The QCD equation of state at finite density, from lattice to neutron stars

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11/11/2020

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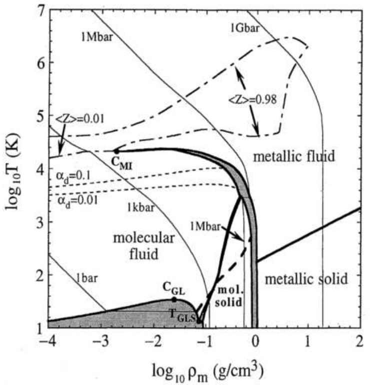
Thanks to:
V. Vovchenko, A. Motornenko, E. Most and H. Stöcker

The features of the QCD phase diagram at high density.

Can we eventually draw a diagram like this for the textbooks?(Hydrogen)



Kitamura H., Ichimaru S., J. Phys. Soc. Japan 67, 950 (1998).

Motivation

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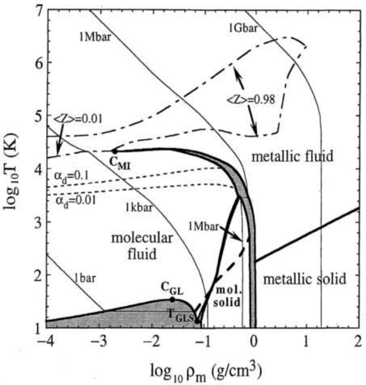

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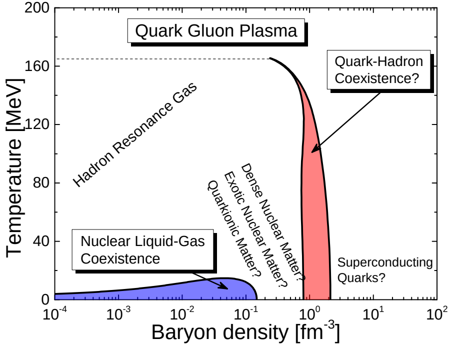
QCD



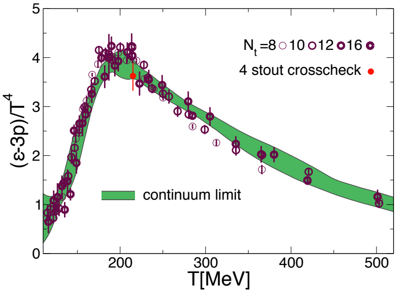
JS , V. Dexheimer, H. Petersen, M. Bleicher, S. Schramm and H. Stoecker, Phys. Rev. C **81**, 044913 (2010)

Robust constraints on the Equation of state from:

- Lattice QCD, for $T \geq 130$ MeV.

Constraints from lQCD:

- The Interaction measure, thermodynamics at $\mu_B = 0$
- Derivatives of the pressure wrt μ_B. Expansion into finite real μ_B.
- Calculations at imaginary μ.



S. Borsanyi, Z. Fodor, C. Hoelbling, S. D. Katz, S. Krieg and K. K. Szabo, Phys. Lett. B **730**, 99 (2014)

A model that uses lattice QCD data in imaginary $\mu_B \rightarrow$ the CEM model

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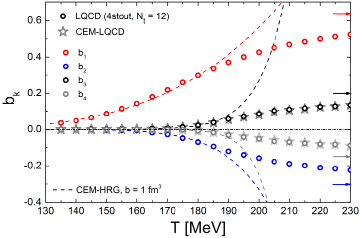
- One can write the density of QCD as a cluster expansion:
- $\frac{\rho_B}{T^3} = \frac{\partial(p/T^4)}{\partial(\mu_B/T)} = \sum_{k=1}^{\infty} b_k(T) \sinh\left(\frac{k\,\mu_B}{T}\right)$



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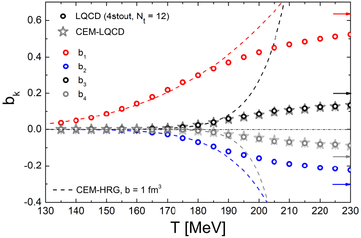
- Assuming the proper SB limit and using only the first two coefficients on can exactly predict finite μ_B thermodynamics
- $b_k(T) = \alpha_k \frac{[b_2(T)]^{k-1}}{[b_1(T)]^{k-2}}$. Use these to calculate χ^n.



