Magnetic field dependence of the NJL coupling from lattice QCD

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Based on: G. Endrődi and GM, JHEP 1908 036 (2019)

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- Intro
- Setting the goals
- NJL details
- Results
- Conclusions & Outlook

Nemzeti Kutatási, Fejlesztési és Innovációs Hivatal

Tempus Közalapítvány

Emmy Noether-Programm DFG Deutsche Forschungsgemeinschaft

Thermodynamics of strongly interacting matter

- The fundamental theory is quantum chromodynamics (QCD)
- **Asymptotic freedom** $\Rightarrow$ perturbative at large energy scales
- **Confinement** $\Rightarrow$ non-perturbative at small energy scales (up to at least a few GeV)
- The **chiral and deconfinement phase transition** are non-perturbative phenomena
- The simplest case is well described now, thanks to lattice simulations[1]
- What changes in the presence of **medium** or **external fields**?
- Correct description is crucial in: heavy ion collisions, evolution of the early universe, compact astrophysical objects, etc.

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[1] Y. Aoki et al., Phys. Lett. B **643**, 46 (2006)

Theoretical methods

Lattice

- First-principle
- Errors are controlled
- Huge numerical effort
- **Importance sampling can break down (sign problem)**, e.g. real time dynamics make the weight non-positive, finite baryonic chemical potential makes the weight complex
- Natural parametrization

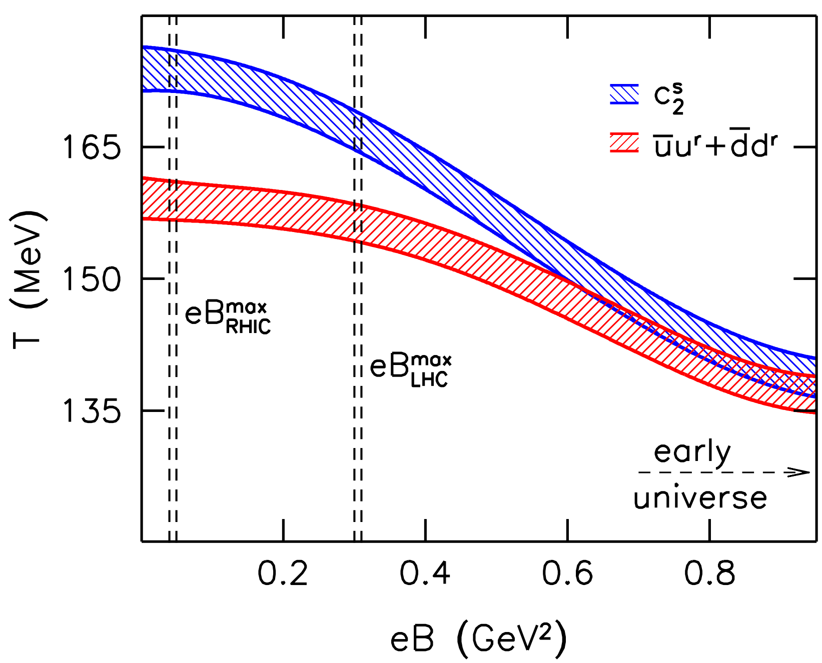
χEFT

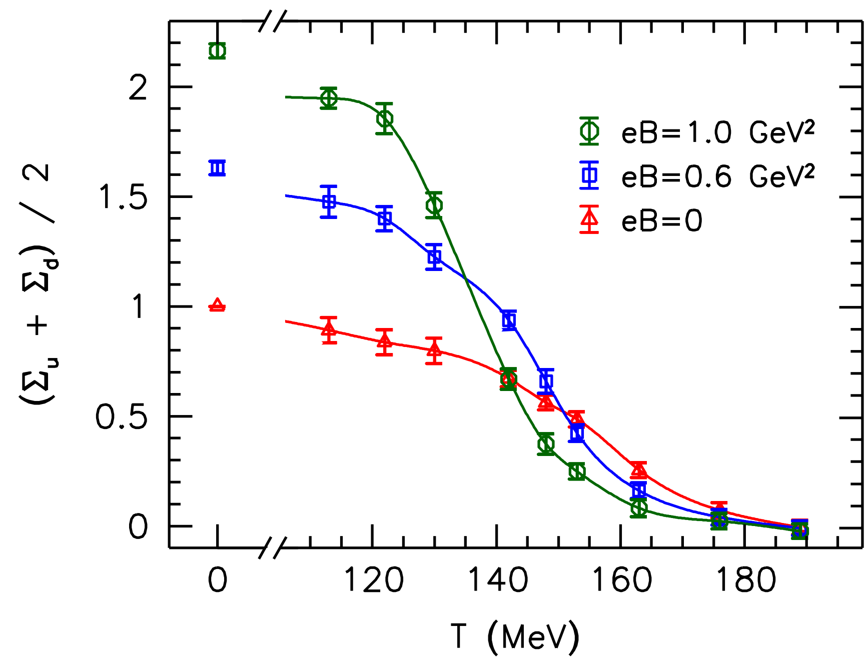
- Fewer degrees of freedom, (over?)simplified field content
- Analytic results may be possible
- Numerically cheap
- **Approximate solutions give rise to uncontrolled errors**
- Parametrization can be problematic

- One big **?** is the critical endpoint in the $\mu_B - T$ phase diagram.
- Effective models give different results, but *more or less* agree on a $\mu_{CEP} > 400$ MeV, and lattice cannot reach it due to the sign problem.
- Can we build effective models with improved applicability region, using lattice results?
- We need a test case: the $B - T$ phase diagram.

