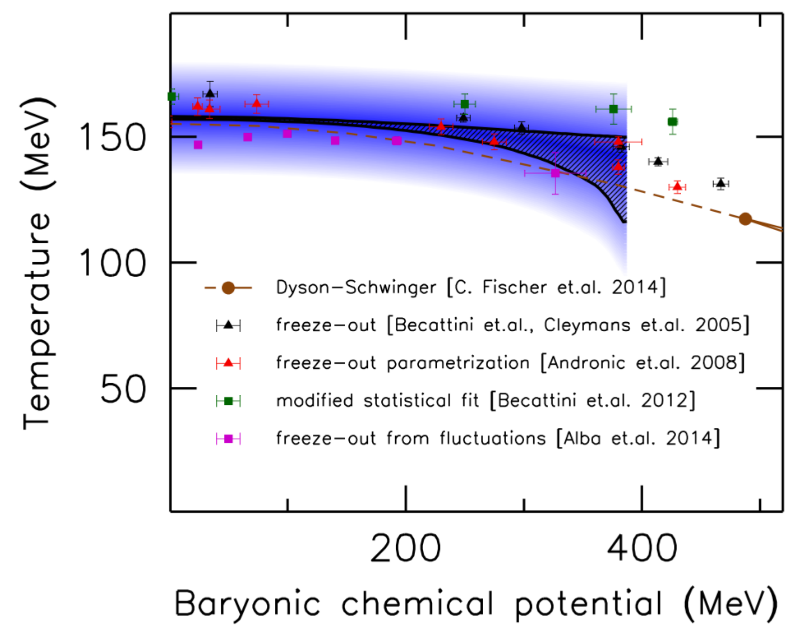
FAIR
Sept. 2020

Fluctuations and the QCD phase diagram from functional methods

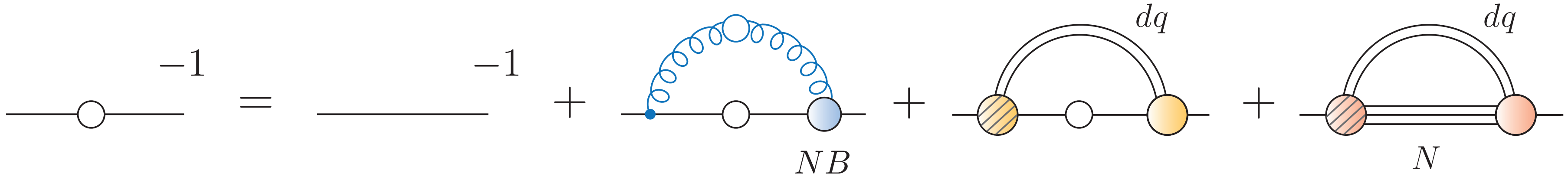
work together with Pascal Gunkel, Philipp Isserstedt

Overview

1.Gluons, quarks and the CEP



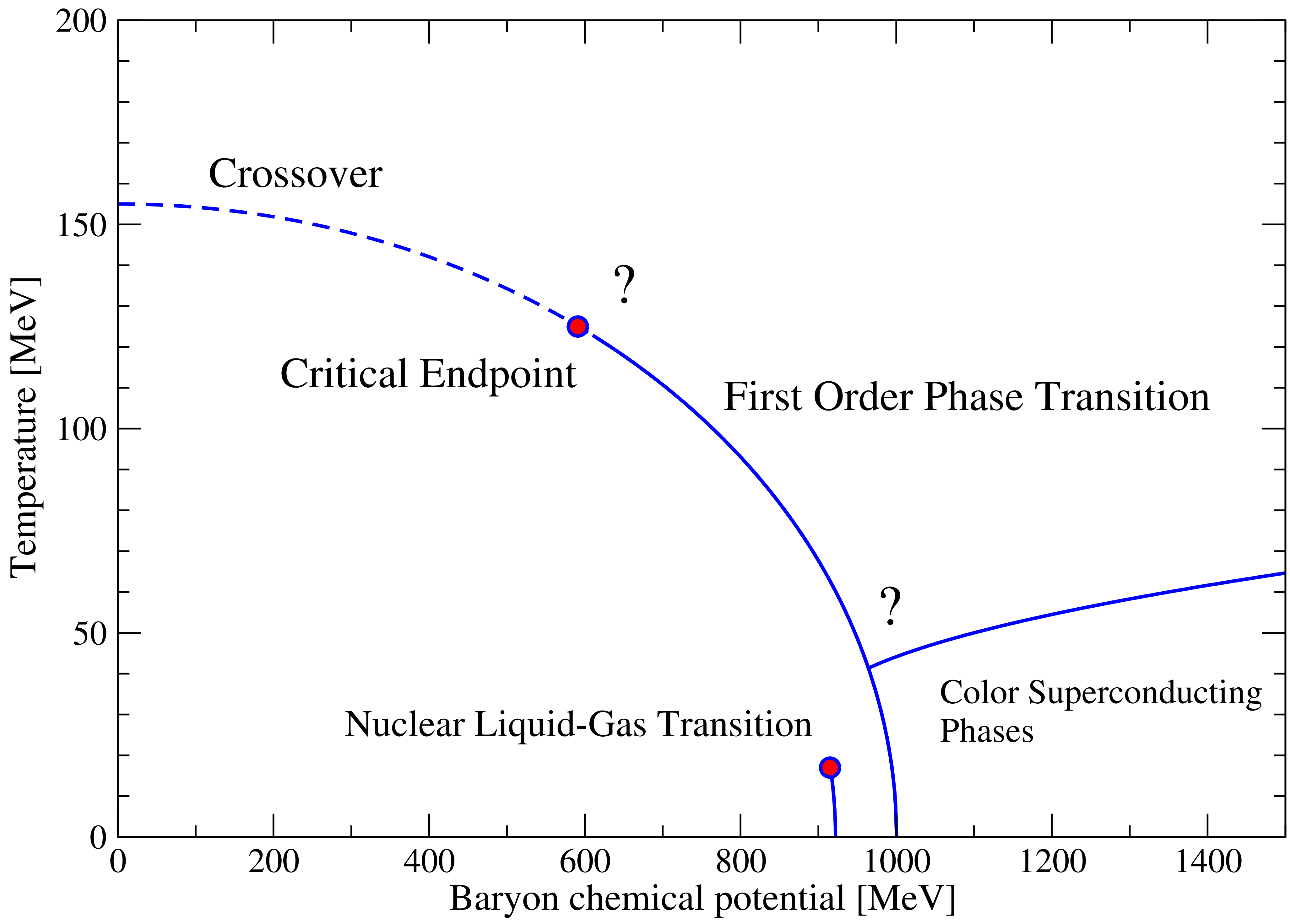
2.Hadron effects



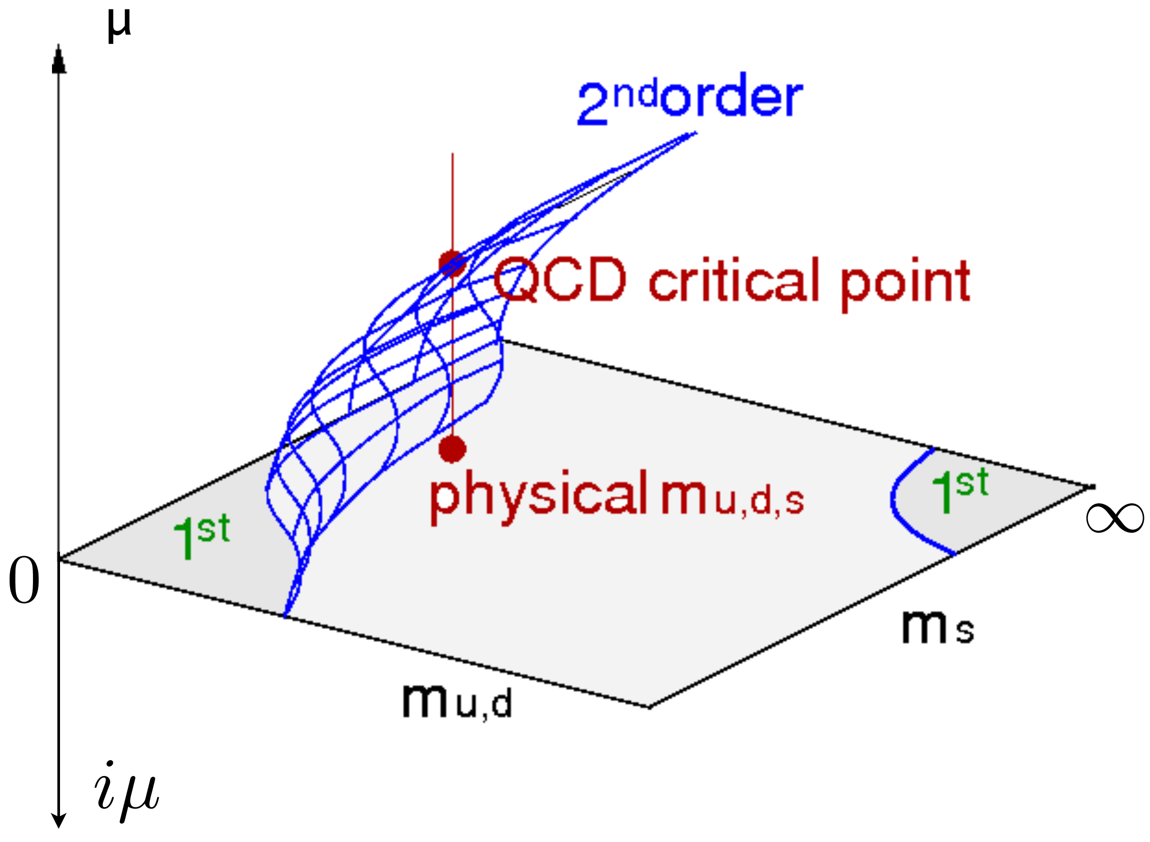
3.Fluctuations

Review: CF, PPNP 105 (2019)

QCD phase transitions



Chiral transition line from analytic continuation



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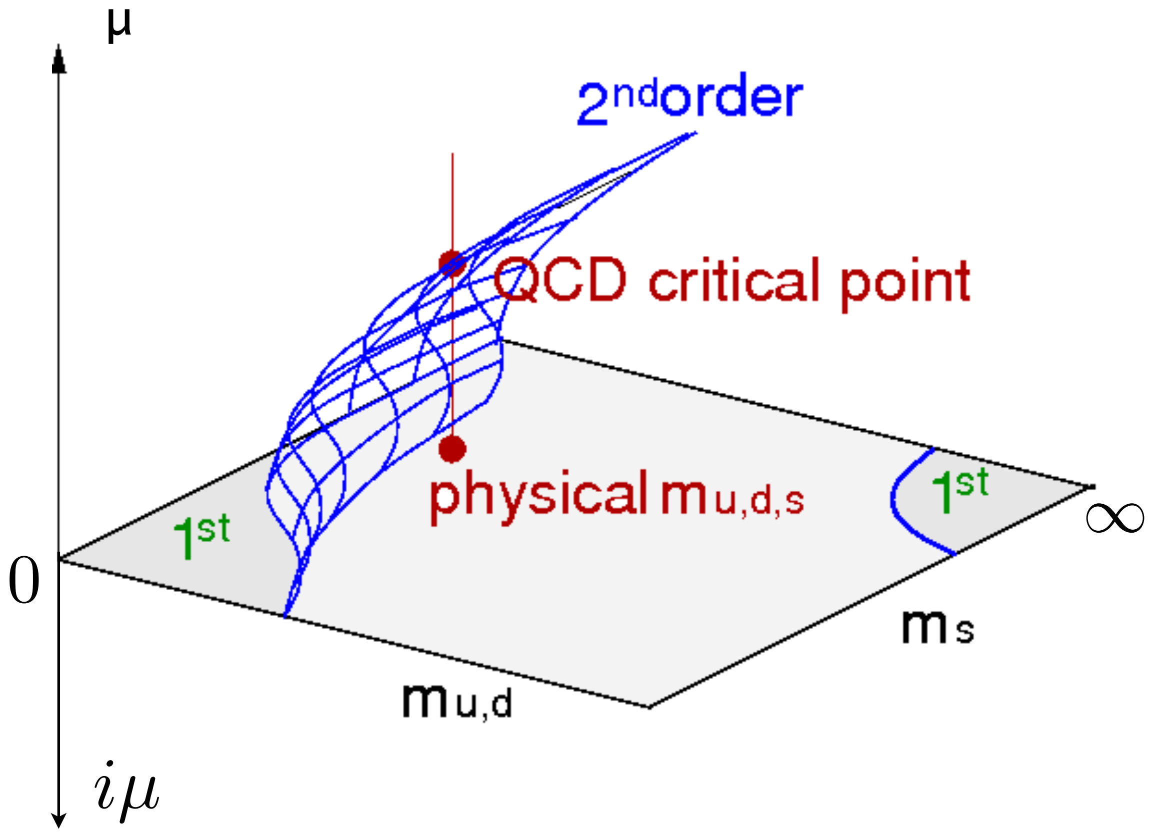

Lattice method:

- Det. crossover at imaginary μ and extrapolate to real μ
- Control systematics

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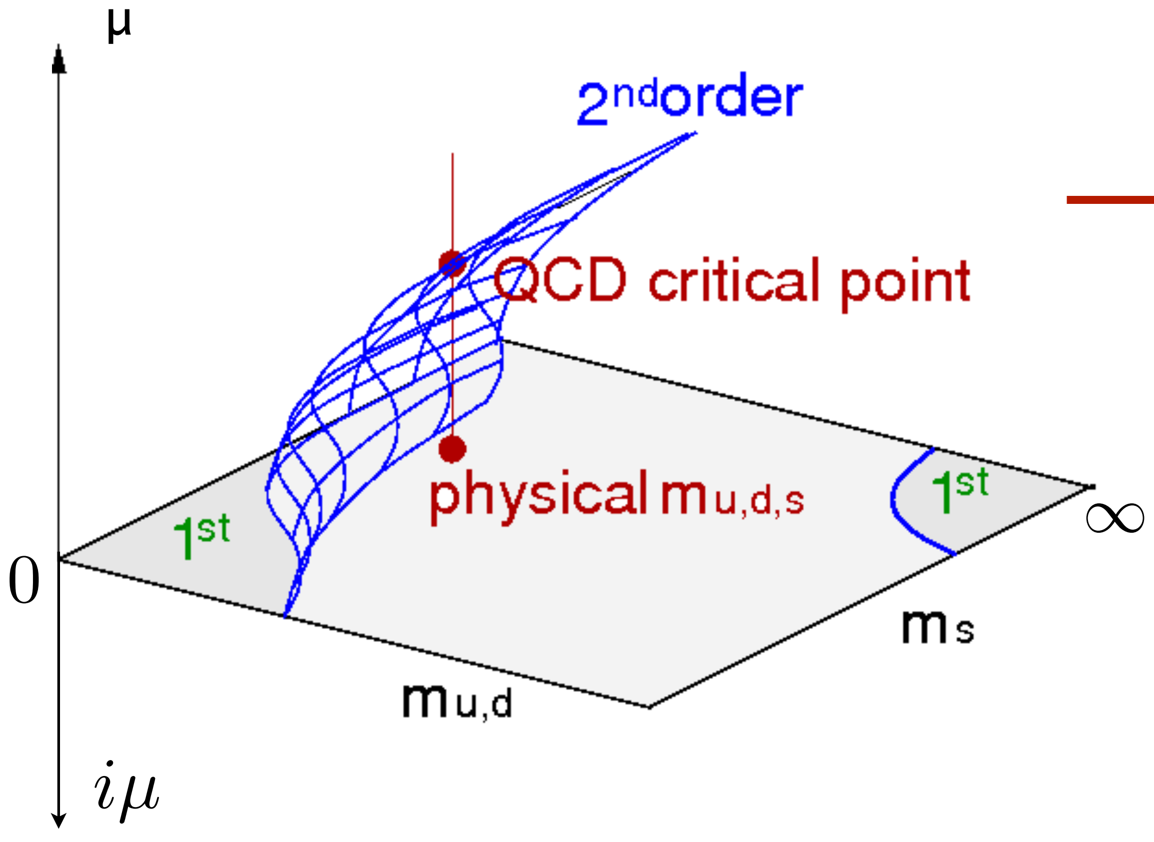

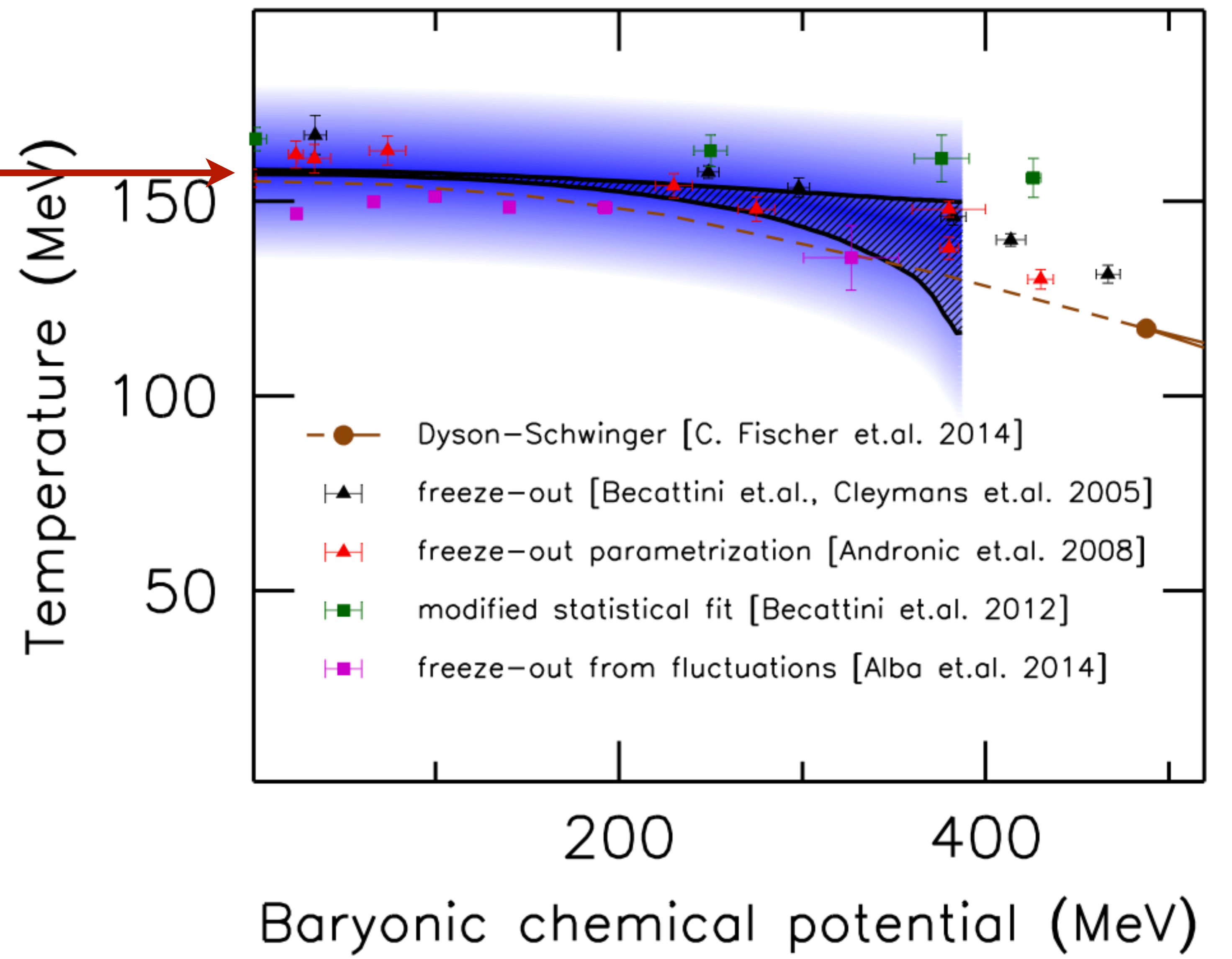

Bellwied, Borsanyi, Fodor, Günther,
Katz, Ratti and Szabo, PLB 751 (2015) 559

HOT-QCD: similar results

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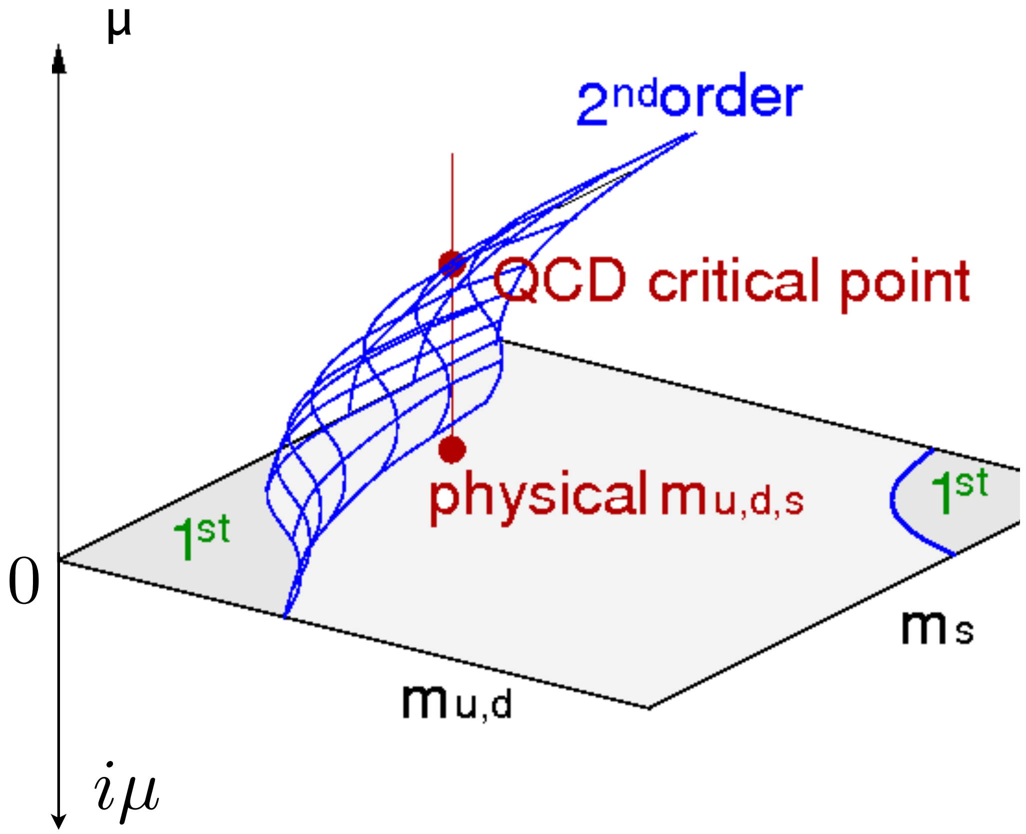

