Nuclear Science Computing Center at CCNU

Correlated Dirac eigenvalues and axial anomaly in chiral symmetric QCD

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Based on arXiv:2010.14836 [hep-lat]

In collaboration with

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Outline

- Motivation
- $\partial^n \rho / \partial m_l^n$ and C_{n+1}
- Lattice Setup
- Results
- Summary

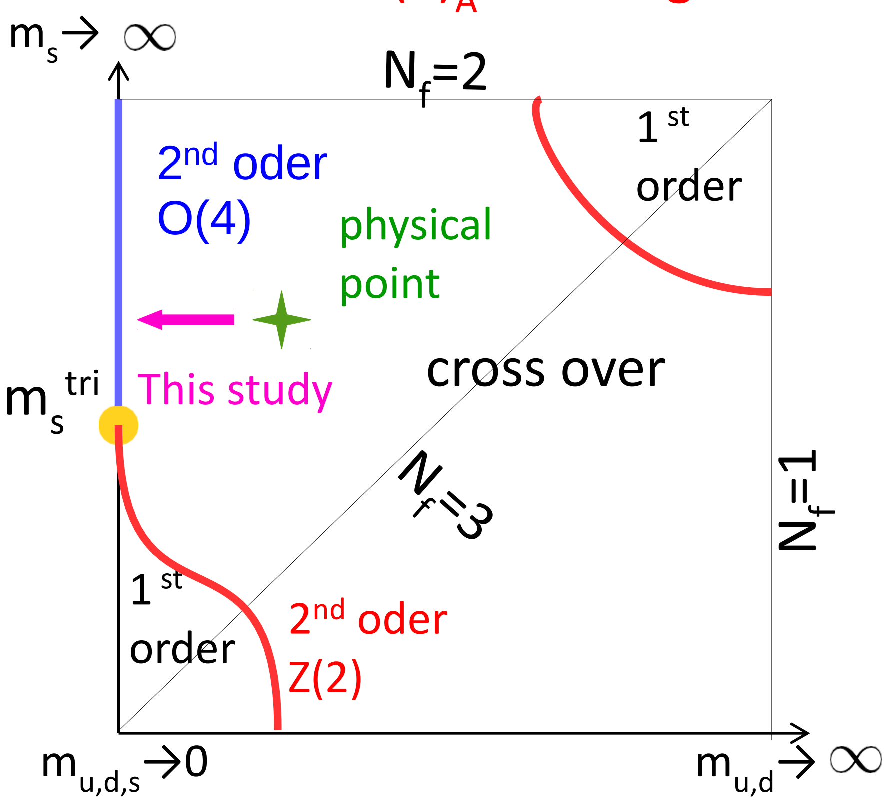
Motivation

At T≥T_c , chiral symmetry is restored.

How about the fate of $U(1)_A$ symmetry?

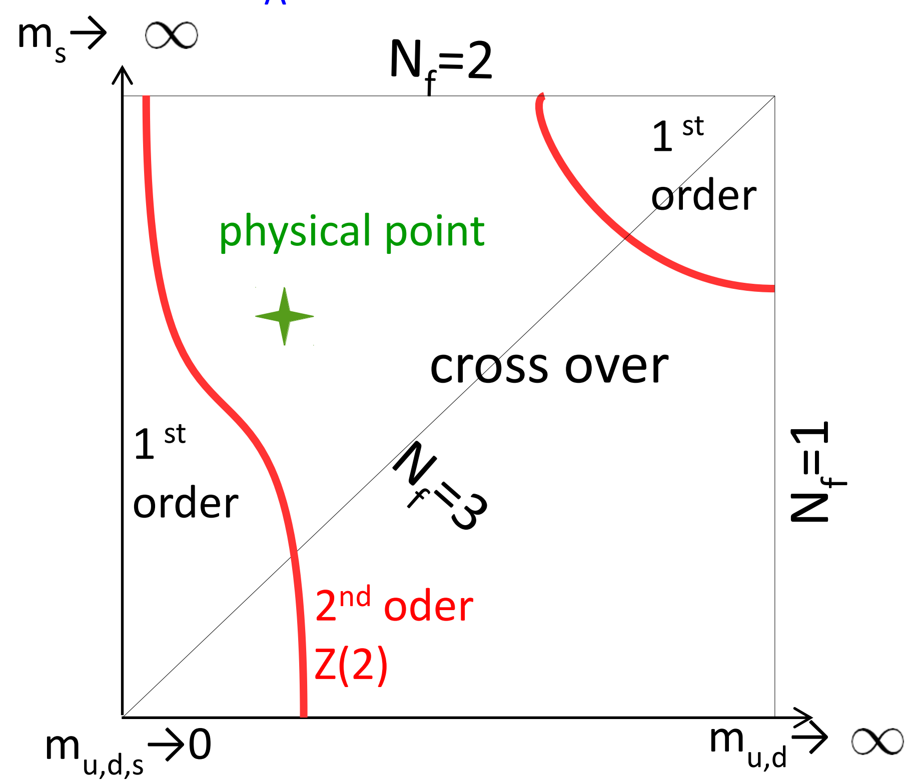
Two possible scenarios:

Substantial $U(1)_A$ breaking



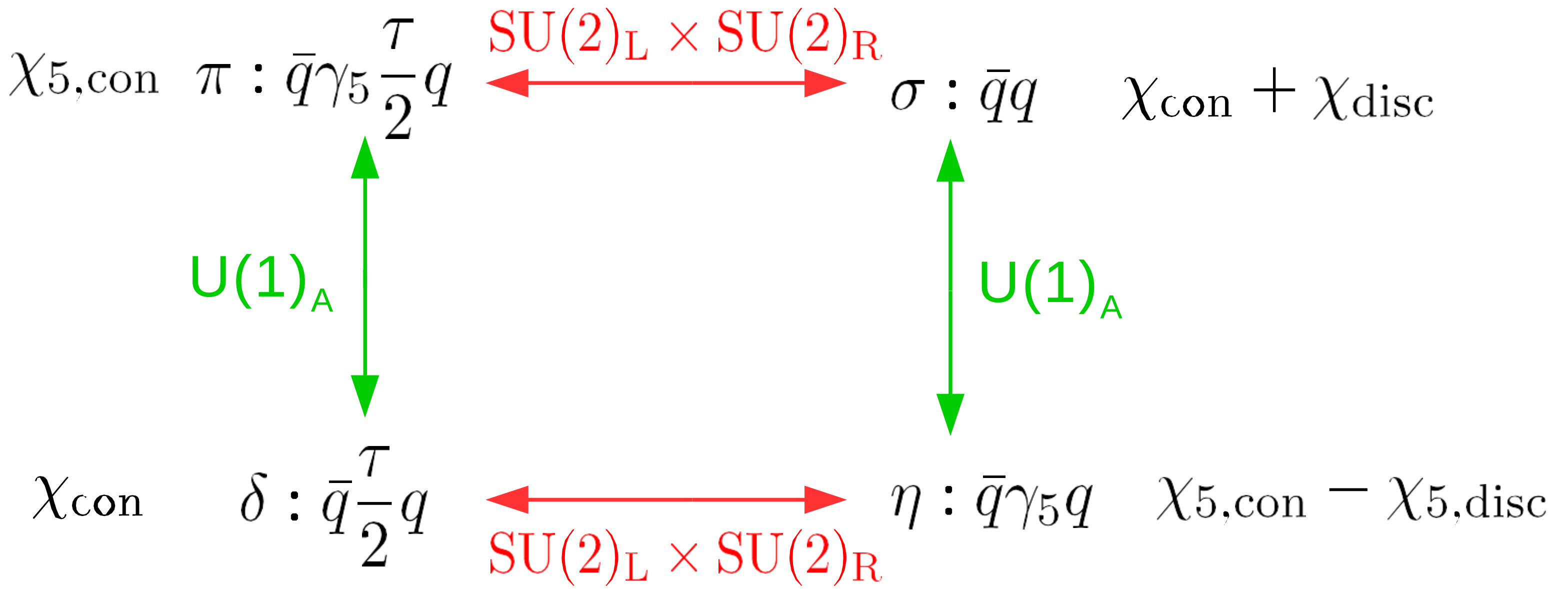
Pisarski, Wilczek (1984)

