

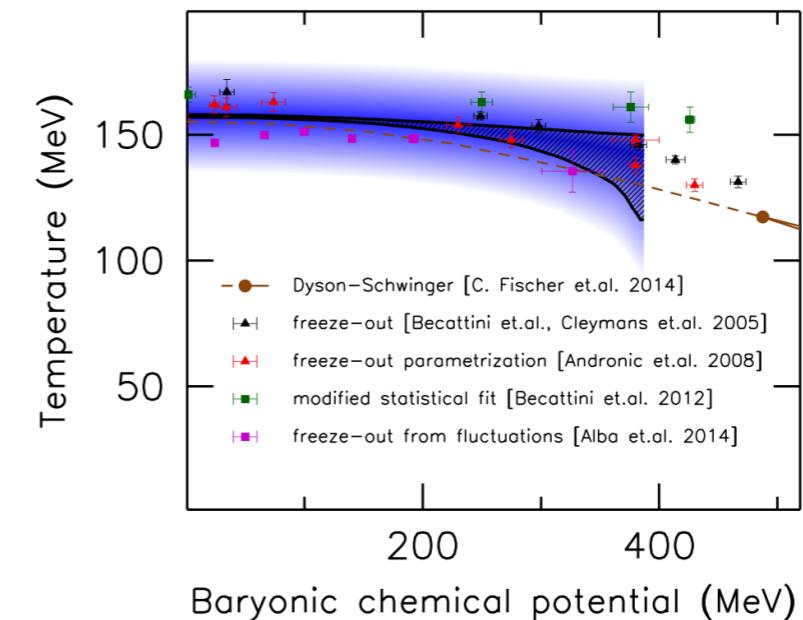


**FAIR**  
Sept. 2020

# Fluctuations and the QCD phase diagram from functional methods

work together with Pascal Gunkel, Philipp Isserstedt

## I. Gluons, quarks and the CEP



## 2. Hadron effects

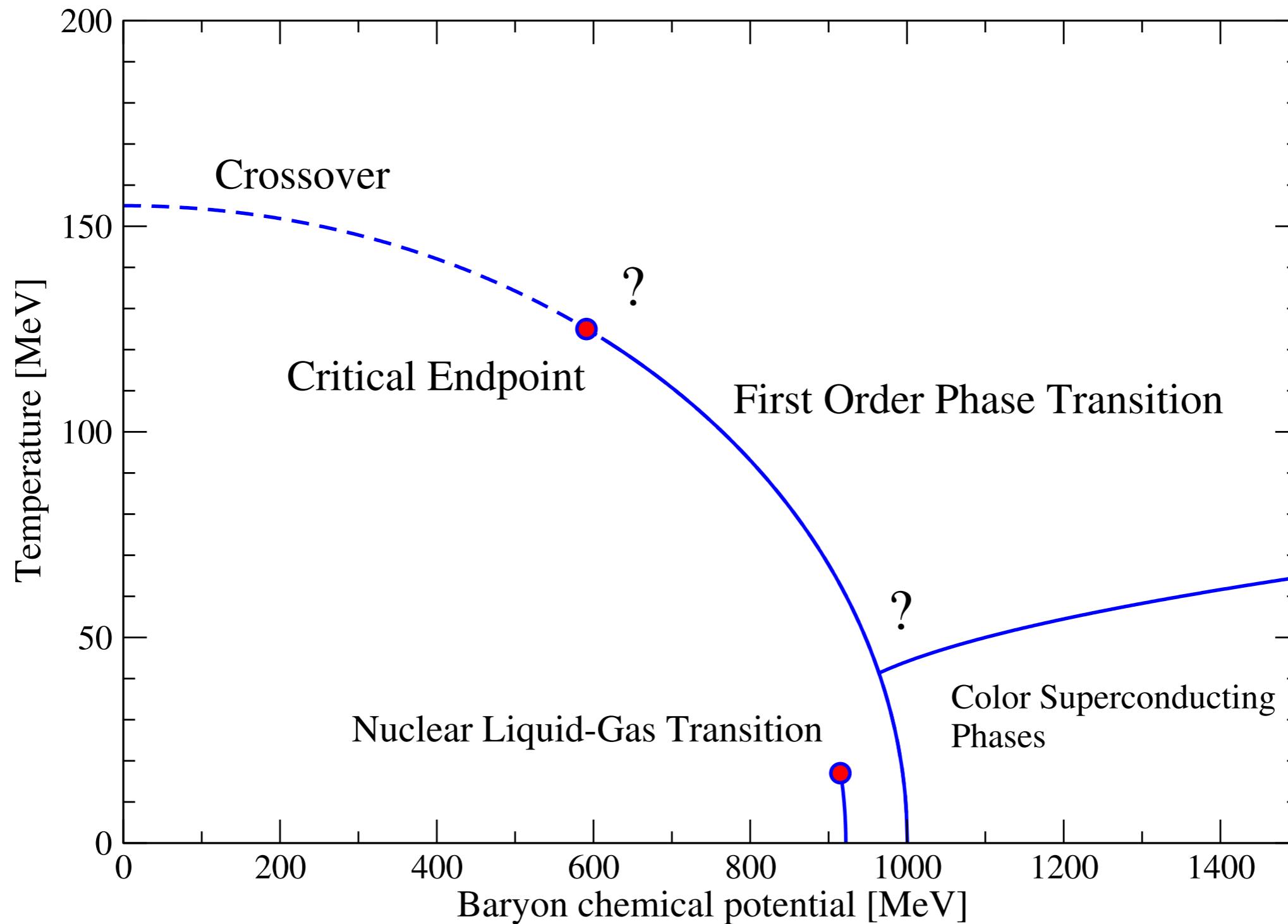
$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---}^{-1}_{NB} + \text{---} \text{---} \text{---} \text{---}^{dq} + \text{---} \text{---} \text{---} \text{---}^{dq}_N$$

The diagram illustrates the perturbative expansion of a loop diagram. It starts with a bare line (---) ending in a circle (○). This is followed by a superscript -1, indicating the inverse of the bare propagator. This is equated to the sum of the bare line plus a loop correction. The loop correction is shown as a sequence of three lines: a bare line (---), a gluon loop (blue wavy line), and another bare line (---). The label NB is placed under the bare line part of the loop. The next term in the expansion is a loop with a gluon loop inside (shaded orange), labeled dq. The final term is a loop with two gluon loops inside (shaded orange), labeled N.

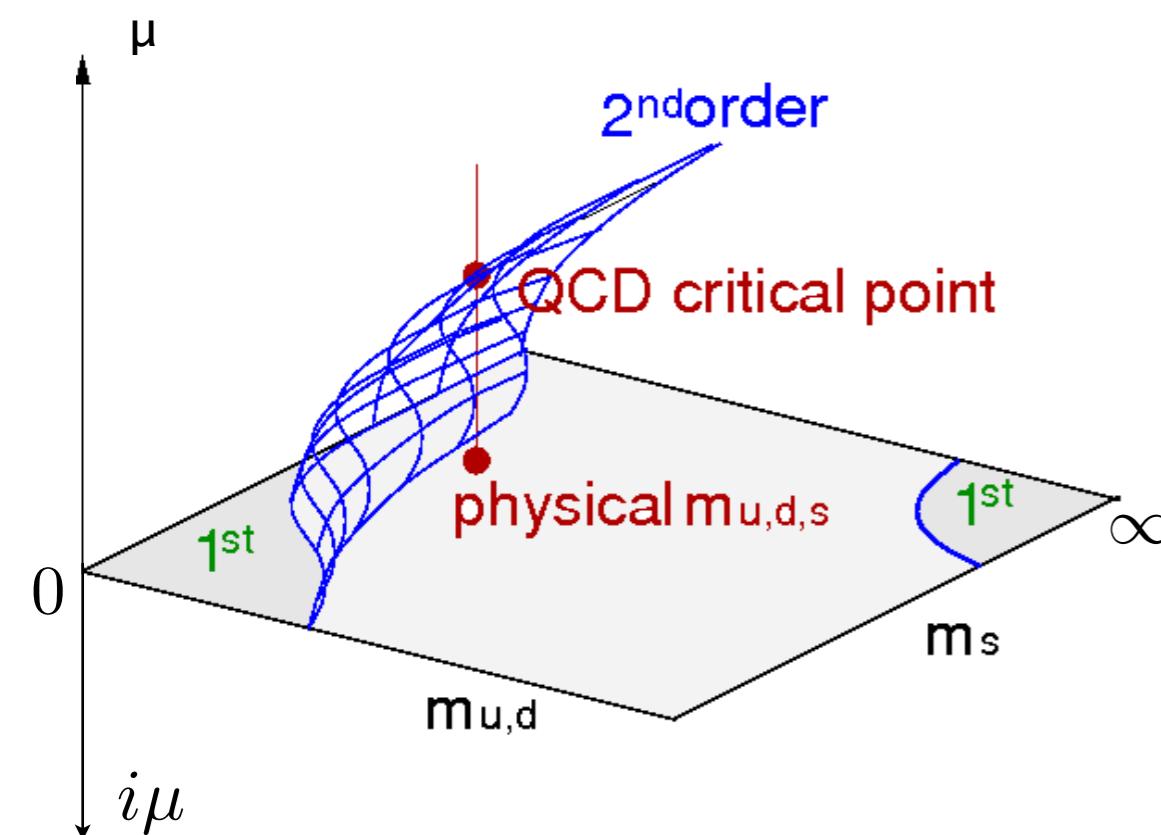
## 3. Fluctuations

Review: CF, PPNP 105 (2019)

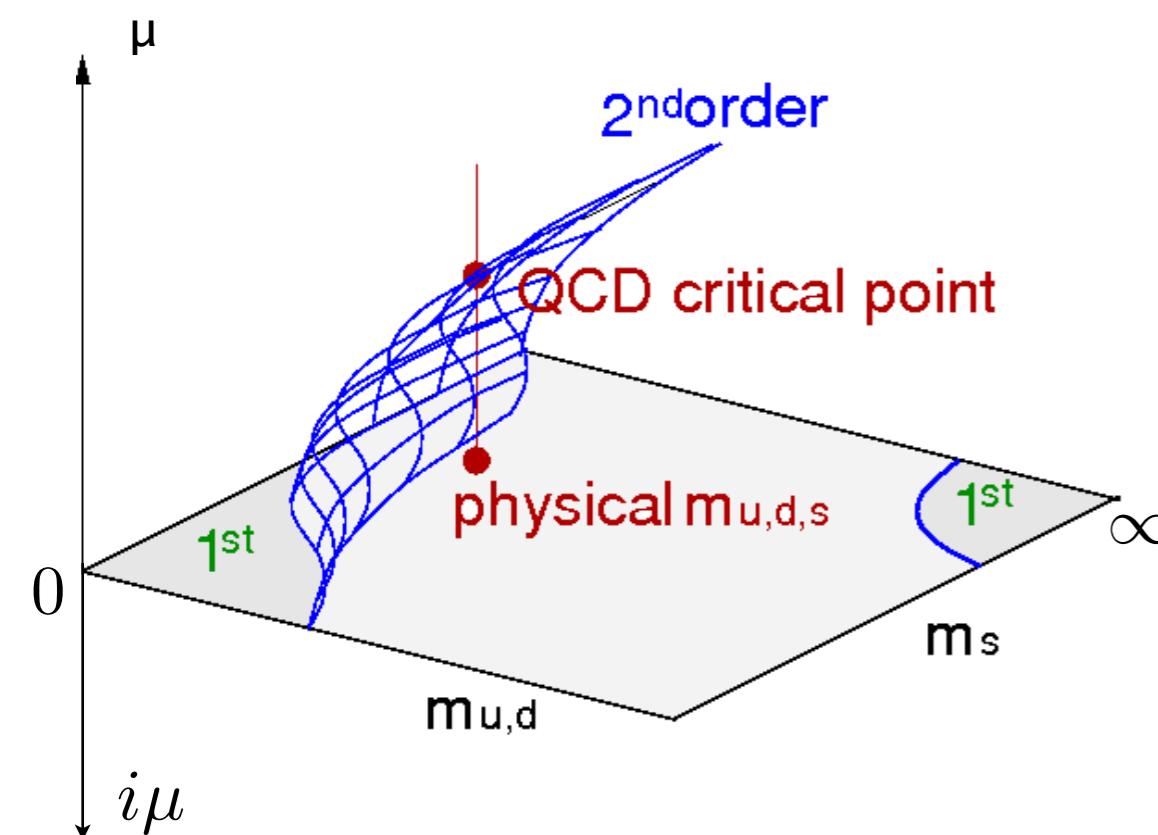
# QCD phase transitions



# Chiral transition line from analytic continuation



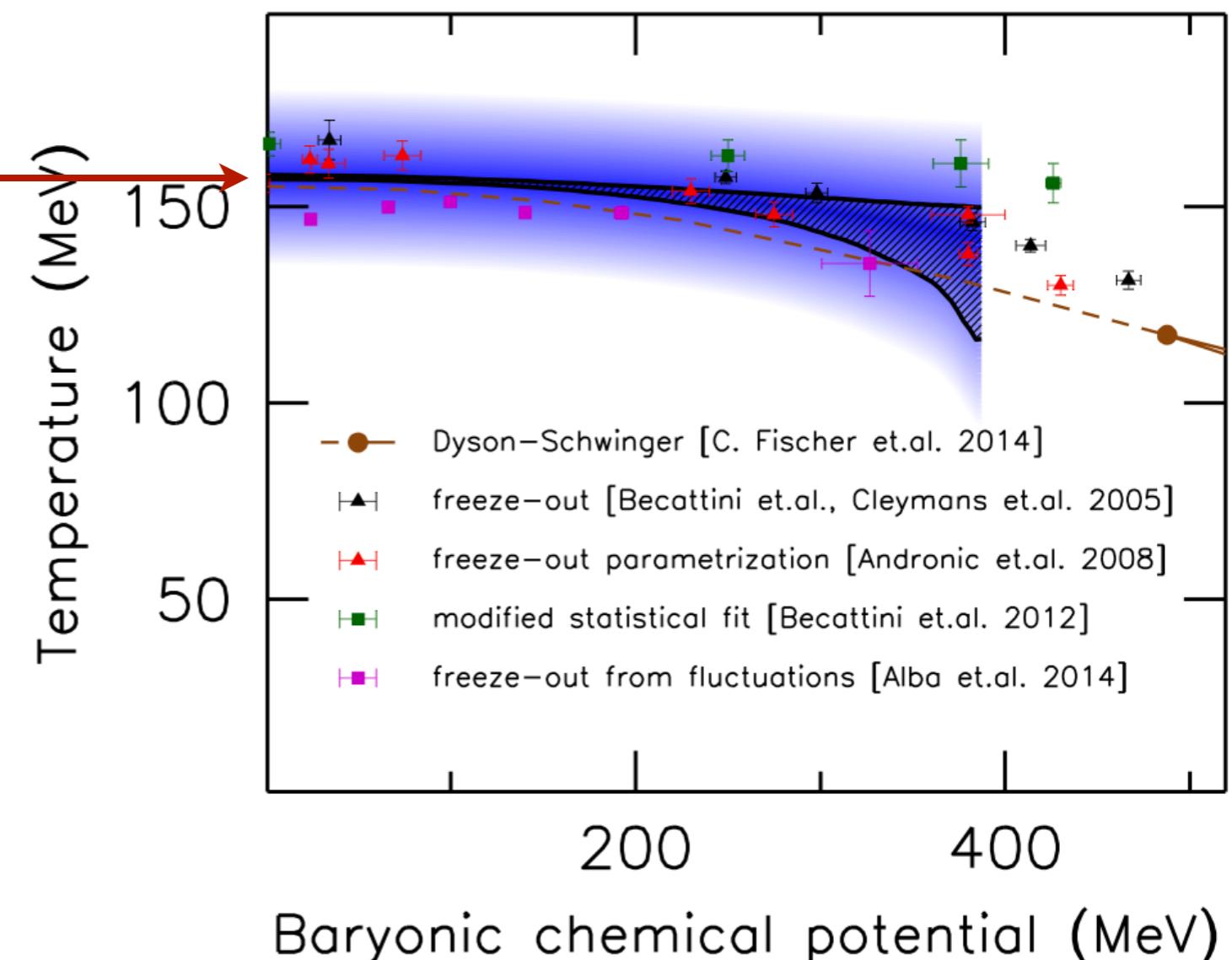
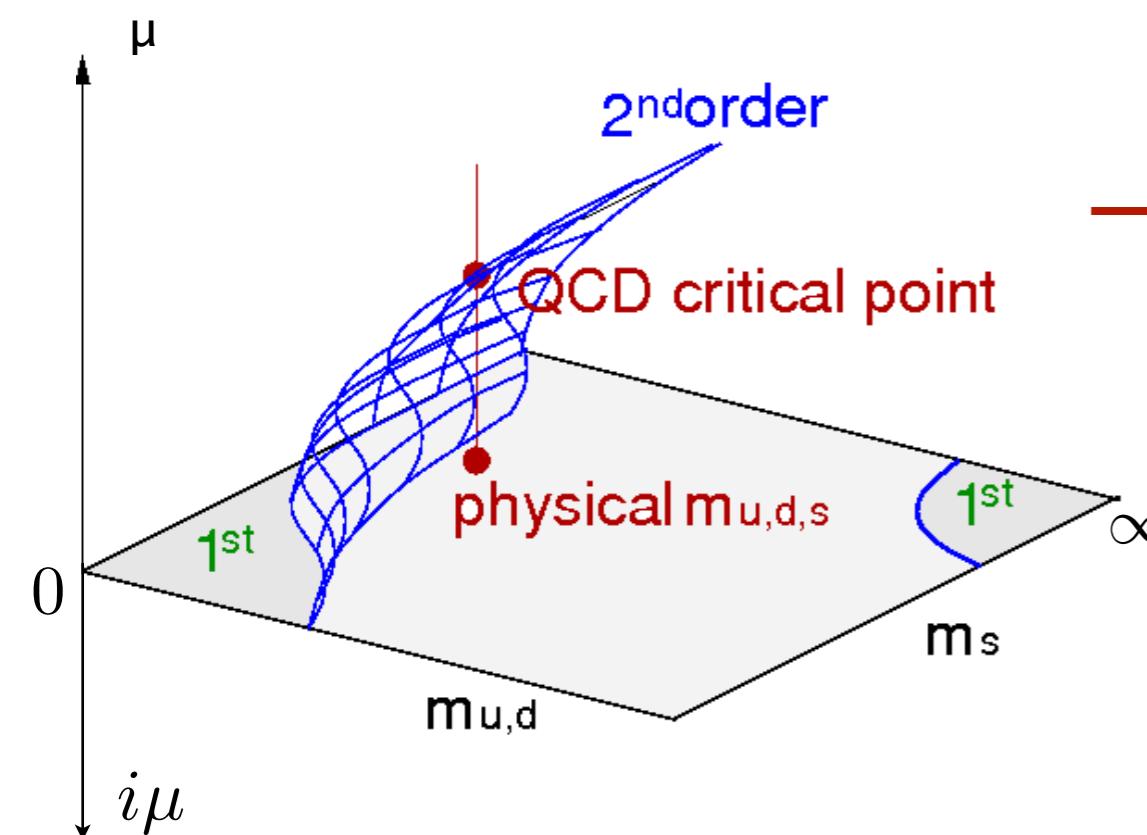
# Chiral transition line from analytic continuation



Lattice method:

- Det. crossover at imaginary  $\mu$  and extrapolate to real  $\mu$
- Control systematics

# Chiral transition line from analytic continuation



Lattice method:

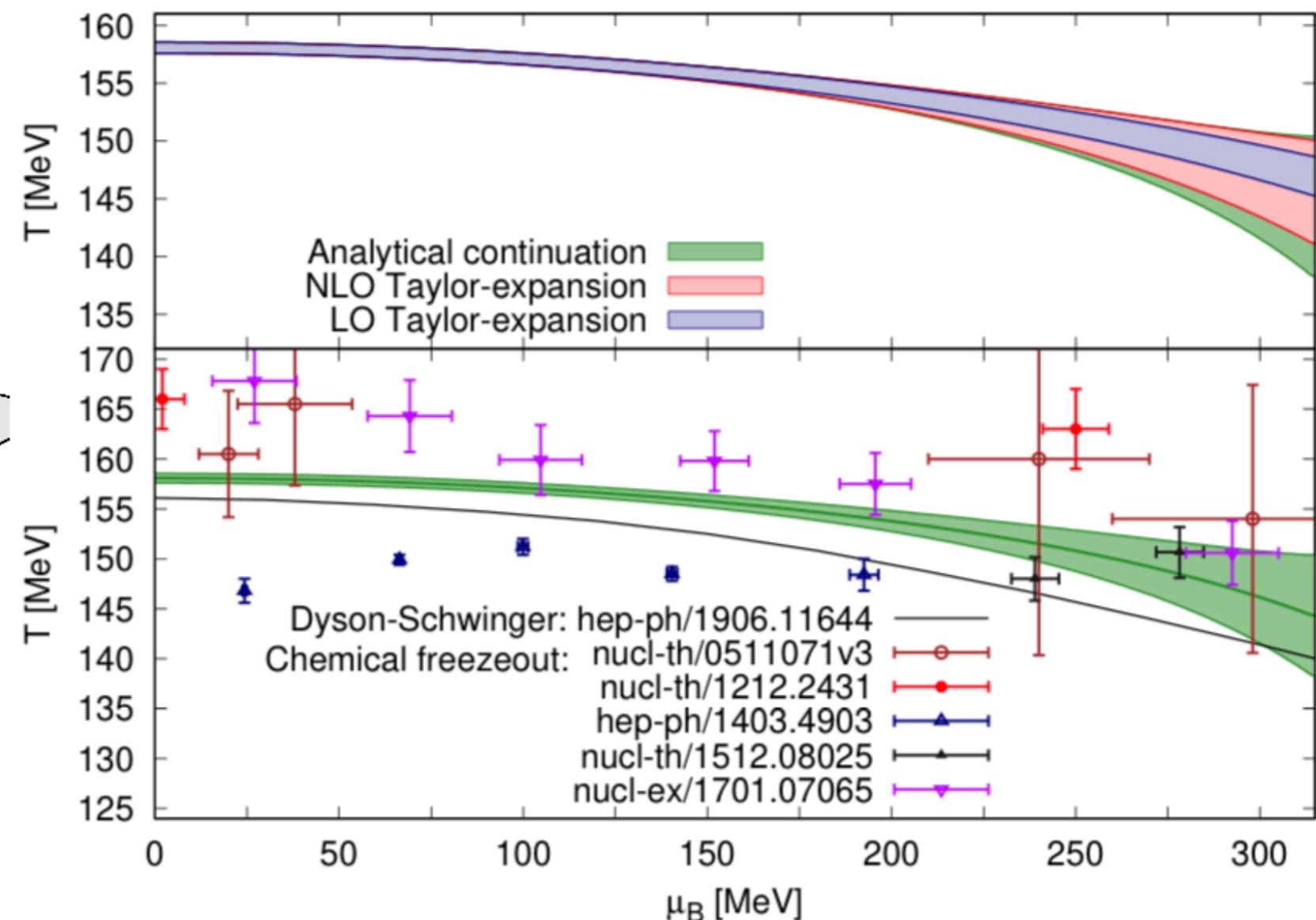
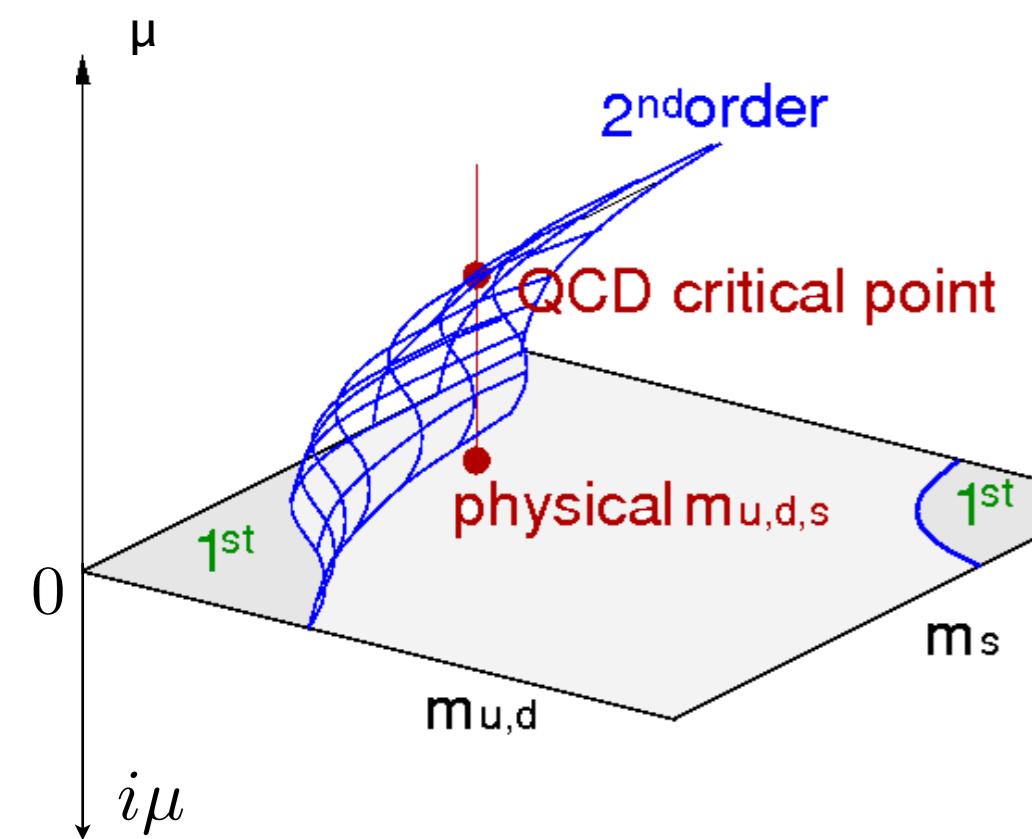
- Det. crossover at imaginary  $\mu$  and extrapolate to real  $\mu$
- Control systematics