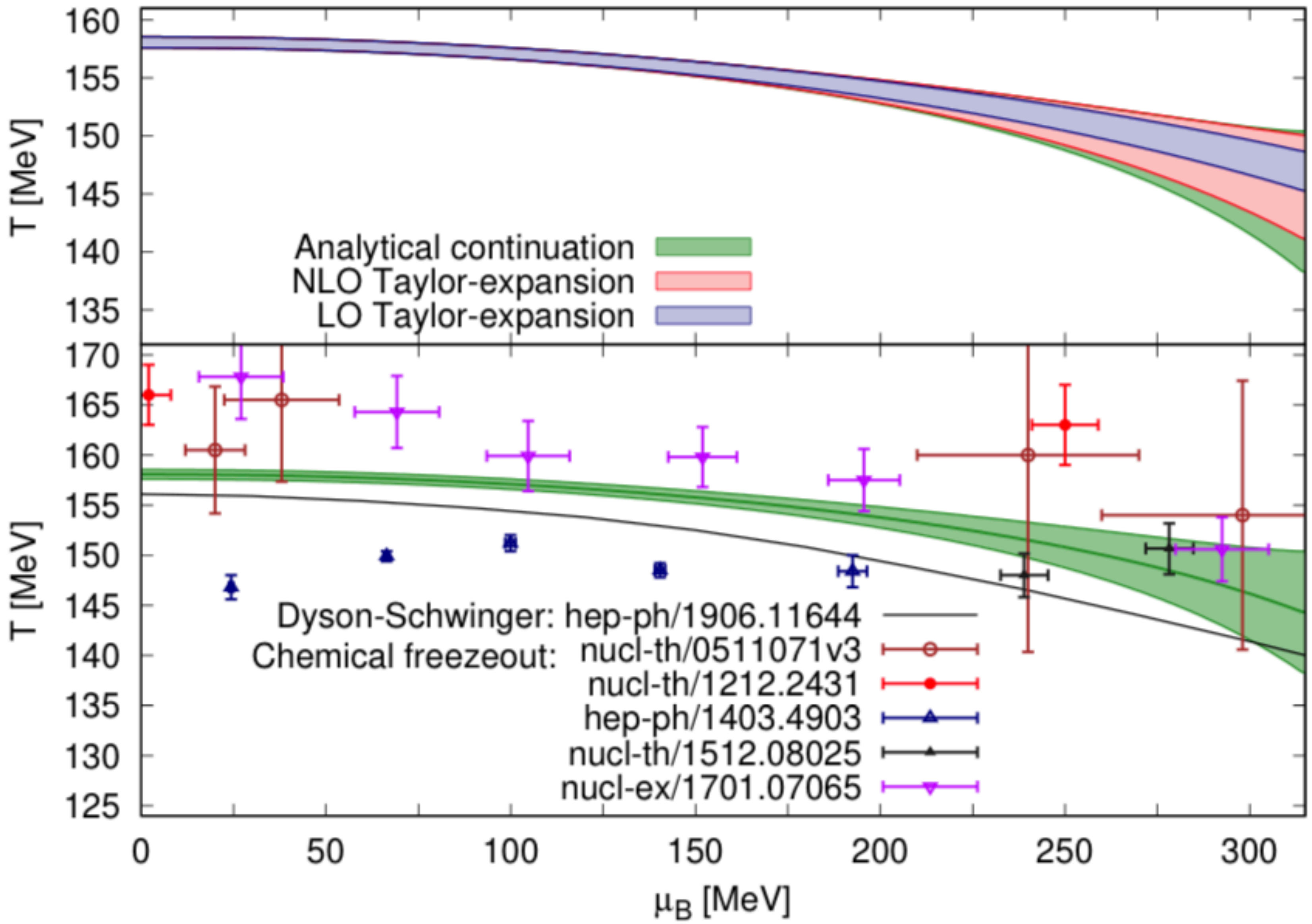

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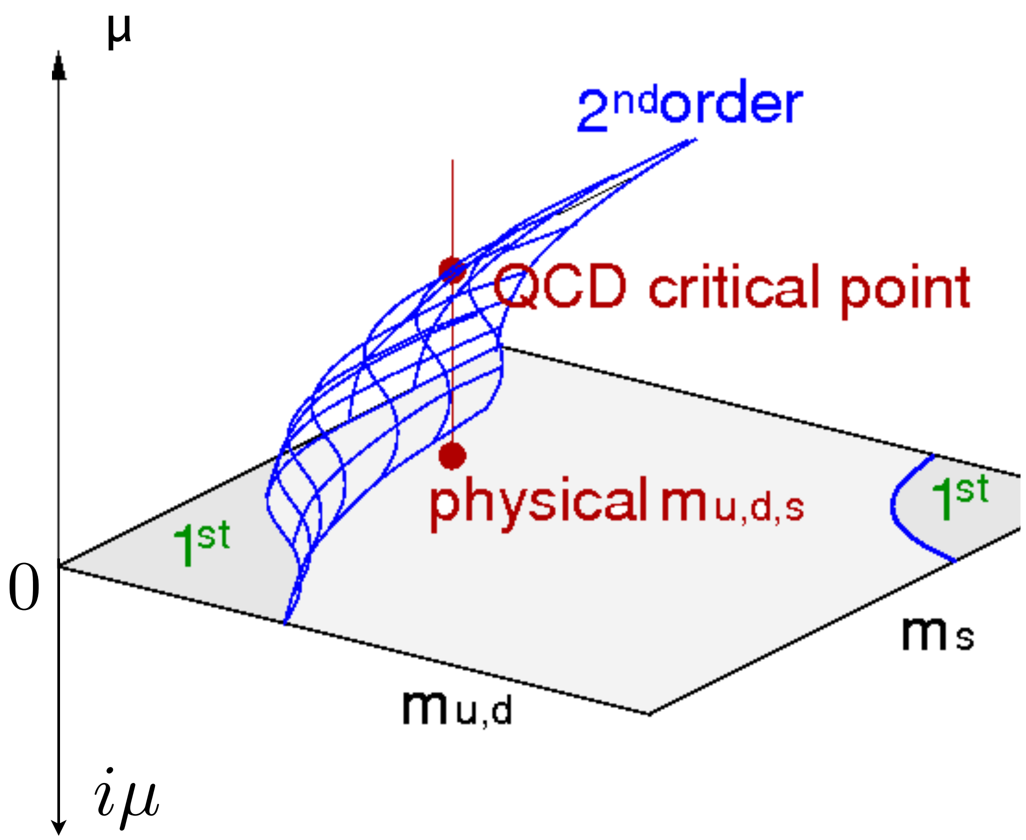
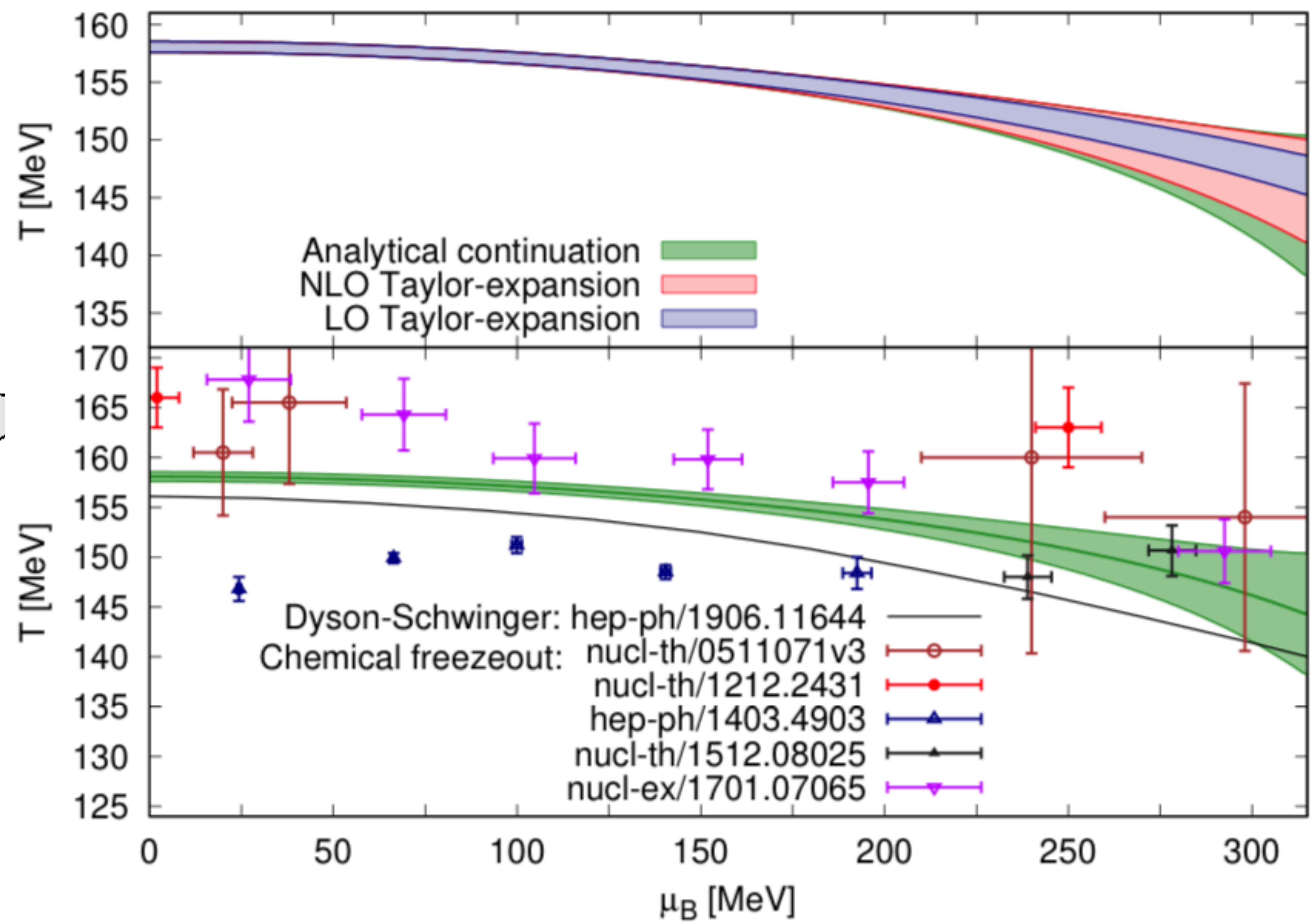
Bellwied, Borsanyi, Fodor, Günther, Katz, Ratti and Szabo, PLB 751 (2015) 559

Borsanyi, Fodor, Günther, Kara, Katz, PRL 125 (2020) no.5, 052001

Chiral transition line from analytic continuation





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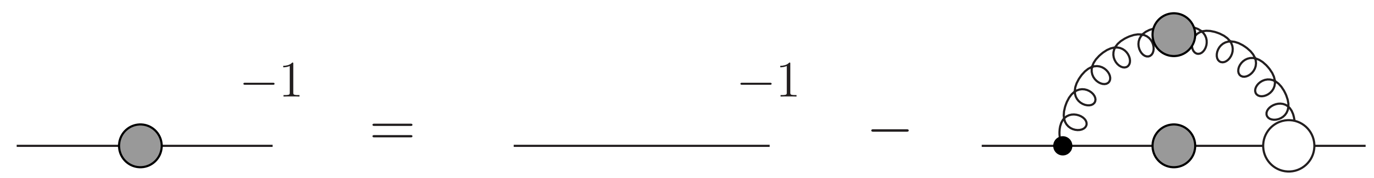
Borsanyi, Fodor, Günther, Kara, Katz, PRL 125 (2020) no.5, 052001

Main result: no CEP for $\mu_B/T < 2\text{-}3$

QCD order parameters from propagators

Chiral order parameter:

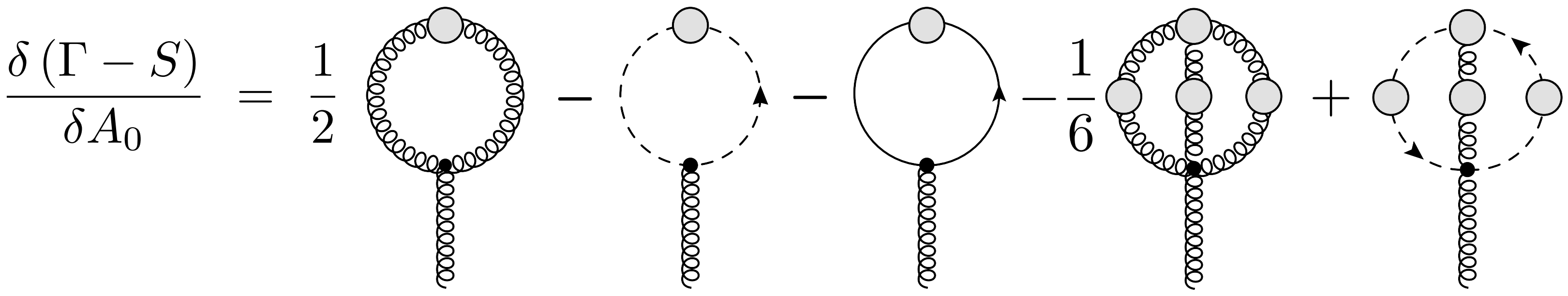
$$\langle\bar{\Psi}\Psi\rangle = Z_2 N_c Tr_D \frac{1}{T}\sum_{\omega}\int\frac{d^3p}{(2\pi)^3}S(\vec{p},\omega)$$



Deconfinement:

- Polyakov loop potential

$$L = \frac{1}{N_c} Tr\, e^{ig\beta A_0}$$



Braun, Gies, Pawlowski, PLB 684, 262 (2010)
Braun, Haas, Marhauser, Pawlowski, PRL 106 (2011)
Fister, Pawlowski, PRD 88 045010 (2013)
CF, Fister, Luecker, Pawlowski, PLB 732 (2013)

The DSE for the quark propagator (finite T,μ)

Two strategies: I. use T-dependent model for gluon

Qin, Chang, Chen, Liu and Roberts, PRL 106 (2011) 172301
Y. Jiang, L.-J. Luo, and H.-S. Zong, JHEP 02 (2011) 066
Gutierrez, Ahmad, Ayala, Bashir and Raya, JPG 41 (2014) 075002
C. Shi, Y.-L. Du, S.-S. Xu, X.-J. Liu, and H.-S. Zong, PRD93 (2016) no. 3, 036006
F. Gao and Y.-x. Liu, PRD 94 (2016) no. 7, 076009
F. Gao and Y.-x. Liu, PRD 94 (2016) no. 9, 094030

→ valuable for exploratory studies

→ not good enough for quantitative and/or systematic studies

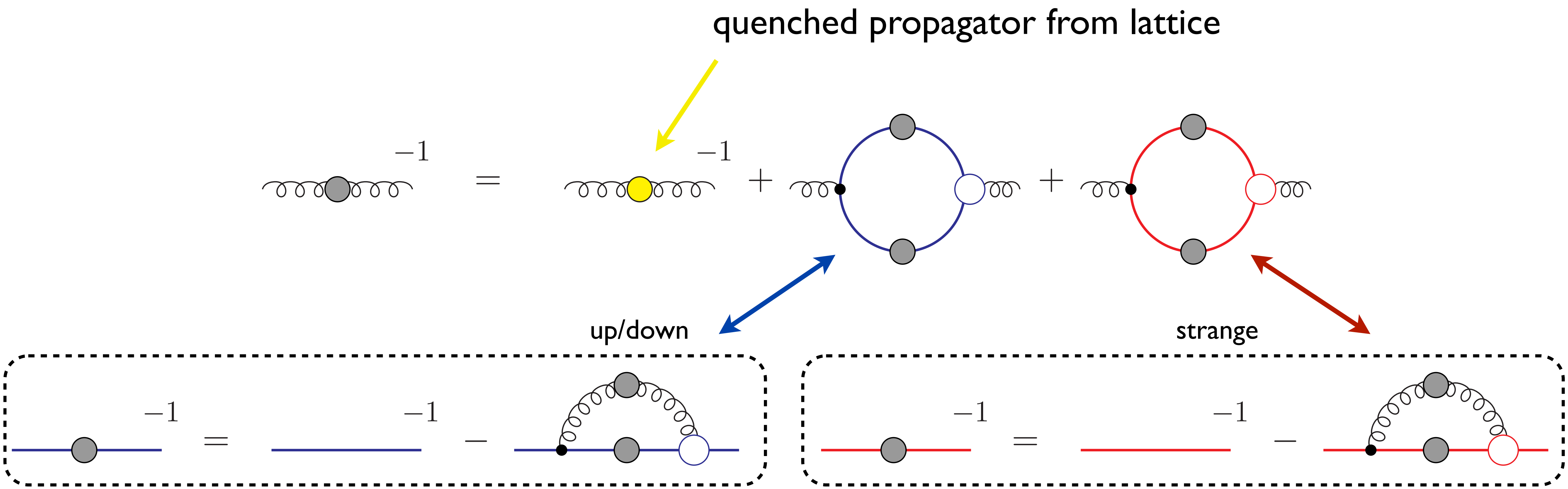
II. treat Yang-Mills sector explicitly

CF, Luecker, Welzbacher, PRD 90 (2014) 034022
Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011
Gao and Pawlowski, PRD 102 (2020) no.3, 034027
Gao and Pawlowski, arXiv:2010.13705 [hep-ph].

SU(2),G(2): Contant and Huber, PRD 96 (2017) no.7, 074002

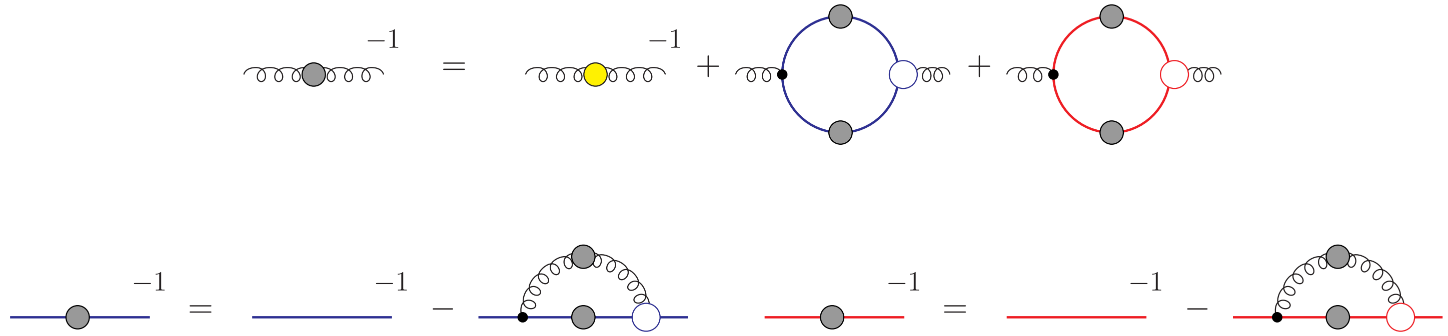
Review: CF, PPNP 105 (2019)

N_f=2+1-QCD with DSEs



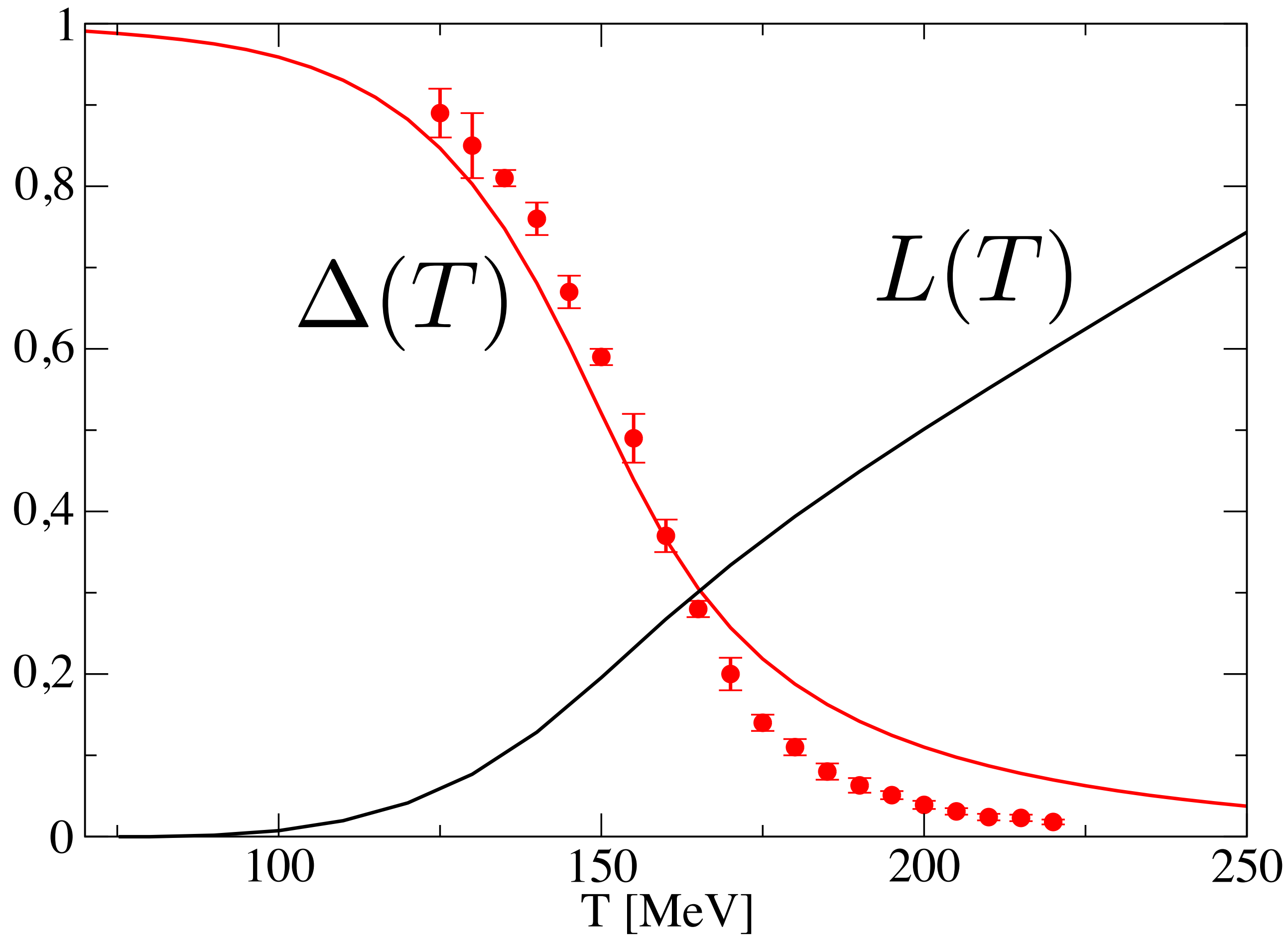
- allows for systematic variation of $m_{u/d}$ and m_s
- quark-gluon vertex:
  ansatz built along STI and known UV/IR behavior
  → T,μ,m-dependent
- $T_c(\mu=0)$ is not predicted, but input

N_f=2+1, zero chemical potential



$$\Delta(T) \sim M(T)$$

$$L(T) \sim \exp(-E/T)$$



Lattice: Borsanyi *et al.* [Wuppertal-Budapest], JHEP 1009(2010) 073
DSE: CF, Luecker, PLB 718 (2013) 1036,
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

