QCD crossover line at finite chemical potential from the Lattice

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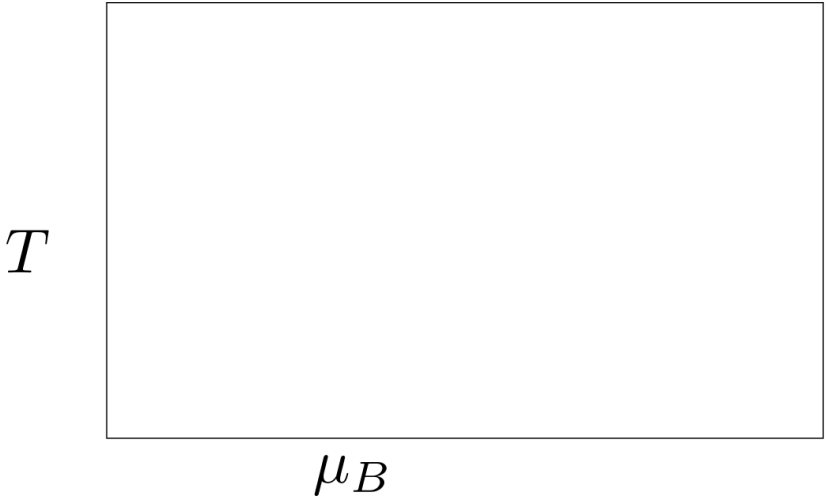
Aix*Marseille université

WB collaboration

BMW collaboration

SFB TR55

The (T, μ_B)-phase diagram of QCD



Our observables: T_c, Equation of state, Fluctuations

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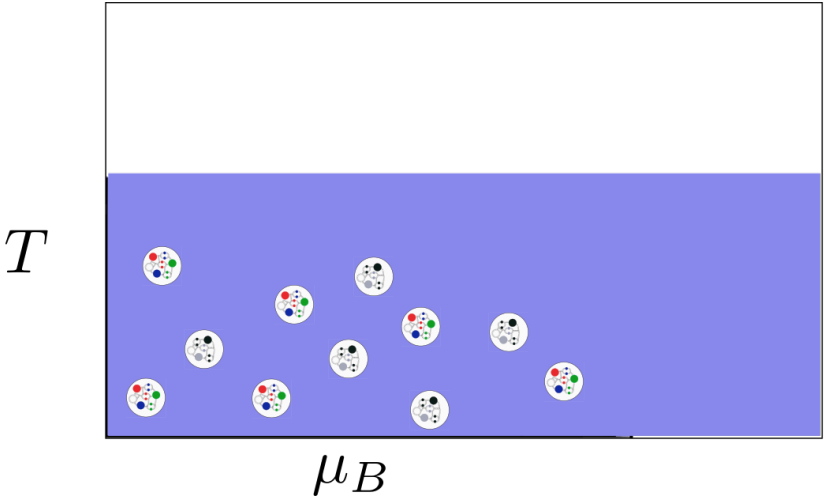

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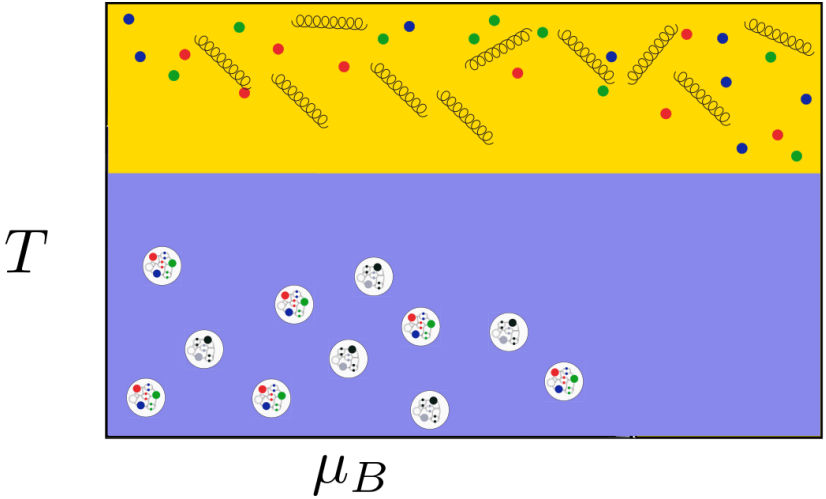

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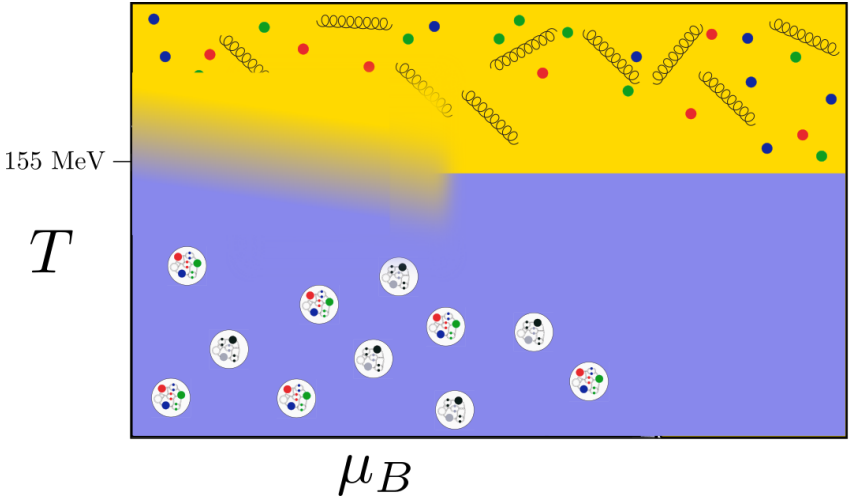

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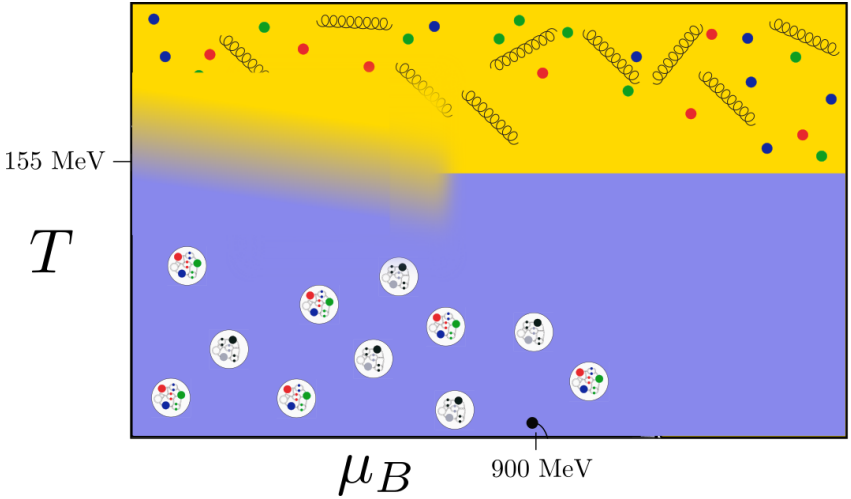

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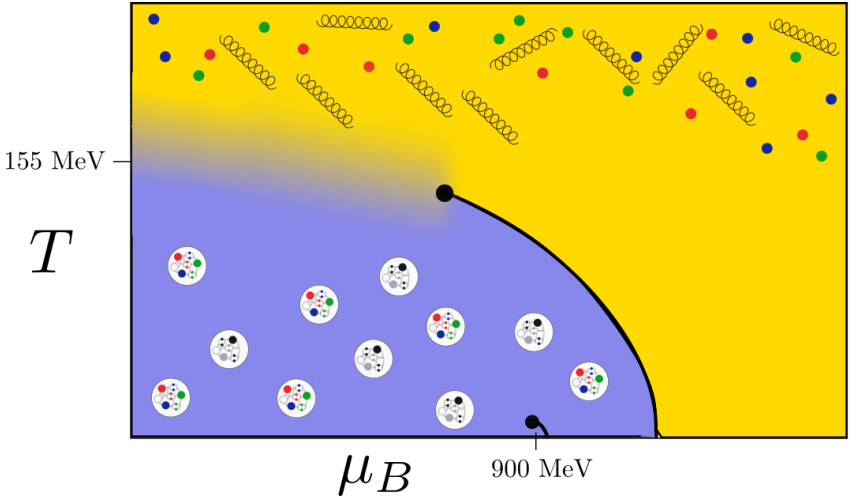

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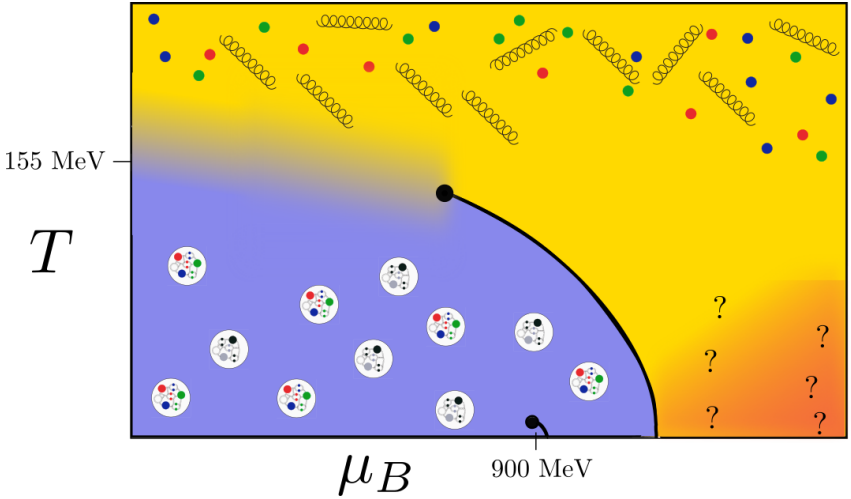

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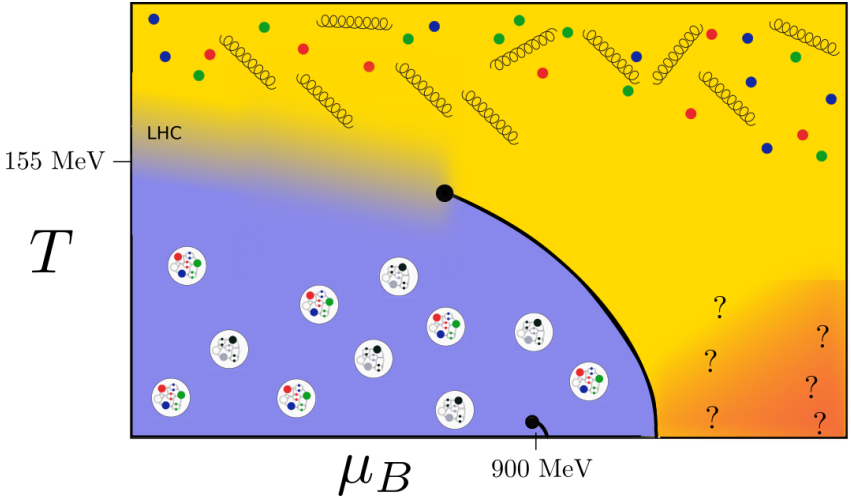

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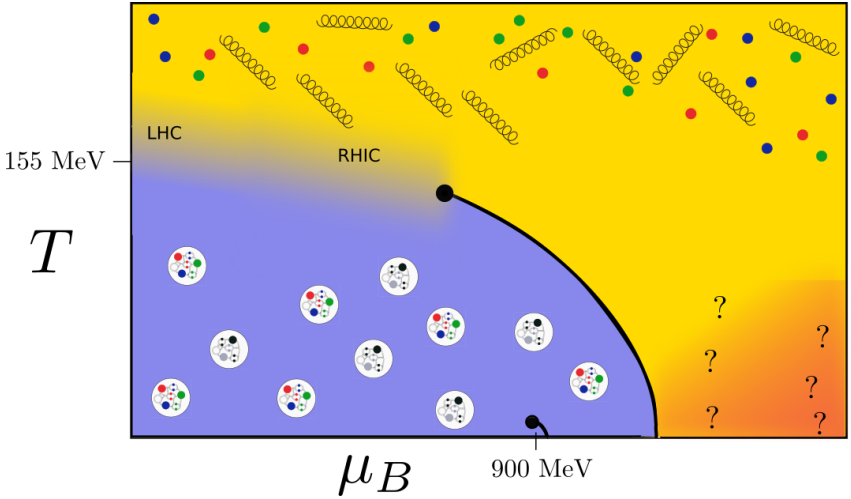

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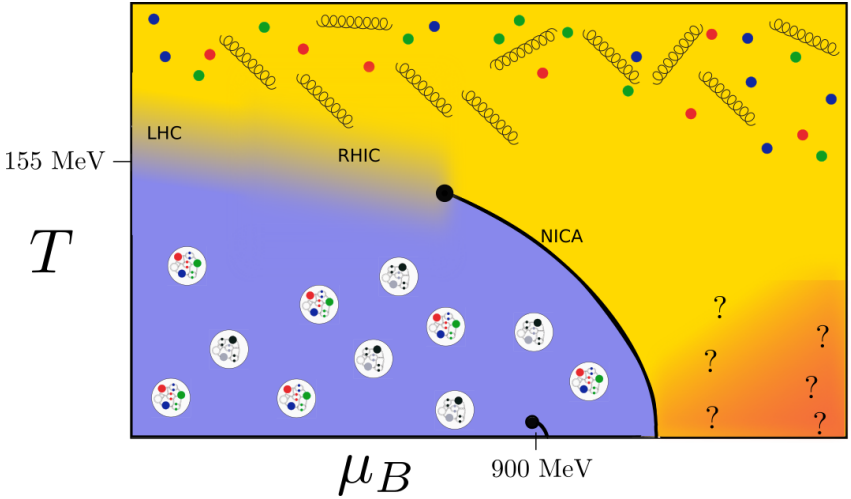

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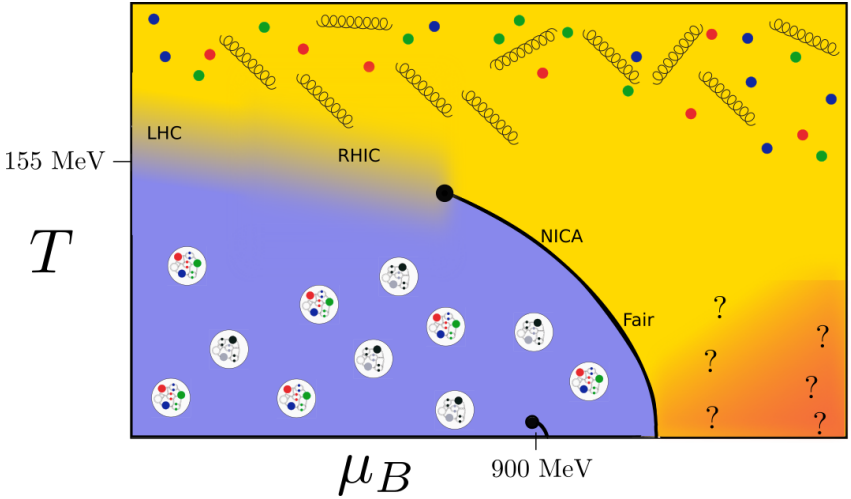

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- R. Bellwied, S. Borsanyi, Z. Fodor, J. N. Guenther, S. Katz, C. Ratti and K. Szabo: *The QCD phase diagram from analytic continuation,* Phys. Lett. B **751** (2015), 559-564 doi:10.1016/j.physletb.2015.11.011 [arXiv:1507.07510 [hep-lat]]

The sign problem

The QCD partition function:

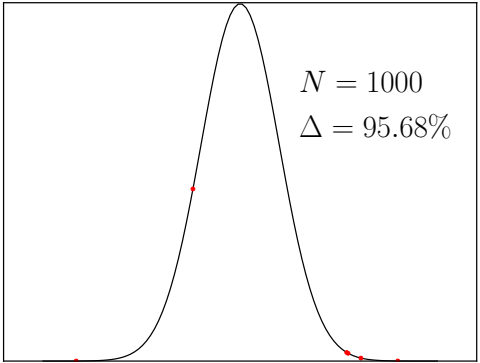
$$Z(V,T,\mu) = \int \mathcal{D}U\mathcal{D}\psi\mathcal{D}\bar{\psi}\ e^{-S_F(U,\psi,\bar{\psi})-\beta S_G(U)}$$
$$= \int \mathcal{D}U\ \det M(U) e^{-\beta S_G(U)}$$

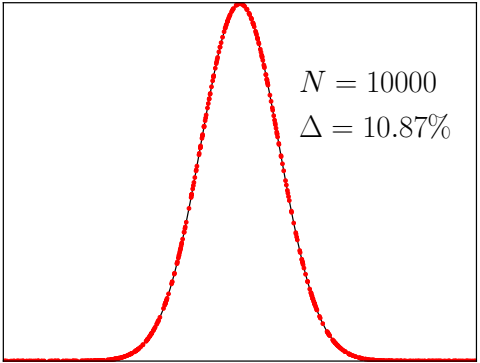
- For Monte Carlo simulations $\det M(U) e^{-\beta S_G(U)}$ is interpreted as Boltzmann weight
- If there is particle-antiparticle-symmetry $\det M(U)$ is real
- If $\mu^2 > 0$ $\det M(U)$ is complex

The sign problem

$$\int_{-\infty}^{\infty}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} \approx \int_{-100}^{100}(100-x^2)\frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N}(100-x_i^2)\frac{e^{-\frac{1}{2}x_i^2}}{\sqrt{2\pi}}\cdot\frac{200}{N}$$

The x_i are drawn from a uniform distribution in the interval $[-100, 100]$

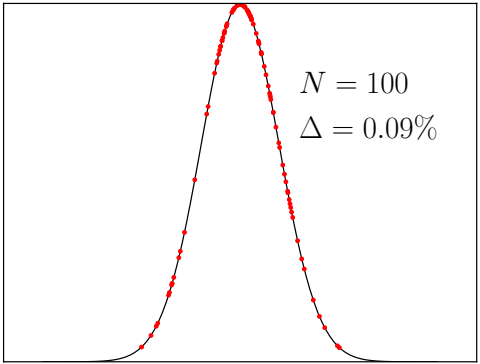


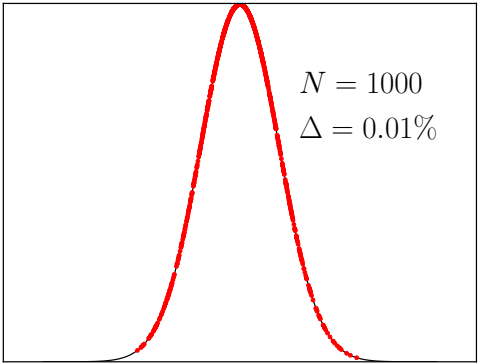


Importance sampling

$$\int_{-\infty}^{\infty} (100 - x^2) \frac{e^{-\frac{1}{2}x^2}}{\sqrt{2\pi}} = \sum_{i=1}^{N} (100 - x_i^2) \cdot \frac{1}{N}$$

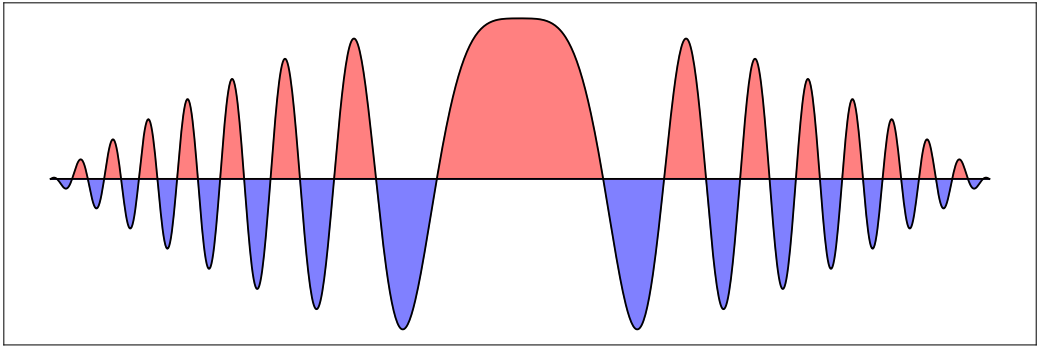
The x_i are drawn from a normal distribution





The sign problem

$$\int_{-\infty}^{\infty} (100 - x^2) \frac{e^{-\frac{\mathrm{i}}{2}x^2}}{\sqrt{2\pi}}$$



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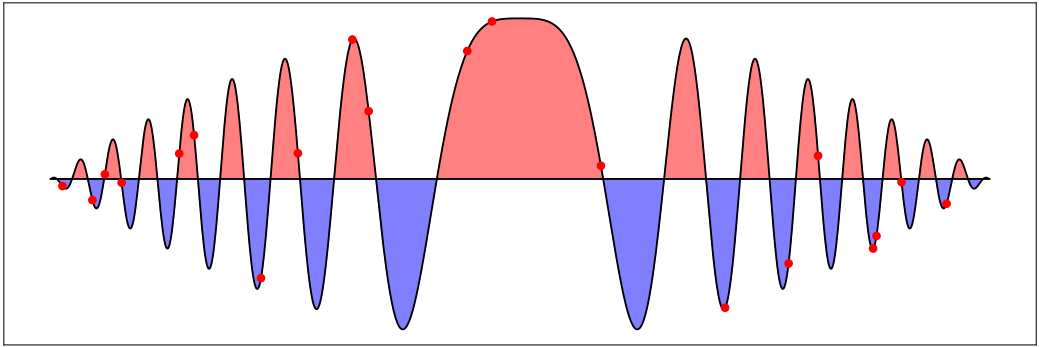

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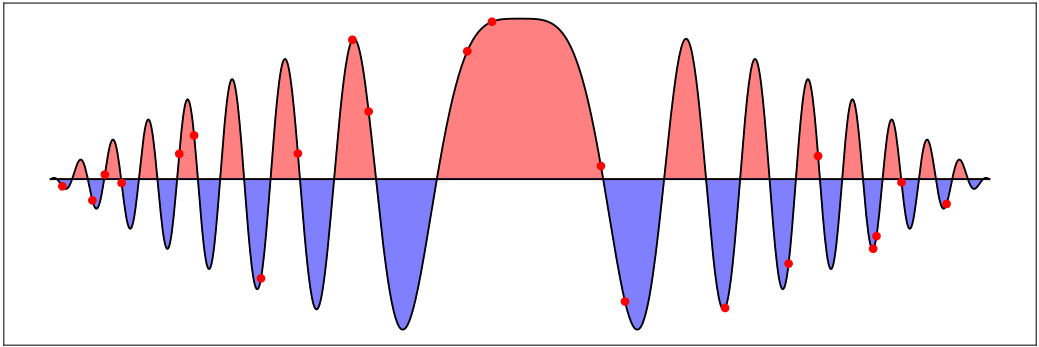
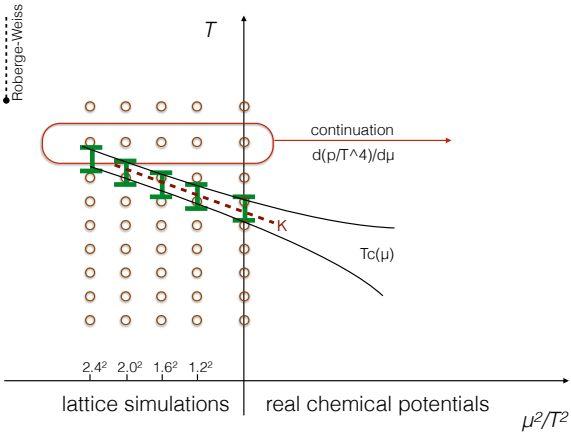


Dealing with the sign problem

- Reweighting techniques
- Canonical ensemble
- Complex Langevin $\longrightarrow$ Talk by Casey Berger on June 18
- Lefshetz Thimble
- Density of state methods
- Dual variables
- ...

- Taylor expansion $\longrightarrow$ [Bazavov et al., Bazavov:2017dus], [Bonati et al., Bonati:2018nut]
- Imaginary μ

Analytic continuation



Common technique:

- [deForcrand:2002hgr]
- [Bonati:2015bha]
- [Cea:2015cya]
- [DElia:2016jqh]
- [Bonati:2018nut]
- . . .

1 Introduction

2 The crossover temperature

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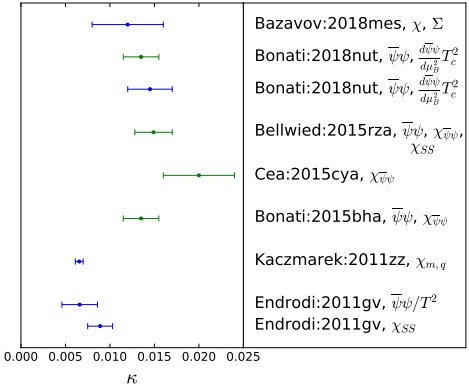
Current status

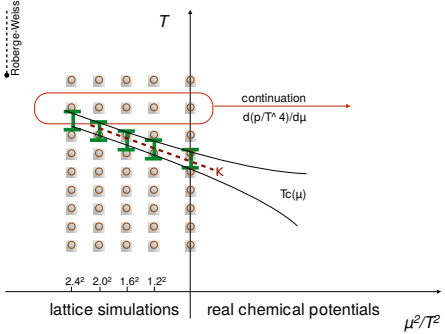
Curvature function:

$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa \left(\frac{\mu_B}{T_c}\right)^2 + \mathcal{O}(\mu_B^4)$$

$$C_1(\hat{\mu}) = 1 + a\hat{\mu}^2 + b\hat{\mu}^4, \;\; C_2(\hat{\mu}) = \frac{1 + a\hat{\mu}^2}{1 + b\hat{\mu}^4}$$

$$C_3(\hat{\mu}) = \frac{1}{1 + a\hat{\mu}^2 + b\hat{\mu}^4}, \;\; \hat{\mu} = \frac{\mu_B}{T}$$





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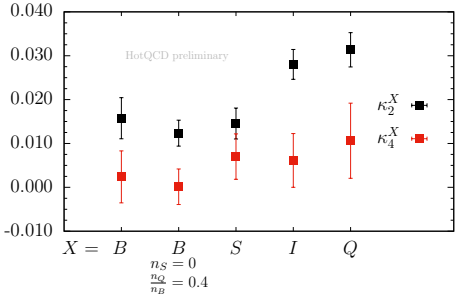
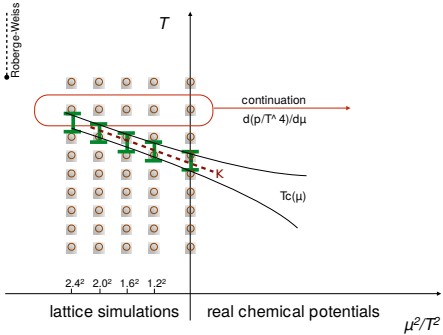
[Steinbrecher:2018phh]





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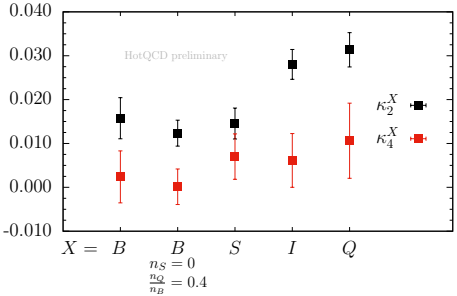

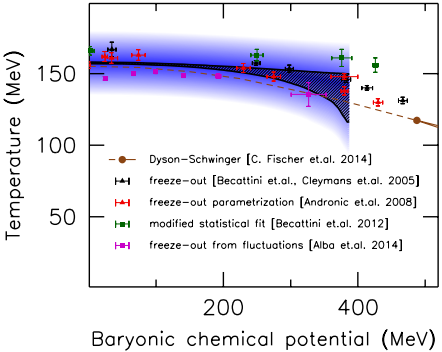

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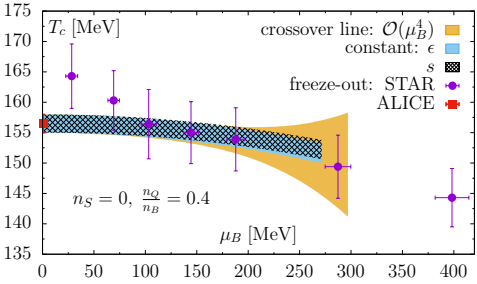
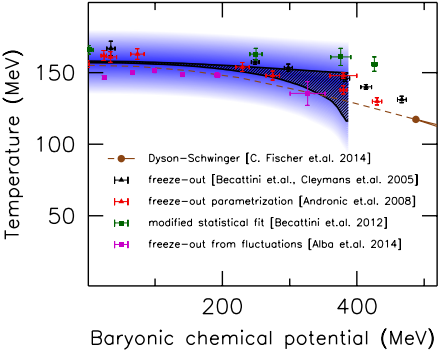
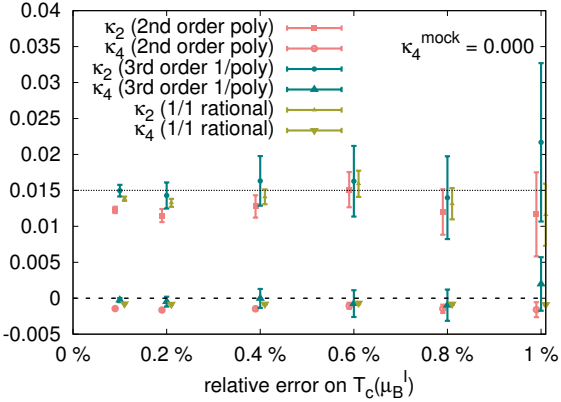
[Bazavov:2018mes]





Mock-Analysis



$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4$$

When reducing the error on T_c we need higher order fit functions for reliable results.

Choice: $C_1(\hat{\mu}) = 1 + a\hat{\mu}^2 + b\hat{\mu}^4 + c\hat{\mu}^6$ and $C_2(\hat{\mu}) = \frac{1}{1+a\hat{\mu}^2+b\hat{\mu}^4+c\hat{\mu}^6}$, $\hat{\mu} = \frac{\mu_B}{T}$

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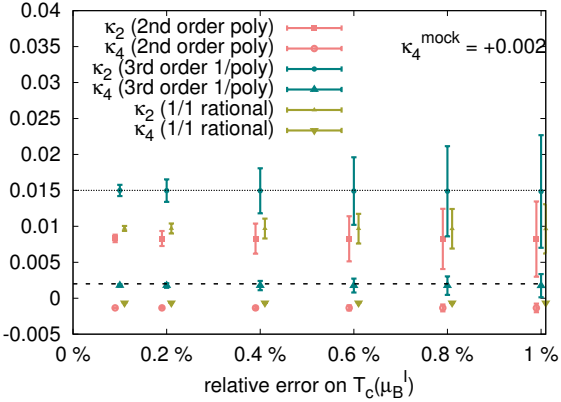

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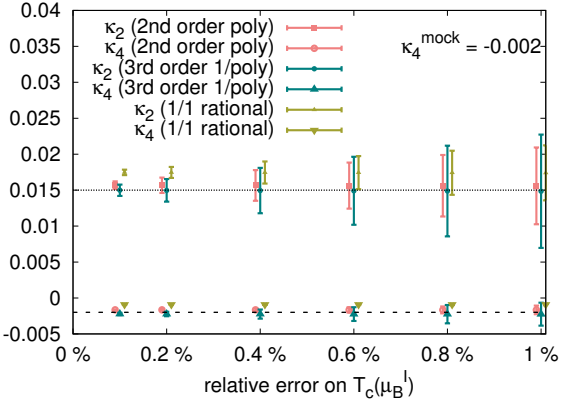

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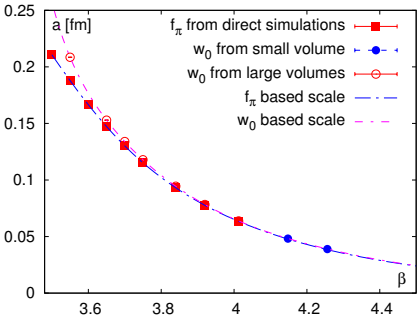
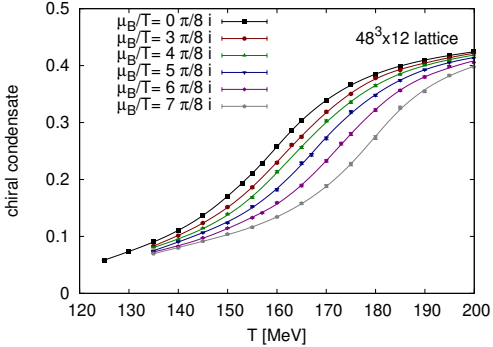
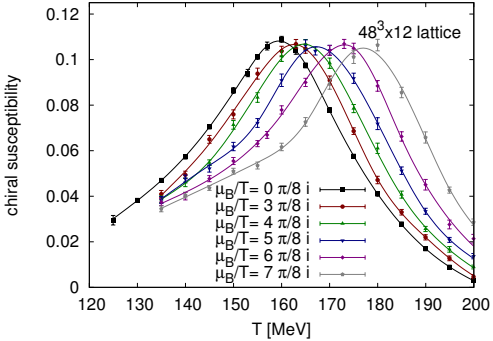
Actual-Analysis



- Action: tree-level Symanzik improved gauge action, with four times stout smeared staggered fermions
- 2+1+1 flavour, on LCP with pion and kaon mass
- Simulation at $\langle n_S \rangle = 0$ and $\langle n_Q \rangle = 0.4 \langle n_B \rangle$
- Continuum extrapolation from lattice sizes: $40^3 \times 10$, $48^3 \times 12$ and $64^3 \times 16$
- $\frac{\mu_B}{T} = \mathrm{i}\frac{j\pi}{8}$ with $j = 0, 3, 4, 5, 6, 6.5$ and 7
- Two methods of scale setting: f_π and w_0, $Lm_\pi > 4$

Observables





Chiral condensate:

$$\langle\bar{\psi}\psi\rangle = \frac{T}{V}\frac{\partial \ln Z}{\partial m_q}$$

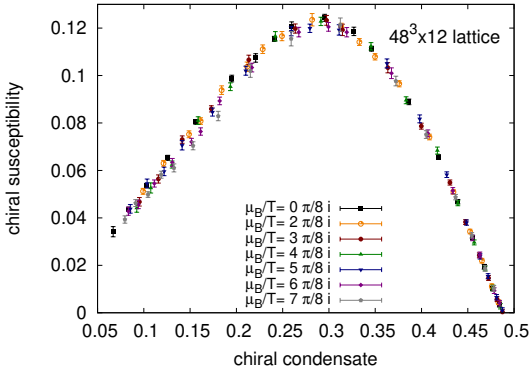
$$\langle\bar{\psi}\psi\rangle^r = \left(\langle\bar{\psi}\psi\rangle(0,\beta) - \langle\bar{\psi}\psi\rangle(T,\beta)\right)\frac{m_l}{f_\pi^4}$$

Chiral susceptibility:

$$\chi_{\bar{\psi}\psi} = \frac{T}{V}\frac{\partial^2 \ln Z}{\partial m_q^2}$$

$$\chi^r_{\bar{\psi}\psi} = \left(\chi_{\bar{\psi}\psi}(T,\beta) - \chi_{\bar{\psi}\psi}(0,\beta)\right)\frac{m_l^2}{f_\pi^4}$$

$\chi_{\bar{\psi}\psi}(\langle\bar{\psi}\psi\rangle)$

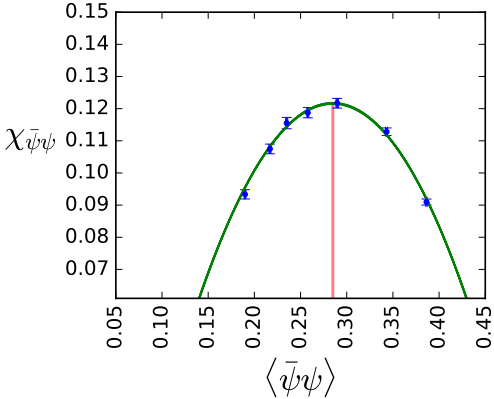


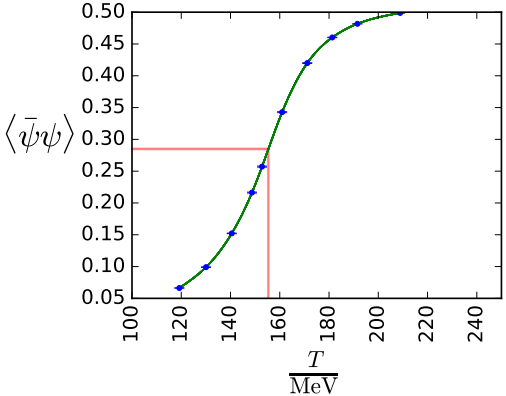
$$\chi_{\bar{\psi}\psi}(\langle\bar{\psi}\psi\rangle) = \sum_{i=0}^{n} \alpha_i \left(1 + \beta_i \left(\frac{\mu_B}{T}\right)^2\right) \langle\bar{\psi}\psi\rangle^i,$$

$$n \in \{2, 3, 4\}$$

Fitting $\chi_{\bar{\psi}\psi}(\langle\bar{\psi}\psi\rangle)$ removes most of the μ_B dependence and allows for a precise determination of the transition value of $\langle\bar{\psi}\psi\rangle$. In a next step this has to be translated into temperature.

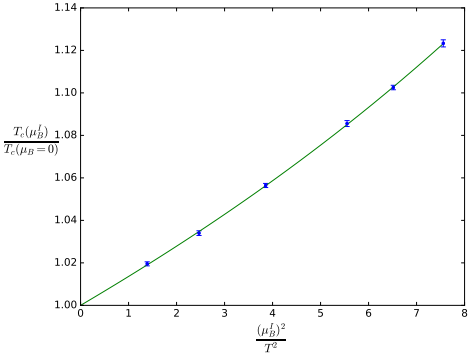
$\langle\bar{\psi}\psi\rangle(T)$

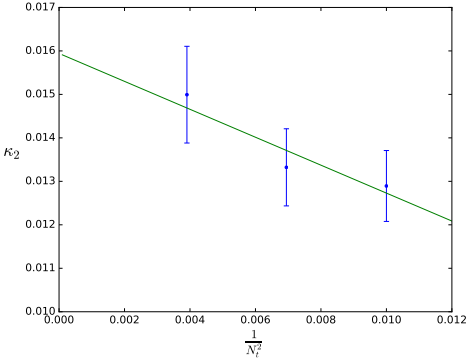




To determine the temperature from the the $\langle\bar{\psi}\psi\rangle$ value we use a spline.

Extrapolation

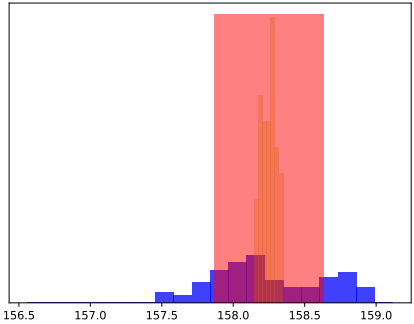




We perform a combined extrapolation in $\frac{\mu_B^2}{T^2}$ and to the continuum, which uses a fully correlated fit.

Systematic Errors

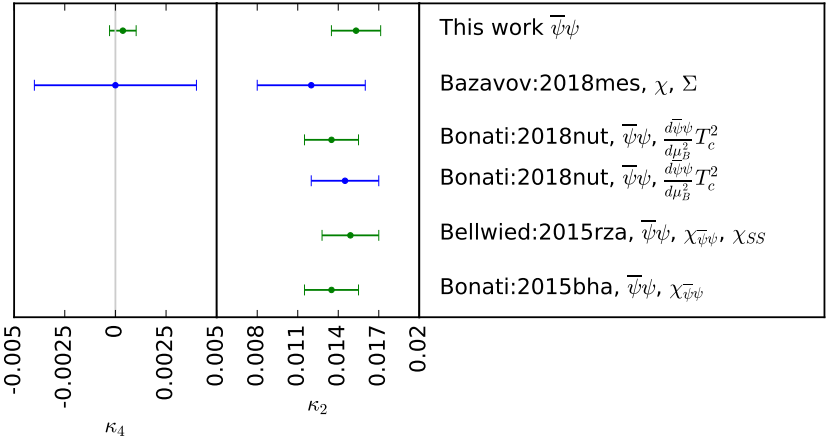
- 2 renormalization fits for $\langle\bar{\psi}\psi\rangle$
- 2 renormalization fits for $\chi_{\bar{\psi}\psi}$
- 4 cuts/fits in $\chi_{\bar{\psi}\psi}$
- 2 two choices of nodepoints for the spline
- 2 functions for the extrapolation
- including or discarding the highest μ_B^I
- scale setting either with w_0 or f_π



histogram for $T_c(\mu_B = 0)$

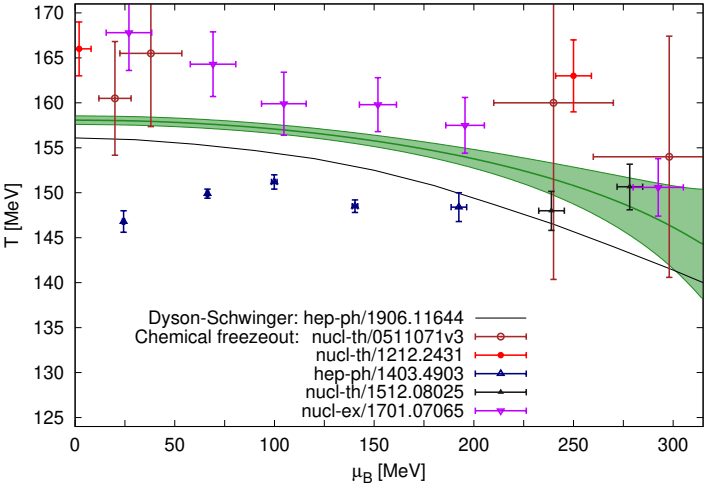
In total we perform 256 analysis. We weight every result with a $Q > 0.1$ uniformly

Result

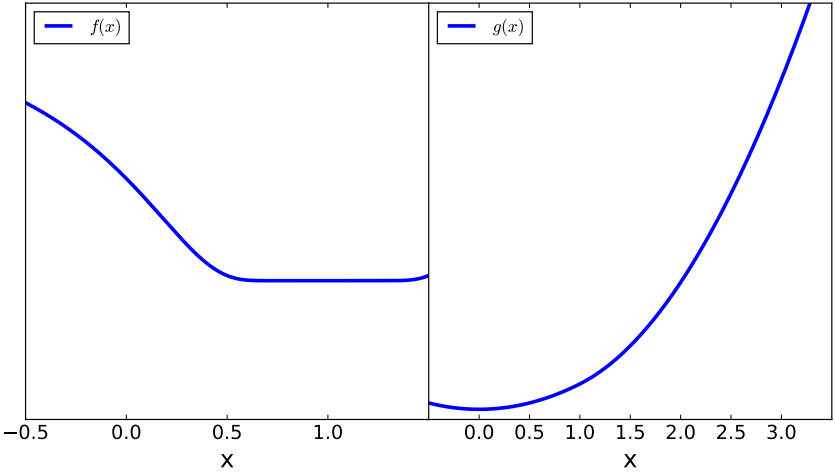


$$\frac{T_c(\mu_B)}{T_c(0)} = 1 - \kappa_2 \left(\frac{\mu_B}{T_c}\right)^2 - \kappa_4 \left(\frac{\mu_B}{T_c}\right)^4 + \mathcal{O}(\mu_B^6),\ \kappa_2 = 0.0153 \pm 0.0018,\ \kappa_4 = 0.00032 \pm 0.00067$$

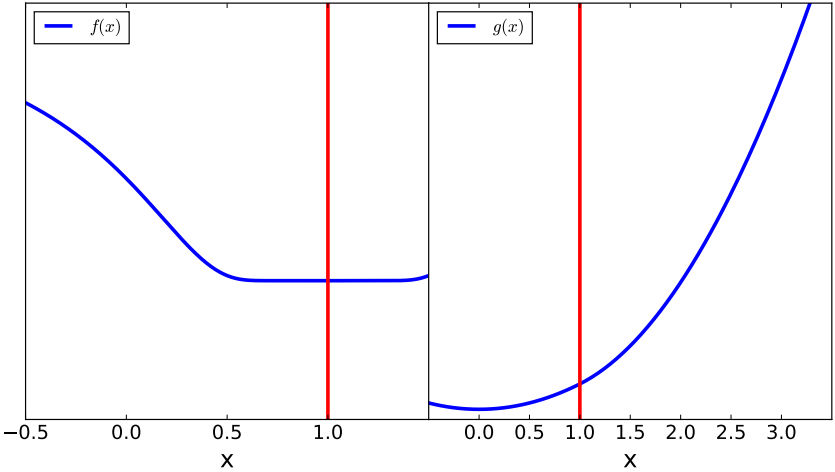
Extrapolation



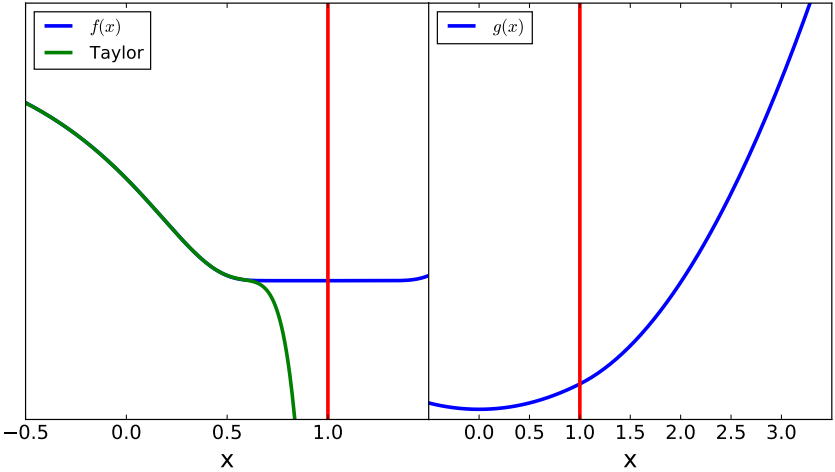
Does that mean there is no critical endpoint?



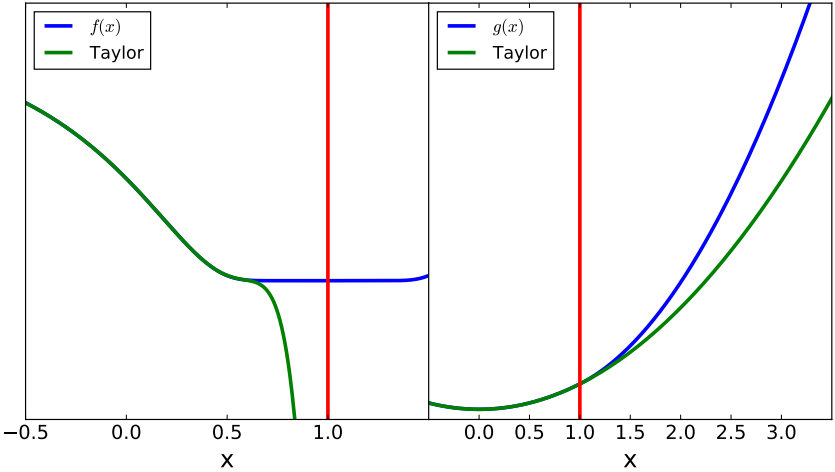
Does that mean there is no critical endpoint?



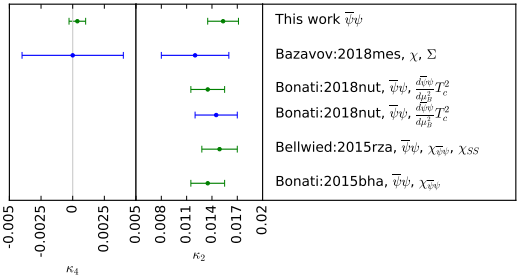
Does that mean there is no critical endpoint?

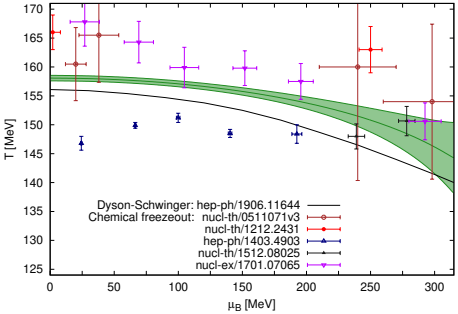


Does that mean there is no critical endpoint?



Summary


This work $\overline{\psi}\psi$
Bazavov:2018mes, χ, Σ
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bonati:2018nut, $\overline{\psi}\psi$, $\frac{d\overline{\psi}\psi}{d\mu_B^2}T_c^2$
Bellwied:2015rza, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$, χ_{SS}
Bonati:2015bha, $\overline{\psi}\psi$, $\chi_{\overline{\psi}\psi}$
-0.005
-0.0025
0
0.0025
0.005
0.008
0.011
0.014
0.017
0.02
κ_4
κ_2


T [MeV]
μ_B [MeV]
Dyson-Schwinger: hep-ph/1906.11644
Chemical freezeout: nucl-th/0511071v3
nucl-th/1212.2431
hep-ph/1403.4903
nucl-th/1512.08025
nucl-ex/1701.07065