. E. Fisher, 1978)

- What do we know about z_c?
  - $z_c = |z_c| \; e^{\pm i \frac{\pi}{2\beta\delta}}$

    for QCD, critical exponents are those of O(4) universality class

    In terms of h: Re $h = 0$. Consequence of $Z(h) = Z(-h)$ and Im $Z(h_c) = 0$.

  - YL edge singularity has its own critical exponent $f_f \sim (z - z_c)^{\sigma+1}$ with $\sigma \approx 0.1$

  - σ is independent of underlying symmetry class (of N for O(N))
    with only exception $\sigma_{N\to\infty} = 1/2$ (same in mean-field approximation)

Yang-Lee edge singularity II

- Field-theoretically, near z_c: ϕ^3 theory with imaginary coupling
- Upper critical dimension is 6; c.f. to 4 near O(N) critical point
- ϵ-expansion around 4 dimensions fails near YL edge singularity $\rightsquigarrow$ **no** input on $|z_c|$
  Due to presence of non-perturbative terms.
- Lattice simulations at imaginary h or complex T: sign problem $\rightsquigarrow$ **no** input on $|z_c|$

See also X. An, D. Mesterhazy, and M. Stephanov, arXiv:1605.06039

Jan Pawlowski's talk

Functional Renormalization Group approach to YL edge singularity I

Skipping technical details

- Consider a theory in the same universality class (O(N) field theory in our case)
- Extract critical exponent to x-check if they coincide with known results for O(N)
- Find non-universal parameters $(T_c, z_0, \ldots)$
- Extract the universal magnetic equation of state $g(z)$ for real z
- By introducing imaginary part to temperature or to symmetry breaking field (our choice), extend $g(z)$ to full complex plane z

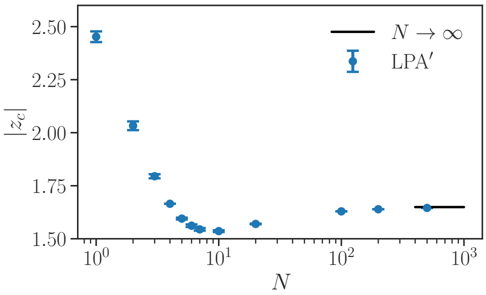
Main difficulty: this doubles the number of FRG equations to be solved

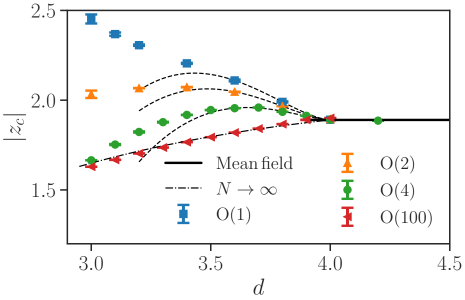
A. Connelly, G. Johnson, F. Rennecke, and V.S.: Phys.Rev.Lett. 125 (2020) 19, 191602

Functional renormalization group approach to YL edge singularity II

- Find z_c
- Check $z_c = |z_c|\, e^{\pm i \frac{\pi}{2\beta\delta}}$: Arg has to be consistent with critical exponents ✓
- Check that z_c as a function of N approaches large N limit (next slide)
- Check that z_c as a function of d approaches mean-field value when $d \to 4$ (next slide)
- Universal location $|z_c| \approx 1.665$ for O(4) scaling function
  $|z_c| \approx 2.032 \pm 0.021$ for O(2) and $|z_c| \approx 2.452 \pm 0.025$ for Z(2)

Functional renormalization group approach to YL edge singularity III





Reproduces

- analytic result in the large N limit
- mean-field result for $d \geq 4$

Dashed curves: fit of non-perturbative terms in $4 - \epsilon$-expansion.

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Radius of convergence I

- Having found z_c and using non-universal parameters from LQCD, one can find the radius of convergence

- Recall

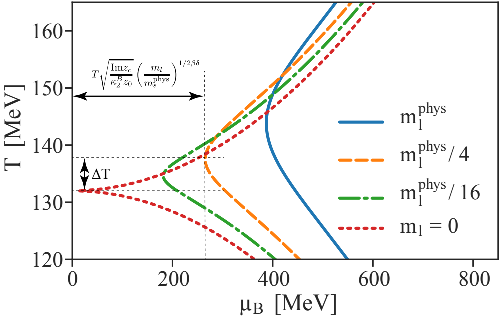
$$p(T,\mu_B)/T^4 = -h^{(2-\alpha)/\beta\delta} f_f(z) - f_{\text{regular}}(T,\mu)$$