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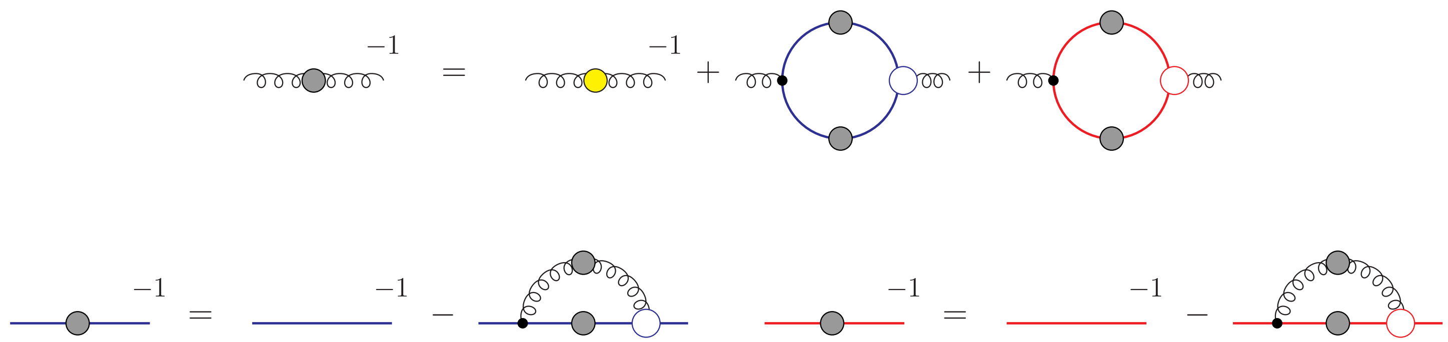

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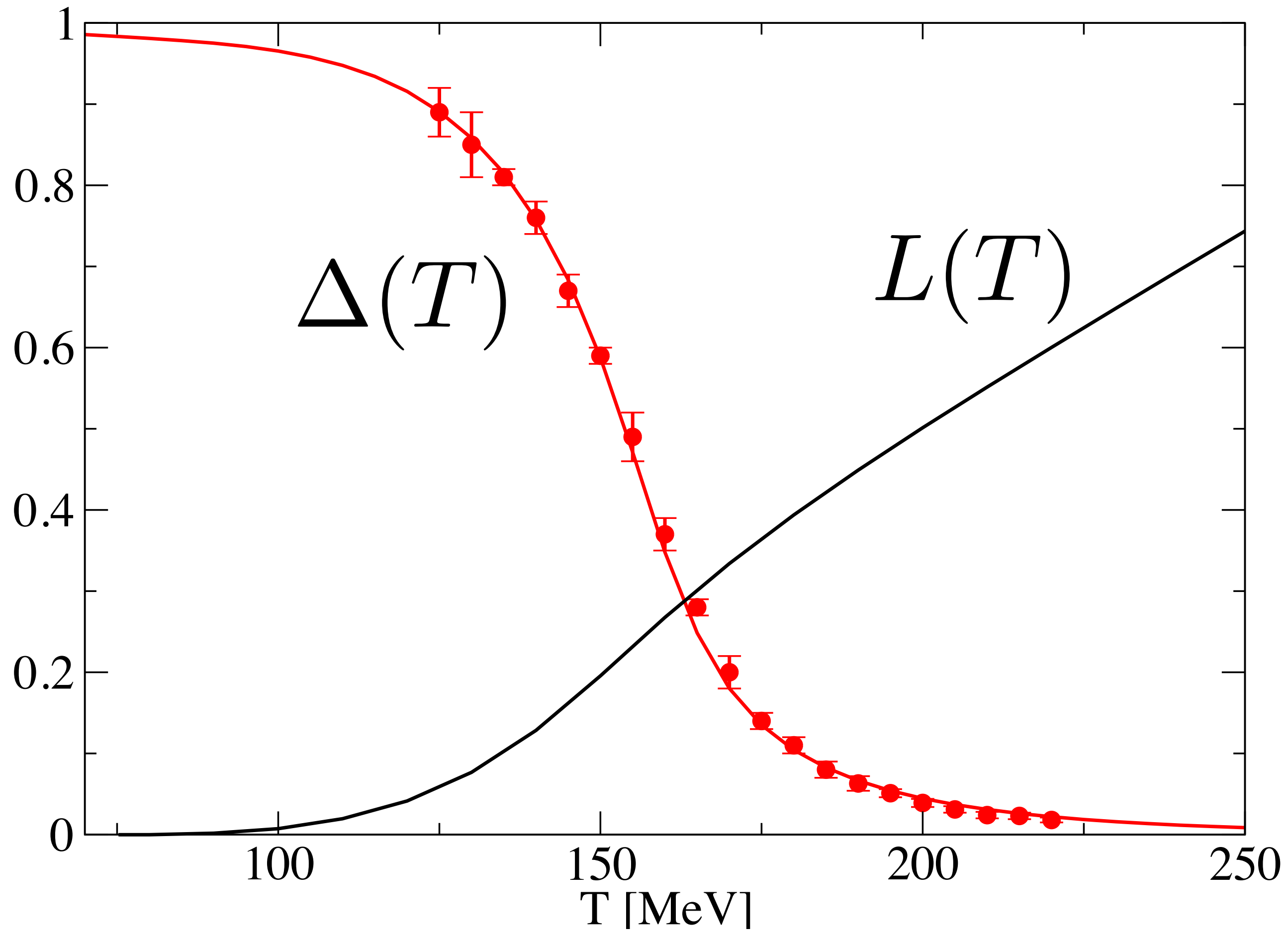

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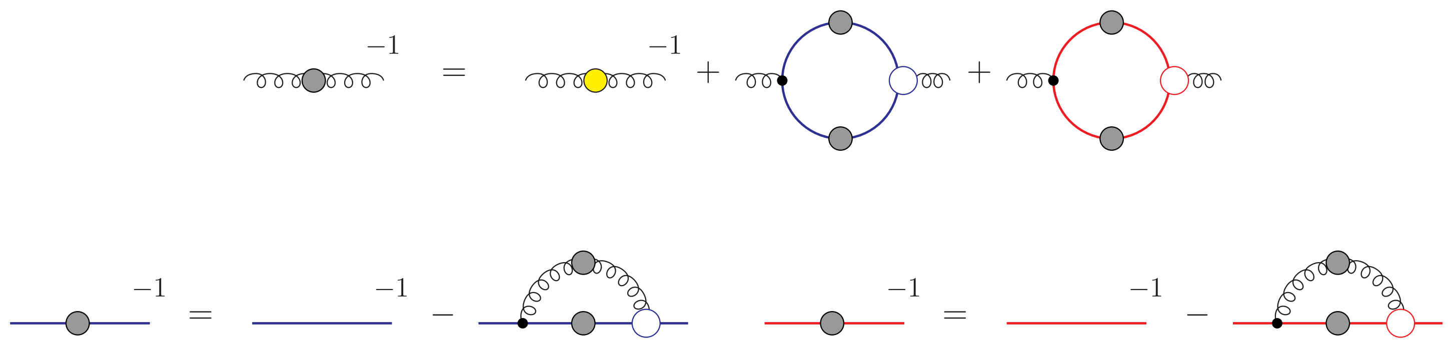

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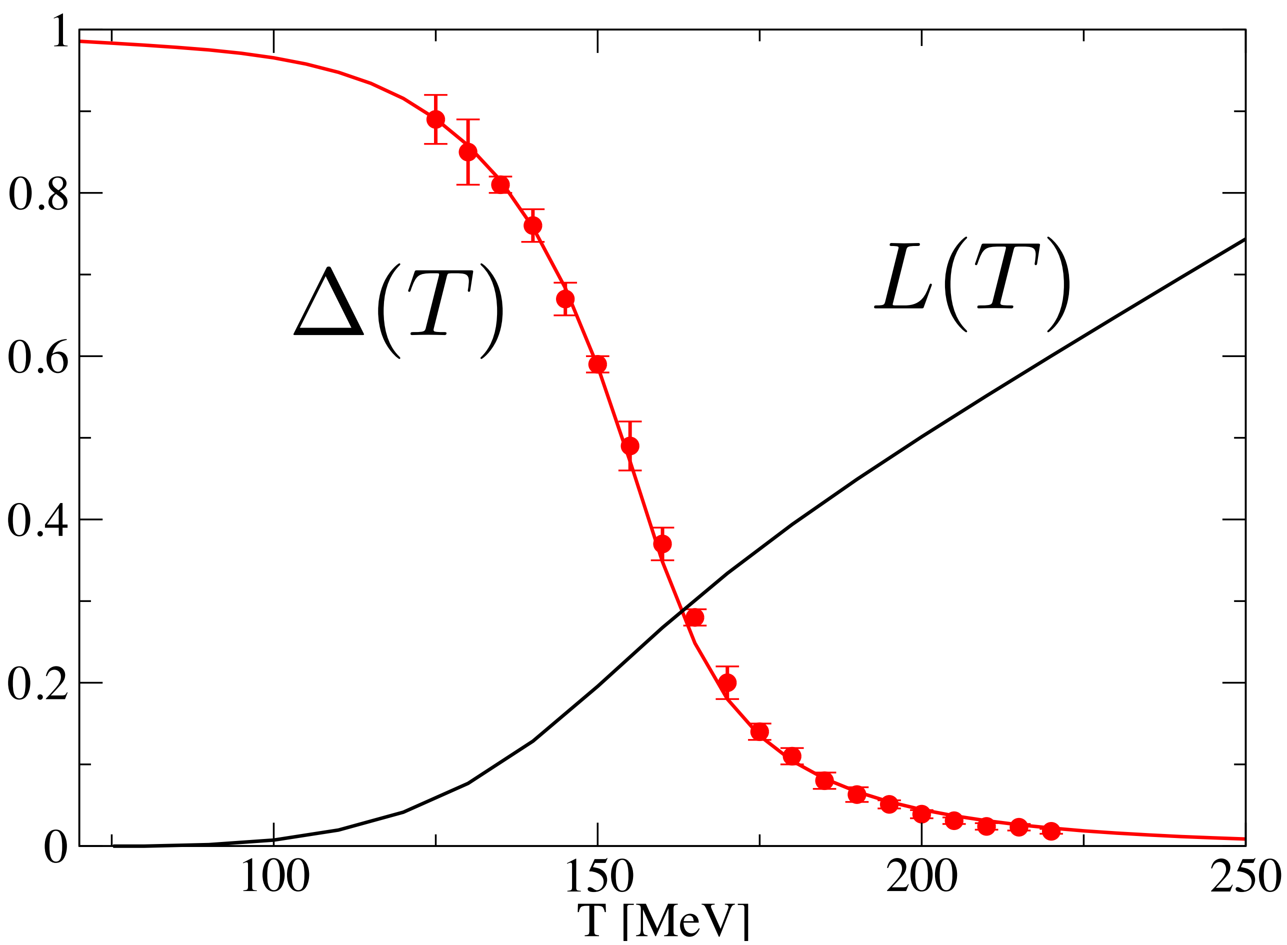

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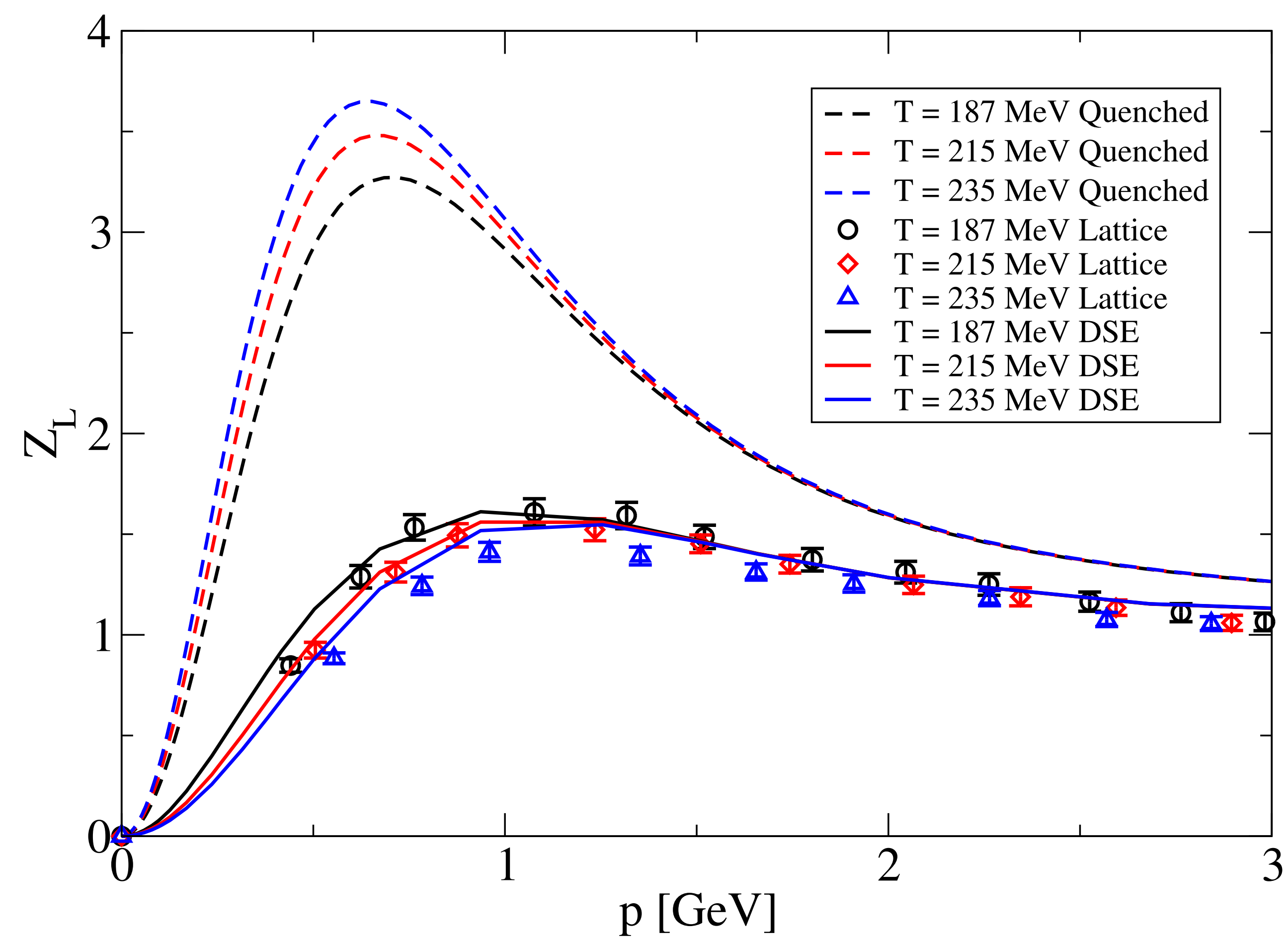
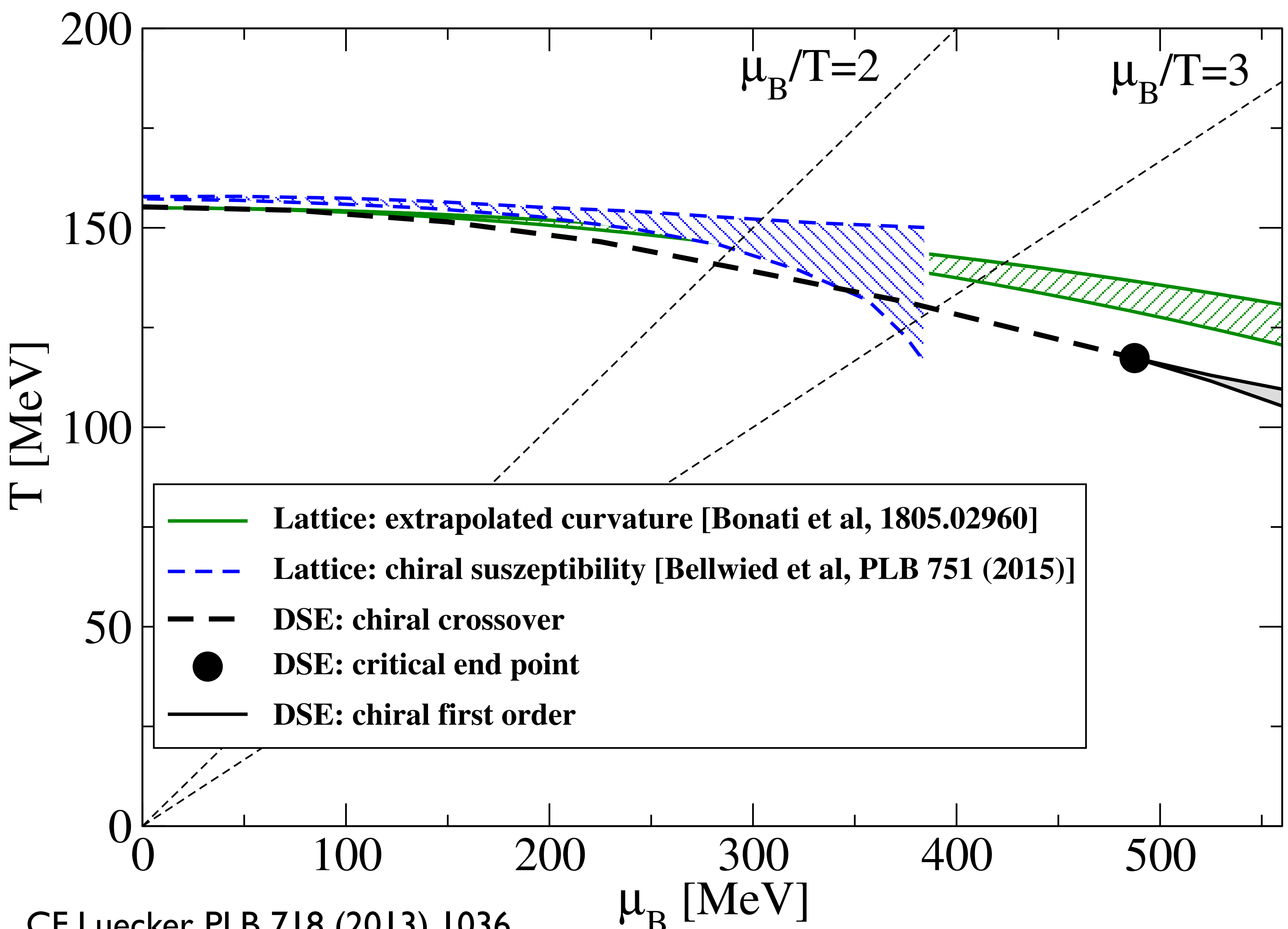


Lattice: Aouane, et al.PRD D87 (2013), [arXiv:1212.1102]
DSE: CF, Luecker, PLB 718 (2013) 1036, [arXiv:1206.5191]

- quantitative agreement: DSE prediction verified by lattice

QCD phase diagram and heavy ion collisions



CF, Luecker, PLB 718 (2013) 1036,
CF, Fister, Luecker, Pawlowski, PLB 732 (2014) 273
CF, Luecker, Welzbacher, PRD 90 (2014) 034022

- combined lattice/DSE evidence: no CEP at $\mu_B/T < 2$-3
- CEP @ $(T, \mu_B)=(117,488)$ MeV
- Yang-Mills sector crucial !

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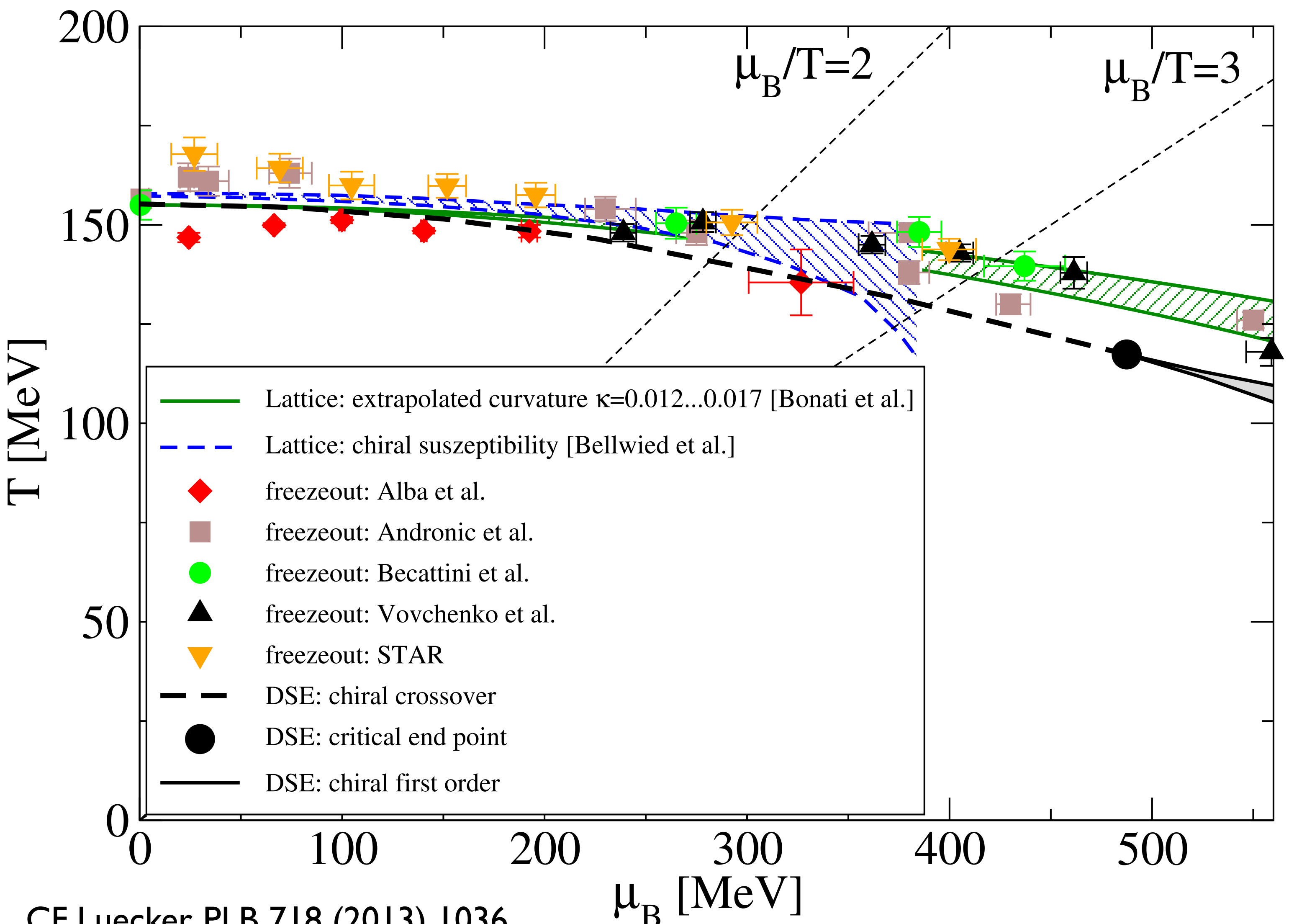