Baryonic chemical potential (MeV)

Bellwied, Borsanyi, Fodor, Günther,  
Katz, Ratti and Szabo, PLB 751 (2015) 559

HOT-QCD: similar results

# Chiral transition line from analytic continuation



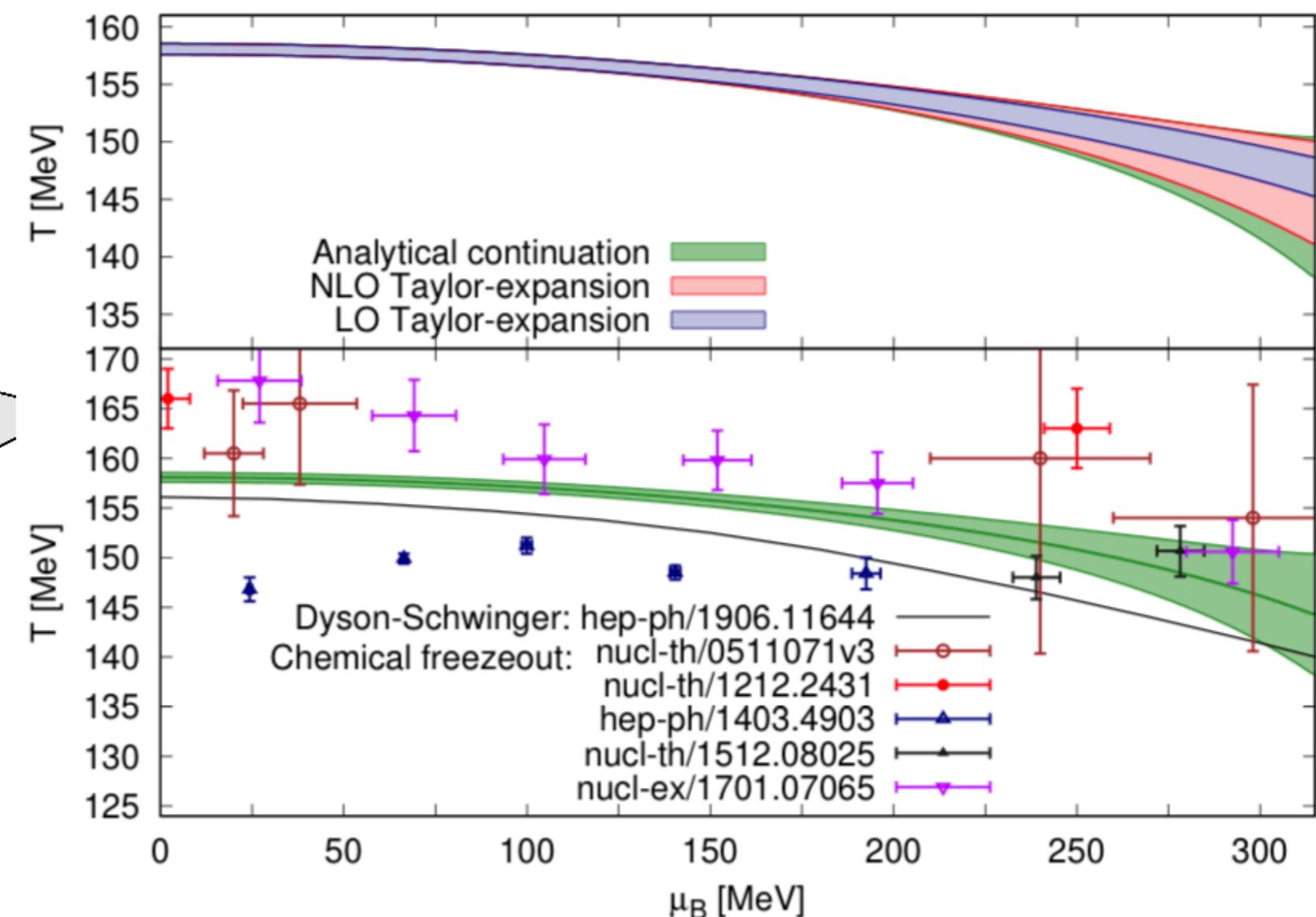
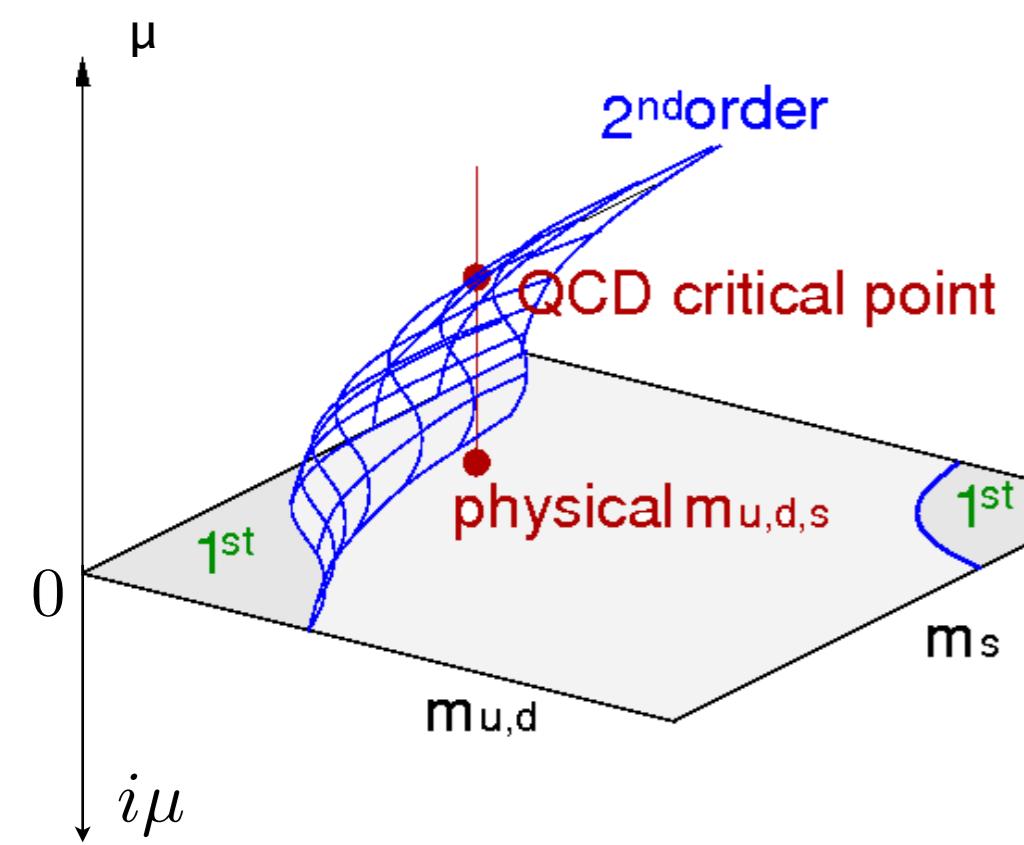
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Borsanyi, Fodor, Günther, Kara, Katz, PRL 125 (2020) no.5, 052001

# Chiral transition line from analytic continuation



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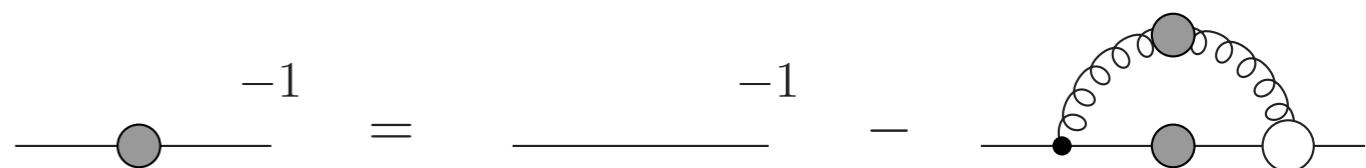
Borsanyi, Fodor, Günther, Kara, Katz, PRL 125 (2020) no.5, 052001

Main result: no CEP for  $\mu_B/T < 2-3$

# QCD order parameters from propagators

Chiral order parameter:

$$\langle \bar{\Psi} \Psi \rangle = Z_2 N_c \text{Tr}_D \frac{1}{T} \sum_{\omega} \int \frac{d^3 p}{(2\pi)^3} S(\vec{p}, \omega)$$



Deconfinement:

• Polyakov loop potential

$$L = \frac{1}{N_c} \text{Tr} e^{ig\beta A_0}$$

$$\frac{\delta (\Gamma - S)}{\delta A_0} = \frac{1}{2} \text{ (loop diagram)} - \text{ (loop diagram)} - \text{ (loop diagram)} - \frac{1}{6} \text{ (loop diagram)} + \text{ (loop diagram)}$$

A series of five Feynman diagrams representing terms in the Polyakov loop potential. They show loops with various internal lines and vertices, some with arrows indicating direction. The first term is a loop with two vertical lines. Subsequent terms involve more complex loop configurations and interactions with external lines.

Braun, Gies, Pawłowski, PLB 684, 262 (2010)

Braun, Haas, Marhauser, Pawłowski, PRL 106 (2011)

Fister, Pawłowski, PRD 88 045010 (2013)

CF, Fister, Luecker, Pawłowski, PLB 732 (2013)

# The DSE for the quark propagator (finite T, $\mu$ )

$$\text{---} \circ \overset{-1}{=} \text{---} \overset{-1}{\text{---}} - \text{---} \bullet \text{---} \circ \text{---}$$

Two strategies: I. use **T-dependent model for gluon**

- Qin, Chang, Chen, Liu and Roberts, PRL 106 (2011) 172301  
Y. Jiang, L.-J. Luo, and H.-S. Zong, JHEP 02 (2011) 066  
Gutierrez, Ahmad, Ayala, Bashir and Raya, JPG 41 (2014) 075002  
C. Shi, Y.-L. Du, S.-S. Xu, X.-J. Liu, and H.-S. Zong, PRD93 (2016) no. 3, 036006  
F. Gao and Y.-x. Liu, PRD 94 (2016) no. 7, 076009  
F. Gao and Y.-x. Liu, PRD 94 (2016) no. 9, 094030

- valuable for exploratory studies
- not good enough for quantitative and/or systematic studies

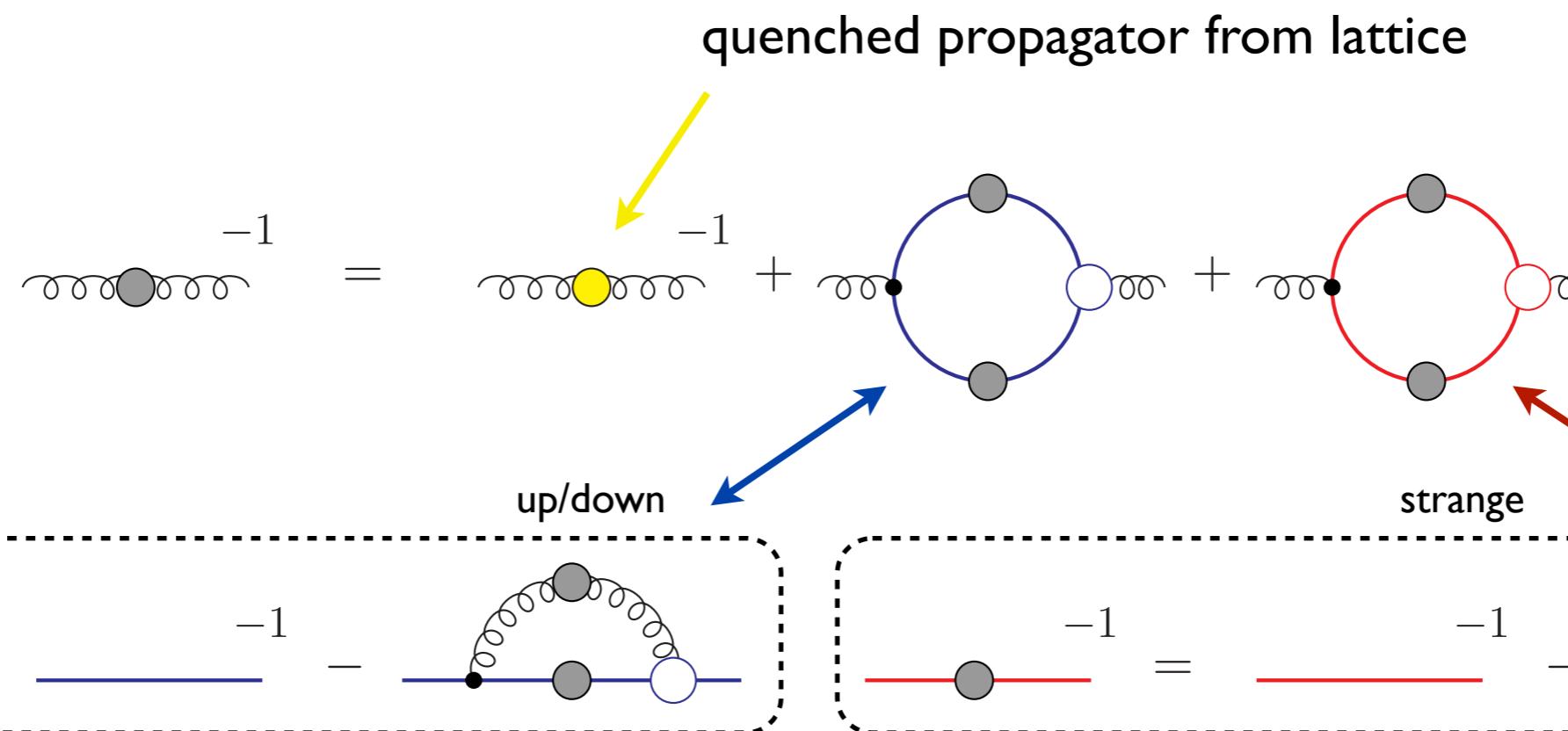
II. treat Yang-Mills sector explicitly

- CF, Luecker, Welzbacher, PRD 90 (2014) 034022  
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011  
Gao and Pawłowski, PRD 102 (2020) no.3, 034027  
Gao and Pawłowski, arXiv:2010.13705 [hep-ph].

SU(2),G(2): Contant and Huber, PRD 96 (2017) no.7, 074002

Review: CF, PPNP 105 (2019)

# $N_f=2+1$ -QCD with DSEs



- allows for systematic variation of  $m_{u/d}$  and  $m_s$
- quark-gluon vertex:  
ansatz built along STI and known UV/IR behavior  
→  $T, \mu, m$ -dependent
- $T_c(\mu=0)$  is not predicted, but input

# $N_f=2+1$ , zero chemical potential

$$\text{Diagrammatic decomposition of } \text{loop}^{-1}$$

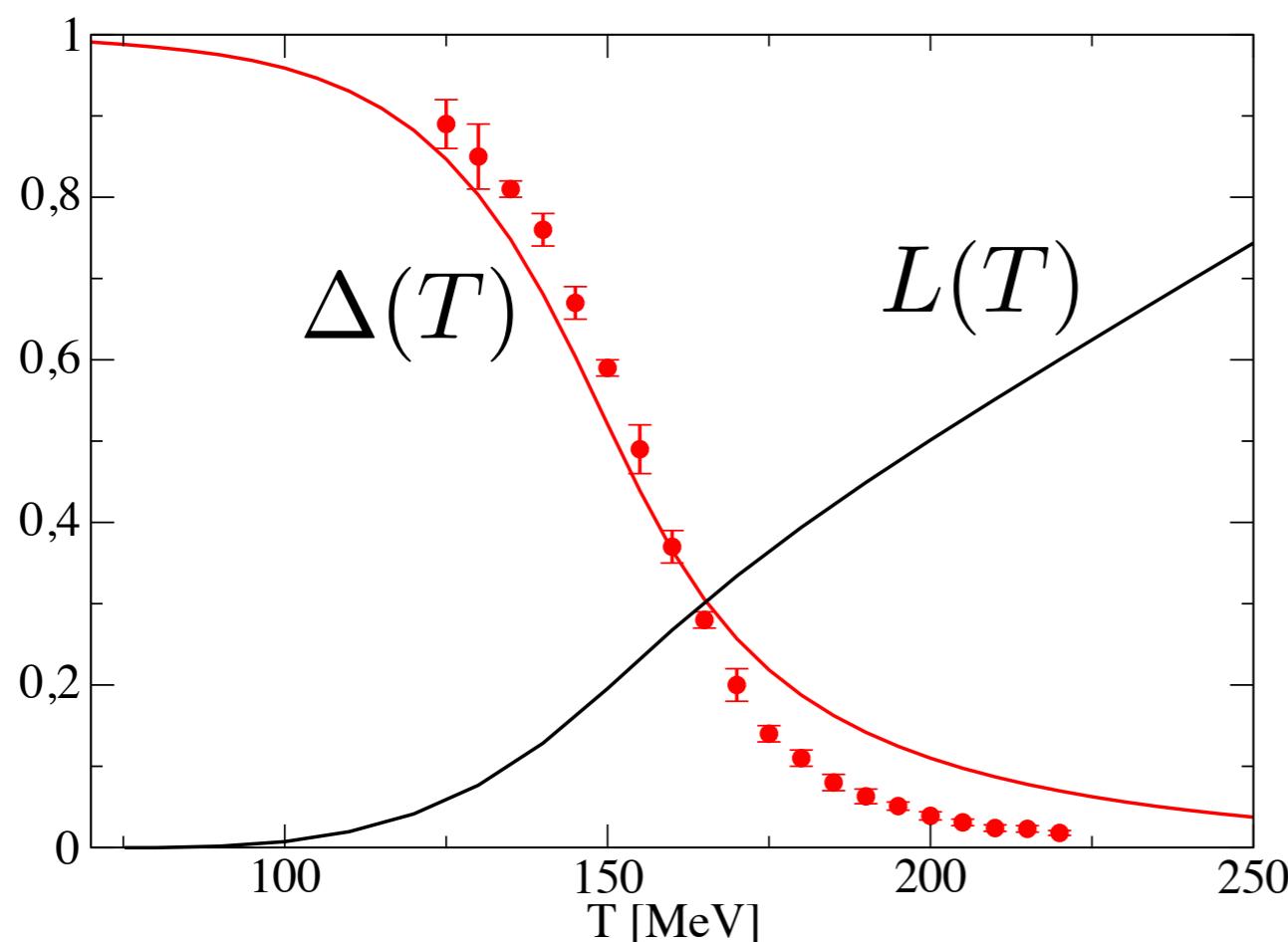
The top row shows the decomposition of a loop diagram into a bare loop and three corrections. The first correction is a yellow circle with a central dot and a wavy line. The second is a blue circle with a central dot and a wavy line. The third is a red circle with a central dot and a wavy line.

$$\text{Diagrammatic decomposition of } \text{line}^{-1}$$

The bottom row shows the decomposition of a bare line into a bare line and two corrections. The first correction is a blue line with a semi-circular loop above it. The second is a red line with a semi-circular loop above it.

$$\Delta(T) \sim M(T)$$

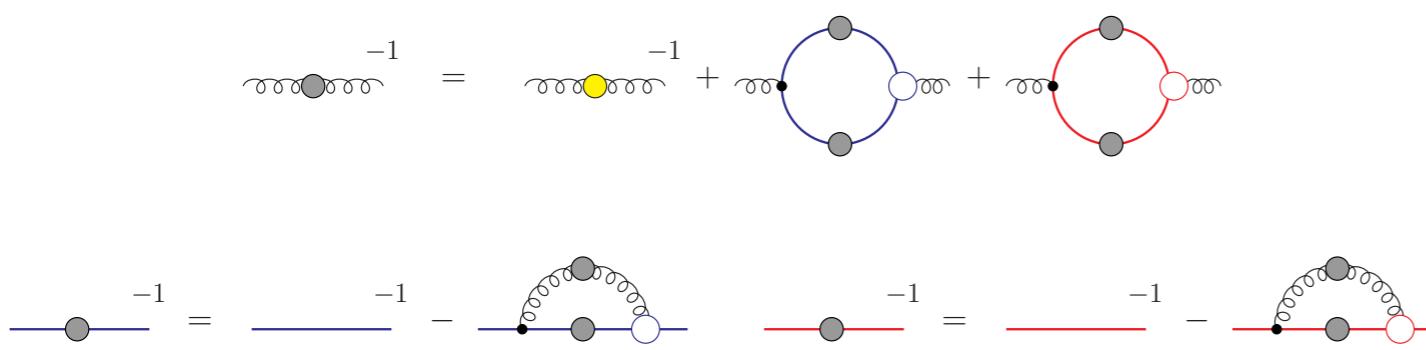
$$L(T) \sim \exp(-E/T)$$



Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073

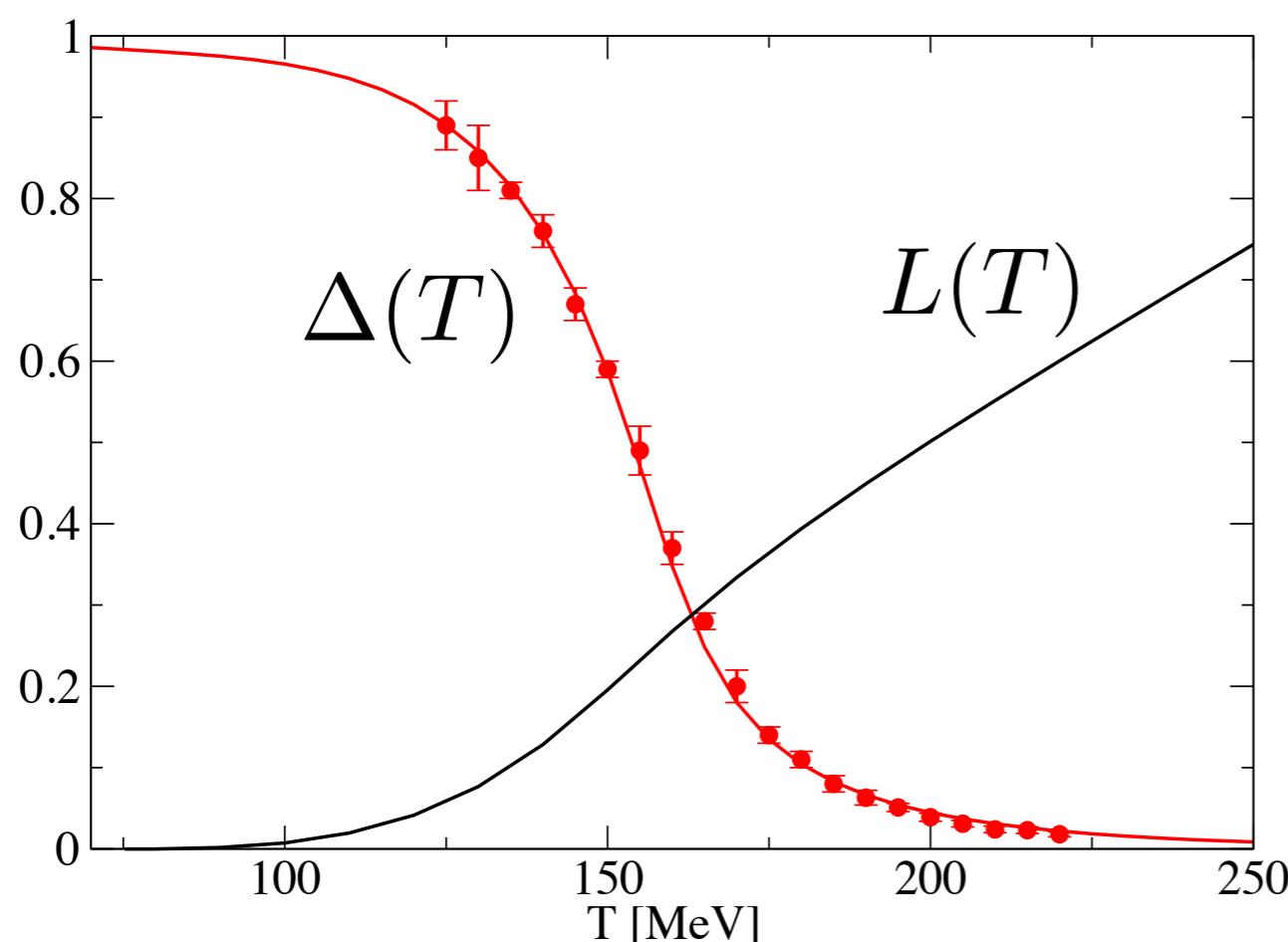
DSE: CF, Luecker, PLB 718 (2013) 1036,  
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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CF, Luecker, Welzbacher, PRD 90 (2014) 034022

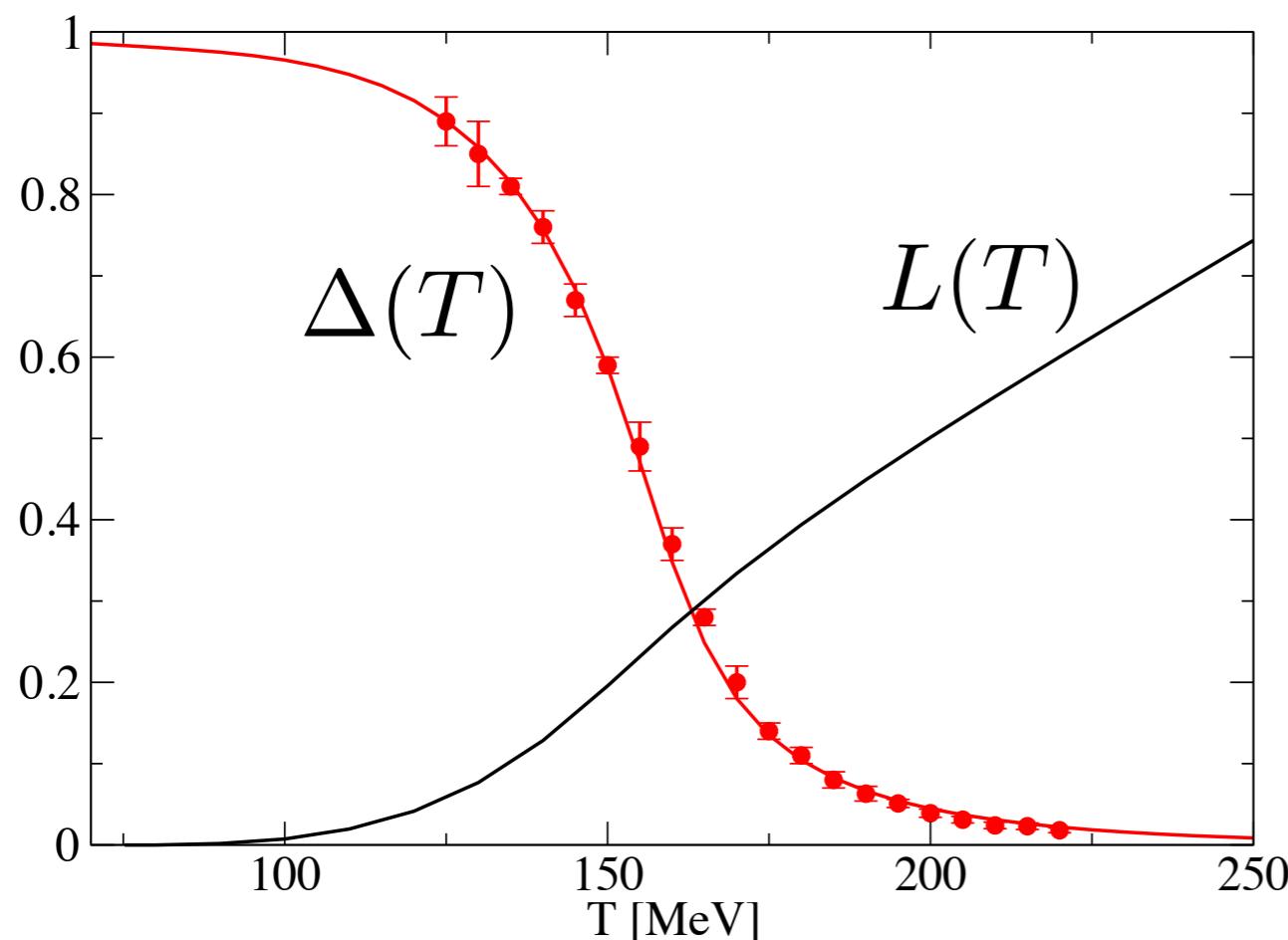
# $N_f=2+1$ , zero chemical potential

$$\text{Diagrammatic equation}$$

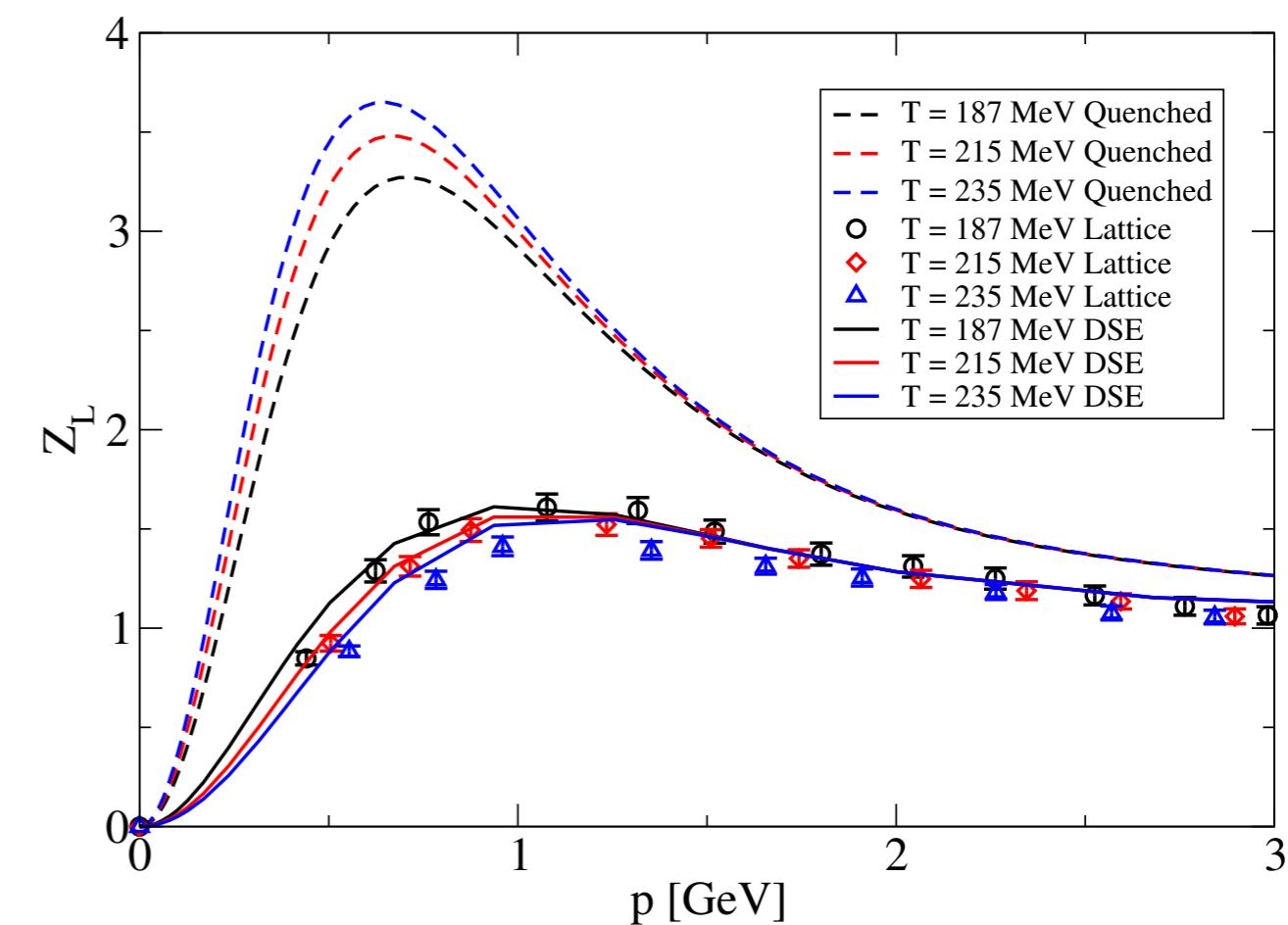
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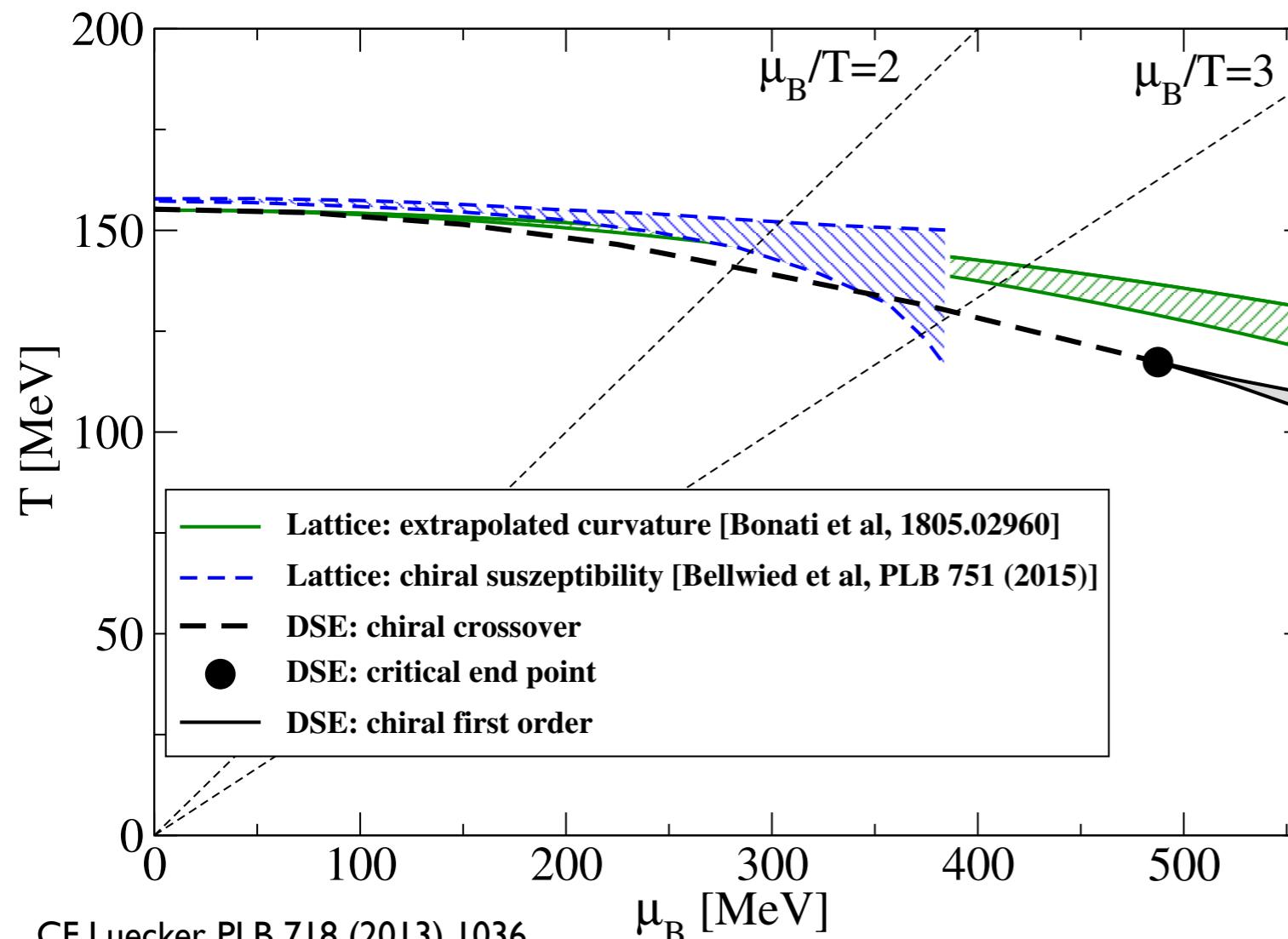
Lattice: Borsanyi et al. [Wuppertal-Budapest], JHEP 1009(2010) 073  
DSE: CF, Luecker, PLB 718 (2013) 1036,  
CF, Luecker, Welzbacher, PRD 90 (2014) 034022



Lattice: Aouane, et al. PRD D87 (2013), [arXiv:1212.1102]  
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

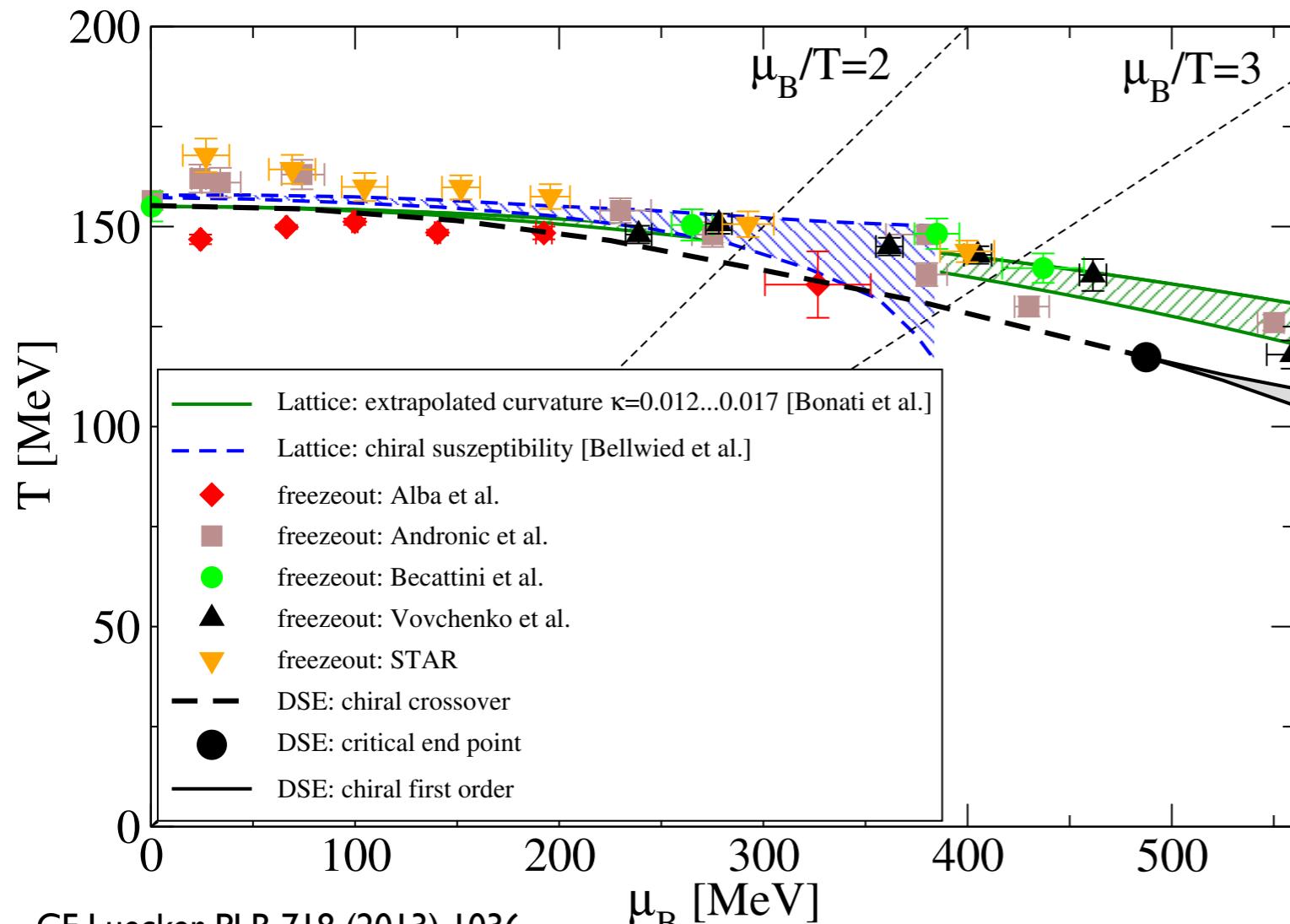
● quantitative agreement: DSE prediction verified by lattice

# QCD phase diagram and heavy ion collisions



- combined lattice/DSE evidence: no CEP at  $\mu_B/T < 2-3$
- CEP @  $(T, \mu_B) = (117, 488)$  MeV
- Yang-Mills sector crucial !

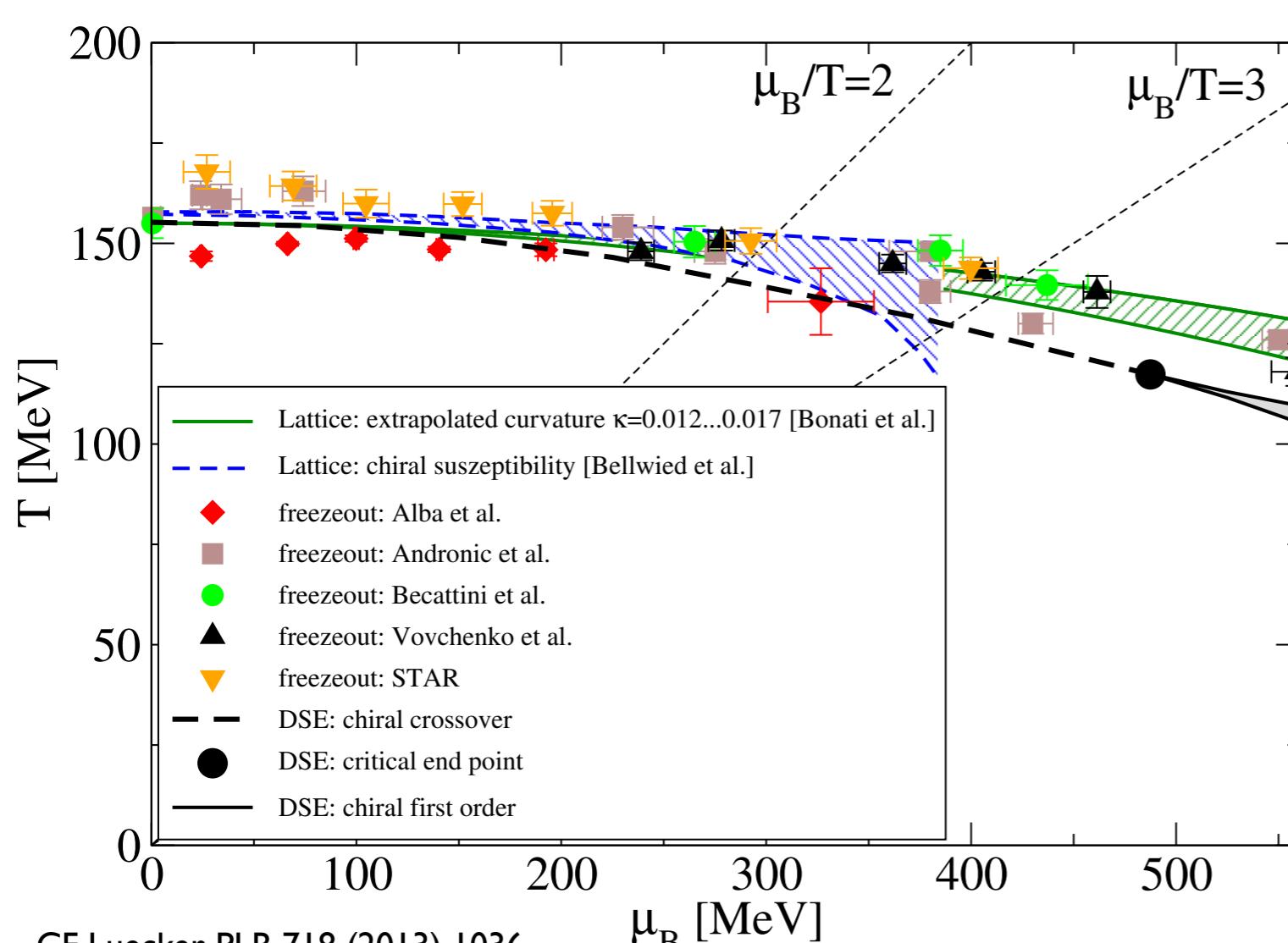
# QCD phase diagram and heavy ion collisions



CF, Luecker, PLB 718 (2013) 1036,  
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273  
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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## heavy ion collisions

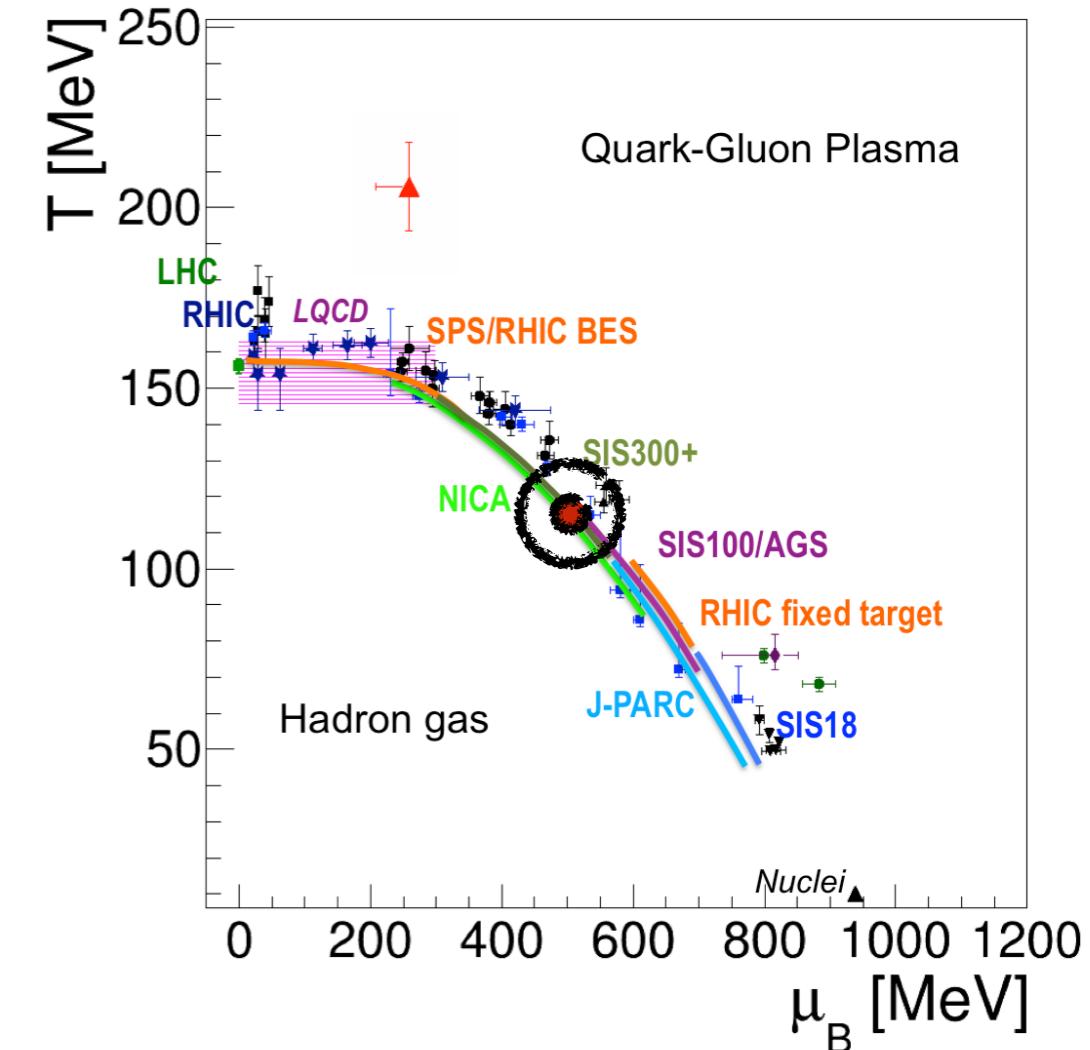
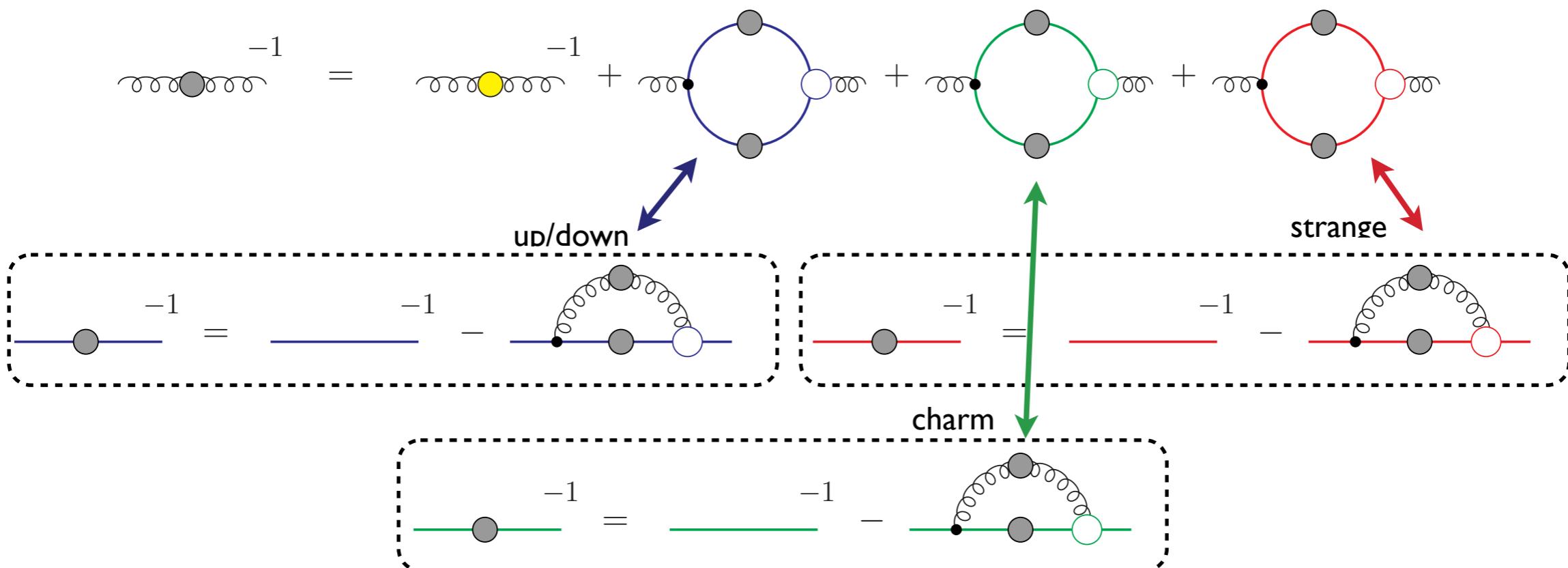


Figure adapted from talk of T. Galatyuk, Erice 2016

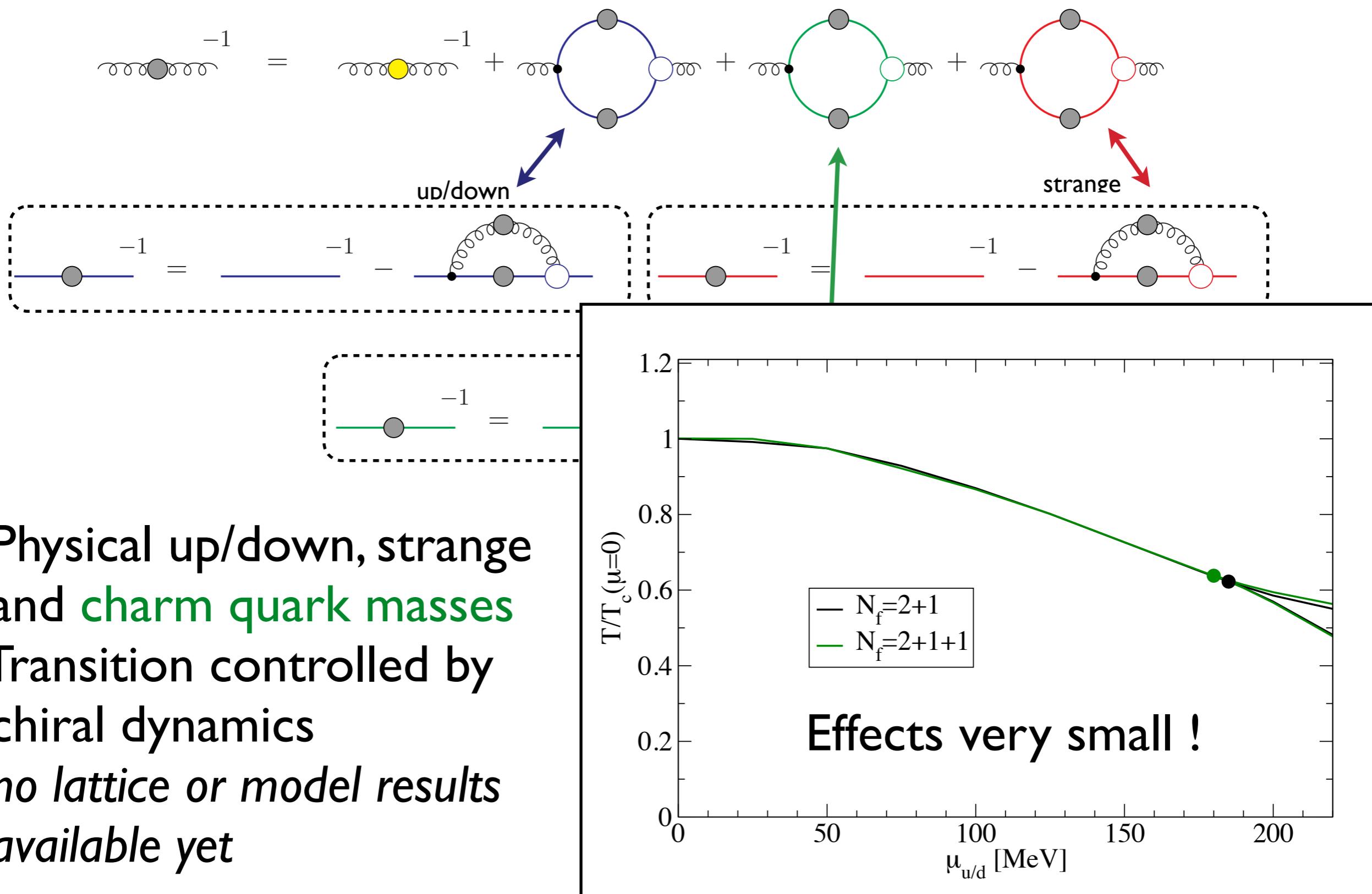
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# $N_f=2+1+1$ : effects of charm



- Physical up/down, strange and **charm quark masses**
- Transition controlled by chiral dynamics
- no lattice or model results available yet*

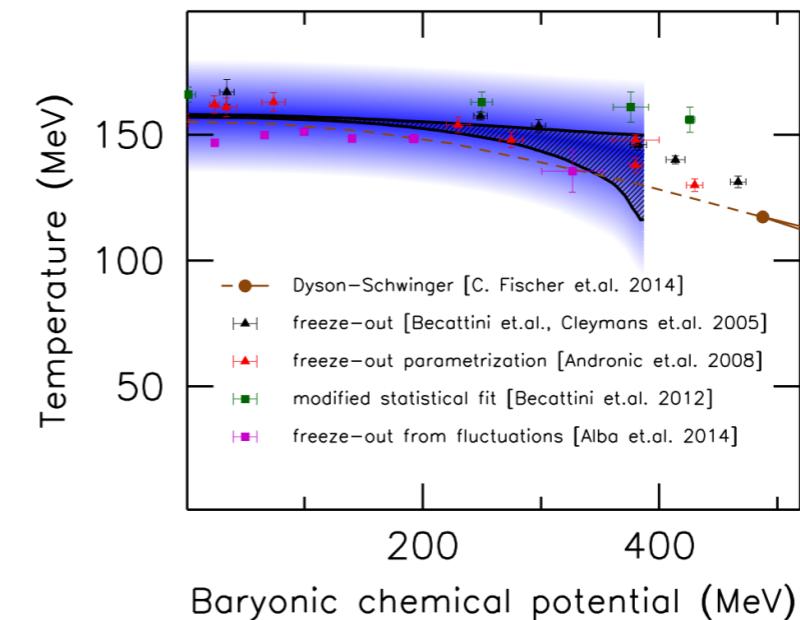
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## I. Gluons, quarks and the CEP



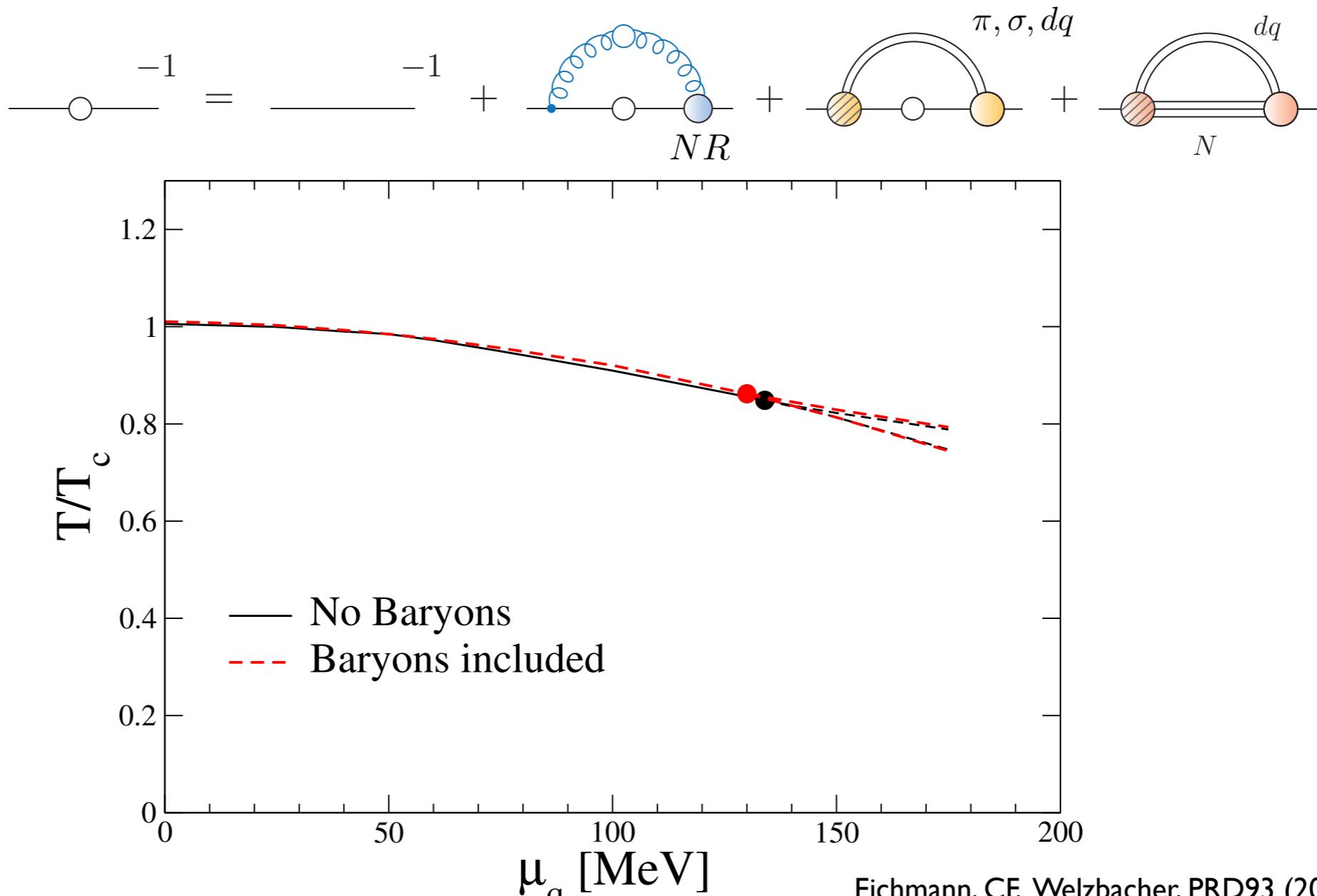
## 2. Hadron effects

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---} \text{---} \text{---} \text{---}^{-1}$$

The diagram illustrates the perturbative expansion of the inverse propagator. The first term is the bare propagator. Subsequent terms show the addition of loop corrections:   
1. A loop with a single gluon line (blue circles) labeled  $NB$ .   
2. A loop with two gluon lines (yellow circles) labeled  $dq$ .   
3. A loop with three gluon lines (orange circles) labeled  $N$ .

## 3. Fluctuations

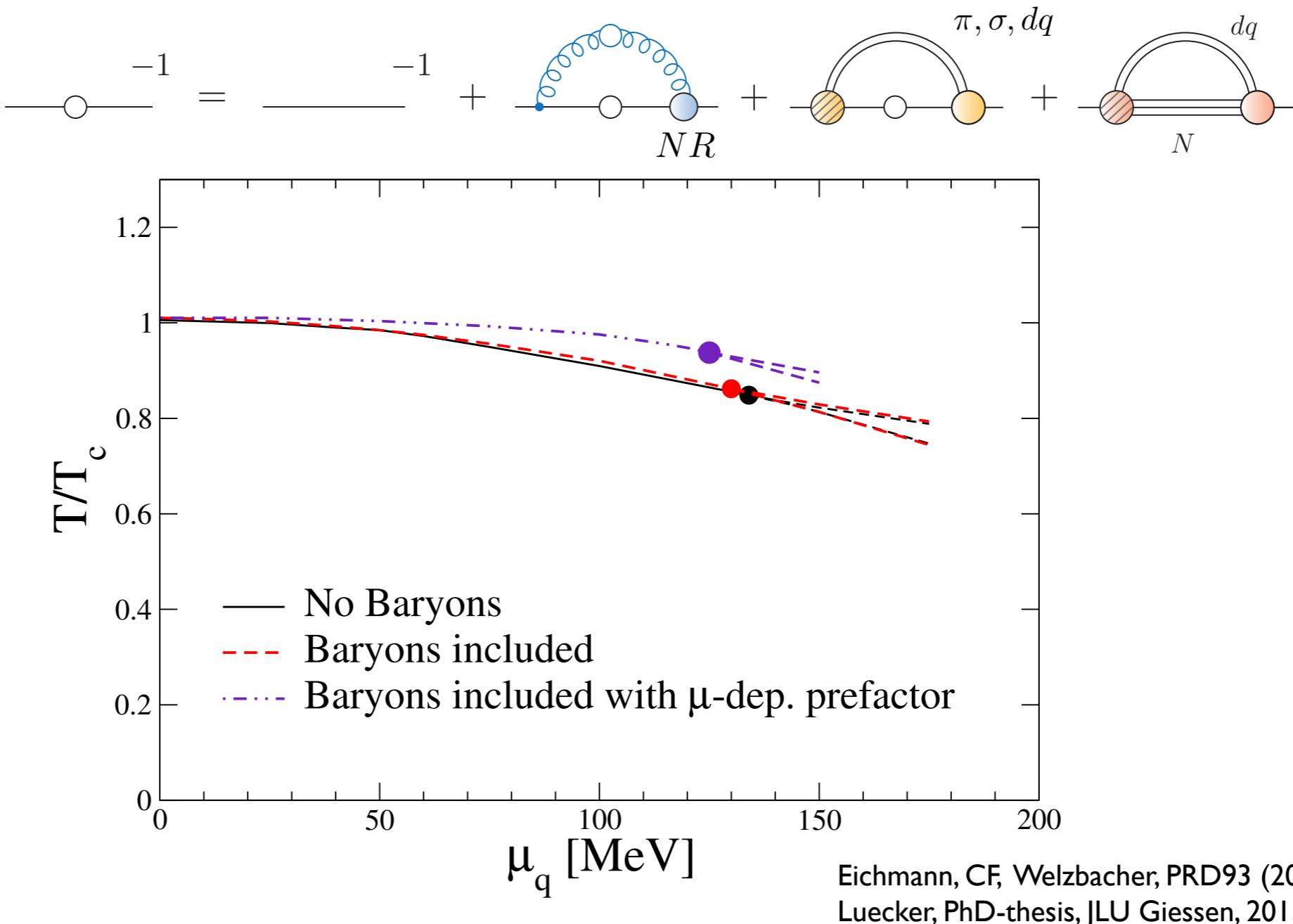
# Baryon effects on the CEP - results ( $N_f=2$ )



Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]  
Luecker, PhD-thesis, JLU Giessen, 2013

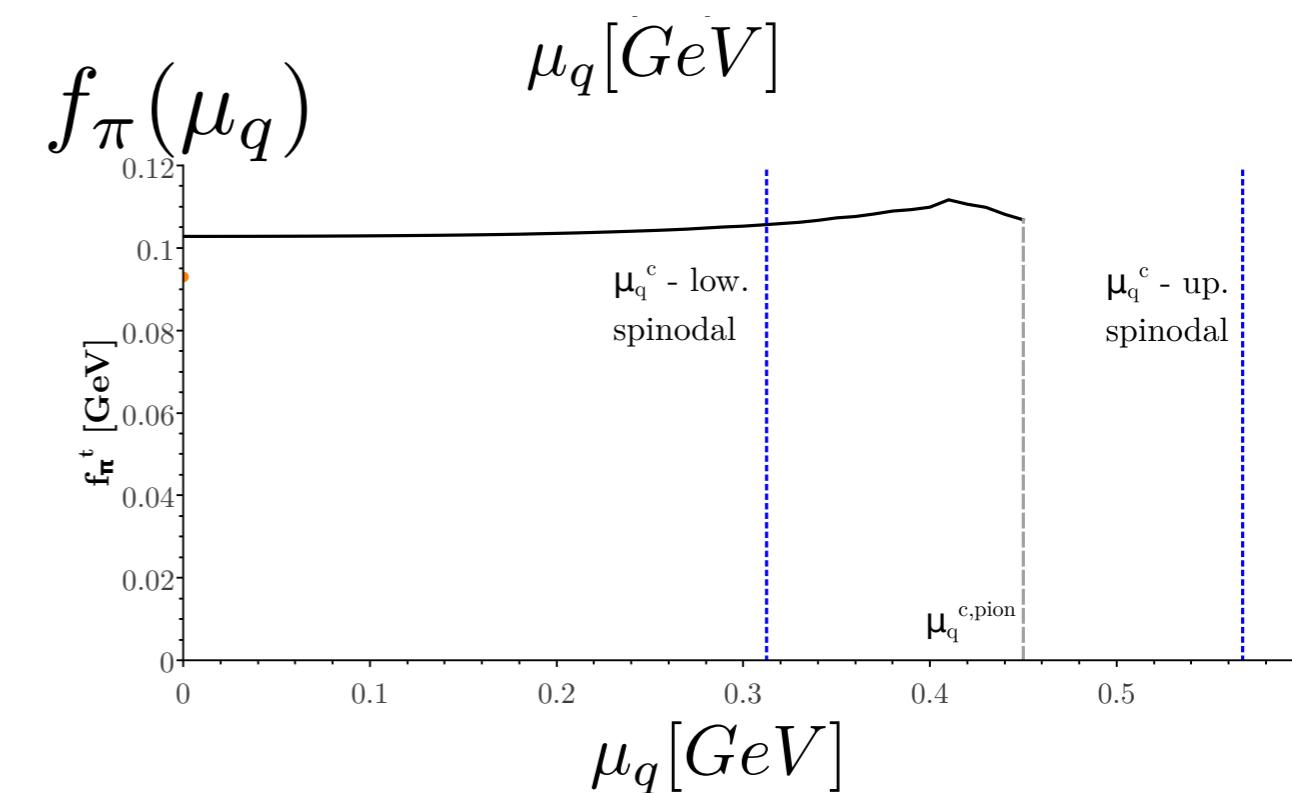
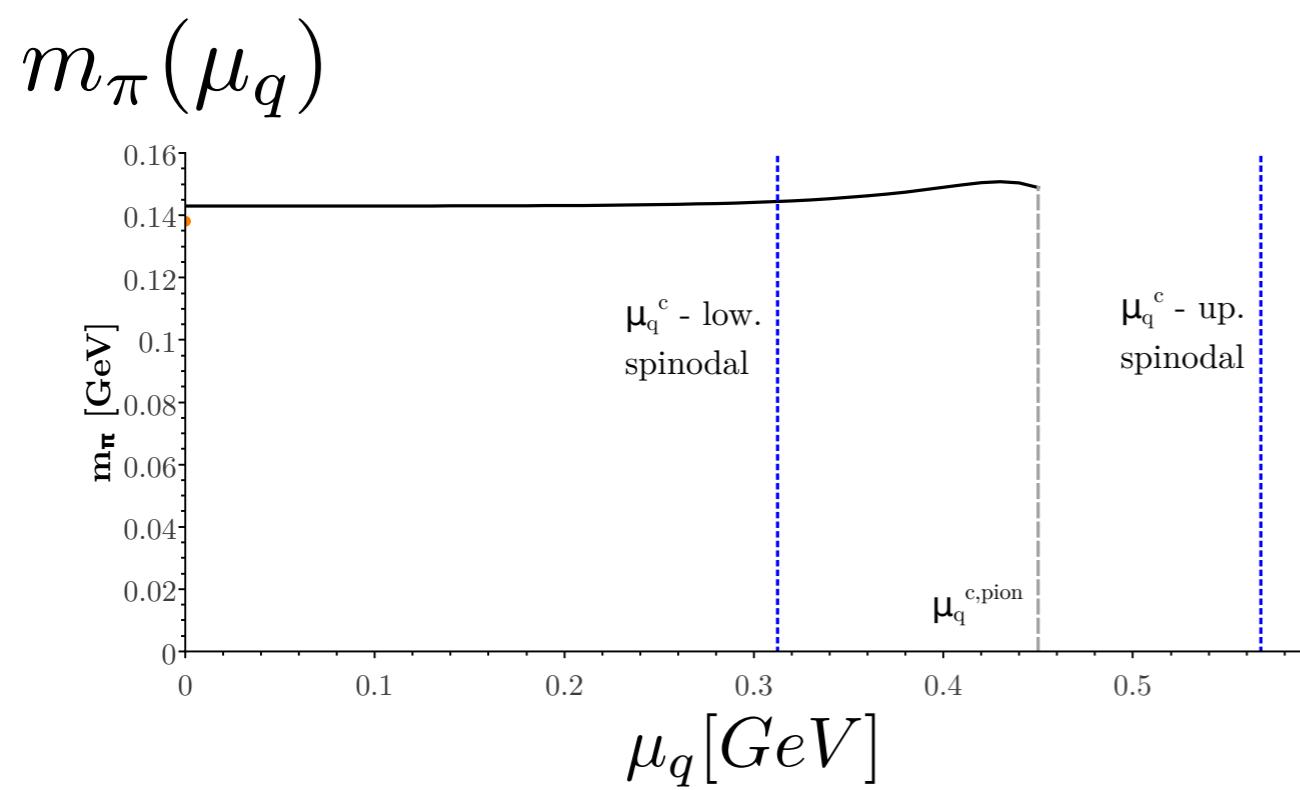
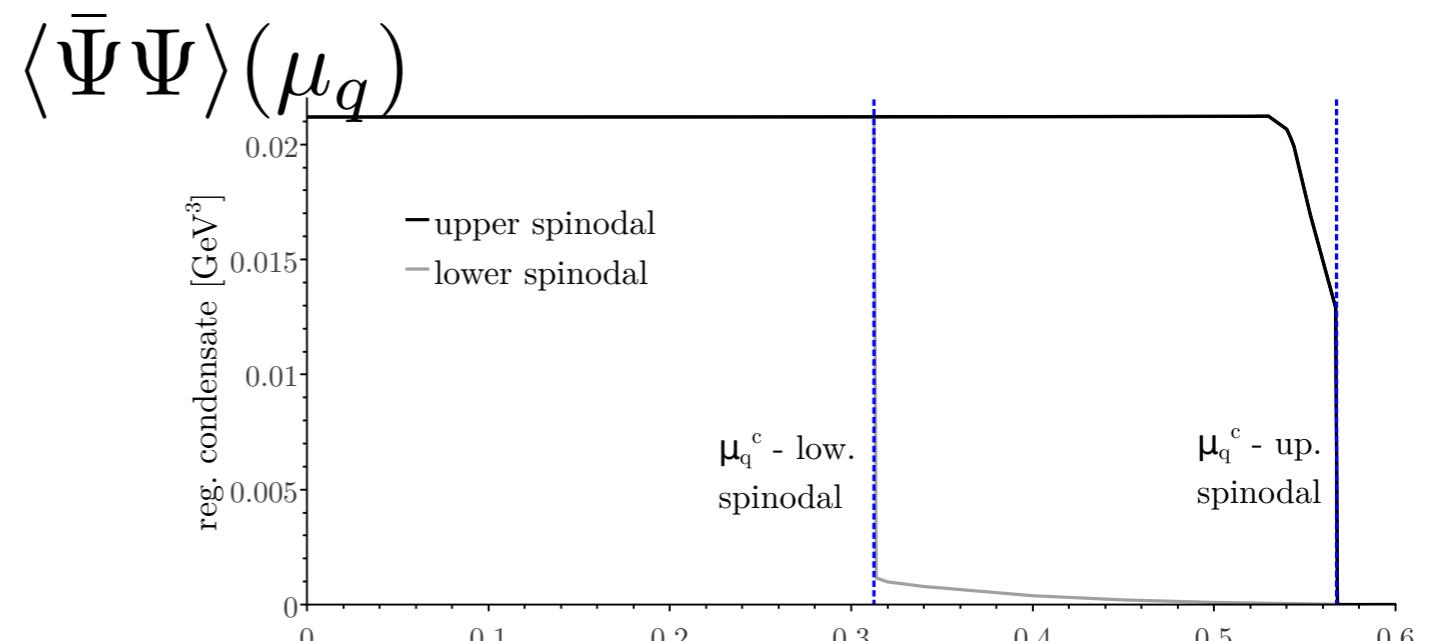
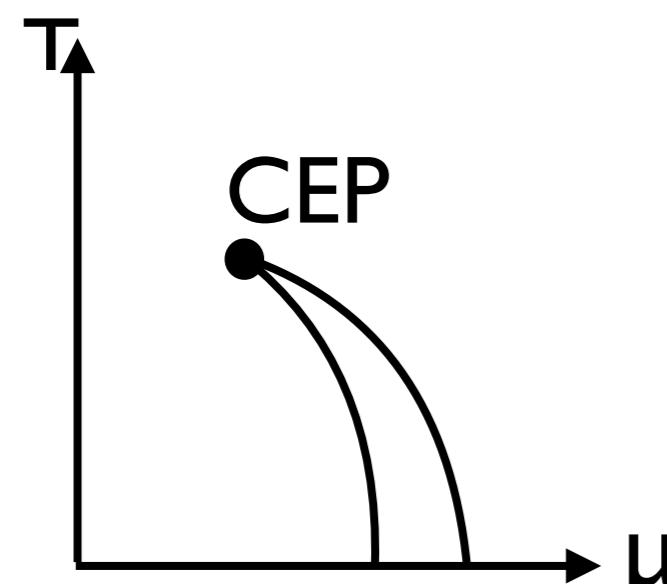
- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)

# Baryon effects on the CEP - results ( $N_f=2$ )



- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)
- But: strong  $\mu$ -dependence of baryon wave function may change situation...

# Meson properties at finite chemical potential



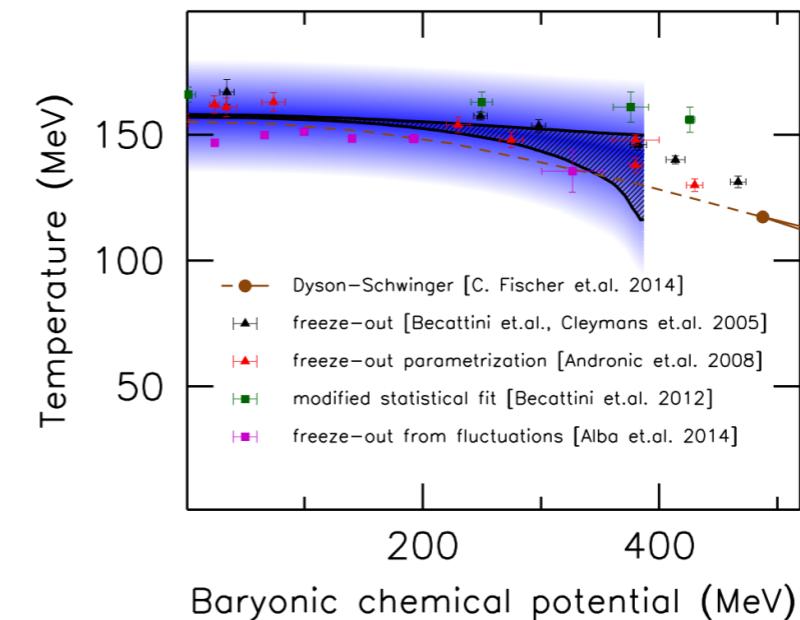
- Silver blaze satisfied

T. D. Cohen, PRL 91 , 222001 (2003)

- But wave functions do change !

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169

## I. Gluons, quarks and the CEP



## 2. Hadron effects

$$\text{---} \circ \text{---}^{-1} = \text{---} \text{---}^{-1} + \text{---} \text{---} \text{---} \text{---}^{-1}_{NB} + \text{---} \text{---} \text{---} \text{---}^{dq} + \text{---} \text{---} \text{---} \text{---}^{dq}_N$$

The equation illustrates the perturbative expansion of a loop diagram. The left side shows a bare loop with a superscript  $-1$ . The right side shows the loop split into a bare part (dashed line) and a correction part (solid line). The correction part is further expanded into three terms: a one-loop correction with a blue shaded loop labeled  $NB$ , a two-loop correction with a yellow shaded loop labeled  $dq$ , and a three-loop correction with an orange shaded loop labeled  $N$ .

## 3. Fluctuations

# Contact with experiment: fluctuations

X.-Luo and N.-Xu, Nucl. Sci. Tech. 28 (2017) no.8, 112 [arXiv:1701.02105 [nucl-ex]].

Quark chemical potentials related to those of conserved charges:

$$\mu_u = \mu_B/3 + 2\mu_Q/3$$

$$\mu_d = \mu_B/3 - \mu_Q/3$$

$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

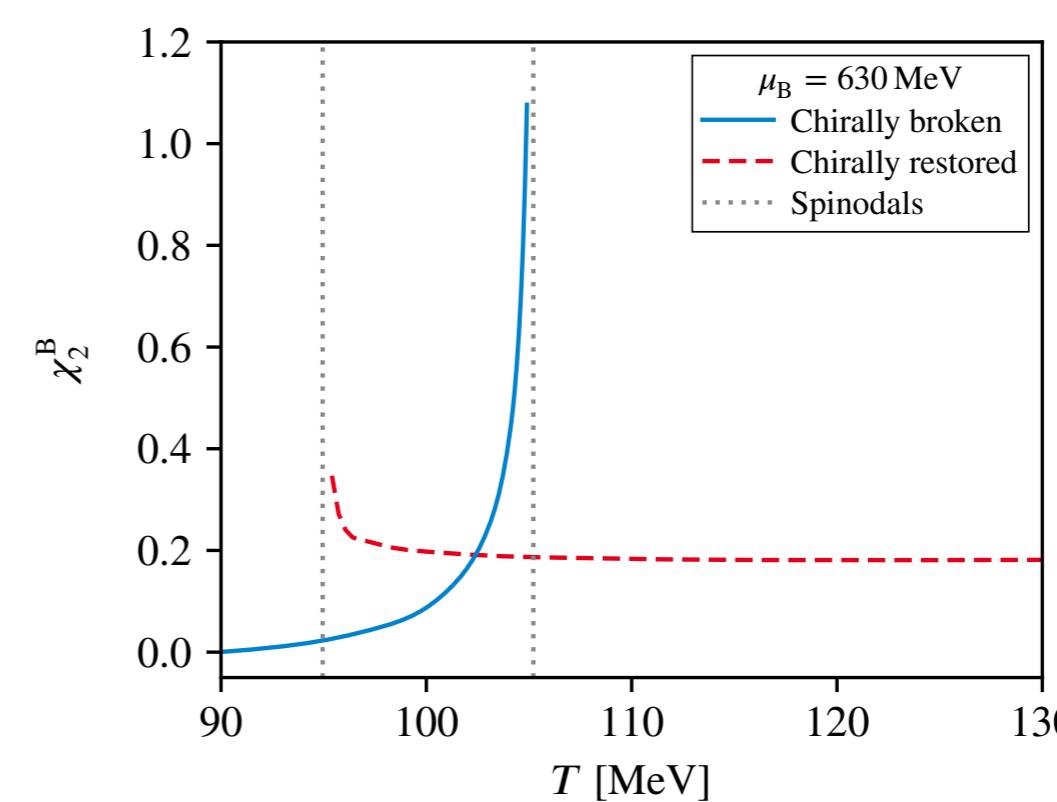
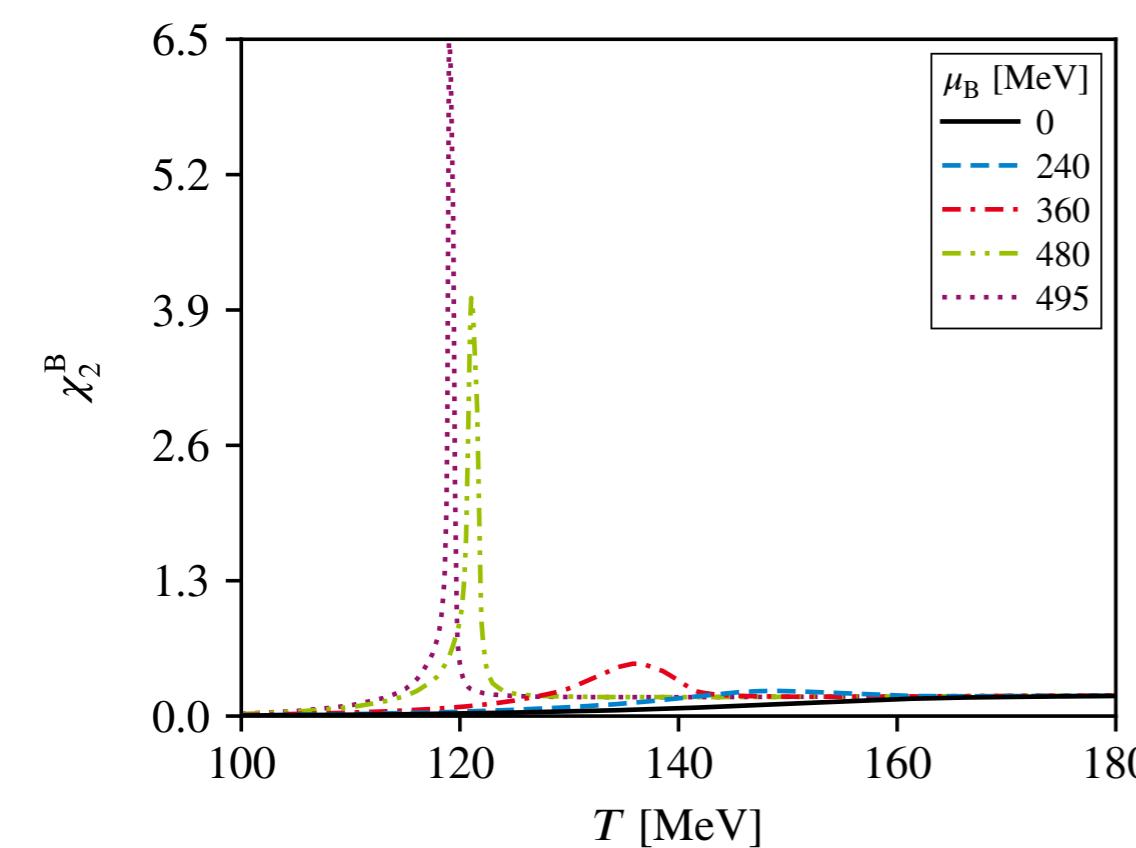
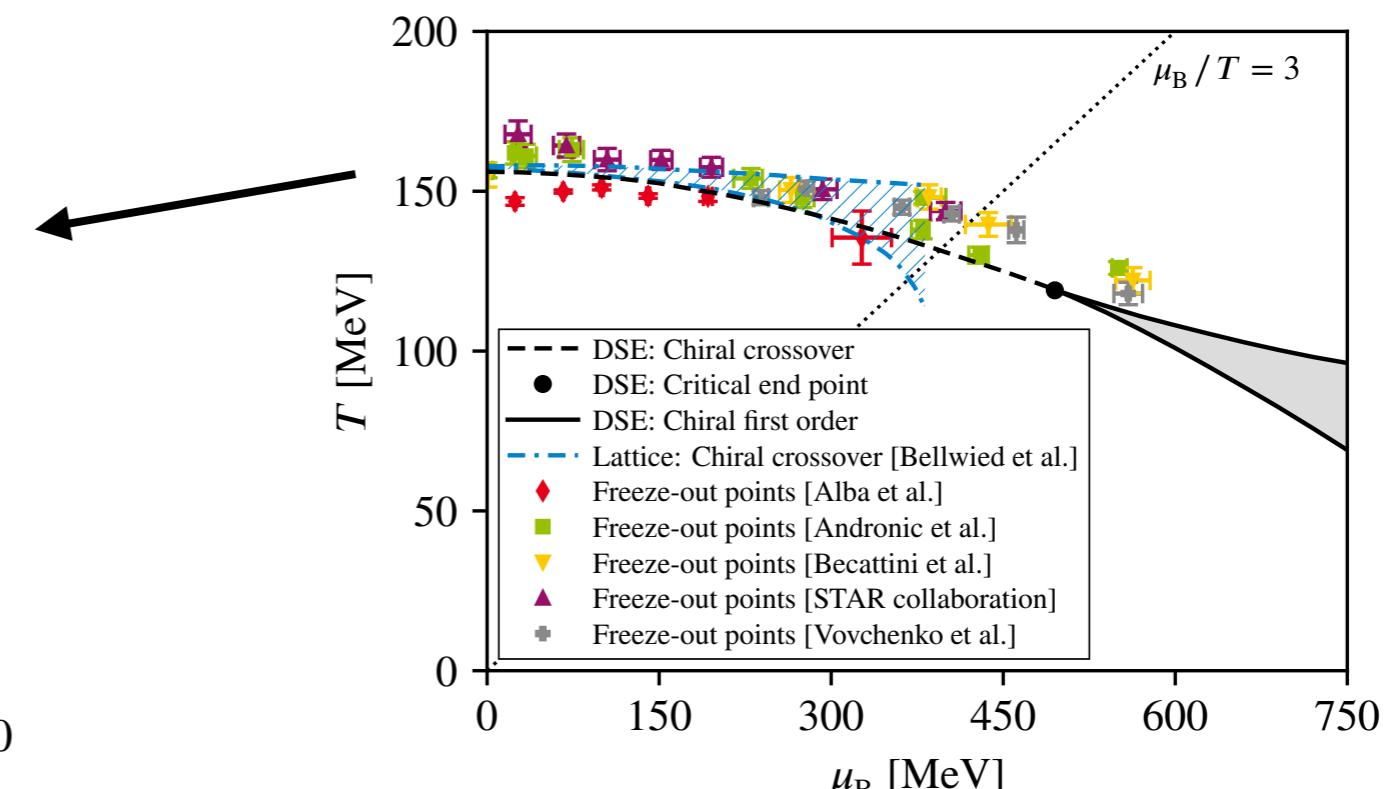
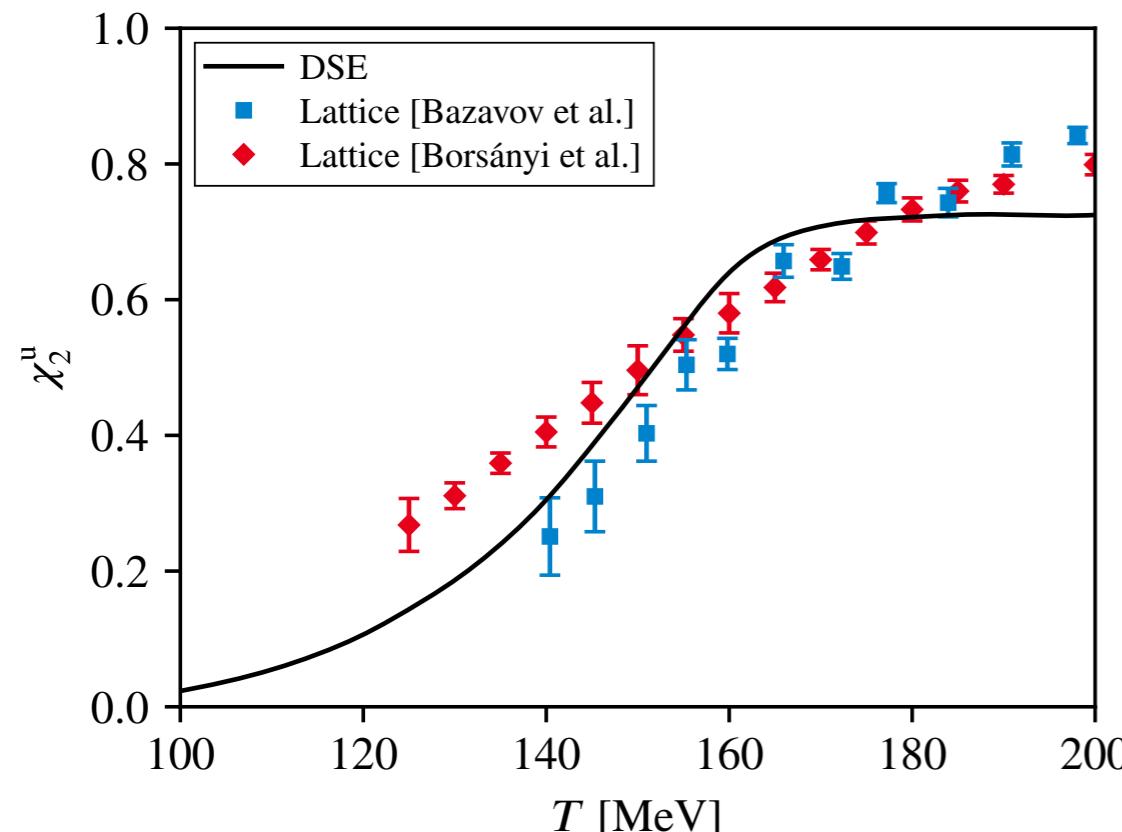
Serve to calculate susceptibilities:

$$\chi_{lmn}^{BSQ} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

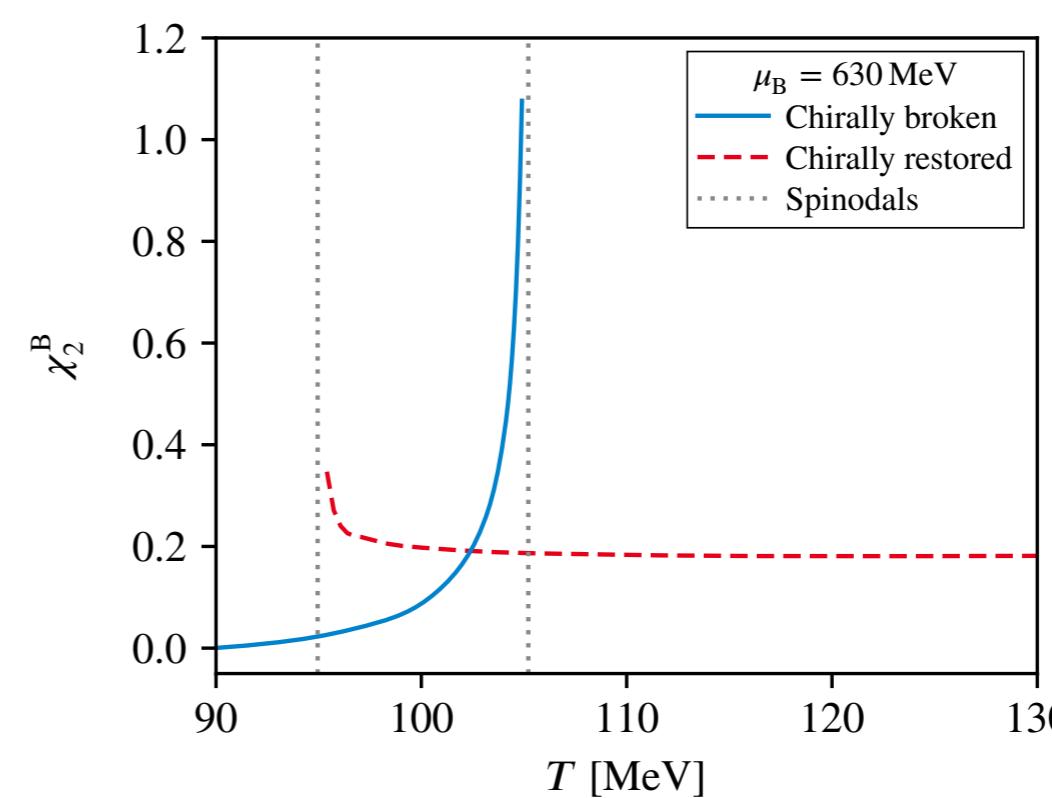
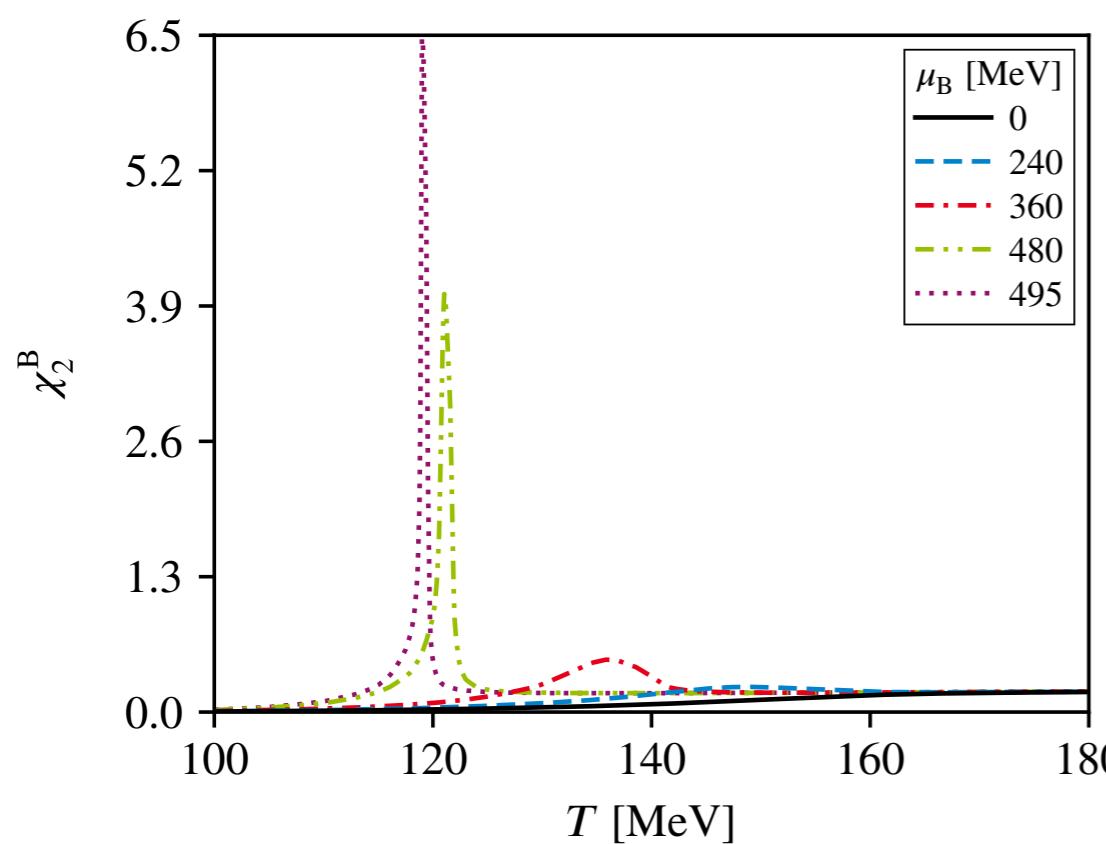
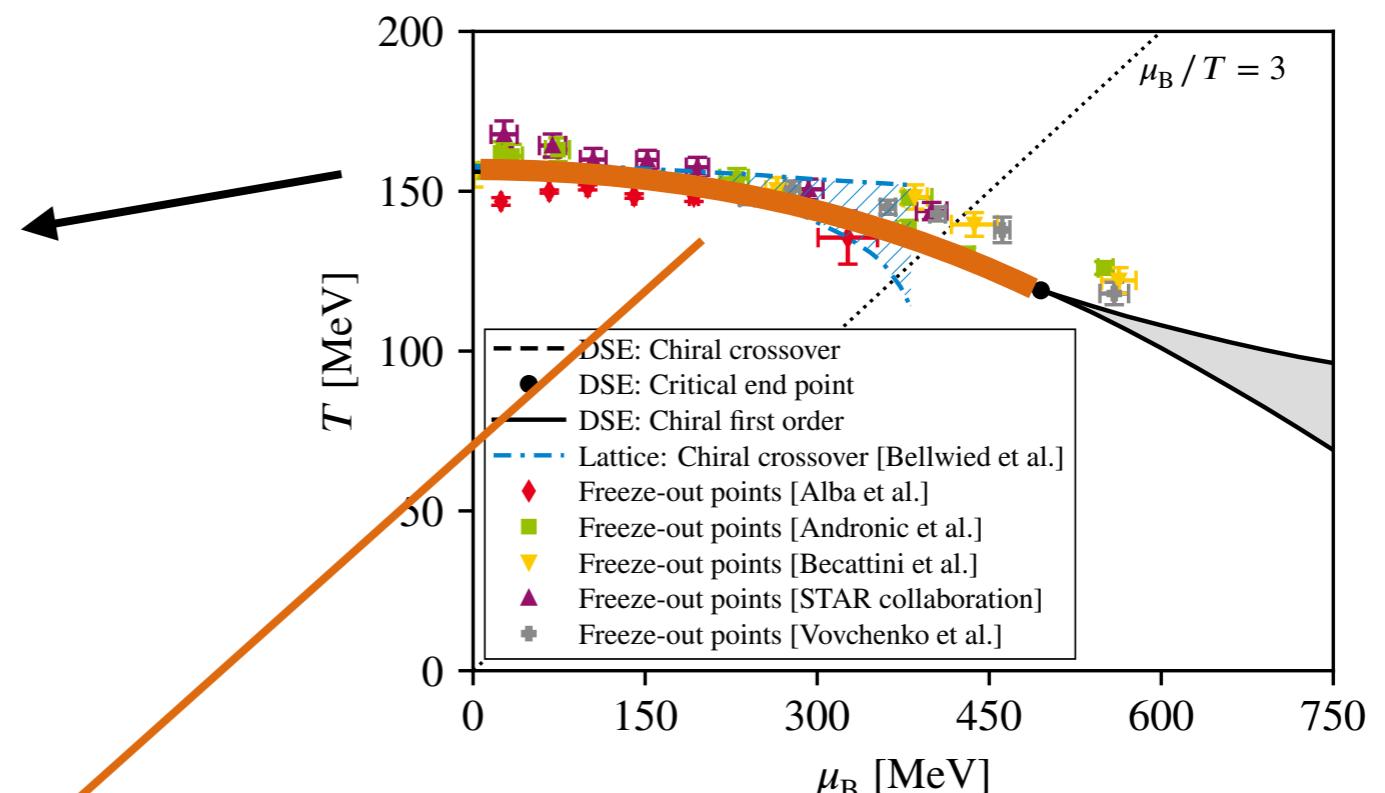
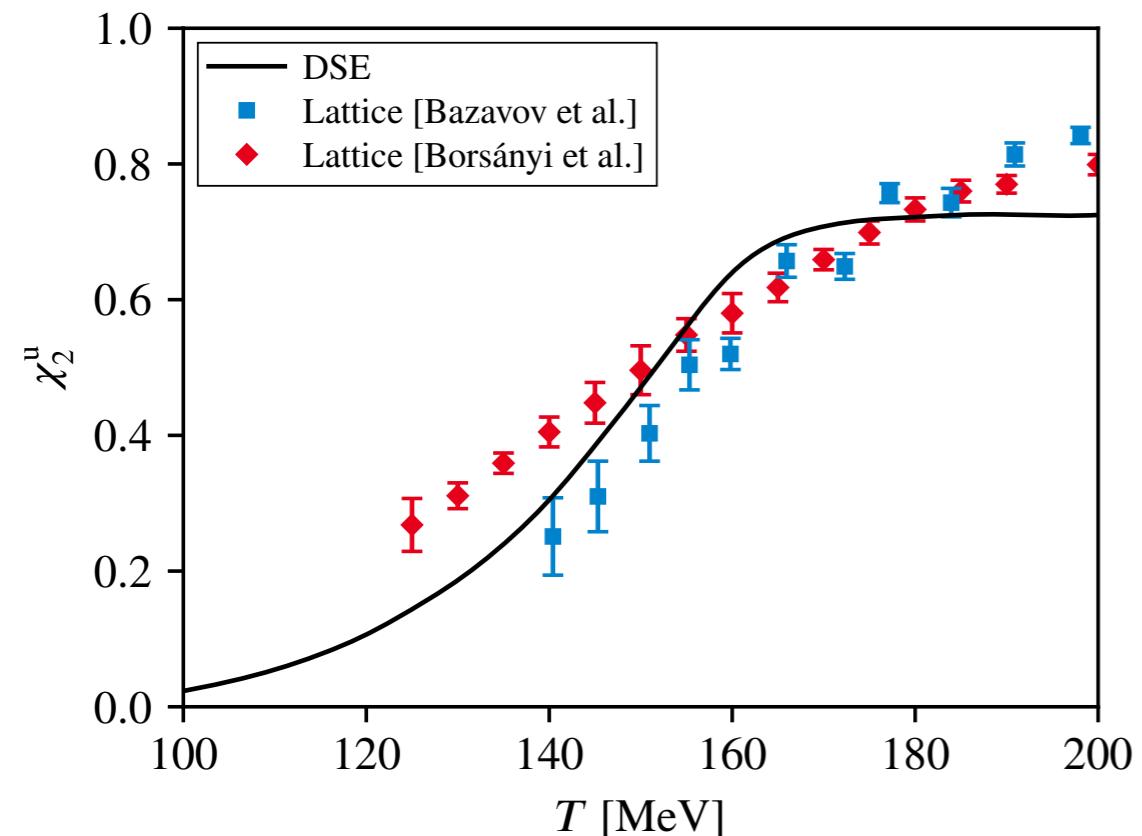
Related to cumulants, which can be extracted from experiment:

$$C_{lmn}^{BSQ} = VT^3 \chi_{lmn}^{BSQ}$$

# Results for fluctuations

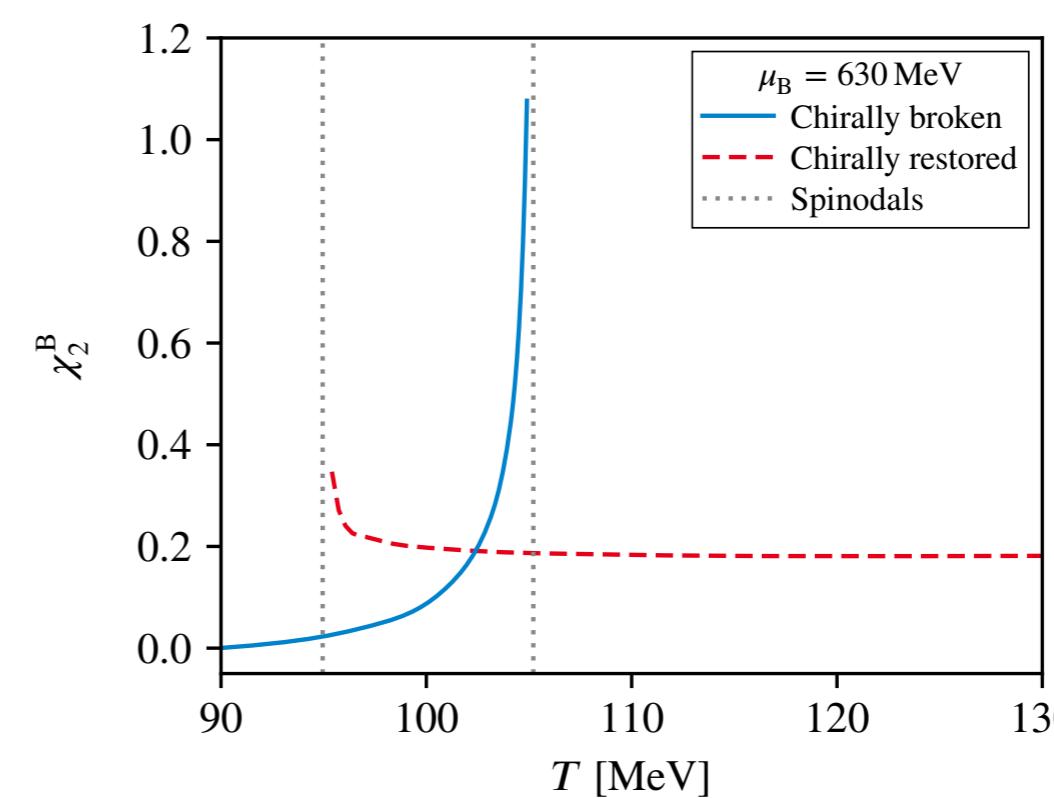
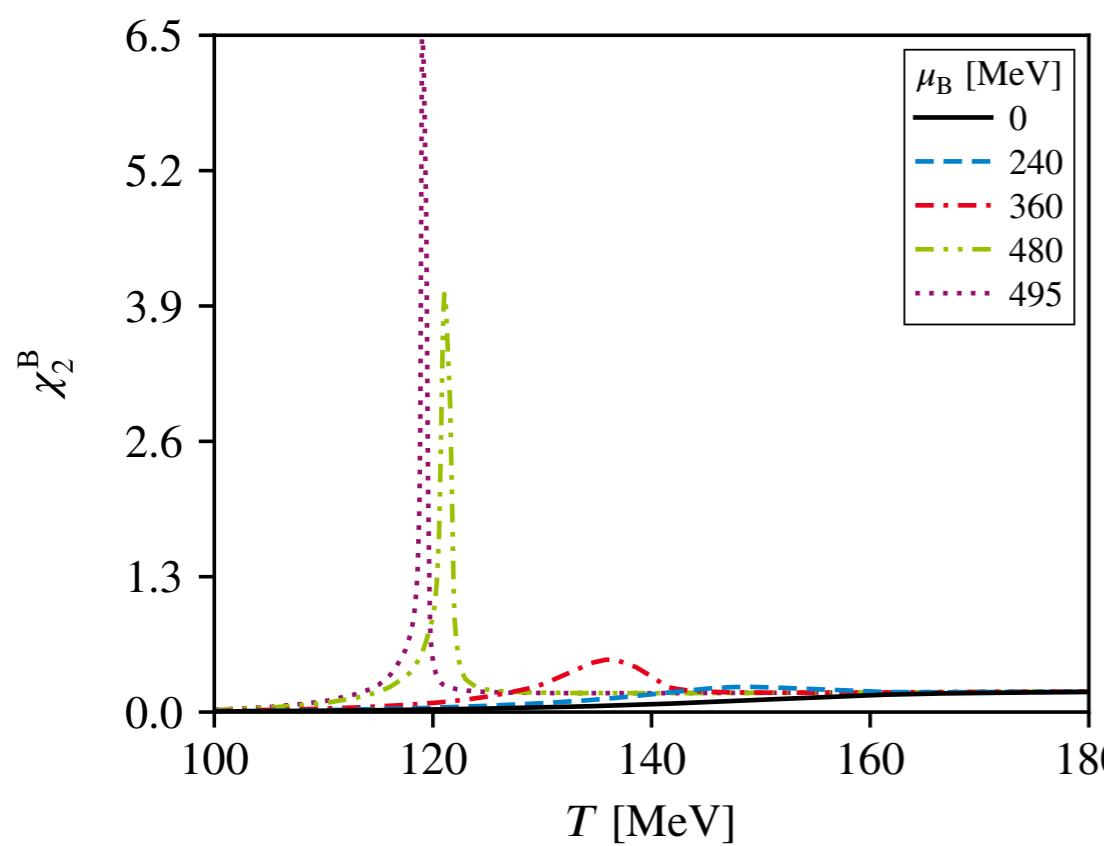
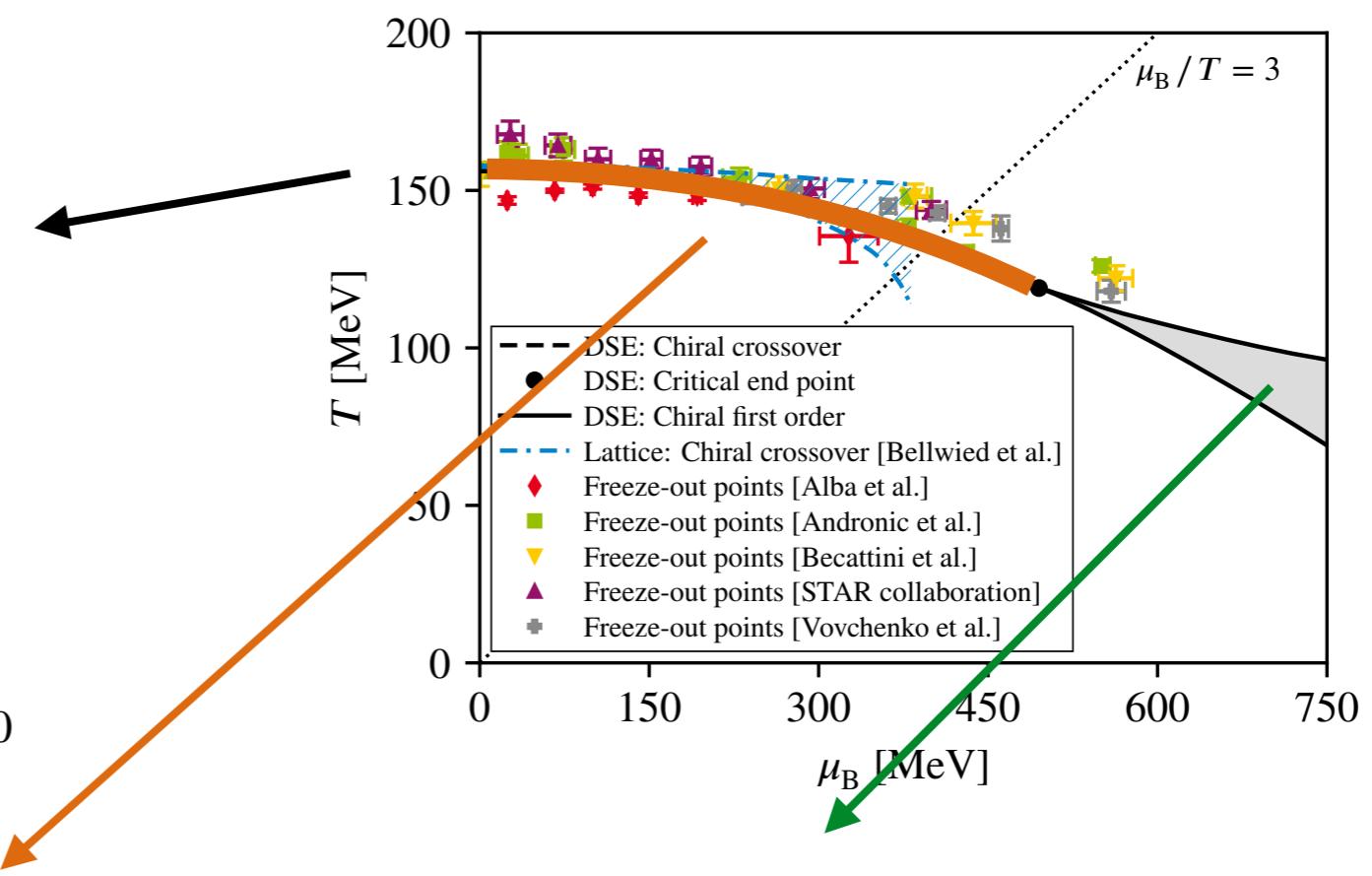
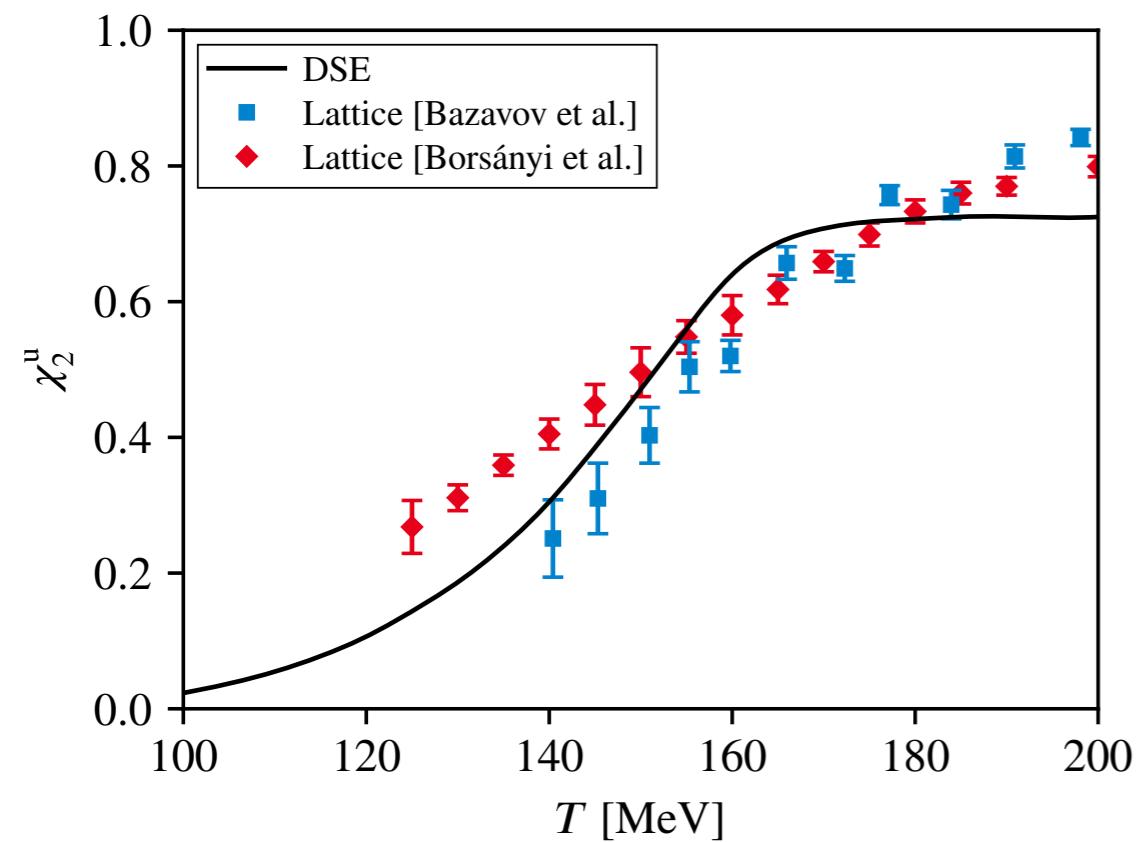


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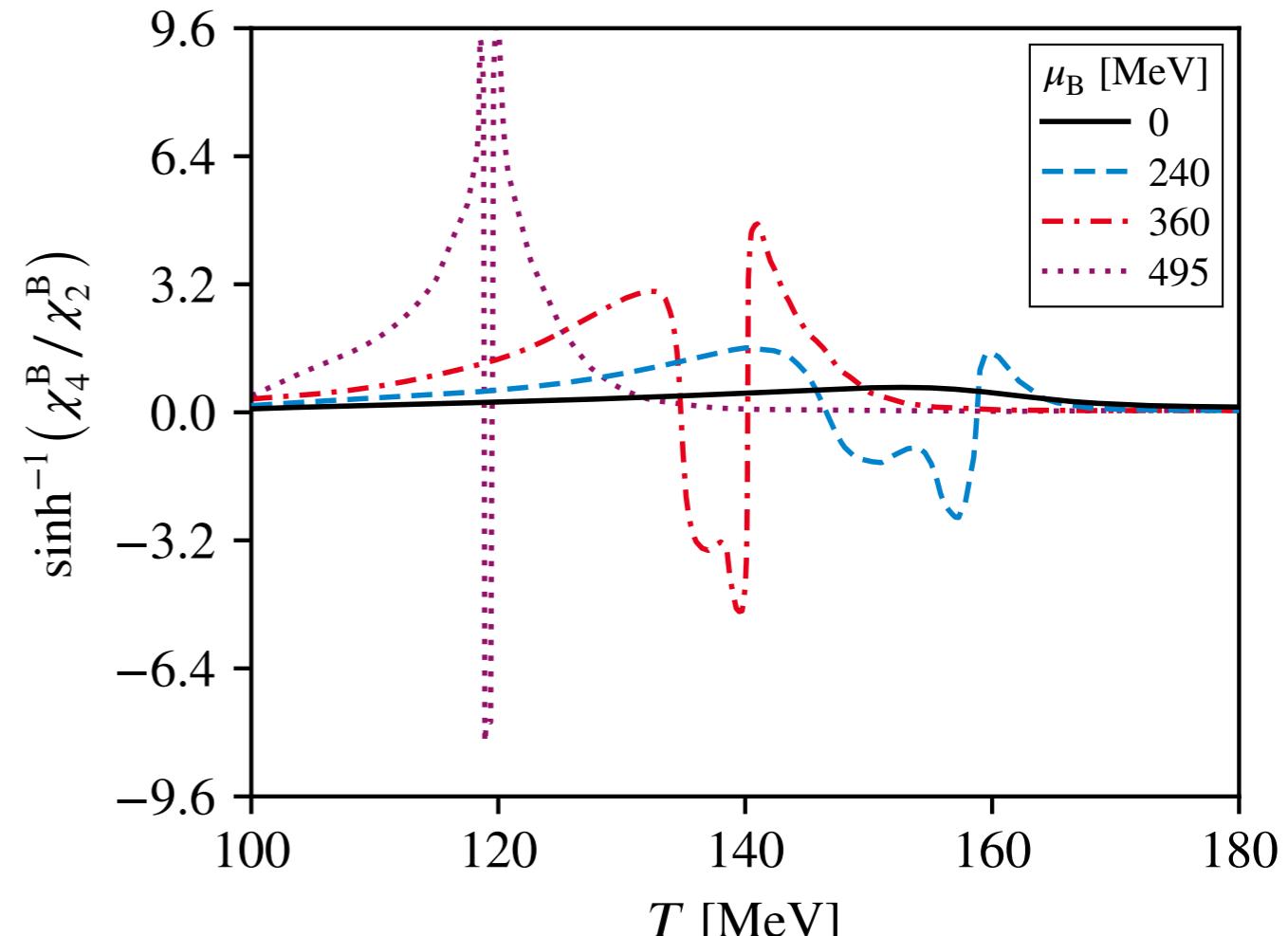
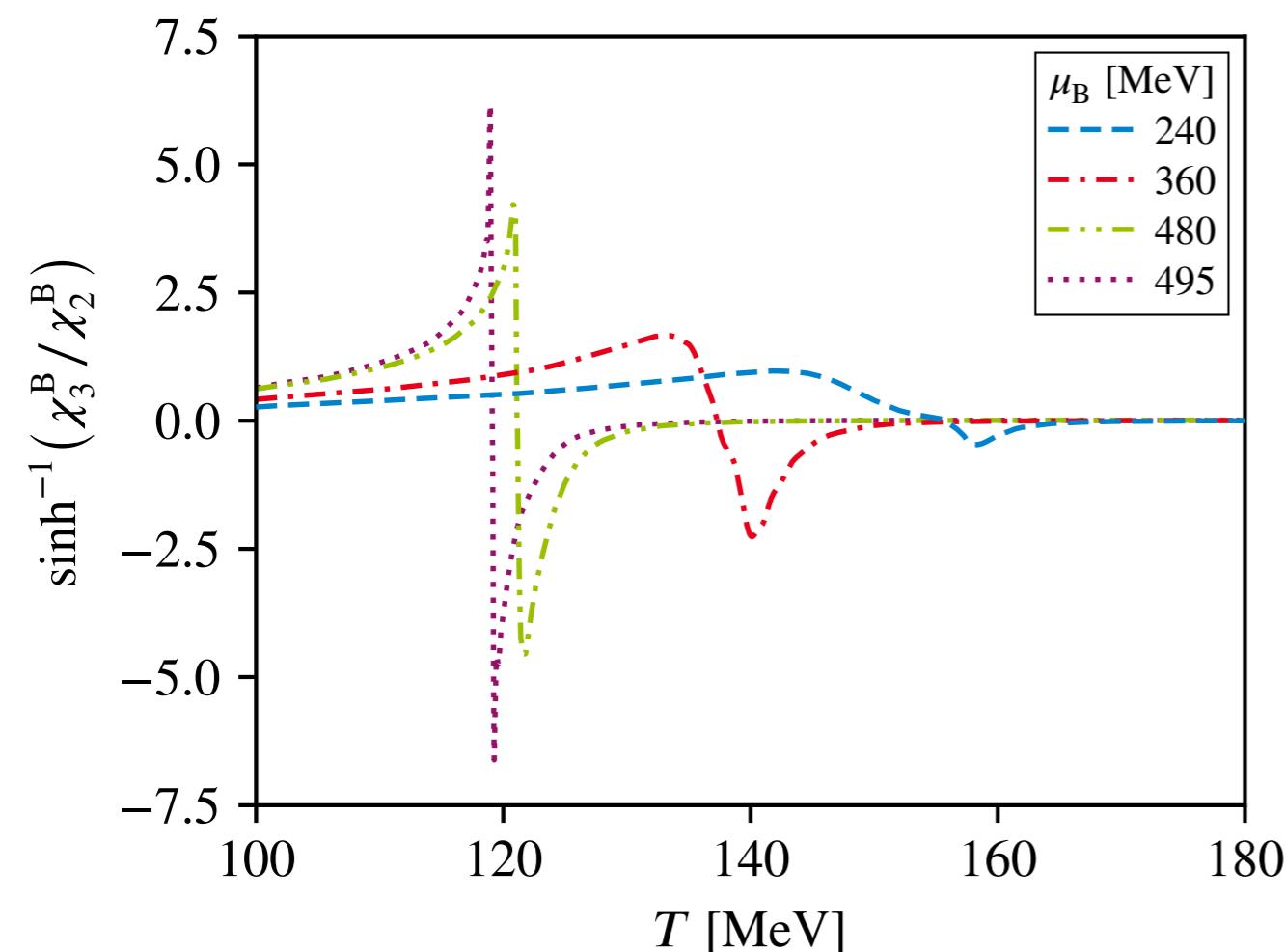


Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

# Results for fluctuations



# Ratios: skewness and kurtosis



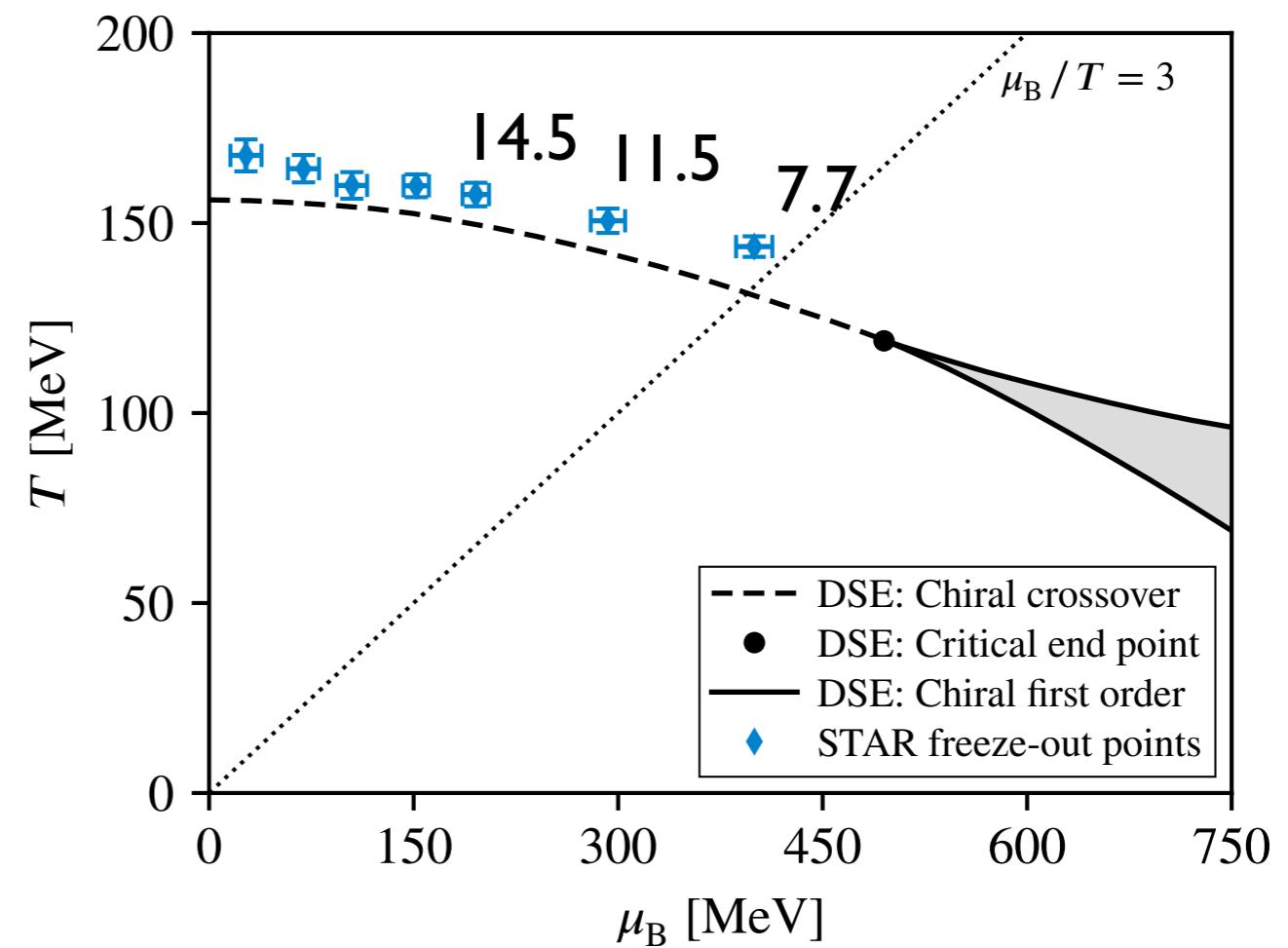
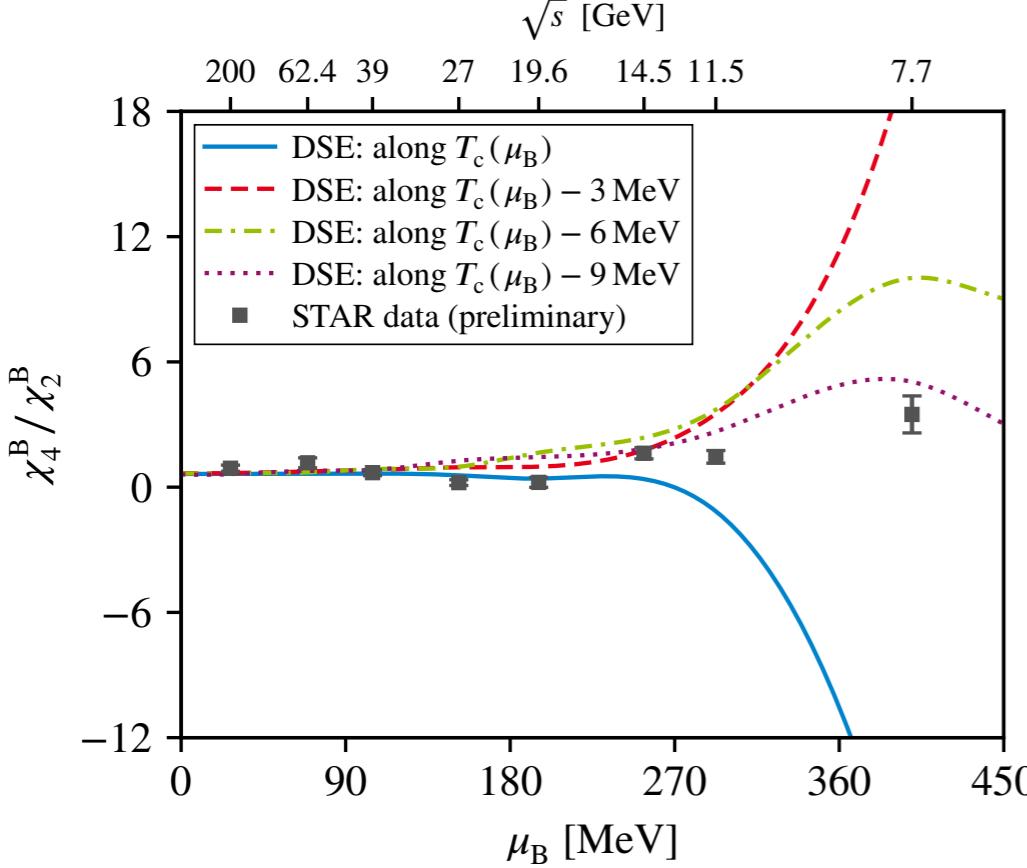
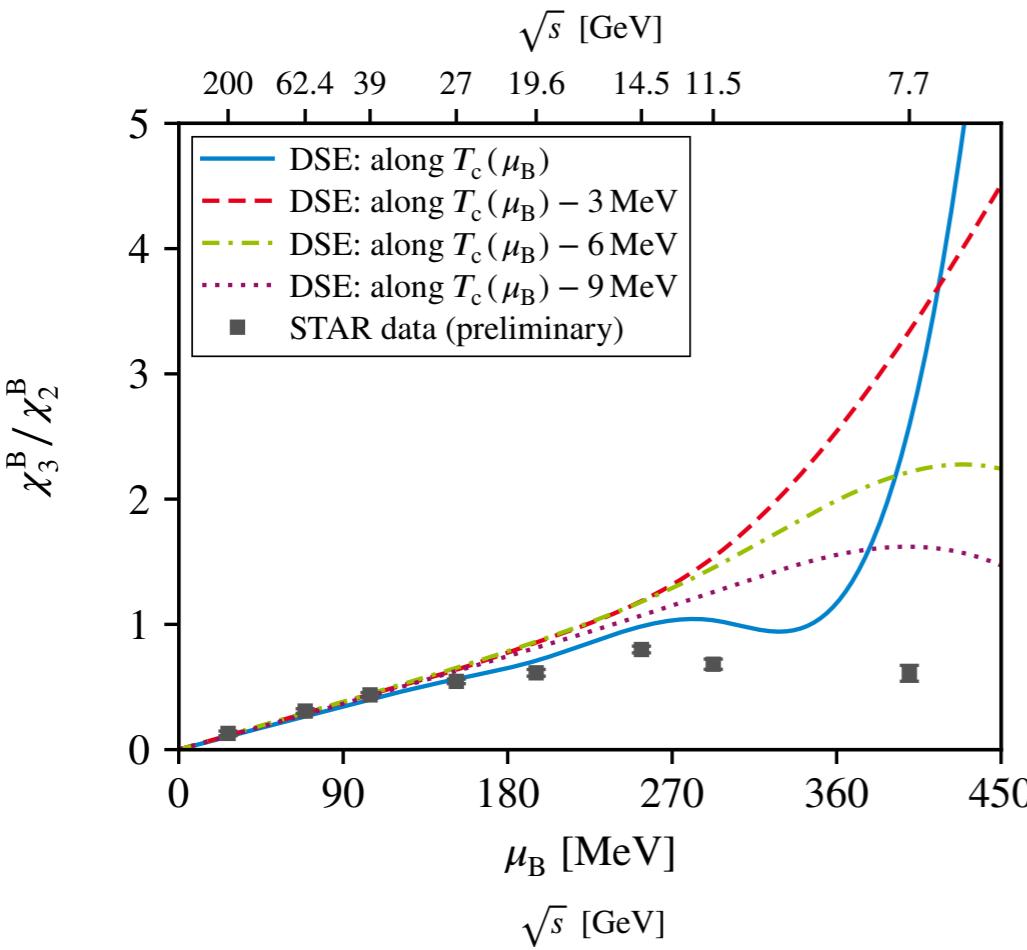
- Clear signals for CEP

Caveats when comparing with experiment:

- so far only flavor-diagonal elements taken into account
- critical region may be too large...
- experimental extraction not without problems

Schaefer and Wambach, PRD 75 (2007) 085015

# Ratios: skewness and curtosis



$\sqrt{s} \geq 14.5$  : good agreement  
 $\sqrt{s} = 11.5$  : trend ok !  
 $\sqrt{s} \leq 7.7$  : freezeout line  
 $\neq$  transition line ?!

## QCD with finite chemical potential:

- back-reaction of quarks onto gluons important
- $N_f=2+1$  and  $N_f=2+1+1$ : CEP at  $\mu_c/T_c > 3$
- charm quark does not influence CEP
- Baryon effects may or may not be significant for CEP...

Work in progress: - mesons and baryons at finite T and  $\mu$

w. Pascal Gunkel

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169

- thermodynamics and fluctuations

w. Phillip Isserstedt

Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

- finite volume effects

w. Julian Bernhard

# Backup Slides

# Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

$$\Gamma^\mu(p, k) = \sum_{i=1,12} \tau_i(p, k) T_i^\mu$$

$$\sim \gamma^\mu \tau(k^2) \quad \text{“approximation” !}$$

$$D^{\mu\nu}(k) = \left( \delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2} \right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

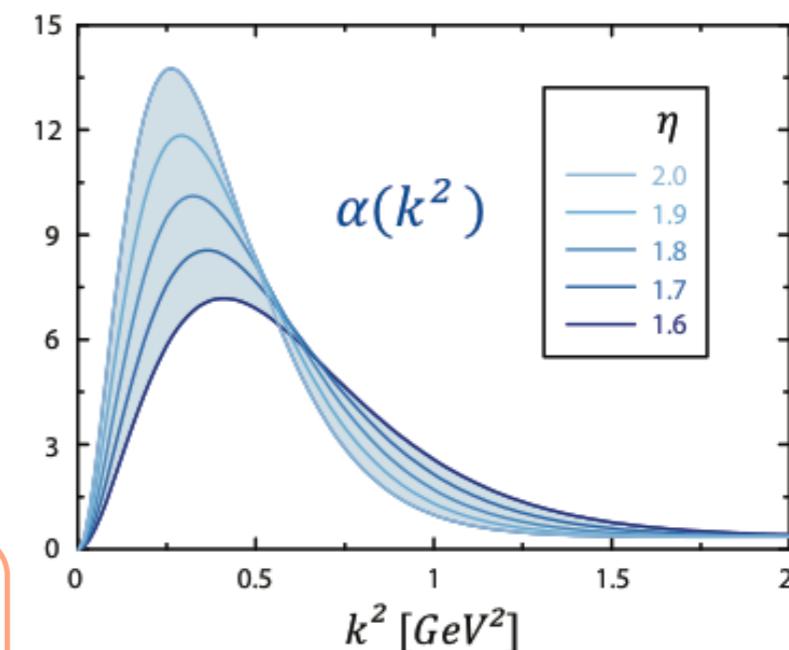
# Rainbow-ladder model for quark-gluon interaction



Combine **gluon** with **quark-gluon vertex**:

effective coupling

$$\alpha(k^2) = \pi \eta^7 \left( \frac{k^2}{\Lambda^2} \right) e^{-\eta^2 \left( \frac{k^2}{\Lambda^2} \right)} + \alpha_{UV}(k^2)$$



Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

- scale  $\Lambda$  from  $f_\pi$ , masses  $m_u = m_d$ ,  $m_s$  from  $m_\pi, m_K$
- $\alpha_{UV}$  from perturbation theory
- parameter  $\eta$ : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.  
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;  
Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035  
Williams, CF Heupel, PRD93 (2016) 034026, and refs. therein

# Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet available...
- Diagrammatics: vertex-DSE (see later...)

explicit solutions at T=0: Mitter, Pawłowski and Strodthoff, PRD 91 (2015) 054035  
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

- Slavnov-Taylor identity: T, μ, m-dependent vertex

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(k) + A(p)}{2} \right) \times \\ \times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

STI

PT

- $d_1$  fixed via  $T_c$
- $d_2$  fixed to match scale of lattice gluon input

# Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet

- Diagrams: vertex-DSE (self-energy)

explicit solutions

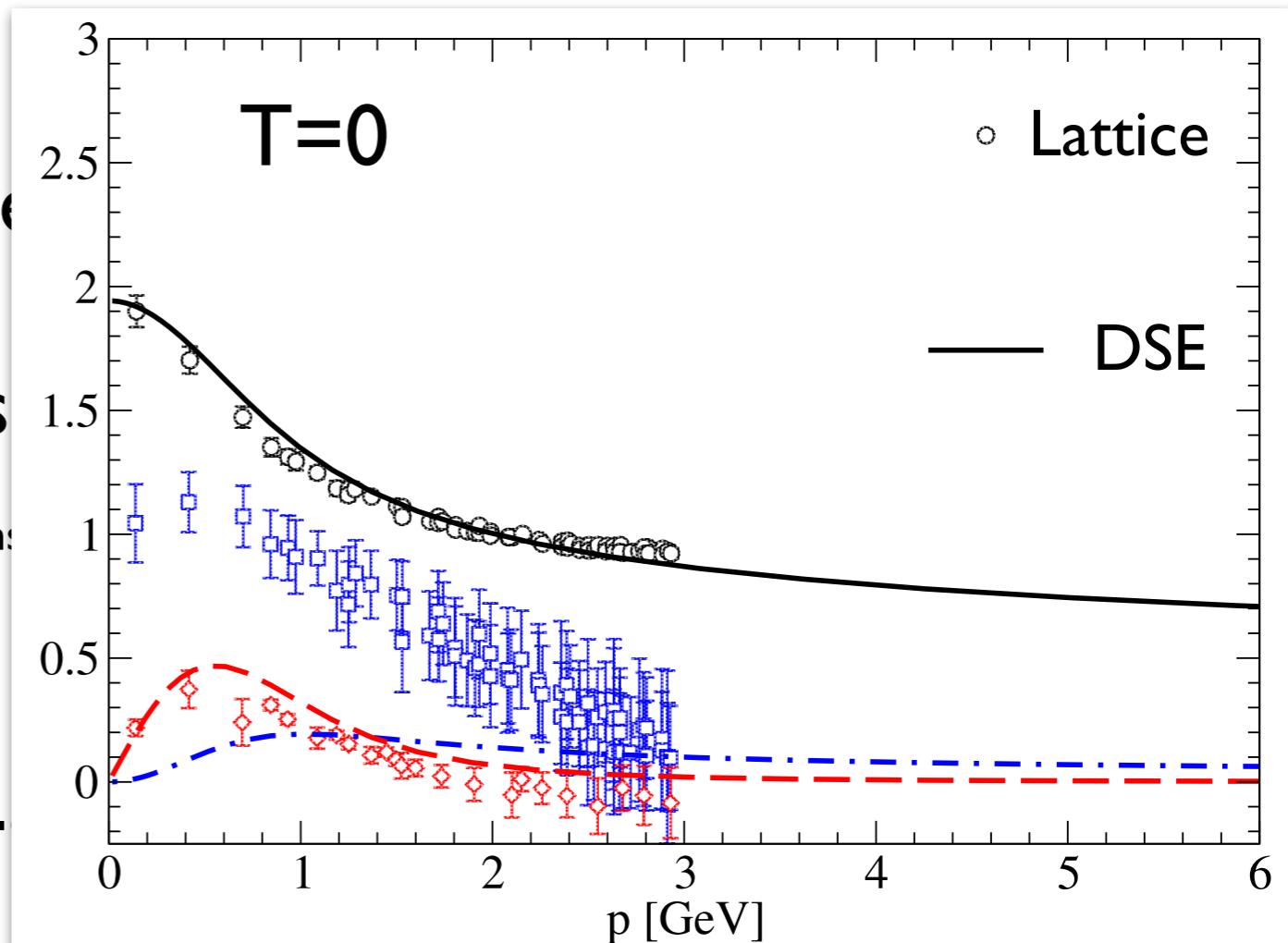
- Slavnov-Taylor identity:  $T, \mu, m$ -

$$\Gamma_\nu(q, k, p) = \tilde{Z}_3 \left( \delta_{4\nu} \gamma_4 \frac{C(k) + C(p)}{2} + \delta_{j\nu} \gamma_j \frac{A(\kappa) + A(p)}{2} \right) \times \left( \frac{d_1}{d_2 + q^2} + \frac{q^2}{\Lambda^2 + q^2} \left( \frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2 + 1]}{4\pi} \right)^{2\delta} \right)$$

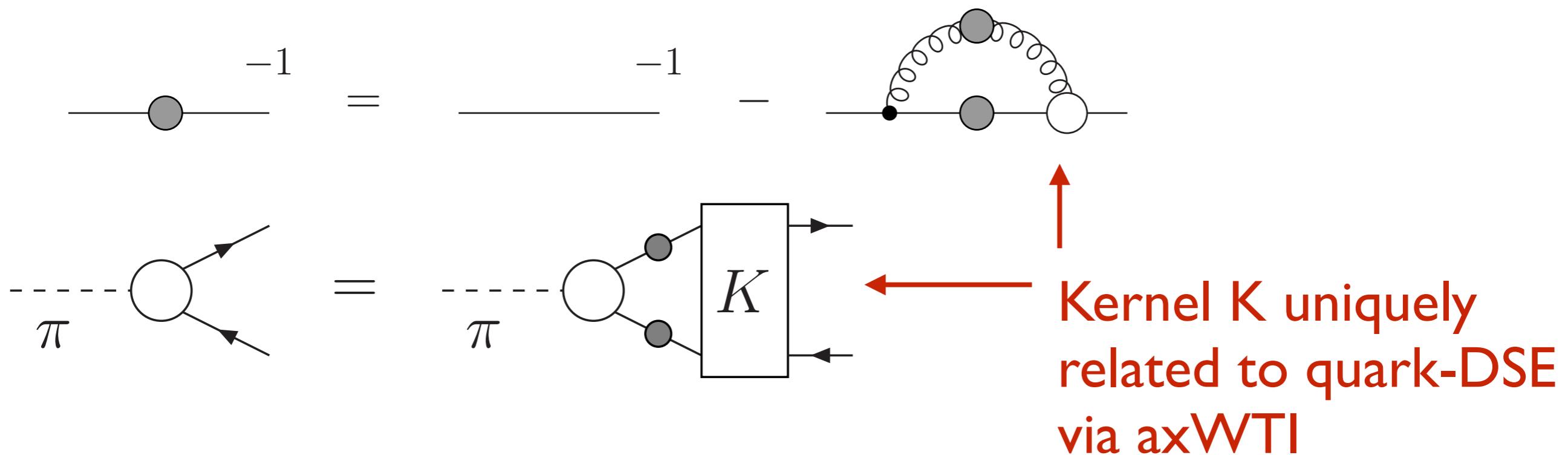
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PT

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- $d_2$  fixed to match scale of lattice gluon input