U(1)$_A$ effectively restored



Philipsen, Pinke, PRD 93 (2016) 114507

Measures of $U(1)_A$ breaking

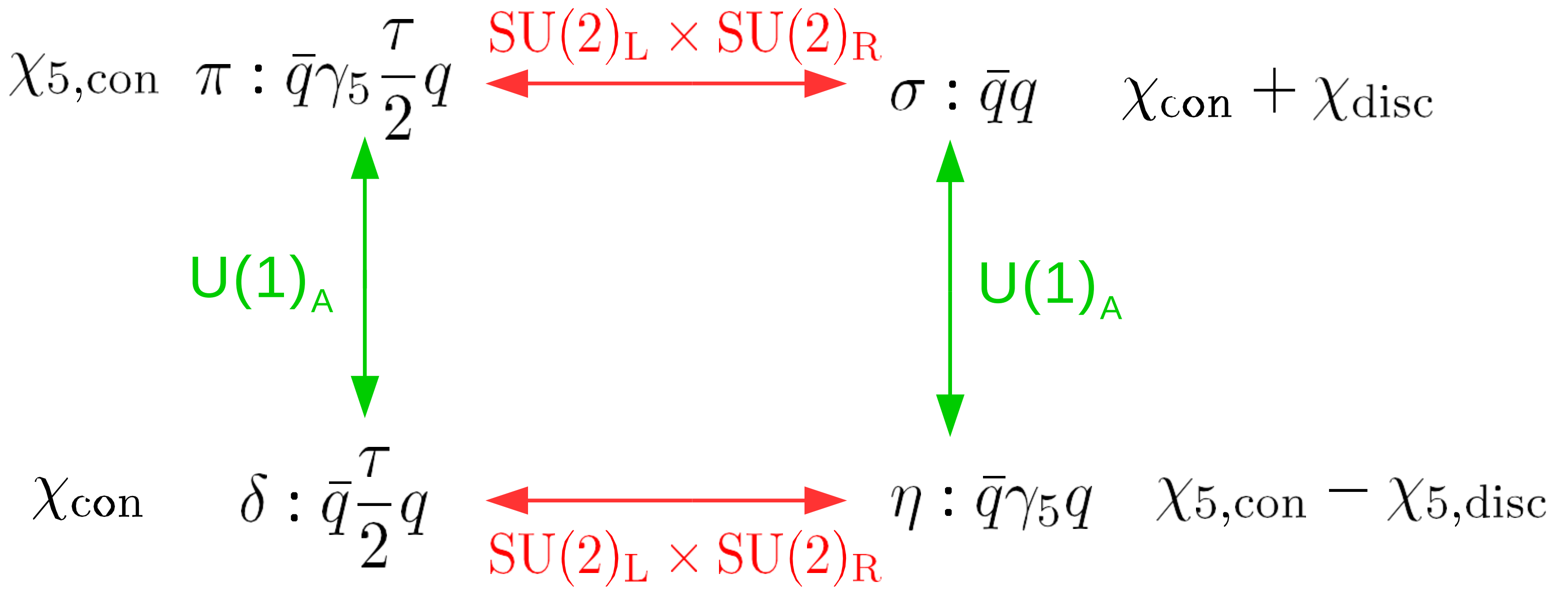


The quantity

$$\chi_\pi - \chi_\delta = \int \mathrm{d}^4x \langle \pi^i(x)\pi^i(0) - \delta^i(x)\delta^i(0)\rangle$$

is zero as $U(1)_A$ is restored

Measures of $U(1)_A$ breaking



For T≥Tc, in the chiral limit: $\chi_\pi - \chi_\delta = \chi_{\mathrm{disc}}$

$$\chi_{\mathrm{disc}} = \frac{T}{V} \int \mathrm{d}^4 x \left\langle [\bar{\psi}(x)\psi(x) - \langle \bar{\psi}(x)\psi(x) \rangle]^2 \right\rangle$$

χ_{disc} can also be used to probe the restoration of the $U(1)_A$ symmetry

Relation with Dirac eigenvalue spectrum

Order parameter of $SU(2)_L \times SU(2)_R$ symmetry :

$$\langle \bar{\psi}\psi \rangle = \int_0^\infty d\lambda \frac{4m_l\rho}{\lambda^2 + m_l^2} \xrightarrow{m_l \to 0} \pi\rho(0)$$

T.Banks and A.Casher (1980)

Two $U(1)_A$ measures:

$$\chi_\pi - \chi_\delta = \int_0^\infty \mathrm{d}\lambda \frac{8m_l^2\rho}{(\lambda^2 + m_l^2)^2}$$

$$\chi_{\mathrm{disc}} = \int_0^\infty \mathrm{d}\lambda \frac{4m_l\, \partial\rho/\partial m_l}{\lambda^2 + m_l^2}$$

The restoration of $U(1)_A$ symmetry:

- A gap in the near-zero modes
- $\rho(\lambda, m_l \to 0) \sim \lambda^3$

Cohen, nucl-th/980106

U(1)$_A$ symmetry and Dirac eigenvalue spectrum

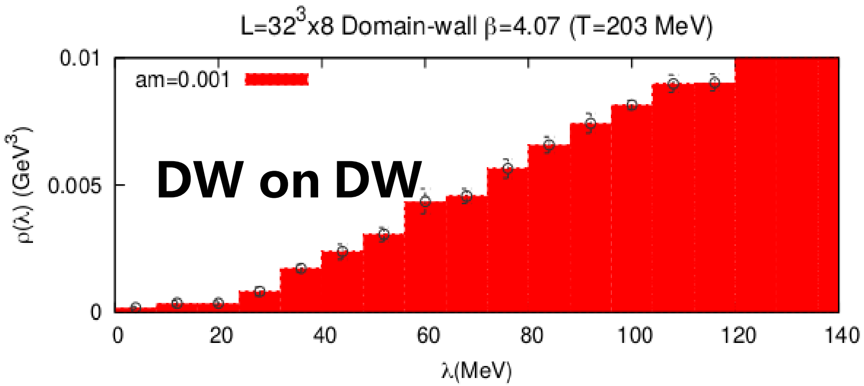
- Weakly interacting instanton gas motivated $\rho \sim m_l^2 \delta(\lambda)$
  $\Longrightarrow$ U(1)$_A$ breaking as $m_l \to 0$ [G. 't Hooft (1976)]

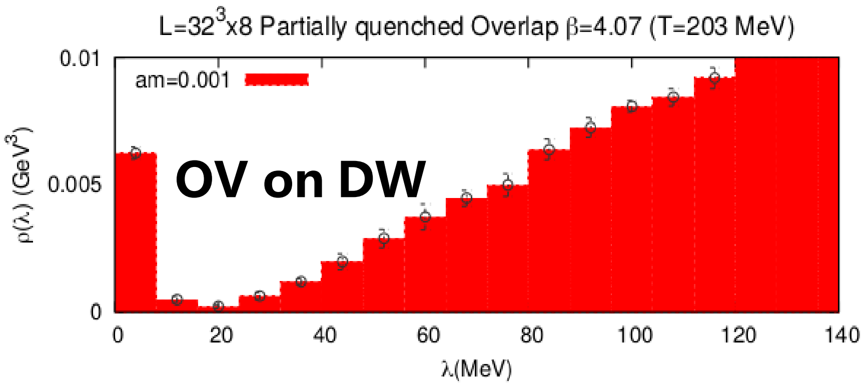
- At high T for the physical m_l , the temperature dependence of χ_t follows dilute instanton gas approximation prediction

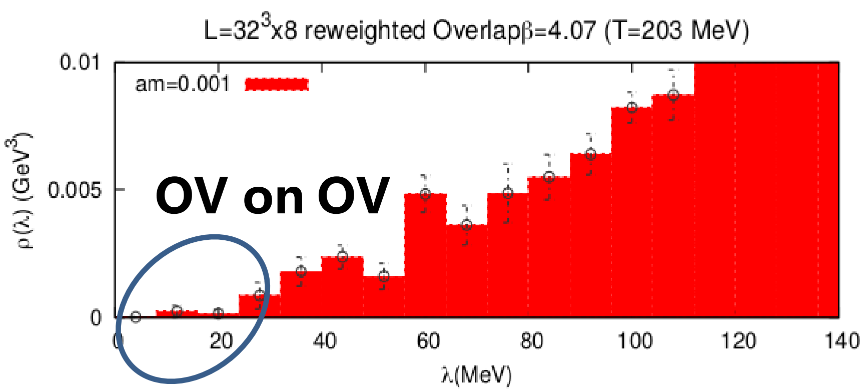
  [See a recent review, M. P. Lombardo & A. Trunin, 2005.06547]

  whether it is due to $\rho \sim m_l^2 \delta(\lambda)$ and what happens for $m_l \to 0$ is unanswered

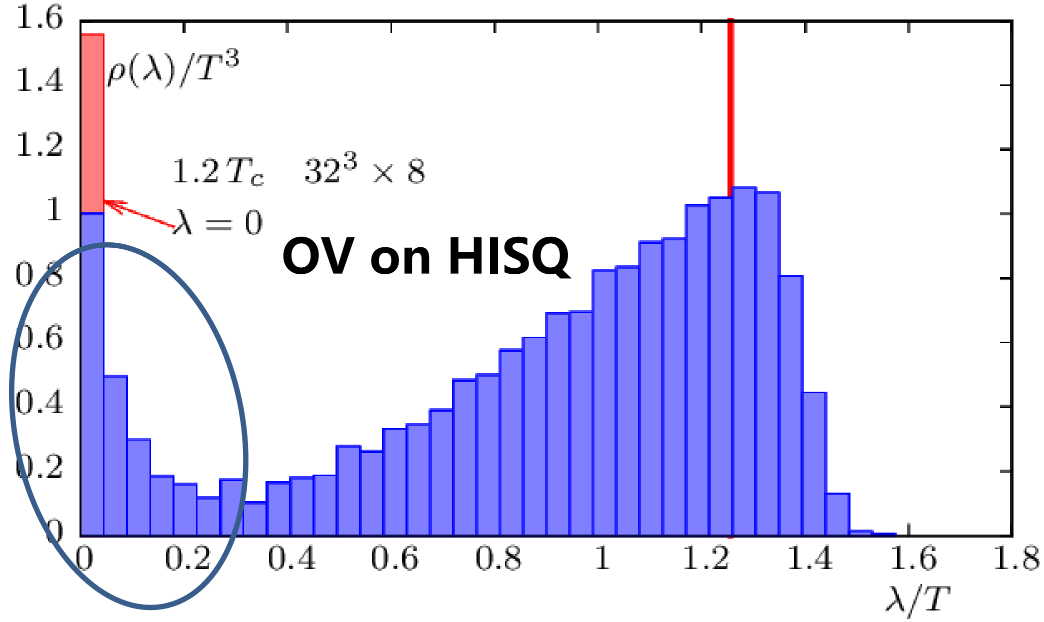
ρ :   wo/w infrared enhancement







A. Tomiya et al PRD [1612.01908]

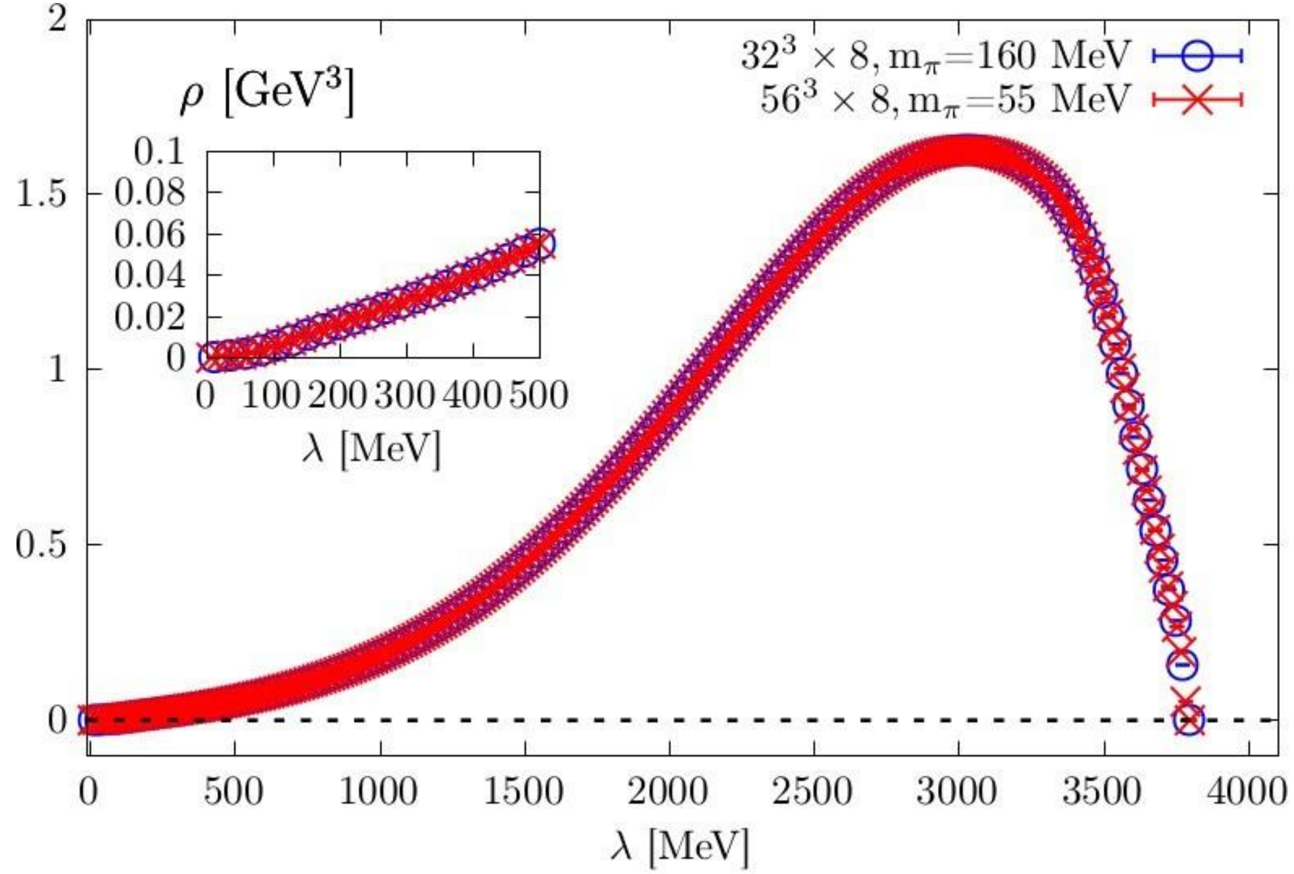


Dick et al PRD [1502.06190]

Partially quench effect needs to be investigated

Underlying mechanism is not clear

Mass dependence of ρ at $T \sim 1.6 T_c$



The mass dependence can be hardly observed from ρ directly

Novel relations between $\partial^n \rho / \partial m_l^n$ and correlation among the eigenvalues

For (2+1)-flavor QCD

$$\rho(\lambda, m_l) = \frac{T}{VZ[U]} \int \mathcal{D}[U] e^{-S_G[U]} \det[\not{D}[U] + m_s] \times (\det[\not{D}[U] + m_l])^2 \rho_U(\lambda),$$

$$\rho_U(\lambda) = \sum_j \delta(\lambda - \lambda_j)$$

$$\frac{V}{T} \frac{\partial \rho}{\partial m_l} = \int_0^\infty \mathrm{d}\lambda_2 \frac{4 m_l \, C_2}{\lambda_2^2 + m_l^2} \,, \tag{1}$$

$$\frac{V}{T} \frac{\partial^2 \rho}{m_l^2} = \int_0^\infty \mathrm{d}\lambda_2 \frac{4(\lambda_2^2 - m_l^2) \, C_2}{(\lambda_2^2 + m_l^2)^2} + \int_0^\infty \mathrm{d}\lambda_2 \mathrm{d}\lambda_3 \frac{(4 m_l)^2 \, C_3}{(\lambda_2^2 + m_l^2)(\lambda_3^2 + m_l^2)} \,, with \tag{2}$$

$$C_n(\lambda_1, \cdots, \lambda_n; m_l) = \left\langle \prod_{i=1}^{n} [\rho_U(\lambda_i) - \langle \rho_U(\lambda_i) \rangle] \right\rangle . \tag{3}$$

In this talk, we want to show you two things:

- How axial anomaly manifest itself at $T \sim 1.6 T_c$?
- What is the underlying mechanism of it ?

Lattice Setup

- **Actions:**

  Tree level improved gauge action

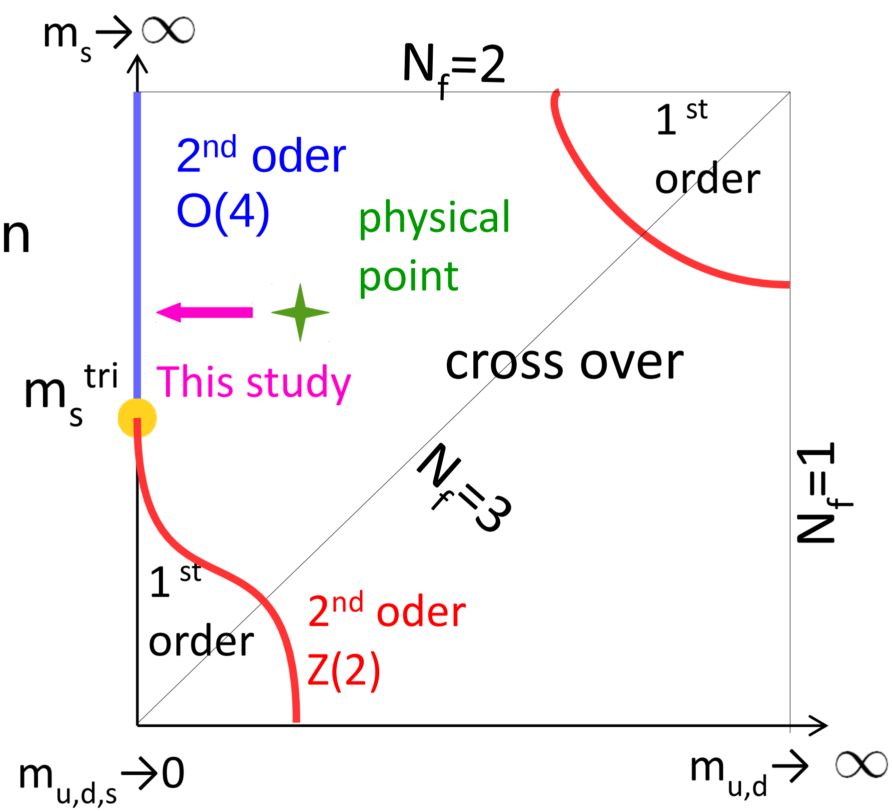
  Highly improved staggered quark action

- **Lattice size:**

  $N_\tau = 8$, $N_\sigma = 32, 40, 56, 72$

  $N_\tau = 12$, $N_\sigma = 48, 60, 72$

  $N_\tau = 16$, $N_\sigma = 64, 80$



- **Quark mass:**

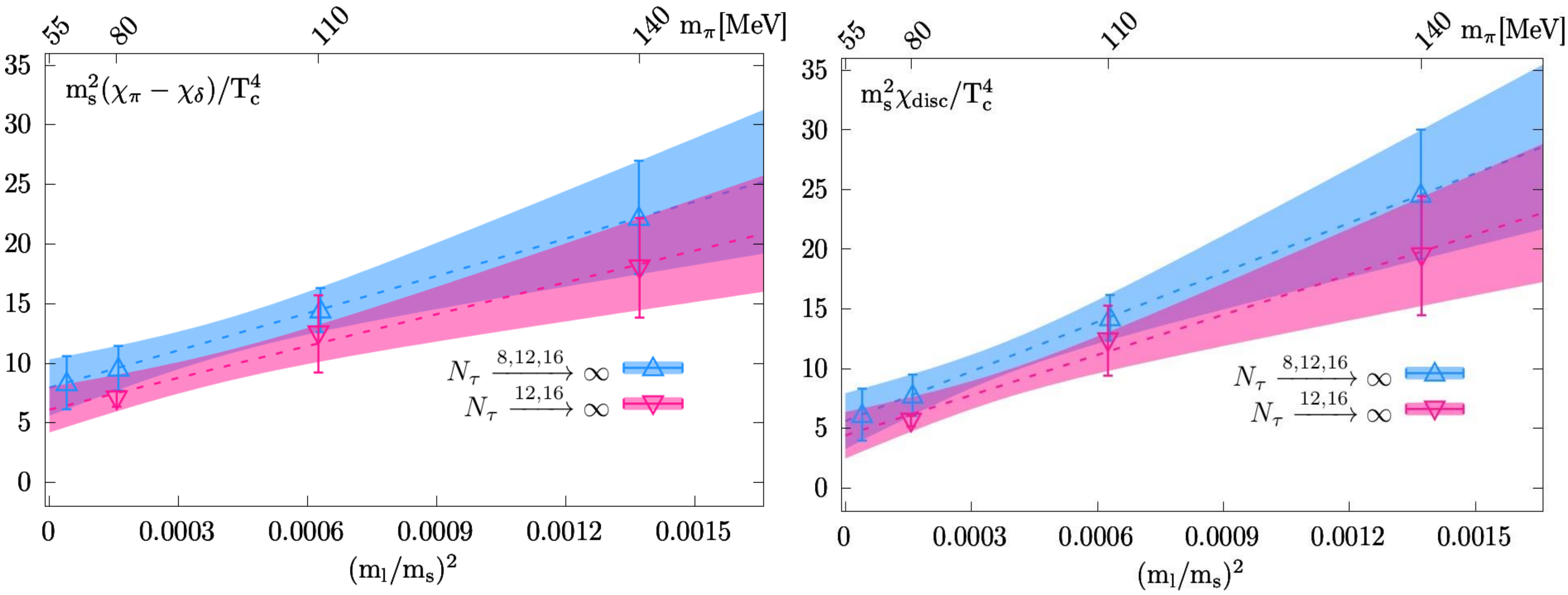
  m_s : set to its physical value

  m_l/m_s =1/20, 1/27, 1/40, 1/80, 1/160 (m_π =160, 140, 110, 80, 55MeV)

- **Temperature:**

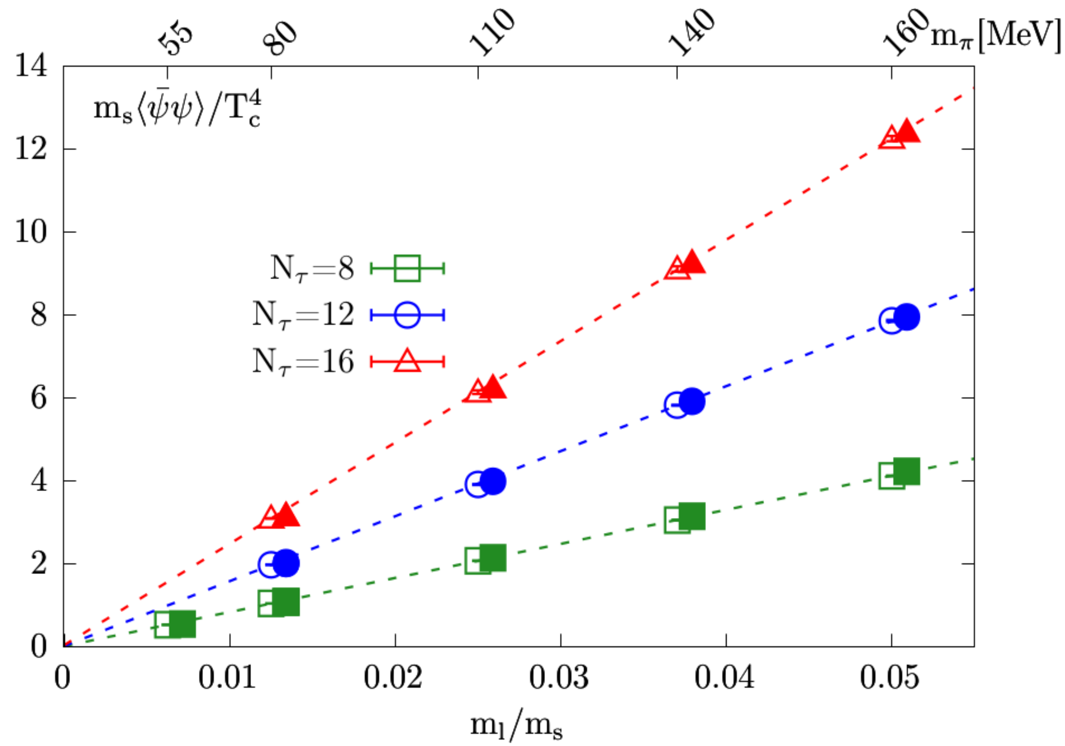
  $T \sim 1.6 T_C$ (205MeV)

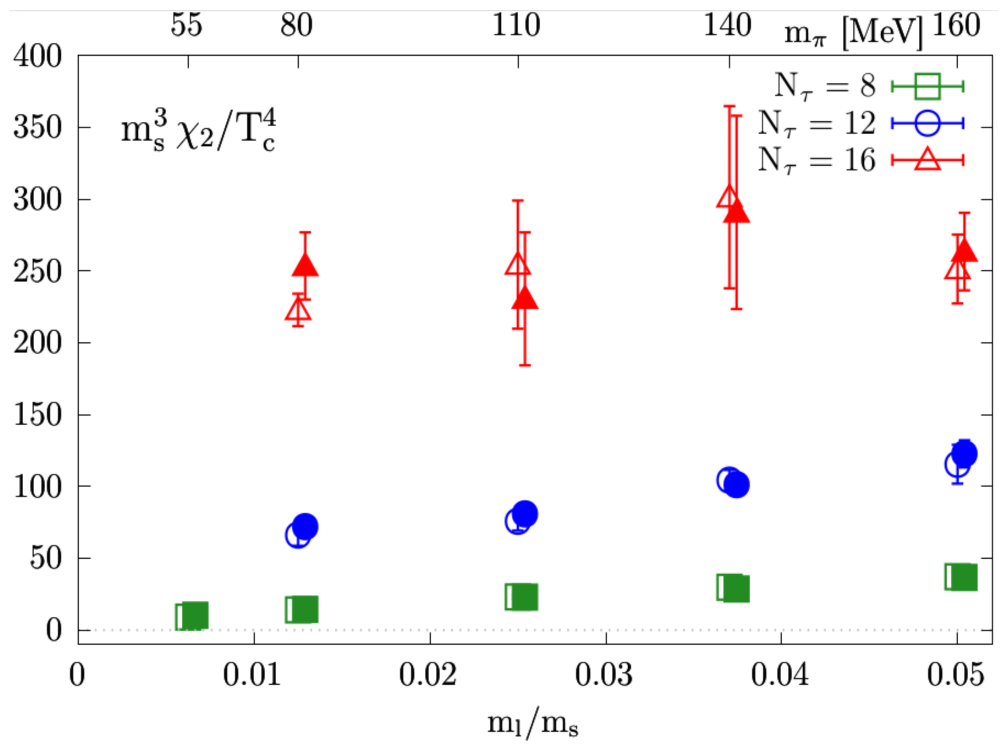
Continuum and chiral extrapolated results for $\chi_\pi - \chi_\delta$ and $\chi_{\rm disc}$



- $\chi_\pi - \chi_\delta \simeq \chi_{\rm disc} \neq 0$ after continuum and chiral extrapolation
- Axial anomaly remains manifested even in the chiral limit at T~1.6T_c

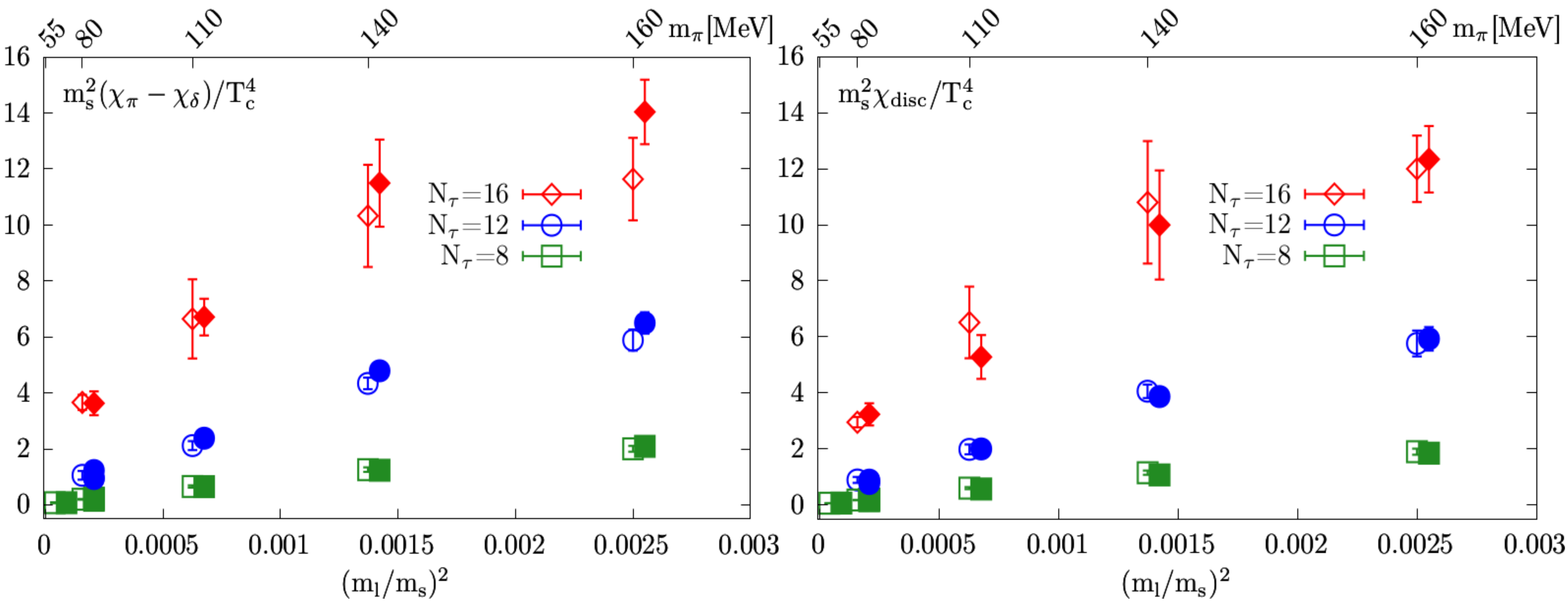
Quantities related to ρ and $\partial^2\rho/\partial m_l^2$





- $\langle\bar{\psi}\psi\rangle$ and χ_2 can be reproduced well from ρ and $\partial^2\rho/\partial m_l^2$
- $\langle\bar{\psi}\psi\rangle$ is linear in quark mass
- In the chiral limit, chiral symmetry is restored at T~1.6T_c

$U(1)_A$ measures from ρ and $\partial\rho/\partial m_l$

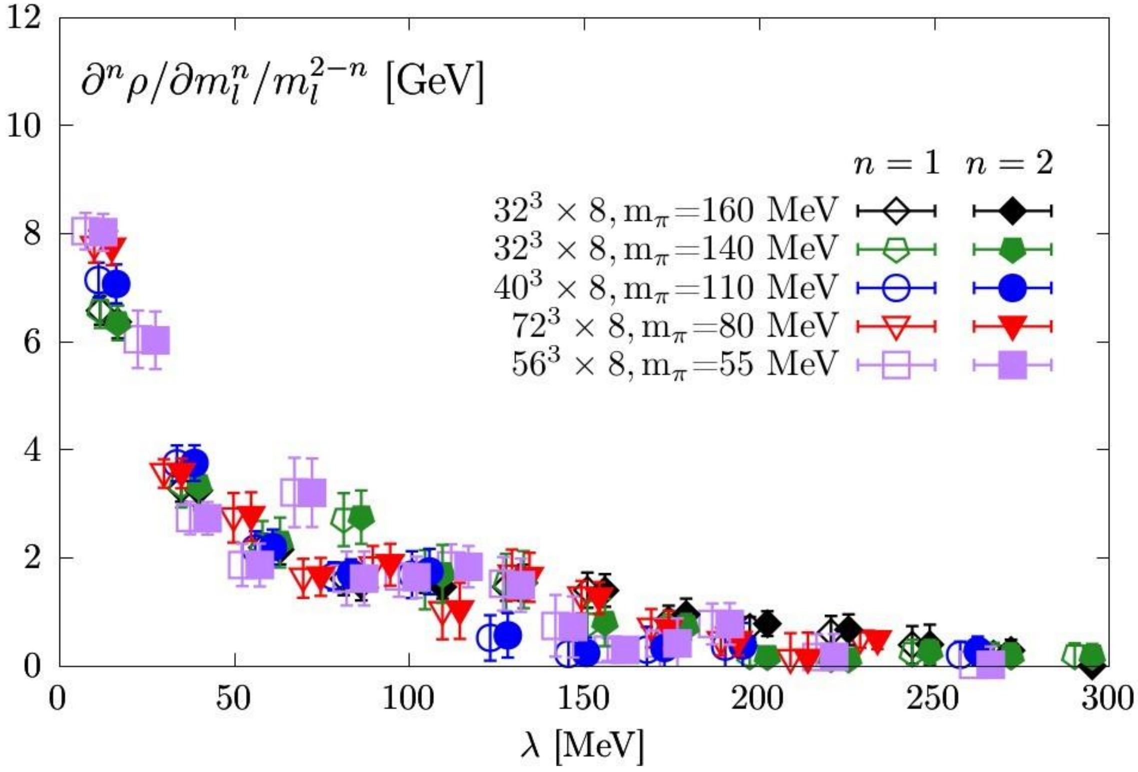


- ρ and $\partial\rho/\partial m_l$ reproduced directly measured $\chi_\pi - \chi_\delta$ and $\chi_{\rm disc}$
- Only infrared part of ρ and $\partial\rho/\partial m_l$ are needed for the reproduction
- Both two $U(1)_A$ measures are linear in m_l^2 at $m_\pi \lesssim 140$ MeV

The consistent results of those quantities from two measurements make us confident to extract information on $\rho, \partial\rho/\partial m_l, \partial^2\rho/\partial m_l^2$

What is the underlying mechanism of axial anomaly ?

Quark mass dependence of $m_l^{-1}(\partial\rho/\partial m_l)$ and $\partial^2\rho/\partial m_l^2$

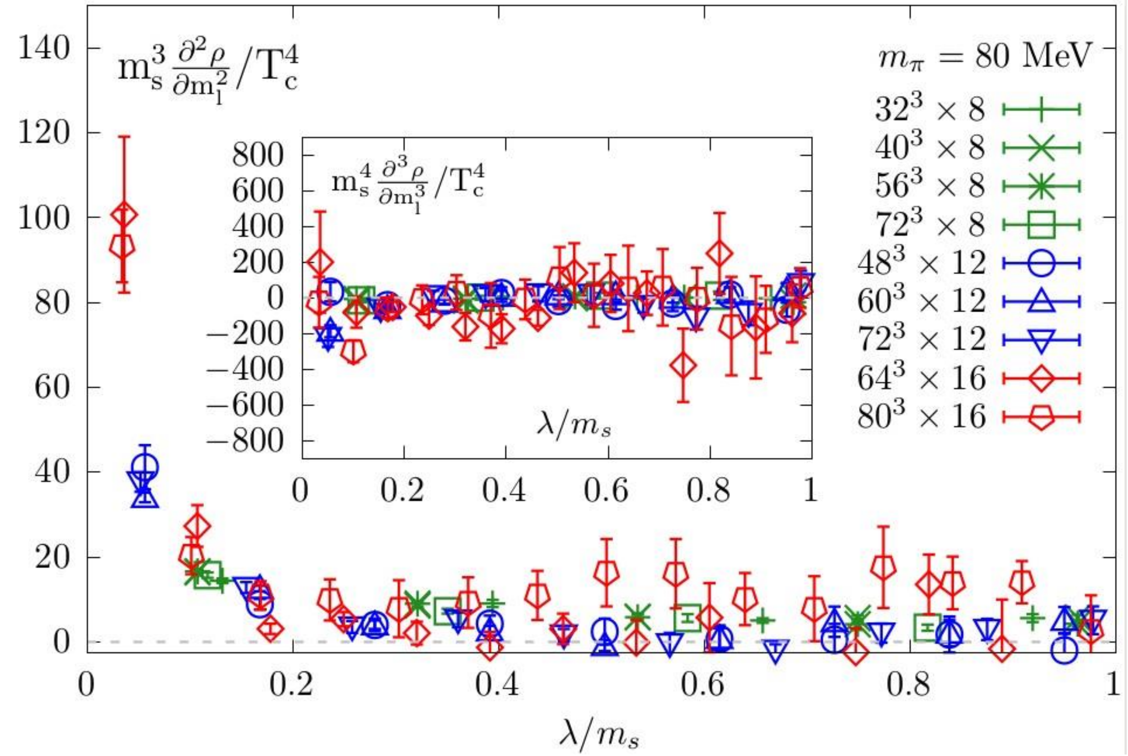


$$m_l^{-1}(\partial\rho/\partial m_l) \approx \partial^2\rho/\partial m_l^2$$

Almost independent of quark mass

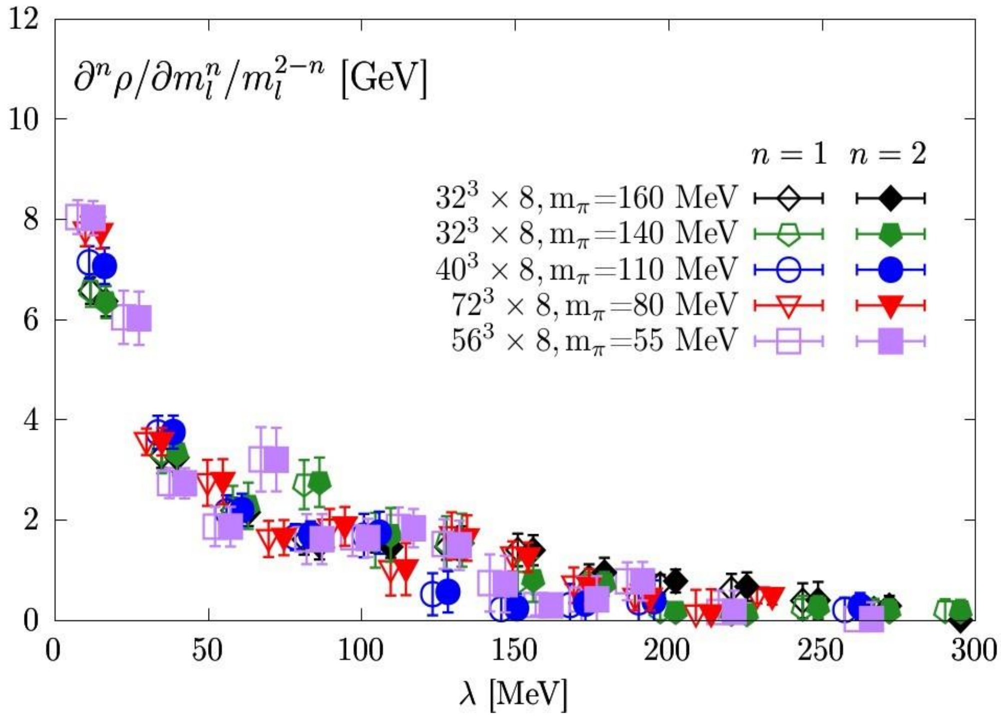
A peak structure developed at $\lambda \to 0$ and drop rapidly towards zero as λ increases

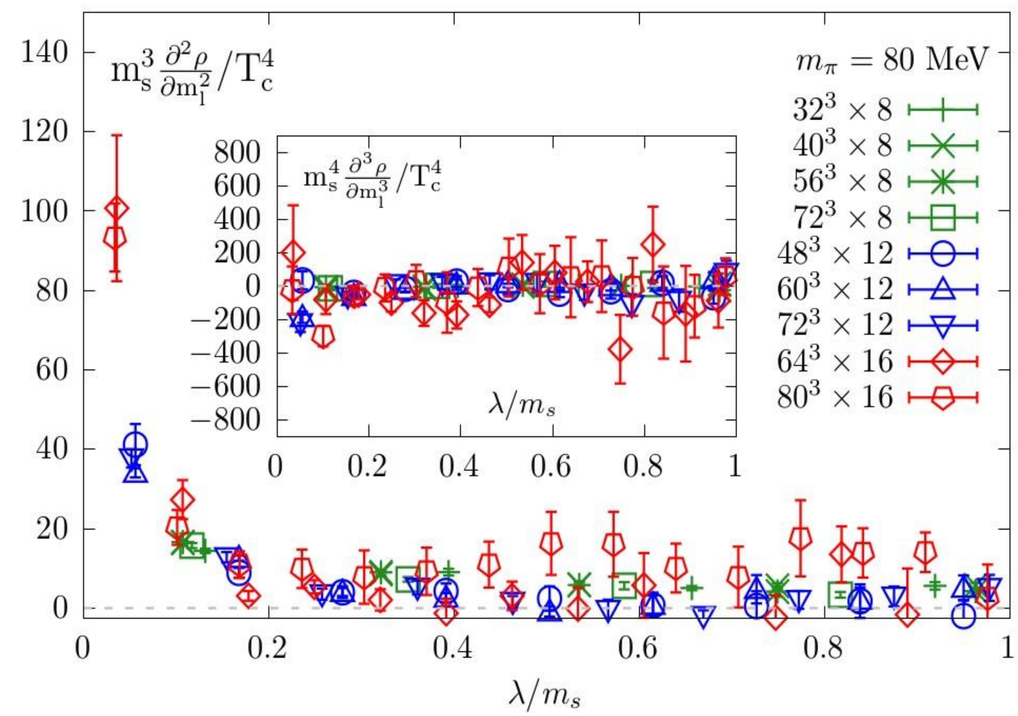
Lattice spacing and volume dependence of $\partial^2\rho/\partial m_l^2$ and $\partial^3\rho/\partial m_l^3$



- The peaked structure in $\partial^2\rho/\partial m_l^2$ within small λ range becomes sharper as $a \to 0$
- $\partial^3\rho/\partial m_l^3 \approx 0$
- Volume dependence is quite small

Quark mass derivatives of ρ

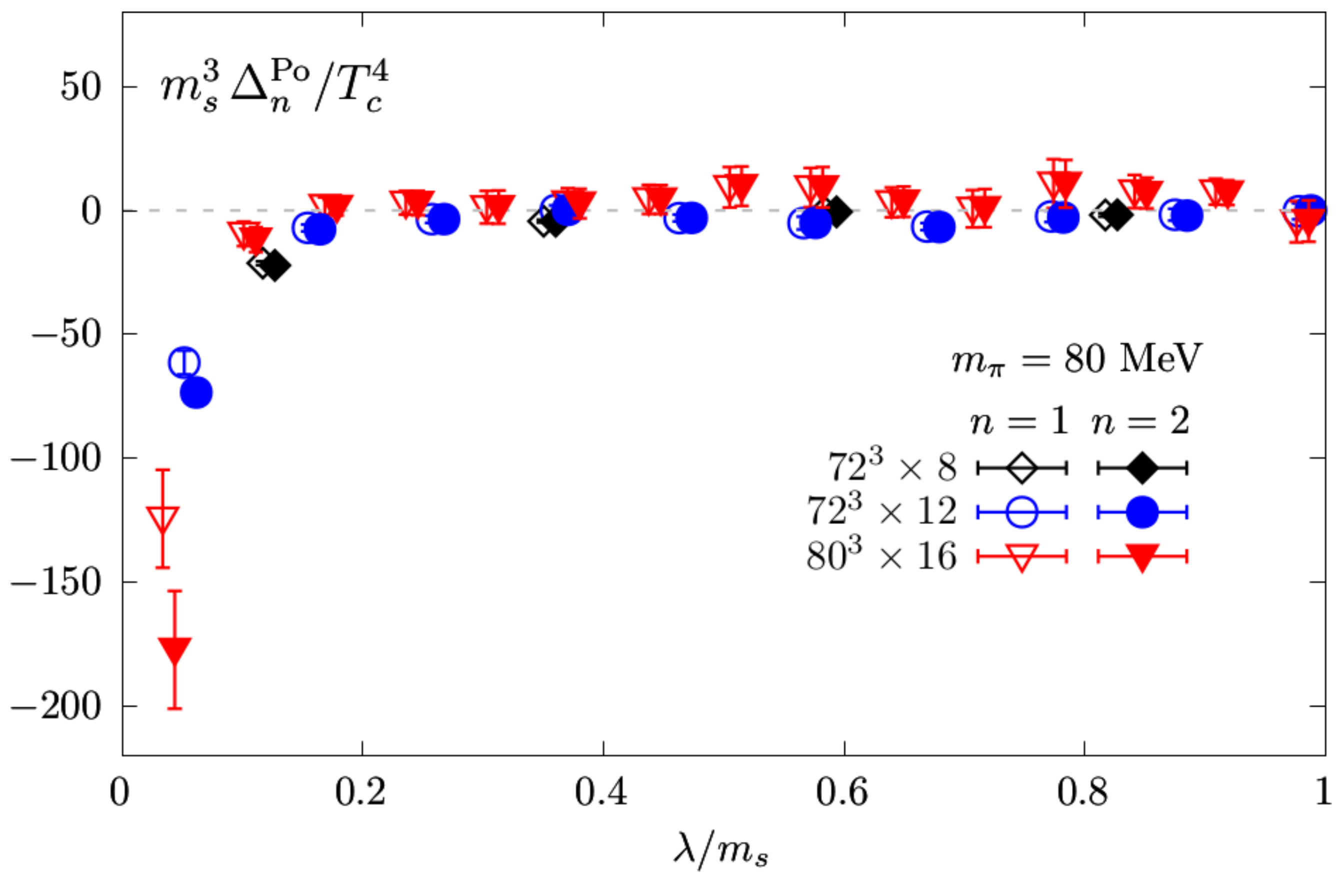




$m_l{}^{-1}(\partial\rho/\partial m_l) \approx \partial^2\rho/\partial m_l^2$ and $\partial^3\rho/\partial m_l^3 \approx 0$ $\Longrightarrow \rho(\lambda \to 0, m_l \to 0) \propto m_l^2$

The differences

$$\Delta_n^{\mathrm{Po}} = m_l^{n-2}[\partial^n \rho/\partial m_l^n - (\partial^n \rho/\partial m_l^n)^{\mathrm{Po}}]$$



The repulsive non-Poisson correlation within the small λ gives rise to the $\rho(\lambda \to 0)$ peak

Summary

- Axial anomaly remains manifested in $\chi_\pi - \chi_\delta$ and $\chi_{\rm disc}$ even in the chiral limit at T~1.6T_c

- The underlying presence of $\rho(\lambda \to 0, m_l)$ leads to manifestations of $U(1)_A$ anomaly in $\chi_\pi - \chi_\delta$ and $\chi_{\rm disc}$

- $\rho(\lambda \to 0, m_l)$ develops a peaked structure due to non-Poisson correlations within small λ, the peak becomes sharper as $a \to 0$, and its amplitude is proportional to m_l^2

Summary

- These suggest that for T≥1.6T_c the microscopic origin of axial anomaly is driven by the weakly interacting instanton gas motivated $\rho(\lambda \to 0, m_l \to 0) \propto m_l^2$

- The chiral phase transition in (2+1)-flavor QCD is 3-dimensional O(4) universality class

backup

Calculation of eigenvalue spectrum

- **Commonlyusedmethod:Lanczosalgorithm tocalculatethe individualow-lyingeigenvalues**

- **HereweutilizedtheChebyshevfilteringtechniquecombined withastochasticestimateofthemodenumber**

mode number:

$$n[s,t] \approx \frac{1}{N_r}\sum_{r=1}^{N_r}\sum_{j=0}^{p} g_j^p \gamma_j \langle \xi_r^\dagger T_j(A) \xi_r \rangle$$

spectrum:

$$\rho(\lambda,\delta) = \frac{1}{V}\frac{\bar{n}[\lambda-\delta/2, \lambda+\delta/2]}{\delta} \qquad (\lambda \geq \delta/2)$$