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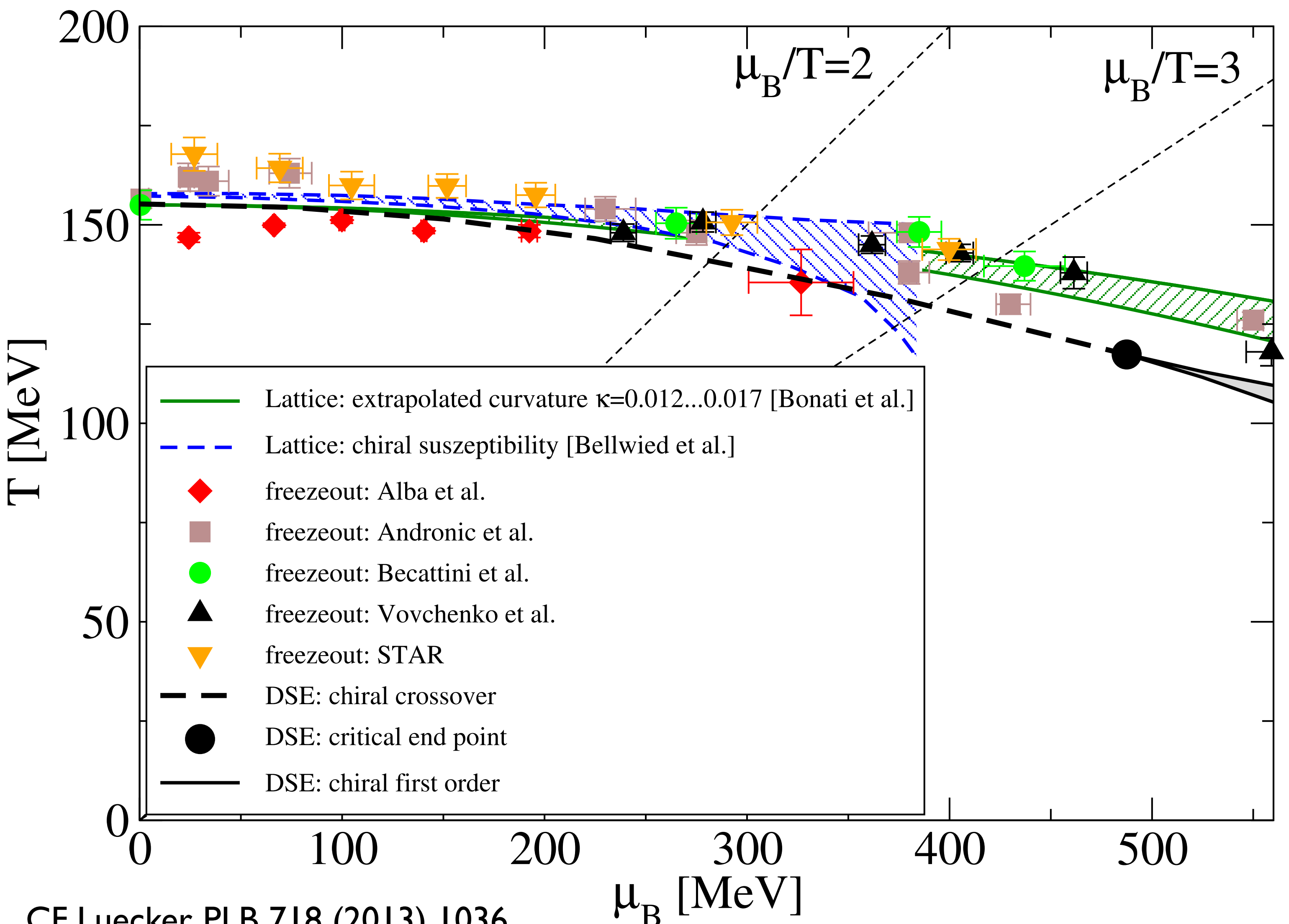

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heavy ion collisions

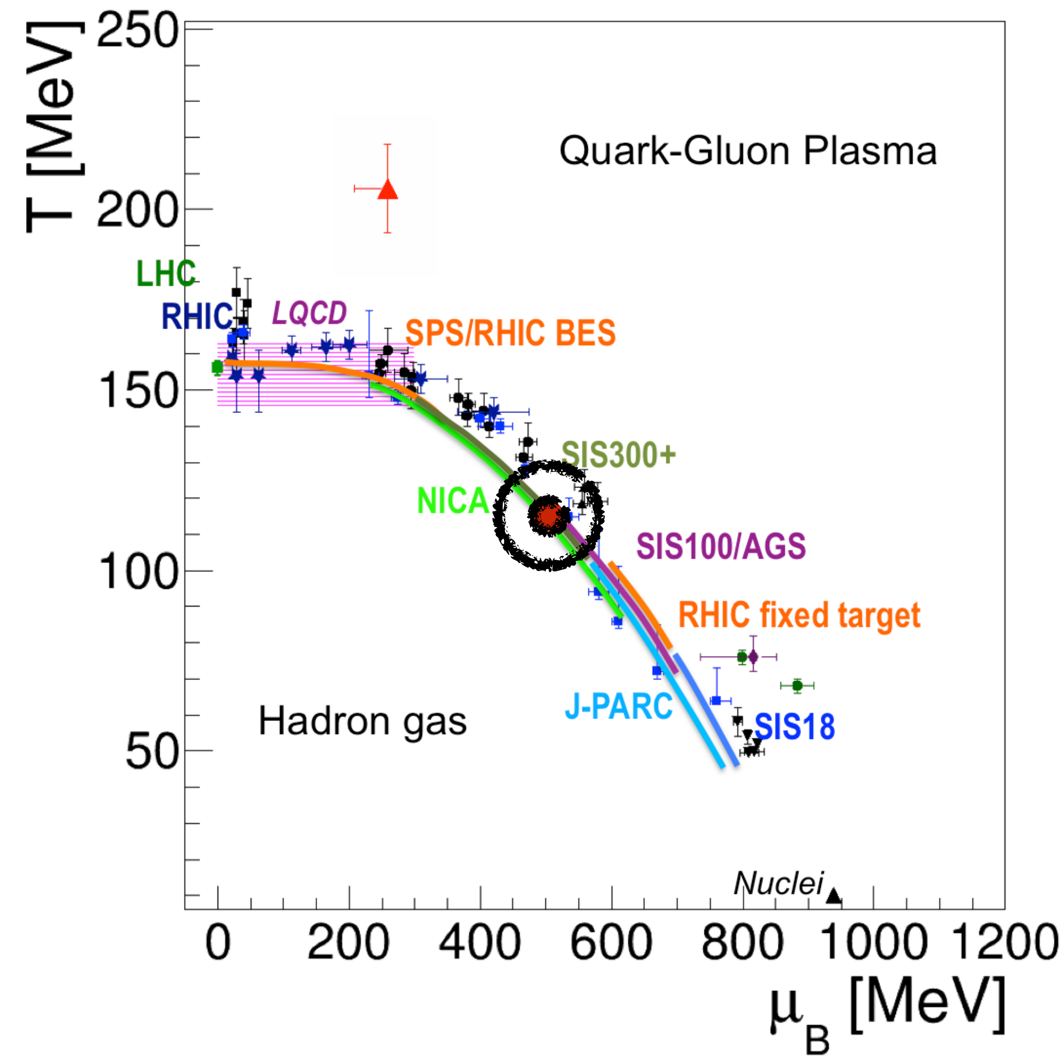

Figure adapted from talk of T. Galatyuk, Erice 2016

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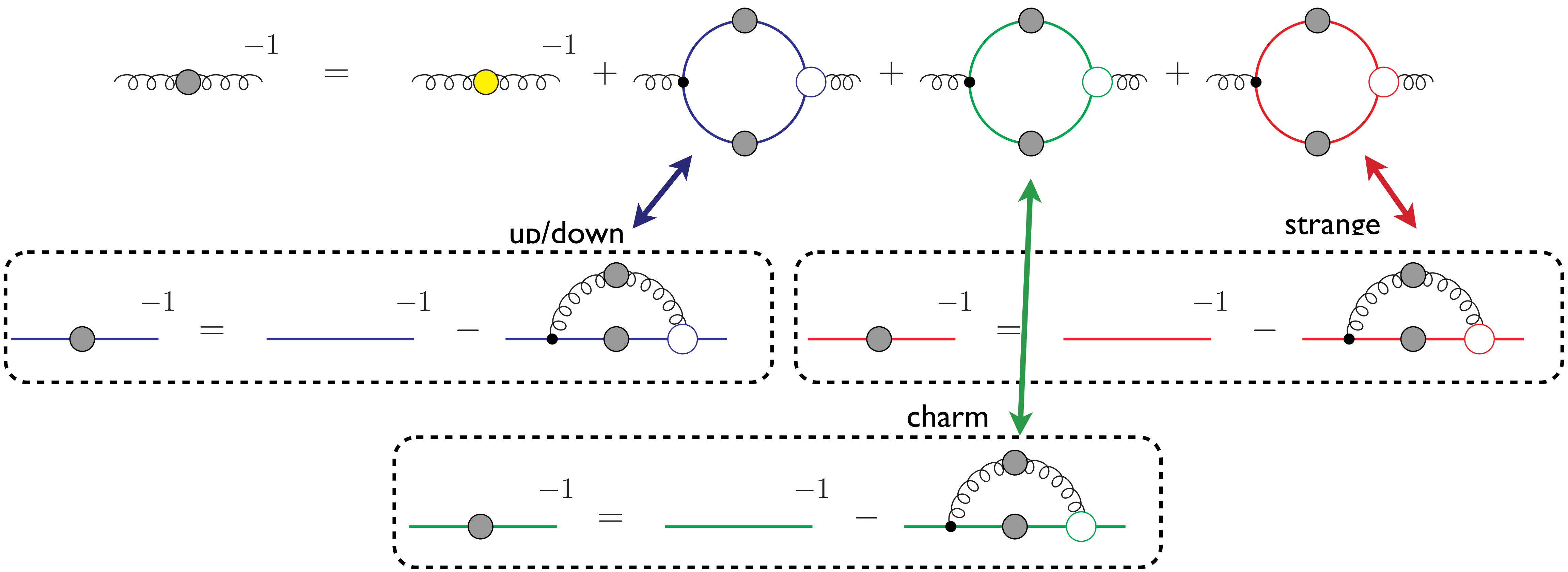
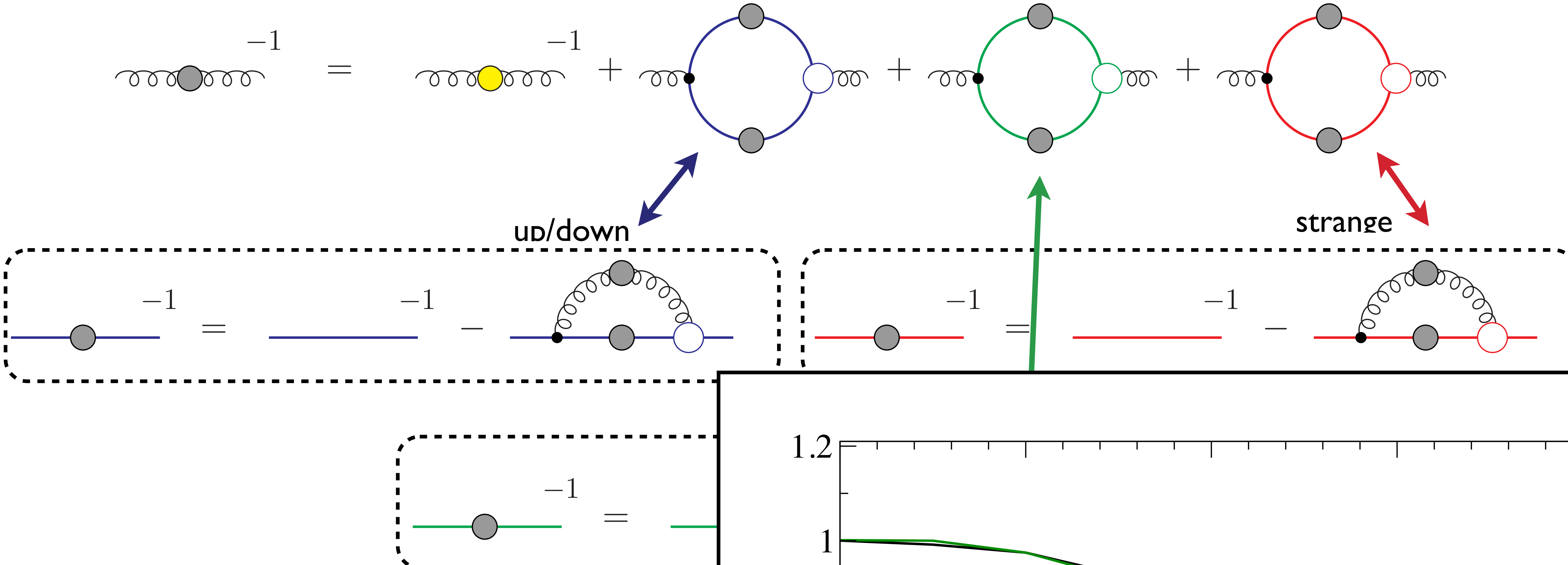
Nf=2+1+1: effects of charm



- Physical up/down, strange and charm quark masses
- Transition controlled by chiral dynamics
- *no lattice or model results available yet*

Nf=2+1+1: effects of charm



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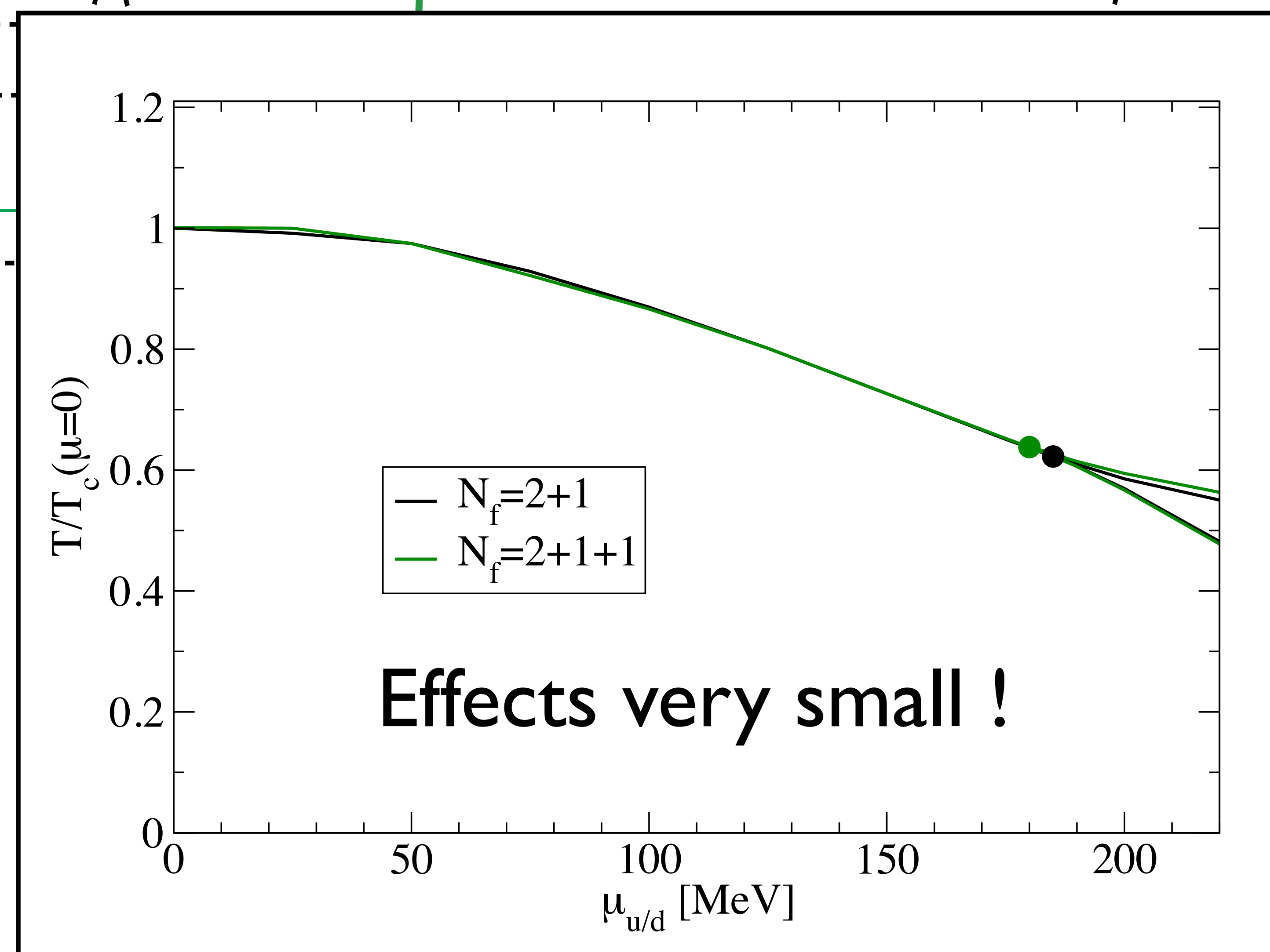

CF, Luecker, Welzbacher, PRD 90 (2014) 034022

Overview

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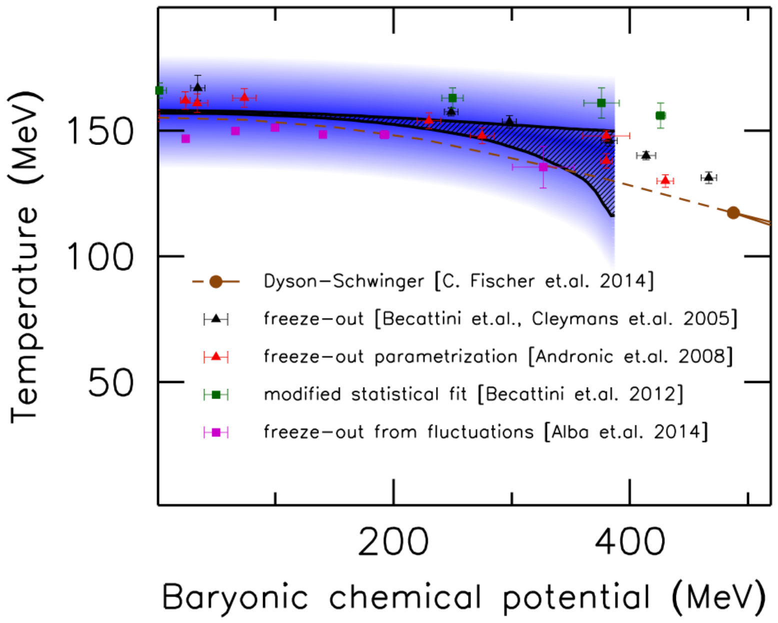

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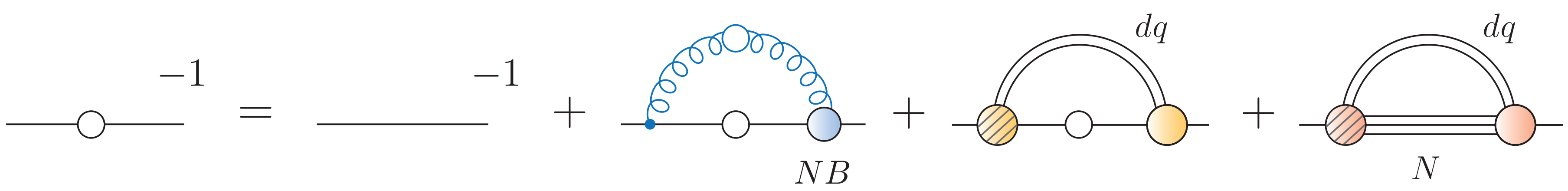
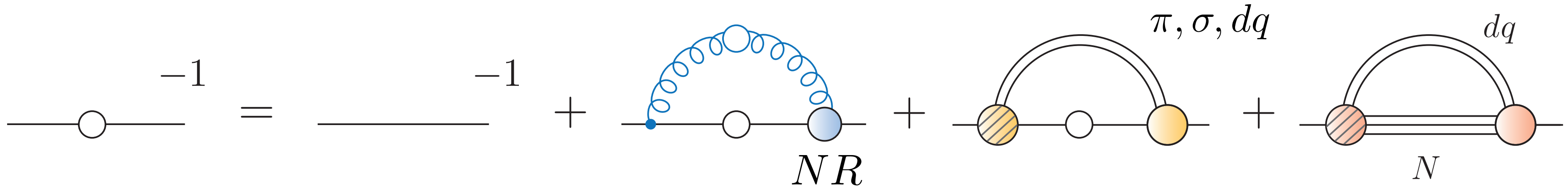
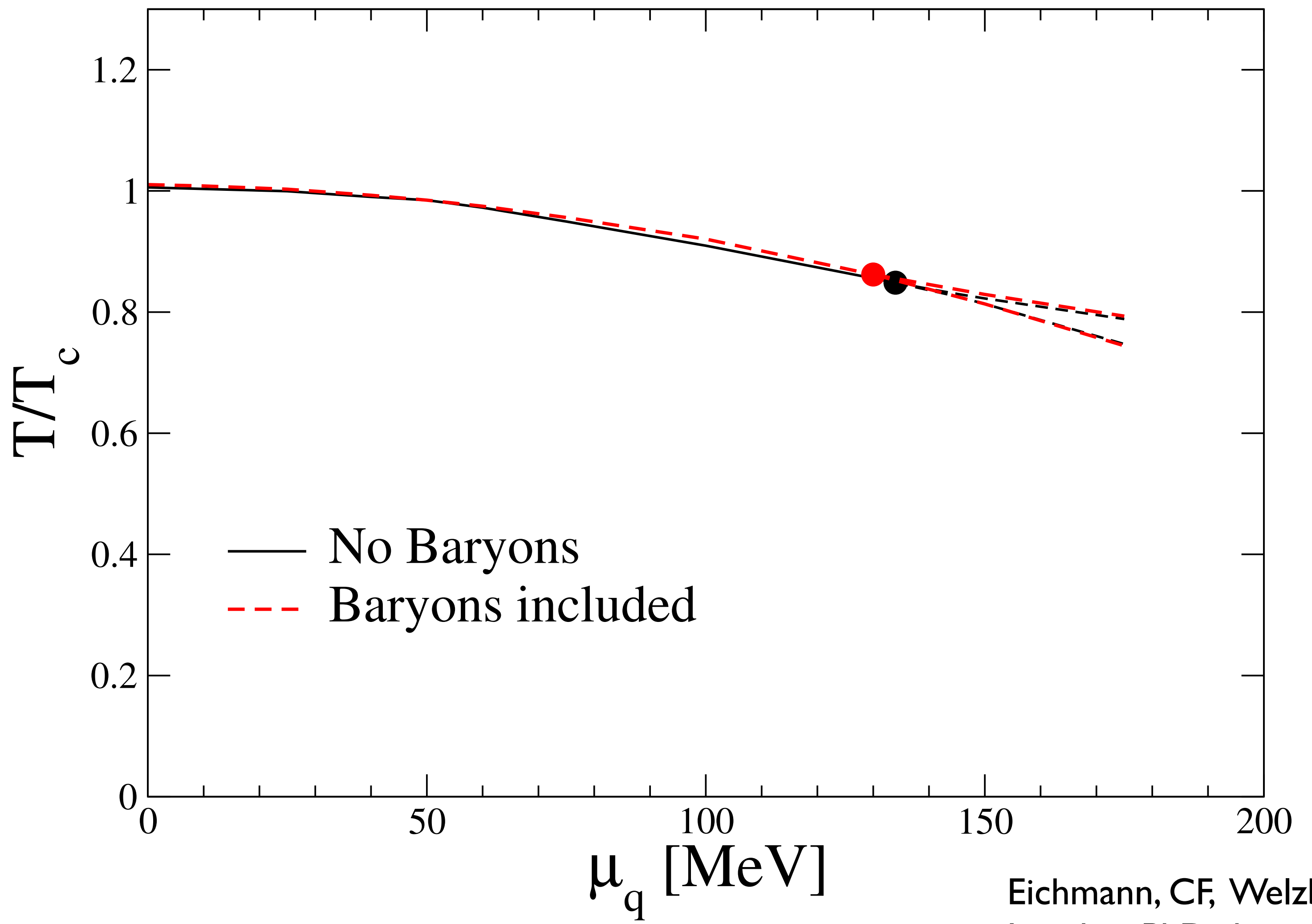