# Theoretical Tools II: DSEs and BSEs

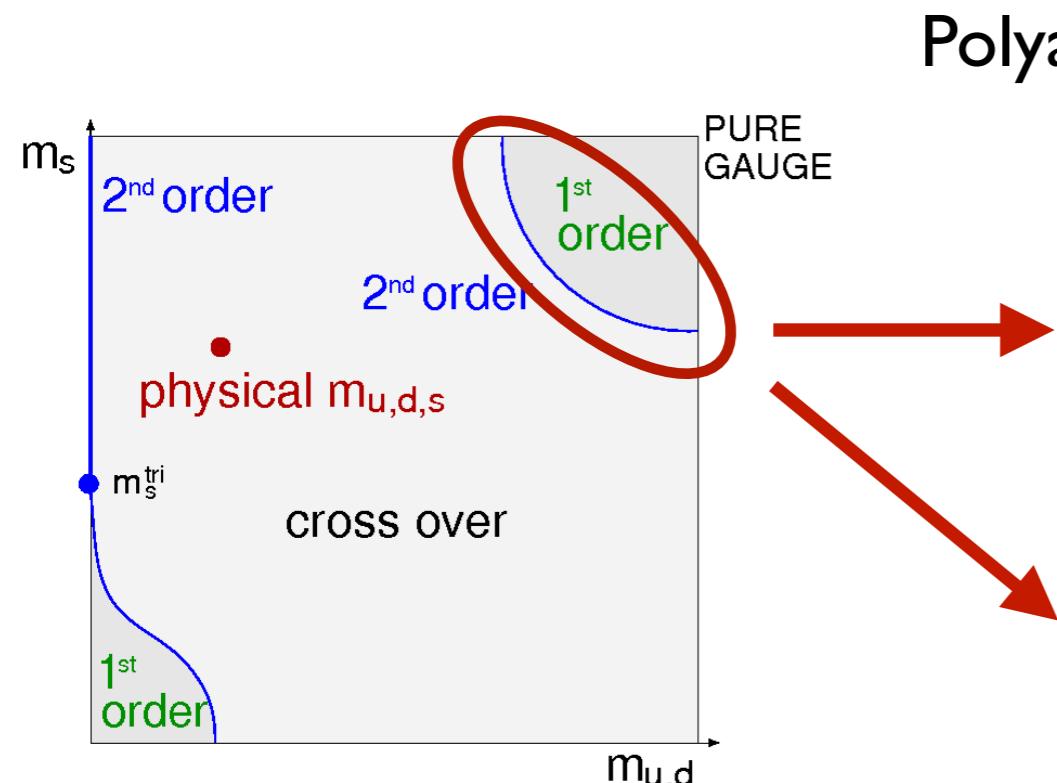


→ Pion is bound state **and** Goldstone boson

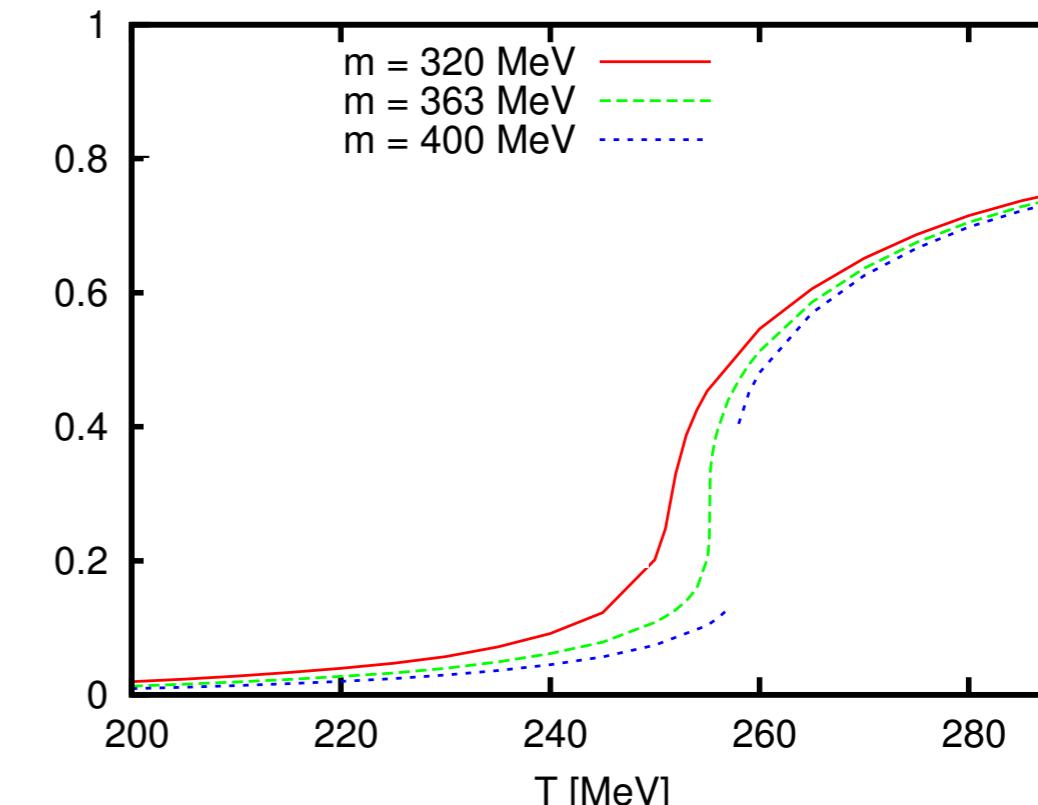
Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex

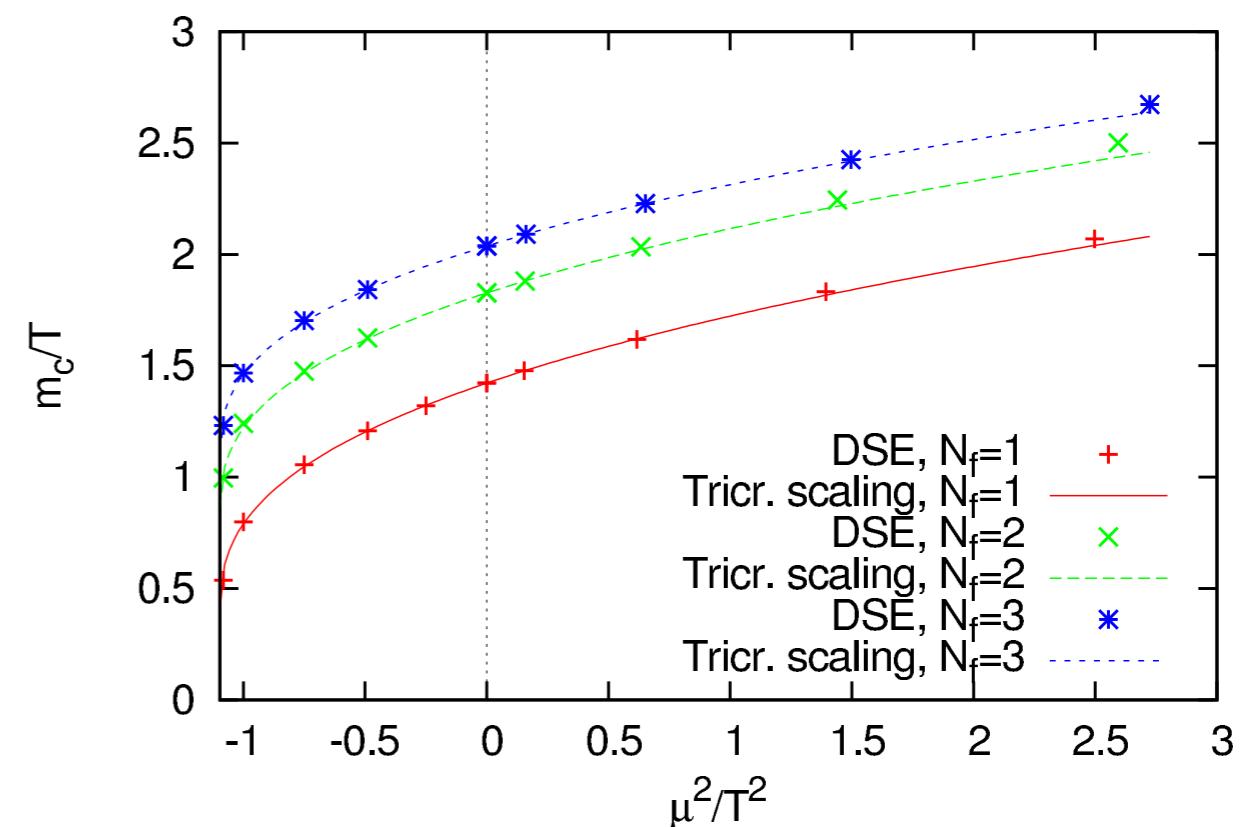
# Critical line/surface for heavy quarks



Polyakov Loop:



- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

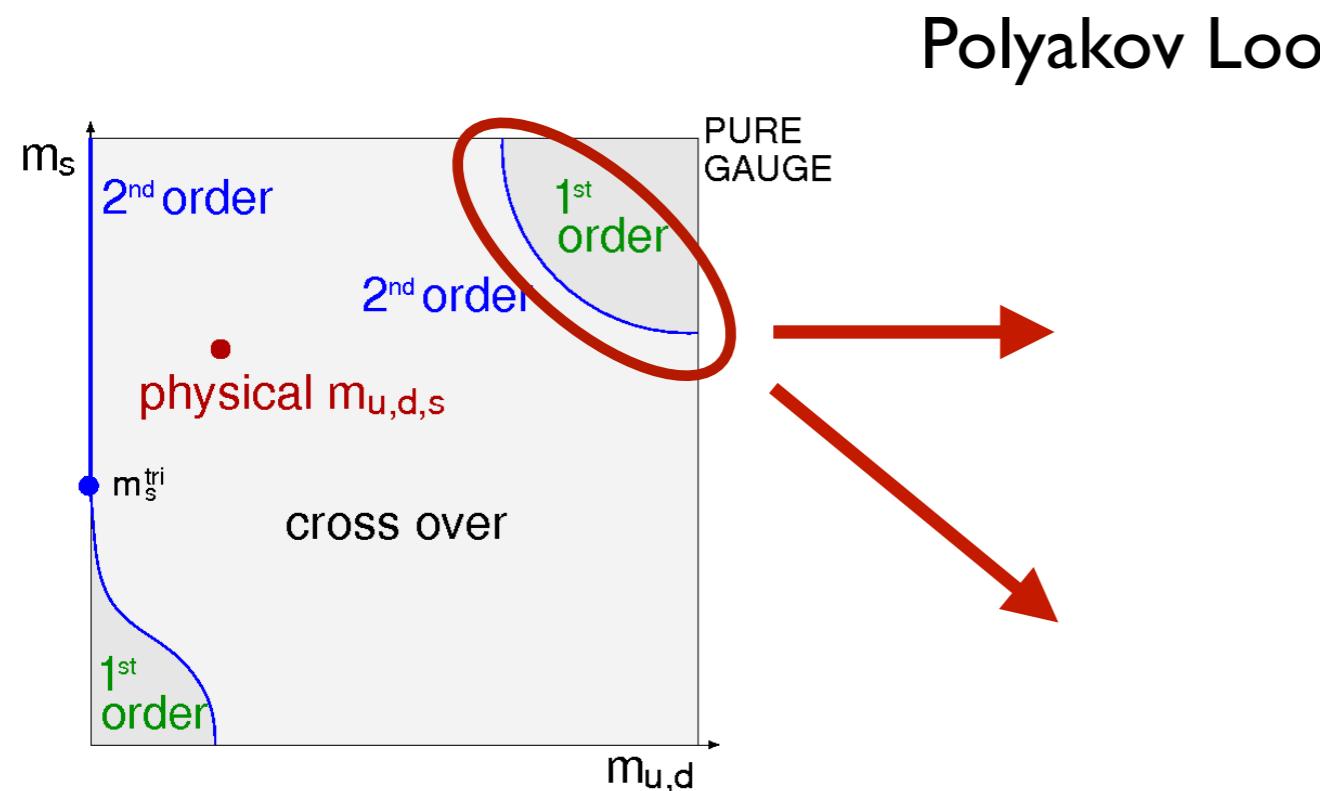


Lattice:

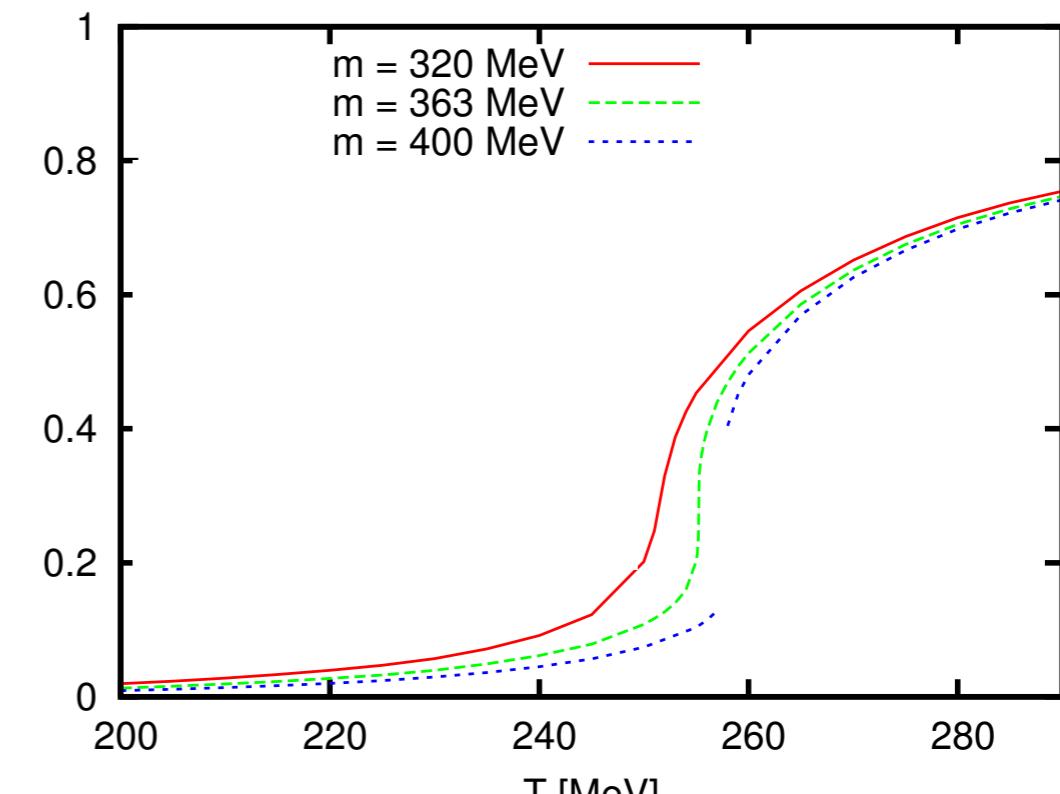
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawłowski, PRD 91 (2015) 1

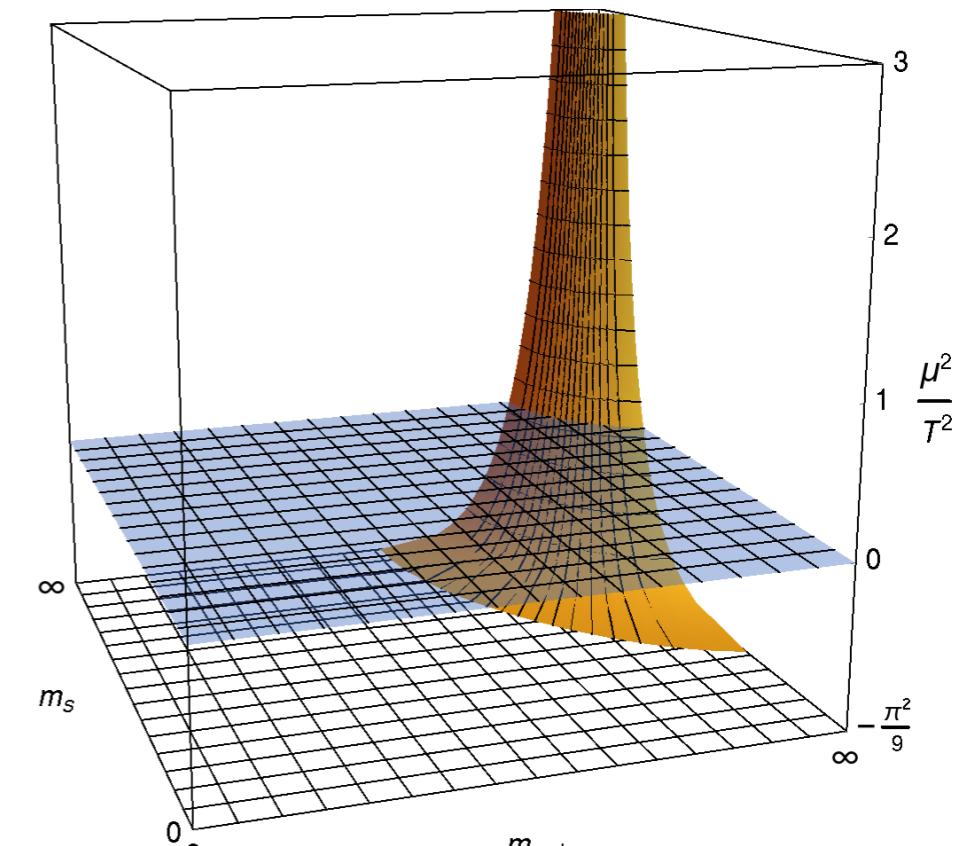
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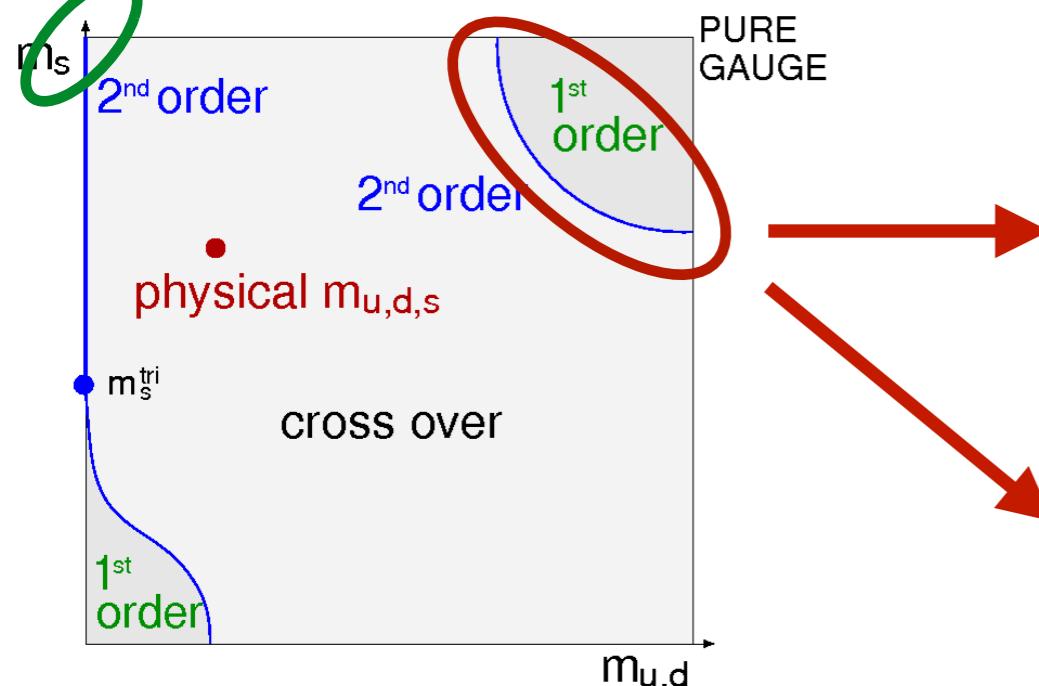
CF, Luecker, Pawłowski, PRD 91 (2015) 1

Lattice:

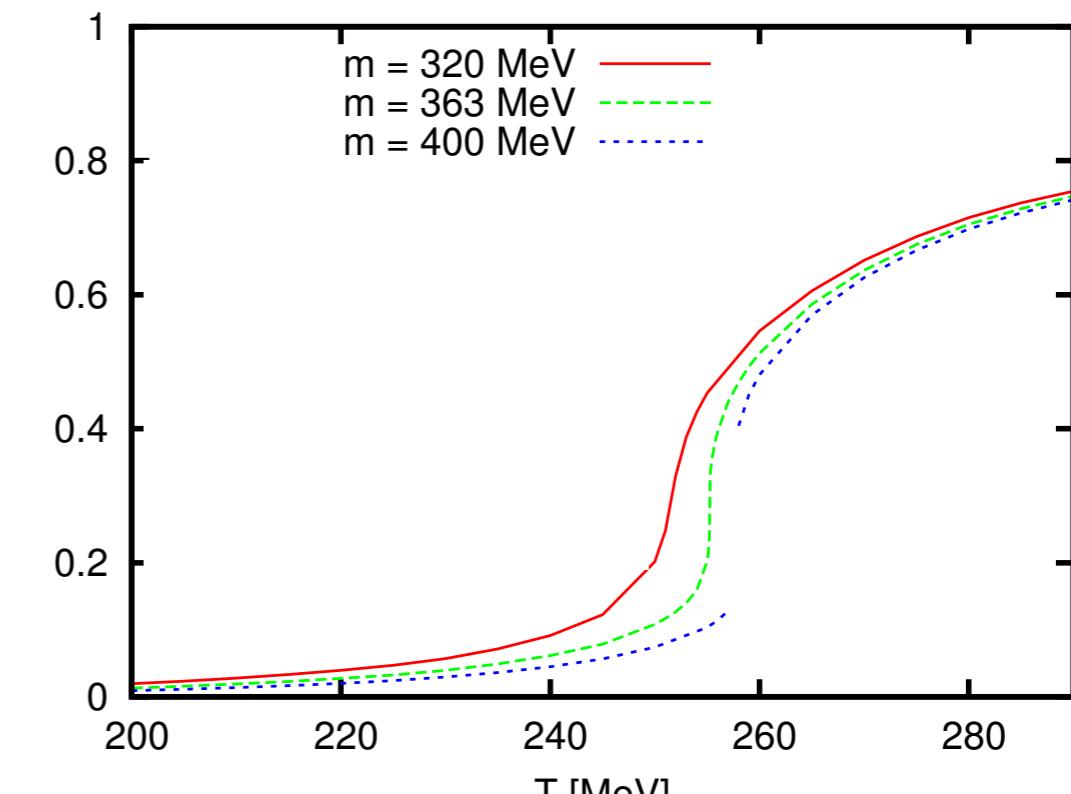
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

# Critical line/surface for heavy quarks

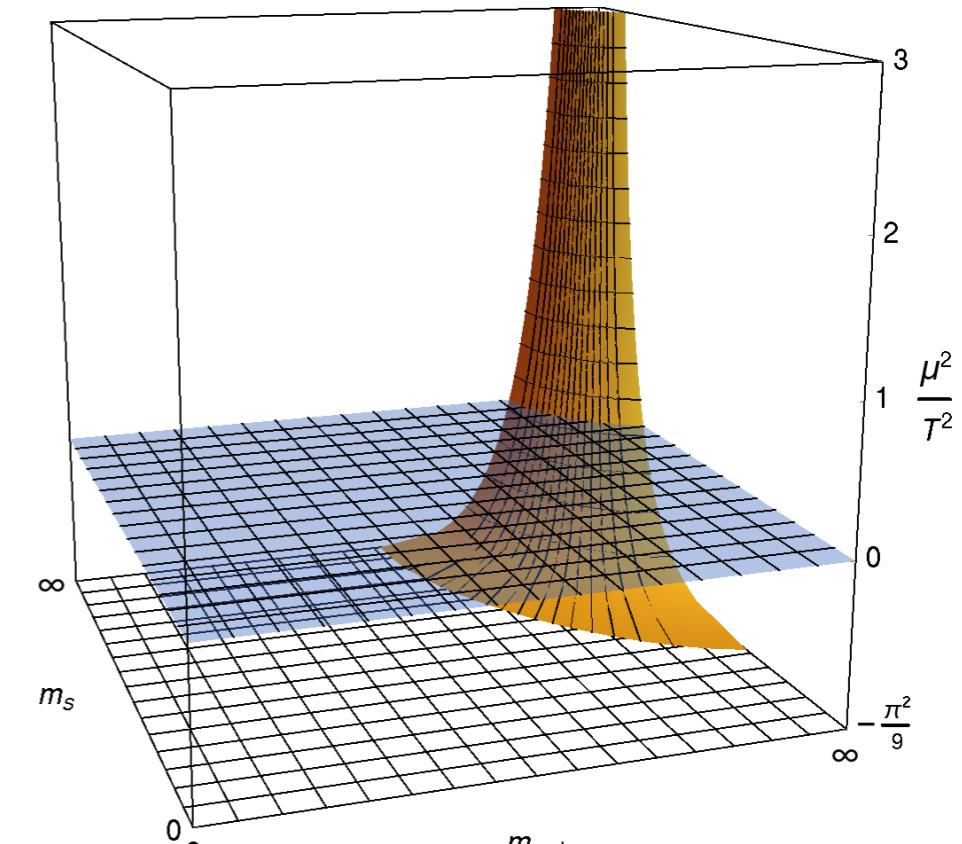
Nf=2: CF and Mueller, PRD 84 (2011) 054013



Polyakov Loop:



- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen



Lattice:

Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawłowski, PRD 91 (2015) 1