$B - T$ phase diagram, lattice results[2]

- No sign problem $\Rightarrow$ Continuum results exist, with physical m_π
- $T_{pc}(B)$ is a decreasing function
- At low temperature magentic catalysis (MC), but at high enough temperature **inverse** magnetic catalysis (IMC) occurs

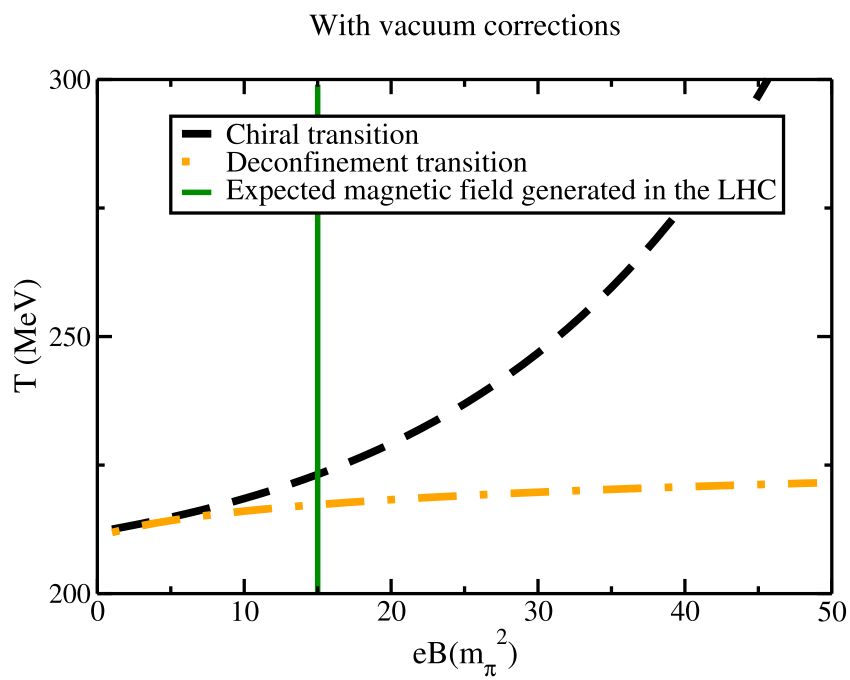


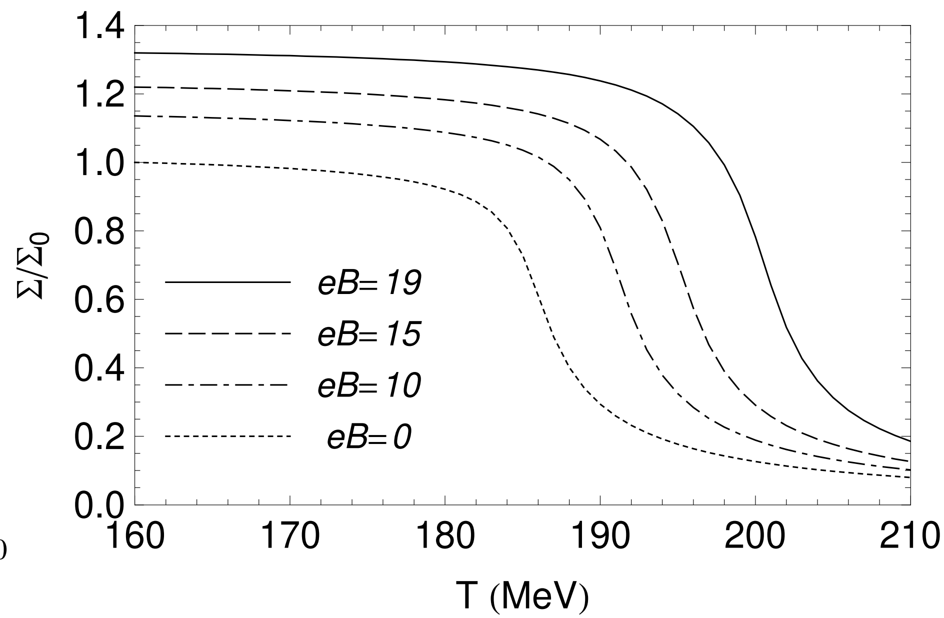


[2] Bali et al., JHEP **1202** 044 (2012)

$B - T$ phase diagram, χEFT results

- PLSQM[3], NJL[4], PNJL[5] (and others) find **increasing** T_{pc}
- and MC at every temperature