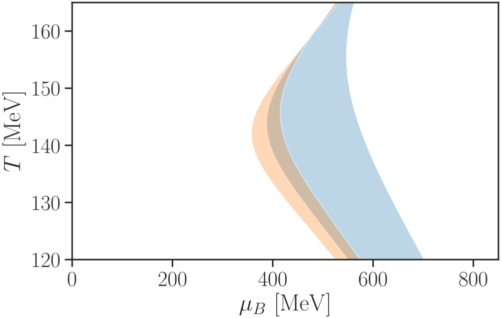
$$z = z_0 \left[\frac{T - T_c}{T_c} + \kappa_B \left(\frac{\mu_B}{T} \right)^2 \right] \left(\frac{m_{u,d}}{m_s} \right)^{-1/\beta\delta}$$

- Both singular and regular parts contribute to Taylor series expansion

- Solve $z = z_c$ to find μ_B^c as a function of T and/or $m_{u,d}/m_s$

Radius of convergence II

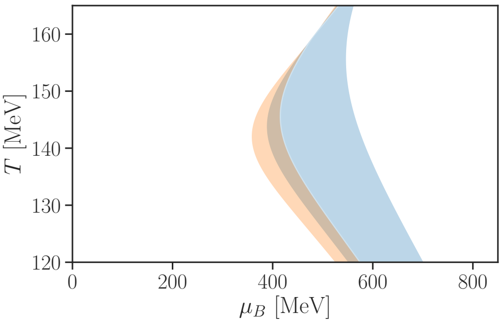


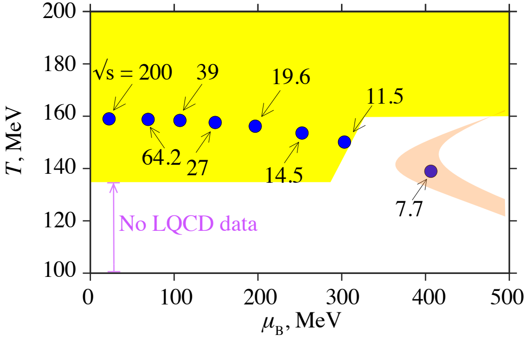


$$\frac{\Delta T}{T_c^0} = \frac{\mathrm{Re}z_c}{z_0}\left(\frac{m_l}{m_s}\right)^{\frac{1}{\beta\delta}}$$

Orange band is for $z_0 = 2$;
it incorporates a 15% uncertainty on the value of $|z_c|$.
Blue band depicts variation of $z_0 = 1 - 2$.

Radius of convergence and BES-II





- Consistent with Taylor series analysis
- Consistent with functional group preliminary results, see Jan Pawlowski's talk

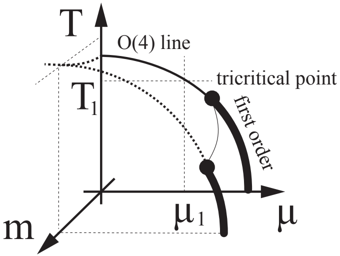
Conclusions

- Location of Yang-Lee singularity is universal, but was unknown for O(N).
  In this talk, $|z_c|$ for O(N) universality class from Functional Renormalization Group

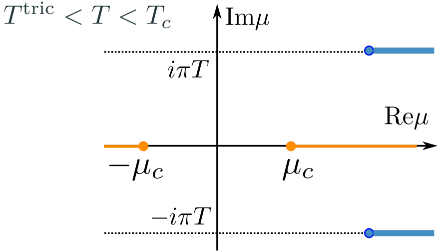
- Input on universal properties and non-universal parameters from lattice QCD $\rightsquigarrow$ radius of convergence in O(4) scaling region

- Implication on location of CP

Outlook: improving parametrization of critical EoS near CEP



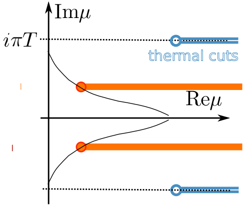
Chiral limit ($m_{u,d} = 0$):



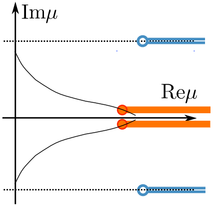
M. Stephanov, hep-lat/0603014; C. Itzykson, et al Nucl.Phys. B220 (1983) 415

$m_{u,d} \neq 0$:

crossover:



above but close to CEP:



at CEP: