Bottomonium suppression and elliptic flow from real-time quantum evolution

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A. Islam and M.S., 2007.10211 (PLB), 2010.05457 (JHEP?)
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

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"Hot problems of Strong Interactions"
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KENT STATE UNIVERSITY

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U.S. DEPARTMENT OF ENERGY

Heavy Quarkonium Suppression

- In a high temperature quark-gluon plasma we expect **weaker color binding** (<u>Debye screening</u> + asymptotic freedom)

E. V. Shuryak, Phys. Rept. 61, 71–158 (1980)
T. Matsui, and H. Satz, Phys. Lett. B178, 416 (1986)
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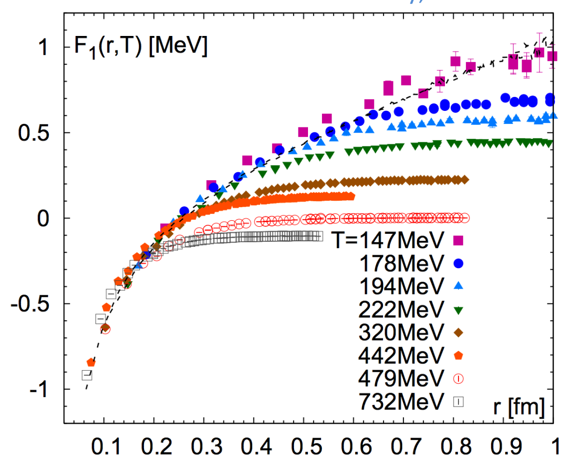
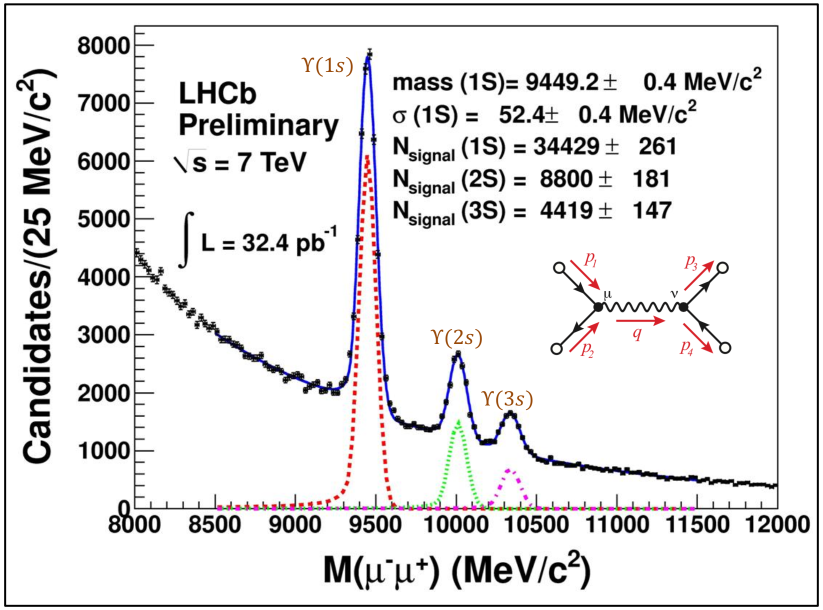
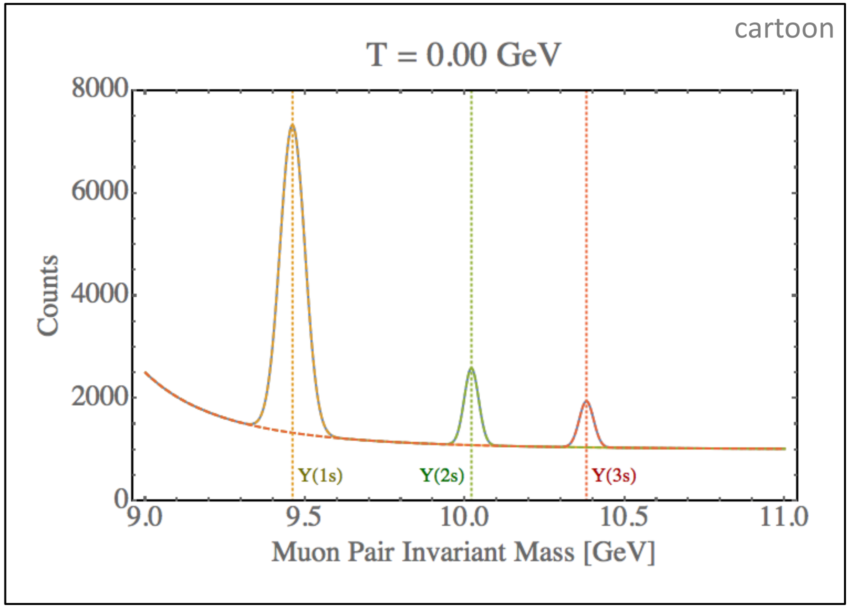
A. Bazavov and P. Petreczky, 1211.5638







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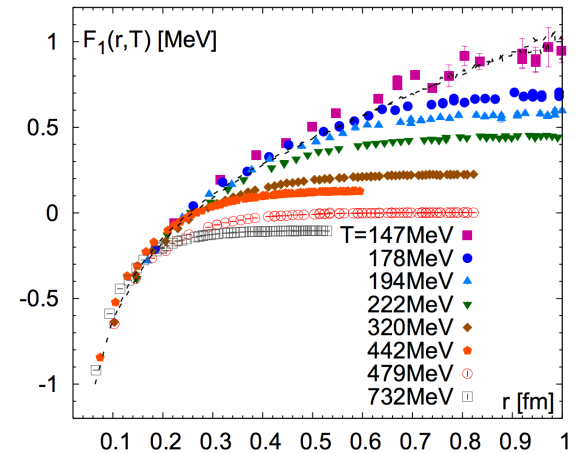

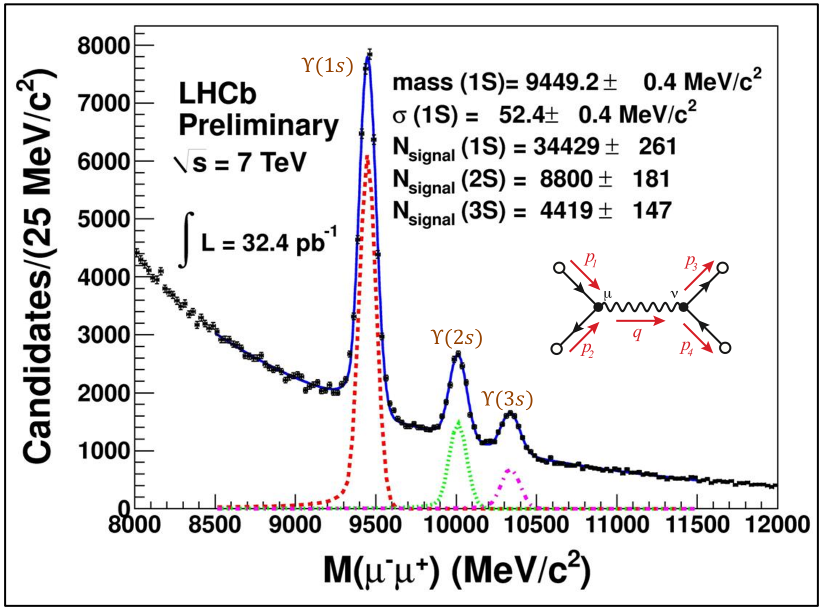

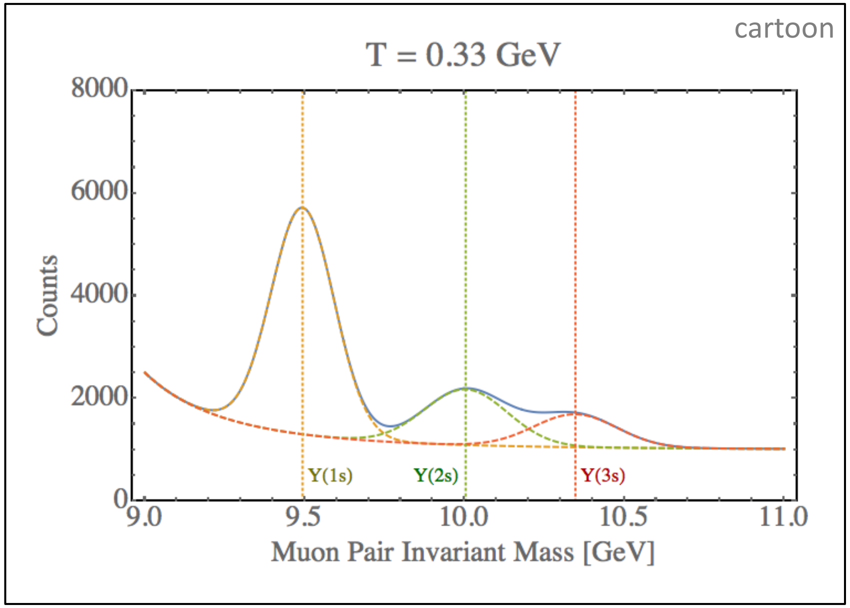

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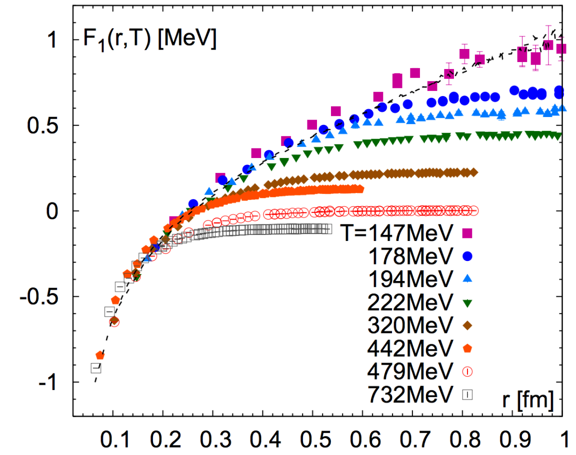

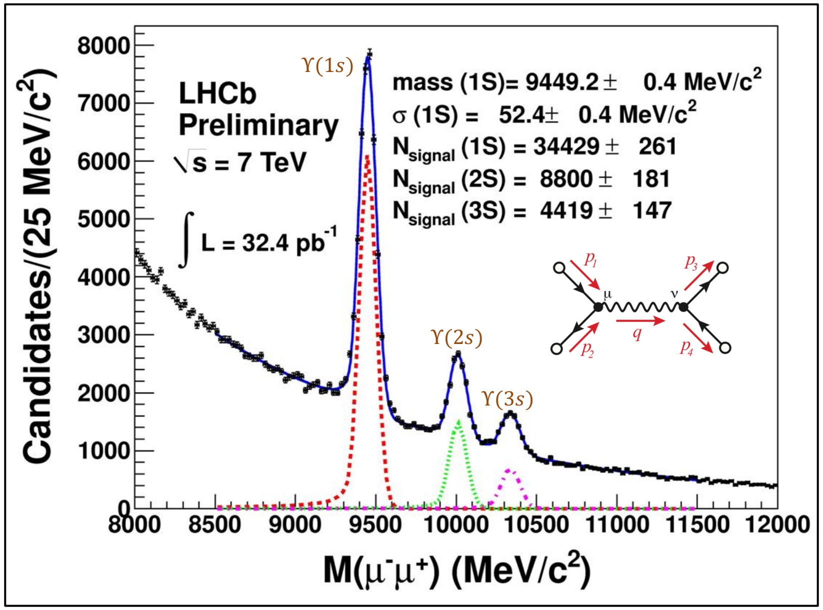

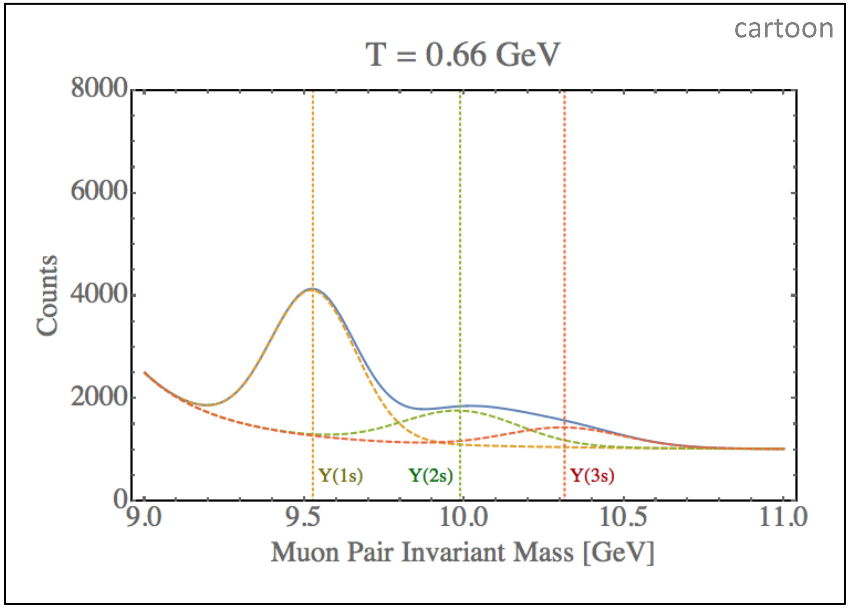

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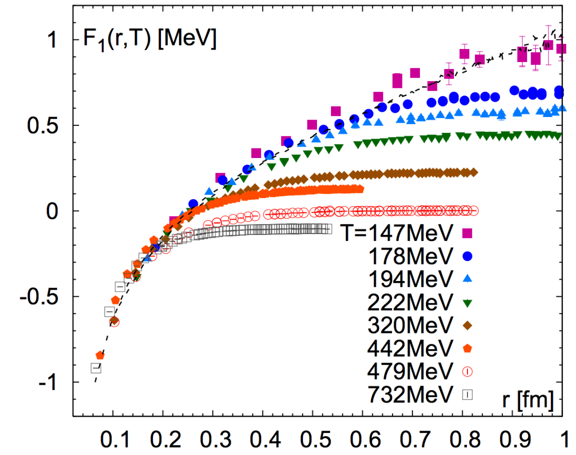
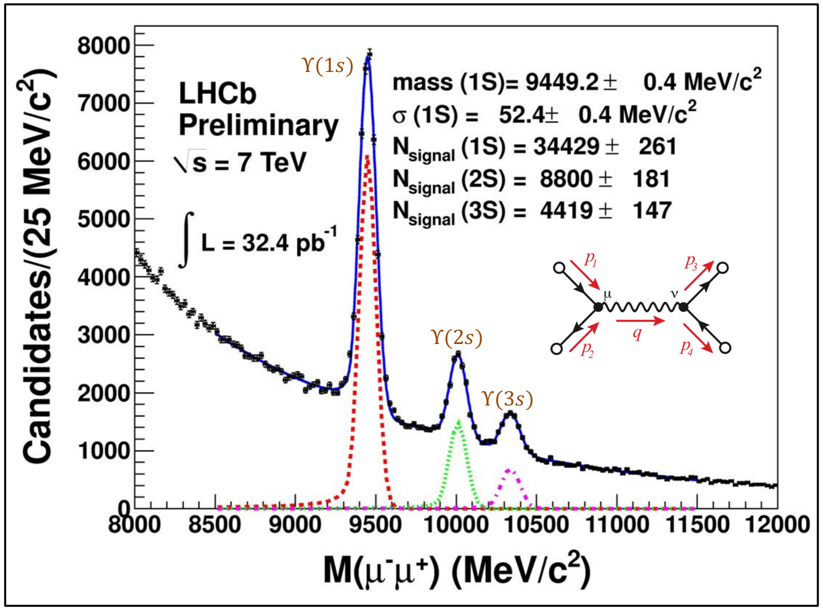
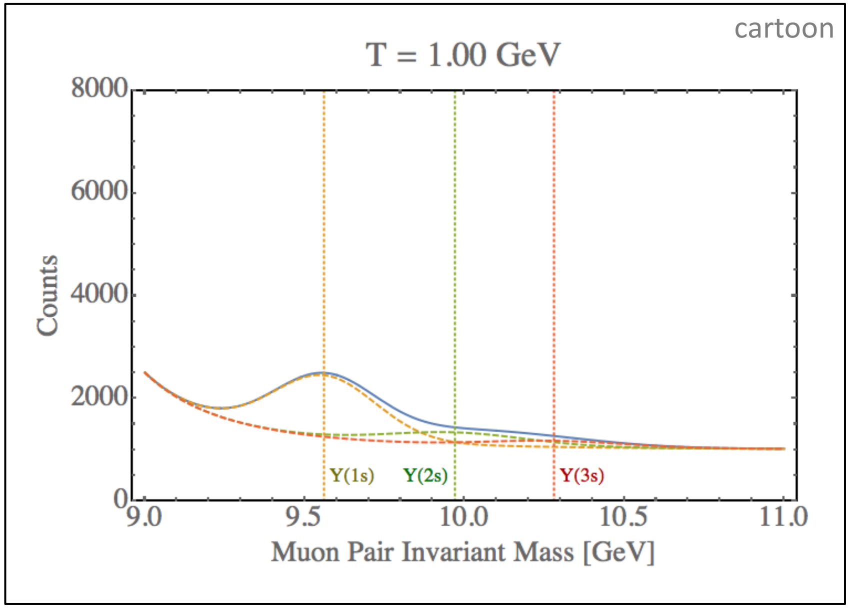
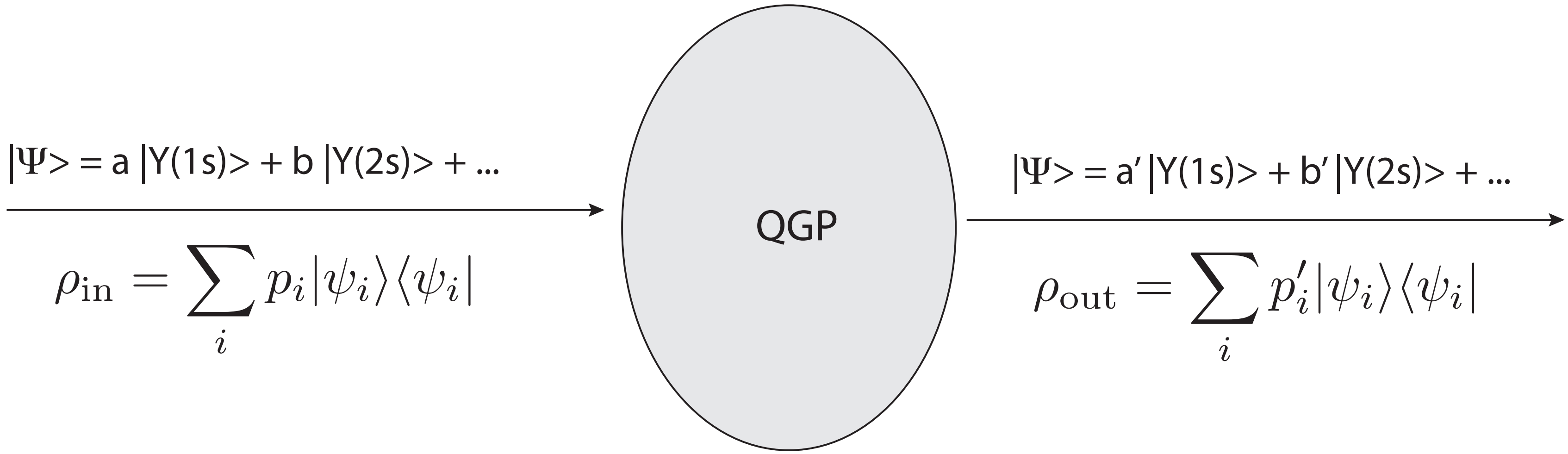




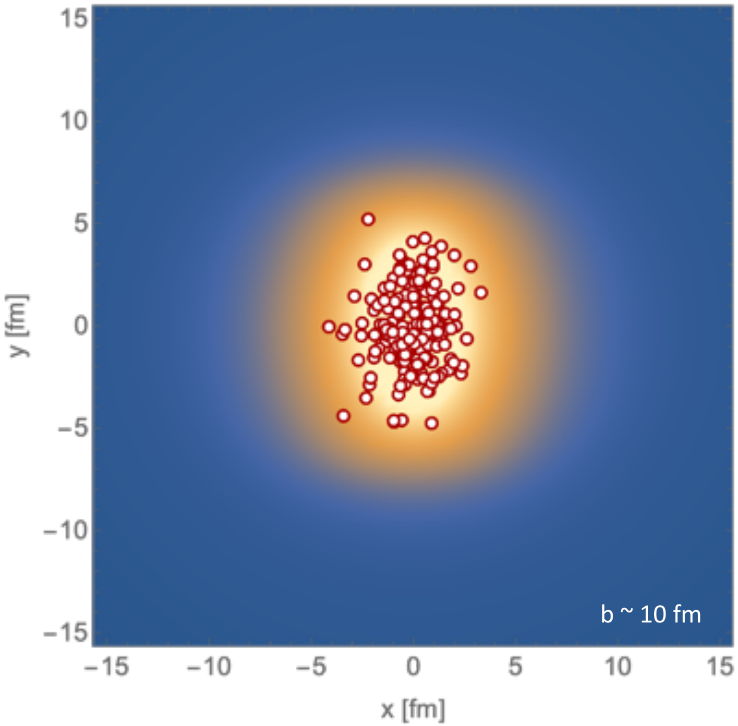


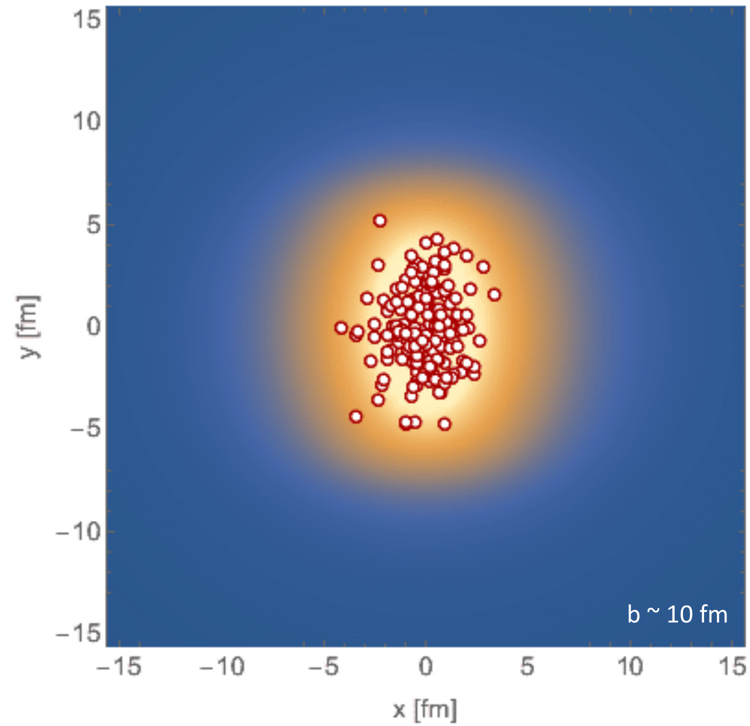
Conceptual problem



- Bottomonium states have a large binding energy and are produced locally at early times in hard collisions ($t < 1$ fm/c).
- They then propagate through the plasma and interact with the medium.
- Bound states can break up and potentially re-form due to in-medium transitions induced by gluon absorption and emission.

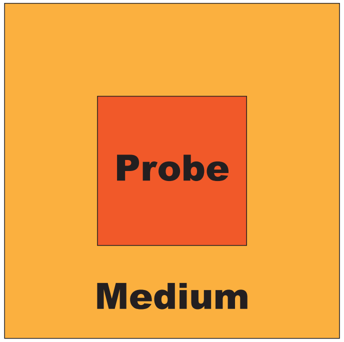
Conceptual problem





- Complicated by the fact that the medium itself is evolving in time.
- Sample initial production points from nuclear binary overlap profile.
- Sample initial momentum from pT distribution's observed in pp collisions.

Open quantum system approach I



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Can treat heavy quarkonium states propagating through QGP using an open quantum system approach

$$H_{\rm tot} = H_{\rm probe} \otimes I_{\rm medium} + I_{\rm probe} \otimes H_{\rm medium} + H_{\rm int}$$

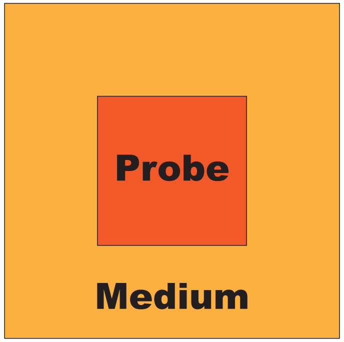
- Total density matrix

$$\rho_{\rm tot} = \sum_k \frac{1}{Z_{\rm tot}} e^{-E_k/T} |E_k\rangle\langle E_k| \longrightarrow \frac{d}{dt}\rho_{\rm tot} = -i[H_{\rm tot}, \rho_{\rm tot}]$$

- Reduced density matrix

$$\rho_{\rm probe} = {\rm Tr}_{\rm medium}[\rho_{\rm tot}] \longrightarrow$$ Evolution equation?

Open quantum system approach II



Probe = heavy-quarkonium state

Medium = light quarks and gluons that comprise the QGP

- Separation of time scales
  - Medium relaxation time scale $\langle \hat{O}_{\rm M}(t)\hat{O}_{\rm M}(0)\rangle \sim e^{-t/t_{\rm M}}$
  - Intrinsic probe time scale $t_P \sim \dfrac{1}{\omega_i - \omega_j}$
  - Probe relaxation time scale $\langle p(t)\rangle \sim e^{-t/t_{\rm rel}}$

Lindblad equation

$t_{\rm rel}\,,\, t_{\rm P} \gg t_{\rm M}$ →

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n\, \rho_{\rm probe}\, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\}\right)$$

G. Lindblad Commun. Math. Phys. 48 (1976) 119
V. Gorini, et.al. J. Math. Phys. 17 (1976) 821

Open quantum system approach III

$$\frac{d\rho_{\rm probe}}{dt} = -i[H_{\rm probe}, \rho_{\rm probe}] + \sum_n \left(C_n \, \rho_{\rm probe} \, C_n^\dagger - \frac{1}{2}\{C_n^\dagger C_n, \rho_{\rm probe}\} \right)$$

- $H_{\rm probe}$ is a Hermitian operator
- C_n are called the **collapse (or jump) operators**
- Partial and total decay widths

$$\Gamma_n = C_n^\dagger C_n \qquad \Gamma = \sum_n \Gamma_n$$

- Can reorganize by defining

$$H_{\rm eff} = H_{\rm probe} - \frac{i}{2}\Gamma$$ ← **Non-Hermitian effective Hamiltonian**

$$\longrightarrow \quad \frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^\dagger + \sum_n C_n \, \rho_{\rm probe} \, C_n^\dagger$$

Collapse operators for heavy quarkonium evolution

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\, \rho_{\rm probe}\, C_n^{\dagger}$$

$$\rho = \begin{pmatrix} \rho_s & 0 \\ 0 & \rho_o \end{pmatrix}$$

$$H = \begin{pmatrix} h_s & 0 \\ 0 & h_o \end{pmatrix} + \frac{r^2}{2}\gamma \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix}$$

$$C_i^0 = \sqrt{\frac{\kappa}{N_c^2-1}}\, r^i \begin{pmatrix} 0 & 1 \\ \sqrt{N_c^2-1} & 0 \end{pmatrix},$$

$$C_i^1 = \sqrt{\frac{(N_c^2-4)\kappa}{2(N_c^2-1)}}\, r^i \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\Gamma = \kappa r^i \begin{pmatrix} 1 & 0 \\ 0 & \frac{N_c^2-2}{2(N_c^2-1)} \end{pmatrix} r^i$$

Six collapse operators cover

- singlet → octet,
- octet → singlet
- octet → octet

$$\gamma \equiv \frac{g^2}{6\,N_c}\,{\rm Im}\int_{-\infty}^{+\infty} ds\, \langle T\, E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

$$\kappa \equiv \frac{g^2}{6\,N_c}\,{\rm Re}\int_{-\infty}^{+\infty} ds\, \langle T\, E^{a,i}(s,\mathbf{0})E^{a,i}(0,\mathbf{0})\rangle$$

N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515

How does one numerically solve these equations?

- If each block of the density matrix in color space is decomposed into orbital angular momentum using

$$\rho^{lm;l'm'} = \int d\Omega(\hat{r})\, d\Omega(\hat{r}')\, Y^{lm}(\hat{r})\, \rho\, Y^{l'm'^*}(\hat{r}')$$

- Upon truncating in angular momentum ($l <= l_{max}$) one can reduce the both the singlet and octet blocks of the reduced density matrix to size $(l_{max}+1)*(l_{max}+1)$. N. Brambilla, M. A. Escobedo, J. Soto and A. Vairo, 1612.07248, 1711.04515
- One can then discretize the wavefunction (NUM = # of points) and evolve the reduced density matrix using standard differential equation solvers and matrix methods → $2*NUM*NUM*(l_{max}+1)*(l_{max}+1)$ matrix size.
- The downside to this approach is that the size of the reduced density matrix for discretized wavefunctions scales is very large. As NUM and l_{max} become larger, the computation becomes very challenging.
- **Need a better/faster method which we can easily parallelize.**

A better way: Quantum Trajectories

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

$$\frac{d\rho_{\rm probe}}{dt} = -iH_{\rm eff}\rho_{\rm probe} + i\rho_{\rm probe}H_{\rm eff}^{\dagger} + \sum_n C_n\,\rho_{\rm probe}\,C_n^{\dagger}$$

Can treat this “quantum jump” term stochastically

- Can be reduced to the solution of a large set of “quantum trajectories” in which we solve a 1D Schrodinger equation with the **non-Hermitian Hamiltonian $\boldsymbol{H_{eff}}$**, subject to **quantum jumps**.
- The evolution with the non-Hermitian $H_{\rm eff}$ preserves the color and angular momentum state of the system (but not norm).
- Collapse/jump operators encode all transitions between different color/angular momentum states (subject to selection rules).
- Jumps are sampled stochastically, and the norm is reset.
- For each **physical trajectory** (path through the QGP) we then must average over a set of **independent quantum trajectories** → **Parallelizable**
- **Can describe all angular momentum states (no cutoff) and the algorithm scales like NUM log(NUM) by using a split-step pseudospectral solver.**

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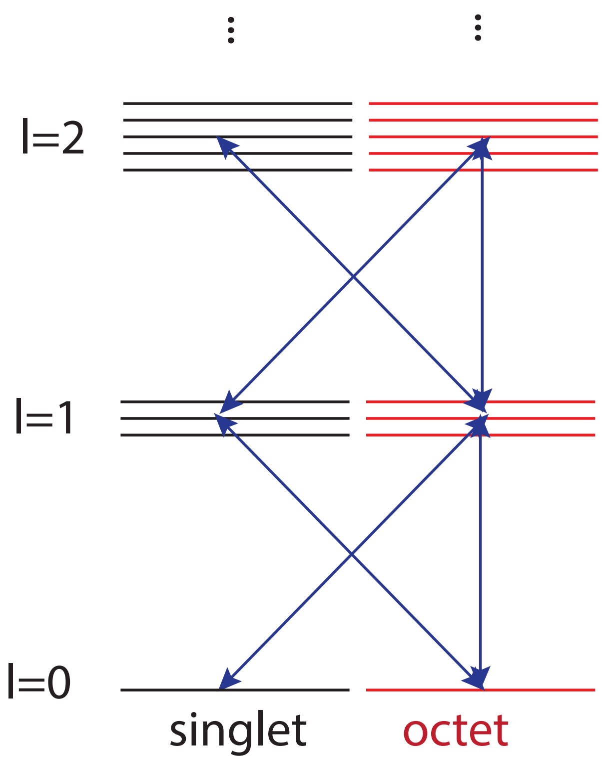

- We find that for bottomonium, the **leading order behavior is well-described by "No Jump" evolution** which corresponds to evolution with the non-Hermitian effective Hamiltonian
- **"Jump" corrections effect excited states** more than the ground state.

No Jump Evolution

A. Islam and MS, 2007.10211 (PLB), 2010.05457

“No jump” quantum evolution

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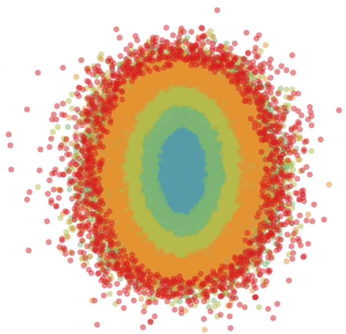

- We solved the real-time Schrodinger equation (SE) with a complex potential and computed the result for **3.6 million physical trajectories** (no quantum jumps).
- We sampled bottomonium production points and momentum using Monte Carlo sampling

$$N_{\rm bottom}(x,y,p_T) \propto \frac{N_{\rm bin}(x,y)}{(p_T^2+M^2)^2}$$

Survival probability

$$SP(n,l) = \frac{|\langle n,l|\psi(t_f)\rangle|^2}{|\langle n,l|\psi(t_0)\rangle|^2}$$

- We then recorded the temperature along each physical trajectory as provided by a 3+1 viscous hydro code (aHydroQP).
- We solved the real-time SE separately for the evolution of the l=0 and l=1 states.
- A realistic in-medium potential was used.

"No jump" quantum evolution

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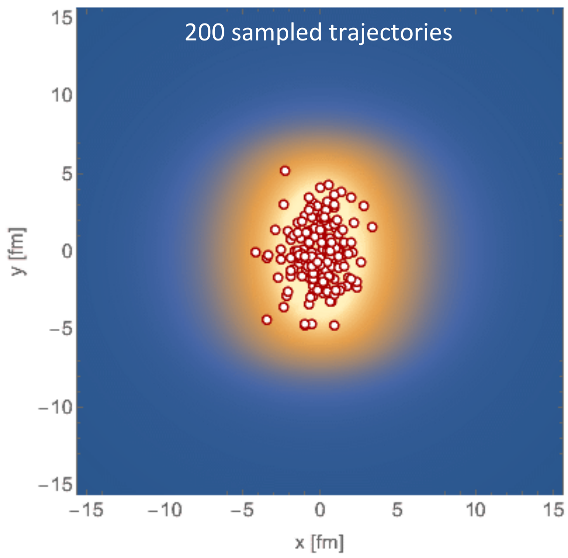

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Initial bottomonium wavefunction

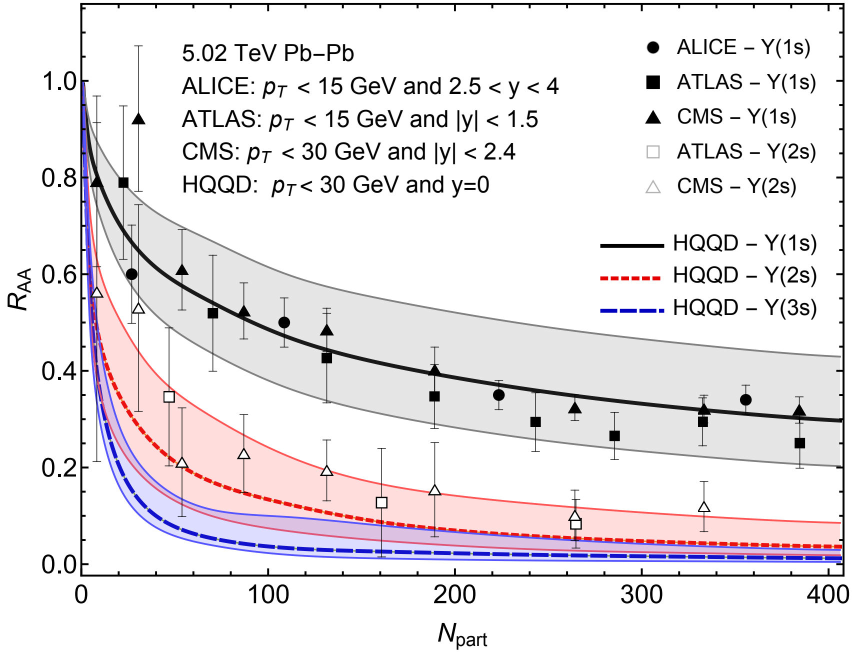
- We took the initial wavefunction to be given by a smeared delta function (local production due to large mass, Δ ~ 1/M) of the form

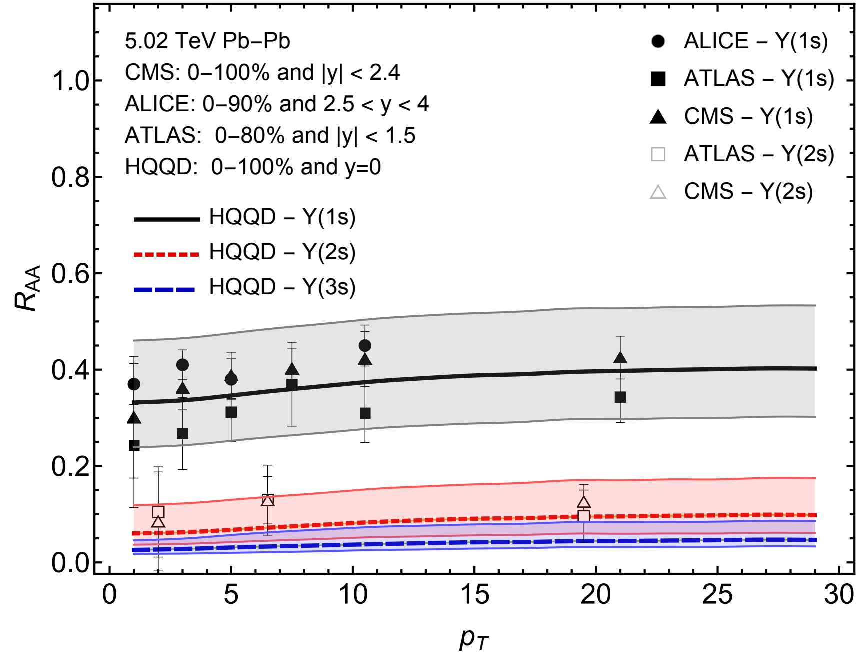
$$u_\ell(r, \tau = 0) \propto r^{\ell+1} \exp(-r^2/\Delta^2)$$

- We start with vacuum potential at τ = 0 and switch on the medium modifications at τ = {0.25,0.4,0.6} fm/c.

- For a given l, the **initial state is a quantum linear superposition** of eigenstates of H.

Comparisons with data: R_{AA}

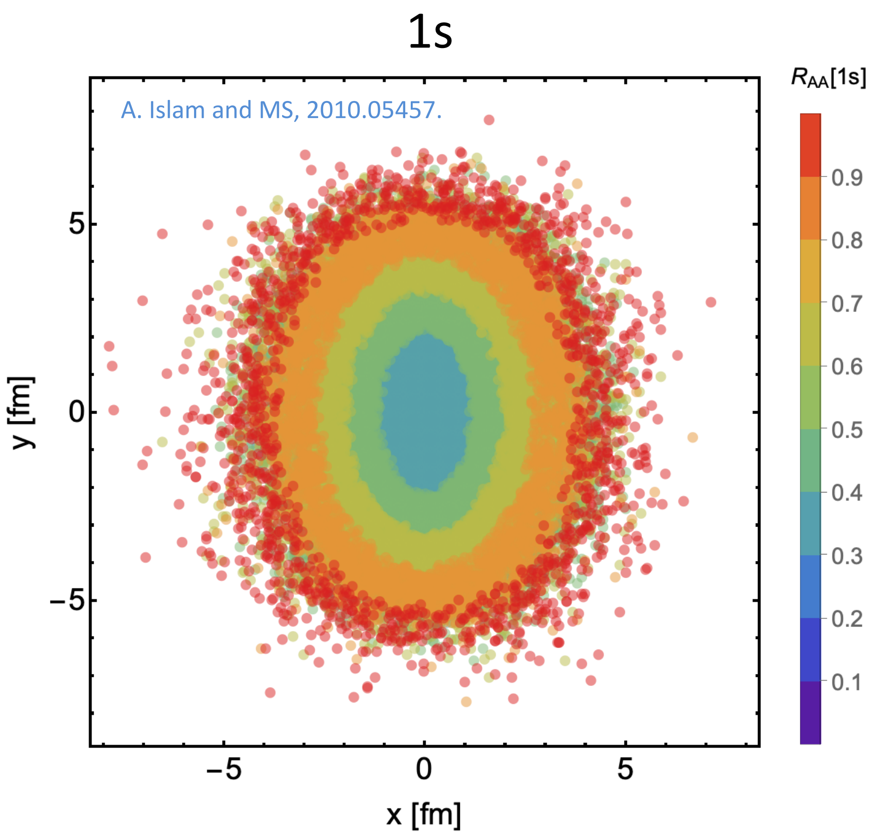




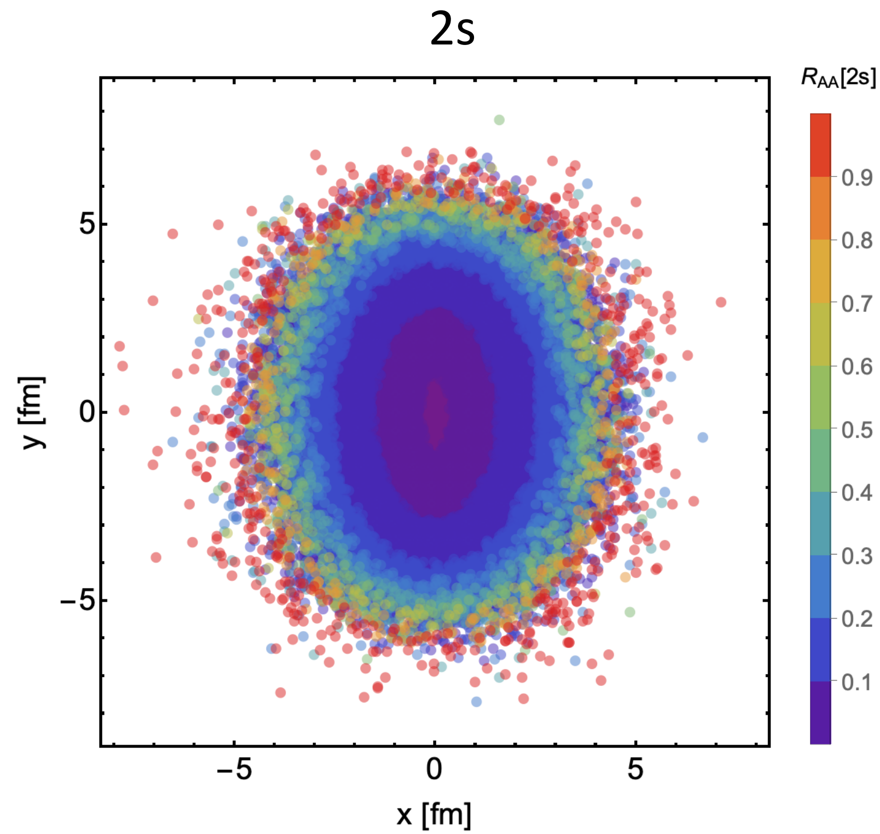
A. Islam and MS, 2007.10211 and 2010.05457

- **Bands show result of varying the Debye mass by 50% around LO pQCD value and t_{med} = {0.25, 0.4, 0.6} fm/c.**
- **Lines are the central case which corresponds to the LO pQCD Debye mass and t_{med} = 0.4 fm/c.**

Tomography



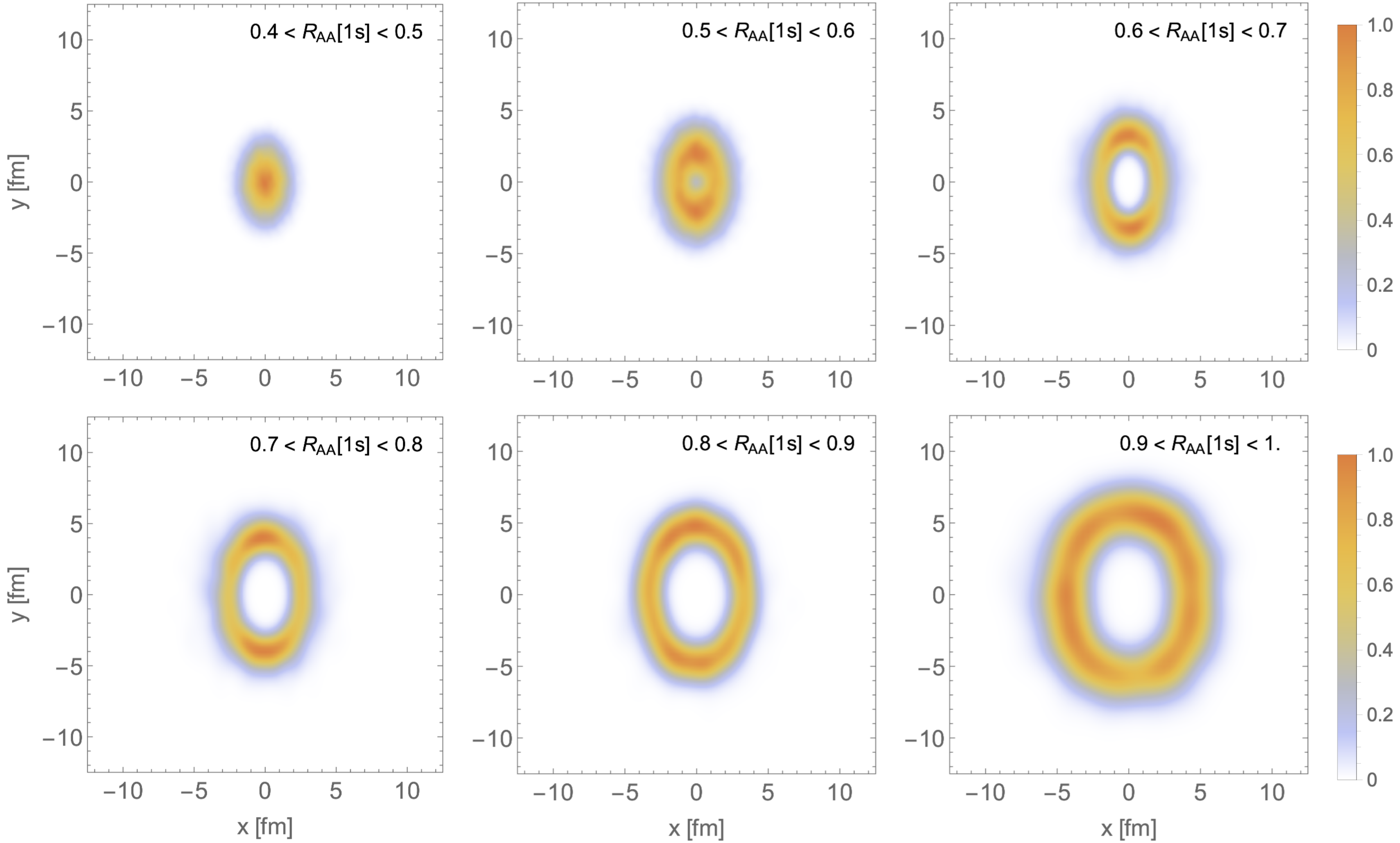
1s
A. Islam and MS, 2010.05457.
R_{AA}[1s]
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
5
0
−5
y [fm]
x [fm]



2s
R_{AA}[2s]
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
5
0
−5
y [fm]
x [fm]

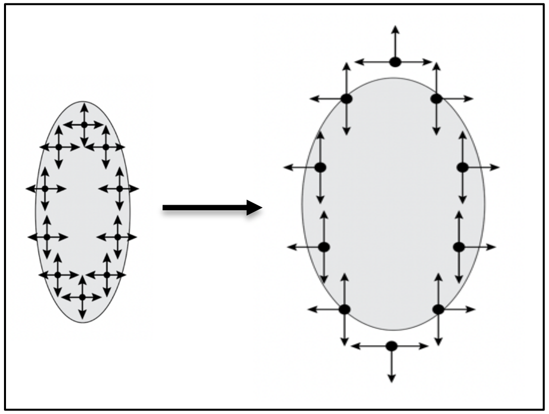
1s Tomography

A. Islam and MS, 2010.05457.

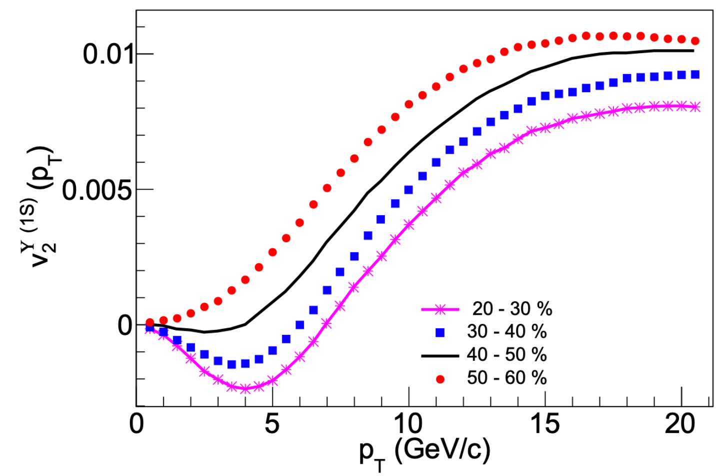


Bottomonium "flow" (aka anisotropic survival)

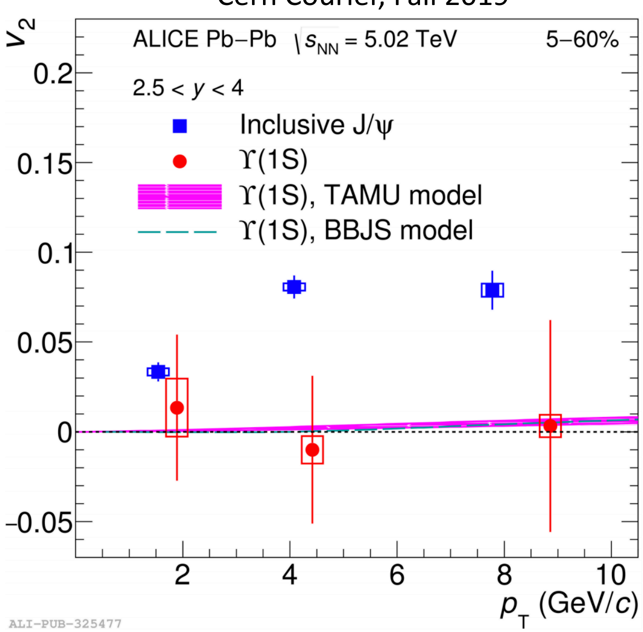
4d flow tomography



Bhaduri, Alqahtani, Borgini, Jaiswal, and MS 2007.03939

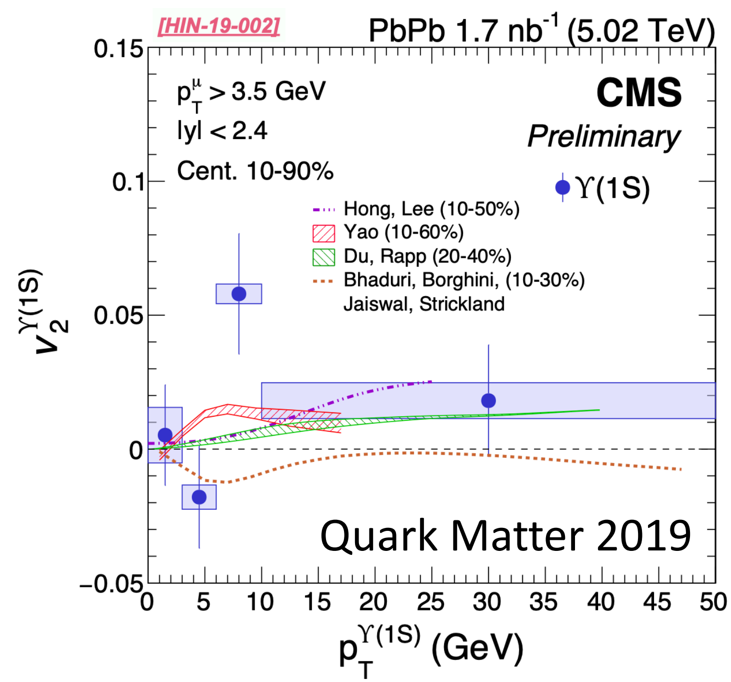


Cern Courier, Fall 2019



TAMU: Phys. Rev. C 96, (2017) 054901
BBJS: arXiv:1809.06235

ALICE Collaboration: arXiv:1907.03169



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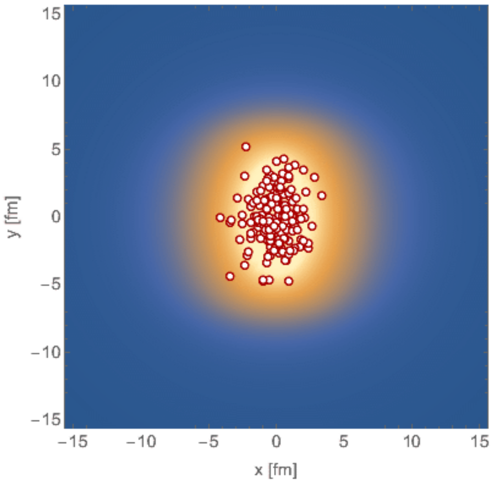

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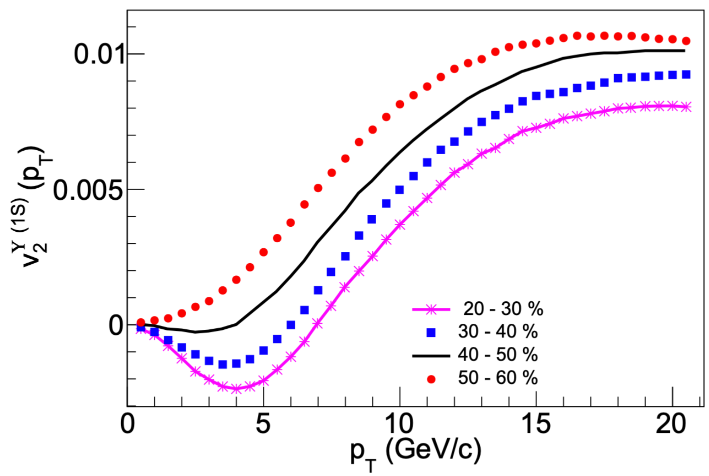

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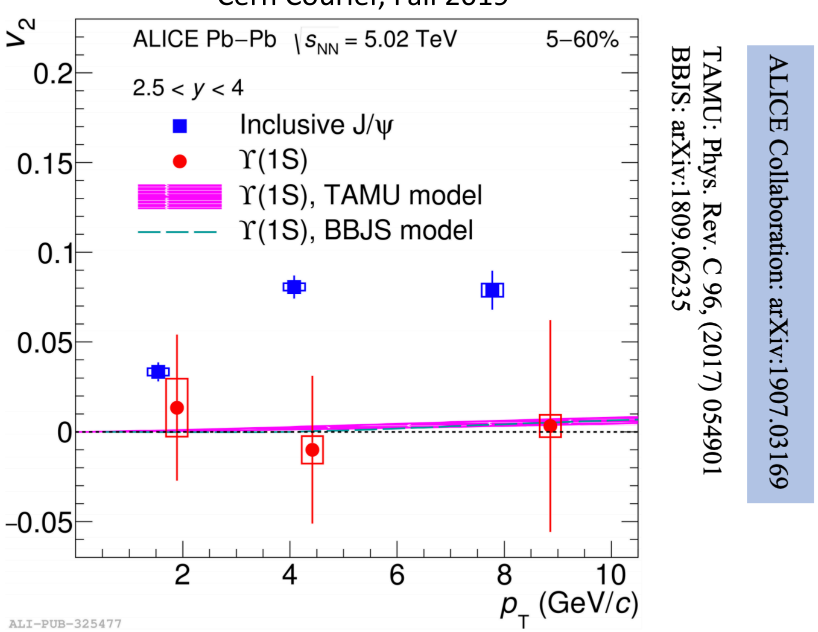
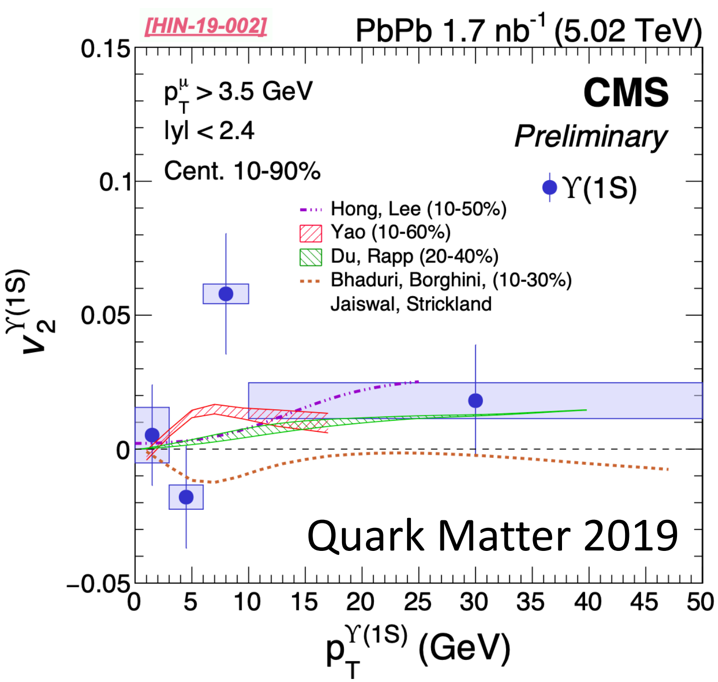
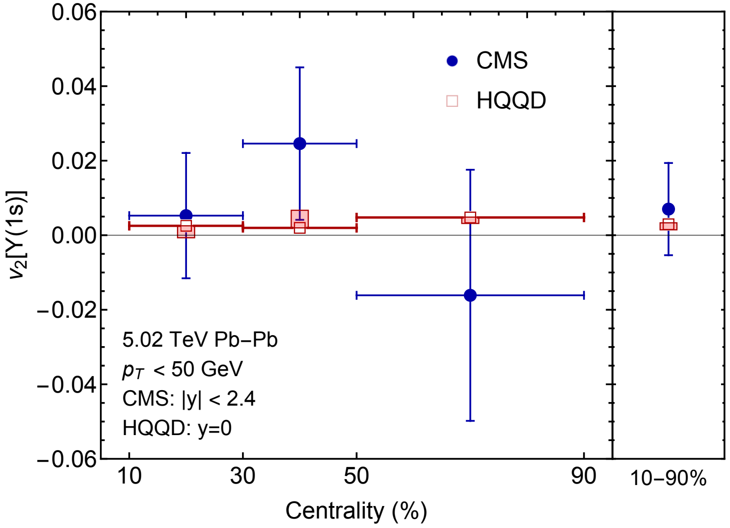
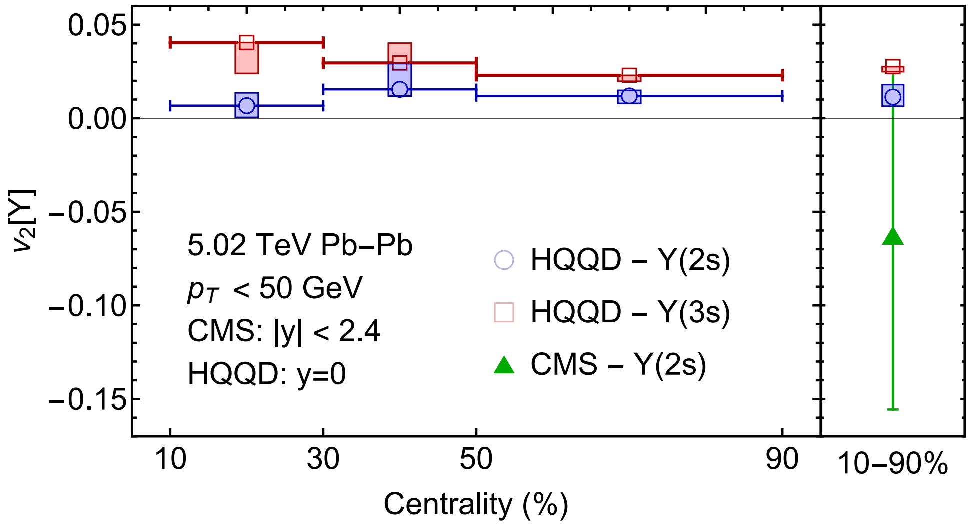


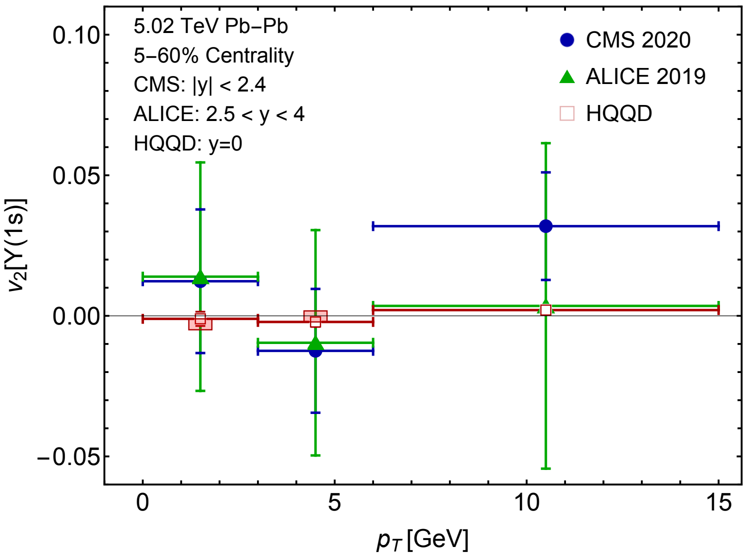


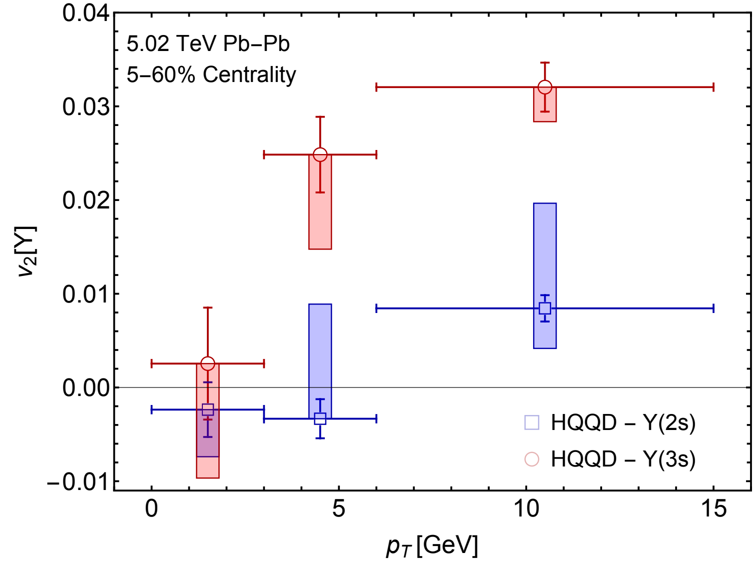
Comparisons with data: Elliptic Flow

A. Islam and MS, 2007.10211 + 2010.05457

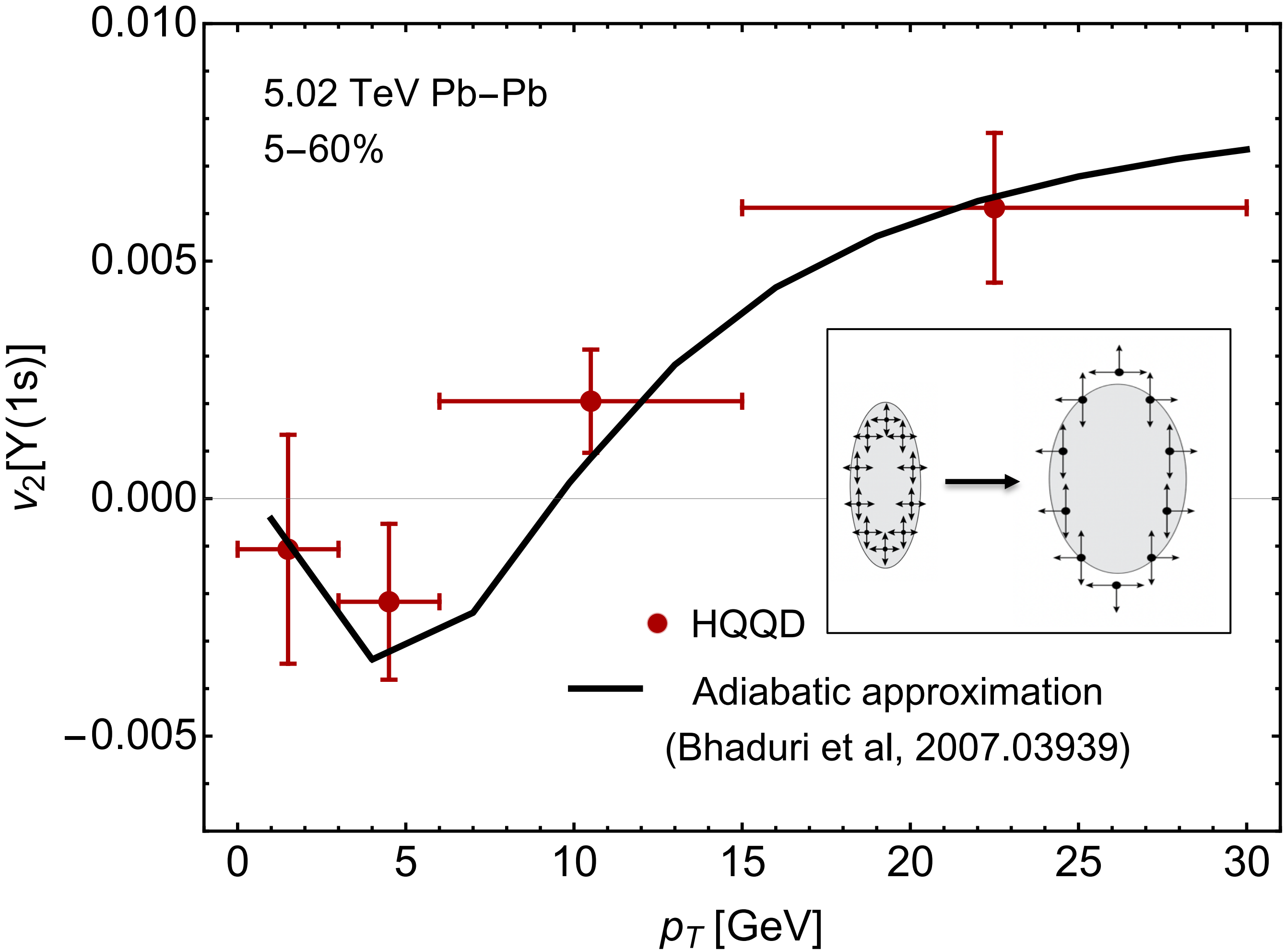








Comparison with adiabatic approx.



Adiabatic = Bhaduri, Alqahtani, Borghini, Jaiswal, and MS 2007.03939

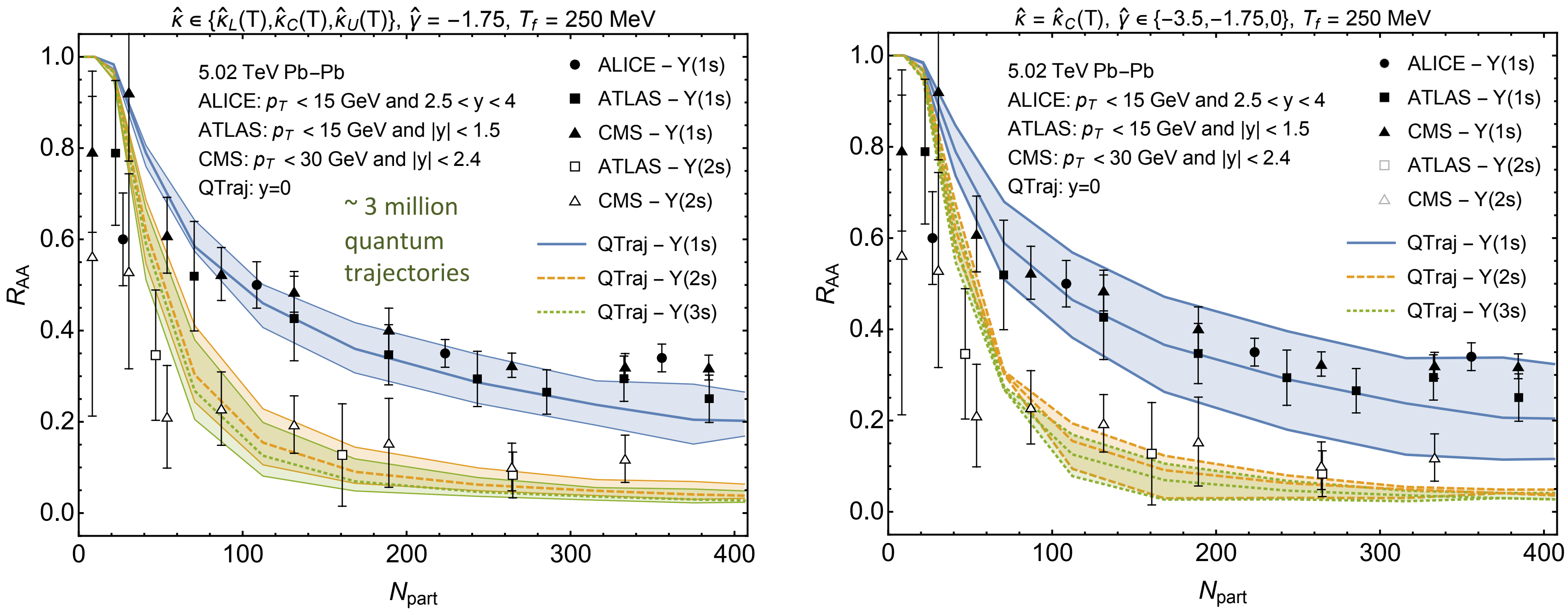
HQQD = A. Islam and MS, 2007.10211 + 2010.05457

Including Quantum Jumps

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming

R_{AA} with quantum jumps turned on

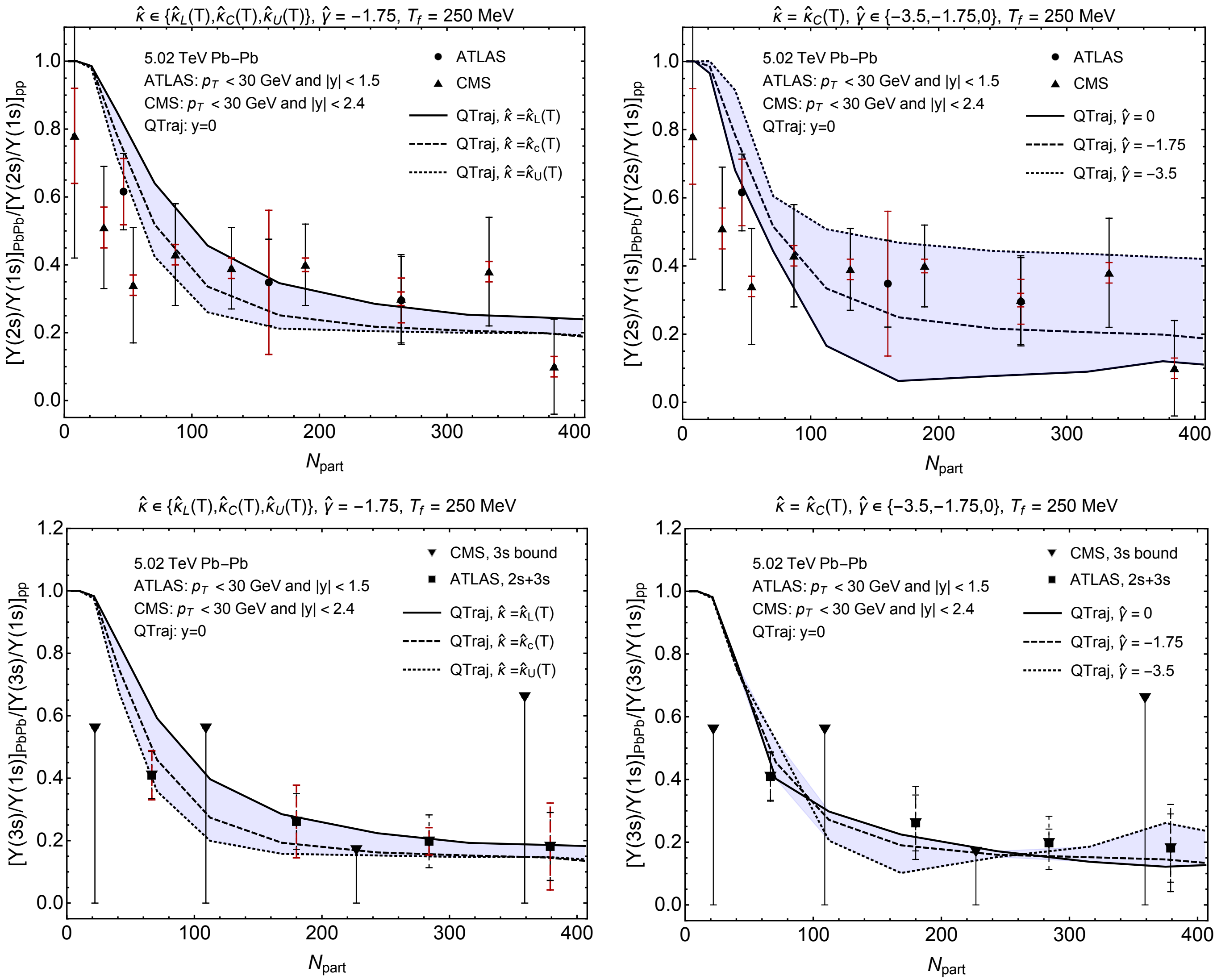
N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming



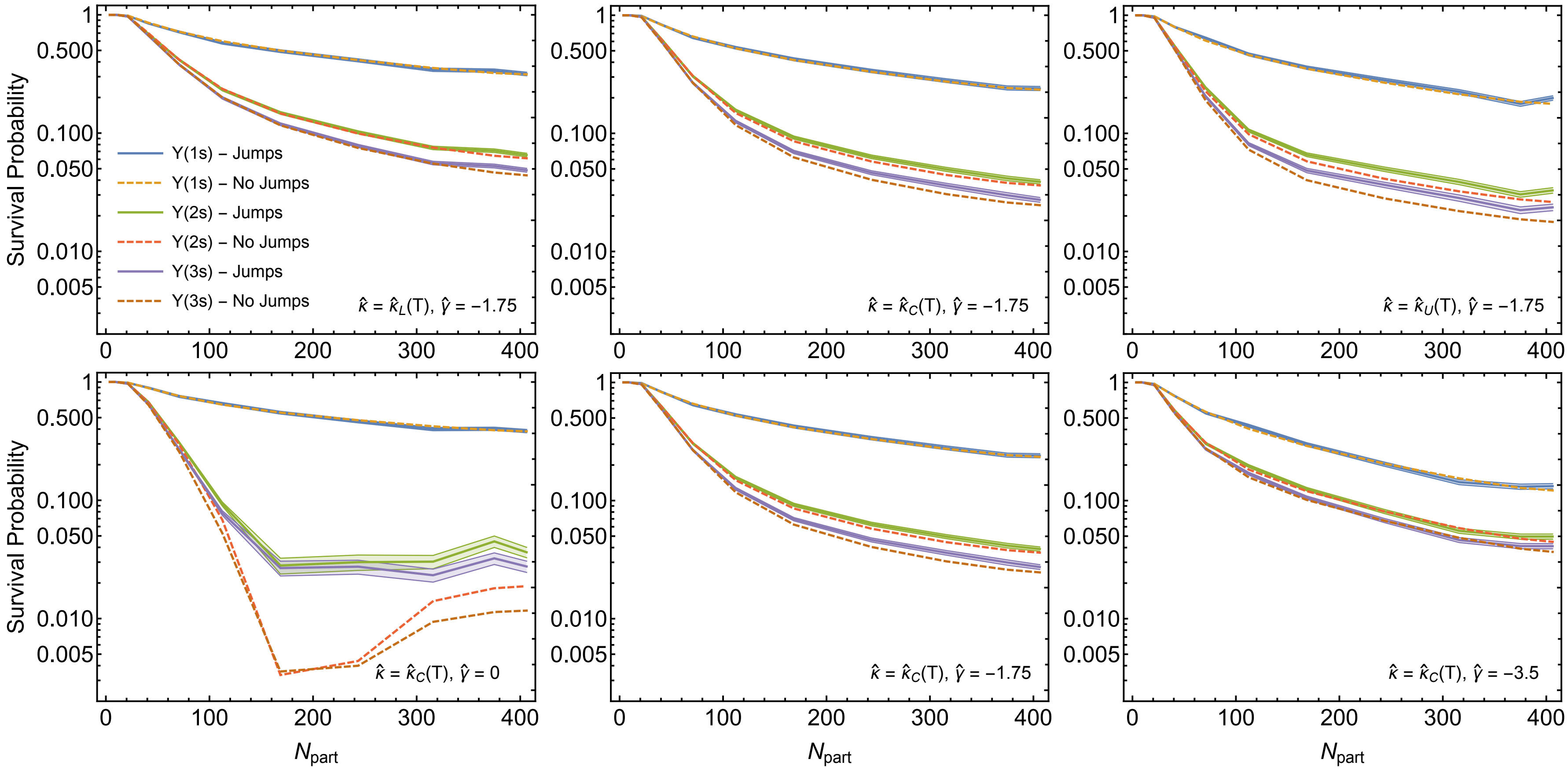
- Left panel shows the final result including feed down, when varying kappa over the theoretical uncertainty.
- Right panel shows the final result including feed down, when varying gamma over the theoretical uncertainty
- Bands also include statistical uncertainty associated with average over quantum trajectories.

2s/1s and 3s/1s double ratios

N. Brambilla, M.-A. Escobedo, M.S., A. Vairo, P. Vander Griend, and J.H. Weber, forthcoming



Effects of jumps



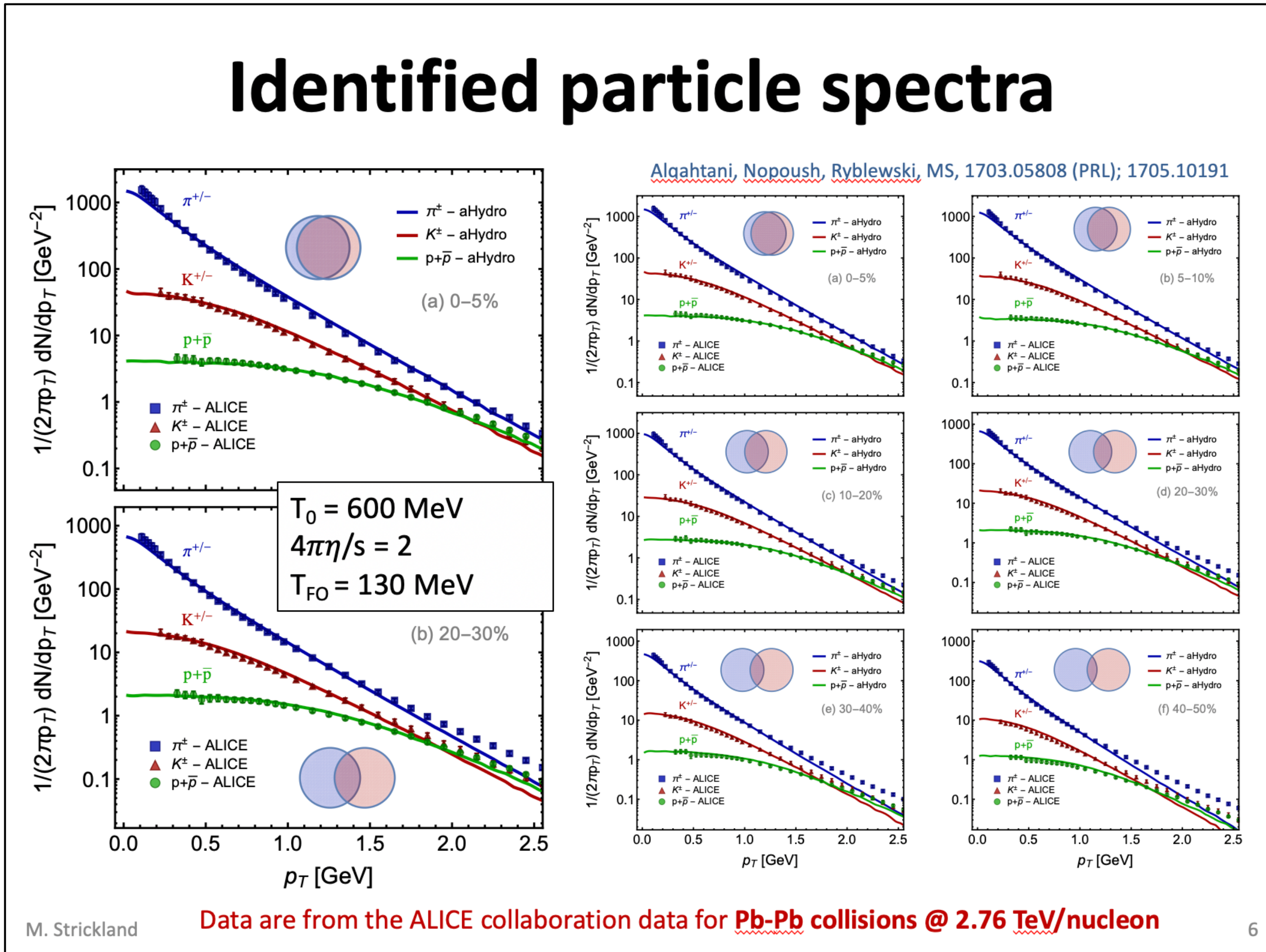
- **Solid lines show result with jumps**
- **Dashed lines show result without jumps (H_{eff} evolution)**

Conclusions and Outlook

- **The suppression of bottomonium is a smoking gun for the creation of the QGP.**
- QCD-based complex-valued potential models work reasonably well to describe the suppression and “flow” seen at LHC → QGP!
- Now able to include effect of quantum jumps between singlet/octet/angular momentum states using a parallelizable **quantum trajectories approach**.
- Firstly, using the “no jump” non-Hermitian evolution, we were able to sample **3.6 million physical trajectories**. Using the central values of our parameter set we found a good description of the data for both R_{AA} and v_2.
- Secondly, I presented forthcoming results from a quantum trajectory approach (**qTraj**) which is, thus far, the most comprehensive study of bottomonium in an open quantum systems framework.
- The qTraj code will be released soon as an **open-source multi-platform package** (GNU Public License).

Backup slides

Hydro background

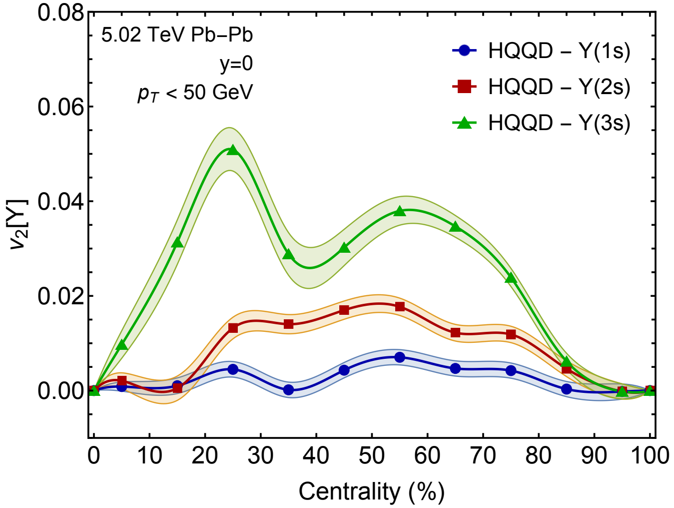


- We use a 3+1D dissipative code for the hydro background (quasiparticle anisotropic hydrodynamics)
- Has been tuned to RHIC and LHC heavy ion collisions
- Reproduces spectra, multiplicities, identified elliptic flow of light hadrons, HBT radii, etc.

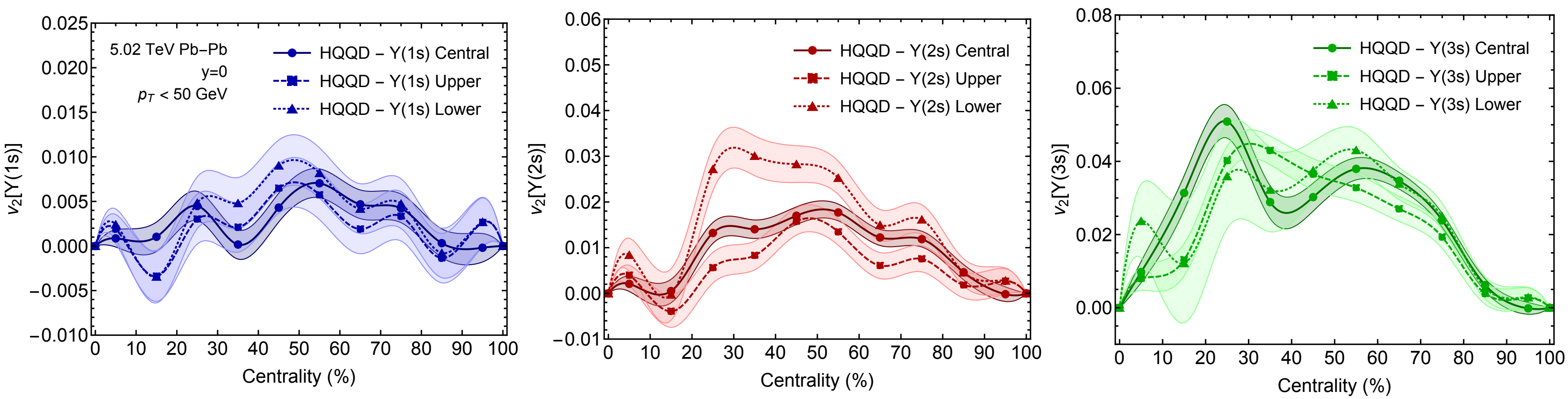
- For 2.76 TeV we use an initial central temperature of T_0 = 600 MeV @ t_0 = 0.25 fm/c
- For 5.02 TeV, T_0 = 630 MeV @ t_0 = 0.25 fm/c

Elliptic Flow (real-time)

A. Islam and MS, 2007.10211 + forthcoming



- Ordering of magnitude of v_2 as expected
- For smooth hydro initial conditions flow should go to zero at zero centrality (check!)
- Bottom panels show systematic variation when changing the Debye mass and medium initialization time.



In-medium heavy quark potential

Using the real-time formalism one can express the potential in terms of the *static* advanced, retarded, and Feynman propagators

$$V(\mathbf{r},\xi) = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \left(e^{i\mathbf{p}\cdot\mathbf{r}} - 1\right) \frac{1}{2} \left(D^*{}^L_R + D^*{}^L_A + D^*{}^L_F \right)$$

Real part can be written as

$$\mathrm{Re}[V(\mathbf{r},\xi)] = -g^2 C_F \int \frac{d^3\mathbf{p}}{(2\pi)^3} \, e^{i\mathbf{p}\cdot\mathbf{r}} \frac{\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2}{(\mathbf{p}^2 + m_\alpha^2 + m_\gamma^2)(\mathbf{p}^2 + m_\beta^2) - m_\delta^4}$$

With direction-dependent masses, e.g.

$$m_\alpha^2 = -\frac{m_D^2}{2p_\perp^2\sqrt{\xi}} \left(p_z^2 \arctan\sqrt{\xi} - \frac{p_z \mathbf{p}^2}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}} \arctan \frac{\sqrt{\xi} p_z}{\sqrt{\mathbf{p}^2 + \xi p_\perp^2}} \right)$$

Anisotropic potential calculation: Dumitru, Guo, and MS, 0711.4722 and 0903.4703
Gluon propagator in an anisotropic plasma: Romatschke and MS, hep-ph/0304092

Complex-valued Potential

- Potential can be parameterized as a Debye-screened potential with a direction-dependent Debye mass
- The potential also has an imaginary part coming from the Landau damping of the exchanged gluon
- This imaginary part also exists in the isotropic case
  Laine et al hep-ph/0611300
- Used this as a model for the free energy (F) and also obtained internal energy (U) from this.

$$V_{\text{screened}}(r,\theta,\xi,\Lambda) = -C_F\alpha_s \frac{e^{-\mu(\theta,\xi,\Lambda)r}}{r}$$

MS, 1106.2571; Bazow and MS, 1112.2761

Internal Energy

$$V_{\rm R}(\mathbf{r}) = -\frac{\alpha}{r}\,(1+\mu\, r)\exp\left(-\mu\, r\right) + \frac{2\sigma}{\mu}\left[1-\exp\left(-\mu\, r\right)\right] - \sigma\, r\,\exp(-\mu\, r) - \frac{0.8\,\sigma}{m_Q^2\, r}$$

Dumitru, Guo, Mocsy, and MS, 0901.1998

$$V_{\rm I}(\mathbf{r}) = -C_F\alpha_s p_{\rm hard}\left[\phi(\hat{r}) - \xi\left(\psi_1(\hat{r},\theta)+\psi_2(\hat{r},\theta)\right)\right]$$

Dumitru, Guo, and MS, 0711.4722 and 0903.4703
Burnier, Laine, Vepsalainen, arXiv:0903.3467 (aniso)

Real-time quantum evolution

A. Islam and MS, 2007.10211

- The final time is chosen to be when the local temperature along a given trajectory falls below the T_c = 155 MeV.
- We compute the survival probability in multiple centrality classes.
- After this stage of the calculation is done, we then do the late time **feed down**.
- Since we have access to individual particles in the simulation, we do the feed down in different manner than previously done.

Numerical Method

- We use a split-step pseudospectral method

Step-1: Update in configuration space using a half-step: $\psi_1 = \exp(-iV\Delta t/2)\psi_0$.

Step-2: Perform Fourier sine transformations ($\mathbb{F}_s$) on real and imaginary parts separately: $\tilde{\psi}_1 = \mathbb{F}_s[\Re\psi_1] + i\mathbb{F}_s[\Im\psi_1]$.

Step-3: Update in momentum space using: $\tilde{\psi}_2 = \exp\left(-i\frac{p^2}{2m}\Delta t\right)\tilde{\psi}_1$.

Step-4: Perform inverse Fourier sine transformations ($\mathbb{F}_s^{-1}$) on real and imaginary parts separately: $\psi_2 = \mathbb{F}_s^{-1}[\Re\tilde{\psi}_2] + i\mathbb{F}_s^{-1}[\Im\tilde{\psi}_2]$.

Step-5: Update in configuration space using a half-step: $\psi_3 = \exp(-iV\Delta t/2)\psi_2$.

Feed-down implementation

A. Islam and MS, 2007.10211

$$N_{\text{final}} = F N_{\text{QGP}}$$

$$F = \begin{pmatrix} 1 & 0.265 & 0.184 & 0.0657 & 0.0650 \\ 0 & 0.735 & 0 & 0.1060 & 0.0946 \\ 0 & 0 & 0.816 & 0 & 0.0047 \\ 0 & 0 & 0 & 0.8283 & 0 \\ 0 & 0 & 0 & 0 & 0.8357 \end{pmatrix}.$$

- N_{QGP} corresponds to $(N_{1s}, N_{2s}, N_{2p}, N_{3s}, N_{3p}, N_{3d})^T$ where, e.g., N_{1s} is the final number of Υ(1s) states that survive the QGP along a given trajectory
- For each state this can be obtained using

$$\langle N_{bin}(b) \rangle * \sigma_{direct} * \text{(Survival probability)}$$

- After feed down, we then normalize the numbers obtained in each row by the total number expected if one scales up pp collisions to AA → R_{AA}.
- We then average over all trajectories in a given centrality/p_T bin to obtain R_{AA} vs centrality and p_T

Details for feed down

State	$\Upsilon(1s)$	$\Upsilon(2s)$	$\chi_b(1p)$	$\Upsilon(3s)$	$\chi_b(2p)$
$\boldsymbol{\sigma_{exp}(nb)}$	57.6	19	13.82	3.36	2.07
$\boldsymbol{\sigma_{primordial}(nb)}$	47.45	24.95	16.92	4.057	2.477

Decay of $\boldsymbol{\Upsilon(2s)}$, $\boldsymbol{\chi_b(1p)}$, and $\boldsymbol{\Upsilon(3s)}$		
Mode	**Fraction (Γ_i/Γ)**	**Feed-down matrix element**
$\Upsilon(2s) \to \Upsilon(1s)\pi^+\pi^-$	17.85%	$f_{12} = (17.85 + 8.6)\% = 0.265$
$\Upsilon(2s) \to \Upsilon(1s)\pi^0\pi^0$	8.6%	
$\chi_{b0}(1p) \to \gamma\Upsilon(1s)$	1.94%	$f_{13} = \frac{1.94\% + 35.2\% + 18.0\%}{3} = 0.184$
$\chi_{b1}(1p) \to \gamma\Upsilon(1s)$	35.2%	
$\chi_{b2}(1p) \to \gamma\Upsilon(1s)$	18.0%	
$\Upsilon(3s) \to \Upsilon(1s)\pi^+\pi^-$	4.37%	$f_{14} = (4.37 + 2.20)\% = 0.0657$
$\Upsilon(3s) \to \Upsilon(1s)\pi^0\pi^0$	2.20%	
$\Upsilon(3s) \to \Upsilon(2s)$ anything	10.6%	$f_{24} = 10.6\% = 0.106$

Decay of $\boldsymbol{\chi_b(2p)}$			
Mode	**Fraction (Γ_i/Γ)**		**Feed-down matrix element**
$\chi_{b0}(2p) \to \gamma\Upsilon(1s)$	3.8×10^{-3}		$f_{15} = \frac{3.8 \times 10^{-3} + 11.53\% + 7.7\%}{3} = 0.065$
$\chi_{b1}(2p) \to \omega\Upsilon(1s)$	1.63%	$(1.63 + 9.9 = 11.53)\%$	
$\chi_{b1}(2p) \to \gamma\Upsilon(1s)$	9.9%		
$\chi_{b2}(2p) \to \omega\Upsilon(1s)$	1.10%	$(1.10 + 6.6 = 7.7)\%$	
$\chi_{b2}(2p) \to \gamma\Upsilon(1s)$	6.6%		
$\chi_{b0}(2p) \to \gamma\Upsilon(2s)$	1.38%		$f_{25} = \frac{1.38\% + 18.1\% + 8.9\%}{3} = 0.0946$
$\chi_{b1}(2p) \to \gamma\Upsilon(2s)$	18.1%		
$\chi_{b2}(2p) \to \gamma\Upsilon(2s)$	8.9%		
$\chi_{b0}(2p) \to \pi\pi\chi_{b0}(1p)$	0		$f_{35} = \frac{0 + 9.1 \times 10^{-3} + 5.1 \times 10^{-3}}{3} = 0.0047$
$\chi_{b1}(2p) \to \pi\pi\chi_{b1}(1p)$	9.1×10^{-3}		
$\chi_{b2}(2p) \to \pi\pi\chi_{b2}(1p)$	5.1×10^{-3}		

A. Islam and MS, forthcoming.

Feed-down implementation

A. Islam and MS, 2007.10211

- We also compute v_2 = <Cos(2ϕ)> by averaging over all trajectories.
- Can compute higher order flow coefficients etc. Only limited by statistics now.