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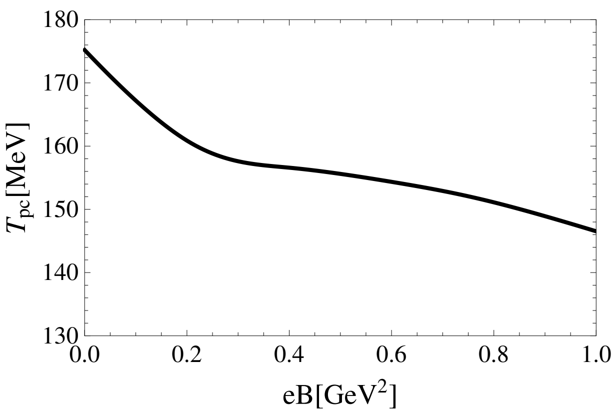
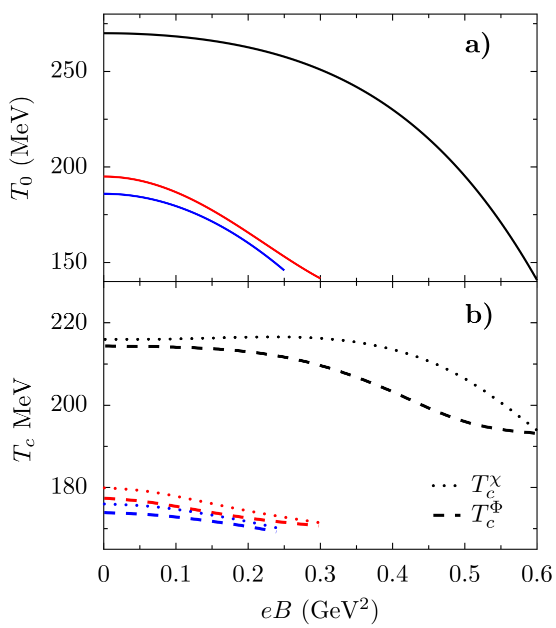
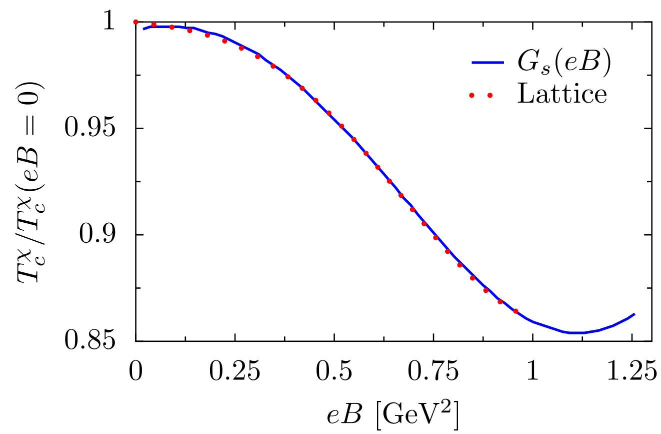
[3] Mizher, Chernodub, Fraga, PRD **82** 105016 (2010)
[4] Klevansky, RevModPhys **64**, 649 (1992)
[5] Gatto, Ruggieri, PRD**83** 034016 (2011)

Why is there a difference?

- Lattice studies[6] shown that IMC is a ”sea effect”, a backreaction of the gluons to the magnetized sea quarks
- This is hardly captured in simple effective models, where the gluons can only enter as a static background (Polyakov loop extensions)
- The model parameters encode those degrees of freedom which are present in QCD but not in χEFTs
- Adding a B dependence **by hand** to model parameters can lead to decreasing $T_{pc}(B)$ curves in (P)NJL[7,8,9]







[6] Bruckmann, Endrődi, Kovács, JHEP **1304** 112 (2013)

[8] Farias et al., PRC**90**, 025203 (2014)

[7] Ferrerira et al., PRD**89** 016002 (2014)

[9] Ferrerira et al., PRD**89** 116011 (2014)

Magnetic field dependent coupling from lattice results

Our program in 4 simple steps:

What?	Why?
We **measure the masses of the baryon octet** on the lattice as a function of B	We need a **physical quantity** from which we can determine the B dependence of the coupling, $\boldsymbol{G}$
We use a simple quantum mechanical quark model to **extract constituent quark masses** from the baryon masses	The NJL model cannot describe full baryons only constituent quarks (or mesons, but those are way harder on lattice)
We **parametrize** the two-flavour NJL model first at $\boldsymbol{T=0}$ & $\boldsymbol{B=0}$, then by a **reparametrization at every $\boldsymbol{B}$** we obtain $\boldsymbol{G}(B)$	The NJL model has **3 parameters** and we have to fix the other two, in order to only let $\boldsymbol{G}$ depend on B
Using $\boldsymbol{G}(B)$ we **solve** the NJL model at $T \neq 0$ to obtain $\boldsymbol{\langle\bar{\psi}\psi\rangle(B,T)}$	We monitor $\langle\bar{\psi}\psi\rangle(B,T)$ to determine $T_{pc}(B)$