3. Fluctuations

Baryon effects on the CEP - results (N_f=2)





Eichmann, CF, Welzbacher, PRD93 (2016) [1509.02082]
Luecker, PhD-thesis, JLU Giessen, 2013

- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)

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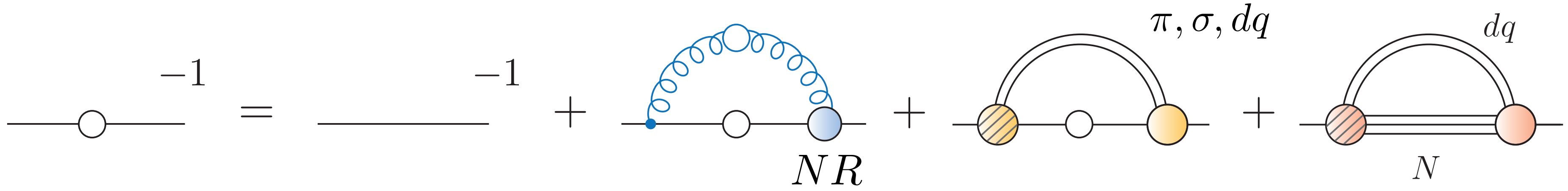

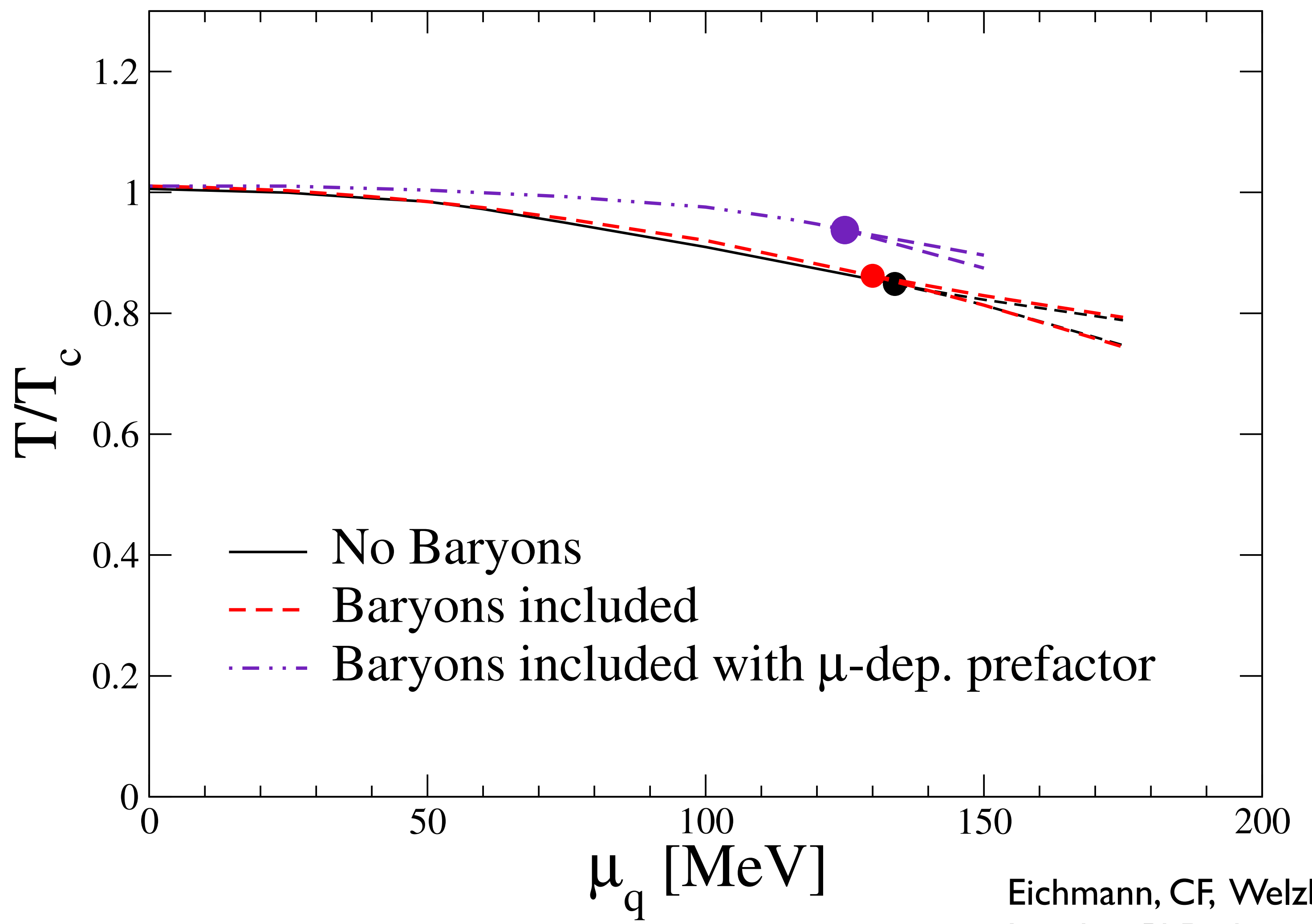

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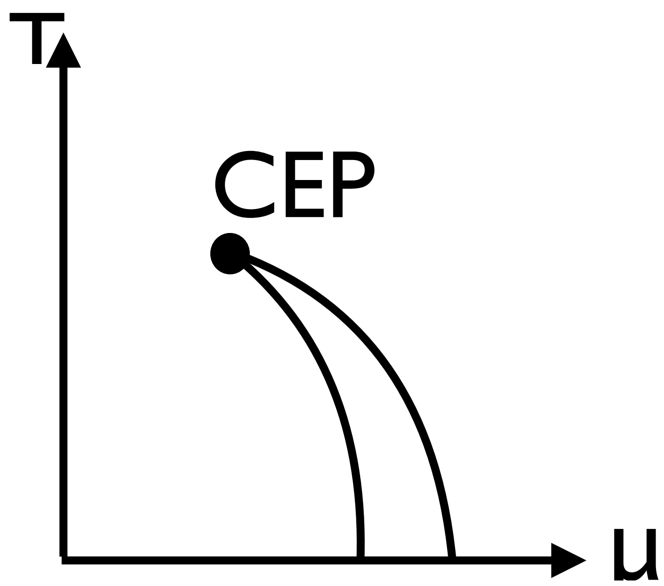
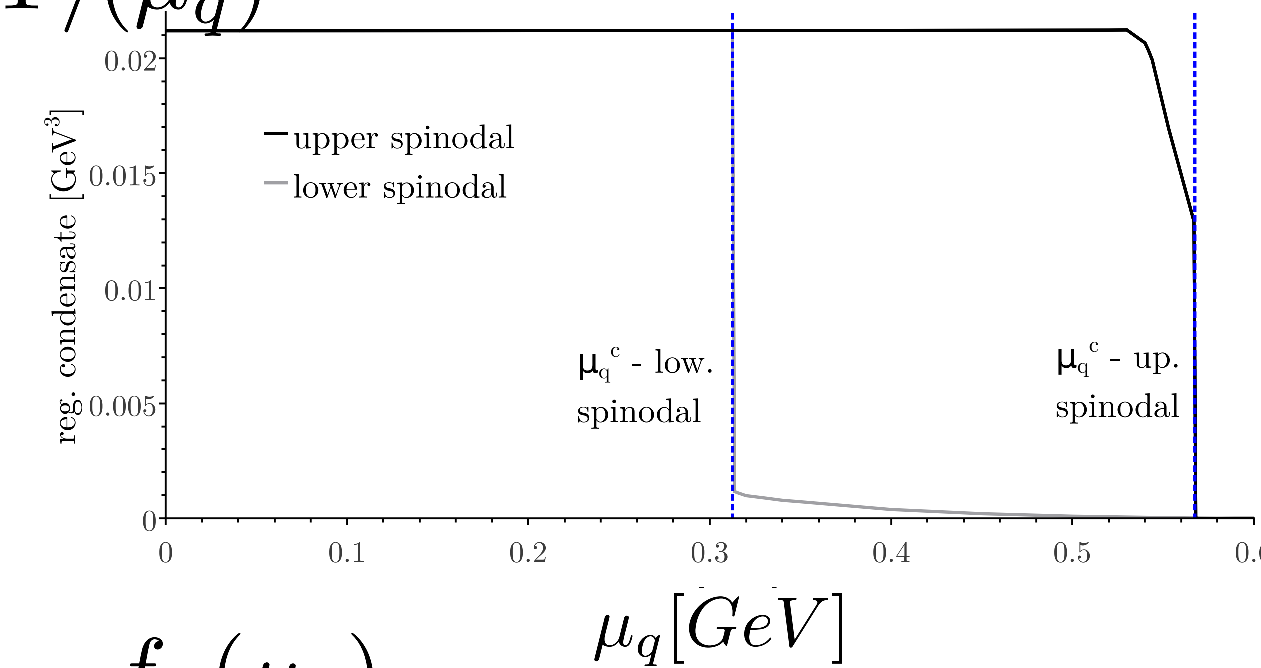
- Zero chemical potential: no effect
- almost no effect on location of CEP (mesons: similar)
- But: strong μ-dependence of baryon wave function may change situation…

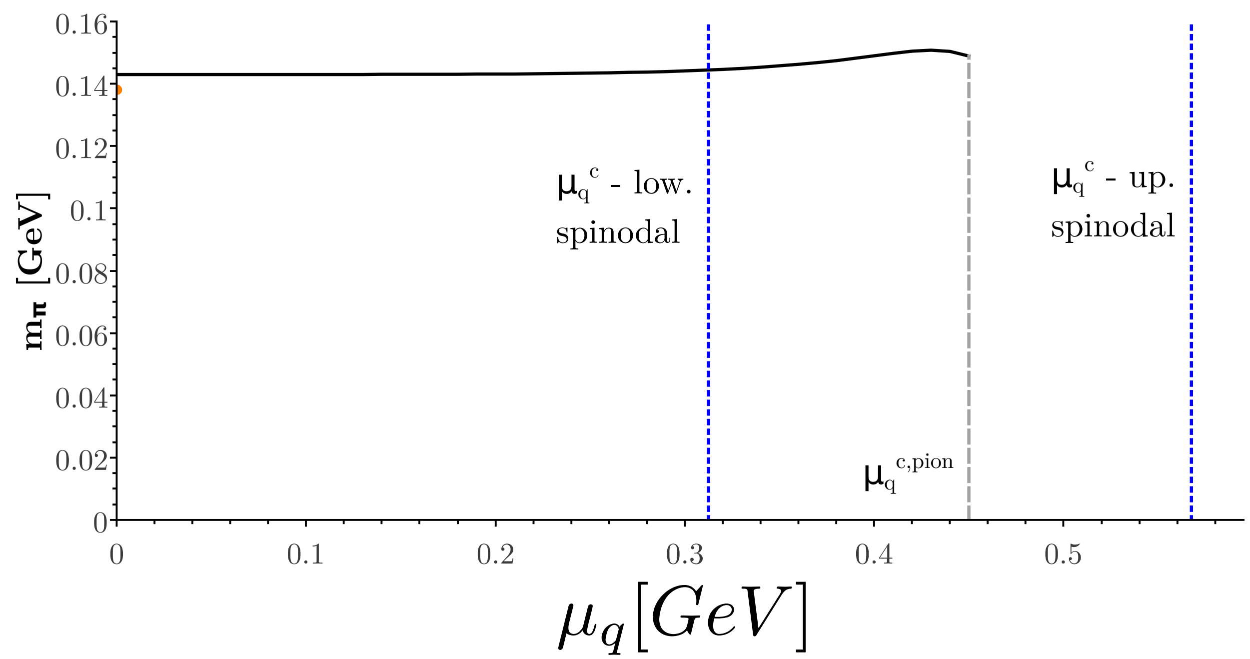
Meson properties at finite chemical potential



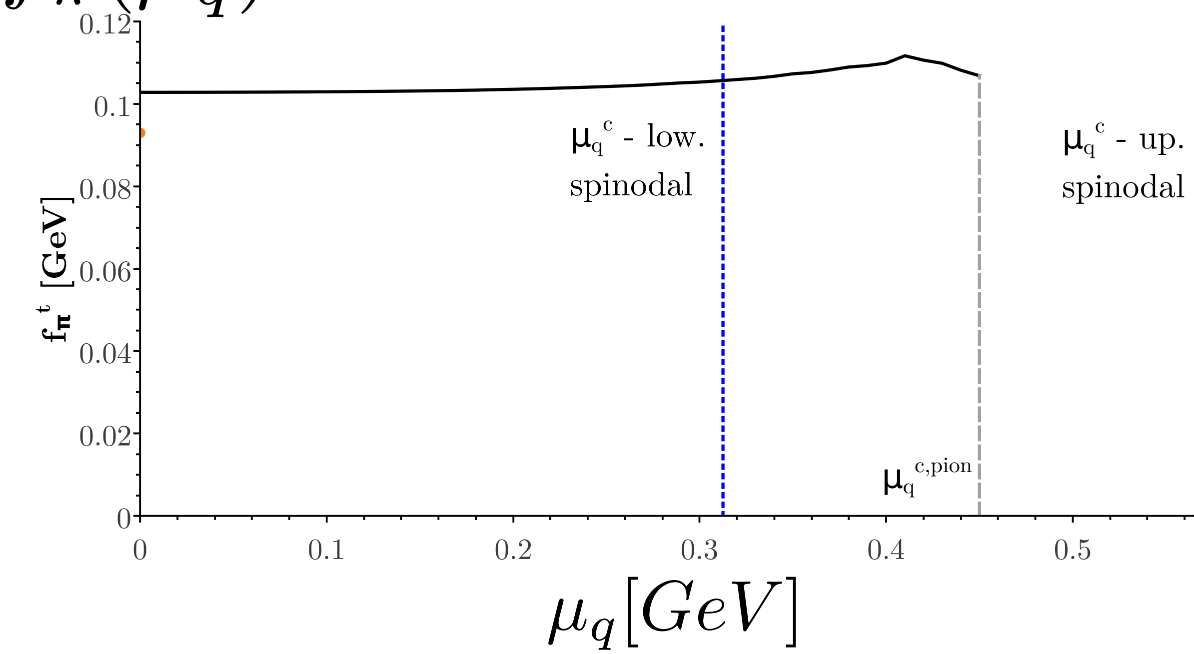
$\langle\bar{\Psi}\Psi\rangle(\mu_q)$



$m_\pi(\mu_q)$



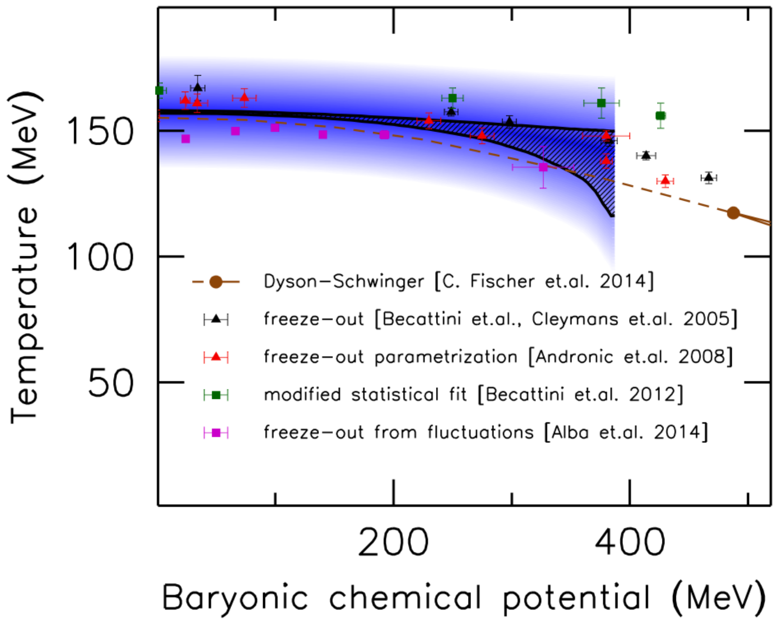
$f_\pi(\mu_q)$



- Silver blaze satisfied T. D. Cohen, PRL 91 , 222001 (2003)
- But wave functions do change !

Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169

Overview



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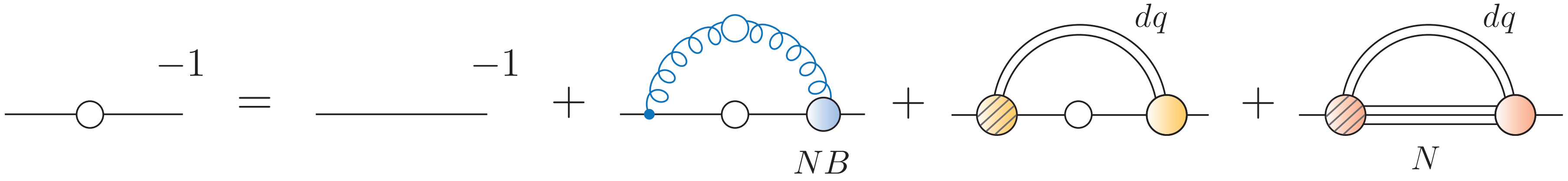


3. Fluctuations

Contact with experiment: fluctuations

X.~Luo and N.~Xu, Nucl. Sci. Tech. 28 (2017) no.8, 112 [arXiv:1701.02105 [nucl-ex]].

Quark chemical potentials related to those of conserved charges:

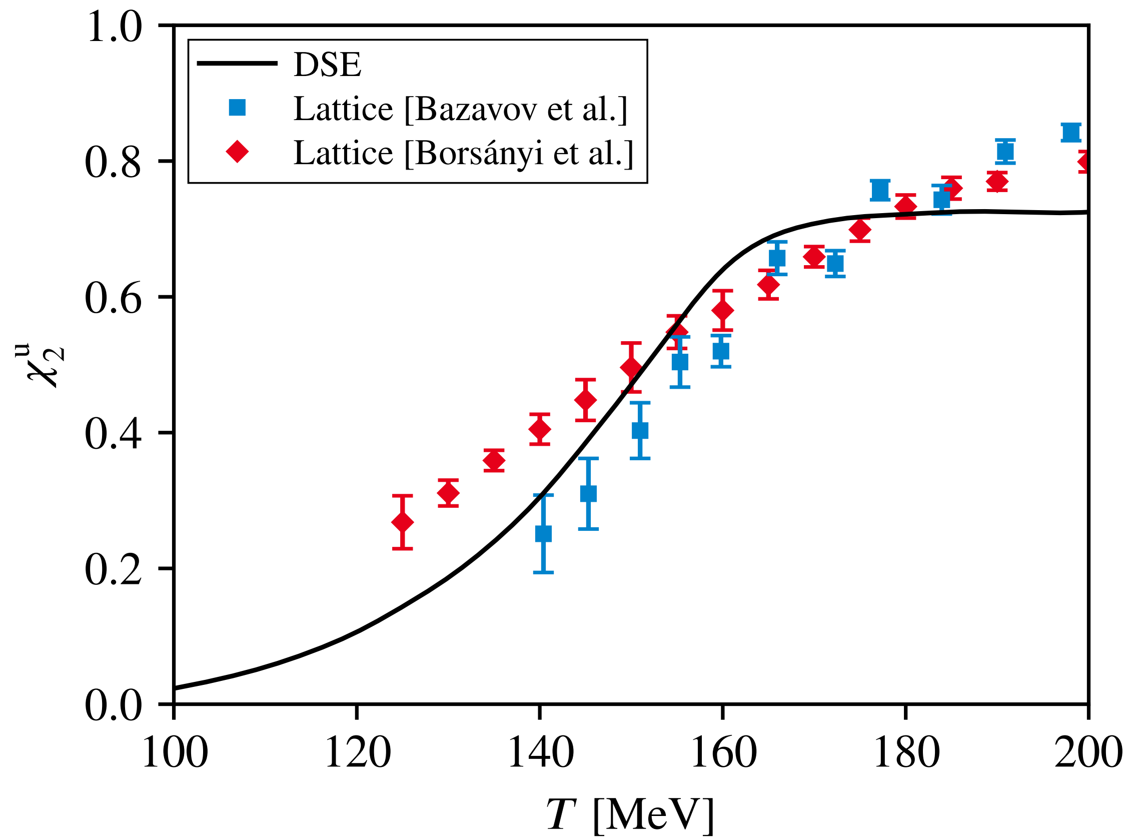
$$\mu_u = \mu_B/3 + 2\mu_Q/3$$
$$\mu_d = \mu_B/3 - \mu_Q/3$$
$$\mu_s = \mu_B/3 - \mu_Q/3 - \mu_S$$

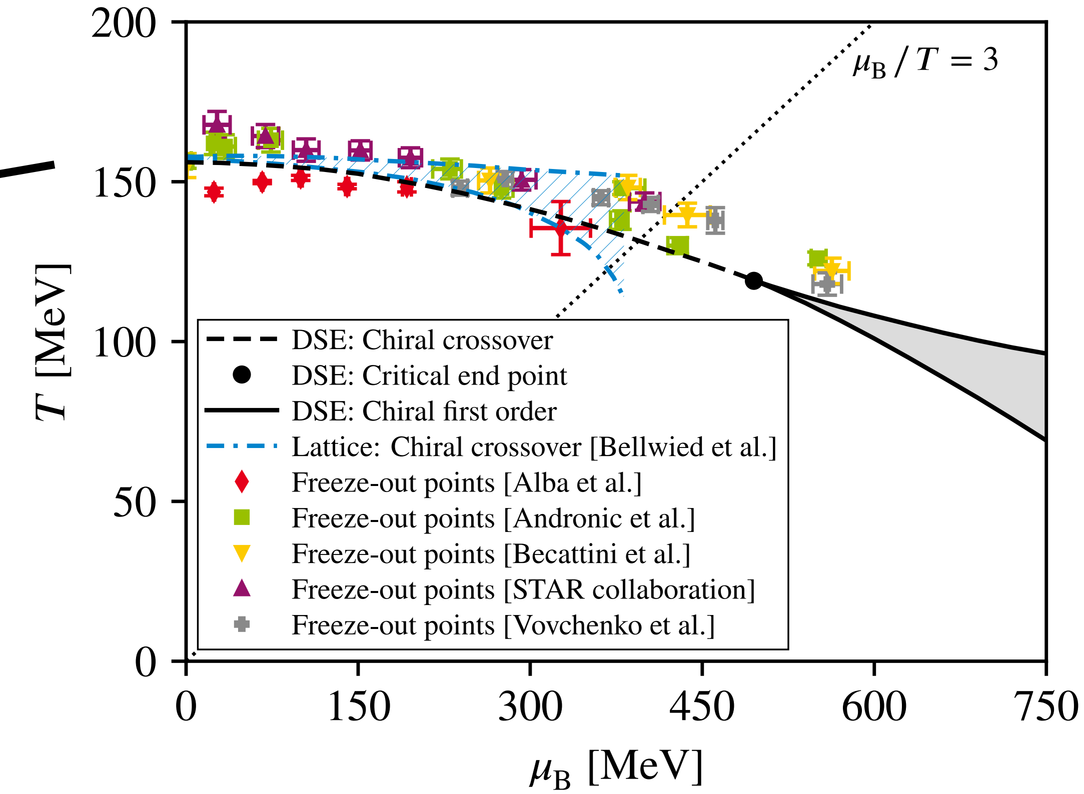
Serve to calculate susceptibilities:

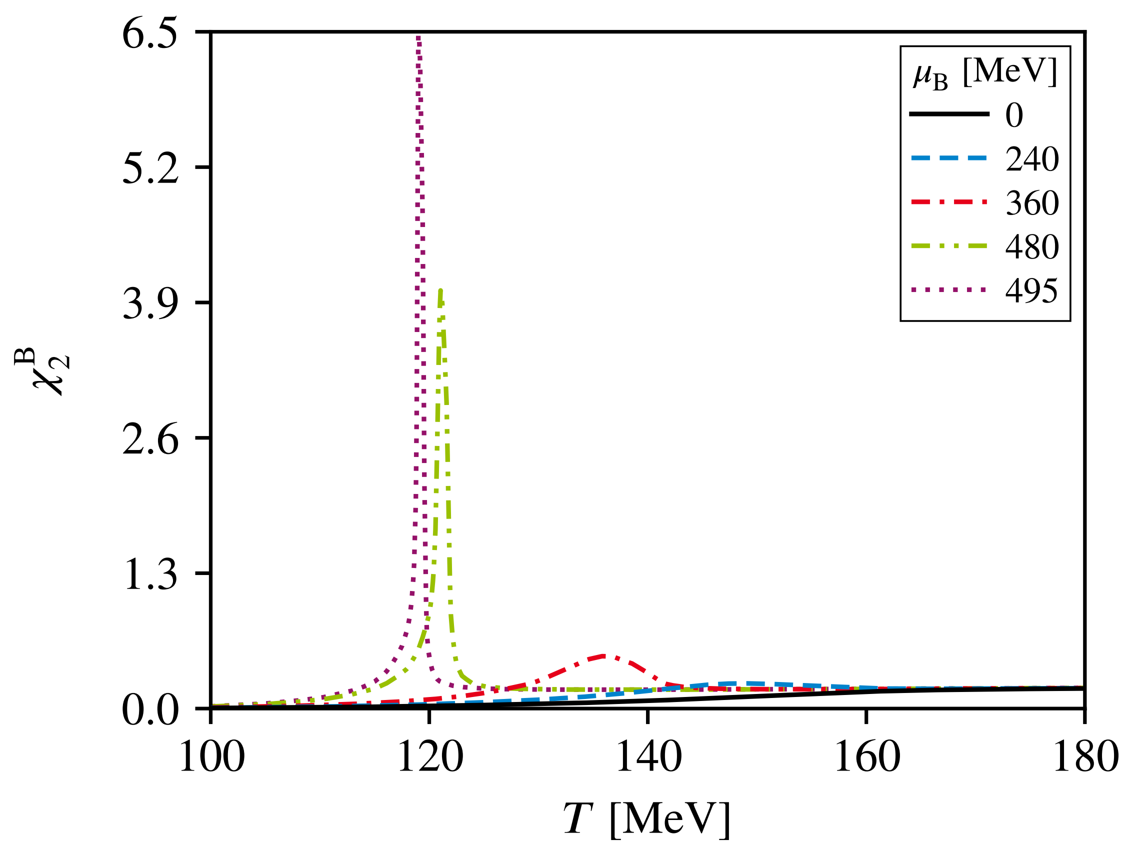
$$\chi^{BSQ}_{lmn} = \frac{\partial^{l+m+n}(p/T^4)}{\partial(\mu_B/T)^l \partial(\mu_S/T)^m \partial(\mu_Q/T)^n}$$

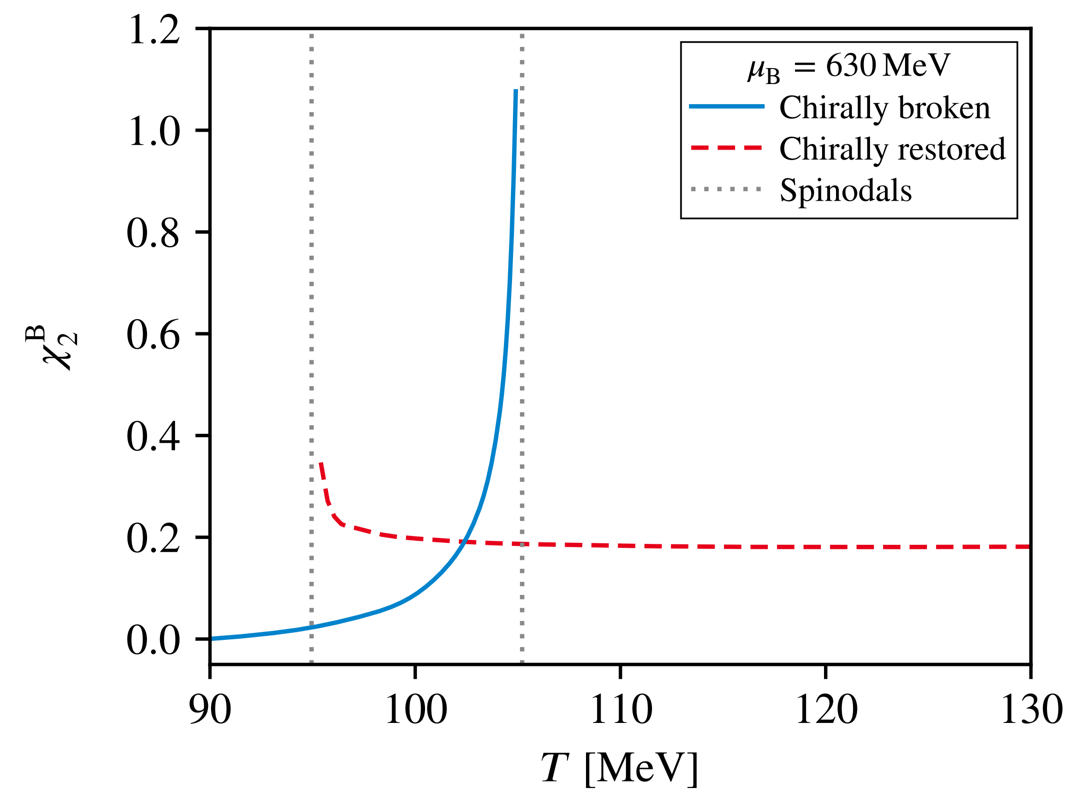
Related to cumulants, which can be extracted from experiment:

$$C^{BSQ}_{lmn} = VT^3\chi^{BSQ}_{lmn}$$



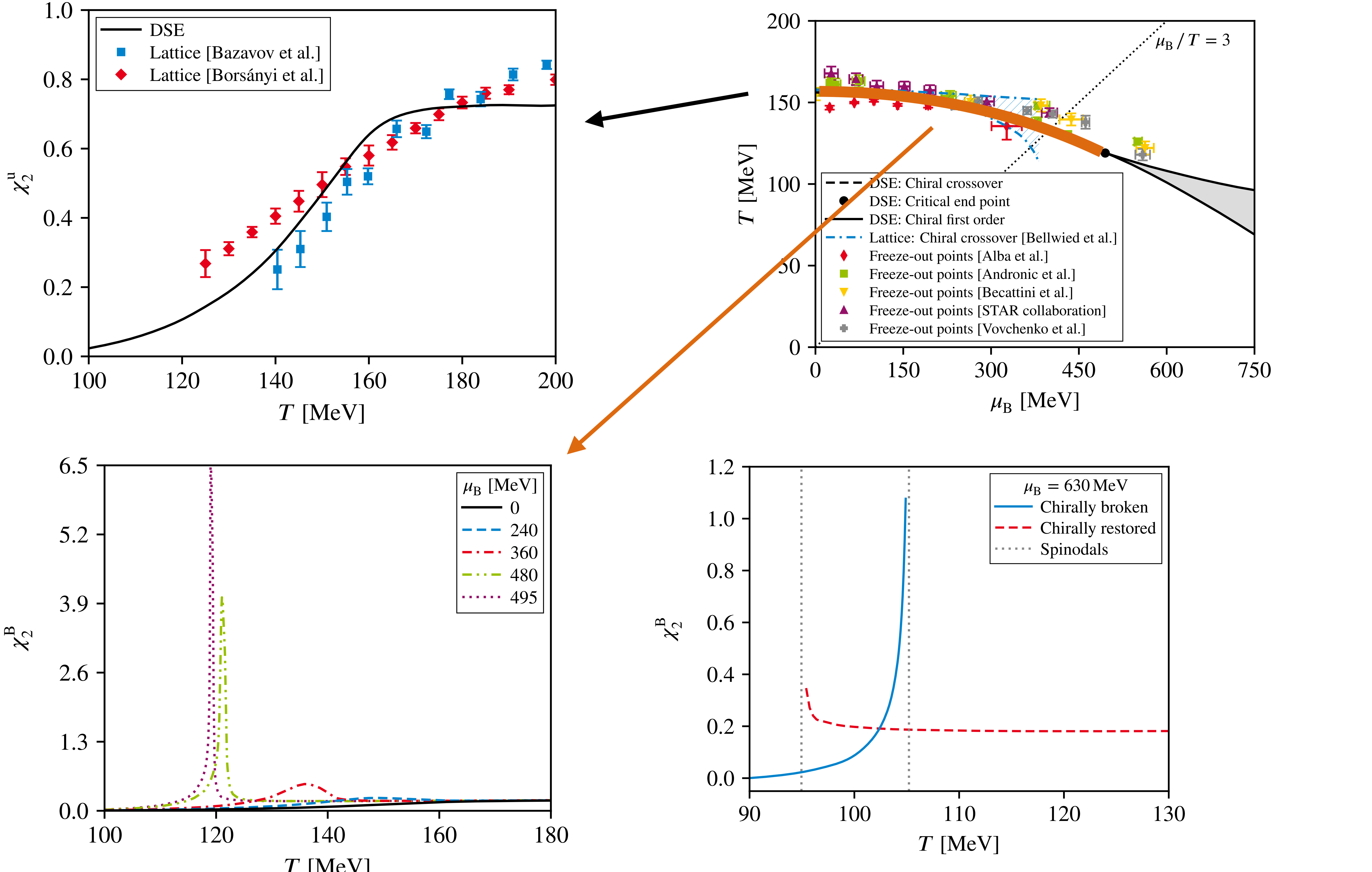






Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Results for fluctuations



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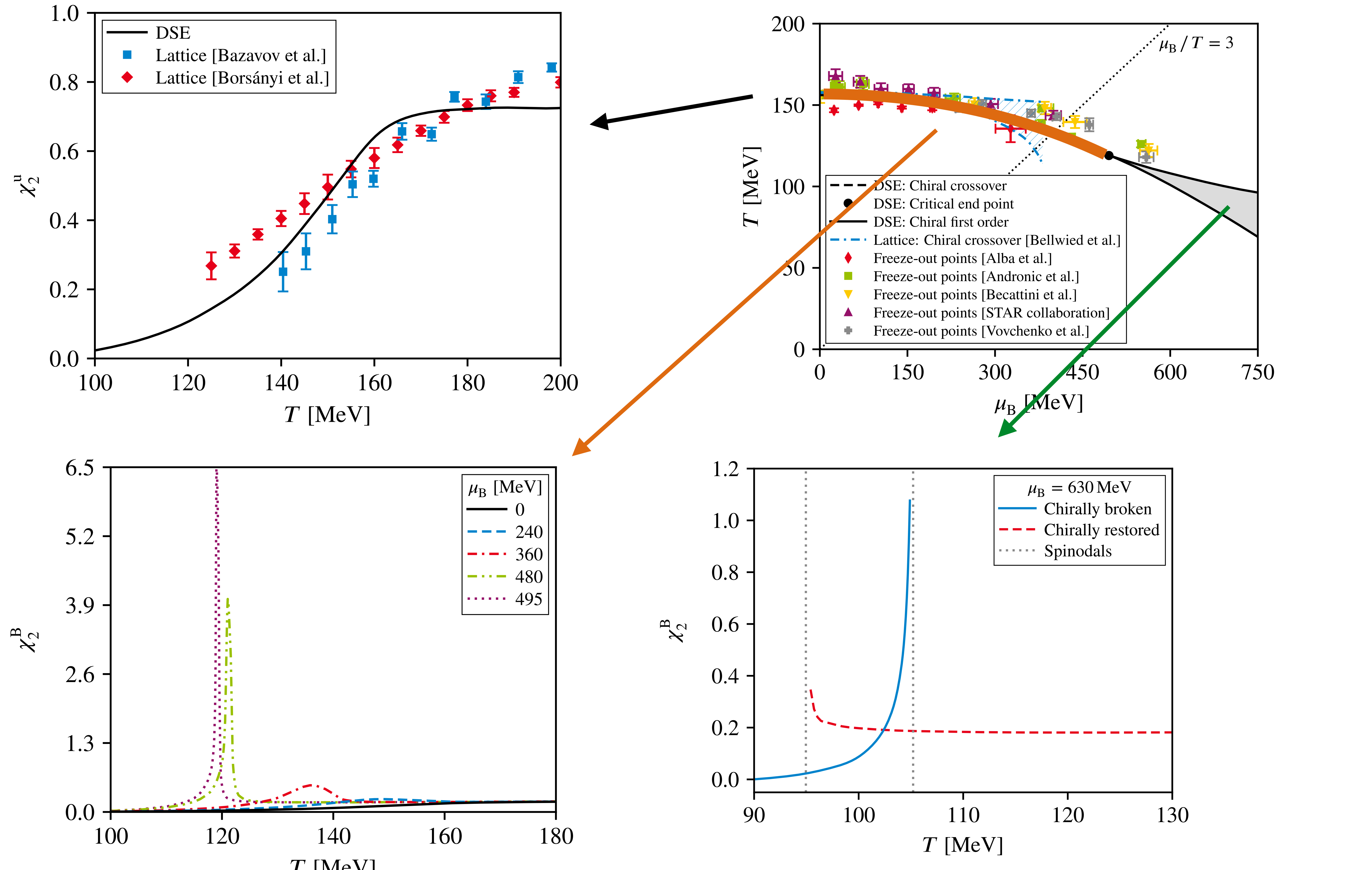
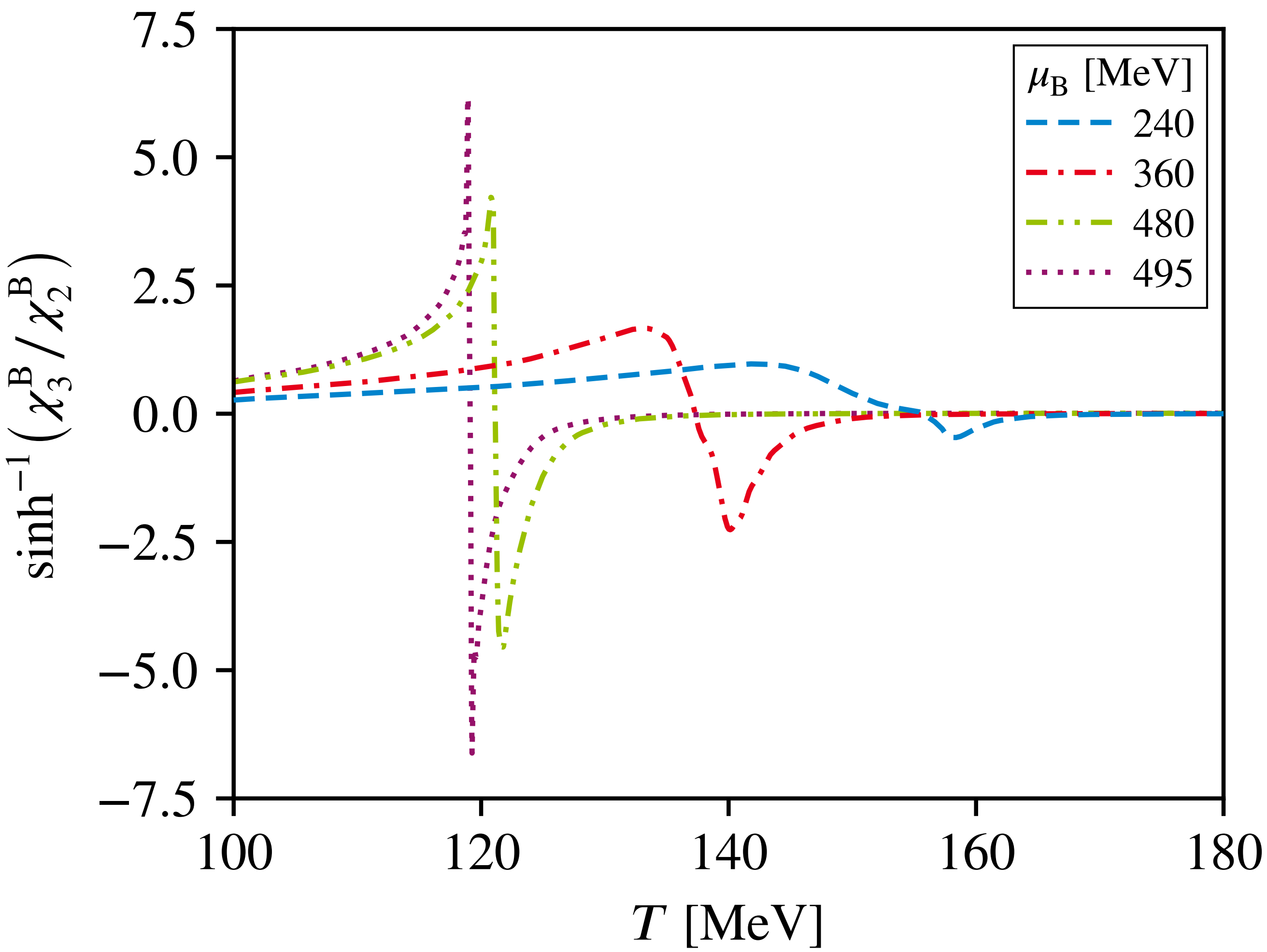
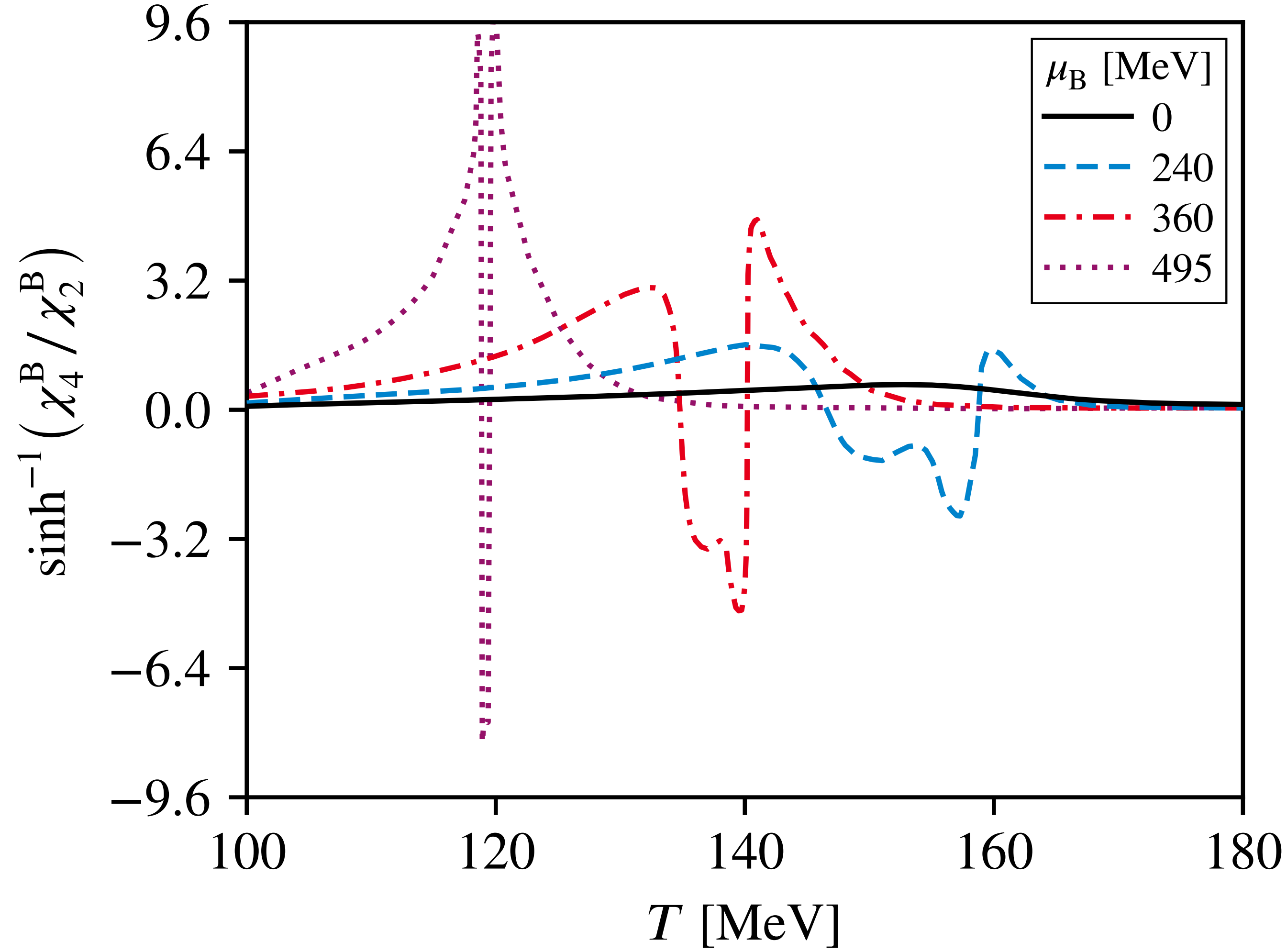


Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Ratios: skewness and kurtosis

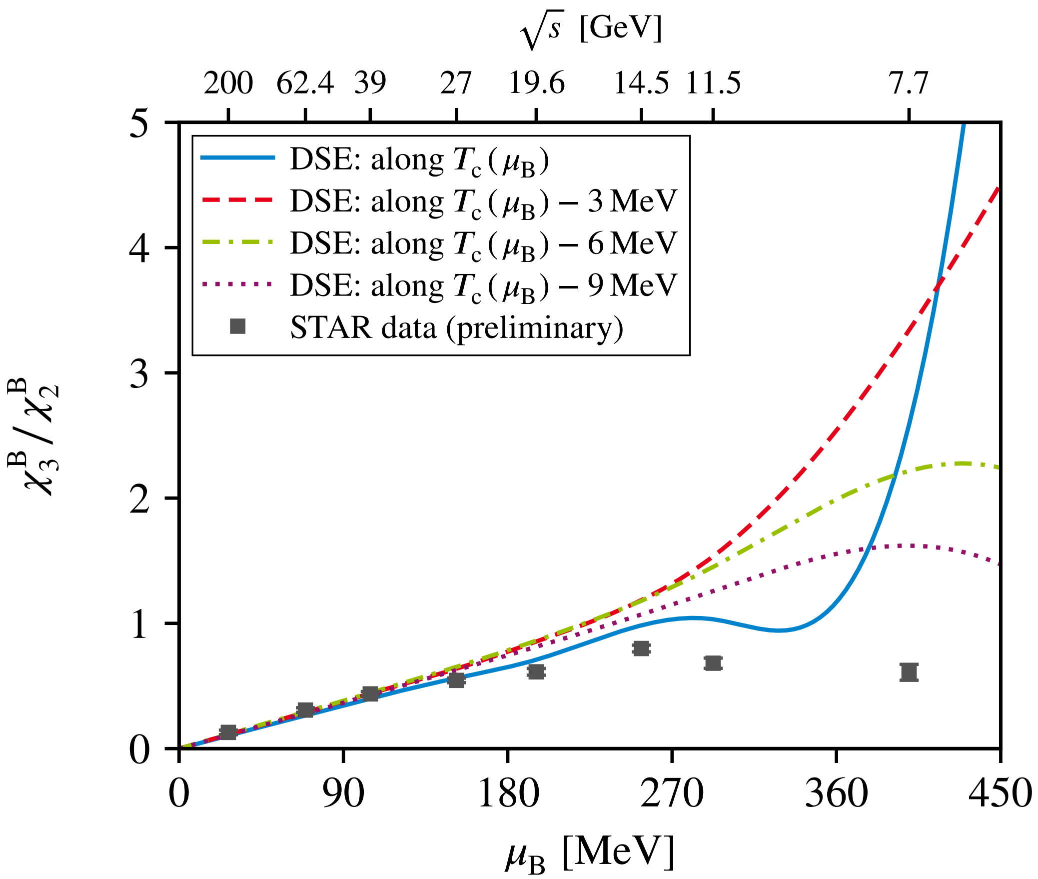


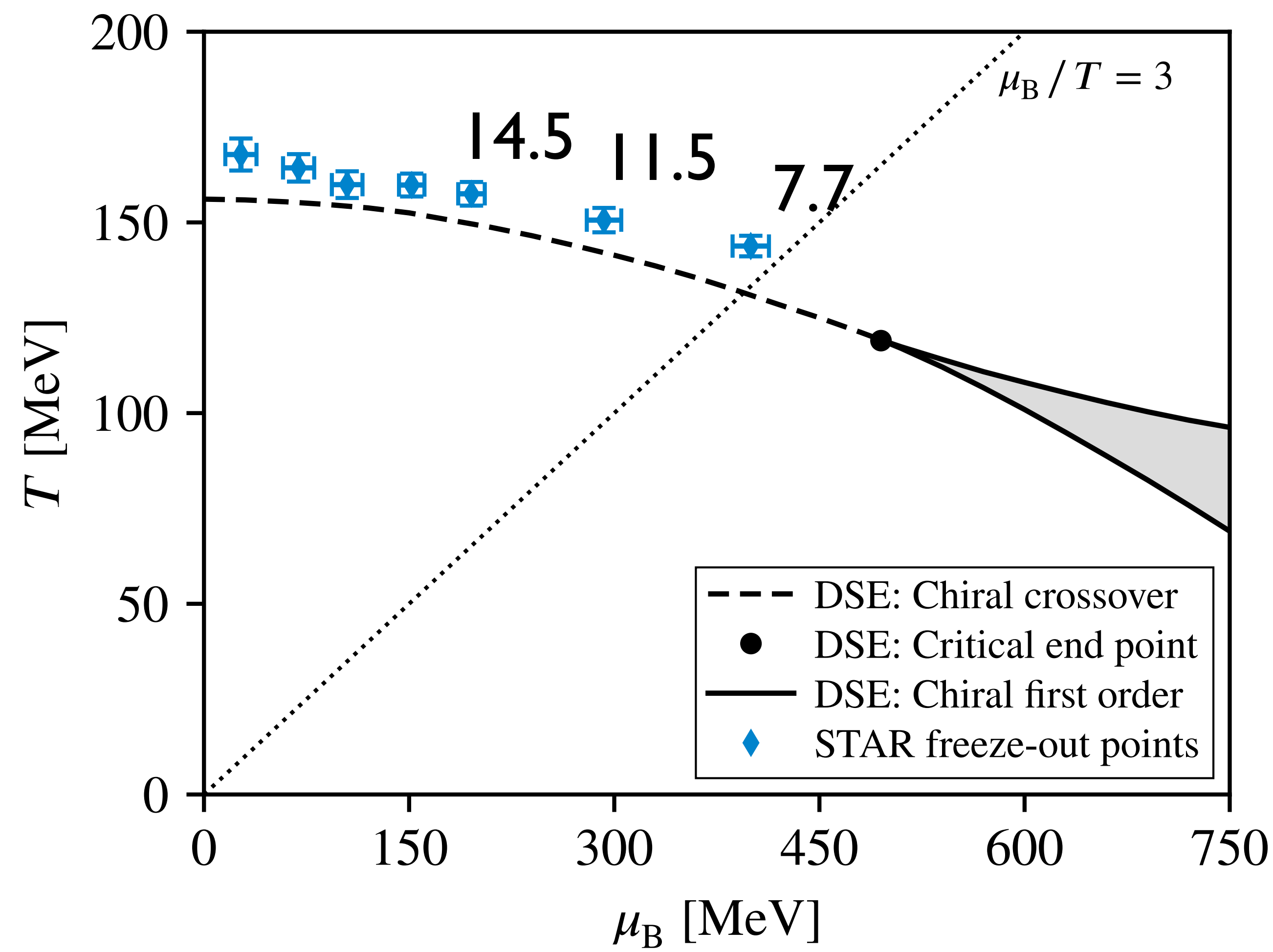


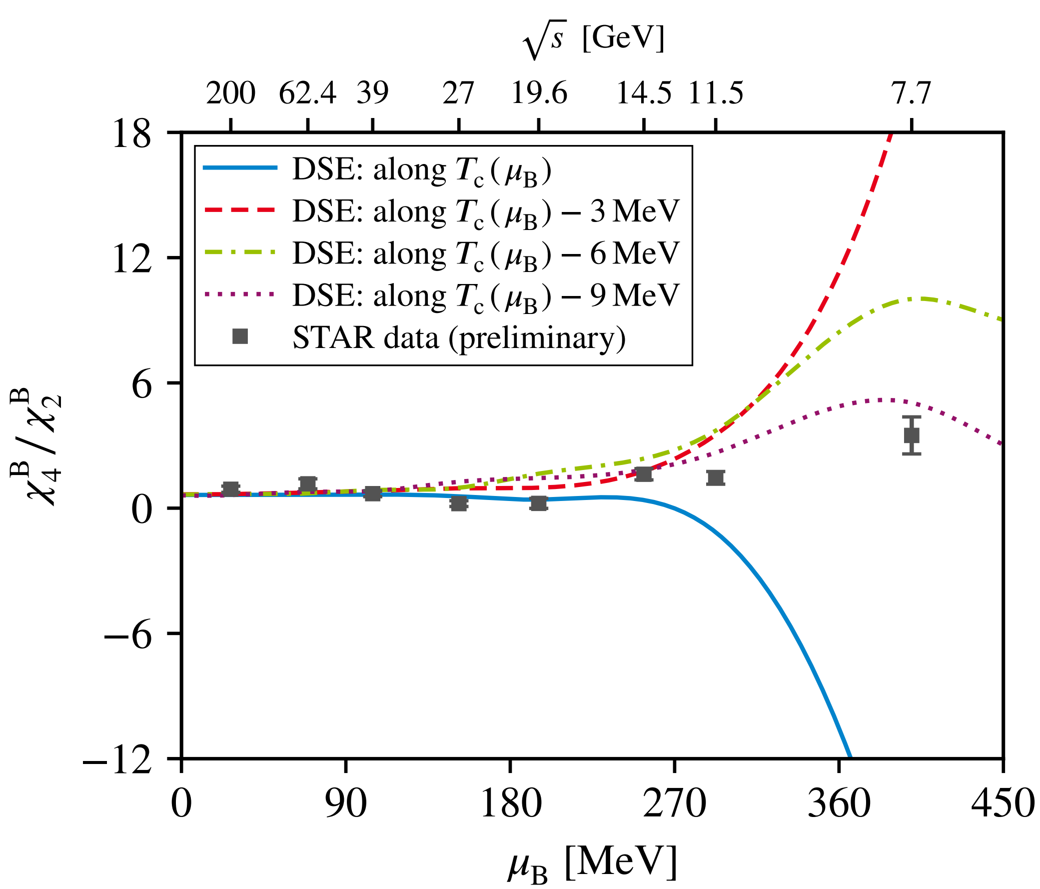
- Clear signals for CEP

Caveats when comparing with experiment:

- so far only flavor-diagonal elements taken into account
- critical region may be too large… Schaefer and Wambach, PRD 75 (2007) 085015
- experimental extraction not without problems







$\sqrt{s} \geq 14.5:$ good agreement

$\sqrt{s} = 11.5:$ trend ok !

$\sqrt{s} \leq 7.7:$ freezeout line $\neq$ transition line ?!

Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

Summary

QCD with finite chemical potential:

- back-reaction of quarks onto gluons important
- N_f=2+1 and N_f=2+1+1: CEP at $\mu_c/T_c > 3$
- charm quark does not influence CEP
- Baryon effects may or may not be significant for CEP…

Work in progress: - mesons and baryons at finite T and μ

w. Pascal Gunkel — Gunkel, CF, Isserstedt, EPJ A 55 (2019) no.9, 169

- thermodynamics and fluctuations

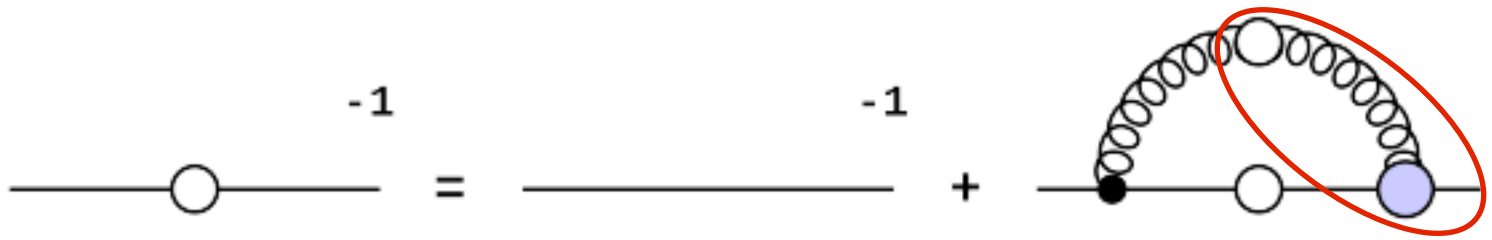
w. Phillip Isserstedt — Isserstedt, Buballa, CF, Gunkel, PRD 100 (2019) no.7, 074011

- finite volume effects

w. Julian Bernhard

Backup Slides

Rainbow-ladder model for quark-gluon interaction



Combine gluon with quark-gluon vertex:

$$\Gamma^\mu(p,k) = \sum_{i=1,12} \tau_i(p,k) T_i^\mu$$

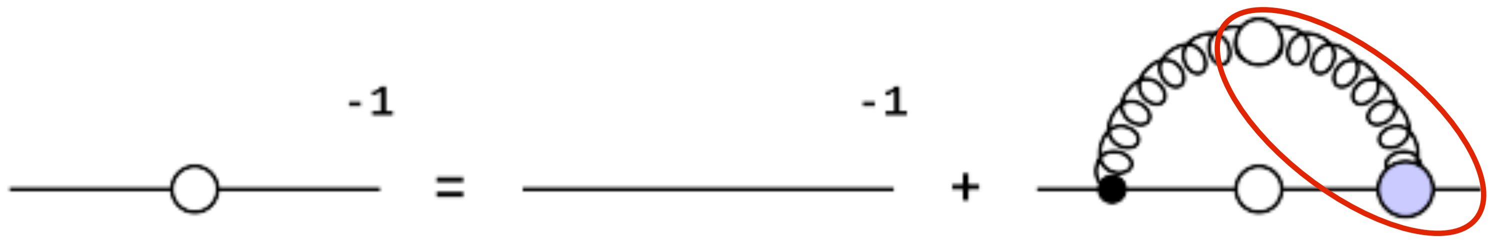
$$\sim \gamma^\mu \tau(k^2)$$

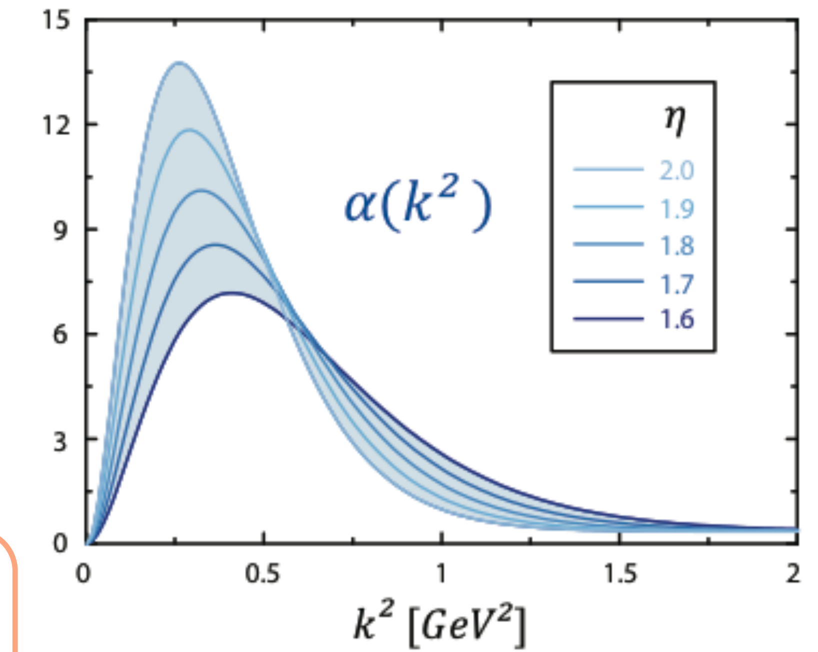
"approximation" !

$$D^{\mu\nu}(k) = \left(\delta^{\mu\nu} - \frac{k^\mu k^\nu}{k^2}\right) \frac{Z(k^2)}{k^2}$$

$$\frac{g^2}{4\pi} \tau(k^2) Z(k^2) \sim \alpha(k^2)$$

Rainbow-ladder model for quark-gluon interaction





Maris, Roberts, Tandy, PRC 56 (1997), PRC 60 (1999)

Combine gluon with quark-gluon vertex:

effective coupling

$$\alpha(k^2) = \pi\eta^7 \left(\frac{k^2}{\Lambda^2}\right) e^{-\eta^2\left(\frac{k^2}{\Lambda^2}\right)} + \alpha_{UV}(k^2)$$

- scale Λ from f_π, masses m_u=m_d, m_s from m_π, m_K
- α_{UV} from perturbation theory
- parameter η : results almost independent
- qualitatively similar to explicit calc.

Williams, EPJA 51 (2015) 5, 57.
Sanchis-Alepuz, Williams, PLB 749 (2015) 592;
Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD93 (2016) 034026, and refs. therein

Approximation for Quark-Gluon interaction

- Lattice input for vertex: not yet available…
- Diagrammatics: vertex-DSE (see later…)

explicit solutions at T=0: Mitter, Pawlowski and Strodthoff, PRD 91 (2015) 054035
Williams, CF, Heupel, PRD PRD 93 (2016) 034026

- Slavnov-Taylor identity: T,μ,m-dependent vertex

$$\Gamma_\nu(q,k,p) = \widetilde{Z}_3 \left(\delta_{4\nu}\gamma_4 \frac{C(k)+C(p)}{2} + \delta_{j\nu}\gamma_j \frac{A(k)+A(p)}{2} \right) \times$$
$$\times \left(\frac{d_1}{d_2+q^2} + \frac{q^2}{\Lambda^2+q^2} \left(\frac{\beta_0 \alpha(\mu) \ln[q^2/\Lambda^2+1]}{4\pi} \right)^{2\delta} \right)$$

STI

PT

- d_1 fixed via T_c
- d_2 fixed to match scale of lattice gluon input

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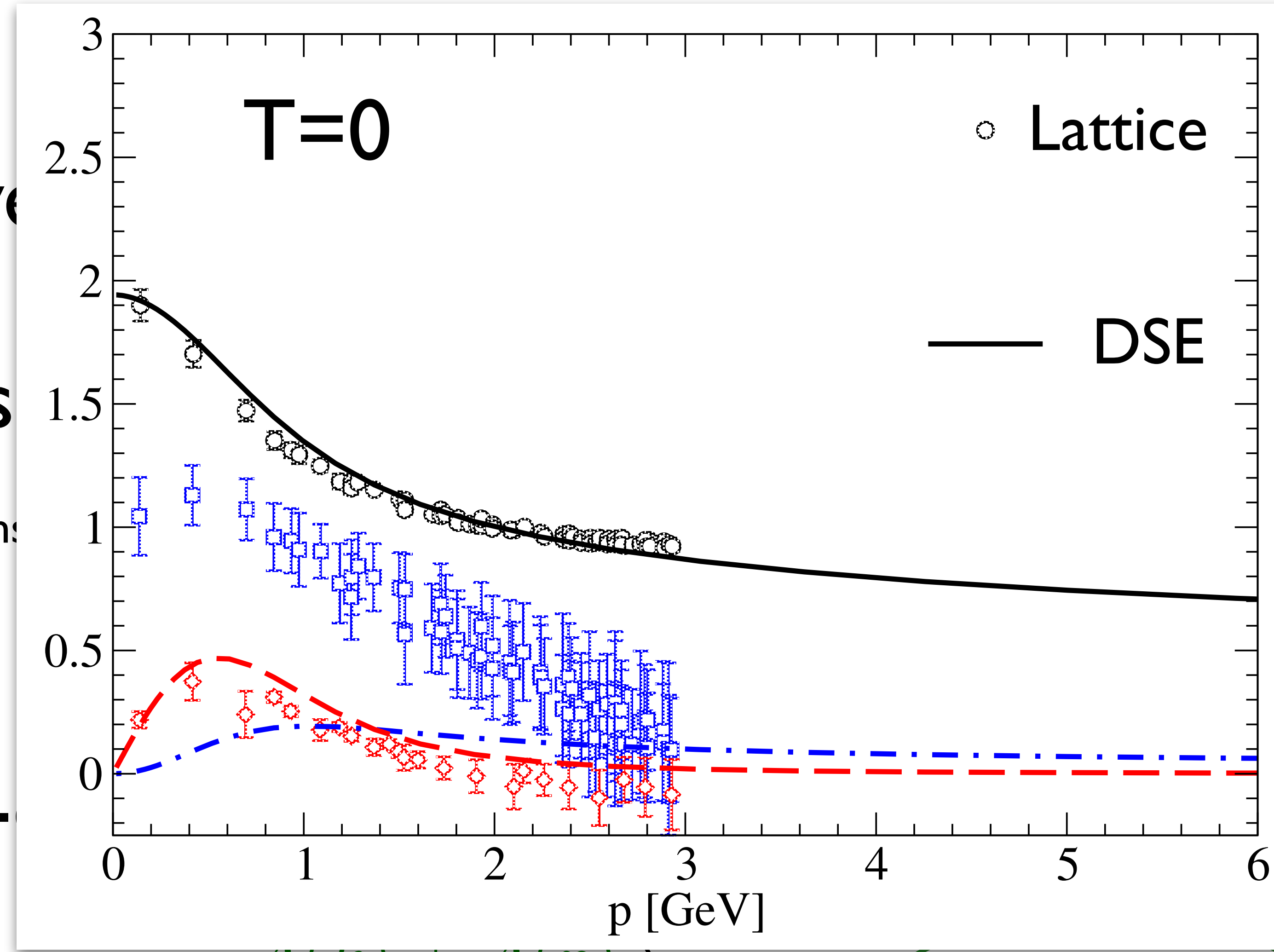

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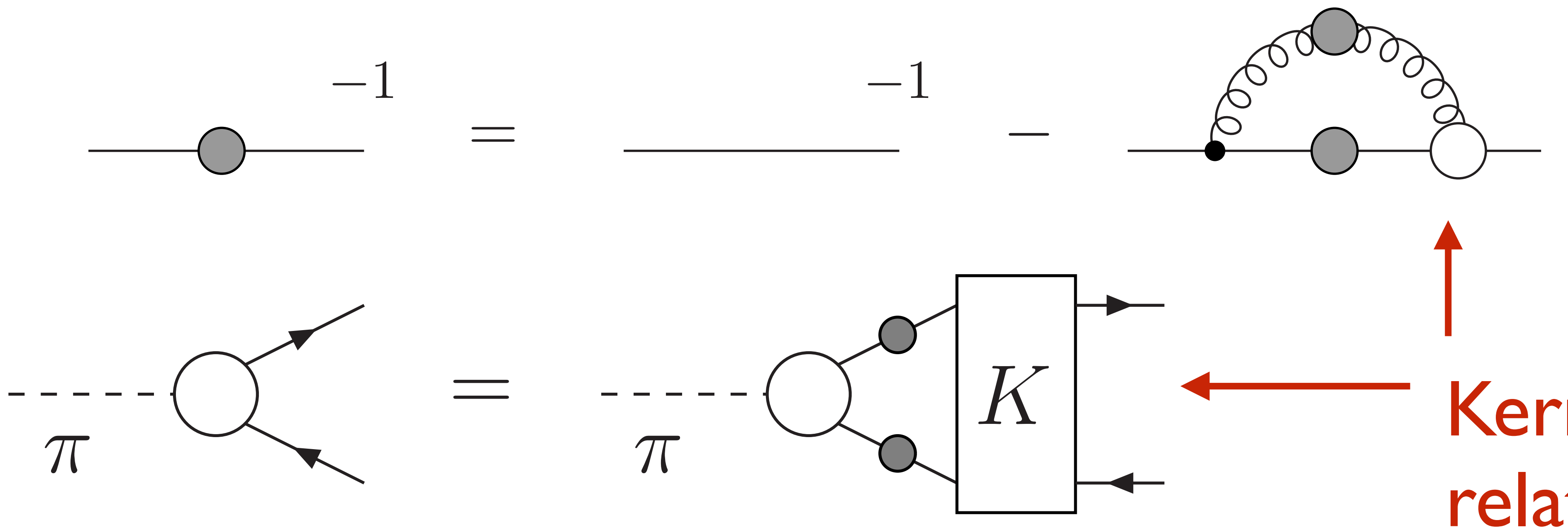
Theoretical Tools II: DSEs and BSEs