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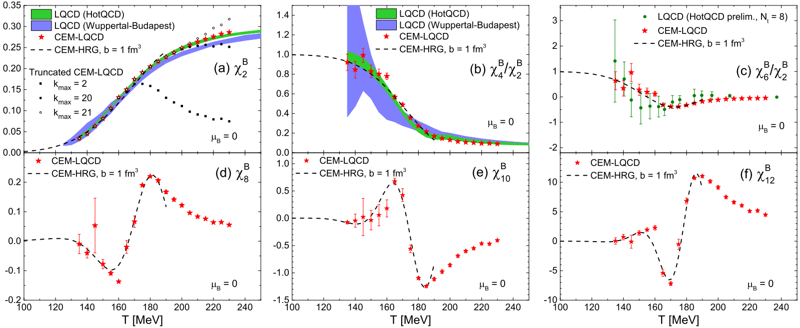

Results on the applicability

Radius of convergence: $\mu_B/T < \pi$

V. Vovchenko, **JS** , O. Philipsen and H. Stoecker, Phys. Rev. D **97**, no.11, 114030 (2018)

Taylor expansion in real μ_B

Instead of expanding in imaginary μ, do a Taylor expansion in real μ_B

- Write the expansion of the pressure using susceptibilities:

$$P = P_0 + T^4 \sum_{i,j,k} \frac{1}{i!j!k!} \chi_{B,Q,S}^{i,j,k} \left(\frac{\mu_B}{T}\right)^i \left(\frac{\mu_Q}{T}\right)^j \left(\frac{\mu_S}{T}\right)^k , \quad (1)$$

A. Bazavov et. al., Phys. Rev. D 95, 054504 (2017)

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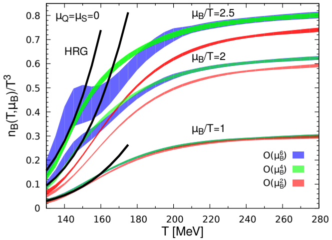

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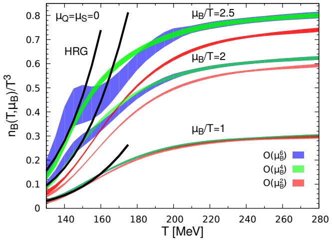

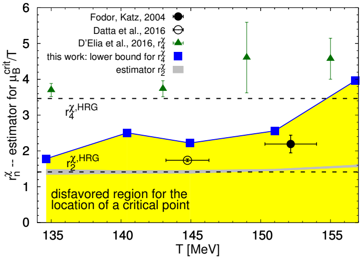

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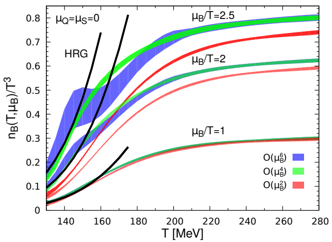

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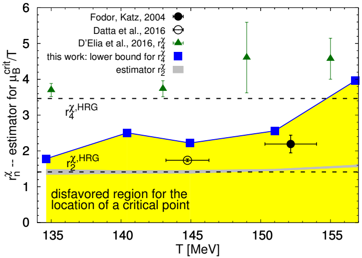
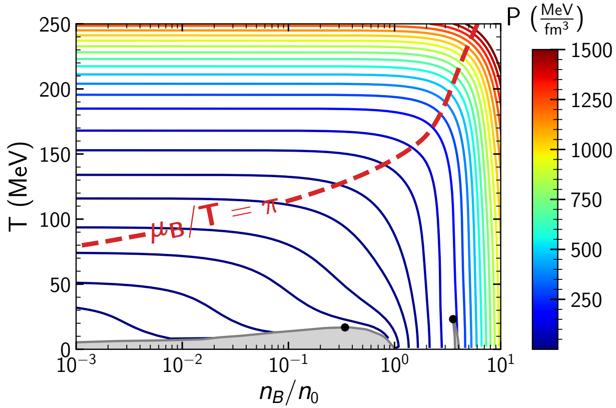


- Artifacts appear around $\mu_B/T > 2.5$
- Radius of convergence $\mu_B/T < 3$
- High T rule out quark-repulsion.

JS and S. Schramm, Phys. Lett. B **736**, 241-245 (2014)

Why the breakdown at $\mu_B/T \approx 3$?



Why do the methods break down?

- Sudden change of isobaric lines at this point.
- From Boson (mesons/gluons) dominated matter to fermionic matter (nucleons/quarks).

A. Motornenko, **JS**, V. Vovchenko, S. Schramm and H. Stöcker, (Quark Matter 2019), Wuhan, China, November 3-9 2019

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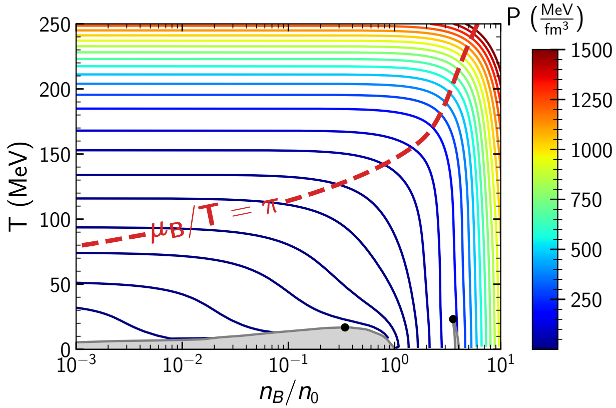

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- First principle calculations seem to fail for fermionic matter.

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Constraints at $T = 0$

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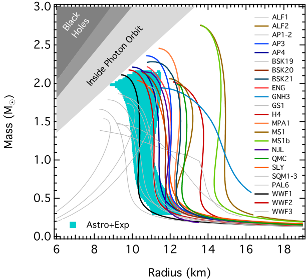

F. Özel and P. Freire, Ann. Rev. Astron. Astrophys.

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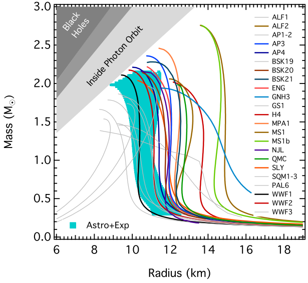

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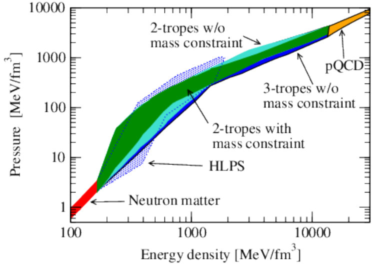

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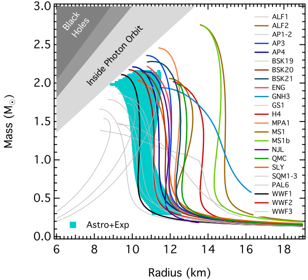

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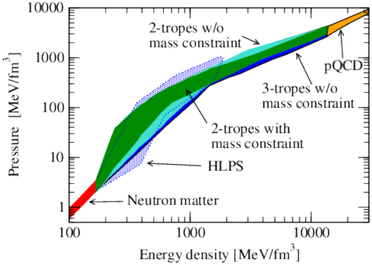

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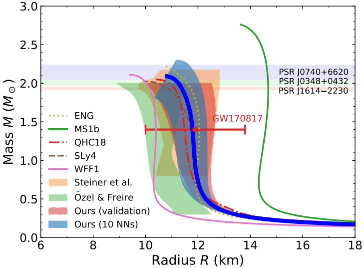
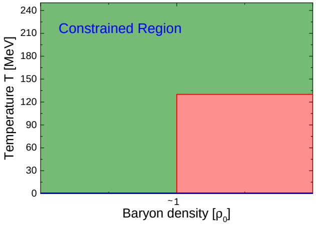


Y. Fujimoto, K. Fukushima and K. Murase, Phys.

Constraints at $T = 0$

- Here we have guidance from measured neutron star masses
- Without Radii no real constraints!
- Add constraints from PQCD.
- Still missing the important region. Extension to finite temperature $\rightarrow$ New degrees of freedom.



The strategy: A phenomenological approach

What can be done to study the EoS at high density?

- Design effective models that match lattice QCD at low μ_B and neutron stars at high density.

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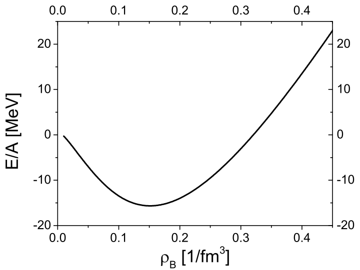
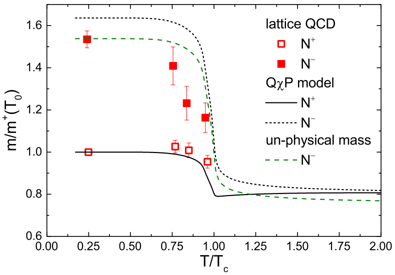
- Design effective models that match lattice QCD at low μ_B and neutron stars at high density.
- Employ these models for heavy ion collisions as well neutron star mergers.
- Find a consistent description
- Possibly new analysis methods that combine many observables and statistical / machine learning methods.

Effective $SU(3)_f$ chiral mean field model based on:

- Chiral symmetry for hadrons via nucleon parity partners: Describes nuclear matter and lattice phenomenology.
- Effective masses for baryons: $m^*_{i\pm} = \sqrt{\left[(g^{(1)}_{\sigma i}\sigma + g^{(1)}_{\zeta i}\zeta)^2 + (m_0 + n_s m_s)^2\right]} \pm g^{(2)}_{\sigma i}\sigma \pm g^{(2)}_{\zeta i}\zeta$.

$$U_{sc} = \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\zeta^2\zeta^2 + \frac{1}{2}k_0 I_2 - k_1 I_2^2 - k_2 I_4 + k_6 I_6 + m_\pi^2 f_\pi \sigma + \left(\sqrt{2}m_K^2 f_K - \frac{1}{\sqrt{2}}m_\pi^2 f_\pi\right)\zeta - k_4 \log\left(\frac{\sigma^2\zeta}{\sigma_0^2\zeta_0}\right) \quad (2)$$





JS, S. Schramm and H. Stöcker, Phys. Rev. C **84**, 045208 (2011)

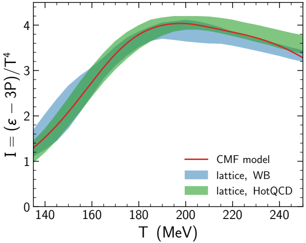
One example: Effective model for this - the CMF

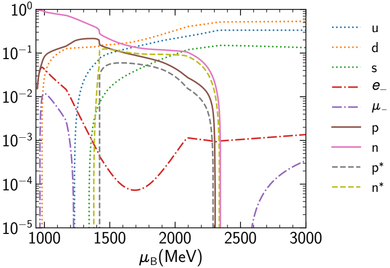
Effective $SU(3)_f$ chiral mean field model based on:

- Deconfined quarks and gluons via effective Polyakov Loop potential and removal of hadrons via excluded volume.

$$\Omega_q = -VT \sum_{i \in Q} \frac{d_i}{(2\pi)^3} \int d^3k \frac{1}{N_c} \ln \left(1 + 3\Phi e^{-(E_i^* - \mu_i^*)/T} + 3\bar{\Phi} e^{-2(E_i^* - \mu_i^*)/T} + e^{-3(E_i^* - \mu_i^*)/T}\right)$$

$$U_{\text{Pol}}(\Phi, \bar{\Phi}, T) = -\frac{1}{2} a(T)\Phi\bar{\Phi} + b(T) \log[1 - 6\Phi\bar{\Phi} + 4(\Phi^3 + \bar{\Phi}^3) - 3(\Phi\bar{\Phi})^2], \, a(T) = a_0 T^4 + a_1 T_0 T^3 + a_2 T_0^2 T^2, \; b(T) = b_3 T_0^4$$



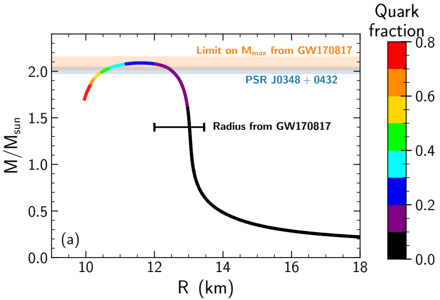


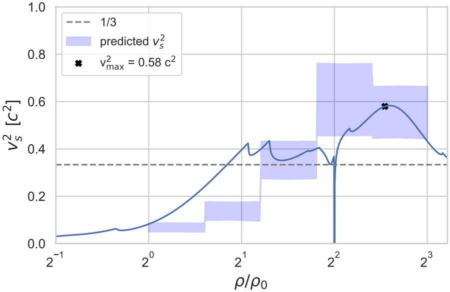
A. Motornenko, **JS**, V. Vovchenko, S. Schramm and H. Stoecker, Phys. Rev. C **101**, no.3, 034904 (2020)

One example: Effective model for this - the CMF

Application for cold compact stars

- Mass radius diagram consistent with astrophysical constraints.





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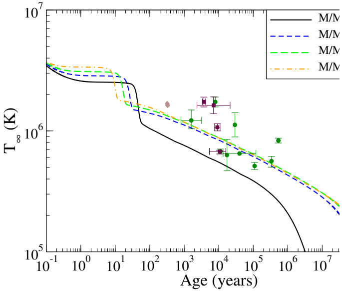
Application for cold compact stars

- Mass radius diagram consistent with astrophysical constraints.
- Interesting effects in supernova and cooling curve: What is the role of the parity partners and quarks in the cooling?

V. Dexheimer, **JS**, R. Negreiros and S. Schramm, Phys. Rev. C **87**, no.1, 015804 (2013)



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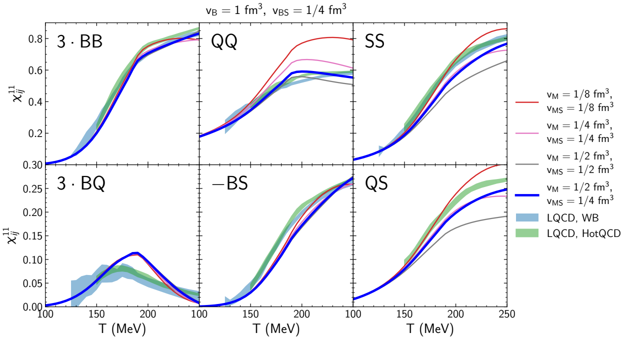
Small caveat

- In principle the model can have an infinite coupling parameters for he hadrons.
- Study suggests they can relate to the susceptibilities.



A. Motornenko, S. Pal, A. Bhattacharyya, **JS** and H. Stoecker,[arXiv:2009.10848 [hep-ph]].

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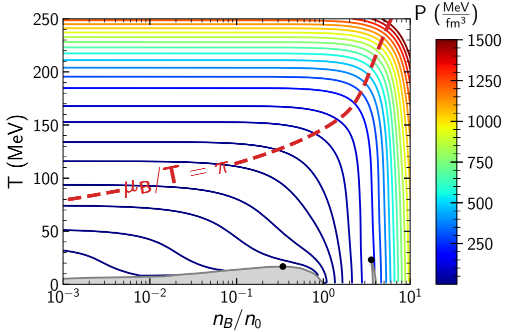
- Study suggests they can relate to the susceptibilities.
- However, resulting phase structure appears mainly insensitive.



Usage in HIC

- This EoS enables us to treat heavy ion collisions and NS mergers on the same footing.
- Relativistic fluid dynamics.
- Can both be consistently described by any model for the EoS?
- In which scenarios do we see effects from a phase transition or EoS which violates our constraints?
- Remember: more constraints possible: e.g. model is still mean field, finite size behavior, etc.

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What is the data situation in HIC?

- In short: what is really measured are fluctuations and correlations in momentum space.
- The downsides of hadrons: freeze-out and rescattering wash out signals
- Implementation of EoS for the fully dynamical description from pre-equilibrum to freeze-out necessary

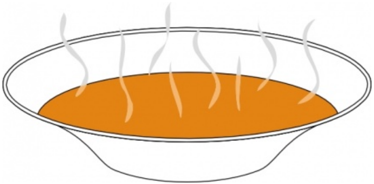
Electromagnetic probes

Electromagnetic probes offer a chance to probe the whole time evolution of the fireball.

In particular di-lepton pairs created by the decay of hadrons or quark annihilation.

- $\rho \rightarrow e^{+} + e^{-}$
- $q + \overline{q} \rightarrow e^{+} + e^{-}$

Process sensitive to the medium in which it takes place (T and ρ_B).



F. Seck, T. Galatyuk, A. Mukherjee, R. Rapp, **JS** and J. Stroth, [arXiv:2010.04614 [nucl-th]].

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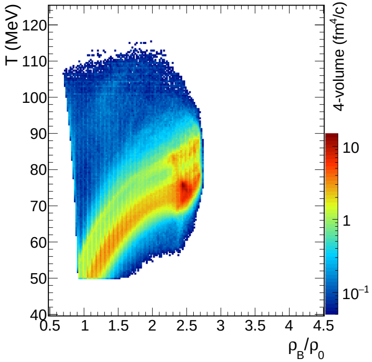
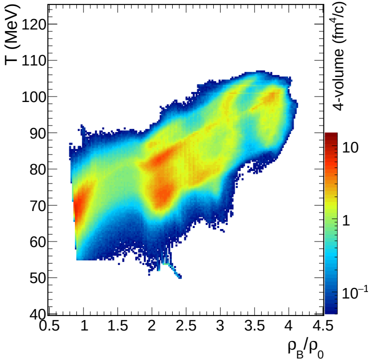


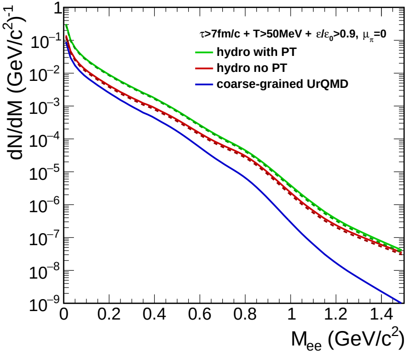


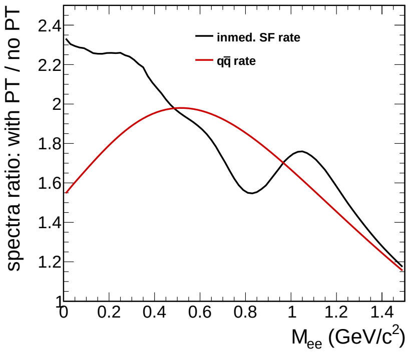
Distinct differences CMF with or without a phase transition

F. Seck, T. Galatyuk, A. Mukherjee, R. Rapp, **JS** and J. Stroth, [arXiv:2010.04614 [nucl-th]].

Indeed di-lepton emission shows a significant effect

- A simulation for Au+Au at the current SIS18 beam energy.
- A factor 2 enhancement of di-lepton emission due to extended 'cooking'.



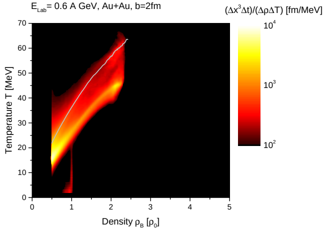


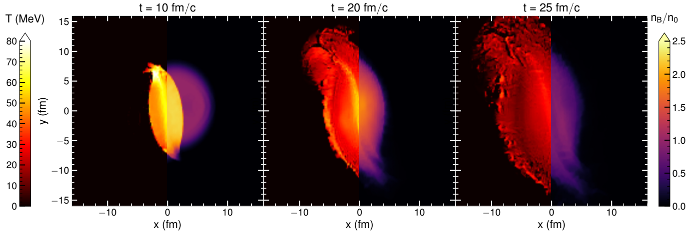
The CMF and neutron star mergers

- What area of the phase diagram are tested by BNSM and what is the overlap with HIC?

The CMF and neutron star mergers

- What area of the phase diagram are tested by BNSM and what is the overlap with HIC?
- Low beam energy HIC $E_{lab} = 600A$ MeV, can be compared to NS merger simulations.
- Both simulations now use the CMF EoS, so a direct comparison is possible!





First results show that BNSM create systems with entropy per baryon $\approx 2 - 3$, comparable to $E_{lab} < 1A$ GeV HIC.

Summary

- Lattice QCD seem to be only useful up to $\mu_B/T \approx 3$,after that fermions become the dominant d.o.f.
- Neutron star properties constrain $T = 0$.
- Small phase transition with low T CeP seems likely.
- Combined/Complex models are necessary to describe the matter in low energy HIC and neutron star mergers.
- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 1$ A GeV) probe complementary region in the phase diagram.

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- We have to take all constraints seriously.
- Neutron star mergers and low energy ($E_{lab} < 1$ A GeV) probe complementary region in the phase diagram.
- Treat both on the same footing $\rightarrow$ Combining QCD thermodynamics, relativistic fluid dynamics and GR.
- Use statistical/ML methods to combine the wealth of data for a consistent picture of the QCD phase diagram.