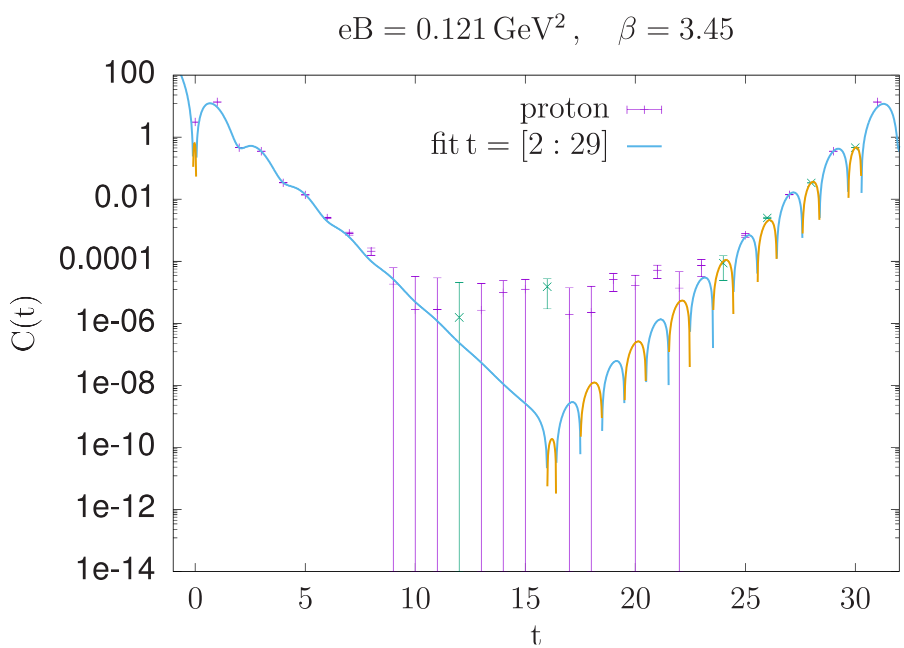
Step 1 - baryon masses

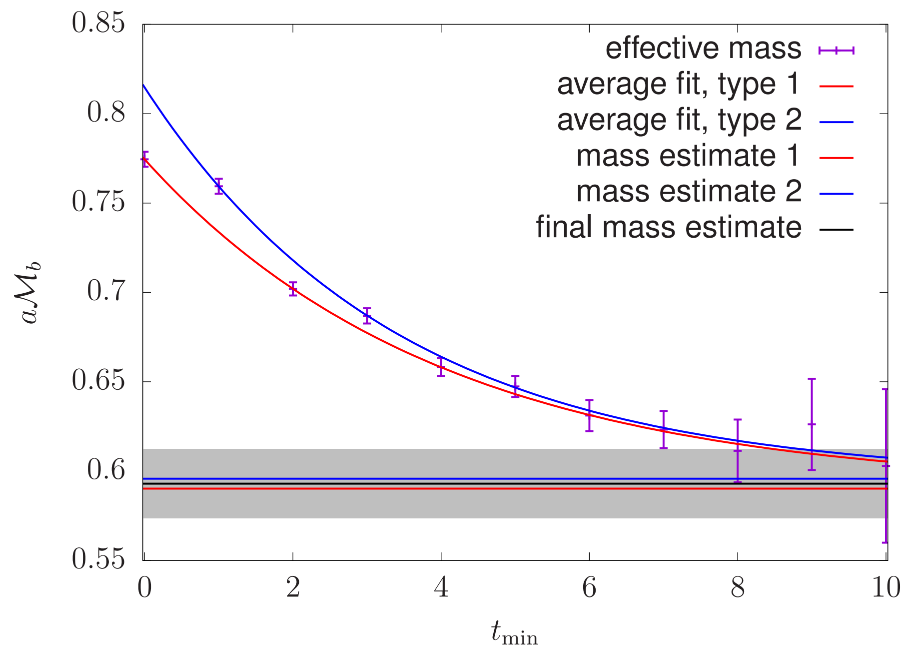
- Staggered fermions, 5 lattices:

$24^3 \times 32$	β, a[fm]	$3.45,\ 0.290$
	N_b	0,1,2,3,4,8,16,24,32
$24^3 \times 32$	β, a[fm]	$3.55,\ 0.215$
	N_b	0,4,8,16,24,32
$32^3 \times 48$	β, a[fm]	$3.67,\ 0.153$
	N_b	0,2,4,6,12,24
$40^3 \times 48$	β, a[fm]	$3.75,\ 0.125$
	N_b	0,2,4,6,12,24
$40^3 \times 48$	β, a[fm]	$3.85,\ 0.098$
	N_b	0,2,4,6,12,24

- Volume slightly too small in the $\beta = 3.85$ simulation, have to be careful when using results
- **5 baryonic correlators** at non-zero magnetic fields (disregard higher spin and cannot access Λ^0)
- fit function **oscillates and mixes** with the parity partner

$$C(t) = A_+ \left(\mathrm{e}^{-\mathcal{M}_+ t} + (-1)^t \, \mathrm{e}^{-\mathcal{M}_+ (N_t - t)} \right) + A_- \left(\mathrm{e}^{-\mathcal{M}_- (N_t - t)} + (-1)^t \, \mathrm{e}^{-\mathcal{M}_- t} \right)$$ [10]





[10] Ishizuka et al., NPB**411** 875 (1994)

Step 1 - baryon masses

The B dependence appears to be less volume dependent than the $B = 0$ value, so in order to be able to use the smallest a:

1. We extrapolate the $\boldsymbol{B = 0}$ **baryon masses** to the continuum **omitting** the $\beta = 3.85$ data

2. And extrapolate the scaled $\boldsymbol{\mathcal{M}(B)/\mathcal{M}(B = 0)}$ **utilizing** the $\beta = 3.85$ data.

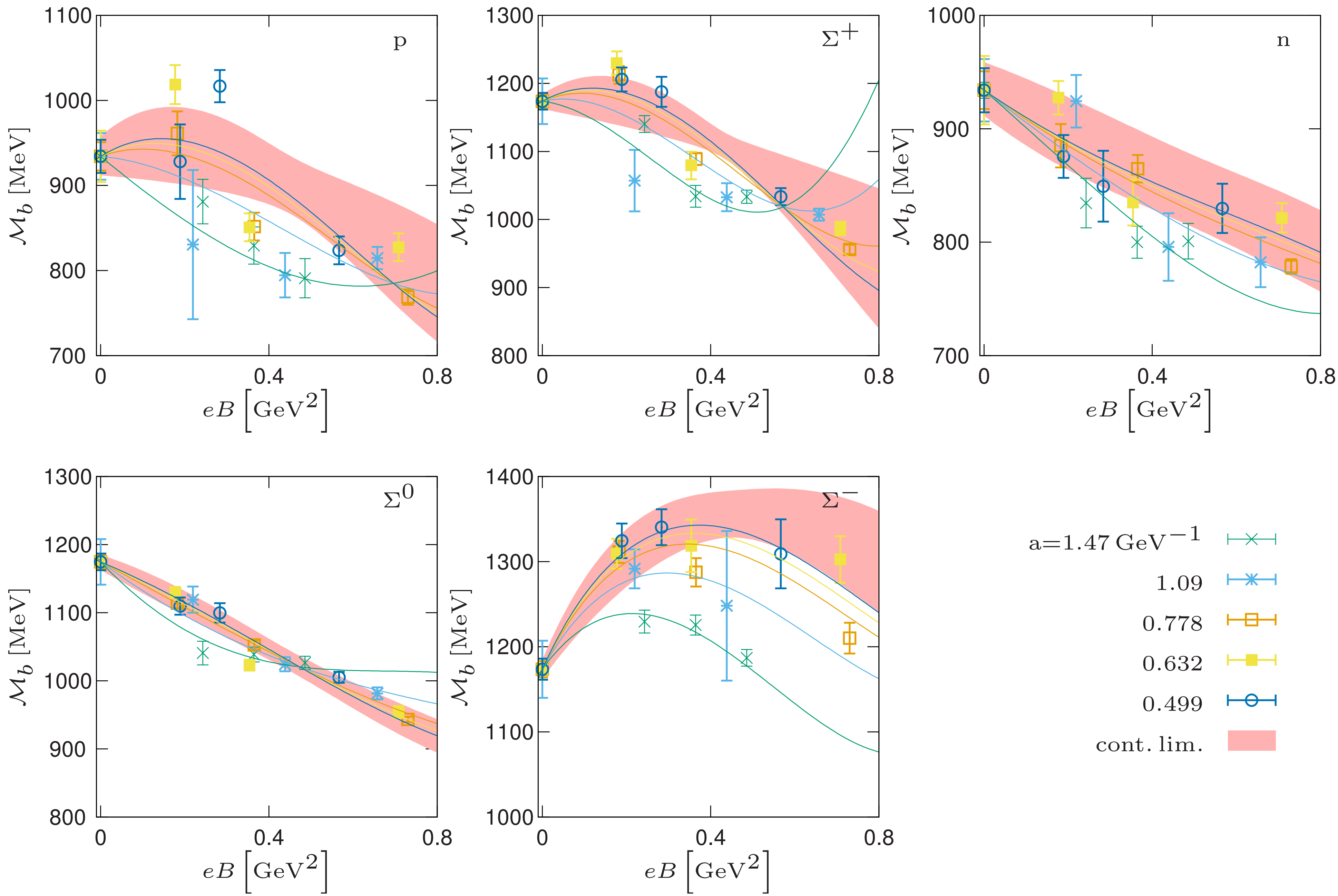
Magnetic **flux is quantized** on the lattice: $B = \frac{6\pi N_b}{a^2 N_s^2}$
$\Rightarrow$ **interpolation is needed** to do continuum extrapolation at any $B \neq 0$. We fit the **surface**

$$\frac{\mathcal{M}_b^2(eB, a)}{\mathcal{M}_b^2(0, a)} = 1 + (c_0 + c_1 a^2) \cdot (eB) + (c_2 + c_3 a^2) \cdot (eB)^2 + (c_4 + c_5 a^2) \cdot (eB)^3 \,, \quad [11]$$

to use all the different a data at once. Systematic errors are estimated by redoing the fit without the $(eB)^3$ term.

[11] Motivated by the B-dependence of a point-like particle.