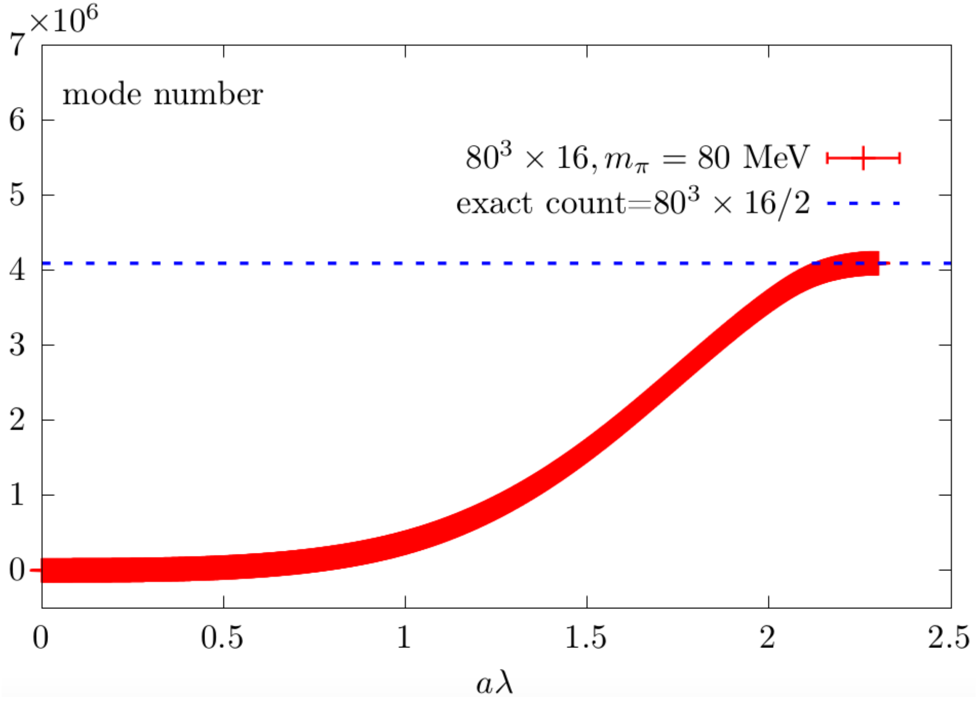
T_j:Chebyshevpolynomial

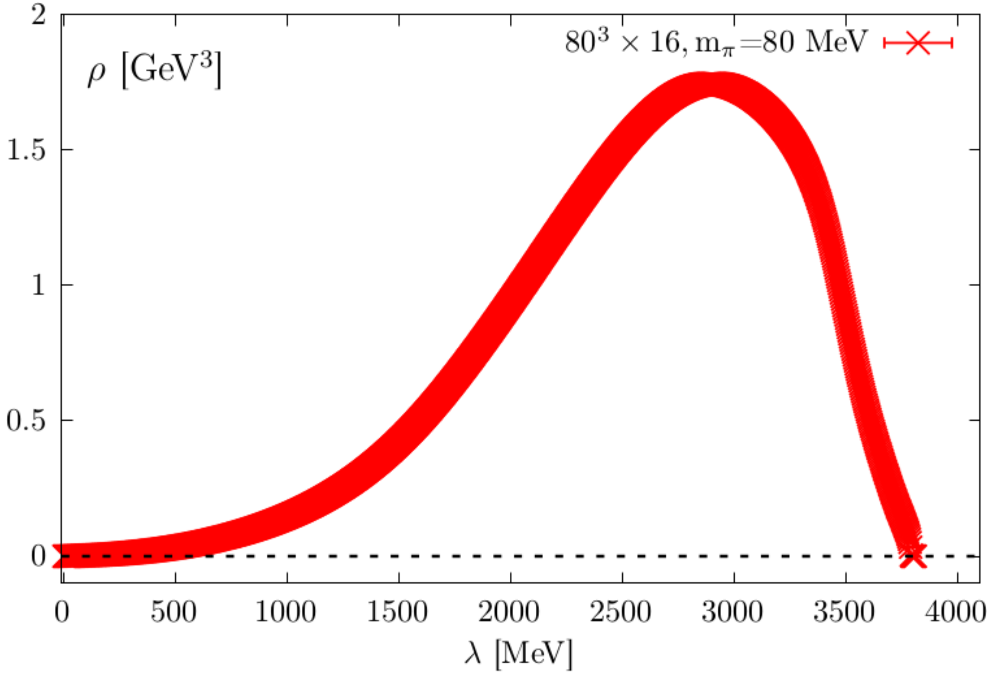
$_j$:coefficient

p:polynomialorder

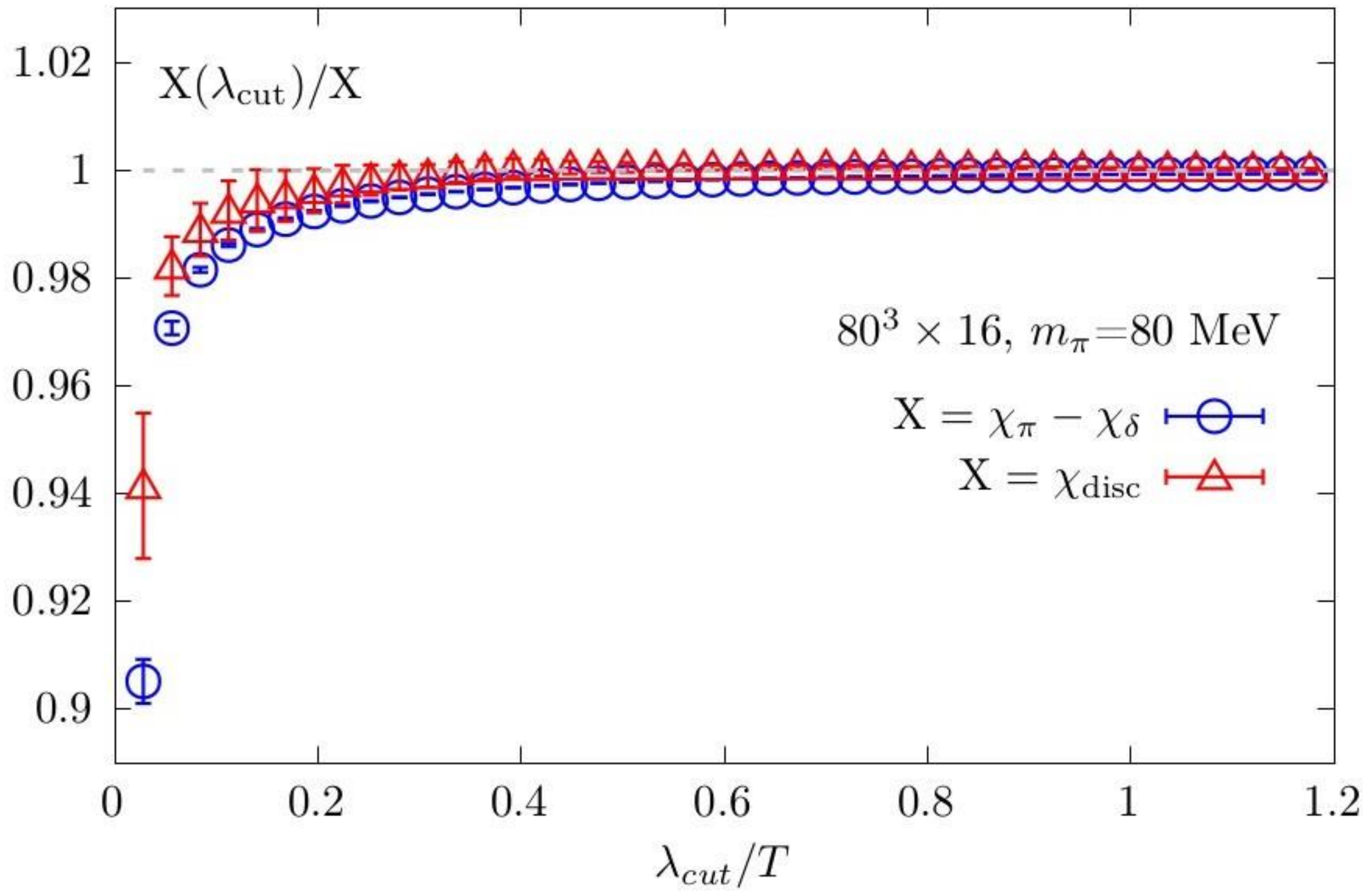
YuZhang,Latice19',arXiv:201.05217
Giusti,Luscher, arXiv: 0812.3638
A.Patela,arXiv:1204.432
DiNapolietal.,arXiv: 1308.4275
Itouetal,arXiv:141.15
Fodoretal.,arXiv:1605.08091
Cosuetal.,arXiv:1601.074

Mode number and eigenvalue spectrum

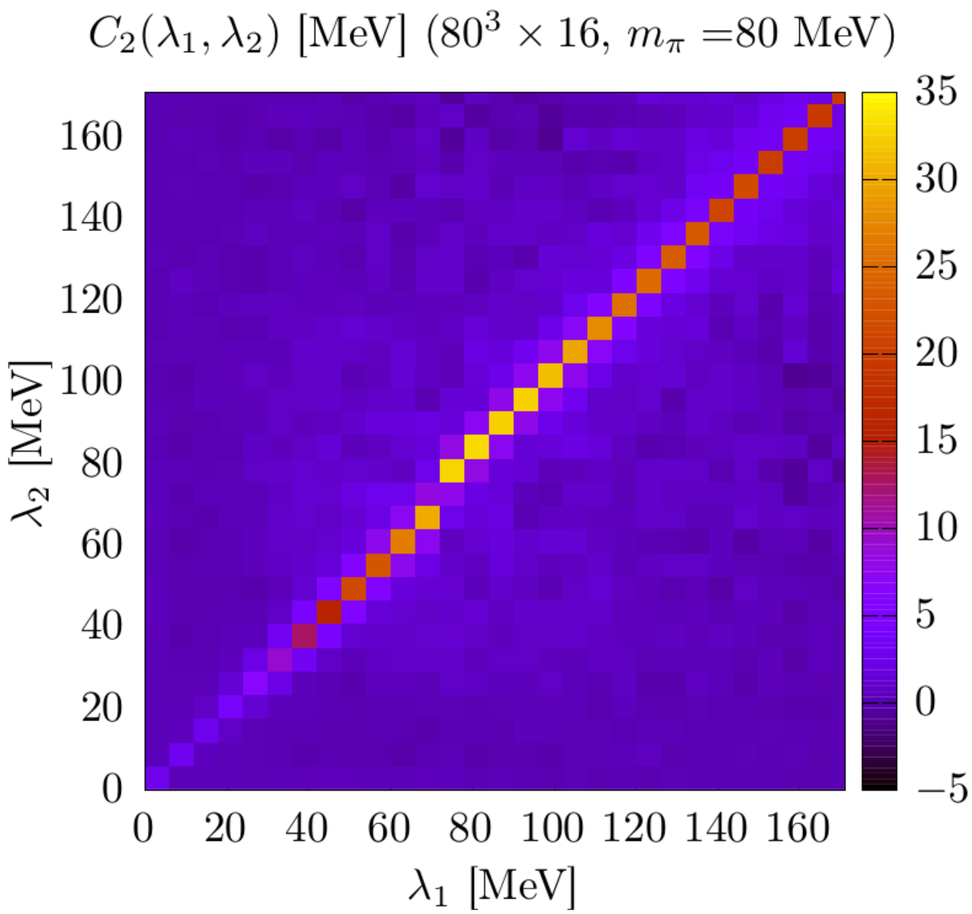




Infrared contribution to $\chi_\pi - \chi_\delta$ and $\chi_{\rm disc}$



Only the infrared parts of ρ **and** $\partial\rho/\partial m_l$ **areneededforthe reproductions of** $\chi_\pi - \chi_\delta$ **and** $\chi_{\rm disc}$


$C_2(\lambda_1, \lambda_2)$ [MeV] ($80^3 \times 16$, m_π =80 MeV)
λ_2 [MeV]
λ_1 [MeV]
0
20
40
60
80
100
120
140
160
35
30
25
20
15
10
5
0
−5

Poisson distribution

$$C_2(\lambda,\lambda') = \langle \rho_u(\lambda)\rho_u(\lambda')\rangle - \langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle$$

$$= (\frac{1}{V})^2 \left\langle \sum_{k=1}^{N} \delta(\lambda-\lambda_k) \sum_{l=1}^{N} \delta(\lambda'-\lambda_l) \right\rangle - \langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle$$

$$= (\frac{1}{V})^2 \left\langle \sum_{k=1}^{N} \delta(\lambda-\lambda_k)\delta(\lambda'-\lambda_k) \right\rangle + (\frac{1}{V})^2 \left\langle \sum_{k=1}^{N} \delta(\lambda-\lambda_k) \sum_{l\neq k} \delta(\lambda'-\lambda_l) \right\rangle - \langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle^{(1)}$$

$$= \frac{1}{V}\langle \rho_u(\lambda)\rangle \delta(\lambda-\lambda') + (\frac{1}{V})^2 \left\langle \sum_{k=1}^{N} \delta(\lambda-\lambda_k) \right\rangle \left\langle \sum_{l\neq k} \delta(\lambda'-\lambda_l) \right\rangle - \langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle$$

$$\frac{1}{V} \left\langle \sum_{l\neq k} \delta(\lambda'-\lambda_l) \right\rangle = \frac{N-1}{N}\langle \rho_u(\lambda')\rangle \qquad (N = V/2)$$

$$C_2(\lambda,\lambda') = \frac{1}{V}\langle \rho_u(\lambda)\rangle \delta(\lambda-\lambda') - \frac{1}{N}\langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle$$

Three point correlations of ρ(λ)

$$C_3(\lambda, \lambda', \lambda'') = \Big\langle \rho_u(\lambda)\rho_u(\lambda')\rho_u(\lambda'')\Big\rangle - \langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rho_u(\lambda'')\rangle$$
$$- \langle \rho_u(\lambda')\rangle\langle \rho_u(\lambda)\rho_u(\lambda'')\rangle - \langle \rho_u(\lambda'')\rangle\langle \rho_u(\lambda)\rho_u(\lambda')\rangle \quad (1)$$
$$+ 2\langle \rho_u(\lambda)\rangle\langle \rho_u(\lambda')\rangle\langle \rho_u(\lambda'')\rangle$$

Time history of the topological charge