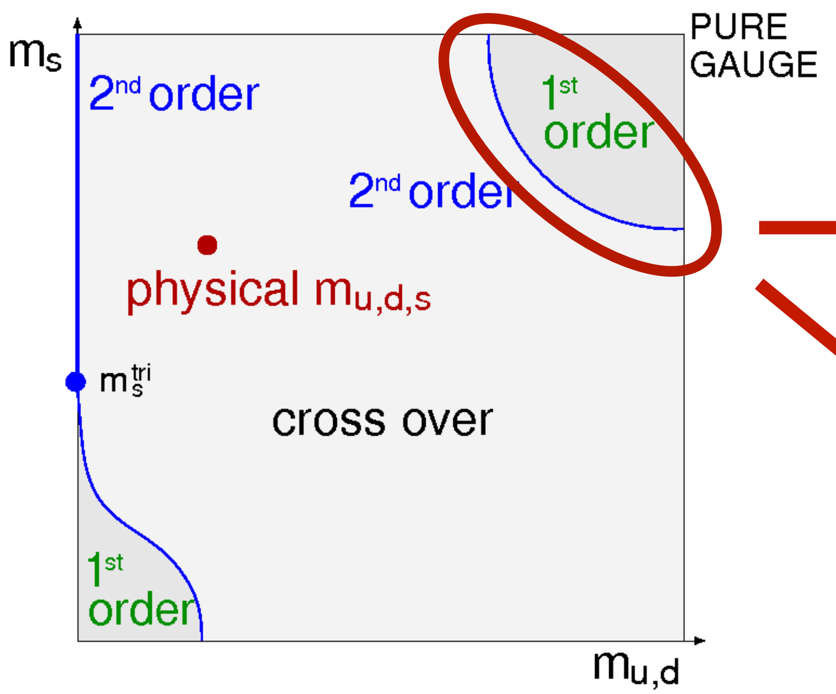


Kernel K uniquely related to quark-DSE via axWTI

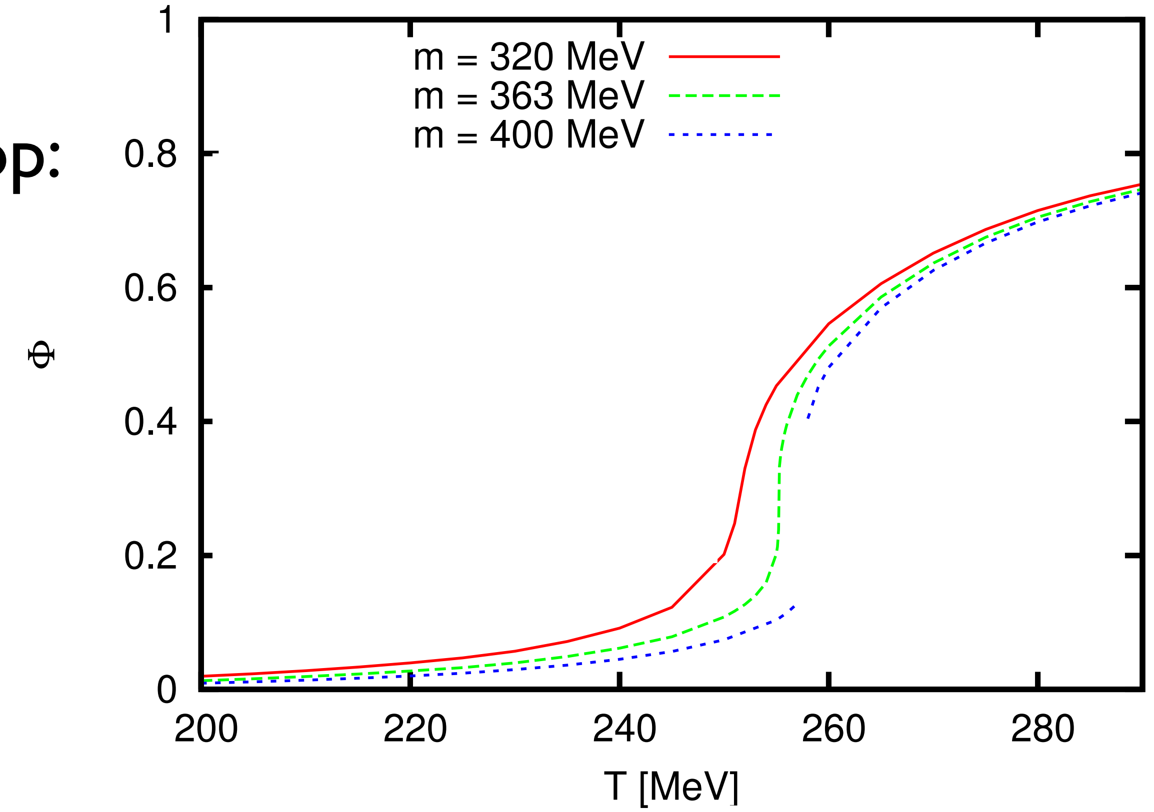
→Pion is bound state **and** Goldstone boson

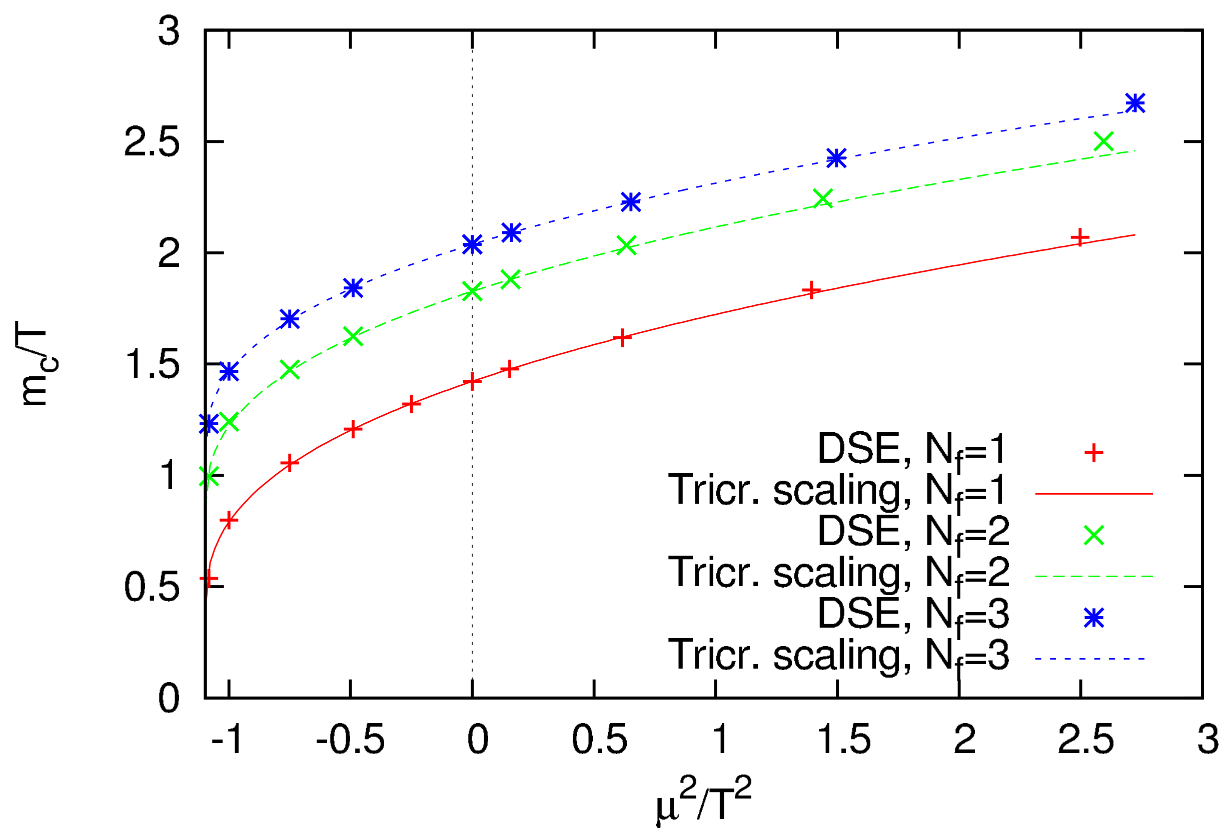
Maris, Roberts, Tandy, PLB 420 (1998) 267

- Determine gauge invariant spectrum from underlying, gauge dependent quark/gluon dynamics
- Need approximations for dressed quark-gluon vertex



Polyakov Loop:



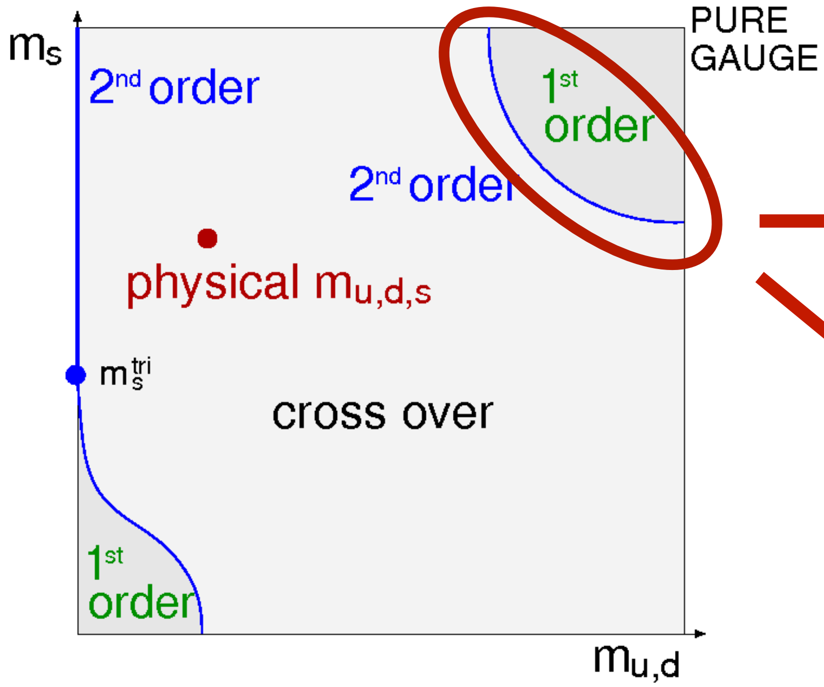


- Deconfinement transition in agreement with lattice QCD
- Correct tricritical scaling
- Roberge-Weiss-transition seen

Lattice:
Fromm, Langelage, Lottini, Philipsen, JHEP 1201 (2012) 042

CF, Luecker, Pawlowski, PRD 91 (2015) 1

Critical line/surface for heavy quarks



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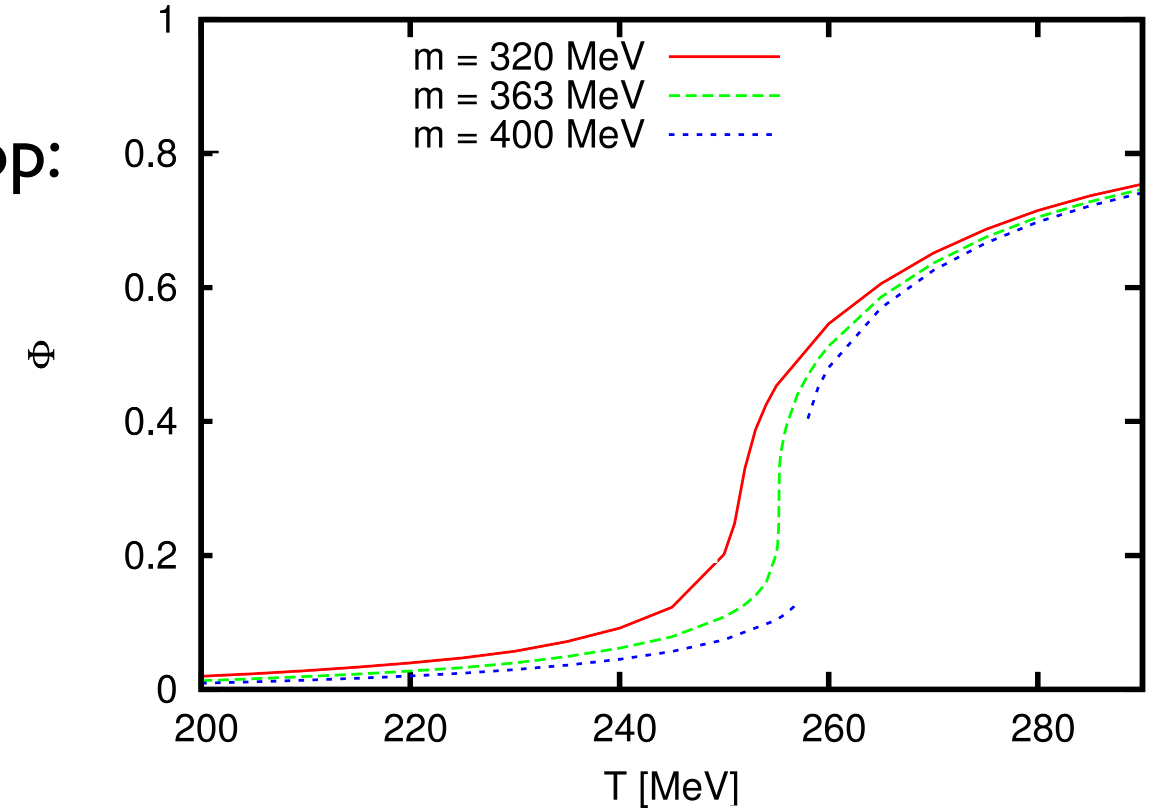

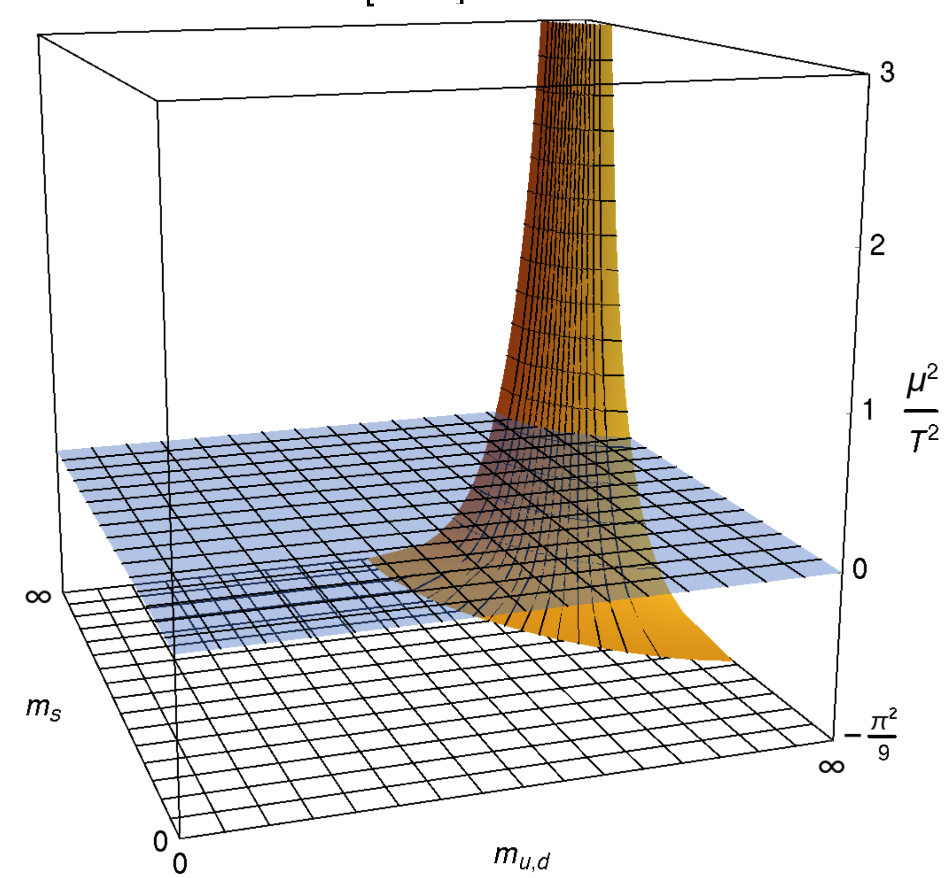

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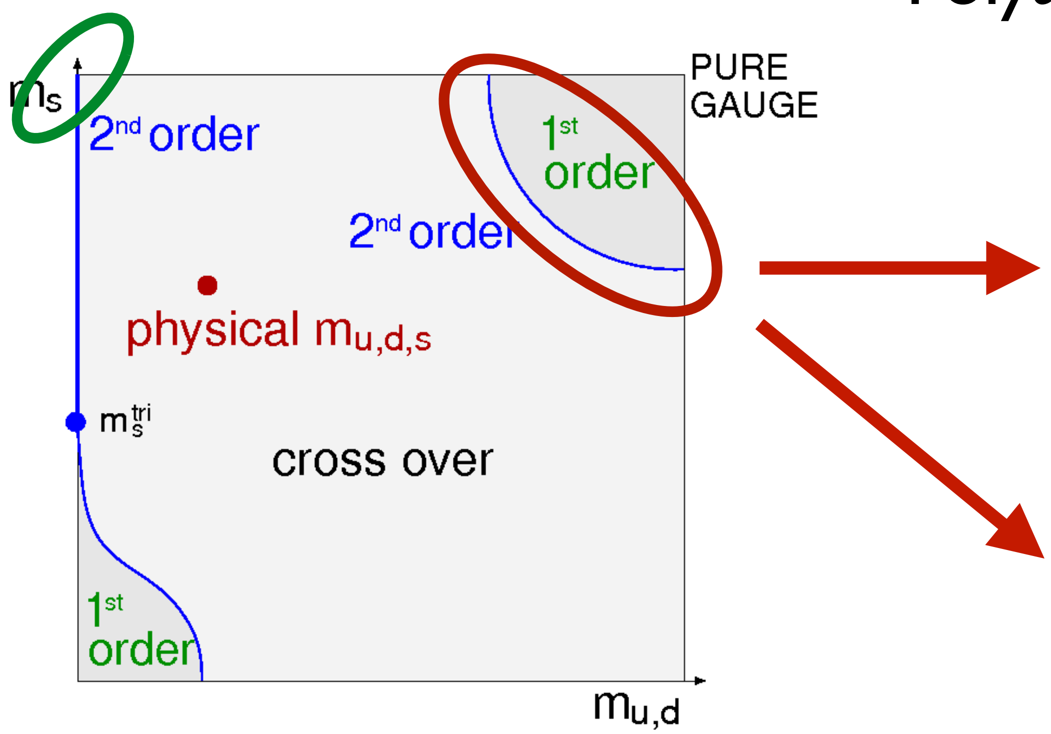
Critical line/surface for heavy quarks

Nf=2: CF and Mueller, PRD 84 (2011) 054013



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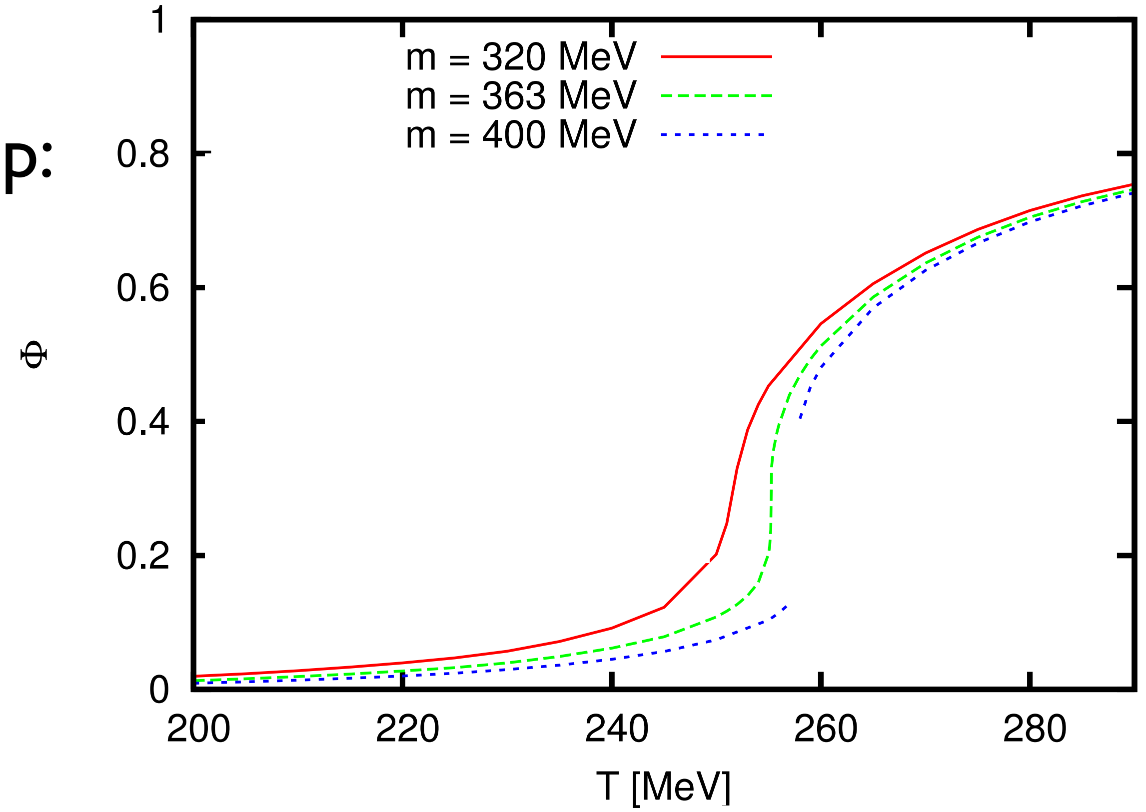

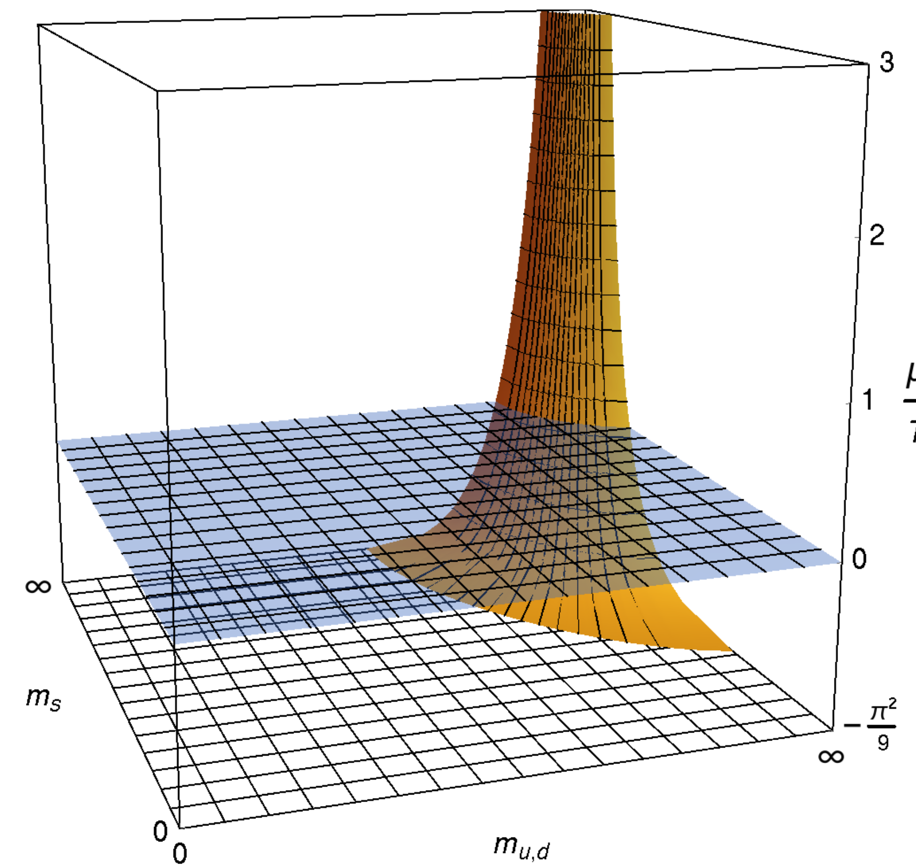

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CF, Luecker, Pawlowski, PRD 91 (2015) 1