Step 1 - baryon masses

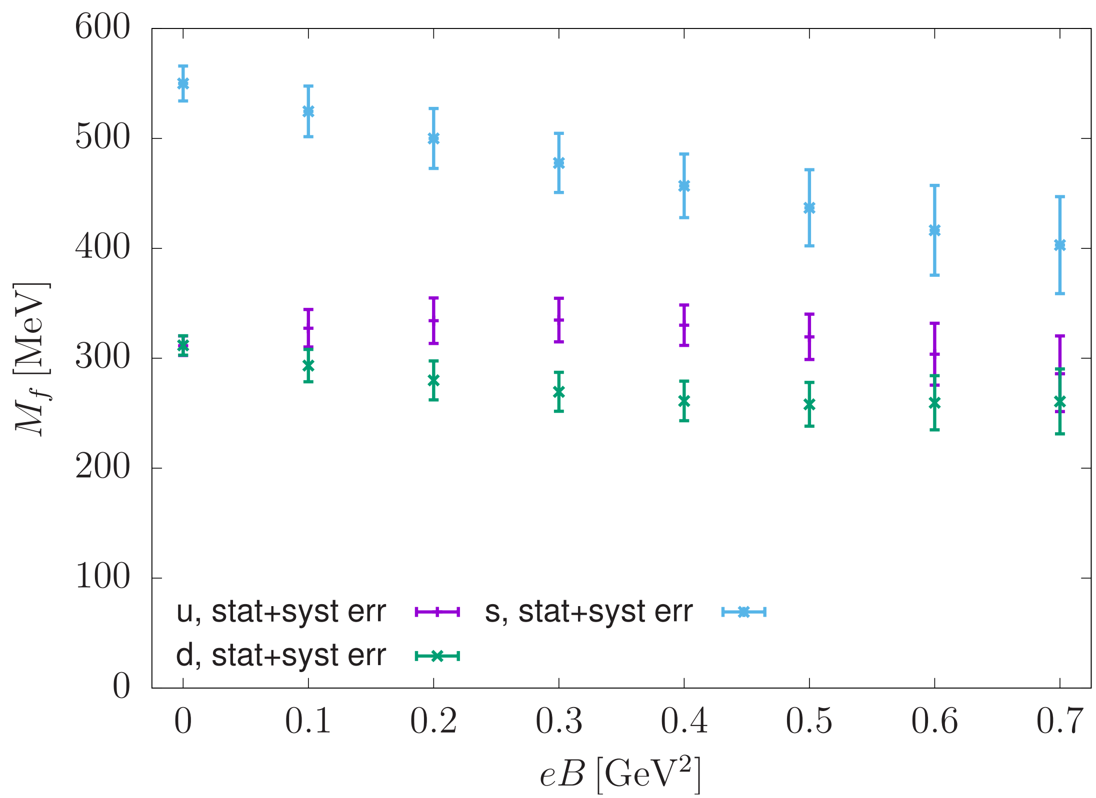


Step 2 - constituent quark masses

We fit the simplest non-relativistic quark model[12] to baryon masses obtained on the lattice for each B.

$$\mathcal{M}_{q_1,q_2,q_3} = M_{q_1} + M_{q_2} + M_{q_3}$$

The Σ^- particle **cannot be in the most energetically favourable spin state** $\rightarrow$ we drop it from the fit.



The input for the NJL is the average light quark mass $(M_u + M_d)/2$.

[12] H. Taya, PRD**92** 014038 (2015)

Step 3 - parametrization of NJL

How do we use the constituent quark mass in the parametrization of the NJL model?

$$\mathcal{L}_2 = \bar{\psi}(i\partial\!\!\!/ - m)\psi + G\left((\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right)$$

2+1 parameters: m, G and regularization scale Λ, bc. of non-renormalizability (G ~GeV^2).
We need three physical quantities calculated within NJL to fix our 3 parameters.

1. Through the **gap equation** the NJL model gives rise to a constituent quark mass $\boldsymbol{M}$:

$$M(T,B) = m - 2G\langle\bar{\psi}\psi\rangle(M(T,B),\Lambda,B,T)$$

2. One pion exchange in a $(ud) \to (du)$ scattering can be described within NJL, leading to a dressed pion propagator, with a **pole** at $\boldsymbol{m_\pi}$.
3. The pion decay constant $\boldsymbol{f_\pi}$ can be calculated from the **one-pion–vacuum matrix element** of the axial vector current:

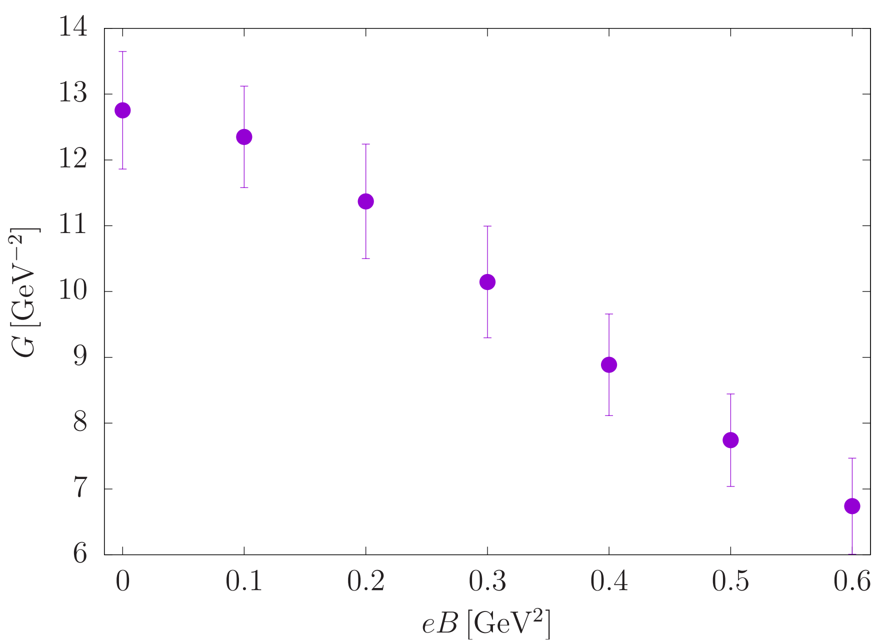
$$\langle 0|J^i_{\mu,5}|\pi^j(k)\rangle = ik_\mu\delta^{ij}f_\pi \, .$$

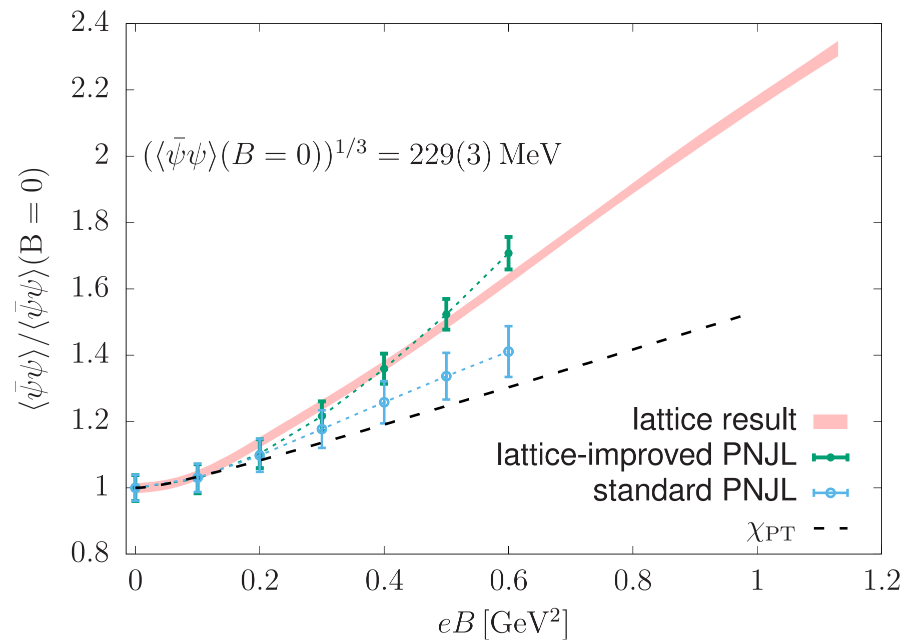
A parametrization is finding values of m, G and Λ in a certain regularization scheme (3D cutoff, 4D cutoff, **Schwinger proper time**, etc.) such that M, m_π and f_π take their physical value (M is usually replaced by $\langle\bar{\psi}\psi\rangle$ and then M is already an output of the model).

Our $\boldsymbol{T=0}$, $\boldsymbol{B=0}$ parameters are
$m = 3.50(5)$ MeV, $G = 12.8(9)$ GeV^2 and $\Lambda = 675(10)$ MeV,
from $M = 311(10)$ MeV, $m_\pi = 138$ MeV and $f_\pi = 93$ MeV.

Step 3 - parametrization of NJL

- For each B we use the gap equation to find G, using fixed m and Λ.
- This results in a decreasing $G(B)$ function.
- Consistency check: B-dependence of the $T = 0$ condensate.





Intermezzo - introducing the Polyakov loop[13]

The **Polyakov loop** is the **order parameter** for the $\mathbb{Z}_3$ centre symmetry of the pure gauge theory. Its definition is

$$\Phi(\vec{x}) = \frac{\mathrm{Tr}_c L(\vec{x})}{N_c}\,, \text{ with} \quad L(x) = \mathcal{P}\exp\left[i\int_0^\beta d\tau\, A_4(\vec{x},\tau)\right]\,.$$

Its thermal expectation value can be connected to the free energy of addig a static quark

$$\langle\Phi\rangle = \frac{Z_q}{Z} = e^{-\beta(F_q - F_0)}\,,$$

from which it follows that at **low temperatures** $\langle\Phi\rangle = 0$ and at **high temperatures** $\langle\Phi\rangle \neq 0$.

An effective descreption of this system can be given by a **classical, homogenous field** ϕ which evolves with the temperature following the **minimum of the potential**

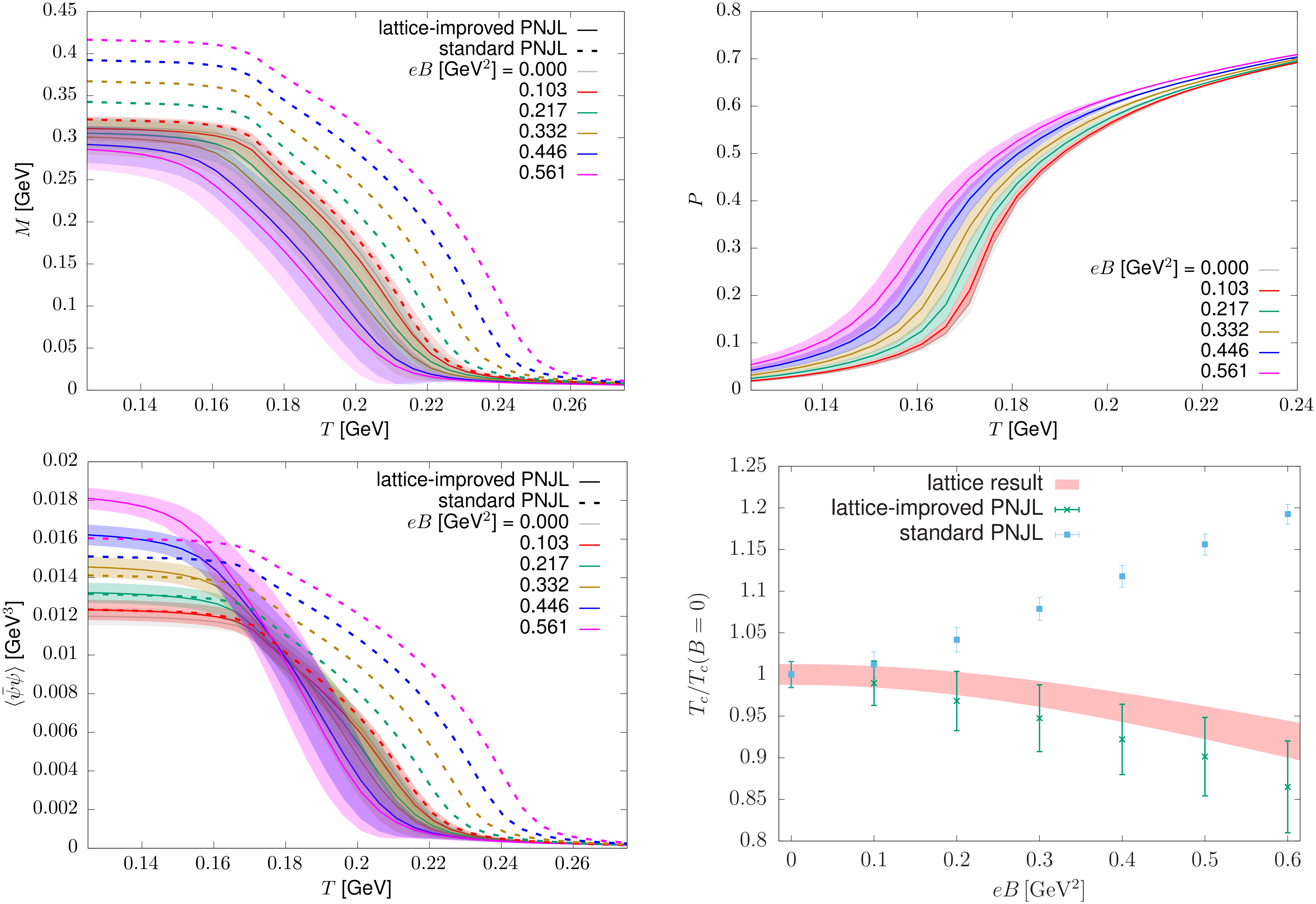
$$\frac{U(\phi)}{T^4} = -\frac{a(T)}{2}\phi^2 + b(T)\ln(1 - 6\phi^2 + 8\phi^3 - 3\phi^4)\,,$$

where $a(T) = a_0 + a_1\left(\frac{T_0}{T}\right) + a_2\left(\frac{T_0}{T}\right)^2$ and $b(T) = b_3\left(\frac{T_0}{T}\right)^3$. The parameters T_0, a_0, a_1, a_2 and b_3 are set to reproduce the pure gauge phase transition measured on the lattice.

Coupling to the NJL model is achieved by considering a homogenous A_4, which is in one-to-one correspondance with ϕ, adding it in a **covariant derivative**. This modifies the **gap equation** as well as the **potential for** ϕ due to **backreaction** of the quarks.

[13] S. Roessner et al., Phys. Rev. D **75**, 034007 (2007).

Step 4 - solve the PNJL at T,B



Conclusions & Outlook

Conclusions

- Lattice results contradict the χEFT results on the $B-T$ phase diagram of strongly interacting matter (behaviour of $T_{pc}(B)$ and IMC vs MC)
- The shortcoming of χEFTs is due to certain QCD d.o.f. are encoded in model parameters.
- A possible solution: B-dependent parameter(s).
- We reparametrize the (P)NJL model in terms of the B-dependent constituent quark masses inferred from baryonic correlators on the lattice.
- Qualitative features are recovered (decreasing transition temperature, IMC at higher T).
- $T_{pc}(B)/T_{pc}(0)$ is consistent with the lattice result.

Outlook

- Try out approaches which are more applicable for the μ_B case.
- Infer a temperature dependent G from lattice results. Combine the two approaches.
- Measure G differently on the lattice, e.g. from the gluon propagator[14]

[14] Braghin, PRD**94** 074030 (2016)